Supplementary Appendix to 'Shifting Currency Mismatch Losses: Effects on Corporate Debt Overhang and Leveraged Banks'

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Mathematical derivations

A: Financially constrained firms borrowing in foreign currency

A1: Solving the financially-constrained firms' profit maximization problem

The firm's borrowing decision depends on the firm's expected working capital needs such that in the beginning of period t the following condition holds:

$$E_{t-1}\left\{S_t\right\}L^*_{i,t}=E_{t-1}\left\{\rho\left(Q_t k_{i,t}+W_t h_{i,t}\right)\right\}$$

where Q_t is the nominal price of capital, W_t is the nominal wage and S_t is the nominal exchange rate. For further derivations we define the domestic consumer price level as P_t.

The firm i born in period t solves the profit maximization problem taking the loan as given. The firm maximizes the expected sum of future revenue from selling goods and depreciated capital subtracted by the second fraction of working capital expenditure together with expenses related to the debt payment. Financial flows received in period t also enter the maximization problem and be summarized as the difference between the loan plus the lump-sum equity injection from the domestic household ($Z_{i,t}$) and working capital expenditure:

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$$\max_{\{k_{i,t},h_{i,t}\}} E_t\beta\Lambda_{t,t+1}\left\{\frac{\left(P^R_{t+1}y^R_{i,t+1}+Q_{t+1}(1-\delta)k_{i,t}-(1-\rho)\left(Q_tk_{i,t}+W_th_{i,t}\right)\right)}{P_{t+1}}\right\}$$
$$-E_t\beta\Lambda_{t,t+1}\left\{\frac{\min\left\{R^{*R}_{i,t}S_{t+1}L^*_{i,t},\quad \kappa\left(P^R_{t+1}y^R_{i,t+1}+Q_{t+1}(1-\delta)k_{i,t}\right)\right\}}{P_{t+1}}\right\}$$
$$+\frac{E_{t-1}\left\{S_t\right\}L^*_{i,t}-\rho\left(Q_tk_{i,t}+W_th_{i,t}\right)+Z_{i,t}}{P_t}$$

s.t.

$$E_{t-1}\left\{\frac{S_t}{P_t}\right\}L^*_{i,t}=E_{t-1}\left\{\rho\frac{Q_tk_{i,t}+W_th_{i,t}}{P_t}\right\}$$

$Z_{i,t}$ stands for the equity injection from the domestic household.

We introduced real variables: $p^R_{t+1}\equiv P^R_{t+1}/P_{t+1}$, $q_{t+1}\equiv Q_{t+1}/P_{t+1}$, $w_t\equiv W_t/P_t$, $L^*_{i,t}\equiv L^*_{i,t}/P^*_t$ and $rer_t\equiv S_tP^*_t/P_t$.

This gives

$$\max_{\{k_{i,t},h_{i,t}\}} E_t\beta\Lambda_{t,t+1}\left\{p^R_{t+1}y^R_{i,t+1}+q_{t+1}(1-\delta)k_{i,t}-(1-\rho)\frac{q_tk_{i,t}+w_th_{i,t}}{\pi_{t+1}}\right\}$$
$$-E_t\beta\Lambda_{t,t+1}\min\left\{\frac{R^{*R}_{i,t}}{\pi^*_{t+1}}rer_{t+1}l^*_{i,t},\quad \kappa\left(p^R_{t+1}y^R_{i,t+1}+q_{t+1}(1-\delta)k_{i,t}\right)\right\}$$
$$+\frac{E_{t-1}\left\{S_tP^*_t\right\}l^*_{i,t}}{P_t}-\rho\left(q_tk_{i,t}+w_th_{i,t}\right)+z_{i,t}$$

s.t.

$$E_{t-1}\left\{rer_t\right\}l^*_{i,t}=E_{t-1}\left\{\rho\left(q_tk_{i,t}+w_th_{i,t}\right)\right\}$$

The corresponding first-order conditions follow:

$$k_{i,t}:\quad E_t\beta\Lambda_{t,t+1}\left\{p^R_{t+1}\frac{\partial y^R_{i,t+1}}{\partial k_{i,t}}+(1-\delta)q_{t+1}-(1-\rho)\frac{q_t}{\pi_{t+1}}\right\}$$
$$-\frac{\partial E_t\beta\Lambda_{t,t+1}E_t\min\left\{\frac{R^{*R}_{i,t}}{\pi^*_{t+1}}rer_{t+1}l_{i,t},\quad \kappa\left(p^R_{t+1}y^R_{i,t+1}+q_{t+1}(1-\delta)k_{i,t}\right)\right\}}{\partial k_{i,t}}$$
$$=\frac{\partial cov\left(\beta\Lambda_{t,t+1},\quad \min\left\{\frac{R^{*R}_{i,t}}{\pi^*_{t+1}}rer_{t+1}l^*_{i,t},\quad \kappa\left(p^R_{t+1}y^R_{i,t+1}+q_{t+1}(1-\delta)k_{i,t}\right)\right\}\right)}{\partial k_{i,t}}$$
$$+\rho q_t$$

$$
\begin{aligned}
h_{i,t}: \quad & E_t \beta \Lambda_{t,t+1} \left\{ p^R_{t+1} \frac{\partial y^R_{i,t+1}}{\partial h_{i,t}} - (1-\rho) \frac{w_t}{\pi_{t+1}} \right\} \\
& - \frac{\partial E_t \beta \Lambda_{t,t+1} E_t \min \left\{ \frac{R^{*R}_{i,t}}{\pi^*_{t+1}} rer_{t+1} l^*_{i,t}, \quad \kappa \left(p^R_{t+1} y^R_{i,t+1} + q_{t+1}(1-\delta) k_{i,t} \right) \right\}}{\partial h_{i,t}} \\
& = \frac{\partial cov \left(\beta \Lambda_{t,t+1}, \quad \min \left\{ \frac{R^{*R}_{i,t}}{\pi^*_{t+1}} rer_{t+1} l^*_{i,t}, \quad \kappa \left(p^R_{t+1} y^R_{i,t+1} + q_{t+1}(1-\delta) k_{i,t} \right) \right\} \right)}{\partial h_{i,t}} \\
& + \rho w_t
\end{aligned}
$$

If we substitute the expression for the expected value of loan repayment (see the next subsection), first-order conditions become functions of default probabilities:

$$
\begin{aligned}
k_{i,t}: \quad & E_t \beta \Lambda_{t,t+1} \left\{ p^R_{t+1} \frac{\partial y^R_{i,t+1}}{\partial k_{i,t}} + q_{t+1}(1-\delta) - (1-\rho) \frac{q_t}{\pi_{t+1}} \right\} \\
& - E_t \beta \Lambda_{t,t+1} \left\{ (1 - \Phi(d_{1,t})) \kappa \left(p^R_{t+1} \frac{\partial y^R_{i,t+1}}{\partial k_{i,t}} + q_{t+1}(1-\delta) \right) \right\} \\
& = \frac{\partial cov \left(\beta \Lambda_{t,t+1}, \quad \min \left\{ \frac{R^{*R}_{i,t}}{\pi^*_{t+1}} rer_{t+1} l^*_{i,t}, \quad \kappa \left(p^R_{t+1} y^R_{i,t+1} + q_{t+1}(1-\delta) k_{i,t} \right) \right\} \right)}{\partial k_{i,t}} \\
& + \rho q_t
\end{aligned}
$$

$$
\begin{aligned}
h_{i,t}: \quad & E_t \beta \Lambda_{t,t+1} \left\{ p^R_{t+1} \frac{\partial y^R_{i,t+1}}{\partial h_{i,t}} - (1-\rho) \frac{w_t}{\pi_{t+1}} \right\} \\
& - E_t \beta \Lambda_{t,t+1} \left\{ (1 - \Phi(d_{1,t})) \kappa \left(p^R_{t+1} \frac{\partial y^R_{i,t+1}}{\partial h_{i,t}} \right) \right\} \\
& = \frac{\partial cov \left(\beta \Lambda_{t,t+1}, \quad \min \left\{ \frac{R^{*R}_{i,t}}{\pi^*_{t+1}} rer_{t+1} l^*_{i,t}, \quad \kappa \left(p^R_{t+1} y^R_{i,t+1} + q_{t+1}(1-\delta) k_{i,t} \right) \right\} \right)}{\partial h_{i,t}} \\
& + \rho w_t
\end{aligned}
$$

where

$$
d_{2,t} \equiv \frac{E_t \ln \left(\kappa \left(p^R_{t+1} y^R_{i,t+1} + q_{t+1}(1-\delta) k_{i,t} \right) \right) - E_t \ln \left(\frac{R^{*R}_{i,t}}{\pi^*_{t+1}} rer_{t+1} l^*_{i,t} \right)}{\sigma_y}, \qquad d_{1,t} = d_{2,t} + \sigma_y
$$

σ_y^2 is given by $var(\pi_{t+1} \kappa \left(p^R_{t+1} y^R_{i,t+1} + q_{t+1}(1-\delta) k_{i,t} \right))$.

A2: Differentiation of the firm's payment function

We need to compute the expected value of the firm's payment function (we abstract from indices i for the sake of brevity):

$$E_t \min \left\{ \frac{R_t^{*R}}{\pi_{t+1}^*} rer_{t+1} l_t^*, \quad \kappa \left(p_{t+1}^R y_{t+1}^R + q_{t+1}(1-\delta)k_t \right) \right\}$$

Then we define $\bar{y}_{t+1} \equiv \frac{\pi_{t+1}^*}{rer_{t+1}} \kappa \left(p_{t+1}^R y_{t+1}^R + q_{t+1}(1-\delta)k_t \right)$, where

$$\bar{y}_{t+1} \sim \text{log-normal} \left(\mu_{\bar{y}_{t+1}}, \sigma_y^2 \right)$$

Then the payment function can be re-written as

$$E_t \min \left\{ R_t^{*R} l_t^*, \quad \bar{y}_{t+1} \right\}$$

Further,

$$\begin{aligned}
E_t \min \left\{ R_t^{*R} l_t^*, \quad \bar{y}_{t+1} \right\} &= R_t^{*R} l_t^* Pr \left(R_t^{*R} l_t^* < \bar{y}_{t+1} \right) + \left(1 - Pr \left(R_t^{*R} l_t^* < \bar{y}_{t+1} \right) \right) E_t \left(\bar{y}_{t+1} \mid \bar{y}_{t+1} < R_t^{*R} l_t^* \right) \\
&= R_t^{*R} l_t^* Pr \left(R_t^{*R} l_t^* < \bar{y}_{t+1} \right) + \left(1 - Pr \left(R_t^{*R} l_t^* < \bar{y}_{t+1} \right) \right) \int_0^{R_t^{*R} l_t^*} \frac{\bar{y}_{t+1} dF\left(\bar{y}_{t+1}\right)}{1 - Pr\left(R_t^{*R} l_t^* < \bar{y}_{t+1}\right)} \\
&= R_t^{*R} l_t^* Pr \left(R_t^{*R} l_t^* < \bar{y}_{t+1} \right) + \int_0^{R_t^{*R} l_t^*} \bar{y}_{t+1} dF\left(\bar{y}_{t+1}\right) \\
&= R_t^{*R} l_t^* \int_{R_t^{*R} l_t^*}^{\infty} dF\left(\bar{y}_{t+1}\right) + \int_0^{R_t^{*R} l_t^*} \bar{y}_{t+1} dF\left(\bar{y}_{t+1}\right) \\
&= R_t^{*R} l_t^* \int_{R_t^{*R} l_t^*}^{\infty} \frac{1}{\bar{y}_{t+1} \sigma_y \sqrt{2\pi}} e^{\frac{-\left(ln\left(\bar{y}_{t+1}\right) - \mu_y\right)^2}{2\sigma_y^2}} d\left(\bar{y}_{t+1}\right) \\
&+ \int_0^{R_t^{*R} l_t^*} \frac{\bar{y}_{t+1}}{\bar{y}_{t+1} \sigma_y \sqrt{2\pi}} e^{\frac{-\left(ln\left(\bar{y}_{t+1}\right) - \mu_y\right)^2}{2\sigma_y^2}} d\left(\bar{y}_{t+1}\right) \\
&= R_t^{*R} l_t^* \Phi \left(\frac{ln\left(\bar{y}_{t+1}\right) - \mu_y}{\sigma_y} \right) |_{R_t^{*R} l_t^*}^{\infty} + \int_0^{R_t^{*R} l_t^*} \frac{1}{\sigma_y \sqrt{2\pi}} e^{\frac{-\left(ln\left(\bar{y}_{t+1}\right) - \mu_y\right)^2}{2\sigma_y^2}} d\left(\bar{y}_{t+1}\right) \\
&= R_t^{*R} l_t^* \left(1 - \Phi \left(\frac{ln\left(R_t^{*R} l_t^*\right) - \mu_y}{\sigma_y} \right) \right) - \frac{1}{2} e^{\mu_y + \frac{\sigma_y^2}{2}} erf \left(\frac{-ln\left(\bar{y}_{t+1}\right) + \mu_y + \sigma_y^2}{\sqrt{2}\sigma_y} \right) |_0^{R_t^{*R} l_t^*} \\
&= R_t^{*R} l_t^* \Phi \left(\frac{\mu_y - ln\left(R_t^{*R} l_t^*\right)}{\sigma_y} \right) + \frac{1}{2} E_t(\bar{y}_{t+1}) \left(erf \left(\frac{ln\left(R_t^{*R} l_t^*\right) - \mu_y - \sigma_y^2}{\sqrt{2}\sigma_y} \right) + 1 \right) \\
&= R_t^{*R} l_t^* \Phi \left(\frac{\mu_y - ln\left(R_t^{*R} l_t^*\right)}{\sigma_y} \right) + E_t(\bar{y}_{t+1}) \Phi \left(\frac{ln\left(R_t^{*R} l_t^*\right) - \mu_y - \sigma_y^2}{\sigma_y} \right) \\
&= R_t^{*R} l_t^* \Phi \left(\frac{\mu_y - ln\left(R_t^{*R} l_t^*\right)}{\sigma_y} \right) + E_t(\bar{y}_{t+1}) \left(1 - \Phi \left(\frac{\mu_y - ln\left(R_t^{*R} l_t^*\right)}{\sigma_y} + \sigma_y \right) \right)
\end{aligned}$$

