Monetary policy along the yield curve: Why can central banks affect long-term real rates?*

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Abstract

Long-term real interest rates are typically seen as pinned down by real forces, with monetary policy having only short-lived effects – just *following* real trends. This paper argues instead that systematic monetary policy can emerge as an (unconscious) driver of low-frequency dynamics in real rates. We first show that temporary demand shocks seem to generate highly persistent movements in forward real rates and estimates of r*. We then demonstrate how such "real rate hysteresis" can arise whenever the central bank overestimates the sensitivity of aggregate demand to permanent real-rate deviations. We show such overestimation can happen when the central bank does not sufficiently recognize the life-cycle drivers of consumption-savings decisions (which can cause asset valuation effects of persistent rate changes to be offset by changes in asset demand). Monetary policy may then end up shaping perceived and actual long-run real rates more than standard models imply.

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1 Introduction

Movements in long-term real interest rates continue to receive considerable attention. Not only as they are thought to affect aggregate outcomes, but also because they influence the distribution of wealth through their impact on asset valuations. The typical explanations for movements in real rates are *real* and *slow moving* in nature, such as productivity growth, demographics, income inequality, and/or changes in the demand and supply of safe assets. One factor that is generally downplayed in explaining real rates over the long term, is monetary policy (which is seen as a mere *follower* of real trends). This reflects the view that real outcomes ought to be invariant to monetary policy beyond horizons long enough to allow prices to be reset.

From this perspective, it is puzzling that long-term real rates (including their "forward" versions) have been found to be rather sensitive to monetary policy shocks.[1] Building on this observation, we first shift focus towards examining how the Federal Reserve's *systematic response* to temporary demand shock affects real interest rates in the longer term. We find that, when the Fed acts to try and offset a temporary demand shock (identified from a sentiment index), long-term real rates move in the same direction; the same holds with respect to a popular estimate for the natural rate of interest, r^*. The observed movements are highly persistent, outlasting the shock's effects on activity and inflation by several years. In light of this evidence, the main part of this paper explores forces that may explain such "real rate hysteresis".

Our starting point is a standard New Keynesian setup from where we look for minimal departures which could produce such a pattern. Our analysis points to one key parameter: the elasticity of aggregate demand with respect to permanent changes in real interest rates. We first show that if the central bank (and households) overestimate this elasticity, this can cause real rates to stay away from r^* for long after the direct effects of a shock have dissipated – *without* large effects on activity and inflation. Such lack of feedback makes it hard to detect the error, giving the misperception the ability to last over time.

In a next step, we examine what could cause agents to have such a misperception. Here, we point to the fact that models used for monetary policy are often focused on short-run issues and therefore may not be sufficiently rich to properly gauge this key elasticity. In particular, we show how the addition of life-cycle forces to a New Keynesian setup can substantially reduce the potency of highly persistent deviations from r^*. If this

[1] We discuss the literature below but see, e.g., Cochrane and Piazzesi (2002) who also nicely voice the standard view (p.91): "Target changes seem to be accompanied by large changes in long-term interest rates (...) Can the Fed really raise the short rate 1 percent for five years or more, without leading to 1 percent lower inflation that would cancel any effect on longer yields?".

is not sufficiently recognized by agents in the model, it can create the type of real rate hysteresis we observe.

Since there are no generally agreed upon empirical estimates for the elasticity of excess demand to permanent changes in real rates, the perception of this elasticity is primarily shaped by models. There, the link between excess demand and real interest rates is typically expressed as:

$$\hat{y}_t = -\mathbb{E}_t \sum_{j=0}^{\infty} \psi_j (r_{t+1+j} - r^*),$$

where $\hat{y}_t$ is the percent deviation in output from its natural level and $\mathbb{E}_t(r_{t+1+j} - r^*)$ captures expected deviations in real interest rates from the natural rate. Such a representation is consistent with – but more general than – a standard log-linearized New Keynesian model. Now suppose the real rate deviates from r^* in a persistent fashion via $(r_t - r^*) = \rho(r_{t-1} - r^*) + \epsilon_t$. Then, the impact effect "$\Psi(\rho)$" on excess demand of a unit interest rate shock equals the persistence-weighted sum of horizon j-specific effects ψ_j, i.e., $\Psi(\rho) = \sum_{j=0}^{\infty} \psi_j \rho^j$. While the literature offers many estimates of $\Psi(\rho)$ for low values of ρ, knowing how $\Psi(\rho)$ behaves as ρ approaches 1 is what is relevant for our purposes. If $\Psi(1)$ is large, persistent deviations from r^* create substantial excess demand and this should be easily noticed. But if $\Psi(1)$ is close to zero, keeping r persistently away from r^* would be difficult to detect as it would not substantially affect activity and inflation.

In most infinitely-lived agent models, the potency of interest rates rises with the shock's persistence ρ due to the compounded power of intertemporal substitution.[2] Hence, it is often believed that $\Psi(1)$ is large. But, when thinking about the impact of very persistent rate changes, forces other than intertemporal substitution are likely important. Persistent rate changes affect working households' desire to accumulate wealth, whilst also changing consumption possibilities of retirees. These life-cycle forces are generally absent from New Keynesian models because they are predominantly used for short-term analyses, where ρ is assumed low. But since $\Psi(1)$ determines what happens when real rates persistently deviate from r^*, it is important to incorporate these lower frequency forces if ones wants to explore why and when real rate hysteresis might arise.

To understand the forces behind $\Psi(\rho)$ when ρ is close to 1, this paper develops a Finitely-Lived Agent New Keynesian (FLANK) model. Such a model yields a rich but concise description of the relation between the future rate path and activity. A key

[2] For the baseline New Keynesian model, $\Psi(1) = \infty$. This has raised issues like the Forward Guidance puzzle, initiating work on the discounted Euler equation (Del Negro et al., 2013; McKay et al., 2016; Gabaix, 2020). But even with a discounted Euler equation, the potency of monetary policy rises with persistence: $\Psi'(\rho) > 0$, meaning that $\Psi(1)$ is still quite large.

insight is that the impact of highly persistent deviations from r^* can be summarized by two effects. First, there is a valuation effect on assets with positive duration, working in the conventional direction (higher rates lowering demand). Second, there is an effect on the marginal propensity to consume (MPC) out of financial wealth. This tends to work in the *unconventional* direction – leaving a net total effect which implies that persistent rate changes might not affect excess demand much (or even with the unconventional sign).

To understand why $\Psi(1)$ may be small, consider a retired household, or one saving to retire in the future. It is not clear they should consume more in response to persistently lower rates, even if that has created capital gains (Auclert, 2019; Moll, 2020; Fagereng et al., 2021; Greenwald et al., 2023): the typical household starts out being "short duration" by having a prospective labor income stream that is of shorter duration than their prospective consumption stream (due to the presence of a retirement phase). As a result, when rates fall persistently, households may see the present discounted value of their liabilities go up by more than that of their assets – making them want to hold *more* assets, to compensate for each unit now yielding less. The existence of such an "interest income effect" implies that the aggregate MPC out of financial wealth might *decrease* when rates fall in a persistent way.[3] This works in the unconventional direction, with lower rates *dampening* demand. Since the asset valuation effect operates in the conventional direction, the competing forces could roughly offset each other.

