

# Collateral matters: Liquidity injection policies and fire sales

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## Abstract

This paper theoretically compares four liquidity-injection policies that an authority can use to bail out banks during a liquidity crisis – unsecured lending, secured lending, buying bank equity and securities purchases – considering ex-ante incentives for liquidity-risk taking and ex-post impacts on the banking system. In 2008 and 2009 these policies exhausted trillions of dollars, most of which was secured lending, whereby banks must collateralise their borrowing with securities. This paper demonstrates that the collateral requirement constrains banks from selling securities, which softens fire-sale price impacts and limits losses on securities banks do sell. Relative to unsecured lending, the selling constraint also lowers incentives for liquidity-risk taking and thus reduces the probability of a crisis occurring, for two reasons. First, it raises the minimum that a bank must borrow to avoid failure, which is costly for banks provided the authority charges penalising interest rates. Second, selling constraints are tighter for banks that need to borrow more, so less-exposed banks benefit more from the limited securities-price declines. A securities-purchase policy incentivises the highest level of liquidity-risk taking because it does not utilise the fact that they have better capacity to pay penalties after the liquidity stress subsides. A policy of purchasing bank equity incentivises liquidity risk more for low-equity than high-equity banks because they face lower ex-post repayments, but is less likely to make low-equity banks insolvent after post-crisis repayments are made. (JEL G01, G11, G21, G28)

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\*Preliminary and incomplete. The views expressed are my own and do not represent those of the Reserve Bank of Australia. The Appendix containing proofs is available by emailing me at [nick.j.garvin@gmail.com](mailto:nick.j.garvin@gmail.com). I would like to thank my advisor Xavier Freixas for extensive guidance, and also Fabrizio Germano and Mark Manning for comments and support. Thanks also to participants at the UPF Finance Lunch Seminar Series, the Reserve Bank of Australia Research Seminar series, and the 2016 BGSE PhD Jamboree.

# Introduction

How should bailouts for liquidity-stressed banks be provided? In late 2008, the world’s largest banking systems experienced a rapid drop of private-sector funding liquidity, alongside fire sales and a dry up of market liquidity for many securities. Banks and shadow banks (henceforth just ‘banks’) with insufficient cash or high-quality liquid assets had difficulty meeting their short-term liabilities, and authorities responded with massive liquidity injections. Leaders were aware that this could damage banks’ incentives to manage their own liquidity risk (for instance see Bernanke (2008)) but considered avoiding imminent and widespread bank failures to be a higher priority.

For example, in the peak-stress period US authorities granted banks upwards of 920 billion USD additional secured lending – also known as collateralised lending or repo (short for ‘repurchase agreement’) – under which the borrower must provide collateral, typically securities, to the lender for the life of the loan. The authorities also subsidised 450 billion USD of banks’ unsecured lending by purchasing their commercial paper and guaranteeing their debt, and spent 265 billion USD on bank-equity purchases.<sup>1</sup> The European Central Bank (ECB) increased secured lending to banks by around 345 billion EUR, and euro area countries guaranteed around 770 billion EUR of banks’ unsecured debt and implemented bank-equity purchase programs with a combined cap of 140 billion EUR.<sup>2</sup> The Bank of England increased secured lending to banks by at least 365 billion GBP and purchased around 100 billion GBP of securities, while the UK government guaranteed up to 250 GBP pounds of banks’ unsecured debt and purchased up to 86 billion GBP of bank equity.<sup>3</sup>

Evidently, authorities have various liquidity-injection policies at their disposal and during 2008 and 2009 heavily relied on expansion of secured lending, also the standard tool of open market operations during normal times. Several questions naturally arise, which this paper seeks to address. Was the heavy use of secured lending justified? Do different means of liquidity provision give banks different incentives regarding liquidity risk? And what are the consequences of taking a large amount of collateral from banks during a crisis?

The model presented here demonstrates that liquidity injection through secured lending can have two key benefits over other forms of funding – limiting fire sales and thus banks’ losses on securities sales, and better disincentivising liquidity-risk taking – aside from the risk protection it affords the authority. If emergency lending policies follow Bagehot (1873)’s suggestion of charging an interest rate high enough to deter unnecessary borrowing, then a collateral requirement forces banks to pledge securities they would otherwise sell and recoup the foregone liquidity with additional borrowing from the authority.<sup>4</sup> The limited selling alleviates downward pressure on securities prices, curbing losses for banks whose borrowing needs are not so high that they must pledge all their securities. Further, by forcing more borrowing, the collateral requirement amplifies the deterrent effect of the policy’s penalising interest rate, better disincentivising liquidity-risk taking and thus reducing the probability of a banking crisis occurring. At the same time, there is less systemic disruption to other entities such as investment funds that are also hurt by fire sales in securities markets.

There is to date not much theoretical work comparing the policies that authorities used to prevent liquidity-driven bank failures during 2008 and 2009. The approach is in part motivated by Farhi and Tirole (2012)’s argument that because an authority’s best response to a banking crisis is to grant banks the funding they need, a policy of withholding emergency intervention, while incentivising prudence, lacks credibility. Acharya and Yorulmazer (2008) show that a policy of subsidising surviving banks reduces pre-crisis incentives for banks to invest in the same assets as each other, relative to a policy of bailing out failed banks, and Acharya, Shin, and Yorulmazer (2011) show that pre-crisis liquidity exposures are minimised when only low-exposure surviving

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<sup>1</sup>The US figures refer to: secured lending through the Term Auction Facility, Primary Dealer Credit Facility and Term Securities Lending Facility; unsecured lending subsidisation through the non-asset-backed component of the Commercial Paper Funding Facility (assuming half of the purchases were of bank paper) and the Temporary Liquidity Guarantee Program; and bank-equity purchases through the Capital Purchase Program and directly to AIG and Bank of America. For each of the Federal Reserve facilities, figures are of peak loans outstanding before April 2009, under-representing total lending across the period.

<sup>2</sup>Secured lending figures refer to the ECB’s main refinancing operations and longer term refinancing operations; other figures are from Panetta, Faeh, Grande, Ho, King, Levy, Signoretti, Taboga, and Zaghini (2009) and Attinasi (2010).

<sup>3</sup>The Bank of England figures are from Cross, Fisher, and Weeken (2010). Secured lending refers to extended-collateral long-term repos and the special liquidity scheme; securities purchases refer to asset-purchase-facility purchases as at mid 2009. Equity purchase and debt guarantee figures are from United Kingdom National Audit Office (2009).

<sup>4</sup>Kohn (2010) explains that the Federal Reserve charged such ‘penalty rates’ during the 2007 to 2009 crisis.

banks are subsidised; neither look at more than one alternative for preventing bank failures. Farhi and Tirole (2012) compare system-wide with bank-specific interventions, but not by the incentives they generate and without the possibility of liquidity-driven failure. Moreover, these studies do not consider effects specific to secured lending, arguably the most used intervention during the 2007 to 2009 liquidity crisis. Ashcraft, Gârleanu, and Pedersen (2011) focus specifically on secured lending, comparing a change in haircuts and a change in interest rates by how they affect required returns on lending and thus economic output; this paper compares a wider range of policies and focuses on incentives and impacts within the banking system.

The model has three defining characteristics: banks choose the liquidity risk of their portfolio; less-liquid assets have shallower markets, i.e. imperfect *market liquidity*; and banks can experience a random outflow of short-term liabilities, i.e. a withdrawal of *funding liquidity*. If a large funding-liquidity withdrawal occurs and banks' assets are not very liquid, the amount of selling required to meet the withdrawal potentially pushes asset prices down to the point where their sales cannot raise enough liquidity. To prevent bank failures, an authority can inject liquidity through either unsecured lending, secured lending, purchasing bank equity, or by buying securities.

Banks' liquidity-exposure decision is formalised by modifying the classic portfolio decision in Diamond and Dybvig (1983) to make assets' market illiquidity endogenous to the aggregate amount of liquidation, similar to Diamond and Rajan (2011)'s analysis of liquidity-risk moral hazard in the absence of bank rescues. Specifically, banks decide how to allocate a liquid endowment, which can be thought of as early returns on investments, between two types of assets. The first – 'cash' – has perfect liquidity and no net return, representing cash or highly liquid securities such as treasuries. The second – 'securities' – have positive net returns unless there is a funding-liquidity withdrawal, in which case the market price is decreasing in the quantity of securities being liquidated. This simplifying dichotomy resembles how liquidity-risk management is interpreted by the Liquidity Coverage Ratio (LCR) of Basel III, which requires banks to hold a sufficient quantity of high-quality assets with low market-liquidity risk.<sup>5</sup> It is also motivated by the fact that banks had high exposures to assets with liquid markets before the crisis, but that became illiquid during the crisis, as documented with respect to private secured-lending markets by Hordahl and King (2008) and Gorton and Metrick (2012), and with respect to asset-backed securities markets by Brunnermeier (2009) and many others.

To endogenise securities prices, the cash-in-the-market pricing of Allen and Gale (1994) is generalised such that the external supply of cash available to buy securities continuously increases as the market price falls further below its arbitrage-free counterpart. Securities-market illiquidity is treated as the key market imperfection; banks would not become distressed if they could liquidate their portfolio at net present value. In contrast, if the supply of cash available to buy securities were fixed, as in Allen and Gale (1994), the equilibrium market price when banks' liquidity needs vary would be uninteresting – either buyers' supply of liquidity weakly exceeds banks' needs and there is no price fall, or it does not and banks fail. A continuously elastic supply also better represents a realistic securities market in which heterogeneous potential buyers each quote a quantity and maximum price at which they are willing to trade.<sup>6</sup> Ample liquidity is assumed to return to the market at a later date, bringing the securities price back to its net present value. Liquidity-driven selling can therefore push prices below their arbitrage-free counterparts, consistent with empirical studies of securities prices such as Coval and Stafford (2007), Hameed, Kang, and Viswanathan (2010) and Longstaff (2010), and with the theory presented in Brunnermeier and Pedersen (2009). This market imperfection permits banks to be solvent yet illiquid – i.e. have positive future net worth but be unable to meet payment obligations – presenting an opportunity for a liquidity-rich authority to prevent inefficient bank closures by

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<sup>5</sup>The Basel Committee for Banking Supervision summarises the LCR as follows: “the objective of the LCR is to promote the short-term resilience of the liquidity risk profile of banks. It does this by ensuring that banks have an adequate stock of unencumbered high-quality liquid assets (HQLA) that can be converted easily and immediately in private markets into cash” (Basel Committee for Banking Supervision, 2013a, paragraph 1). The approach also matches Saunders and Cornett (2007)'s textbook definition of liability-side liquidity risk management: “When liability holders demand cash by withdrawing deposits, the [financial institution (FI)] needs to borrow additional funds or sell assets to meet the withdrawal. The most liquid asset is cash; FIs use this asset to pay claim holders who seek to withdraw funds. However, FIs tend to minimize their holdings of cash reserves as assets because those reserves pay no interest. To generate interest revenues, most FIs invest in less liquid and/or longer maturity assets. While most assets can be turned into cash eventually, for some assets this can be done only at a high cost when the asset must be liquidated immediately. The price the asset holder must accept for immediate sale may be far less than it would receive with a longer horizon over which to negotiate a sale” (pages 493-494).