The expression can be simplified as

$$E_t \min \left\{ R_t^{*R} l_t^*, \quad \bar{y}_{t+1} \right\} = \left(1 - \Phi(d_{1,t})\right) E_t\left(\bar{y}_{t+1}\right) + \Phi(d_{2,t}) R_t^{*R} l_t^*$$

where

$$d_{2,t} \equiv \frac{\mu_y - \ln\left(R_t^{*R} l_t^*\right)}{\sigma_y}, \qquad d_{1,t} \equiv d_{2,t} + \sigma_y$$

and

$$\mu_y \equiv E_t \ln\left(\bar{y}_{t+1}\right)$$

Recall that $\bar{y}_{t+1} \equiv \frac{\pi_{t+1}^*}{rer_{t+1}} \kappa \left(p_{t+1}^R y_{t+1}^R + q_{t+1}(1-\delta)k_t\right)$ so it can be substituted back to get complete expressions.

To solve for the first-order conditions, we differentiate the expected loan payment w.r.t. k_t:

$$\begin{aligned}
\frac{\partial E_t \min\left\{R_t^{*R} l_t^*, \quad \bar{y}_{t+1}\right\}}{\partial k_t} &= \left(1 - \Phi(d_{1,t})\right) \frac{\partial E_t \bar{y}_{t+1}}{\partial k_t} \\
&- E_t \bar{y}_{t+1} \frac{\partial \Phi(d_{1,t})}{\partial d_{1,t}} \frac{\partial d_{1,t}}{\partial k_t} + R_t^{*R} l_t^* \frac{\partial \Phi(d_{2,t})}{\partial d_{2,t}} \frac{\partial d_{2,t}}{\partial k_t} \\
&= \left(1 - \Phi(d_{1,t})\right) \frac{\partial E_t \bar{y}_{t+1}}{\partial k_t}
\end{aligned}$$

where the proof of the last expression comes from by using $\frac{\partial d_{1,t}}{\partial k_t} = \frac{\partial d_{2,t}}{\partial k_t}$ and computing the following:

$$\begin{aligned}
& -E_t\left(\bar{y}_{t+1}\right) \Phi'(d_{1,t}) + R_t^{*R} l_t^* \Phi'(d_{2,t}) \\
=\ & -e^{\ln(E_t \bar{y}_{t+1})} \Phi'(d_{1,t}) + e^{\ln(R_t^{*R} l_t^*)} \Phi'(d_{2,t}) \\
=\ & -e^{\ln(E_t \bar{y}_{t+1})} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} d_{1,t}^2} + e^{\ln(R_t^{*R} l_t^*)} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} d_{2,t}^2} \\
=\ & -e^{\ln(E_t \bar{y}_{t+1})} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(d_{2,t}^2 + 2 d_{2,t}\sigma_y + \sigma_y^2\right)} + e^{\ln(R_t^{*R} l_t^*)} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} d_{2,t}^2} \\
=\ & -e^{\ln(E_t \bar{y}_{t+1})} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} d_{2,t}^2} e^{-\left(d_{2,t}\sigma_y + \frac{1}{2}\sigma_y^2\right)} + e^{\ln(R_t^{*R} l_t^*)} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} d_{2,t}^2} \\
=\ & -e^{\ln(E_t \bar{y}_{t+1})} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} d_{2,t}^2} e^{-E_t\left(\ln \bar{y}_{t+1}\right) - \ln(R_t^{*R} l_t^*) + \frac{1}{2}\sigma_y^2} + e^{\ln(R_t^{*R} l_t^*)} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} d_{2,t}^2} \\
=\ & \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} d_{2,t}^2} \left[-e^{\ln(E_t \bar{y}_{t+1})} e^{-\left(\ln(E_t \bar{y}_{t+1}) - \frac{1}{2}\sigma_y^2 - \ln(R_t^{*R} l_t^*) + \frac{1}{2}\sigma_y^2\right)} + e^{\ln(R_t^{*R} l_t^*)}\right] \\
=\ & -\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} d_{2,t}^2} e^{\ln(R_t^{*R} l_t^*)} + e^{\ln(R_t^{*R} l_t^*)} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} d_{2,t}^2} \\
=\ & 0,
\end{aligned}$$

where such expressions are used as

$$E_t \ln\left(\bar{y}_{t+1}\right) = \ln\left(E_t \bar{y}_{t+1}\right) - \frac{1}{2}\sigma_y^2$$

and the definition of the variable $d_{1,t}$. Substituting a definition for $\bar{y}_{t+1}$ back gives

$$\frac{\partial E_t \min\left\{R_t^{*R} l_t^*, \quad \frac{\pi_{t+1}^*}{rer_{t+1}} \kappa \left(p_{t+1}^R y_{t+1}^R + q_{t+1}(1-\delta)k_t\right)\right\}}{\partial k_t} = \left(1 - \Phi(d_{1,t})\right) \frac{\partial E_t \left(\frac{\pi_{t+1}^*}{rer_{t+1}} \kappa \left(p_{t+1}^R y_{t+1}^R + q_{t+1}(1-\delta)k_t\right)\right)}{\partial k_t} \tag{A2.1}$$

Similarly it can be showed that

$$\frac{\partial E_t \min\left\{R_t^{*R} l_t^*, \quad \frac{\pi_{t+1}^*}{rer_{t+1}}\kappa\left(p_{t+1}^R y_{t+1}^R + q_{t+1}(1-\delta)k_t\right)\right\}}{\partial h_t} = (1-\Phi(d_{1,t}))\frac{\partial E_t\left(\frac{\pi_{t+1}^*}{rer_{t+1}}\kappa\left(p_{t+1}^R y_{t+1}^R + q_{t+1}(1-\delta)k_t\right)\right)}{\partial h_t} \tag{A2.2}$$

B: Solving banks' problem

Domestic households own all banks that operate in the domestic economy and lend to financially constrained domestic firms. We assume that there is a continuum of these banks and every period there is a probability ω that a bank continues operating. Otherwise, the net worth is transferred to the owners of the bank, domestic households. We assume that banks give foreign currency loans out of accumulated nominal equity N_t, nominal domestic deposits D_t and nominal foreign debt D_t^* that is expressed in units of foreign currency. Consequently, as long as foreign debt is larger than zero, banks are subject to currency mismatch. Lending in foreign currency hedges the open currency position: currency mismatch decreases. The balance sheet constraint of a bank j, expressed in units of domestic goods, is given by

$$N_{j,t} + D_{j,t} + S_t D_{j,t}^* = S_t L_{j,t}^*$$

Banks pay a nominal domestic interest rate R_t on deposits and a nominal foreign interest rate $R_t^* \xi_t$ on foreign debt. R_t^* follows a stationary AR(1) process. ξ_t denotes a premium on bank foreign debt. To ensure stationarity in the model, we assume that the premium depends on the level of foreign bank debt (as in Schmitt-Grohé and Uribe, 2003):

$$\xi_t = \exp\left(\kappa_\xi \frac{(S_t D_t^* - S \cdot D^*)}{S \cdot D^*} + \frac{\zeta_t - \zeta}{\zeta}\right) \tag{1}$$

where ζ_t is an exogenous shock that follows a stable AR(1) process.

Banks are subject to an agency problem as in Gertler and Karadi (2011). At the end of every period every bank can divert a fraction λ^L of divertable assets. Creditors take this possibility into account and lend only up to the point where the continuation value of the bank is still larger or equal to what can be diverted. This condition acts as an incentive constraint for the bank and eventually limits expansion of the balance sheet.

The only asset on the banks' balance sheet is foreign currency loans to domestic financially constrained firms, thus, the expected nominal return of the bank j is defined as $R_{j,t}^{*L}$ and given by:

$$E_t\left\{R_t^{*L} S_{t+1} L_{j,t}^*\right\} \equiv E_t\left\{(1-\Phi(d_{1,t}))\kappa\left(P_{t+1}^R y_{j,t+1}^R + (1-\delta)Q_{t+1}k_{j,t}\right) + \Phi(d_{2,t})R_{j,t}^{*R} S_{t+1} L_{j,t}^*\right\}$$

Then the optimization problem of the bank j can be written as:

$$V_{j,t} = \max_{\left\{D_{j,t}, D_{j,t}^*, L_{j,t}^*\right\}} \quad E_t\left[\beta \Lambda_{t,t+1}\left\{(1-\omega)\frac{N_{j,t+1}}{P_{t+1}} + \omega V_{j,t+1}\right\}\right]$$

s.t.

$$V_{j,t} \geq \lambda^L \frac{S_t L_{j,t}^*}{P_t}, \qquad \textit{(Incentive constraint)}$$

$$\frac{N_{j,t} + D_{j,t} + S_t D_{j,t}^*}{P_t} = \frac{S_t L_{j,t}^*}{P_t}, \qquad \textit{(Balance sheet constraint)}$$

$$\frac{N_{j,t}}{P_t} = \frac{R^{*L}_{j,t-1}}{P_t} S_t L^*_{j,t-1} - \frac{R_{t-1}}{P_t} D_{j,t-1} - \frac{R^*_{t-1}\xi_{t-1}}{P_t} S_t D^*_{j,t-1} \qquad \textit{(LoM of net worth)}$$

Define $rer_t \equiv P^*_t S_t / P_t$, $d^*_{j,t} \equiv D^*_{j,t}/P^*_t$, $d_{j,t} \equiv D_{j,t}/P_t$, $l^*_{j,t} \equiv L^*_{j,t}/P^*_t$, and $n_{j,t} \equiv N_{j,t}/P_t$. It follows that

$$V_{j,t} = \max_{\{d_{j,t}, d^*_{j,t}, l^*_{j,t}\}} \quad E_t \left[\beta \Lambda_{t,t+1} \left\{(1-\omega) n_{j,t+1} + \omega V_{j,t+1}\right\}\right]$$

s.t.