The above discussion is illustrative. Our model enables us to quantitatively analyze under what conditions $\Psi(1)$ may be small. This will be shown to depend on several factors, including the expected duration of working and retirement phases, and average asset duration. But a key parameter is the elasticity of intertemporal substitution (EIS). For $EIS \geq 1$, FLANK behaves much like standard infinitely-lived agent models, with the potency of interest rates rising with persistence and $\Psi(1)$ being quite large. Real rates then cannot depart from r^* for long without creating strong inflation or deflation. In contrast, for $EIS < 1$ (a case with strong empirical support; Yogo, 2004, Best et al., 2020; Ring, 2024) the MPC out of wealth becomes *increasing* in the real rate, thus countering valuation effects. Very persistent rate changes may then have only small effects

[3] This aligns with Ring (2024), who empirically finds that wealth taxation (lowering the rate of return) *raises* savings; some studies report the opposite (Jakobsen et al., 2020) but, as argued in Brülhart et al. (2022), this may be due to tax evasion/avoidance. That income effects may dominate substitution also aligns with the observation that retirees do not dissave much (De Nardi et al., 2016; Fella et al., 2024; Auclert et al., 2024), mainly consuming the return on their savings (Daniel et al., 2021; Crawley, 2025). Rajan (2013) already worried that persistently low rates post-GFC might not be expansionary because "savers put more money aside as rates fall in order to meet the savings they think they will need when they retire". Nabar (2011), Aizenman et al. (2019), Van den End et al. (2020), and Ahmed et al. (2024) find support for this in aggregate data.

on activity and inflation. These variables, which shape monetary policy by their presence in typical central bank mandates, then become very "forgiving" towards a central bank working with a wrong view of r^*. Instead, *the central bank's perception of* r^* can emerge as an important driver of real rates over time, allowing for real rate hysteresis to emerge.

Related literature. Our paper relates to work analyzing the impact of monetary policy shocks on long-term real rates. Cochrane and Piazzesi (2002), Piazzesi (2005) document such sensitivity in U.S. data, with Skinner and Zettelmeyer (1995) reporting similar evidence for Germany, France, and the U.K. Hillenbrand (2025) notes that nearly all of the post-1980 decline in long-term U.S. rates occurred in a narrow window around FOMC meetings. Nakamura and Steinsson (2018), Hansen, McMahon and Tong (2019), and Hillenbrand (2025) explain these findings via a central bank information effect, while Rungcharoenkitkul and Winkler (2023) allow for two-sided learning (with markets not just learning from the central bank, but the reverse occurring as well). Hanson and Stein (2015) allude to the impact of monetary policy on the term premium, Bianchi et al. (2022) and Pflueger and Rinaldi (2022) focus on the impact on the equity premium, while Beaudry et al. (2024) develop a model featuring r^*-multiplicity. We do not wish to deny that these factors might play a role, but propose a novel mechanism that has different implications. Our explanation aligns well with empirical results in Rigon (2022) and Hofmann et al. (2025), who find that Hillenbrand's (2025) finding mostly runs through changes in expected (real) short rates – not through information effects or term premiums.

We also build on papers that have enriched the New Keynesian model with additional, asset-price driven transmission mechanisms (Caballero and Simsek, 2024; Caballero et al., 2025) and agent heterogeneity, such as the "TANK/HANK" literature (Galí et al., 2007; Bilbiie, 2008; Oh and Reis, 2012; Gornemann et al., 2014; Werning, 2015; Guerrieri and Lorenzoni, 2017; Ravn and Sterk, 2017; Den Haan et al., 2018; Kaplan et al., 2018; Debortoli and Galí, 2024; Auclert et al., 2025).[4] We also build on Auclert (2019), who analyzes the impact of transitory rate changes – showing how unhedged interest rate exposure, distinguishing solely between net assets that pay "today" versus "in the future", is sufficient for the first-order response of consumption to shocks. For persistent rate changes, the exact timing of cash flows matters. In light of this, Greenwald et al. (2023) develop a life-cycle model to understand how the observed, persistent decline in real

[4]Given this literature's focus on idiosyncratic income risk, consideration of retirement risk seems a natural complement as this can be seen as the (high) risk of becoming "long-term unemployed" late in life, potentially due to an adverse health shock. Borella et al. (2025) find that some 80% of U.S. wealth holdings are driven by retirement, health care, and bequest motives; wage risk accounts for about 10%.

rates has affected wealth inequality, documenting how lower rates contract consumption possibilities for “the young” who do not yet hold many financial assets with positive duration.

Our work furthermore links to papers asking whether lower rates are always expansionary. Bilbiie (2008) features “inverted aggregate demand logic” due to limited asset market participation. In Mian et al. (2021) monetary stimulus promotes debt accumulation, giving a short-term boost at the expense of longer-run drag. In Abadi et al. (2023), Eggertsson et al. (2024), and Cavallino and Sandri (2023) rate cuts can be contractionary due to an adverse impact on the banking sector or capital flows.

Gertler’s (1999) OLG structure, which we deploy, has also been used to analyze issues related to monetary policy by, among others, Sterk and Tenreyro (2016) and Galí (2021). Sterk and Tenreyro focus on a redistribution channel of monetary policy when prices are fully flexible, while Galí’s work analyzes the conduct of monetary policy under bubble-driven fluctuations. Fujiwara and Teranishi (2008) examine the impact of demographics on r^* alongside the distributional impact monetary policy may have on workers versus retirees. Bielecki et al. (2022) offer a more general OLG model to analyze the heterogeneous impact monetary policy can have across generations; Eggertsson et al. (2019) and Rachel and Summers (2019) use OLG models to formalize thinking about “secular stagnation”. Our paper, in contrast, focuses on the impact that a retirement savings motive has on the monetary transmission mechanism and the resulting powers of central banks over long-run outcomes and interest rates.[5]

Outline. After documenting the persistent imprint left by the systematic part of monetary policy on long-term real rates in Section 2, Section 3 discusses how this pattern could result from a central bank overestimating the potency of highly persistent deviations from r^*. Section 4 tractably extends the New Keynesian model with life-cycle forces and uses the model to examine the effects of having real rates deviate from r^* for long periods. Our main finding is that the key elasticity may be much lower than conventionally thought (possibly even close to zero). Section 5 combines elements of Sections 3 and 4 to show how they jointly can explain the apparent “long arm of monetary policy”. Section 6 concludes.

[5]Del Negro et al. (2013) bring finite lives to a New Keynesian setup via the Blanchard-Yaari route, abstracting from retirement. To mitigate the Forward Guidance puzzle, they need high mortality risk – yielding an expected lifetime of about 11 years (but, as they point out, OLG models can be seen as proxying models of agents hitting liquidity constraints). Thanks to the presence of retirees (whose behavior is quite different to that of workers) our model can be calibrated to data on expected working/retired lives *and still* solve various puzzles.

2 The long arm of monetary policy

As noted in our Introduction, there is ample evidence that monetary policy *shocks* affect long-term real rates. One explanation is a central bank information effect, with markets updating their beliefs about the economy in response to monetary surprises (Nakamura and Steinsson, 2018; Hillenbrand, 2025). Here, we however present evidence that the "monetary imprint" on long-term rates extends to the *endogenous* response of central banks to other shocks (with this systematic part being the main driver of monetary policy in practice). This suggests there are forces at play *beyond* the information effect. Moreover, we document that the resulting imprint is highly persistent – still visible several years out.