<sup>6</sup>Diamond and Rajan (2011) achieve a similar outcome by assuming external buyers have an outside lending option that has decreasing returns to scale.

providing liquidity in exchange for later repayment.

A crisis is represented by a withdrawal of banks' funding liquidity that drains market liquidity to the point of bank distress. The funding liquidity withdrawal is a random outflow of cash, similar to much of the banking literature including Diamond and Dybvig (1983), which can be interpreted as short-term creditors not rolling over debt, withdrawing depositors, credit-line draw downs, or, similar to the immediate cause of AIG's liquidity distress in 2008, unexpected margin requirements. Funding liquidity and market liquidity are linked by the constraint that if a bank's outflow of short-term liabilities cannot be funded by cash or interbank borrowing, it must be met by liquidating assets. This link is why the LCR requires banks to hold a quantity of high-quality liquid assets that depends on their short-term liabilities, and is empirically documented by Nyborg and Östberg (2014), who term this securities liquidation as 'liquidity pullback', and Fontaine and Garcia (2011). Combining it with cash-in-the market pricing gives the definition of a liquidity crisis adopted here, being that funding liquidity scarcity diminishes securities-market liquidity to the point where a bank cannot meet immediate liabilities with immediately available assets. This mechanism, also modelled by Diamond and Rajan (2011), resembles the 'liquidity spiral' in Brunnermeier and Pedersen (2009) whereby to meet a tightened capital constraint, investors must sell assets, pushing down the asset price and further reducing their capital value.

This relatively simple liquidity-crisis anatomy yields some important distinctions between secured lending and other forms of liquidity injection, notwithstanding risk taken by the authority, that are reasonably intuitive but have not been highlighted by previous literature. First, securities-market prices should be higher under a secured lending policy than other funding policies because banks sell less. Ashcraft et al. (2011) also show that a central bank's expansion of secured lending can raise market prices, because it is cheaper to fund a security's purchase by using it as collateral to borrow from the central bank, compared to funding it with one's own capital; this paper assumes liquidity rather than capital is constrained, and shows a similar result occurs by alleviating fire sales. A necessary condition for modelling this outcome is that the authority charges a 'penalty' interest rate sufficient to deter unneeded borrowing, which is also a requirement for preventing banks from ex-ante maximising their liquidity exposures. The penalty rate implies that in an unsecured-lending policy banks only borrow what they need after liquidating securities, which is more liquidation than under a secured-lending policy, where banks must post some securities as collateral.

In essence, the collateral requirement forces banks to treat emergency borrowing and securities-market sales as substitutes. The more scarce is liquidity, the greater is the role played by the authority's balance sheet – which is immune to liquidity spirals – in replacing the liquidity provision function of securities markets. In contrast, under an unsecured lending policy, banks treat emergency borrowing and securities markets as complements, only borrowing to cover the part of the liquidity withdrawal that cannot be funded by sales. In this sense, a secured lending policy supports market liquidity whereas an unsecured lending policy supports funding liquidity.

For banks to sell securities when more emergency funding is available, the interest rate must be high relative to cost of selling securities in a depressed market. This model demonstrates that if an authority intervenes by lending, such a 'penalty' rate is also required to disincentivise banks against ex-ante maximising their liquidity exposures, as also argued by, for example, Fischer (1999). Without penalty rates, emergency funding is cheaper than private liquidity sources, so the ex-post marginal cost of a liquidity exposure is lower conditional on requiring a bailout than conditional on a bailout not being available. Since banks can raise the probability of requiring a bailout by increasing their liquidity exposure, a lack of penalty rates implies that the expected cost of liquidity exposures (given a withdrawal) can be reduced by taking more liquidity risk.

Incidentally, the rule mirrors Bagehot's advice for emergency lending that "loans should be made at a very high rate of interest. This will operate as a heavy fine on unreasonable timidity, and will prevent the greatest number of applications by persons who do not require it." Bagehot argued that banks should exhaust private liquidity sources before using emergency funding because this would minimise depletion of the Bank of England's limited quantity of reserves and, in turn, the risk that it would be ran on by private banks. Bagehot's 'very high rate' suggestion has sparked a thorough debate; for example see Freixas, Parigi, and Rochet (2004), Rochet and Vives (2004) and Castiglionesi and Wagner (2012). Rochet and Vives (2004)'s modelling suggests that a penalty rate threatens solvency by lowering profits, and thus raises the risk of a

run on funding liquidity. This paper concludes differently because it assumes a bank’s funding outflows can be detached from its solvency position – caused by, for example, conditions in other parts of the financial system, or a shift in creditors’ risk preferences.

The model also shows that when penalty rates are charged, a secured lending policy better deters liquidity exposures than an unsecured lending policy, because for a given liquidity shortage, the collateral requirement makes banks sell fewer securities, and therefore borrow more at the penalty rate. Nevertheless, the penalty rate and larger quantity of borrowing do not necessarily leave banks with lower profit after the crisis, because they receive a higher price for the securities they do sell. Importantly, the higher market price offsets the profit effect but not the disincentive effect of the higher liquidity cost, because it accrues at least as much to banks with relatively low liquidity exposures, whose securities sales are less restricted by collateral obligations, than high-exposure banks that require more borrowing. It is therefore possible for a secured-lending equilibrium to have lower ex-ante liquidity exposures, that is, lower probability of bank failures, as well as higher expected ex-post profits, than an unsecured-lending equilibrium. The larger quantity of borrowing implies that, for a given liquidity exposure, a secured-lending policy involves greater use of the authority’s balance sheet (although likely with less risk); however, this is to some extent offset by banks holding lower liquidity exposures.

For a secured lending policy to prevent bank failures, the model highlights that it must permit banks to obtain more liquidity from pledging their securities than can be obtained by selling; that is, it must raise the securities’ *liquidity value*. This parallels Ashcraft et al. (2011)’s argument that to lower funding costs, the authority’s haircut must improve on what is offered in private markets. More specifically, it requires that the amount that can be borrowed against a given value of collateral – one minus the *haircut* – is sufficiently above what that collateral can be sold for – one minus the percentage fall in the market price. Cechetti (2009) argues that this was observed during the crisis, stating “the Fed is taking collateral at a price that is almost surely above its market price” (page 67). While central banks did not appear to adjust haircuts during the crisis, many – including the US Federal Reserve, the European Central Bank, the Bank of England and the Reserve Bank of Australia – increased the range of securities they accepted as collateral, often as a first resort and multiple times. This can be interpreted as reducing haircuts from 100 per cent on those securities, and the fact that banks heavily utilised the looser collateral requirements indicates that the central banks’ haircuts improved on what was being offered in private markets.

While secured and unsecured lending appeared to be the most utilised liquidity-injection policies in 2008 and 2009, central banks and governments also carried out large quantities of bank-equity and securities purchases. Under a bank-equity injection, the authority provides a distressed bank with cash in return for a share of its ex-post profits, most likely proportional to the amount of cash provided. The model presented here demonstrates that an equity-purchase policy more heavily penalises banks with higher profits, incentivising banks with low-quality assets to take higher liquidity risk than other banks.

Under a securities purchase policy, the authority supports the market liquidity of banks’ securities, preventing prices from falling far enough to cause widespread distress. The modelling here shows that under such a policy banks are indiscriminately incentivised to carry high liquidity exposures. This occurs because banks’ implicit payment for the liquidity assistance – receiving a purchase price below the fundamental value – is part of the immediate transaction, during the liquidity stress, rather than afterward as is possible for loan or equity repayments. During the liquidity stress, illiquid yet solvent banks are constrained in what they can pay without failing, whereas once the stress subsides and longer-term assets are liquid, these banks can afford to pay higher penalties. Therefore under a securities purchase policy an authority is handicapped in its ability to impose penalties, which for banks raises the expected return on securities by lowering the cost of illiquidity.

Multiple equilibria cannot be ruled out under a securities purchase policy: the more the authority lets the price fall before intervening, the fewer securities banks want to hold; and the fewer securities banks hold, the more willing the authority is to let the price fall. Nevertheless, as in Farhi and Tirole (2012), the authority cannot credibly induce a low-exposure equilibrium, because if banks’ expectations lead them to hold high exposures, the authority’s priority of avoiding bank failures leads it to act in line with banks’ expectations. This paper therefore demonstrates that if the crisis is characterised by prices departing from their fundamental values – a temporary widening of the separation of solvency and liquidity for asset holders – the credibility problem is diminished under policies that permit banks to repay after the liquidity stress

subsidies.

The rest of this paper has five sections. Section 1 describes the general model, containing all the assumptions made. Section 2 sets up a bank's payoff function and subsection 2.1 applies it to an unsecured lending policy, which is used as a benchmark for comparison for the analyses of secured lending, bank-equity injections and securities purchases, in subsections 2.2, 2.3 and 2.4, respectively. Section 3 relaxes three stylised features of the model – that the authority minimises its haircuts on secured lending, that all banks experience the same liquidity withdrawal, and that the authority uses only one policy at a time – to show that the results are not fundamentally changed. Section 4 discusses the liquidity-injection policies adopted by US and European authorities around late 2008 in light of the model's conclusions, and section 5 concludes.

## 1 Model setup

There are three dates,  $t = 0, 1, 2$ , and a measure-one continuum of risk-neutral banks, indexed by  $i$ , that maximise their date 2 (expected) payoff. The three other types of agents are banks' external (i.e. non bank) creditors, securities buyers, and the authority, which all behave according to exogenously defined rules.