$$V_{j,t} \geq \lambda^L rer_t l^*_{j,t}, \qquad \textit{(Incentive constraint)}$$

$$n_{j,t} + d_{j,t} + rer_t d^*_{j,t} = rer_t l^*_{j,t}, \qquad \textit{(Balance sheet constraint)}$$

$$n_{j,t} = \frac{R^{*L}_{j,t-1}}{\pi^*_t} rer_t l^*_{j,t-1} - \frac{R_{t-1}}{\pi_t} d_{j,t-1} - \frac{R^*_{t-1}\xi_{t-1}}{\pi^*_t} rer_t d^*_{j,t-1} \qquad \textit{(LoM of net worth)}$$

Lagrangian of the problem can be formulated as:

$$\begin{aligned} L = \quad & (1+\nu_{1,t}) E_t \beta \Lambda_{t,t+1} \left\{ (1-\omega) \left(\frac{R^{*L}_{j,t}}{\pi^*_{t+1}} rer_{t+1} l^*_{j,t} - \frac{R_t}{\pi_{t+1}} d_{j,t} - \frac{R^*_t \xi_t}{\pi^*_{t+1}} rer_{t+1} d^*_{j,t} \right) + \omega V_{j,t+1} \right\} \\ & - \nu_{1,t} \lambda^L rer_t l^*_{j,t} \\ & + \nu_{2,t} \left(\frac{R^{*L}_{j,t-1}}{\pi^*_t} rer_t l^*_{j,t-1} - \frac{R_{t-1}}{\pi_t} d_{j,t-1} - \frac{R^*_{t-1}\xi_{t-1}}{\pi^*_t} rer_t d^*_{j,t-1} - rer_t l^*_{j,t} + d_{j,t} + rer_t d^*_{j,t} \right) \end{aligned}$$

This gives the first-order conditions:

$$l^*_{j,t}: \quad (1+\nu_{1,t}) \beta E_t \Lambda_{t,t+1} \left\{ (1-\omega) \left(\frac{R^{*L}_{j,t}}{\pi^*_{t+1}} rer_{t+1} \right) + \omega \frac{\partial V(.)}{\partial l^*_{j,t}} \right\} = \lambda^L \nu_{1,t} rer_t + \nu_{2,t} rer_t$$

$$d_{j,t}: \quad (1+\nu_{1,t}) \beta E_t \Lambda_{t,t+1} \left\{ (1-\omega) \left(\frac{R_t}{\pi_{t+1}} \right) - \omega \frac{\partial V(.)}{\partial d_{j,t}} \right\} = \nu_{2,t}$$

$$d^*_{j,t}: \quad (1+\nu_{1,t}) \beta E_t \Lambda_{t,t+1} \left\{ (1-\omega) \left(\frac{R^*_t \xi_t}{\pi^*_{t+1}} rer_{t+1} \right) - \omega \frac{\partial V(.)}{\partial d^*_{j,t}} \right\} = \nu_{2,t} rer_t$$

with complementary slackness conditions:

$$\nu_{1,t}: \quad \nu_{1,t} \left(V_{j,t} - \lambda^L rer_t l^*_{j,t} \right) = 0$$

$$\nu_{2,t}: \quad \nu_{2,t} \left(\frac{R^{*L}_{j,t-1}}{\pi^*_t} rer_t l^*_{j,t-1} - \frac{R_{t-1}}{\pi_t} d_{j,t-1} - \frac{R^*_{t-1}\xi_{t-1}}{\pi^*_t} rer_t d^*_{j,t-1} - rer_t l^*_{j,t} + d_{j,t} + rer_t d^*_{j,t} \right) = 0$$

Further, the first-order conditions can be expressed as

$$l^*_{j,t}: \quad (1+\nu_{1,t}) \beta E_t \Lambda_{t,t+1} \left\{ (1-\omega) + \omega \nu_{2,t+1} \right\} \left(\frac{R^{*L}_{j,t}}{\pi^*_{t+1}} \frac{rer_{t+1}}{rer_t} \right) = \lambda^L \nu_{1,t} + \nu_{2,t}$$

$$d_{j,t}: \quad (1+\nu_{1,t})\beta E_t \Lambda_{t,t+1}\left\{(1-\omega)+\omega\nu_{2,t+1}\right\}\left(\frac{R_t}{\pi_{t+1}}\right)=\nu_{2,t}$$

$$d^*_{j,t}: \quad (1+\nu_{1,t})\beta E_t \Lambda_{t,t+1}\left\{(1-\omega)+\omega\nu_{2,t+1}\right\}\left(\frac{R^*_t\xi_t}{\pi^*_{t+1}}\frac{rer_{t+1}}{rer_t}\right)=\nu_{2,t}$$

Besides these first-order conditions, the set of equilibrium conditions includes the law of motion for aggregate net worth of banks and the bank incentive constraint. First, we formulate the law of motion for aggregate net worth. We assume that aggregate net worth consists of the net worth of non-bankrupted banks and the new worth of new banks. The new equity is injected by domestic households and is assumed to be of the size ιn. Then

$$n_t=\omega\left(\frac{R^{*L}_{j,t-1}}{\pi^*_t}rer_t l^*_{t-1}-\frac{R_{t-1}}{\pi_t}d_{t-1}-\frac{R^*_{t-1}\xi_{t-1}}{\pi^*_t}rer_t d^*_{t-1}\right)+\iota n$$

To include the incentive constraint in the equilibrium conditions, we have to redefine it by using the value of marginal utility from increasing assets by one unit and the value of marginal disutility from increasing debt by one unit. It follows from the previously derived results that the value of the bank j can also be defined as:

$$\begin{aligned} V_{j,t} &= \left(\lambda^L\tfrac{\nu_{1,t}}{1+\nu_{1,t}}+\tfrac{\nu_{2,t}}{1+\nu_{1,t}}\right)rer_t l^*_{j,t}-\tfrac{\nu_{2,t}}{1+\nu_{1,t}}d_{j,t}-\tfrac{\nu_{2,t}}{1+\nu_{1,t}}rer_t d^*_{j,t} \\ &= \tfrac{\nu_{2,t}}{1+\nu_{1,t}}\left(rer_t l^*_{j,t}-d_{j,t}-rer_t d^*_{j,t}\right)+\lambda^L\tfrac{\nu_{1,t}}{1+\nu_{1,t}}rer_t l^*_{j,t} \end{aligned}$$

$$\Rightarrow \quad V_{j,t}=\frac{\nu_{2,t}}{1+\nu_{1,t}}n_{j,t}+\lambda^L\frac{\nu_{1,t}}{1+\nu_{1,t}}rer_t l^*_{j,t}$$

Then we can modify the incentive constraint as

$$\frac{\nu_{2,t}}{1+\nu_{1,t}}n_{j,t}+\lambda^L\frac{\nu_{1,t}}{1+\nu_{1,t}}rer_t l^*_{j,t}\geq\lambda^L rer_t l^*_{j,t}$$

$$\Rightarrow \quad \nu_{2,t}n_{j,t}\geq\lambda^L rer_t l^*_{j,t}$$

C: Retail firms

Retail firms constitute a continuum of mass one. They buy homogenous goods y^R_t at the market price P^R_t and use them in the production of differentiated goods $y^H_t(j)$. Differentiated goods are sold in a monopolistically competitive market subject to the demand of the domestic final goods producer.

Domestic retail firms are subject to sticky prices (Calvo, 1983), so every period $(1-\omega^H)$ of them adjust prices to the optimal reset price $P^\#_t(j)$. Then the profit of a retail firm j that is allowed to adjust its price in period t is thus given by $\left(P^\#_t(j)-P^R_t\right)y^H_t(j)$. The fraction ω^H of remaining firms adjust past prices by the rate $\pi^{adj}_t=\pi$.

Then the aggregate price level of domestic retail goods is

$$P^H_t=\left((1-\omega^H)\left(P^\#_t\right)^{1-\epsilon_H}+\omega^H\left(P^H_{t-1}\pi^{adj}_t\right)^{1-\epsilon_H}\right)^{1/(1-\epsilon_H)}$$

Define

$$\tilde{p}_t^H \equiv \frac{P_t^{\#}}{P_t^H} \tag{B.1}$$

It follows that

$$\Rightarrow \quad 1 = (1-\omega^H)\left(\tilde{p}_t^H\right)^{1-\epsilon_H} + \omega^H \left(\frac{P_{t-1}^H \pi_t^{adj}}{P_t^H}\right)^{1-\epsilon_H}$$

Re-writing in terms of relative prices with respect to the price level of domestic final goods P_t such that $p_t^H \equiv P_t^H / P_t$ gives

$$\Rightarrow \quad 1 = (1-\omega^H)\left(\tilde{p}_t^H\right)^{1-\epsilon_H} + \omega^H \left(\frac{p_{t-1}^H \pi_t^{adj}}{\pi_t p_t^H}\right)^{1-\epsilon_H}$$

As a result, a domestic retail firm j solves the problem how to set the optimal price $P_t^{\#}(j)$ conditional on not changing it in the future:

$$\max_{P_t^{\#}(j)} E_t \sum_{s=0}^{\infty} \left(\omega^H\right)^s \beta^s \Lambda_{t,t+s} \frac{\left(P_t^{\#}(j)\left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj}\right) - P_{t+s}^R\right)}{P_{t+s}} y_{t+s}^H(j)$$

s.t. the demand for domestic retail goods (equation (3))

$$y_t^H(j) = \left(\frac{P_t^{\#}(j)\left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj}\right)}{P_t^H}\right)^{-\epsilon_H} y_t^H$$

Define $p_t^R \equiv \frac{P_t^R}{P_t}$:

$$\max_{P_t^{\#}(j)} E_t \sum_{s=0}^{\infty} \left(\omega^H\right)^s \beta^s \Lambda_{t,t+s} \left(\frac{P_t^{\#}(j)}{P_{t+s}}\left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj}\right) - p_{t+s}^R\right) y_{t+s}^H(j)$$

s.t. the demand for domestic retail goods

$$y_t^H(j) = \left(\frac{P_t^{\#}(j)\left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj}\right)}{P_t^H}\right)^{-\epsilon_H} y_t^H$$

$$\Rightarrow \max_{P_t^{\#}(j)} E_t \sum_{s=0}^{\infty} \left(\omega^H\right)^s \beta^s \Lambda_{t,t+s} \left(\frac{P_t^{\#}(j)\left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj}\right)}{P_{t+s}} \left(\frac{P_t^{\#}(j)\left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj}\right)}{P_{t+s}^H}\right)^{-\epsilon_H} y_{t+s}^H - p_{t+s}^R \left(\frac{P_t^{\#}(j)\left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj}\right)}{P_{t+s}^H}\right)^{-\epsilon_H} y_{t+s}^H\right)$$

$$\Rightarrow \max_{P_t^{\#}(j)} E_t \sum_{s=0}^{\infty} \left(\omega^H\right)^s \beta^s \Lambda_{t,t+s} \left(p_{t+s}^H \left(\frac{P_t^{\#}(j)\left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj}\right)}{P_{t+s}^H}\right)^{1-\epsilon_H} y_{t+s}^H - p_{t+s}^R \left(\frac{P_t^{\#}(j)\left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj}\right)}{P_{t+s}^H}\right)^{-\epsilon_H} y_{t+s}^H\right)$$