As monetary policy is thought to be most reactive to demand shocks,[6] we start by constructing demand-side "sentiment shocks" from the U.S. sentiment index of Shapiro et al. (2020). We estimate a five-variable VAR for the U.S. featuring (i) real GDP growth, (ii) core PCE inflation, (iii) the Federal funds rate, (iv) the sentiment index, and (v) the 5-year/5-year (5y5y) forward real rate.[7] Data are monthly (for the sentiment index we take a monthly average of daily data) and the baseline sample spans 1993m1 (when the monthly real GDP growth series starts) to 2019m12 (to exclude Covid dynamics). Our baseline estimation has 12 lags. Applying a Cholesky decomposition to a VAR with this ordering orthogonalizes sentiment with respect to current and lagged values of growth, inflation, and the Federal funds rate, lagged values of sentiment itself and the 5y5y forward real rate, whilst allowing this forward rate to respond to a sentiment shock on impact. (But very similar results obtain when ordering the sentiment index first, or last.)

Figure 1 shows the associated Impulse Response Functions (IRFs) to a 1 standard deviation, positive shock to sentiment. They look like responses to a classic temporary demand shock. Inflation and growth first rise. The Fed responds by tightening (by some 25 basis points at its peak) with a degree of persistence which exceeds that of other variables. Subsequently, sentiment, inflation, and growth all return to trend at the 2-3 year horizon.

[6] Central banks often "look through" supply shocks, which limits the possibility of real rate hysteresis arising. However, very big supply shocks – which do induce rate changes, like that associated with Covid – might lead to a similar form of hysteresis. We leave the exploration of this possibility for future work.

[7] We use the growth rate in S&P Global's monthly real GDP index (available at www.spglobal.com/market-intelligence/en/solutions/products/us-monthly-gdp-index) but results are robust to analysis at the quarterly frequency, where we can use the official BEA series. From FRED, we use PCEPILFE (to calculate core PCE inflation), DFF (the effective Federal funds rate), THREEFY5 (the 5-year Treasury yield), and THREEFY10 (the 10-year Treasury yield); from these latter two, one can calculate the 5y5y forward rate (where we use inflation expectations from the Cleveland Fed to calculate ex-ante real rates).

More striking, however, is the response of the 5y5y forward real rate (plotted in the same panel as the Federal funds rate to aid comparison). Since this is the current expectation for the 5-year real Treasury yield in 5 years' time, standard theory suggests that it should be tightly linked to r^* and insulated from the Fed's stabilization efforts (which shouldn't affect outcomes at such horizons). But this is not what Figure 1 shows. Instead, the 5y5y forward rate shows a significant response – moving in the same direction as Fed policy. It still stands at a significantly higher level after 5 years. Quantitatively, it is interesting how the Federal funds rate and the 5y5y forward rate end up in a similar place (but note that the former IRF is surrounded by greater uncertainty). All in response to a sentiment shock, 5 years prior, which otherwise appears to trigger purely transitory effects and which should not have any direct causal impact on long rates. (Importantly, the shock has no long-run effect on growth – which works against the hypothesis of r^* having risen due to a greater desire to borrow against higher future income.) This points to a degree of hysteresis in interest rates that is difficult to square with conventional logic.

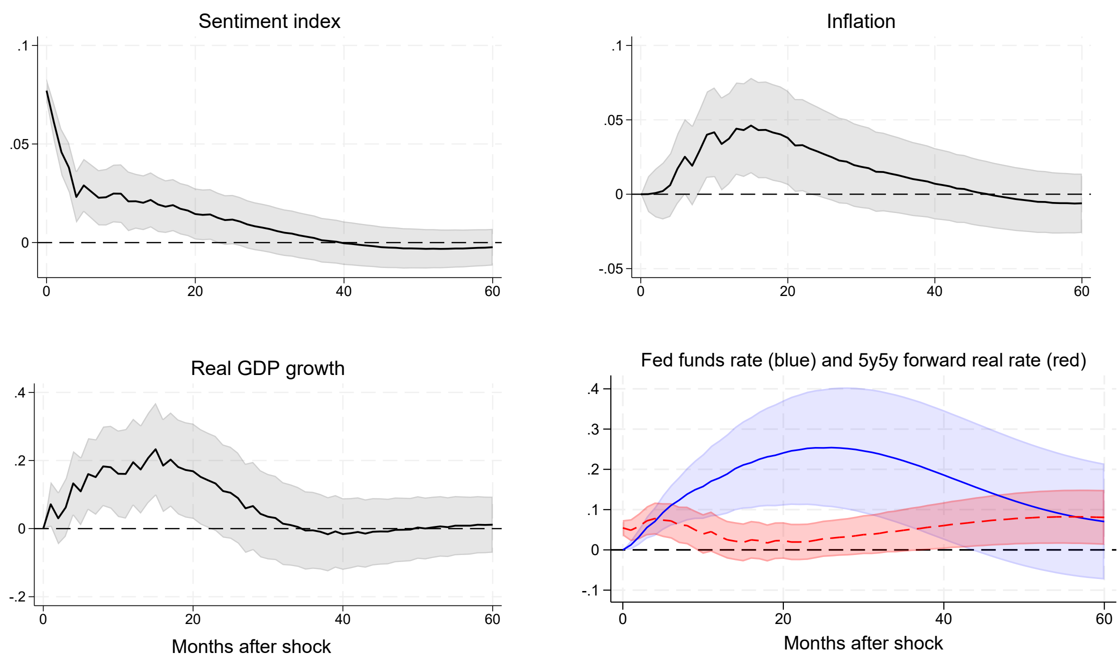

Figure 1: Baseline IRFs to a 1 standard deviation sentiment shock. Shaded areas represent 95% confidence bands.

In Appendix A we document similar results when first regressing the sentiment index on other variables – in particular unemployment and the VIX (to ensure our sentiment shocks do not capture fluctuations in uncertainty, which are distinct). There, we take a two-step approach – first orthogonalizing the sentiment index with respect to the unemployment rate and VIX, followed by using the residuals as an internal instrument (Plagborg-Møller and Wolf, 2021) in a VAR containing the same variables as Figure 1. Near-identical results follow from using the residuals as an external instrument in a proxy-SVAR (Stock and Watson, 2012; Mertens and Ravn, 2013).