At date 0 bank  $i$  has an endowment of illiquid real assets that cannot be sold at date 1, and at date 2 return cash  $a_i > 0$ . Illiquid assets  $a_i$  can vary across  $i$  and aside from this banks are homogeneous. At date 0 banks also have a liquid endowment  $l$ , which they can allocate between two types of *liquid assets* – securities ' $s$ ' and cash ' $c$ '. Denote bank  $i$ 's securities choice by  $s_i$ . The date 0 price of securities is normalised to 1 so  $c_i + s_i = l$  for all  $i$ .

Denote the set of choices of any unit measure of banks that excludes bank  $i$  as  $s_{-i}$ , and the set of all banks' choices as  $s$ .<sup>7</sup> The aggregate mass of securities held by banks is defined as  $S = \int_0^1 s_i di$ . It follows that if  $s_{-i} = S$ , there is no positive measure of banks choosing values different to  $S$ . Conversely,  $s_{-i} \neq S$  implies that there is a positive measure of banks choosing values different to  $S$ . The statement  $s = S$  implies a fully symmetric choice outcome.

At date 1, with probability  $1 - \lambda$  such that  $0 < \lambda < 1$ , nothing happens and at date 2 securities are worth  $1 + r_s > 1$ . With probability  $\lambda$  there is a liquidity withdrawal by *external creditors*, whereby each bank has the same cash obligation  $b$  that is randomly drawn from positive continuous density  $f(b)$  defined on  $[0, l]$ . In this case external creditors also take liquidity from the securities market by selling  $\gamma b$  value of securities, where  $\gamma > 0$ .

Given a liquidity withdrawal, if bank  $i$  has  $c_i < b$  it must satisfy its cash shortfall by selling securities to *securities buyers* or banks with spare cash. The date-1 equilibrium market price is equal to  $1 - m^*$  where  $m^*$  is termed *market illiquidity*, which ensures the net mass value of securities sales by banks and external creditors equals the supply of liquidity from securities buyers. Securities buyers' liquidity supply  $L_S(m)$  is assumed to have the following properties: i)  $L_S(0) = 0$  and  $L_S$  is perfectly elastic for any  $m \leq 0$ , i.e. buyers are inactive if the price is 1 and ensure the price does not exceed 1; ii) when positive,  $L_S$  is increasing and continuously differentiable, i.e. the amount of buying is smoothly increasing in the profitability of securities purchases; and iii)  $L_S(1) > l(1 + \gamma)$ , i.e. liquidity supply necessarily exceeds liquidity demand if the price falls sufficiently close to zero.

Define the demand by banks and external creditors for liquidity from the securities market as the function  $L_D(b, s, m)$ , whose specific form depends on the policy implemented. Date 1 securities market equilibrium requires that  $m^*$  satisfies

$$L_S(m^*) = L_D(b, s, m^*)$$

where  $b$  is exogenous and banks choose  $s$  at date 0. To simplify notation,  $m^*$  will typically be characterised by the inverted supply function  $M$  such that  $M(L_D) \equiv L_S^{-1}(L_D)$ .<sup>8</sup> The assumptions on  $L_S$  imply that  $M(L_D) = 0$  for all  $L_D \leq 0$ ,  $M(l + \gamma l) < 1$ , and that  $M$  has a first derivative, sometimes denoted as  $M'$ , that

<sup>7</sup>The set of choices  $s$  is restricted to cases where a reordering of  $i$  permits  $s_i : [0, 1] \rightarrow [0, l]$  to be Riemann integrable, and  $i$  is always treated as ordered in such a way. This does not rule out any economically interpretable cases.

<sup>8</sup> $L_S$  is injective for all  $m \geq 0$  so the inverse supply function is well defined.

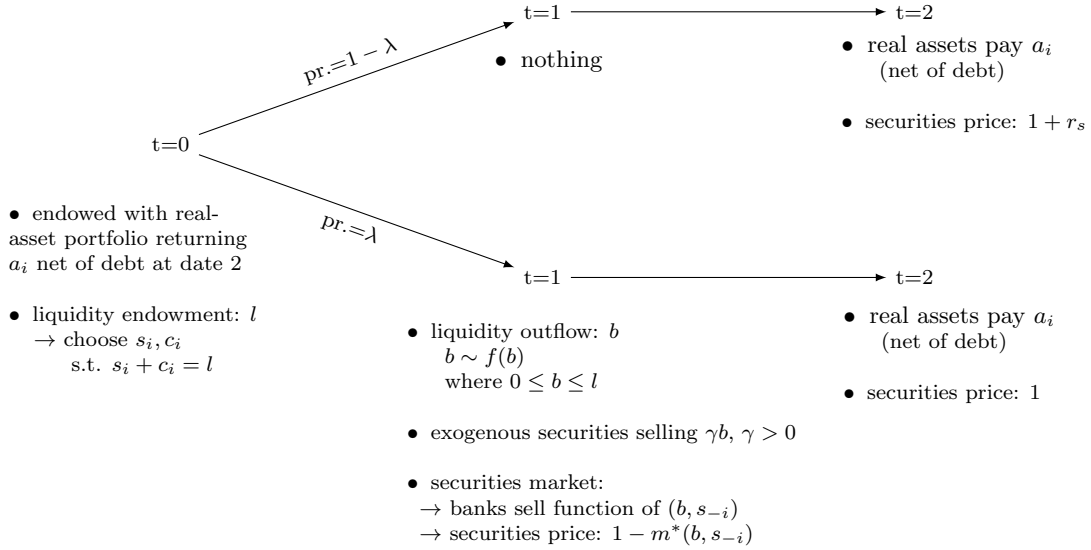
is continuous and positive for all  $L_D > 0$ . In other words, the market price  $1 - m^*$  is weakly decreasing in the net mass value of securities sold, such that

$$m^* = M(L_D(b, s, m^*)). \quad (1.1)$$

The general expression for equilibrium market illiquidity used throughout this paper is  $m^*(b, s_{-i})$ , highlighting the fact that no zero measure of banks can affect the equilibrium price. The implicit function  $m^*(b, s_{-i})$  will be shown to be well defined in each case examined.

Figure 1 illustrates the timeline of events.

**Figure 1: Model timeline without intervention**



## 1.1 Date 1 outcomes

The date 1 value of bank  $i$ 's liquid assets is  $c_i + s_i(1 - m^*) = l - s_i m^*$ . Define  $\bar{b}_i = \bar{b}(s_i, s_{-i})$  as the liquidity withdrawal that expends precisely all of bank  $i$ 's liquid assets.<sup>9</sup> The implicit function  $\bar{b}_i$  is defined by

$$\bar{b}_i \equiv l - s_i m^*(\bar{b}_i, s_{-i}). \quad (1.2)$$

If  $b \leq \bar{b}_i$ , the net value of securities that bank  $i$  sells is equal to  $b - c_i$ ; throughout it is assumed that at date 1 banks buy if they have spare cash and are indifferent between buying and selling. If  $b > \bar{b}_i$  bank  $i$  is *liquidity deficient* and the authority intervenes to prevent its failure. Consider the case with  $s_{-i} = S$  and banks selling all their securities when liquidity deficient. Market illiquidity takes the form

$$m^* = \begin{cases} M(b + S - l + \gamma b) & \text{when banks are not liquidity deficient, defined as } m_G^*(b, S) \\ M(S(1 - m^*) + \gamma b) & \text{when banks are liquidity deficient, defined as } m_U^*(b, S). \end{cases} \quad (1.3)$$

If  $s_{-i} \neq S$  then for some  $b$  and  $S$ , the liquidation value  $L_D$  depends on how much of  $S$  is held by banks that are liquidity deficient at that  $b$ . It follows that  $s_{-i}$  cannot be fully represented by  $S$ , so in general market illiquidity is referred to as  $m^*(b, s_{-i})$ .

As an example, consider a linear market illiquidity function  $M(L_D) = \max\{\alpha L_D, 0\}$  where  $0 < \alpha < 1/(l + \gamma l)$  (so that  $M(l + \gamma l) < 1$ ) and the case characterised by expression 1.3. Provided that  $S > 0$ , the unit measure of banks is liquidity deficient at all  $b > \bar{b}_S$  such that

$$\bar{b}_S = \frac{l + \alpha S(l - S)}{1 + \alpha S(1 + \gamma)}.$$

<sup>9</sup> $\bar{b}_i$  will be shown to be unique in each case examined.

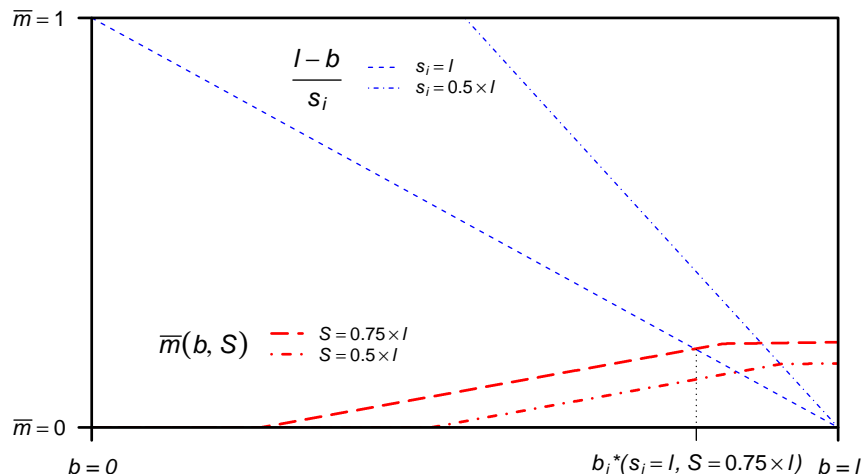
At  $b \leq \bar{b}_S$  market illiquidity takes the form  $m_G^* = \alpha(b+S-l+\gamma b)$  and at  $b > \bar{b}_S$  it is  $m_U^* = \alpha(S(1-m_U^*)+\gamma b)$ , which yields

$$m_U^* = \frac{\alpha S + \gamma b}{\alpha S + 1}.$$

Figure 1.1 plots this specification of market illiquidity for two values of  $S$ . The market illiquidity functions are equal to zero at  $b$  less than  $l - S$ . If  $b$  is raised past this point, the net mass value of selling, and hence market illiquidity, becomes positive. The second kink in each function occurs at  $\bar{b}_S$ ; at any  $b > \bar{b}_S$  banks are selling all their securities and the positive slope is driven by  $\gamma b$ . An increase in  $S$  raises market illiquidity at all  $b > l - S$  and shifts both kinks left.