We take a derivative w.r.t. $P_t^{\#}(j)$ and rearrange terms:

$$E_t \sum_{s=0}^{\infty} \left(\omega^H\right)^s \beta^s \Lambda_{t,t+s} \left((1-\epsilon_H) p^H_{t+s} \left(P^{\#}_t(j)\right)^{-\epsilon_H} \left(\frac{\prod_{j=1}^{j=s} \pi^{adj}_{t+j}}{P^H_{t+s}} \right)^{1-\epsilon_H} - \epsilon_H p^R_{t+s} \left(P^{\#}_t(j)\right)^{-\epsilon_H - 1} \left(\frac{\prod_{j=1}^{j=s} \pi^{adj}_{t+j}}{P^H_{t+s}} \right)^{-\epsilon_H} \right) y^H_{t+s}$$

$$\Rightarrow E_t \sum_{s=0}^{\infty} \left(\omega^H\right)^s \beta^s \Lambda_{t,t+s} \left((1-\epsilon_H) p^H_{t+s} P^{\#}_t(j) \left(P^H_{t+s}\right)^{\epsilon_H - 1} \left(\prod_{j=1}^{j=s} \pi^{adj}_{t+j} \right)^{1-\epsilon_H} - \epsilon_H p^R_{t+s} \left(P^H_{t+s}\right)^{\epsilon_H} \left(\prod_{j=1}^{j=s} \pi^{adj}_{t+j} \right)^{-\epsilon_H} \right) y^H_{t+s} = 0$$

$$\Rightarrow P^{\#}_t(j) = \frac{\epsilon_H}{(\epsilon_H - 1)} \frac{E_t \sum_{s=0}^{\infty} \left(\omega^H\right)^s \beta^s \Lambda_{t,t+s} \left(p^R_{t+s} \left(P^H_{t+s}\right)^{\epsilon_H} \left(\prod_{j=1}^{j=s} \pi^{adj}_{t+j} \right)^{-\epsilon_H} y^H_{t+s} \right)}{E_t \sum_{s=0}^{\infty} \left(\omega^H\right)^s \beta^s \Lambda_{t,t+s} \left(p^H_{t+s} \left(P^H_{t+s}\right)^{\epsilon_H - 1} \left(\prod_{j=1}^{j=s} \pi^{adj}_{t+j} \right)^{1-\epsilon_H} y^H_{t+s} \right)} = 0$$

$$\Rightarrow \frac{P^{\#}_t(j)}{P^H_t} = \frac{\epsilon_H}{(\epsilon_H - 1)} \frac{E_t \sum_{s=0}^{\infty} \left(\omega^H\right)^s \beta^s \Lambda_{t,t+s} \left(p^R_{t+s} \left(\frac{P^H_{t+s}}{P^H_t}\right)^{\epsilon_H} \left(\prod_{j=1}^{j=s} \pi^{adj}_{t+j} \right)^{-\epsilon_H} y^H_{t+s} \right)}{E_t \sum_{s=0}^{\infty} \left(\omega^H\right)^s \beta^s \Lambda_{t,t+s} \left(p^H_{t+s} \left(\frac{P^H_{t+s}}{P^H_t}\right)^{\epsilon_H - 1} \left(\prod_{j=1}^{j=s} \pi^{adj}_{t+j} \right)^{1-\epsilon_H} y^H_{t+s} \right)}$$

Since $\tilde{p}^H_t \equiv P^{\#}_t / P^H_t$,

$$\tilde{p}^H_t = \frac{\epsilon_H}{(\epsilon_H - 1)} \frac{E_t \sum_{s=0}^{\infty} \left(\omega^H\right)^s \beta^s \Lambda_{t,t+s} \left(p^R_{t+s} \left(\frac{P^H_{t+s}}{P^H_t}\right)^{\epsilon_H} \left(\prod_{j=1}^{j=s} \pi^{adj}_{t+j} \right)^{-\epsilon_H} y^H_{t+s} \right)}{E_t \sum_{s=0}^{\infty} \left(\omega^H\right)^s \beta^s \Lambda_{t,t+s} \left(p^H_{t+s} \left(\frac{P^H_{t+s}}{P^H_t}\right)^{\epsilon_H - 1} \left(\prod_{j=1}^{j=s} \pi^{adj}_{t+j} \right)^{1-\epsilon_H} y^H_{t+s} \right)}$$

$$\Rightarrow \tilde{p}^H_t = \frac{\epsilon_H}{(\epsilon_H - 1)} \frac{E_t \sum_{s=0}^{\infty} \left(\omega^H\right)^s \beta^s \Lambda_{t,t+s} \left(p^R_{t+s} \left(\frac{p^H_{t+s}\pi_{t+s}}{p^H_t}\right)^{\epsilon_H} \left(\prod_{j=1}^{j=s} \pi^{adj}_{t+j} \right)^{-\epsilon_H} y^H_{t+s} \right)}{E_t \sum_{s=0}^{\infty} \left(\omega^H\right)^s \beta^s \Lambda_{t,t+s} \left(p^H_{t+s} \left(\frac{p^H_{t+s}\pi_{t+s}}{p^H_t}\right)^{\epsilon_H - 1} \left(\prod_{j=1}^{j=s} \pi^{adj}_{t+j} \right)^{1-\epsilon_H} y^H_{t+s} \right)}$$

$$\Rightarrow \tilde{p}^H_t = \frac{\epsilon_H}{(\epsilon_H - 1)} \frac{F^H_{1,t}}{F^H_{2,t}}$$

where

$$F^H_{1,t} = p^R_t y^H_t + E_t \omega^H \beta \Lambda_{t,t+1} \left(\frac{p^H_{t+1}\pi_{t+1}}{p^H_t \left(\prod_{j=1}^{j=s} \pi^{adj}_{t+j} \right)} \right)^{\epsilon_H} F^H_{1,t+1}$$

and

$$F^H_{2,t} = p^H_t y^H_t + E_t \omega^H \beta \Lambda_{t,t+1} \left(\frac{p^H_{t+1}\pi_{t+1}}{p^H_t \left(\prod_{j=1}^{j=s} \pi^{adj}_{t+j} \right)} \right)^{\epsilon_H - 1} F^H_{2,t+1}$$

D: Importers

We assume that there is a continuum of monopolistically competitive importers. They buy a variety j of foreign goods $y_t^F(j)$ at the foreign price P_t^* and sell it to the final goods producer at a nominal price $P_t^F(j)$, expressed in domestic currency.

Every period there is a fraction $(1-\omega^F)$ of importers who can adjust their prices. The set of importers who can adjust the price choose it such that their profits are maximized. The fraction ω^F of remaining importing firms adjust past prices by the rate $\pi_t^{adj} = \pi$. As a result, an importer j solves the problem how to set the optimal price $P_t^{\#F}(j)$ conditional on not changing it in the future:

$$\max_{P_t^{\#F}(j)} E_t \sum_{s=0}^{\infty} \left(\omega^F\right)^s \beta^s \Lambda_{t,t+s} \frac{\left(P_t^{\#F}(j)\left(\prod_{j=1}^{j=s}\pi_{t+j}^{adj}\right) - S_{t+s}P_{t+s}^*(j)\right)}{P_{t+s}} y_{t+s}^F(j) \tag{2}$$

s.t.

$$y_t^F(j) = \eta\left(\frac{P_t^{\#F}(j)\left(\prod_{j=1}^{j=s}\pi_{t+j}^{adj}\right)}{P_t^F}\right)^{-\epsilon} y_t^F$$

Since $rer_t \equiv S_t P_t^*/P_t$,

$$\max_{P_t^{\#F}(j)} E_t \sum_{s=0}^{\infty} \left(\omega^F\right)^s \beta^s \Lambda_{t,t+s} \left(\frac{P_t^{\#F}(j)\left(\prod_{j=1}^{j=s}\pi_{t+j}^{adj}\right)}{P_{t+s}} - rer_{t+s}\right) y_{t+s}^F(j)$$

s.t.

$$y_t^F(j) = \eta\left(\frac{P_t^{\#F}(j)\left(\prod_{j=1}^{j=s}\pi_{t+j}^{adj}\right)}{P_t^F}\right)^{-\epsilon} y_t^F$$

In analogy to the problem of domestic retail firms, we maximize and rearrange terms. Since all importers who can adjust their price set the same optimal price, $P_t^{\#F}(j) = P_t^{\#F} \forall j$. After introducing a variable $\tilde{p}_t^F$ which is defined as

$$\tilde{p}_t^F \equiv P_t^{\#F}/P_t^F, \tag{B.2}$$

we can show that the optimal price-setting equation follows as

$$\Rightarrow \tilde{p}_t^F = \frac{\epsilon_F}{(\epsilon_F - 1)} \frac{E_t \sum_{s=0}^{\infty} \left(\omega^F\right)^s \beta^s \Lambda_{t,t+s} \left(rer_{t+s}\left(\frac{p_{t+s}^F \pi_{t+s}}{p_t^F}\right)^{\epsilon_F} \left(\prod_{j=1}^{j=s}\pi_{t+j}^{adj}\right)^{-\epsilon_F} y_{t+s}^F\right)}{E_t \sum_{s=0}^{\infty} (\omega^F)^s \beta^s \Lambda_{t,t+s} \left(p_{t+s}^F \left(\frac{p_{t+s}^F \pi_{t+s}}{p_t^F}\right)^{\epsilon_F - 1} \left(\prod_{j=1}^{j=s}\pi_{t+j}^{adj}\right)^{1-\epsilon_F} y_{t+s}^F\right)}$$

$$\Rightarrow \tilde{p}_t^F = \frac{\epsilon_F}{(\epsilon_F - 1)} \frac{F_{1,t}^F}{F_{2,t}^F}$$

where

$$F_{1,t}^F = rer_t y_t^F + E_t \omega^F \beta \Lambda_{t,t+1} \left(\frac{p_{t+1}^F \pi_{t+1}}{p_t^F \left(\prod_{j=1}^{j=s}\pi_{t+j}^{adj}\right)}\right)^{\epsilon_F} F_{1,t+1}^F$$

and

$$F_{2,t}^F = p_t^F y_t^F + E_t \omega^F \beta \Lambda_{t,t+1} \left(\frac{p_{t+1}^F \pi_{t+1}}{p_t^F \left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj} \right)} \right)^{\epsilon_F - 1} F_{2,t+1}^F$$

The price index for imported goods is given by: $1 = (1-\omega^F) \left(\tilde{p}_t^F \right)^{1-\epsilon_F} + \omega^F \left(\frac{p_{t-1}^F \pi_t^{adj}}{p_t^F \pi_t} \right)^{1-\epsilon_F}$.

E: Price dispersion

We define the price dispersion of domestic retail goods as

$$D_t^H \equiv \int_0^1 \left(\frac{P_t^H(j)}{P_t^H} \right)^{-\epsilon_H} dj$$

Noting that $(1-\omega^H)$ of firms will update to the same optimal price $P_t^{\#}$, and ω^H of firms will leave the last period's price gives

$$\begin{aligned}
D_t^H &= \int_0^{1-\omega^H} \left(\frac{P_t^{\#}}{P_t^H} \right)^{-\epsilon_H} dj + \int_{1-\omega^H}^1 \left(\frac{P_{t-1}^H(j) \left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj} \right)}{P_t^H} \right)^{-\epsilon_H} dj \\
&= \int_0^{1-\omega^H} \left(\frac{P_t^{\#}}{P_t^H} \right)^{-\epsilon_H} dj + \int_{1-\omega^H}^1 \left(\frac{P_{t-1}^H(j)}{P_{t-1}^H} \right)^{-\epsilon_H} \left(\frac{P_{t-1}^H \left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj} \right)}{P_t^H} \right)^{-\epsilon_H} dj \\
&= (1-\omega^H) \left(\tilde{p}_t^H \right)^{-\epsilon_H} + \int_{1-\omega^H}^1 D_{t-1}^H \left(\frac{P_{t-1}^H \left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj} \right)}{P_t^H} \right)^{-\epsilon_H} dj \\
&= (1-\omega^H) \left(\tilde{p}_t^H \right)^{-\epsilon_H} + \omega^H \left(\frac{p_{t-1}^H \left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj} \right)}{\pi_t p_t^H} \right)^{-\epsilon_H} D_{t-1}^H
\end{aligned}$$

In analogy, the price dispersion of importers' goods is given by

$$D_t^F \equiv \int_0^1 \left(\frac{P_t^F(j)}{P_t^F} \right)^{-\epsilon_F} dj$$

and it follows a rule

$$D_t^F = (1-\omega^F) \left(\tilde{p}_t^F \right)^{-\epsilon_F} + \omega^F \left(\frac{p_{t-1}^F \left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj} \right)}{\pi_t p_t^F} \right)^{-\epsilon_F} D_{t-1}^F$$

F: Domestic final goods producer

The domestic final goods producer combines domestic composite goods and foreign composite goods into domestic final goods and sells them to the household, the government and capital goods producers. We define the supply of domestic final goods as y_t^C. The demanded amount of production inputs, namely, domestic composite goods and foreign composite goods, is denoted as x_t^H and x_t^F respectively.