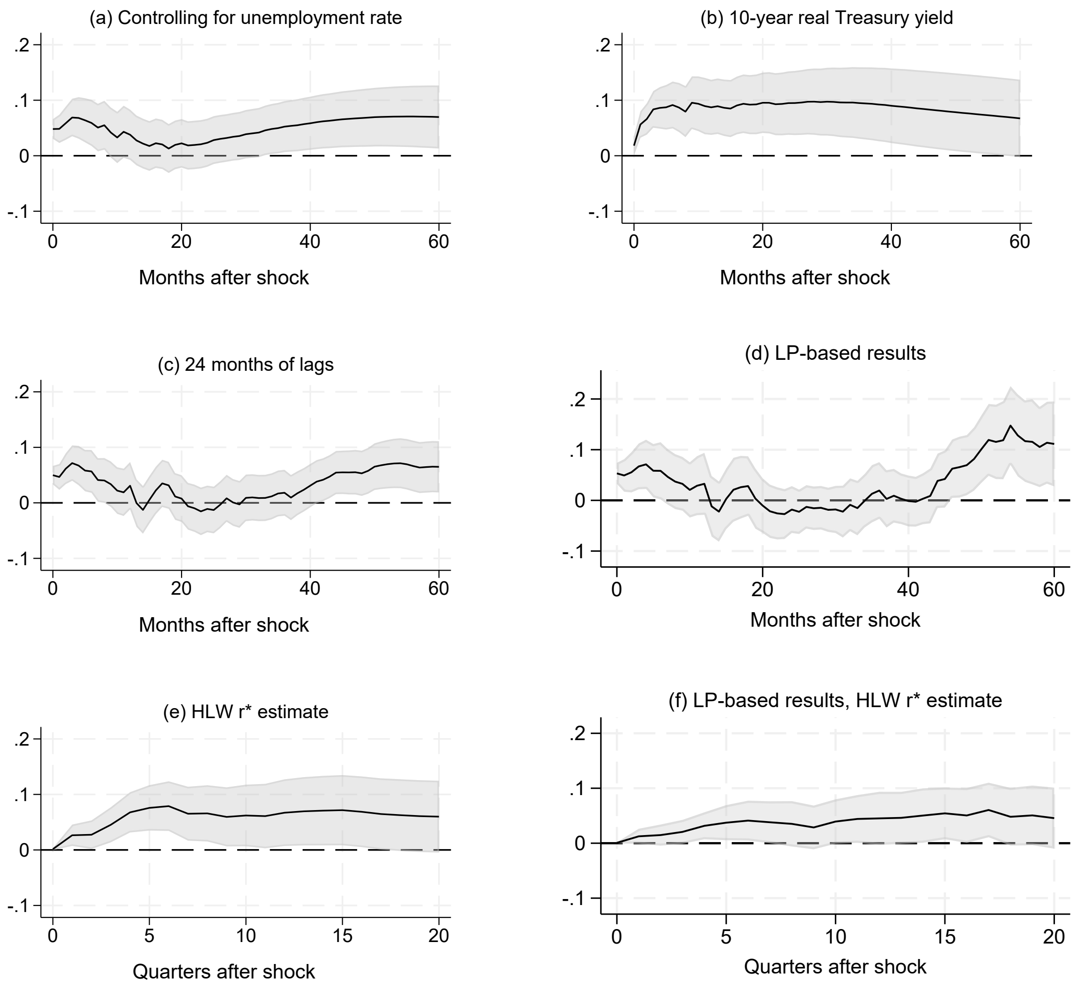

Figure 2: Robustness checks. Responses of the 5y5y forward real rate, unless mentioned otherwise (and see the text for further descriptions). Shaded areas represent 95% confidence bands.

Figure 2 shows further robustness. Since responses of sentiment, the Federal funds rate, inflation, and activity are all much like those in Figure 1, we focus on long-term rates. Panel (a) shows a similar headline result when using the unemployment rate (FRED code: UNRATE) as real activity indicator (instead of growth). Results are also robust to using the 10-year real Treasury yield (FRED code: REAINTRATREARAT10Y); see panel (b). Panel (c) replicates our main result (back to the 5y5y forward real rate) when doubling the VAR's lag length to 24 months; panel (d) documents robustness to estimation via Local Projections (using 12 months of lags). Panels (e) and (f) show that the r^*estimate of Holston, Laubach, and Williams (2017, "HLW") behaves similarly in response to the shock. This r^* estimate is available at the quarterly frequency since 1961, so we can exploit a longer sample (1980q1-2019q4, where the starting date is determined by the availability of the sentiment index). Panel (e) displays the result for our baseline VAR, while panel (f) uses Local Projections; in both cases, estimation takes place at the quarterly frequency and includes 8 lags.

In the remainder of this paper we develop a model in which short-run stabilization efforts by the central bank can leave an imprint on long-term real rates. This ultimately stems from the existence of a "persistence-potency trade-off" (arising from a need to save for retirement in our framework) which is overlooked by agents in the model (as they are guided by standard business cycle models, such as RANK or HANK, which abstract from life-cycle forces).

3 What can cause real rate hysteresis?

The previous section documented that temporary demand shocks appear to have rather persistent effects on interest rates – noticeable long after the shock's effects on growth and inflation have dissipated. While there may be several explanations, this section examines conditions under which this can result from a feedback between (i) how interest rates affect the economy and (ii) how the central bank views and sets rates.

A main challenge for central banks is disentangling rather temporary shocks, e.g. to demand, from shocks to the "natural", long-run equilibrium rate (r^*), which tend to be more persistent. This begs the question: is there a risk that a central bank misdiagnosing the situation (erroneously interpreting a temporary shock as a shock to r^*) ends up keeping rates away from their natural level for long?

To clarify when this may occur, consider a stylized setup featuring only temporary demand shocks (e.g., shocks to the household's discount factor), but the central bank thinks there may be shocks to both demand and to r^*. The central bank furthermore believes that r^* follows a random walk; demand shocks are known to follow an autoregressive process: $\varepsilon_t^d = \rho_d \varepsilon_{t-1}^d + \epsilon_t^d$, with $0 < \rho_d < 1$. The output gap is determined via:

$$\hat{y}_t = -\mathbb{E}_t \sum_{j=0}^{\infty} \psi_j (r_{t+1+j} - r^*) + \mathbb{E}_t \sum_{j=0}^{\infty} \omega_j \varepsilon_{t+j}^d, \tag{1}$$

which is a rather general formulation, for example nesting the popular "discounted" Euler equation (McKay et al., 2016; 2017).[8] The coefficient sequences $\{\psi_j\}_{j=0}^{\infty}$ and $\{\omega_j\}_{j=0}^{\infty}$ capture the sensitivity of the output gap to, respectively, the expected real rate gap and the expected value of the demand process. Define $\Psi(\rho_d) \equiv \sum_{j=0}^{\infty} \psi_j \rho_d^j$ and $\Omega(\rho_d) \equiv \sum_{j=0}^{\infty} \omega_j \rho_d^j$. Crucially, the central bank doesn't know the true values of these coefficients. Instead, it works with *perceived* sequences, $\{\psi_j^{CB}\}_{j=0}^{\infty}$ and $\{\omega_j^{CB}\}_{j=0}^{\infty}$ (super-scripted "CB"), where at least some parameters ψ_j^{CB} and ω_j^{CB} are not equal to their true values ψ_j and ω_j.

Assuming the central bank aims to close the output gap in the face of demand shocks, it will set its policy rate according to:

$$r_t = r_t^{*,CB} + \frac{\Omega^{CB}(\rho_d)}{\Psi^{CB}(\rho_d)}\varepsilon_t^d, \tag{2}$$

with $r_t^{*,CB}$ being the central bank's time-t perception of r^*. The ratio $\frac{\Omega^{CB}(\rho_d)}{\Psi^{CB}(\rho_d)}$ captures the perceived differential effect of demand versus interest rates on $\hat{y}_t$; in standard models (where a monetary policy shock is just a particular type of demand shock) this ratio equals unity.

Households are assumed to know the central bank's policy rule (2) and its view about r^*, so that household expectations are identical to those of the central bank. Accordingly, and in contrast to models featuring a central bank information effect, there is no asymmetry of information about r^* (or the future path of rates) between the central bank and the public.

If the true natural rate were known perfectly ($r_t^{*,CB} = r^*$, $\forall t$), a central bank implementing (2) would succeed in keeping the output gap at 0. But since r^* is unobservable, and believed to be time-varying, the central bank will try to infer r^* from output gap realizations. Given the central bank's view about the functioning of the economy, its optimal updating rule reads:[9]

$$r_{t+1}^{*,CB} = r_t^{*,CB} + \frac{1}{\Psi^{CB}(1)}\hat{y}_t, \tag{3}$$

[8]Starting from $\hat{y}_t = \alpha \mathbb{E}_t \hat{y}_{t+1} - \frac{1}{\sigma}(r_{t+1} - r^*) + \frac{1}{\sigma}\varepsilon_t^d$, where $\alpha < 1$ captures some discounting and σ is the inverse elasticity of substitution, its forward solution yields (1) with particular values for ψ_j and ω_j.