Also shown is bank  $i$ 's capacity to survive market illiquidity  $(l - b)/s_i$  for two different values of  $s_i$ . Bank  $i$  avoids liquidity deficiency if and only if  $(l - b)/s_i \geq m^*$  so, given  $s_i$  and  $S$ , its  $\bar{b}_i$  is defined by the intersection of the upward and downward sloping lines. At  $b = l$  if bank  $i$  has  $s_i > 0$  it cannot survive any market illiquidity because the liquidity withdrawal requires its full date 0 liquid endowment. An increase in  $s_i$  pivots bank  $i$ 's illiquidity capacity anti-clockwise around  $(l, 0)$ , lowering its  $\bar{b}_i$  and reducing its probability of avoiding liquidity deficiency for any given  $S$ .

**Figure 2: Market illiquidity and a bank's capacity to handle it**



In this illustration  $m(L_D) = \max\{\alpha L_D, 0\}$  where  $0 < \alpha < 1/(l + \gamma l)$ . Banks are assumed to choose symmetrically, i.e.  $s_{-i} = S$ , and sell all their securities if  $b > \bar{b}_i$ . Parameter assumptions are  $l = 1$ ,  $\alpha = 0.3$  and  $\gamma = 0.1$ , chosen for illustration clarity.

## 1.2 Liquidity injection policies

The authority has full information and declares its policy at the start of date 0. When implementing the policy at date 1, it minimises its expenditure subject to the requirement that banks do not fail. Banks are assumed to utilise an offered intervention rather than fail. Four policies are compared:

1. **Unsecured lending ( $\mathcal{U}$ ):** the authority lends each bank enough to avoid failure, with repayment at date 2 of principal plus interest at rate  $r_{\mathcal{U}}$ .
2. **Secured lending (or repo) ( $\mathcal{R}$ ):** the authority lends each bank enough to avoid failure, provided that the bank collateralises its borrowing with a sufficient quantity of securities, with repayment at date 2 of principal plus interest at rate  $r_{\mathcal{R}}$ .
3. **Equity injection ( $\mathcal{E}$ ):** the authority injects enough cash into each bank to avoid failure and requires repayment of  $\phi$  proportion of the bank's date 2 profits, where  $\phi$  is increasing in the size of the cash injection.
4. **Securities purchase ( $\mathcal{S}$ ):** the authority determines an acceptable schedule of market illiquidity  $m^S(b)$  and purchases sufficient securities such that the date 1 market price is at least  $1 - m^S(b)$ .



## 2 Payoffs under different policies

Under policy  $\mathcal{P}$  a bank's payoff  $\Pi_i^{\mathcal{P}}(s_i, s_{-i})$  depends on its own securities choice  $s_i$  and the set of choices of other banks  $s_{-i}$ . It is the gross return on real assets  $a_i$  plus the expected gross return on liquid assets across three possible outcomes: no liquidity withdrawal, a liquidity withdrawal that the bank can satisfy with its liquid assets, or a liquidity withdrawal with  $b$  high enough that the bank is liquidity deficient and requires an intervention.

$$\begin{aligned} \Pi_i^{\mathcal{P}}(s_i, s_{-i}) = & a_i + (1 - \lambda)(l + s_i r_s) \\ & + \lambda \left( E_b \left[ \underbrace{(\text{liquidity surplus})}_{l - s_i m^*(b, s_{-i}) - b} * \underbrace{(\text{return on liquidity})}_{\frac{1}{1 - m^*(b, s_{-i})}} \middle| \underbrace{\text{sufficient liquidity}}_{b \leq \bar{b}(s_i, s_{-i})} \right] * pr(\text{sufficient liquidity}) \right. \\ & \left. + E_b \left[ \underbrace{(\text{liquidity surplus})}_{l - s_i m^*(b, s_{-i}) - b} * \underbrace{(\text{return on liquidity})}_{\text{determined by authority}} \middle| \underbrace{\text{liquidity deficient}}_{b > \bar{b}(s_i, s_{-i})} \right] * pr(\text{liquidity deficient}) \right) \end{aligned} \quad (2.1)$$

In general, a bank's gross return on liquid assets given a liquidity withdrawal is its *liquidity surplus* (or deficit if negative) times the *return on liquidity*. The return on liquidity is the value of cash at date 1, and the liquidity surplus can be thought of as the difference between two components: the funding liquidity position  $l - b$ , equal to the net of cash inflows and outflows; and the market liquidity loss  $sm^*$ , being the effect of market illiquidity on the bank's date 1 access to liquidity. If the funding liquidity position is less than the market liquidity loss – i.e. if  $b \leq \bar{b}_i$  – the return on liquidity is  $1/(1 - m^*)$ , which is the quantity of securities purchased (if  $b < c_i$ ) or not sold (if  $b \geq c_i$ ) per unit of liquidity surplus, multiplied by the date 2 value of each security (i.e. 1). If the liquidity surplus is negative, the return on liquidity is the cost of sourcing liquidity from the authority and is proportional to the size of the deficit.<sup>10</sup>

Each bank chooses  $s_i$  by comparing the returns that securities give in the normal state  $r_s$  against the expected costs given a liquidity withdrawal. These costs depend on how many securities other banks hold because higher  $S$  (holding its composition  $s_{-i}$  constant) causes more selling pressure and market illiquidity whenever  $b$  is high enough such that  $m^* > 0$ . Higher market illiquidity lowers the expected marginal return to securities, since depressed prices raise the value of spare cash if  $b < c_i$  and the cost of having to sell securities if  $b > c_i$ . Accordingly, as  $S$  increases, securities' expected returns decrease, typically permitting a fixed point where optimal  $s_i$  given  $S$  is equal to  $S$ , so that acting in line with other banks is the best response to the aggregate effect of their choices.

Denote by  $s_{i,\mathcal{P}}^*(s_{-i})$  an optimal choice of securities for bank  $i$  given the choices of other banks; that is,

$$s_{i,\mathcal{P}}^*(s_{-i}) \in \arg \max_{s_i} \Pi_i^{\mathcal{P}}(s_i, s_{-i}).$$

**Definition** For policy  $\mathcal{P}$ , a (Nash) equilibrium set of securities choices  $s_{\mathcal{P}}^*$ , with corresponding aggregate securities holdings  $S_{\mathcal{P}}^*$ , is one that satisfies  $s_i = s_{i,\mathcal{P}}^*(s_{-i}^*)$  for all  $i$ .

The  $\mathcal{P}$  subscripts will be dropped in some cases where the policy being referred to is obvious.

Banks' only ex-ante heterogeneity comes from  $a_i$  which drops out of marginal considerations (until the equity injection policy is analysed in section 2.3). Accordingly, asymmetric pure-strategy choices, with some banks not choosing  $s_i = S$ , can be equivalently interpreted as symmetric mixed strategies, whereby the continuum of banks ensures that the distribution of choices observed corresponds exactly to their probabilities

<sup>10</sup>As outlined in section 1.2, under the unsecured and secured lending policies the authority is assumed to only lend banks the minimum they need to avoid failure, forcing them to sell all their securities even if they would prefer to use more emergency funding instead. Without this assumption, there would be two possible outcomes: the interest rate is high enough that banks still minimise borrowing, or it is not, and banks borrow enough to reduce their securities selling to the point where the cost of selling at  $1 - m^*$  is equal to the borrowing cost imposed by the authority. This paper focuses on situations in which banks would choose to minimise their borrowing, where the assumption is of no consequence. Corollary 3 considers the alternative. While its point – that a low interest rate on emergency borrowing leads to corner solutions with maximum liquidity exposures – would likely be strengthened by removing this assumption, the solution is more complicated because a bank's ex-post payoffs would discontinuously increase at the  $b$  at which emergency lending becomes available.

under the mixed strategies. The following uses the terminology ‘a/symmetric’ rather than mixed and pure strategies.

## 2.1 Unsecured lending

Under an unsecured lending policy the authority chooses only the interest rate  $r_U$ . Bank  $i$ 's payoff function given a liquidity withdrawal is the expectation across  $b$  of its date 2 portfolio value. First  $m^*(b, s_{-i})$  is shown to be well defined and  $\bar{b}_i$  to be unique.

**Lemma 1** *If each individual bank's selling is Lipschitz continuous in  $m^*$  and  $b$ , nonincreasing in  $m^*$ , and both partial derivatives are continuous at all but one point, then  $m^*(b, s_{-i})$  is unique, continuous in  $b$ , and almost everywhere continuously differentiable in  $b$ .*

**Proof.** See Appendix.

Uniqueness of  $\bar{b}_i$  (defined in equation 1.2) then follows from  $L_D$  and thus  $m^*$  being weakly increasing in  $b$ . This permits the payoff function to be expressed as

$$\begin{aligned} \Pi_i^U(s_i, s_{-i}) &= (1 - \lambda)(a_i + l + s_i r_s) \\ &+ \lambda \left[ \int_0^{\bar{b}(s_i, s_{-i})} \left( a_i + \frac{l - b - s_i m^*(b, s_{-i})}{1 - m^*(b, s_{-i})} \right) f(b) db + \int_{\bar{b}(s_i, s_{-i})}^l [a_i - (b + s_i m^*(b, s_{-i}) - l)(1 + r_U)] f(b) db \right]. \end{aligned} \quad (2.2)$$

Banks necessarily find sourcing liquidity from the authority more expensive than through the securities market if

$$r_U \geq \frac{M(l + \gamma l)}{1 - M(l + \gamma l)} \quad (2.3)$$

where  $M(l + \gamma l)$  is the least upper bound on market illiquidity, corresponding to  $S = b = l$ . Condition 2.3 ensures that if a bank marginally increases its securities holdings, which raises the probability that it needs to borrow from the authority, its ex-ante marginal return to securities decreases. In other words, it ensures that a bank cannot increase its marginal return to securities by increasing its liquidity risk  $s_i$ .

**Proposition 2** *If condition 2.3 is satisfied then the unsecured lending regime has a unique (symmetric) equilibrium  $s = S_U^*$  that is interior for intermediate values of  $(1 - \lambda)r_s$ .*

**Proof.** See Appendix.

Banks' equilibrium securities holdings reflect the typical risk-return tradeoff: increasing in non-crisis expected returns (relative to cash); decreasing in securities' liquidity risk, represented in the model by how much  $m^*$  increases with  $b$ , and in the interest rate charged by the authority on emergency borrowing. Condition 2.3 is sufficient but not necessary for Proposition 2 because in equilibrium  $m^*(\bar{b}_S, S)$  is necessarily less than  $M(l + \gamma l)$ , and for many parameter values by a non-negligible difference; however, stating the weakest sufficient condition requires specifying  $f(b)$  and accounting for equilibrium  $s$ .