Domestic composite goods. Domestic composite goods y_t^H result from assembling retailers' production $y_t^H(j)$ for $j \in [0,1]$, each bought at price $P_t^H(j)$, expressed in domestic currency, and with no additional costs incurred. Let the aggregate price level of retail goods be $P_t^H \equiv \left(\int_0^1 \left(P_t^H(j)\right)^{1-\epsilon_H} dj\right)^{1/(1-\epsilon_H)}$, expressed in domestic currency. Then it follows that the demand for retail goods is given as a solution to the problem

$$\max_{y_t^H(j)} \left\{ P_t^H y_t^H - \int_0^1 P_t^H(j) y_t^H(j) dj \right\}$$

subject to the assembling technology

$$y_t^H = \left(\int_0^1 y_t^H(j)^{1-\frac{1}{\epsilon_H}} dj \right)^{\frac{\epsilon_H}{\epsilon_H - 1}}$$

and to the market clearing constraint that says that domestic composite goods are also used to satisfy foreign demand ex_t, all expressed in units of domestic final goods:

$$y_t^H = x_t^H + ex_t$$

As a result, optimal demand for retail goods is given by

$$y_t^H(j) = \left(\frac{P_t^H(j)}{P_t^H} \right)^{-\epsilon_H} y_t^H \tag{3}$$

and domestic demand for domestic composite goods satisfies the clearing condition $x_t^H = y_t^H - ex_t$.

Foreign composite goods. Foreign composite goods y_t^F result from assembling importers' production $y_t^F(j)$ for $j \in [0,1]$, each bought at price $P_t^F(j)$, expressed in domestic currency, and with no additional costs incurred. Let the aggregate price level of importers' goods be $P_t^F \equiv \left(\int_0^1 \left(P_t^F(j)\right)^{1-\epsilon_F} dj\right)^{1/(1-\epsilon_F)}$, expressed in domestic currency. Then it follows that the demand for retail goods is given as a solution to the problem

$$\max_{y_t^F(j)} \left\{ P_t^F y_t^F - \int_0^1 P_t^F(j) y_t^F(j) dj \right\}$$

subject to the assembling technology

$$y_t^F = \left(\int_0^1 y_t^F(j)^{1-\frac{1}{\epsilon_F}} dj \right)^{\frac{\epsilon_F}{\epsilon_F - 1}}$$

and to the market clearing constraint that says that all foreign composite goods are used to satisfy the demand of the final goods producer:

$$y_t^F = x_t^F$$

As a result, optimal demand for importers' production is given by

$$y_t^F(j) = \left(\frac{P_t^F(j)}{P_t^F} \right)^{-\epsilon_F} y_t^F \tag{4}$$

and demand for foreign composite goods clears $x_t^F = y_t^F$.

Domestic final goods. Given inputs x_t^H and x_t^F, domestic final goods are assembled with the aggregation technology

$$y_t^C \equiv \left[(1-\eta)^{\frac{1}{\epsilon}}(x_t^H)^{\frac{\epsilon-1}{\epsilon}} + \eta^{\frac{1}{\epsilon}}(x_t^F)^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}} \tag{5}$$

where ϵ stands for elasticity of substitution between domestically produced goods and imported goods.

The domestic final goods producer operates in a perfectly competitive market, so she maximizes profits $P_t y_t^C - P_t^H x_t^H - P_t^F x_t^F$ subject to the technology (5). This boils down to two demand conditions:

$$x_t^H = (1-\eta)\left(\frac{P_t^H}{P_t}\right)^{-\epsilon} y_t^C$$

and

$$x_t^F = \eta\left(\frac{P_t^F}{P_t}\right)^{-\epsilon} y_t^C$$

where a parameter η proxies for openness of the home economy. Further, we introduce relative prices $p_t^H \equiv P_t^H/P_t$ and $p_t^F \equiv P_t^F/P_t$ and get

$$x_t^H = (1-\eta)\left(p_t^H\right)^{-\epsilon} y_t^C \tag{6}$$

and

$$x_t^F = \eta\left(p_t^F\right)^{-\epsilon} y_t^C \tag{7}$$

G: Capital producers

Capital producers operate the technology that allows them to combine depreciated capital with investment i_t and increase the capital stock. They sell capital at the nominal competitive price Q_t to risky firms. Capital producers maximize profits, expressed in units of domestic final goods, subject to the production technology by choosing an optimal level of investment. The technology is given by

$$k_t = (1-\delta)k_{t-1} + \left(1-\Gamma\left(\frac{i_t}{i_{t-1}}\right)\right)i_t \tag{8}$$

where

$$\Gamma\left(\frac{i_t}{i_{t-1}}\right) = \frac{\gamma}{2}\left(\frac{i_t}{i_{t-1}}-1\right)^2$$

are investment adjustment costs. Profit maximization problem can be written in units of domestic final goods as

$$\max_{i_t}\{q_t k_t - q_t(1-\delta)k_{t-1} - i_t\} \tag{9}$$

s.t.

$$k_t = (1-\delta)k_{t-1} + \left(1-\Gamma\left(\frac{i_t}{i_{t-1}}\right)\right)i_t \tag{10}$$

Optimizing gives the demand function for investment:

$$\frac{1}{q_t} = 1 - \frac{\kappa}{2}\left(\frac{i_t}{i_{t-1}}-1\right)^2 - \kappa\left(\frac{i_t}{i_{t-1}}-1\right)\frac{i_t}{i_{t-1}} + \kappa\beta E_t\Lambda_{t,t+1}\frac{q_{t+1}}{q_t}\left(\frac{i_{t+1}}{i_t}-1\right)\left(\frac{i_{t+1}}{i_t}\right)^2 \tag{11}$$

H: Exporters

We assume that all economies are identical: foreign economies also do assemble goods with the very same technologies and no additional costs incurred (no trade barriers in the model). Hence, the rest of the world demands ex_t units of domestic composite goods at a price $P_t^{H*} = P_t^H / S_t$, which is the price of domestic composite goods expressed in units of foreign currency. Demand for exports can be solved from the foreign household's problem, where the foreign household chooses between goods produced in the home economy ex_t and other goods y_t^*:

$$\max_{ex_t} \left\{ P_t^* y_t^* - P_t^{H*} ex_t \right\}$$

$$\max_{ex_t} \left\{ P_t^* y_t^* - \frac{P_t^H}{S_t} ex_t \right\}$$

The share of goods produced in the home economy in total foreign consumption is described by two new parameters: η^* proxies for preferences of the foreign household for goods produced in the home economy and ϵ_* stands for elasticity of substitution between goods produced in the home economy and other goods. Also we define the foreign demand for goods other than goods produced in the home economy as an exogenous variable x^{*other}. Then total demand in the rest of the world is given by

$$y_t^* \equiv \left[(1-\eta^*)^{\frac{1}{\epsilon_*}} (x_t^{*other})^{\frac{\epsilon_*-1}{\epsilon_*}} + (\eta^*)^{\frac{1}{\epsilon_*}} (ex_t)^{\frac{\epsilon_*-1}{\epsilon_*}} \right]^{\frac{\epsilon_*}{\epsilon_*-1}} \tag{12}$$

Maximizing profits $P_t^* y_t^* - P_t^{H*} ex_t$, expressed in units of foreign currency, subject to the aggregation technology (12) boils down to the foreign demand condition for goods produced in the home economy:

$$ex_t = \eta^* \left(\frac{P_t^H}{S_t P_t^*} \right)^{-\epsilon_*} y_t^*$$

Since $rer_t \equiv S_t P_t^* / P_t$,

$$ex_t = \eta^* \left(\frac{p_t^H}{rer_t} \right)^{-\epsilon_*} y_t^*$$

I: Government

The government collects lump-sum taxes T_t from the household and issues domestic bonds B_t to finance a stochastic stream of nominal government expenditure, G_t. Therefore, it satisfies the budget constraint:

$$G_t + R_{t-1} B_{t-1} = T_t + B_t \tag{13}$$

which, given $g_t \equiv G_t / P_t$, $b_t \equiv B_t / P_t$ and $t_t \equiv T_t / P_t$, can be expressed in units of domestic final goods as

$$g_t + \frac{R_{t-1}}{\pi_t} b_{t-1} = t_t + b_t \tag{14}$$

Taxes in units of domestic final goods are assumed to follow this tax rule:

$$t_t = \bar{t} + \kappa_b \left(b_{t-1} - \bar{b} \right) + \tau_t, \qquad \tau_t \sim NID(0, \sigma_\tau^2) \tag{15}$$

J: Current account and its components

First, we derive an expression for aggregate nominal imports M_t in units of domestic currency. We aggregate importers' demand $y_t^F(j)$ that is priced at P_t^* and express the sum in units of domestic currency:

$$M_t = \int_0^1 S_t P_t^* y_t^F(j) dj$$

Further we use the derived demand function (4) to get

$$M_t = \int_0^1 S_t P_t^* \left(\frac{P_t^F(j)}{P_t^F} \right)^{-\epsilon_F} y_t^F$$

Define the price dispersion of importers' goods as $D_t^F \equiv \int_0^1 \left(\frac{P_t^F(j)}{P_t^F} \right)^{-\epsilon_F} dj$ (more details on the price dispersion are in Appendix B2). Then

$$M_t = S_t P_t^* D_t^F y_t^F \tag{16}$$

which in units of domestic final goods is given by

$$m_t \equiv \frac{M_t}{P_t} = rer_t D_t^F y_t^F \tag{17}$$

Second, we define nominal exports EX_t, expressed in units of domestic currency. Since exports are purchased at the price P_t^{H*}, expressed in foreign currency, nominal exports EX_t, expressed in units of domestic currency, is given by

$$EX_t = S_t P_t^{H*} ex_t = P_t^H ex_t \tag{18}$$

The trade balance TB_t in nominal domestic terms evolves as

$$TB_t = EX_t - M_t$$

Recall definitions for nominal exports and nominal imports in units of domestic currency (equations (18) and (16)). Then the trade balance in units of domestic final goods can be expressed as

$$tb_t \equiv \frac{TB_t}{P_t} = \frac{P_t^H ex_t}{P_t} - \frac{S_t P_t^* D_t^F y_t^F}{P_t}$$

$$\Rightarrow \qquad tb_t = p_t^H ex_t - rer_t D_t^F y_t^F$$

Since $m_t \equiv rer_t D_t^F y_t^F$,

$$tb_t = p_t^H ex_t - m_t$$

The domestic household owns banks that borrow from the foreign household. As a result, a current account in nominal domestic terms is given by the sum of nominal trade balance and nominal net income from abroad, both expressed in domestic currency:

$$CA_t = TB_t + NI_t \tag{19}$$

In our case nominal net income from abroad is negative and equal to minus payments on bank foreign debt:

$$NI_t = -\left(R_{t-1}^* \xi_{t-1} - 1\right) S_t D_{t-1}^*$$

Substituting the last expression in equation (19) gives a more detailed expression for the nominal current account:

$$CA_t = TB_t - \left(R^*_{t-1}\xi_{t-1} - 1\right) S_t D^*_{t-1}$$

Further, we express the current account in units of domestic final goods as ca_t ($ca_t \equiv CA_t/P_t$):

$$ca_t = tb_t - \left(R^*_{t-1}\xi_{t-1} - 1\right) \frac{S_t D^*_{t-1}}{P_t}$$

$$\Rightarrow \quad ca_t = tb_t - \left(R^*_{t-1}\xi_{t-1} - 1\right) rer_t \frac{d^*_{t-1}}{\pi^*_t} \tag{20}$$

In equilibrium the current account has to equal the capital account balance CP_t. In our case the capital account balance is given by the change in stocks of bank foreign debt:

$$CP_t = -\left(S_t D^*_t - S_t D^*_{t-1}\right)$$

We express the capital account balance in units of domestic final goods as cp_t ($cp_t \equiv CP_t/P_t$):

$$cp_t = -\left(rer_t d^*_t - rer_t \frac{d^*_{t-1}}{\pi^*_t}\right)$$

Then, next to the current account definition (20), we impose an additional restriction that enters the set of equilibrium equations:

$$ca_t = -\left(rer_t d^*_t - rer_t \frac{d^*_{t-1}}{\pi^*_t}\right) \tag{21}$$

K: Model with domestic currency loans

K1: Financially constrained firms

The first difference from the main model occurs in the firm's loan demand function. The firm's borrowing decision depends on the firm's expected working capital needs such that in the beginning of period t the following condition holds:

$$L_{i,t} = E_{t-1}\left\{\rho\left(Q_t k_{i,t} + W_t h_{i,t}\right)\right\}$$

where Q_t is the nominal price of capital and W_t is the nominal wage.