[9]This assumes that the central bank aims to close the output gap (by applying (2)) and updates its view of r^* based on subsequent output gap realizations. Assuming instead that the central bank targets inflation, and updates its views about r^* based on inflation outturns, has similar implications when inflation obeys a standard New Keynesian Phillips curve. More generally, the Divine Coincidence applies with respect to demand shocks in a New Keynesian setup, meaning that inflation stabilization and output gap stabilization are equivalent.

where $\Psi^{CB}(1) \equiv \sum_{j=0}^{\infty} \psi_j^{CB}$ plays a crucial role in what follows. It captures the perceived effect, on the output gap, of keeping interest rates permanently away from r^*; analogously, $\Psi(1) \equiv \sum_{j=0}^{\infty} \psi_j$ captures the true strength of this effect.

In this setup, output gap realizations feed into policy, which in turn affects the output gap. If the central bank's view of the economy is perfectly in line with how the economy actually functions, this feedback loop works well and the central bank learns the true value of r^* within one period. In this case, a demand shock would not have any effect on a central bank's perception of the natural rate, or its policy stance in the long run.

But this no longer applies when there is some misperception (i.e., when some perceived coefficients ψ_j^{CB} and ω_j^{CB} differ from their true values ψ_j and ω_j). Given the central bank's policy rule (2) and its updating equation (3), the central bank's perception of r^* will now start interacting with a richer determination of the output gap. In particular, the central bank will end up partially misattributing the effects of demand shocks to changes in r^*. This in turn affects the output gap, which will influence anew the central bank's perception of the natural rate, in turn affecting the policy stance via (2). This feedback loop yields the following autoregressive process for the central bank's perception error with respect to r^*:[10]

$$(r_{t+1}^{*,CB} - r^*) = \rho_1 (r_{t+1}^{*,CB} - r^*) + \rho_2 \varepsilon_t^d, \text{ with} \tag{4}$$
$$\rho_1 = 1 - \frac{\Psi(1)}{\Psi^{CB}(1)}; \, \rho_2 = \frac{1}{\Psi^{CB}(1)} \left[\Omega(\rho_d) - \frac{\Omega^{CB}(\rho_d)}{\Psi^{CB}(\rho_d)} \Psi(\rho_d) \right].$$

From the expression for ρ_2 we see that the central bank's view of r^* is only insulated from demand shocks if the actual functioning of the economy is in line with the central bank's perceptions. But if there are some misperceptions, $\rho_2 \neq 0$ and the central bank will start misattributing some of the effects of demand shocks to changes in r^*. For example: following an adverse demand shock, a central bank overestimating the potency of its policy stance might conclude that r^* has fallen, when noting that its stimulus has not produced as big a boom as expected.

The first term in (4) tells us that, when ρ_1 is equal or close to 0, such errors would only last while demand shocks are directly affecting activity. But with ρ_1 close to 1 (i.e., when $\frac{\Psi(1)}{\Psi^{CB}(1)}$ is small), demand shocks continue affecting interest rates for long after the direct effects of the shock have essentially died out.

The key element therefore is how the true $\Psi(1)$ compares to the central bank's per-

[10] This follows from first noting that, given (2), the output gap will obey $\hat{y}_t = -\Psi(1)(r_t^{*,CB} - r^*) + \left[\Omega(\rho_d) - \frac{\Omega^{CB}(\rho_d)}{\Psi^{CB}(\rho_d)} \Psi(\rho_d) \right] \varepsilon_{t+j}^d$. Combining this with (3) yields (4).

ception of this object, $\Psi^{CB}(1)$. When $\frac{\Psi(1)}{\Psi^{CB}(1)}$ is small, the central bank *overestimates* the potency of permanent rate changes and will update its beliefs towards the true r^* only very slowly. This type of slow learning happens for two reasons. First, when $\Psi(1)$ is small, being wrong about r^* will not be very apparent as the economy becomes very "forgiving" towards a central bank setting policy based on a mistaken view of r^*. This gives rise to a muted system in which learning is rather difficult. Second, when $\Psi^{CB}(1)$ is relatively big, an observed output gap only triggers minor updates to the central bank's perception of r^* (because a big $\Psi^{CB}(1)$ means that the central bank believes $\hat{y}_t$ to be very sensitive to mistaken views on r^*); see equation (3). Since $r_t^{*,CB}$ is the intercept to the central bank's policy rule (2), any associated misperceptions end up affecting the policy stance – potentially thus in a rather persistent way. This can give the central bank some lasting (yet unconscious) control over interest rate trends.

While the above model is highly simplified, the resulting intuition is likely more general. In particular, if the central bank does not have an exact knowledge of the economy, it is reasonable to expect that demand shocks might be confused with shocks to r^*. But that by itself would not explain the observations presented in Section 2. The most important insight from this illustrative model is that inference errors regarding r^* can become long-lasting if the actual effect of the misperception, as governed by $\Psi(1)$, is small. Since $\Psi(1)$ represents how the economy reacts to permanent changes in interest rates, this points to the importance of understanding its driving forces. This is what we turn to next, when showing how life-cycle forces – which are neglected by the standard (discounted) Euler equation approach to demand – can lead to $\Psi(1)$ being rather small.

4 A life-cycle model for monetary policy

This section describes our model.[11] As we adopt a common production setup – monopolistically competitive firms facing price adjustment frictions – and combine this with life-cycle consumption-savings decisions, one can refer to this model as "FLANK", for Finitely-Lived Agent New Keynesian model. We model all households as optimizers. This may not seem realistic given the evidence on the presence of "hand-to-mouth" households. But, as we discuss in Section 6, we do not think this modelling choice hinders the model's main insights (even if we agree that optimizers represent only a fraction of the population). Our analysis is done within a closed economy (Cesa-Bianchi et al. (2023), Obstfeld

[11] The real side of the model shares features with the continuous time model in Beaudry et al. (2024) but differs by being stochastic, set in discrete time, allowing for long-term debt, and being embedded in a New Keynesian setup.

(2023), and Auclert et al. (2024) offer open economy considerations) and is therefore best thought of as applying to a rather large economy, like the U.S.

Environment. There is a measure one of households, subject to a life cycle as in Gertler (1999, which – in turn – builds on Yaari (1965) and Blanchard (1985)). Each household starts life in a working state and transits out with Poisson probability δ_1 – either due to being sent to retire, or because of a health shock preventing further work. At this transition, the household enters retirement where it faces Poisson death probability δ_2. Deceased households are immediately replaced by new, working households, implying that the share of workers is constant at $\vartheta = \frac{\delta_2}{\delta_1+\delta_2}$.