**Corollary 3** *If there is an equilibrium  $s^*$  with  $0 < s_i^*(s_{-i}^*) < l$  for all  $i$ ,  $S^*$  is strictly decreasing in  $r_U$ . Moreover, for  $r_U$  and  $s$  such that*

$$r_U < \frac{m^*(\bar{b}(s_i, s_{-i}), s_{-i})}{1 - m^*(\bar{b}(s_i, s_{-i}), s_{-i})} \quad (2.4)$$

*the marginal return to  $s_i$  is increasing in  $s_i$ .*

**Proof.** See Appendix.

Condition 2.4 ensures that a bank prefers to maximise borrowing rather than liquidate any securities at  $b \geq \bar{b}_i$ . A stronger version of its complement

$$r_U > \frac{m^*(\bar{b}(l, l), l)}{1 - m^*(\bar{b}(l, l), l)}$$

ensures that at all  $b$  banks prefer to minimise their borrowing, which aligns neatly with Bagehot’s recommendation that the interest rate on emergency lending should be high enough that banks do not borrow more than they need. Bagehot’s recommendation therefore also ensures that banks cannot raise their marginal benefits from liquidity exposures by increasing their exposures, an outcome that would lend itself to excessive liquidity-risk taking and high likelihoods of banking crises occurring.

## 2.2 Secured lending

Under a secured lending regime, a liquidity-deficient bank must use some of its securities as collateral for borrowing from the authority, and therefore sells less. At an individual-bank level, the constraint on selling can generate an opportunity cost of borrowing, on top of the interest cost, raising the return on liquidity and consequently the cost of being liquidity deficient. In aggregate, the limited selling reduces market illiquidity, having a positive effect on banks’ ex-post payoffs by raising the selling price of securities that can be sold.

The secured lending policy works by raising the amount of liquidity that a bank can get from its securities, which will be referred to as the securities’ *liquidity value*, relative to if it sells them. A security’s liquidity value if sold is  $1 - m^*$ , and if used as collateral is  $1 - h$  where  $h$  is the haircut set by the authority. For the policy to raise securities’ liquidity value enough to save banks,  $h$  must be sufficiently below  $m^*$ . For the sake of exposition the following assumes that the authority sets the lowest  $h$  that eliminates its principal risk, being  $h = 0$ , so securities retain full liquidity value if used as collateral. Higher haircuts are examined in section 3.

When liquidity deficient, bank  $i$ ’s allocation of securities to collateral  $s_{ri}$  and to sales  $s_{mi}$  is determined by the authority’s preference for minimising lending subject to the requirement that the bank survives. That is, the authority ensures that  $s_{ri}$  is at the lowest value that satisfies

$$s_{ri} + s_{mi}(1 - m^*) \geq b - c_i \quad (2.5)$$

such that  $s_i \geq s_{ri} + s_{mi}$ ,  $s_{ri} \geq 0$  and  $s_{mi} \geq 0$ . The bank’s liquidity deficiency  $s_i(1 - m^*) < b - c_i$  then implies that the quantity of securities used as collateral satisfies

$$s_{ri} = s_i - s_{mi} = s_i - \frac{l - b}{m^*}. \quad (2.6)$$

Penalty rates would also imply that borrowing is minimised, giving the same value for  $s_{ri}$ . Either way, the quantity of securities sold  $(l - b)/m^*$  does not depend on securities held  $s_i$ , because each bank sells the maximum such that its total loss of liquidity value  $s_{mi}m^*$  does not exceed the loss that its funding liquidity position  $l - b$  can absorb. Securities sold  $s_{mi}$  is therefore determined only by market conditions and withdrawing creditors, neither of which the bank controls.

For  $b > \bar{b}_i$ , securities sales  $s_{mi}$  are decreasing in  $b$  because the lower a bank’s funding liquidity position, the more it must maximise its securities’ liquidity value by using them as collateral. Market illiquidity is therefore potentially decreasing in the liquidity withdrawal size, as large collateral requirements mean few securities are left over to be sold. An equilibrium would become difficult to solve if it was possible that at high  $b$  market illiquidity was low enough for a bank’s liquidity position to return to surplus. Lemma 4 shows that the positive, indirect effect of  $b$  on a bank’s liquidity position  $l - b - s_i m^*$ , through  $m^*$ , cannot exceed by such an extent the negative, direct effect.<sup>11</sup>

**Lemma 4** *Under the secured lending policy for any bank  $i$  there is at most a single value of  $\bar{b}_i$  that separates liquidity sufficiency from deficiency.*

**Proof.** See Appendix.

The intuition of Lemma 4 is clearest for the symmetric case  $s = S$ . Considering continually higher  $b$ , if  $m^*$  were to fall to the point  $(l - b)/s_i$  at which banks were liquid again, expression 2.6 shows that  $s_{mi}$  would approach  $s_i$ , that is, banks would be selling all their securities, which contradicts that  $m^*$  has fallen below the level at which banks became illiquid.

<sup>11</sup>The assumption in section 1 that  $\gamma > 0$  is made to ensure this result.

Lemma 4 implies that bank  $i$ 's objective function is

$$\begin{aligned} \Pi_i^{\mathcal{R}}(s_i, s_{-i}) &= (1 - \lambda)(a_i + l + s_i r_s) \\ &+ \lambda \left[ \int_0^{\bar{b}(s_i, s_{-i})} \left( a_i + \frac{l - b - s_i m^*(b, s_{-i})}{1 - m^*(b, s_{-i})} \right) f(b) db + \int_{\bar{b}(s_i, s_{-i})}^l \left[ a_i - (b + s_i m^*(b, s_{-i}) - l) \frac{r_{\mathcal{R}}}{m^*(b, s_{-i})} \right] f(b) db \right]. \end{aligned} \quad (2.7)$$

Given liquidity deficiency, a bank's date 2 payoff is affected positively by the value of securities it still owns  $s_{ri}$  and negatively by the date 2 repayment  $s_{ri}(1 + r_{\mathcal{R}})$ , leaving the net loss at  $s_{ri}r_{\mathcal{R}}$ . Substituting expression 2.6 into  $s_{ri}r^r$  shows that the return on liquidity is  $r_{\mathcal{R}}/m^*$ . It is the interest cost on necessary borrowing, because collateralising one unit of borrowing means losing  $1 - m^*$  liquidity through foregone sales, so to obtain one more unit of liquidity, borrowing must increase by  $1/m^*$ . This imposes  $(1 + r_{\mathcal{R}})/m^*$  repayment cost and saves  $1/m^*$  on securities used as collateral rather than sold.

**Proposition 5** *If a secured lending policy satisfies*

$$r_{\mathcal{R}} \geq \frac{M(l + \gamma l)}{1 - M(l + \gamma l)} \quad (2.8)$$

*then it has a unique equilibrium  $s = S_{\mathcal{R}}^*$ , and if the equilibrium in an unsecured lending regime with  $r_{\mathcal{U}} = r_{\mathcal{R}}$  satisfies condition 2.3 and  $0 < S_{\mathcal{U}}^* < l$ , then  $S_{\mathcal{R}}^* < S_{\mathcal{U}}^*$ .*

**Proof.** See Appendix.

The marginal cost of  $s_i$  given liquidity deficiency is  $r_{\mathcal{R}}$ , being the net cost of an extra unit of borrowing – every extra security held means one less unit of cash, and to cover this the security must be used as collateral for one more unit of borrowing. In comparison, the marginal cost of  $s_i$  given liquidity deficiency under unsecured lending is  $m^*(1 + r_{\mathcal{U}})$ , being the loss from selling the security  $m^*$  and the cost of the borrowing required to make up for the remaining shortfall  $m^*r_{\mathcal{U}}$ . With  $r_{\mathcal{R}} = r_{\mathcal{U}}$ , these marginal costs would be the same if covering a liquidity need by only borrowing imposed the same cost as covering it by selling some and borrowing only the remainder; that is, if the collateral constraint  $s_{ri} \leq s_i - s_{mi}$  was just binding at the optimal choice of  $s_{mi}$ . By condition 2.8, however, the cost of borrowing is higher than that of selling, so the secured lending policy imposes a higher cost of liquidity deficiency and results in lower equilibrium securities holdings.

**Proposition 6** *For any pair of interest rates  $r_{\mathcal{U}}$  and  $r_{\mathcal{R}}$  that induce equal symmetric equilibria  $0 < S_{\mathcal{U}}^* = S_{\mathcal{R}}^* < l$ , the expected payoff is higher under the secured lending policy than under the unsecured lending policy.*

**Proof.** See Appendix.

Propositions 5 and 6 imply that there exist pairs of interest rates under which the secured lending policy induces both lower liquidity exposures and higher expected payoffs than the unsecured lending policy. For instance, if  $r_{\mathcal{R}}$  is slightly higher than that which induces  $\tilde{S}^u = \tilde{S}^r$ , Proposition 6 shows that banks' ex-post equity given a liquidity withdrawal is non-negligibly higher for all  $b > \bar{b}_i$  under the secured policy. This is because the mitigated market illiquidity means banks can raise more funds through securities sales and hence borrow less, but importantly, this benefit does not raise incentives to hold securities, because a bank's securities sales given liquidity deficiency is not increasing in  $s_i$ , as already explained. The benefit of reduced market illiquidity therefore does not offset the disincentives for liquidity exposures generated by the collateral constraint and associated increased borrowing needs.

The model also shows that secured lending has two opposing effects on how much balance-sheet the authority uses, relative to unsecured lending. Holding  $s$  constant, banks borrow more from the authority under secured lending because they raise less liquidity from securities sales. Still, Proposition 5 shows that banks are likely to hold fewer securities ex ante, which lowers their expected liquidity needs. While it is not clear which effect is larger, it is clear that gauging a policy ex-post by how much balance sheet it has used could be misleading. That is, an alternative policy could save banks using less balance sheet, but it may create incentives that necessitate larger interventions in the next crisis. Furthermore, balance-sheet use is arguably an indirect measure of how much risk the authority takes with public funds, so it is also important to consider how much collateral the authority has backing its assets.