Let the matured nominal loan in units of domestic currency be $R^R_{i,t} L_{i,t}$, where $R^R_{i,t}$ is the nominal interest rate on the loan. The contracted collateral is a share κ ($0 < \kappa \leq 1$) of firms' revenue from selling goods and depreciated capital in the next period, $P^R_{t+1} y^R_{i,t+1} + Q_{t+1}(1-\delta)k_{i,t}$. $PM_{i,t+1}$ stands for a default decision of the financially constrained firm i born in period t:

$$PM_{i,t+1} = \min\left\{R^R_{i,t} L_{i,t}, \quad \kappa\left(P^R_{t+1} y^R_{i,t+1} + Q_{t+1}(1-\delta)k_{i,t}\right)\right\}$$

where $P^R_{t+1} y^R_{i,t+1} = P^R_{t+1} A_{t+1} \theta_{i,t+1} k^\alpha_{i,t} h^{1-\alpha}_{i,t}$.

Therefore, after shocks take place, the generation of firms t will solve the same profit maximization problem taking the loan as given:

$$\max_{\{k_{i,t},h_{i,t}\}} E_t\beta\Lambda_{t,t+1}\left\{\frac{\left(P^R_{t+1}y^R_{i,t+1}+Q_{t+1}(1-\delta)k_{i,t}-PM_{i,t+1}-(1-\rho)\left(Q_tk_{i,t}+W_th_{i,t}\right)\right)}{P_{t+1}}\right\}$$
$$+\frac{L_{i,t}-\rho\left(Q_tk_{i,t}+W_th_{i,t}\right)+Z_{i,t}}{P_t}$$

$Z_{i,t}$ stands for the equity injection from the domestic household.

Define $p^R_{t+1}\equiv P^R_{t+1}/P_{t+1}$, $q_{t+1}\equiv Q_{t+1}/P_{t+1}$, $w_t\equiv W_t/P_t$ and $l_{i,t}\equiv L_{i,t}/P_t$.

This, after substituting for $PM_{i,t+1}$, gives

$$\max_{\{k_{i,t},h_{i,t}\}} E_t\beta\Lambda_{t,t+1}\left\{p^R_{t+1}y^R_{i,t+1}+q_{t+1}(1-\delta)k_{i,t}-(1-\rho)\frac{q_tk_{i,t}+w_th_{i,t}}{\pi_{t+1}}\right\}$$
$$-E_t\beta\Lambda_{t,t+1}\min\left\{\frac{R^R_{i,t}}{\pi_{t+1}}l_{i,t},\quad \kappa\left(p^R_{t+1}y^R_{i,t+1}+q_{t+1}(1-\delta)k_{i,t}\right)\right\}$$
$$+l_{i,t}-\rho\left(q_tk_{i,t}+w_th_{i,t}\right)+z_{i,t}$$

s.t.

$$l_{i,t}=E_{t-1}\left\{\rho\left(q_tk_{i,t}+w_th_{i,t}\right)\right\}$$

The corresponding first-order conditions:

$$k_{i,t}:\quad E_t\beta\Lambda_{t,t+1}\left\{p^R_{t+1}\frac{\partial y^R_{i,t+1}}{\partial k_{i,t}}+(1-\delta)q_{t+1}-(1-\rho)\frac{q_t}{\pi_{t+1}}\right\}$$
$$-\frac{\partial E_t\beta\Lambda_{t,t+1}E_t\min\left\{\frac{R^R_{i,t}}{\pi_{t+1}}l_{i,t},\quad \kappa\left(p^R_{t+1}y^R_{i,t+1}+q_{t+1}(1-\delta)k_{i,t}\right)\right\}}{\partial k_{i,t}}$$
$$=\frac{\partial cov\left(\beta\Lambda_{t,t+1},\quad \min\left\{\frac{R^R_{i,t}l_{i,t}}{\pi_{t+1}},\quad \kappa\left(p^R_{t+1}y^R_{i,t+1}+q_{t+1}(1-\delta)k_{i,t}\right)\right\}\right)}{\partial k_{i,t}}$$
$$+\rho q_t$$

$$h_{i,t}:\quad E_t\beta\Lambda_{t,t+1}\left\{p^R_{t+1}\frac{\partial y^R_{i,t+1}}{\partial h_{i,t}}-(1-\rho)\frac{w_t}{\pi_{t+1}}\right\}$$
$$-\frac{\partial E_t\beta\Lambda_{t,t+1}E_t\min\left\{\frac{R^R_{i,t}}{\pi_{t+1}}l_{i,t},\quad \kappa\left(p^R_{t+1}y^R_{i,t+1}+q_{t+1}(1-\delta)k_{i,t}\right)\right\}}{\partial h_{i,t}}$$
$$=\frac{\partial cov\left(\beta\Lambda_{t,t+1},\quad \min\left\{\frac{R^R_{i,t}l_{i,t}}{\pi_{t+1}},\quad \kappa\left(p^R_{t+1}y^R_{i,t+1}+q_{t+1}(1-\delta)k_{i,t}\right)\right\}\right)}{\partial h_{i,t}}$$
$$+\rho w_t$$

If we substitute the expression for the expected value of loan repayment, we get:

$$
\begin{aligned}
k_{i,t}: \quad & E_t \beta \Lambda_{t,t+1} \left\{ p^R_{t+1} \frac{\partial y^R_{i,t+1}}{\partial k_{i,t}} + q_{t+1}(1-\delta) - (1-\rho) \frac{q_t}{\pi_{t+1}} \right\} \\
& - E_t \beta \Lambda_{t,t+1} \left\{ (1-\Phi(d_{1,t}))\kappa \left(p^R_{t+1} \frac{\partial y^R_{i,t+1}}{\partial k_{i,t}} + q_{t+1}(1-\delta) \right) \right\} \\
& = \frac{\partial cov \left(\beta \Lambda_{t,t+1}, \quad \min \left\{ \frac{R^R_{i,t} l_{i,t}}{\pi_{t+1}}, \quad \kappa \left(p^R_{t+1} y^R_{i,t+1} + q_{t+1}(1-\delta) k_{i,t} \right) \right\} \right)}{\partial k_{i,t}} \\
& + \rho q_t
\end{aligned}
$$

$$
\begin{aligned}
h_{i,t}: \quad & E_t \beta \Lambda_{t,t+1} \left\{ p^R_{t+1} \frac{\partial y^R_{i,t+1}}{\partial h_{i,t}} - (1-\rho) \frac{w_t}{\pi_{t+1}} \right\} \\
& - E_t \beta \Lambda_{t,t+1} \left\{ (1-\Phi(d_{1,t}))\kappa \left(p^R_{t+1} \frac{\partial y^R_{i,t+1}}{\partial h_{i,t}} \right) \right\} \\
& = \frac{\partial cov \left(\beta \Lambda_{t,t+1}, \quad \min \left\{ \frac{R^R_{i,t} l_{i,t}}{\pi_{t+1}}, \quad \kappa \left(p^R_{t+1} y^R_{i,t+1} + q_{t+1}(1-\delta) k_{i,t} \right) \right\} \right)}{\partial h_{i,t}} \\
& + \rho w_t
\end{aligned}
$$

where

$$
d_{2,t} \equiv \frac{E_t \ln \left(\kappa \left(p^R_{t+1} y^R_{i,t+1} + q_{t+1}(1-\delta) k_{i,t} \right) \right) - E_t \ln \left(\frac{R^R_{i,t}}{\pi_{t+1}} l_{i,t} \right)}{\sigma_y}, \qquad d_{1,t} = d_{2,t} + \sigma_y
$$

σ^2_y is given by $var(\pi_{t+1}\kappa \left(p^R_{t+1} y^R_{i,t+1} + q_{t+1}(1-\delta) k_{i,t} \right))$.

The first-order conditions w.r.t. capital, labour demand and the postulated loan demand enter the set of equilibrium equations.

K2: Banks

We assume that banks give domestic currency loans out of accumulated equity N_t, domestic deposits D_t and foreign debt D^*_t. The balance sheet constraint of a bank j, expressed in units of domestic currency, is given by

$$
N_{j,t} + D_{j,t} + S_t D^*_{j,t} = L_{j,t}
$$

where S_t is the nominal exchange rate. Banks are subject to an agency problem as in Gertler and Karadi (2011). At the end of every period every bank can divert a share λ^L of their divertable assets. Creditors take this possibility into account and lend only up to the point where the continuation value of the bank is still larger or equal to what can be diverted. This condition acts as an incentive constraint for the bank and eventually limits expansion of the balance sheet.

The only asset on the banks' balance sheet is domestic currency loans to domestic financially constrained firms, thus, the nominal expected return of the bank j is defined as $R^L_{j,t}$ and given by:

$$E_t\left\{R^L_t L_{j,t}\right\} \equiv E_t\left\{(1-\Phi(d_{1,t}))\kappa\left(P^R_{t+1}y^R_{j,t+1}+(1-\delta)Q_{t+1}k_{j,t}\right)+\Phi(d_{2,t})R^R_{j,t}L_{j,t}\right\}$$

Then the optimization problem of the bank j can be written as:

$$V_{j,t}=\max_{\left\{D_{j,t},D^*_{j,t},L_{j,t}\right\}} \quad E_t\left[\beta\Lambda_{t,t+1}\left\{(1-\omega)\frac{N_{j,t+1}}{P_{t+1}}+\omega V_{j,t+1}\right\}\right]$$

s.t.

$$V_{j,t}\geq\lambda^L\frac{L_{j,t}}{P_t}, \qquad \textit{(Incentive constraint)}$$

$$\frac{N_{j,t}+D_{j,t}+S_tD^*_{j,t}}{P_t}=\frac{L_{j,t}}{P_t}, \qquad \textit{(Balance sheet constraint)}$$

$$\frac{N_{j,t}}{P_t}=\frac{R^L_{j,t-1}}{P_t}L_{j,t-1}-\frac{R_{t-1}}{P_t}D_{j,t-1}-\frac{R^*_{t-1}\xi_{t-1}}{P_t}S_tD^*_{j,t-1} \qquad \textit{(LoM of net worth)}$$

Define $rer_t\equiv P^*_tS_t/P_t$, $d^*_{j,t}\equiv D^*_{j,t}/P^*_t$, $d_{j,t}\equiv D_{j,t}/P_t$, $l_{j,t}\equiv L_{j,t}/P_t$, and $n_{j,t}\equiv N_{j,t}/P_t$. It follows that

$$V_{j,t}=\max_{\left\{d_{j,t},d^*_{j,t},l_{j,t}\right\}} \quad E_t\left[\beta\Lambda_{t,t+1}\left\{(1-\omega)n_{j,t+1}+\omega V_{j,t+1}\right\}\right]$$

s.t.

$$V_{j,t}\geq\lambda^L l_{j,t}, \qquad \textit{(Incentive constraint)}$$

$$n_{j,t}+d_{j,t}+rer_td^*_{j,t}=l_{j,t}, \qquad \textit{(Balance sheet constraint)}$$

$$n_{j,t}=\frac{R^L_{j,t-1}}{\pi_t}l_{j,t-1}-\frac{R_{t-1}}{\pi_t}d_{j,t-1}-\frac{R^*_{t-1}\xi_{t-1}}{\pi^*_t}rer_td^*_{j,t-1} \qquad \textit{(LoM of net worth)}$$