Retired Households. A retired household derives income from its financial wealth, reflecting past savings and a possible lump-sum public pension payment. Retirees invest their wealth in a portfolio of short- and long-term bonds. Short-term bonds are one-period assets whose gross nominal return, i_t, is set by the central bank. Their real return is $r_{t+1} \equiv i_t/\pi_{t+1}$, where π denotes the gross inflation rate. We model long-term bonds as real perpetuities with coupons decaying geometrically at rate μ (Woodford, 2001). A bond issued in period t then pays $(1-\mu)^h$ units of consumption $h+1$ periods later; hence, the bond's duration decreases in μ ($\mu = 1$ reduces this bond to a one-period instrument). The gross return on the long-term bond is:

$$r^b_{t+1} = \frac{1 + (1-\mu)\, q_{t+1}}{q_t},$$

where q_t is the long-term bond's price. The optimization problem faced by a retired household j with CRRA-preferences (where σ is the coefficient of relative risk aversion, making $1/\sigma$ the EIS) reads:

$$V^r_t\left(\tilde{a}^j_t\right) = \max_{c^j_t, \alpha^j_t, \tilde{a}^j_{t+1}} \left\{ \frac{\left(c^j_t\right)^{1-\sigma}}{1-\sigma} + (1-\delta_2)\,\beta_t \mathbb{E}_t\left[V^r_{t+1}\left(\tilde{a}^j_{t+1}\right)\right] \right\},$$

$$s.t.\ \tilde{a}^j_{t+1} = r^j_{t+1}\left(\tilde{a}^j_t - c^j_t\right), \tag{5}$$

$$r^j_{t+1} = r_{t+1} + \left(r^b_{t+1} - r_{t+1}\right)\alpha^j_t \tag{6}$$

where c^j_t is consumption, $\alpha^j_t \equiv \left(q_t b^j_t\right)/a^j_t$ is the share of wealth invested in long-term bonds "b", and $\tilde{a}^j_t \equiv r^j_t a^j_{t-1}$ is the beginning-of-period t stock of wealth held by household j, such that the real rate of return r^j_t works on whatever is left after period-$(t-1)$ consumption has been financed, i.e., on $a^j_{t-1} = \tilde{a}^j_{t-1} - c^j_{t-1}$. Finally, $\beta_t \equiv \beta e^{\varepsilon^\beta_t}$, where ε^β_t is a demand shifter. Optimal consumption satisfies:

$$\left(c_t^j\right)^{-\sigma} = (1-\delta_2)\,\beta_t\mathbb{E}_t\left[\frac{dV^r\left(\tilde{a}_{t+1}^j\right)}{d\tilde{a}_{t+1}^j} r_{t+1}^j\right], \tag{7}$$

with the portfolio optimality condition:

$$0 = \mathbb{E}_t\left[\frac{dV^r\left(\tilde{a}_{t+1}^j\right)}{d\tilde{a}_{t+1}^j}\left(r_{t+1}^b - r_{t+1}\right)\right]. \tag{8}$$

By the envelope theorem, $\frac{dV_t^r\left(\tilde{a}_t^j\right)}{d\tilde{a}_t^j} = \left(c_t^j\right)^{-\sigma}$, so that (8) boils down to:

$$0 = \mathbb{E}_t\left[\left(c_{t+1}^j\right)^{-\sigma}\left(r_{t+1}^b - r_{t+1}\right)\right].$$

Guessing that $V_t^r\left(\tilde{a}_t^j\right) \equiv \frac{\left(\tilde{a}_t^j\right)^{1-\sigma}}{1-\sigma}\left(\Gamma_t^j\right)^{-\sigma}$, with Γ_t^j conjectured to be a function of the future path of r_t^j and independent of $\tilde{a}_t^j$, this gives:

$$\frac{dV_t^r\left(\tilde{a}_t^j\right)}{d\tilde{a}_t^j} = \left(\tilde{a}_t^j\Gamma_t^j\right)^{-\sigma}. \tag{9}$$

By using the envelope theorem in (9) we obtain:

$$\left(c_t^j\right)^{-\sigma} = \left(\tilde{a}_t^j\Gamma_t^j\right)^{-\sigma} \Leftrightarrow c_t^j = \tilde{a}_t^j\Gamma_t^j, \tag{10}$$

which we can plug into (5) to yield:

$$\tilde{a}_{t+1}^j = r_{t+1}^j\tilde{a}_t^j\left(1-\Gamma_t^j\right). \tag{11}$$

Finally, plugging (9), (10), and (11) into (7) gives a non-linear difference equation for Γ_t:

$$\left[\left(\Gamma_t^j\right)^{-1} - 1\right]^{\sigma} = (1-\delta_2)\,\beta_t\mathbb{E}_t\left[r_{t+1}\left(\Gamma_{t+1}^j r_{t+1}^j\right)^{-\sigma}\right]. \tag{12}$$

This verifies our guess that Γ_t^j is independent of $\tilde{a}_t^j$, confirming that it is only a function of expected future rates and demand shocks. Note from (10) that Γ_t^j equals the MPC out of (beginning of period) financial wealth for retirees, which plays an important role to the interpretation of our findings later on.

We can thus write the utility of retirees as $V^r\left(\tilde{a}_t^j, \Gamma_t^j\right) = (1-\sigma)^{-1}\left(\tilde{a}_t^j\right)^{1-\sigma}\left(\Gamma_t^j\right)^{-\sigma}$, where V^r depends both on the stock of assets with which the household enters retirement ($\tilde{a}_t^j$) as well as on the entire future path of interest rates working over that stock (captured via Γ_t). Given the value of assets $\tilde{a}_t^j$, retirees are better off when rates are expected to be high, as this offers them a superior interest income stream.

Let $c_t^r \equiv \int_{\mathbf{R}_{r,t}} c_t^j dj / (1-\vartheta)$ be the consumption of the representative retiree and define $a_t^r \equiv \int_{\mathbf{R}_{r,t}} a_t^j dj / (1-\vartheta)$ as their (end of period) financial wealth, where $\mathbf{R}_{r,t}$ denotes the set of retired households at time t. Since all retired households choose the same asset portfolio, that is $\alpha_t^j = \alpha_t^r$ for all $j \in \mathbf{R}_{r,t}$, this implies $\Gamma_t^j = \Gamma_t$ for all $j \in \mathbf{R}_{r,t}$. Therefore:

$$c_t^r = a_t^r \left[\left(\Gamma_t^j \right)^{-1} - 1 \right]^{-1},$$

where $\left[\left(\Gamma_t^j \right)^{-1} - 1 \right]^{-1}$ reflects the MPC out of (end of period) financial wealth of the representative retiree, with a_t^r evolving as:

$$a_{t+1}^r = \left[(1-\delta_2) a_t^r r_{t+1}^r + \delta_2 \left(a_t^w r_{t+1}^w + \tau_{t+1}^r \right) \right] \left(1 - \Gamma_{t+1}^j \right).$$

where τ^r is the lump-sum transfer received by households upon retirement. It can be seen as a public pension transfer that is paid once to the household upon retiring, and thereafter managed by the household.

Working Households. Next, consider a working household. It receives a real wage w_t for any labor input ℓ_t it provides, plus transfers from good-producing firms and transfers from/to the government. Workers face a δ_1 probability of moving into retirement next period. Their decision problem reads:

$$V_t^w \left(\tilde{a}_t^j \right) = \max_{c_t^j, \ell_t^j, \alpha_t^j, \tilde{a}_{t+1}^j} \left\{ \frac{\left(c_t^j \right)^{1-\sigma}}{1-\sigma} - \frac{\left(\ell_t^j \right)^{1+\varphi}}{1+\varphi} + \beta_t \mathbb{E}_t \left[(1-\delta_1) V_{t+1}^w \left(\tilde{a}_{t+1}^j \right) + \delta_1 V_{t+1}^r \left(\tilde{a}_{t+1}^j + \tau_{t+1}^r \right) \right] \right\},$$
$$s.t.\ \tilde{a}_{t+1}^j = r_{t+1}^j \left(\tilde{a}_t^j - c_t^j + \ell_t^j w_t + z_t^j + \tau_t^w + \tau_t^n \right),$$
$$r_{t+1}^j = r_{t+1} + \left(r_{t+1}^b - r_{t+1} \right) \alpha_t^j$$