## 2.3 Equity injection

Under an equity injection policy, the authority gives a liquidity deficient bank the minimum funds it needs to avoid failure, and receives back a proportion  $\phi \leq 1$  of that bank's date 2 assets. Assume that  $\phi$  is strictly and continuously increasing in the amount of funds provided, and zero if no funds are provided. Bank  $i$ 's payoff function is

$$\Pi_i^{\mathcal{E}}(s_i, s_{-i}) = (1 - \lambda)(a_i + l + s_i r_s) + \lambda \left[ \int_0^{\bar{b}(s_i, s_{-i})} \left( a_i + \frac{l - b - s_i m^*(b, s_{-i})}{1 - m^*(b, s_{-i})} \right) f(b) db + \int_{\bar{b}(s_i, s_{-i})}^l [1 - \phi(b + s_i m^*(b, s_{-i}) - l)] a_i f(b) db \right]$$

**Proposition 7** *If  $S_{\mathcal{U}}^* > 0$  then an equity injection policy cannot have a symmetric equilibrium unless  $S_{\mathcal{E}}^* = l$ , and any asymmetric equilibria are characterised by  $s_i^*(s_{-i}^*)$  being weakly decreasing in  $a_i$ .*

**Proof.** See Appendix.

**Corollary 8** *The equity-injection policy cannot leave a bank with a negative ex-post payoff whereas the unsecured lending policy can. Further, if  $\phi$  is linear, for any given  $s$  and  $b$ , there is a threshold  $\underline{a}$  such bank  $i$  has a lower ex-post payoff under the unsecured lending policy than under the equity-injection policy if and only if  $a_i > \underline{a}$ .*

**Proof.** See Appendix.

Proposition 7 and Corollary 8 demonstrate that for banks with low capital, an equity purchase policy has little capacity to punish excessive liquidity-risk taking, and for the same reason, it is less likely to leave these banks ex-post insolvent. In this paper the authority is assumed to care only about illiquidity-driven failures; however, as argued by for example Morris and Shin (2009), illiquidity during a crisis can be driven by short-term creditors' reactions to post-crisis solvency concerns. Similar to the argument in Rochet and Vives (2004), if a bank's short-term creditors believe it has insufficient long-term assets to later repay penalising interest rates, an authority's ex-ante claim to refrain from purchasing equity in this bank may lack credibility.

Refraining from equity injections would be more credible if the authority is perceived as willing to let a small number of low-equity banks fail. Previous work on 'gambling for resurrection' suggests that ex-post, failure of a low-profitability bank is socially preferable over failure of a high-profitability bank, because low-profitability banks are more likely to take excessive risks (for some discussion see Freixas and Rochet (2008) chapter 9, and for some empirical evidence see Jiménez, Ongena, Peydró, and Saurina (2014)). However, if the bank failure occurs during a crisis, the bank's systemic importance becomes more relevant for evaluating the social costs or benefits, and it is unclear that systemic importance should be negatively correlated with its asset quality. For instance, as part of the Basel III reforms, Basel Committee for Banking Supervision (2013b) declares that global systemically important banks should be identified based on five characteristics, none of which are necessarily related to capital: size, interconnectedness, availability of substitutes for their services, global activity and complexity (paragraphs 15 to 16).

## 2.4 Securities purchase

The authority can also ensure the liquidity value of banks' portfolios remains sufficiently high by purchasing securities to support their price. Consider a policy whereby the authority sets a schedule  $m^s(b)$  such that if without intervention the market illiquidity  $m^*$  would be above  $m^s(b)$ , it purchases enough securities to ensure the market price is  $1 - m^s(b)$ . The higher is  $s$ , the more securities will need to be purchased to achieve a given  $m^s(b)$ .

In contrast to the other interventions analysed, the authority saves banks indirectly, through the securities market, and so cannot condition the quantity of liquidity it provides on individual banks' needs. Therefore the authority must choose a schedule  $m^s(b)$  that either ensures no bank can fail, or that saves only banks with low enough  $s_i$ . This section first considers the former, then formulates a bank's payoff given failure to consider the latter.

The most price reduction a bank can handle without failure is  $(l - b)/s_i$ , so the first securities-purchase

policy involves ensuring the market illiquidity does not exceed  $1 - b/l$ , which requires purchases at all  $b > \underline{b}$  such that  $\underline{b} = l[1 - M(\underline{b} + S - l + \gamma \underline{b})]$ . For  $b \leq \underline{b}$ , the market illiquidity is  $M(b + S - l + \gamma b)$  which is defined here as  $M(b, S)$ . Bank  $i$ 's payoff function is

$$\begin{aligned} \Pi_{0i}^S(s_i, s_{-i}) &= (1 - \lambda)(a_i + l + s_i r_s) \\ &+ \lambda \left[ \int_0^{\underline{b}} \left( a_i + \frac{l - b - s_i M(b, S)}{1 - M(b, S)} \right) f(b) db + \int_{\underline{b}}^l \left( a_i + \frac{(l - b)(l - s_i)}{b} \right) f(b) db \right]. \end{aligned} \quad (2.9)$$

**Proposition 9** *If  $0 < S_{\mathcal{U}}^* < l$  then a securities purchase policy that does not allow any banks to fail has no symmetric equilibrium at  $S \leq S_{\mathcal{U}}^*$  and a symmetric equilibrium at  $S > S_{\mathcal{U}}^*$ .*

**Proof.** See Appendix.

To relax the policy such that it is possible for banks to fail, assume that a bank's cost of failure is  $z > 0$  multiplied by its liquidity deficit upon failure, making its payoff conditional on  $b > \bar{b}_i$  equal to  $a_i - (b + s_i m^* - l)z$ . This setup could be motivated by a reputational cost for the bank's management that is proportional to how much harm the bank's failure would cause the financial system.

An alternative would be that the payoff function is constant given liquidity deficiency. In this case, absent intervention a moral hazard problem would arise, akin to the effect of limited liability discussed by, for example, Diamond and Rajan (2011). If it was optimal for a bank to take any risk of failure, it would be optimal to maximise the risk. The interventions examined in sections 2.1, 2.2 and 2.3, on the other hand, would attenuate this moral hazard – the costs of emergency funding imposed by the authority generate a negative ex-post marginal return to securities whenever  $b > \bar{b}_i$ , which in turn means lower ex-ante marginal returns than if the ex-post marginal return is zero. This section takes the view that bank rescues are more likely to augment than attenuate moral hazard, consistent with Farhi and Tirole (2012) and Acharya and Yorulmazer (2008), which supports using a proportional rather than fixed payoff given failure.

Define a securities purchase policy that permits bank failure as  $m_1^s(b)$ . The authority targets  $\bar{S}$  such that any bank with  $s_i \leq \bar{S}$  is saved by the policy, by ensuring market illiquidity does not exceed  $(l - b)/\bar{S}$ . The lowest  $b$  at which the authority intervenes therefore satisfies  $b = l - \bar{S}m^*(b, s_{-i})$ . Call an equilibrium *effective* if the policy prevents any bank acting in line with the aggregate, i.e. that chooses  $s_i = S$ , from failing, and does so without purchasing more securities than necessary.

**Proposition 10** *A securities purchase policy  $m_1^s(b)$  either: i) if  $z$  is not less than a specific threshold (that is greater than one), has an effective symmetric equilibrium  $S_S^* > S_{\mathcal{U}}^*$ ; or ii) otherwise, can only have effective symmetric equilibria at  $S_S^* = l$ .*

**Proof.** See Appendix.

The model illustrates two important features of a securities purchase policy. First, the policy generates higher-exposure equilibria than an unsecured lending policy, because the authority cannot penalise liquidity deficient banks by requiring a date 2 repayment. At date 2, banks can afford to pay penalties because their securities and other assets are liquid. In contrast, under the securities purchase policy, banks' only payment to the authority occurs at date 1 when the securities transactions occur, and the size of this payment is constrained by their liquidity deficiency.

Second, because a bank's ex-ante marginal return to securities drops if its exposure is higher than what is guaranteed to be safe by the policy – because failure and its associated costs become a possibility – it is likely that the authority could induce a lower equilibrium by setting lower  $\bar{S}$ . Furthermore, if the cost of failure  $z$  is sufficiently deterrent, the authority could likely induce an equilibrium at  $\bar{S}$  as banks maximise their securities holdings subject to avoiding the risk of failure, so the policy would also be effective. However, any policy under which the mass of banks fails is not credible, and moreover, even an effective equilibrium lacks credibility, because if banks coordinate on some higher  $S$ , the authority's preference for avoiding bank failure means it would raise  $\bar{S}$  in response. Farhi and Tirole (2012) show a similar result.

Together, these two results reveal a mitigating factor to Farhi and Tirole (2012)'s argument that inducing a low-risk equilibrium by promising a low level of intervention is not credible. In Farhi and Tirole (2012), asset illiquidity, captured by the 'agency wedge' between the value of assets and their pledgeability (discussed further in Tirole (2011)), does not vary over time. However, if banks' liquidity and solvency positions are not so tightly connected – for instance if markets for banks' assets experience a temporary period of diminished liquidity – authorities have greater capacity to implement penalising interventions without harming the financial system, as long as the penalties occur once banks' liquidity positions have improved. This capacity lends credibility to an authority's promised intervention.

### 3 Extensions and applications

#### 3.1 Positive haircuts on secured lending

Section 2.2 assumes that the authority requires no haircut on secured lending, because there is no uncertainty around the date 2 value of securities. Here, positive haircuts are shown to tend to support the results in section 2.2.

Section 2.2 defines haircuts  $h$  such that 1 security can be used as collateral to borrow  $1 - h$  funds. For the liquidity injection to be capable of saving banks, it must be the case that  $s_i h \leq l - b$ . Assume that the haircut is set at  $h = \psi(l - b)/l$ , where  $0 \leq \psi < 1$ . This functional form has three appealing characteristics: it is a simple generalisation of the secured lending setup in section 2.2 with  $\psi$  representing the authority's conservativity in setting the haircut ( $\psi = 0$  gives the model in section 2.2), it ensures that any bank can stay liquid, and it does not make the haircut negatively related to  $s_i$  which would favour riskier banks. Refer now to the interest rate as  $r_{\mathcal{R}}^h$ .

Conditional on being liquidity deficient, bank  $i$ 's binding liquidity constraint is

$$c_i + s_{ri}(1 - h) + s_{mi}(1 - m^*) \geq b$$

which gives the quantity of securities used as collateral

$$s_{ri} = \frac{s_i m^* + b - l}{m^* - h}$$

and the quantity of securities sold

$$s_{mi} = s_i - s_{ri} = \frac{l - b - s_i h}{m^* - h}.$$

Securities sales given liquidity deficiency  $s_{mi}$  are now decreasing in  $s_i$ . A positive haircut means that if one more security is held, it cannot collateralise enough borrowing to cover the forgone unit of cash, so holding more securities requires posting disproportionately more as collateral, and therefore selling less.