Lagrangian of the problem can be formulated as:

$$\begin{aligned}L=\quad&(1+\nu_{1,t})E_t\beta\Lambda_{t,t+1}\left\{(1-\omega)\left(\frac{R^L_{j,t}}{\pi_{t+1}}l_{j,t}-\frac{R_t}{\pi_{t+1}}d_{j,t}-\frac{R^*_t\xi_t}{\pi^*_{t+1}}rer_{t+1}d^*_{j,t}\right)+\omega V_{j,t+1}\right\}\\&-\nu_{1,t}\lambda^Ll_{j,t}\\&+\nu_{2,t}\left(\frac{R^L_{j,t-1}}{\pi_t}l_{j,t-1}-\frac{R_{t-1}}{\pi_t}d_{j,t-1}-\frac{R^*_{t-1}\xi_{t-1}}{\pi^*_t}rer_td^*_{j,t-1}-l_{j,t}+d_{j,t}+rer_td^*_{j,t}\right)\end{aligned}$$

This gives the first-order conditions:

$$l_{j,t}:\quad(1+\nu_{1,t})\beta E_t\Lambda_{t,t+1}\left\{(1-\omega)\left(\frac{R^L_{j,t}}{\pi_{t+1}}\right)+\omega\frac{\partial V(.)}{\partial l_{j,t}}\right\}=\lambda^L\nu_{1,t}+\nu_{2,t}$$

$$d_{j,t}:\quad(1+\nu_{1,t})\beta E_t\Lambda_{t,t+1}\left\{(1-\omega)\left(\frac{R_t}{\pi_{t+1}}\right)-\omega\frac{\partial V(.)}{\partial d_{j,t}}\right\}=\nu_{2,t}$$

$$d^*_{j,t}:\quad(1+\nu_{1,t})\beta E_t\Lambda_{t,t+1}\left\{(1-\omega)\left(\frac{R^*_t\xi_t}{\pi^*_{t+1}}rer_{t+1}\right)-\omega\frac{\partial V(.)}{\partial d^*_{j,t}}\right\}=\nu_{2,t}rer_t$$

with complementary slackness conditions:

$$\nu_{1,t}: \quad \nu_{1,t}\left(V_{j,t} - \lambda^L l_{j,t}\right) = 0$$

$$\nu_{2,t}: \quad \nu_{2,t}\left(\frac{R^L_{j,t-1}}{\pi_t} l_{j,t-1} - \frac{R_{t-1}}{\pi_t} d_{j,t-1} - \frac{R^*_{t-1}\xi_{t-1}}{\pi^*_t} rer_t d^*_{j,t-1} - l_{j,t} + d_{j,t} + rer_t d^*_{j,t}\right) = 0$$

Further, the first-order conditions can be expressed as

$$l_{j,t}: \quad (1+\nu_{1,t})\beta E_t \Lambda_{t,t+1}\left\{(1-\omega) + \omega\nu_{2,t+1}\right\}\left(\frac{R^L_{j,t}}{\pi_{t+1}}\right) = \nu_{1,t}\lambda^L + \nu_{2,t}$$

$$d_{j,t}: \quad (1+\nu_{1,t})\beta E_t \Lambda_{t,t+1}\left\{(1-\omega) + \omega\nu_{2,t+1}\right\}\left(\frac{R_t}{\pi_{t+1}}\right) = \nu_{2,t}$$

$$d^*_{j,t}: \quad (1+\nu_{1,t})\beta E_t \Lambda_{t,t+1}\left\{(1-\omega) + \omega\nu_{2,t+1}\right\}\left(\frac{R^*_t\xi_t}{\pi^*_{t+1}} \frac{rer_{t+1}}{rer_t}\right) = \nu_{2,t}$$

Besides these first-order conditions, the set of equilibrium conditions would include the law of motion for aggregate net worth of banks and the incentive constraint. First, we formulate the law of motion for aggregate net worth. We assume that aggregate net worth consists of the net worth of non-bankrupted banks and the new worth of new banks. The new equity is injected by domestic households and is assumed to be of the size ιn. Then

$$n_t = \omega\left(\frac{R^L_{j,t-1}}{\pi_t} l_{t-1} - \frac{R_{t-1}}{\pi_t} d_{t-1} - \frac{R^*_{t-1}\xi_{t-1}}{\pi^*_t} rer_t d^*_{t-1}\right) + \iota n$$

To include the incentive constraint in the equilibrium conditions, we have to redefine it by using the value of marginal utility from increasing assets by one unit and the value of marginal disutility from increasing debt by one unit. It follows from the previously derived results that the value of the bank j can also be defined as:

$$\begin{aligned} V_{j,t} &= \left(\lambda^L \tfrac{\nu_{1,t}}{1+\nu_{1,t}} + \tfrac{\nu_{2,t}}{1+\nu_{1,t}}\right) l_{j,t} - \tfrac{\nu_{2,t}}{1+\nu_{1,t}} d_{j,t} - \tfrac{\nu_{2,t}}{1+\nu_{1,t}} rer_t d^*_{j,t} \\ &= \tfrac{\nu_{2,t}}{1+\nu_{1,t}}\left(l_{j,t} - d_{j,t} - rer_t d^*_{j,t}\right) + \lambda^L \tfrac{\nu_{1,t}}{1+\nu_{1,t}} l_{j,t} \end{aligned}$$

$$\Rightarrow \quad V_{j,t} = \frac{\nu_{2,t}}{1+\nu_{1,t}} n_{j,t} + \lambda^L \frac{\nu_{1,t}}{1+\nu_{1,t}} l_{j,t}$$

Then we can modify the incentive constraint as

$$\frac{\nu_{2,t}}{1+\nu_{1,t}} n_{j,t} + \lambda^L \frac{\nu_{1,t}}{1+\nu_{1,t}} l_{j,t} \geq \lambda^L\left(l_{j,t} - \lambda^D d_{j,t}\right)$$

$$\Rightarrow \quad \nu_{2,t} n_{j,t} \geq \lambda^L l_{j,t}$$

L: Model with flexible labour demand

This model differs from the main model in two main ways. First, the only input for financially constrained firms' production is capital. Second, there emerges a new layer of production firms that combine financially constrained firms' production with labour and sell goods to domestic retail firms. The latter type of firms is not subject to financial frictions.

Then, in case of domestic currency loans, the financially constrained firm's problem changes accordingly. The firm's borrowing decision depends on the firm's expected working capital needs such that in the beginning of period t the following condition holds:

$$L_{i,t} = E_{t-1}\left\{\rho\left(Q_t k_{i,t}\right)\right\}$$

where Q_t is the nominal price of capital. Definition of $P^R_{t+1}y^R_{i,t+1}$ changes in the following way: $P^R_{t+1}y^R_{i,t+1} = P^R_{t+1}A_{t+1}\theta_{i,t+1}k_{i,t}$.

After shocks take place, the generation of firms t will solve the same profit maximization problem taking the loan as given:

$$\begin{aligned}\max_{\{k_{i,t},h_{i,t}\}} & E_t\beta\Lambda_{t,t+1}\left\{\frac{\left(P^R_{t+1}y^R_{i,t+1} + Q_{t+1}(1-\delta)k_{i,t} - PM_{i,t+1} - (1-\rho)\left(Q_t k_{i,t}\right)\right)}{P_{t+1}}\right\} \\ & + \frac{L_{i,t} - \rho\left(Q_t k_{i,t}\right) + Z_{i,t}}{P_t}\end{aligned}$$

$Z_{i,t}$ stands for the equity injection from the domestic household.

Define $p^R_{t+1} \equiv P^R_{t+1}/P_{t+1}$, $q_{t+1} \equiv Q_{t+1}/P_{t+1}$ and $l_{i,t} \equiv L_{i,t}/P_t$.

This, after substituting for $PM_{i,t+1}$, gives

$$\begin{aligned}\max_{\{k_{i,t},h_{i,t}\}} & E_t\beta\Lambda_{t,t+1}\left\{p^R_{t+1}y^R_{i,t+1} + q_{t+1}(1-\delta)k_{i,t} - (1-\rho)\frac{q_t k_{i,t}}{\pi_{t+1}}\right\} \\ & - E_t\beta\Lambda_{t,t+1}\min\left\{\frac{R^R_{i,t}}{\pi_{t+1}}l_{i,t}, \quad \kappa\left(p^R_{t+1}y^R_{i,t+1} + q_{t+1}(1-\delta)k_{i,t}\right)\right\} \\ & + l_{i,t} - \rho\left(q_t k_{i,t}\right) + z_{i,t}\end{aligned}$$

s.t.

$$l_{i,t} = E_{t-1}\left\{\rho\left(q_t k_{i,t}\right)\right\}$$

The corresponding first-order condition is:

$$
\begin{aligned}
k_{i,t}: \quad & E_t \beta \Lambda_{t,t+1} \left\{ p^R_{t+1} \frac{\partial y^R_{i,t+1}}{\partial k_{i,t}} + (1-\delta) q_{t+1} - (1-\rho) \frac{q_t}{\pi_{t+1}} \right\} \\
& - \frac{\partial E_t \beta \Lambda_{t,t+1} E_t \min \left\{ \frac{R^R_{i,t}}{\pi_{t+1}} l_{i,t}, \quad \kappa \left(p^R_{t+1} y^R_{i,t+1} + q_{t+1}(1-\delta) k_{i,t} \right) \right\}}{\partial k_{i,t}} \\
& = \frac{\partial cov \left(\beta \Lambda_{t,t+1}, \quad \min \left\{ \frac{R^R_{i,t} l_{i,t}}{\pi_{t+1}}, \quad \kappa \left(p^R_{t+1} y^R_{i,t+1} + q_{t+1}(1-\delta) k_{i,t} \right) \right\} \right)}{\partial k_{i,t}} \\
& + \rho q_t
\end{aligned}
$$

If we substitute the expression for the expected value of loan repayment, we get:

$$
\begin{aligned}
k_{i,t}: \quad & E_t \beta \Lambda_{t,t+1} \left\{ p^R_{t+1} \frac{\partial y^R_{i,t+1}}{\partial k_{i,t}} + q_{t+1}(1-\delta) - (1-\rho) \frac{q_t}{\pi_{t+1}} \right\} \\
& - E_t \beta \Lambda_{t,t+1} \left\{ (1-\Phi(d_{1,t})) \kappa \left(p^R_{t+1} \frac{\partial y^R_{i,t+1}}{\partial k_{i,t}} + q_{t+1}(1-\delta) \right) \right\} \\
& = \frac{\partial cov \left(\beta \Lambda_{t,t+1}, \quad \min \left\{ \frac{R^R_{i,t} l_{i,t}}{\pi_{t+1}}, \quad \kappa \left(p^R_{t+1} y^R_{i,t+1} + q_{t+1}(1-\delta) k_{i,t} \right) \right\} \right)}{\partial k_{i,t}} \\
& + \rho q_t
\end{aligned}
$$

where

$$
d_{2,t} \equiv \frac{E_t \ln \left(\kappa \left(p^R_{t+1} y^R_{i,t+1} + q_{t+1}(1-\delta) k_{i,t} \right) \right) - E_t \ln \left(\frac{R^R_{i,t}}{\pi_{t+1}} l_{i,t} \right)}{\sigma_y}, \qquad d_{1,t} = d_{2,t} + \sigma_y
$$

σ_y^2 is given by $var(\pi_{t+1} \kappa \left(p^R_{t+1} y^R_{i,t+1} + q_{t+1}(1-\delta) k_{i,t} \right))$.

Homogenous goods produced by financially constrained firms are purchased as inputs by a new layer of competitive producers, called intermediate producers. Intermediate producers hire labour and combine it with homogenous goods produced by financially constrained firms by using the following technology:

$$
y^I_t = \left(y^R_t \right)^\alpha h_t^{1-\alpha}
$$

Recall that financially constrained firms' aggregate production function now is given by: $y^R_t = A_t k_{t-1}$. Produced goods are sold to domestic retail firms at the nominal price P^I_t immediately after production takes place. This gives two equilibrium conditions that can be derived from profit maximization with respect to inputs::

$$
\begin{aligned}
y^R_t: \quad & p^R_t = p^I_t \alpha \left(y^R_t \right)^{\alpha-1} h_t^{1-\alpha} \\
h_t: \quad & w_t = p^I_t (1-\alpha) \left(y^R_t \right)^\alpha h_t^{-\alpha}
\end{aligned}
$$

In derivations we defined the following relative prices: $p_t^I \equiv P_t^I/P_t$, $p_t^R \equiv P_t^R/P_t$ and $w_t \equiv W_t/P_t$.