where z_t^j represents dividends received from good-producing firms. τ_t^w and τ_t^n both represent tax/transfer schemes. τ_t^w is a tax used by the government to pay expenditures and interest on debt. τ_t^n is tax or transfer scheme which ensures that the inheritance received by newly-born households allows them to resemble existing working households – implying that we can consider a representative working household. The optimality conditions give rise to the following Euler equation:

$$\left(c_t^j\right)^{-\sigma} = \beta_t \left\{ (1-\delta_1) \mathbb{E}_t \left[\left(c_{t+1}^j\right)^{-\sigma} r_{t+1} \right] + \delta_1 \mathbb{E}_t \left[\left(\tilde{a}_{t+1}^j + \tau_{t+1}^n\right)^{-\sigma} \Gamma_{t+1}^{-\sigma} r_{t+1} \right] \right\}, \tag{13}$$

supplemented by the portfolio decision and the labor supply schedule:

$$\begin{aligned} 0 &= \mathbb{E}_t \left[\left\{ (1-\delta_1) \left(c_{t+1}^j\right)^{-\sigma} + \delta_1 \frac{dV_{t+1}^r\left(\tilde{a}_{t+1}^j\right)}{d\tilde{a}_{t+1}^j} \right\} \left(r_{t+1}^b - r_{t+1}\right) \right], \\ w_t &= \left(c_t^j\right)^{\sigma} \left(\ell_t^j\right)^{\varphi}. \end{aligned}$$

Note how the Euler equation for working households (13) features two terms on the RHS: the first term is familiar from models without retirement and implies that a lower interest rate, *ceteris paribus*, decreases the household's desire to save; this is standard intertemporal substitution. The second term stems from the introduction of the prospect of retirement and shows how consumption is driven by wealth ($\tilde{a}_{t+1}^j$) adjusted for the expected path of interest rates (as captured by $\Gamma_{t+1}^{-\sigma} r_{t+1}$).

Since the assets of new and existing working households are equalized via the transfer τ^n, workers can be treated as homogeneous. Let c_t^w denote the consumption of the representative worker and a_t^w its end-of-period financial wealth. Then, c_t^w solves:

$$\left(c_t^w\right)^{-\sigma} = \beta_t \left\{ (1-\delta_1) \mathbb{E}_t \left[\left(c_{t+1}^w\right)^{-\sigma} r_{t+1} \right] + \delta_1 \mathbb{E}_t \left[\left(a_t^w r_{t+1}^w \Gamma_{t+1}\right)^{-\sigma} r_{t+1} \right] \right\},$$

where a_t^w evolves as:

$$a_{t+1}^w = (1-\delta_1) a_t^w r_{t+1}^w + \delta_1 a_t^r r_{t+1}^r - c_{t+1}^w + \ell_{t+1} w_{t+1} + z_{t+1} + \tau_{t+1}^w.$$

Good-producing firms. Each working household $j \in \mathbf{R}_{w,t}$ owns a firm that produces a differentiated good using the technology $y_t^j = A\ell_t^j$. Upon retiring, households liquidate their firms which are replaced by new ones owned by new working households. Firms are monopolistically competitive and set prices subject to a quadratic adjustment cost (Rotemberg, 1982). Let P_t^j be the price chosen by firm j at time t and $\pi_t^j \equiv P_t^j / P_{t-1}^j$ be its growth rate. Then, the firm pays adjustment cost $\Theta\left(\pi_t^j\right) = y_t^j \frac{\theta}{2} \left(\pi_t^j - \bar{\pi}\right)^2$, where $\bar{\pi}$ is the inflation target and θ governs the cost of adjusting prices. The resulting Phillips curve takes the standard form (which, to a first-order approximation, has the same reduced form as under Calvo-pricing; Roberts (1995)):

$$(\pi_t - \bar{\pi})\,\pi_t = \lambda\left(\frac{\epsilon}{\epsilon-1}mc_t - 1\right) + \mathbb{E}_t\left[\Lambda^w_{t,t+1}\,(\pi_{t+1} - \bar{\pi})\,\pi_{t+1}\frac{y_{t+1}}{y_t}\right],$$

where $\lambda \equiv (\epsilon - 1)/\theta$ represents the slope of the Phillips curve and ϵ is the elasticity of substitution between product varieties (with households consuming the CES aggregate), $y_t = \int_{\mathbf{R}_{w,t}} y^j_t dj$ denotes aggregate output, while $\Lambda^w_{t,t+1}$ is the stochastic discount factor of the representative working household:

$$\Lambda^w_{t,t+1} = \beta_t \frac{(1-\delta_1)\left(c^w_{t+1}\right)^{-\sigma} + \delta_1\left(a^w_t r^w_{t+1}\Gamma_{t+1}\right)^{-\sigma}}{\left(c^w_t\right)^{-\sigma}}.$$

This captures that households place more weight on the future when their marginal utility is high, but it features the additional forces stemming from retirement: households now place more weight on the future when they hold fewer assets a^w_t or when the interest rate path is lower (as captured via Γ).

The real marginal cost of production is $mc_t = (1-\tau_t)\,w_t/A$, where τ_t is a wage subsidy financed through lump-sum taxes levied directly on firms. We use this subsidy to undo the steady-state markup and to eliminate the impact of labor supply wealth effects on inflation, such that $mc_t = \frac{\epsilon-1}{\epsilon}\left(\frac{y_t}{\vartheta A}\right)^{1+\varphi}$. Since firms are identical, the real dividend generated by each firm is $z_t = \frac{y_t}{\vartheta}\left[1 - \frac{\theta}{2}(\pi_t - \bar{\pi})^2\right] - \ell_t w_t$.

Government. The government's budget constraint reads:

$$s^g_t + q_t b^g_t = q_{t-1} b^g_{t-1} r^b_t + s^g_{t-1} r_t + \vartheta \tau^w_t + \vartheta \delta_1 \tau^r_t,$$

where s^g_t and b^g_t are the supply of short- and long-term government bonds, respectively. Without loss of generality, we take the limit for $s^g_t \downarrow 0$ and assume $b^g_t = b^g$, for all $t \geq 0$. This implies that tax policy must satisfy $\vartheta\tau^w_t + \vartheta\delta_1\tau^r_t = -b^g(1-\mu q_t)$.

The central bank sets monetary policy according to the following rule:

$$i_t = r^*\bar{\pi}\left(\frac{\mathbb{E}_t[\pi_{t+1}]}{\bar{\pi}}\right)^{1+\phi} e^{\varepsilon^i_t}, \tag{14}$$

where $\phi > 0$ governs the central bank's responsiveness to expected inflation-deviations from target ($\bar{\pi}$), r^* is the steady-state real interest rate, and ε_t is a monetary policy shock.

Market clearing . Market clearing requires:

$$\vartheta c_t^w + (1-\vartheta)\, c_t^r = y_t \left[1 - \frac{\theta}{2}\left(\pi_t - \bar{\pi}\right)^2\right],$$
$$\vartheta a_t^w + (1-\vartheta)\, a_t^r = q_t b^g,$$
$$\vartheta b_t^w + (1-\vartheta)\, b_t^r = b^g,$$

where $b_t^r \equiv \int_{\mathbf{R}_{r,t}} b_t^j dj / (1-\vartheta)$ and $b_t^w \equiv \int_{\mathbf{R}_{w,t}} b_t^j dj / \vartheta$ are the long-term bond holdings of the representative retiree and the representative worker, respectively.