The date 2 payoff is  $a_i$  minus the repayment from borrowing  $b - c_i - (s_i - s_{ri})(1 - m^*)$ , plus the securities not sold  $s_{ri}$ . The proof of Proposition 11 shows that  $\bar{b}_i$  is unique, and given uniqueness of  $\bar{b}_i$ , the payoff function assuming  $s_{-i} = S$  is

$$\begin{aligned} \Pi_{hi}^{\mathcal{R}}(s_i, S) &= (1 - \lambda)(a_i + l + s_i r_s) \\ &+ \lambda \left[ \int_0^{\bar{b}(s_i, S)} \left( a_i + (l - s_i - b) \frac{m^*(b, S)}{1 - m^*(b, S)} \right) f(b) db + \int_{\bar{b}(s_i, S)}^l [a_i - (b + s_{mi} m^* - l)(1 + r_{\mathcal{R}}^h) - s_{ri} r_{\mathcal{R}}^h] f(b) db \right]. \end{aligned}$$

**Proposition 11** *If condition 2.8 holds for  $r_{\mathcal{R}}^h$  then for any secured lending policy with  $\psi \in [0, 1]$ :*

1. *for any given  $s_{-i}$  and  $b \in (\bar{b}_i, l)$  market illiquidity is decreasing in  $\psi$ ;*
2. *holding  $r_{\mathcal{R}}^h$  fixed, the highest equilibrium is strictly decreasing in  $\psi$  unless at a corner; and*
3. *there are pairs of policies such that one has  $\psi > 0$  and the other has  $\psi = 0$  that give the same equilibrium  $S^*$ , where the first policy has a higher expected payoff.*

**Proof.** See Appendix.

In section 2.2 the marginal cost of holding securities given liquidity deficiency is independent of the securities price, because holding one more security simply leads to one unit more borrowed. Because the payoff is affected positively by a higher securities price, this independence permits a deterrence to holding securities that does not imply a lower overall payoff. With  $\psi > 0$ , the marginal cost of holding securities given liquidity deficiency is *positively* related to the securities price, because higher  $s_i$  means less selling and therefore less benefit from a better price. This strengthens the notion that secured lending can give two seemingly opposing benefits – lower probability and expected severity of a liquidity crisis, and higher returns on assets with liquidity risk.

This extension also raises some points of discussion around how authorities should set haircuts during a crisis. First, the haircut size, which determines how much liquidity banks can access, should be decreasing in the severity of the crisis, which determines how much liquidity banks need. During the 2007-2009 liquidity crisis many central banks extended the list of securities they would accept as collateral. This was essentially a reduction of haircuts on the newly eligible securities from 100 per cent to a level below what the market is offering, in line with the model’s implicit recommendation.

Second, the model suggests that it is beneficial to raise haircuts to the maximum that still avoids bank failures. This should be interpreted loosely, because the model omits two factors that would make this maximum difficult to determine – banks may have varying degrees of liquidity deficiency, and the authority may not have perfect information regarding how much liquidity banks need. If haircuts were set too high, bank failure would not be prevented, and moreover, banks on the verge of failure might liquidate all their securities, reversing the policy’s positive effects on the securities market.

### 3.2 Heterogeneous liquidity withdrawals and an interbank market

The model has also assumed that all banks experience the same liquidity withdrawal  $b$ , which leads to symmetric liquidity positions and therefore no interbank transactions. This section generalises the model to permit heterogeneous withdrawals, giving rise to an active interbank market in equilibrium, to show that the results are maintained.

Assume that  $\delta > 0$  proportion of banks experience the liquidity withdrawal  $b$  and the rest have no funding liquidity outflow, and at date 0 banks do not know which group they will be in. Banks with no liquidity withdrawal always have  $l - s_i$  spare cash, which they can use to buy securities from banks that experience withdrawals.

**Proposition 12** *Propositions 2 to 11 are maintained if  $\delta \in (0, 1)$  randomly selected banks experience liquidity withdrawals.*

In both the homogeneous and heterogeneous forms of the model, each bank is influenced by other banks only through their effect on the aggregate liquidity available. This effect is represented by their quantity of buying or selling of securities, and transmitted through  $m^*$ . The fact that when distressed, bank  $i$  can source some of its liquidity from other banks, rather than the authority, therefore simply reduces bank  $i$ ’s need to obtain funds from the authority, but does not change the per-unit cost of emergency funding that it still requires. For this reason the model results are robust to heterogeneous liquidity positions and an interbank market.

### 3.3 Combining policies

During the 2008 financial crisis authorities typically combined secured lending with unsecured lending, equity injections and sometimes securities purchases. This section extends the model by assuming that at date 1, some proportion of securities become worthless, so that under a secured lending policy with nonnegative haircuts, banks’ liquidity needs can exceed the borrowing that their securities can collateralise. In line with the results in section 2, a policy that uses secured lending as much as possible and unsecured lending for banks’ remaining liquidity needs has higher market prices and lower ex-ante incentives for liquidity-risk taking than a policy using only unsecured lending.



Assume that when a liquidity withdrawal occurs, a fixed proportion  $\kappa$  of randomly selected securities experience a decline in fundamental value to zero. This reduces bank  $i$ 's liquidity position by  $s_i\kappa$  so that the highest liquidity withdrawal it can survive without intervention is

$$\bar{b}_i = l - s_i[\kappa - (1 - \kappa)m^*(\bar{b}_i, s_{-i})]$$

Under the unsecured lending policy, there is little change aside from the lower liquidity position.

Under the secured lending policy with zero haircut banks can borrow at most  $s_i(1 - \kappa)$ . Consider a policy combination whereby if  $b > l - s_i\kappa$ , the authority lends banks  $b - l - s_i\kappa$  on an unsecured basis. This policy does not in general maintain the result of Lemma 4, but under the restriction  $s_{-i} = S$ , the threshold shock  $\bar{b}_i$  can be shown to be unique with assumptions on  $M$  and  $\gamma$ .<sup>12</sup> Consistent with the results in section 2.2, a combination of secured and unsecured lending better deters liquidity-risk taking than a policy of only unsecured lending.

**Proposition 13** *If condition 2.3 is satisfied, a policy with only unsecured lending has a unique symmetric equilibrium that is interior for intermediate values of  $(1 - \lambda)r_s$ . If the equilibrium is interior, a policy combination of secured and unsecured lending as described above, with  $r_{\mathcal{R}} = r_{\mathcal{U}}$  and assuming condition 2.3, can only have equilibria below the equilibrium for the unsecured lending policy.*

**Proof.** See Appendix.

An authority could also combine the secured lending policy with equity purchases when  $b > l - s_i\kappa$ . As an equity-purchase policy generates heterogeneity of marginal returns, there cannot be an interior symmetric equilibrium. As discussed in section 2.3, such a combination may at date 1 be preferred to a combination of secured and unsecured lending if there are concerns about banks' date 2 solvency that could intensify the liquidity distress.

**Corollary 14** *Given fixed symmetric  $s$  and assuming  $r_{\mathcal{R}} = r_{\mathcal{U}}$ , for  $a_i$  below some threshold, combining secured lending with unsecured lending as described above gives a lower ex-post payoff than a combination with equity purchases.*

**Proof.** The result follows immediately from the payoff functions. ■

On contrast, a securities purchase policy cannot usefully be combined with the secured lending policy because banks cannot sell their securities and pledge them as collateral. Such a combination would therefore only improve financial-system liquidity if the authority purchases securities that it does not accept as collateral. For example, the authority could provide liquidity to a specific bank or group of banks by purchasing securities that those but few other other banks hold. Assuming they are not acceptable as collateral because they are low quality, combining this policy with secured lending is likely to be similar to combining secured lending with an equity purchase policy that has very low repayment requirements  $\phi$ .

## 4 Discussion of liquidity injection policies implemented in the US and Europe

To provide more context to the arguments presented so far, this section reviews some of the largest liquidity injection policies adopted by authorities in Europe and the US around late 2008. Both regions utilised secured lending, bank-equity purchases, and unsecured lending subsidisation. Unsecured lending subsidisation was mostly through government guarantees on banks' unsecured debt, with also some direct loans to banks. The US relied most heavily on secured lending, while in Europe government guarantees on banks' unsecured debt were more prominent. In both regions, rates charged on some of the emergency lending facilities were possibly too low to penalise banks for their prior liquidity exposures.<sup>13</sup>

ECB's standard tools for open market operations are its main refinancing operations (MRO) and longer-term refinancing operations (LTRO), under which banks borrow from ECB against eligible collateral. MROs

<sup>12</sup>For example,  $M(x) = x/(l + \gamma l)$  for  $x \geq 0$  and  $\gamma = l/5$ .

<sup>13</sup>Data on usage of the secured lending facilities in the US and Europe are available at the respective central banks' websites.

are conducted weekly with one-week maturity and LTROs are typically conducted monthly with maturities of one month and longer. In March 2008, ECB announced it would run a series of LTROs with six-month maturities, compared to maturities of three months in previous LTROs. Prior to October 2008, ECB priced the MROs and LTROs by taking bids from banks comprising interest rates and corresponding quantities, auctioning a predetermined total amount at the lowest successful interest-rate bid, which was targeted to be a certain level above the ‘deposit rate’ that ECB pays banks on their overnight cash holdings. On 15 October 2008 for MROs and 30 October 2008 for LTROs, ECB switched to a fixed rate tender with full allotment, whereby it fixed the interest rate and banks could borrow any amount requested. On 23 October ECB also substantially widened the range of eligible collateral, accepting more corporate debt instruments and lowering the required credit rating of collateral from A- to BBB-. The most rapid and substantial increase in MRO and LTRO lending occurred between late September and late October 2008, rising from 480 billion to 820 billion EUR. Over the same period the interest rate on ECB’s lending declined from around 4.7 to 3.75 per cent, above the interbank EONIA rate which declined from 3.70 to 3.55 per cent.