Marginal costs of the domestic retail firms changes from being the price of financially constrained firms' production to the price of intermediate goods' production.

M: Equilibrium equations of the main model with foreign currency debt

The model for the case with fixed exchange rate regime is described by 45 endogenous variables:

$$\Big\{\lambda_t, c_t, h_t, w_t, R_t, d_{1,t}, d_{2,t}, R_t^{*R}, l_t^*, \pi_t, \Lambda_{t,t+1}, p_t^R, k_t, i_t, q_t, p_t^H, \tilde{p}_t^H, D_t^H, y_t^H, x_t^H, F_{1,t}^H, F_{2,t}^H,$$
$$y_t^C, p_t^F, y_t^F, x_t^F, m_t, ex_t, \tilde{p}_t^F, D_t^F, F_{1,t}^F, F_{2,t}^F, R_t^{*L}, d_t^*, d_t, n_t, \nu_{1,t}, \nu_{2,t}, t_t, b_t, rer_t, S_t, tb_t, ca_t, \xi_t\Big\}$$

They are given by 45 equilibrium equations below.

Households

$$\lambda_t = \left(c_t - \frac{\chi\,(h_t)^{1+\varphi}}{1+\varphi}\right)^{-\gamma} \tag{E.1}$$

$$w_t = \chi\,(h_t)^{\varphi} \tag{E.2}$$

$$\Lambda_{t,t+1} \equiv \frac{\lambda_{t+1}}{\lambda_t} \tag{E.3}$$

$$1 = E_t\beta\Lambda_{t,t+1}\frac{R_t}{\pi_{t+1}} \tag{E.4}$$

Financially constrained firms

$$E_t\beta\Lambda_{t,t+1}\left\{(1-(1-\Phi(d_{1,t}))\kappa)\left(\alpha p_{t+1}^R A_{t+1}k_t^{\alpha-1}h_t^{1-\alpha} + q_{t+1}(1-\delta)\right) - (1-\rho)\frac{q_t}{\pi_{t+1}}\right\} = \rho q_t \tag{E.5}$$

$$E_t\beta\Lambda_{t,t+1}\left\{(1-(1-\Phi(d_{1,t}))\kappa)\,(1-\alpha)p_{t+1}^R A_{t+1}k_t^{\alpha}h_t^{-\alpha} - (1-\rho)\frac{w_t}{\pi_{t+1}}\right\} = \rho w_t \tag{E.6}$$

$$E_{t-1}\left\{rer_t\right\}l_t^* = E_{t-1}\left\{\rho\left(q_t k_t + w_t h_t\right)\right\} \tag{E.7}$$

$$d_{2,t} \equiv \frac{E_t \ln\left(\frac{\pi_{t+1}^*}{rer_{t+1}}\left(p_{t+1}^R A_{t+1}k_t^{\alpha}h_t^{1-\alpha} + q_{t+1}(1-\delta)\right)\right) - \ln\left(R_t^{*R}l_t^*\right)}{\sigma_y} \tag{E.8}$$

$$d_{1,t} \equiv d_{2,t} + \sigma_y \tag{E.9}$$

Capital producers

$$k_t = (1-\delta)k_{t-1} + \left(1-\Gamma\left(\frac{i_t}{i_{t-1}}\right)\right)i_t \tag{E.10}$$

$$\frac{1}{q_t} = 1 - \frac{\kappa}{2}\left(\frac{i_t}{i_{t-1}}-1\right)^2 - \kappa\left(\frac{i_t}{i_{t-1}}-1\right)\frac{i_t}{i_{t-1}} + \kappa\beta E_t\Lambda_{t,t+1}\frac{q_{t+1}}{q_t}\left(\frac{i_{t+1}}{i_t}-1\right)\left(\frac{i_{t+1}}{i_t}\right)^2 \tag{E.11}$$

Retail firms

$$1 = (1-\omega^H)\left(\tilde{p}_t^H\right)^{1-\epsilon_H} + \omega^H \left(\frac{p_{t-1}^H\left(\prod_{j=1}^{j=s}\pi_{t+j}^{adj}\right)}{p_t^H \pi_t}\right)^{1-\epsilon_H} \tag{E.12}$$

$$D_t^H = (1-\omega^H)\left(\tilde{p}_t^H\right)^{-\epsilon_H} + \omega^H \left(\frac{p_{t-1}^H\left(\prod_{j=1}^{j=s}\pi_{t+j}^{adj}\right)}{p_t^H \pi_t}\right)^{-\epsilon_H} D_{t-1}^H \tag{E.13}$$

$$\tilde{p}_t^H = \frac{\epsilon_H}{(\epsilon_H - 1)}\frac{F_{1,t}^H}{F_{2,t}^H} \tag{E.14}$$

$$F_{1,t}^H = p_t^R y_t^H + E_t \omega^H \beta \Lambda_{t,t+1}\left(\frac{p_{t+1}^H \pi_{t+1}}{p_t^H\left(\prod_{j=1}^{j=s}\pi_{t+j}^{adj}\right)}\right)^{\epsilon_H} F_{1,t+1}^H \tag{E.15}$$

$$F_{2,t}^H = p_t^H y_t^H + E_t \omega^H \beta \Lambda_{t,t+1}\left(\frac{p_{t+1}^H \pi_{t+1}}{p_t^H\left(\prod_{j=1}^{j=s}\pi_{t+j}^{adj}\right)}\right)^{\epsilon_H - 1} F_{2,t+1}^H \tag{E.16}$$

$$D_t^H y_t^H = A_t \theta_t F\left(k_{t-1}, n_{t-1}\right) \tag{E.17}$$

Final goods producer

$$y_t^C \equiv \left[(1-\eta)^{\frac{1}{\epsilon}}\left(x_t^H\right)^{\frac{\epsilon-1}{\epsilon}} + \eta^{\frac{1}{\epsilon}}\left(x_t^F\right)^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}} \tag{E.18}$$

$$x_t^H = (1-\eta)\left(p_t^H\right)^{-\epsilon} y_t^C \tag{E.19}$$

$$x_t^F = \eta\left(p_t^F\right)^{-\epsilon} y_t^C \tag{E.20}$$

Exporters

$$ex_t = \eta^*\left(\frac{p_t^H}{rer_t}\right)^{-\epsilon_*} y_t^* \tag{E.21}$$

Definition of the real exchange rate

$$\frac{rer_t}{rer_{t-1}} = \frac{S_t}{S_{t-1}}\frac{\pi_t^*}{\pi_t} \tag{E.22}$$

Importers

$$1 = (1-\omega^F)\left(\tilde{p}_t^F\right)^{1-\epsilon_F} + \omega^F \left(\frac{p_{t-1}^F\left(\prod_{j=1}^{j=s}\pi_{t+j}^{adj}\right)}{p_t^F \pi_t}\right)^{1-\epsilon_F} \tag{E.23}$$

$$D_t^F = (1-\omega^F)\left(\tilde{p}_t^F\right)^{-\epsilon_F} + \omega^F \left(\frac{p_{t-1}^F\left(\prod_{j=1}^{j=s}\pi_{t+j}^{adj}\right)}{\pi_t p_t^F}\right)^{-\epsilon_F} D_{t-1}^F \tag{E.24}$$

$$\tilde{p}_t^F = \frac{\epsilon_F}{(\epsilon_F - 1)}\frac{F_{1,t}^F}{F_{2,t}^F} \tag{E.25}$$

$$F_{1,t}^F = rer_t y_t^F + E_t \omega^F \beta \Lambda_{t,t+1}\left(\frac{p_{t+1}^F \pi_{t+1}}{p_t^F\left(\prod_{j=1}^{j=s}\pi_{t+j}^{adj}\right)}\right)^{\epsilon_F} F_{1,t+1}^F \tag{E.26}$$

$$F_{2,t}^F = p_t^F y_t^F + E_t \omega^F \beta \Lambda_{t,t+1} \left(\frac{p_{t+1}^F \pi_{t+1}}{p_t^F \left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj} \right)} \right)^{\epsilon_F - 1} F_{2,t+1}^F \tag{E.27}$$

$$m_t = rer_t D_t^F y_t^F \tag{E.28}$$

Banks

$$E_t \left\{ \frac{R_t^{*L}}{\pi_{t+1}^*} rer_{t+1} l_t^* \right\} \equiv E_t \left\{ (1 - \Phi(d_{1,t})) \kappa \left(p_{t+1}^R y_{t+1}^R + (1-\delta) q_{t+1} k_t \right) + \Phi(d_{2,t}) \frac{R_{j,t}^{*R}}{\pi_{t+1}^*} rer_{t+1} l_t^* \right\} \tag{E.29}$$

$$(1 + \nu_{1,t}) \beta E_t \Lambda_{t,t+1} \left\{ (1-\omega) + \omega \nu_{2,t+1} \right\} \left(\frac{R_t^{*L}}{\pi_{t+1}^*} \frac{rer_{t+1}}{rer_t} \right) = \lambda^L \nu_{1,t} + \nu_{2,t} \tag{E.30}$$

$$(1 + \nu_{1,t}) \beta E_t \Lambda_{t,t+1} \left\{ (1-\omega) + \omega \nu_{2,t+1} \right\} \left(\frac{R_t}{\pi_{t+1}} \right) = \nu_{2,t} \tag{E.31}$$

$$(1 + \nu_{1,t}) \beta E_t \Lambda_{t,t+1} \left\{ (1-\omega) + \omega \nu_{2,t+1} \right\} \left(\frac{R_t^* \xi_t}{\pi_{t+1}^*} \frac{rer_{t+1}}{rer_t} \right) = \nu_{2,t} \tag{E.32}$$

$$n_t = \omega \left(\frac{R_{j,t-1}^{*L}}{\pi_t^*} rer_t l_{t-1}^* - \frac{R_{t-1}}{\pi_t} d_{t-1} - \frac{R_{t-1}^* \xi_{t-1}}{\pi_t^*} rer_t d_{t-1}^* \right) + \iota n \tag{E.33}$$

$$\nu_{2,t} n_t \geq \lambda^L rer_t l_t^* \tag{E.34}$$

$$n_t + d_t + rer_t d_t^* = rer_t l_t^* \tag{E.35}$$

Monetary policy

$$\frac{R_t}{\bar{R}} = \left(\frac{R_{t-1}}{\bar{R}} \right)^{\gamma_R} \left(\frac{y_t^H}{\bar{y^H}} \right)^{(1-\gamma_R)\gamma_Y} \left(\frac{p_t^H / p_{t-1}^H \pi_t}{\bar{\pi}} \right)^{(1-\gamma_R)\gamma_\pi} \exp(mp_t) \tag{E.36}$$

Government

$$g_t + \frac{R_{t-1}}{\pi_t} b_{t-1} = t_t + b_t \tag{E.37}$$

$$t_t = \bar{t} + \kappa_b \left(b_{t-1} - \bar{b} \right) + \tau_t \tag{E.38}$$

Aggregate domestic demand has to equal aggregate supply of domestic final goods

$$y_t^C = c_t + i_t + g_t \tag{E.39}$$

Aggregate demand for domestic composite goods and demand for exports clears with production of domestic *composite goods*

$$y_t^H = x_t^H + ex_t \tag{E.40}$$

Aggregate domestic demand for foreign composite goods clears with imports

$$y_t^F = x_t^F \tag{E.41}$$

Trade balance

$$tb_t = p_t^H ex_t - m_t \tag{E.42}$$

Current account

$$ca_t = tb_t - \left(R_{t-1}^* \xi_{t-1} - 1\right) rer_t \frac{d_{t-1}^*}{\pi_t^*} \tag{E.43}$$

$$ca_t = -\left(rer_t d_t^* - rer_t \frac{d_{t-1}^*}{\pi_t^*}\right) \tag{E.44}$$

$$\xi_t = \exp\left(\phi \frac{(rer_t d_t^* - rer \cdot d^*)}{rer \cdot b^*} + \frac{\zeta_t - \zeta}{\zeta}\right) \tag{E.45}$$

There are 10 exogenous variables:

$$\{A_t, \theta_t, \pi_t^*, R_t^*, \zeta_t, y_t^*, mp_t, g_t, \tau_t\}$$