In mid October several European countries offered government guarantees on banks’ newly issued debt, guaranteeing around 770 billion EUR, and often charging prices estimated to be close to market rates in normal times. Around the same time, some of these countries engaged in bank recapitalisation schemes, with a combined cap of 140 billion EUR, and purchased or guaranteed around 43 billion EUR of banks’ assets.<sup>14</sup> A large component of this was the Swiss National Bank’s (SNB) transaction with UBS, under which SNB created a ‘bad bank’ fund, owned and mostly funded by SNB, that purchased around 30 billion EUR of assets from UBS with an arrangement that SNB would receive UBS shares if the bad bank eventually made a loss. The transaction therefore had similarities to an equity purchase. In November 2013 SNB sold the last of the fund back to UBS and announced that it made around 2.8 billion EUR capital gains on top of interest payments.

Throughout the crisis the US Federal Reserve introduced a number of new facilities for collateralised open market operations, including the term auction facility (TAF), the primary dealer credit facility (PDCF) and the term securities lending facility (TSLF).<sup>15</sup> The TAF, introduced in March 2007, lent to a wide range of depository institutions – in contrast to the standard open market operations that only transact with the 20 or so primary dealers – for terms of one or three months, via single price auctions each of a fixed total amount. The largest monthly increase in TAF lending was from 125 to 390 billion USD between early October and early November 2008. The PDCF, introduced in March 2008, lent funds without limit to primary dealers on an overnight basis, at the Fed’s overnight policy rate and with an additional frequency-based fee for each loan to a borrower that had used the facility more than 45 times. The TSLF, also introduced in March 2008, made one-month loans of Treasury securities to primary dealers, collateralised by other securities, through single price auctions. In mid September 2008 the Fed widened its acceptable collateral for the PDCF – from investment-grade securities to those typically accepted in private repo markets – and the TSLF – from certain types of AAA securities to any investment-grade debt instruments. From mid September to early October TSLF loans outstanding rose from 135 billion to 275 billion USD, and overnight lending under the PDCF rose from zero to 155 billion USD.

US authorities also injected substantial liquidity using unsecured-debt subsidisation and bank-equity purchases. On 7 October 2008 the Fed introduced the commercial paper funding facility (CPFF) to purchase newly issued commercial paper of three-month maturity from a wide variety of companies, with a substantial proportion from the banking sector. It purchased unsecured commercial paper, essentially making unsecured loans, charging the overnight index swap rate (OIS) plus 100 basis points, and asset-backed commercial paper (ABCP), charging OIS plus 300 basis point. By the end of October the Fed owned 157 billion USD of unsecured commercial paper and 94 billion of ABCP. On 14 October 2008 the Federal Deposit Insurance Corporation (FDIC) implemented the temporary liquidity guarantee program (TLGP), which guaranteed without limit banks’ newly issued unsecured debt, charging a 75 basis point fee for any loan that applied the guarantee. TLGP-guaranteed debt outstanding reached 224 billion by the end of 2008, later peaking at 336 billion USD.

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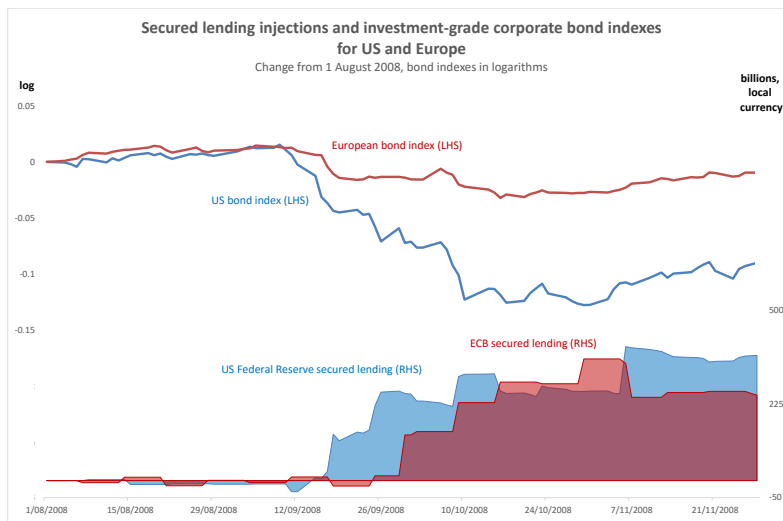
<sup>14</sup>These figures are from Attinasi (2010) and Panetta et al. (2009).

<sup>15</sup>Some facilities not discussed are: the term asset-backed securities loan facility (TALF) and the large scale asset purchases (LSAP), aimed at stimulating lending to borrowers outside the financial system; and the money market investor funding facility (MMIFF), targeted at liquidity problems in the money market fund sector.

Bank equity purchases in the US were made under the Troubled Assets Relief Program (TARP). The largest part of TARP was the Capital Purchase Program (CPP), through which on 28 October 2008 the US Treasury purchased 115 billion USD of equity and warrants from eight of the largest US banks, and by February 2009 had disbursed a total of 194 billion USD to 317 different financial entities. Additional TARP funds were spent on institution-specific purchases, providing AIG 40 billion on 10 November 2008 – which it used to partially repay a senior unsecured loan of 85 billion USD from the Fed made on 15 September – and Bank of America 20 billion on 16 January. Treasury made positive returns on the CPP and both institution-specific purchases, which with hindsight suggests that for the troubled entities illiquidity was a larger concern than solvency.

Some of the policy decisions were most likely driven by political or legal constraints; for instance, the Federal Reserve’s unsecured loan to AIG was made around three weeks before TARP permitted the US Treasury to purchase bank capital. Nevertheless, a key contrast between the US and European policies is the difference in reliance on secured lending relative to subsidising unsecured lending. In the US, debt guarantees were implemented only after the largest increases in secured lending, and were significantly smaller than the secured lending injections, whereas in Europe the two types of policies were utilised around the same time and debt guarantees were noticeably larger than secured lending injections. Figure 3 shows investment-grade corporate bond prices and increases in secured loans outstanding since 1 September for both regions. Although there were other important influencing factors, notably the large coordinated interest rate cut by several major central banks on 8 October, the data are consistent with securities prices ending their decline once secured lending reached a certain level, and with US authorities dealing with more severe fire selling than European authorities.

**Figure 3: Secured lending injections and investment-grade corporate bond indexes for US and Europe**



Secured lending in the US refers to lending under the TAF, PDCF and TSLF and in Europe to MRO and LTRO lending. The corporate bond indexes are the S&P US issued and Eurozone investment grade corporate bond indexes.

Both in the US and Europe, some of the lending policies were priced through auctions and others with fixed prices or fees. Single price auctions are likely to result in prices close to the marginal bidder’s outside option, being the cost of sourcing liquidity in private markets, consistent with the results in section 2.1. When ECB switched to fixed rate lending, the fixed rate remained above the market price of liquidity as measured by the EONIA, possibly indicating that the rates remained penalising. However, the dysfunctioning interbank markets could have made the EONIA an unreliable measure of market conditions, and moreover, the fact

that EONIA subsequently fell to the deposit rate around mid November, attributed by Trichet (2010) to the liquidity injection policies, may suggest that banks were not minimising their borrowing. In the US, the TAF and TSLF used auction-determined prices, although the PDCF, charging the Fed's policy rate, was likely too cheap to impose penalties. The prices charged on debt guarantees in Europe, based on market rates in normal times, and in the US, being 0.75 per cent compared to a TED spread of 4.6 per cent just prior to the policy introduction, were likely also too low to impose penalties.

Authorities in both regions combined unsecured-debt guarantees with bank-equity purchases. Section 2.3 argues that for a sufficiently profitable but illiquid bank, a bank-equity purchase provides a stronger punishment for illiquidity than an unsecured lending policy. For tractability, the model assumes that the funding liquidity withdrawal is exogenous, although in reality banks' funding liquidity is likely to depend on creditors' perceptions about the banks' long-term solvency, as argued by Rochet and Vives (2004) and others. Combining these two arguments suggests that if creditors' perceptions of bank solvency are more pessimistic than the authority's, bank-equity purchases might more effectively alleviate liquidity distress while also generating better incentives for future liquidity-risk management. For both regions, this may explain the authorities' clear preference for bank-equity purchases over direct unsecured lending. Unsecured lending subsidies through government guarantees, however, were also widely adopted, particularly in Europe. Since debt guarantees protect creditors that provide funding liquidity, the argument that equity injections may better protect funding liquidity is less applicable than to direct unsecured lending. Further, it may be the case that debt guarantees are favoured over direct unsecured lending because the latter do not entail government balance sheet expansion, making the risk taken by the authority less visible to the public. As discussed in section 2.2, if government losses are a bigger concern than balance-sheet expansion itself, this would have been a suboptimal response.

## 5 Conclusion

This paper's main argument is that if an authority needs to inject liquidity into the banking system to avoid bank failures, secured lending has benefits over unsecured lending and other policies aside from the risk protection afforded to the authority. Banks' liquidity-risk decisions are modelled as a portfolio choice between risk-free zero-return cash, and securities that have positive expected return but carry liquidity risk. The liquidity risk crystallises if a creditor withdrawal subsequently occurs that forces banks to sell securities, because the securities market price is assumed decreasing in the aggregate value of selling. If the price falls too low, banks cannot meet their creditor obligations and fail unless the authority injects liquidity, through unsecured lending, secured lending, bank-equity purchases or securities purchases.

A secured lending policy can charge high enough interest rates to deter unnecessary borrowing while also alleviating fire selling in securities markets, because banks are forced to post their securities as collateral rather than selling them. The higher securities-market prices counteracts the costs imposed on banks of the high interest rates, but without reducing the effectiveness of high interest rates as a deterrent to ex-ante liquidity-risk taking, because higher securities prices more strongly benefit banks that have lower borrowing and collateral requirements, which are more free to participate in securities markets. Indeed, high interest rates are a more effective deterrent to risk taking under a secured lending than unsecured lending policy, because the inability to sell securities posted as collateral forces banks to borrow more than they otherwise would.

Bank-equity purchases are shown to be less punishing for low-equity banks than high-equity banks, which gives low-equity banks higher incentives to take ex-ante liquidity risk, but also implies that the post-crisis repayments are less likely to make them insolvent. A securities purchase policy does not require banks to make post-crisis repayments, and must be done on terms favourable enough to provide them the liquidity they need during the crisis. This generates strong incentives for liquidity risk taking. The authority could potentially induce a low-risk equilibria by ex-ante promising minimal securities-market intervention, but such a promise lacks credibility because if banks take higher risk, the policy must accommodate them in order to for the liquidity injection to work.

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# A Appendix

Proofs are available by emailing me at [nick.j.garvin@gmail.com](mailto:nick.j.garvin@gmail.com).