Exogenous Processes. We allow the model to be hit by two types of shocks: first, a standard monetary policy shock "ε_t^i" to the Taylor rule (14) and, second, a demand shock to β, ε_t^β. The exogenous variables ε_t^i and ε_t^β are assumed to follow AR(1) processes:

$$\varepsilon_t^i = \rho_i \varepsilon_{t-1}^i + \sigma_i \epsilon_t^i, \tag{15}$$
$$\varepsilon_t^\beta = \rho_\beta \varepsilon_{t-1}^\beta + \sigma_\beta \epsilon_t^\beta, \tag{16}$$

with the innovations "ϵ^i" and "ϵ^β" following a standard-normal distribution (σ_i and σ_β scale the shocks' standard deviations).

We furthermore assume a zero inflation target ($\bar{\pi} = 1$). The equilibrium and steady-state equations of our full model can be found in Appendix B.

5 Model properties: analytical and quantitative

To highlight how retirement preoccupations affect monetary policy transmission, we simplify our model to derive analytical results that clarify the key mechanisms. Our simplifying assumptions lead to a compact system that can be handled almost as easily as the standard New Keynesian model, while simultaneously capturing forces stemming from life-cycle considerations. We then derive a "term structure representation" of the Euler equation, which shows how interest rates at different horizons affect activity differently. This enables us to discuss when and why our framework implies that $\Psi(1)$ may be close to 0. As discussed in Section 3, this has important implications for how monetary policy may end up driving dynamics in long-term real rates.

5.1 Simplifying the model

To provide a model which can be easily compared to a standard New Keynesian model, we assume that the transfer received by households upon retirement, τ^r, is designed to keep the distribution of financial wealth between workers and retirees constant at its steady-state level.[12] This enables us to obtain analytical solutions, while we shall later show that it is not driving the model's implications – neither qualitatively nor quantitatively. We set the level of government debt, b^g, so that the steady-state real interest rate (r^*) equals $1/\beta$. This ensures that the log-linearized system nests the standard representative agent New Keynesian ("RANK") model for $\delta_1 = 0$ (when every household remains in its working state *ad infinitum*). Finally, for the main propositions, we will focus on the case where $\delta_2 < \mu$, which implies that the expected duration of retirement is greater than the average duration offered by bonds.[13] This means that, in equilibrium, households' saving efforts in the asset with positive duration cannot fully close their negative duration gap (stemming from the need to finance consumption in retirement).[14]

With these simplifications, the log-linearized equilibrium reads:

$$\hat{y}_t = (1-\gamma)\,\hat{c}^w_t + \gamma \hat{c}^r_t \tag{17}$$

$$\hat{c}^r_t = \hat{q}_t + \left[\beta\,(1-\delta_2)^{\frac{1}{\sigma}}\right]^{-1} \hat{\Gamma}_t \tag{18}$$

$$\hat{c}^w_t = (1-\delta_1)\left(\mathbb{E}_t \hat{c}^w_{t+1} - \frac{1}{\sigma}\mathbb{E}_t \hat{r}_{t+1}\right) + \delta_1\left(\hat{q}_t + \left[\beta\,(1-\delta_2)^{\frac{1}{\sigma}}\right]^{-1}\hat{\Gamma}_t\right) - \frac{1-\delta_1}{\sigma}\varepsilon^\beta_t \tag{19}$$

$$\hat{\Gamma}_t = \beta\,(1-\delta_2)^{\frac{1}{\sigma}}\left[\mathbb{E}_t\hat{\Gamma}_{t+1} + \frac{\sigma-1}{\sigma}\mathbb{E}_t\hat{r}_{t+1} - \frac{1}{\sigma}\varepsilon^\beta_t\right] \tag{20}$$

$$\hat{q}_t = \beta\,(1-\mu)\,\mathbb{E}_t\hat{q}_{t+1} - \mathbb{E}_t\hat{r}_{t+1} \tag{21}$$

$$\hat{\pi}_t = \kappa\hat{y}_t + \beta\mathbb{E}_t\hat{\pi}_{t+1} \tag{22}$$

[12] To simplify the algebra, the time-varying nature of the transfer is unexpected, so that workers do not anticipate receiving a transfer that varies with the state of the economy. This assumption is not necessary for our main results, but does make the presentation more transparent.

[13] Our propositions technically only require the weaker condition that $(1-\delta_2)^{1/\sigma} > 1-\mu$ (which is easily satisfied when σ is large), but imposing the stronger condition $\delta_2 < \mu$ eases exposition.

[14] This is clear to pension funds (to whom many have outsourced their retirement savings): they often have negative duration gaps of about 10 years, which forced many to increase premiums over the zero-interest rate era, asking for greater saving efforts from their members. See, e.g., https://macrosynergy.com/research/low-for-long-rates-pressure-on-pensions-and-insurances/. As a concrete example, ABP (the largest Dutch pension fund) issued a statement back in 2019 (www.abp.nl/content/dam/abp/nl/documents/persbericht%20premie-indexatie%202020.pdf) saying "Pensions are becoming increasingly expensive [...] With the current pension ambition and the expectation that interest rates will remain low for a long time, higher premiums will be needed."

with

$$\mathbb{E}_t\hat{r}_{t+1} = \hat{i}_t - \mathbb{E}_t\hat{\pi}_{t+1} - \varrho$$
$$\hat{i}_t = \varrho + (1+\phi)\,\mathbb{E}_t\hat{\pi}_{t+1} + \varepsilon_t^i$$

where $\varrho \equiv \log r^*$, $\kappa \equiv \lambda(1+\varphi)$, and $\gamma \equiv \delta_1/[1+\delta_1-(1-\delta_2)^{\frac{1+\sigma}{\sigma}}]$ is the steady-state consumption share of retirees. Hats denote deviations from steady state (except for $\hat{i}_t$, which denotes the log of i_t).

From (19) one can see how the workers' Euler equation incorporates both the standard force of intertemporal substitution, as captured by the first RHS term, and a second term which captures wealth-related factors associated with retirement. As the probability of retiring (δ_1) goes up, the weight on wealth-related factors increases relative to intertemporal substitution. Greater retirement preoccupations thus imply that wealth-related factors will be more central to consumption decisions and the monetary transmission mechanism.

Note from (18) and (19) that wealth-related factors consist of two parts: an effect via the asset price, $\hat{q}_t$, and an effect stemming from the impact on retirees' MPC out of wealth, $\hat{\Gamma}_t$. Regarding the former, (21) shows that a higher expected rate path depresses the price q of the long-term bond. Via equations (18) and (19) this lowers consumption demand. We call this the "asset valuation channel". It works as a pure financial wealth effect and in the conventional direction, with rate hikes weighing on activity (see Caramp and Silva (2023) for a RANK model featuring this effect, also via long-term bonds).

When $\sigma > 1$, retirees' MPC out of wealth is positively related to the expected rate path, bringing a countervailing force. The reason is that, given the value of assets, a higher rate path implies that these assets will come with a superior income stream. This reduces the need to hold as many assets for retirement, lowering asset demand, stimulating goods demand. We call this the "asset demand channel". Since working households care about the retirement state when $\delta_1 > 0$, $\hat{\Gamma}_t$ shows up in (19) too (just weighted by the retirement probability δ_1). This channel works in the unconventional direction when $\sigma > 1$, with higher rates *boosting* activity.

5.2 How the effect of interest rates on activity varies along the yield curve

To see these effects differently, note that both q_t and Γ_t can be expressed as function of current and future interest rates – leading to a term structure representation for the Euler equation. Disregarding ε_t^β for a moment, the workers' Euler equation reads:

$$\hat{c}_t^w = (1-\delta_1)\,\mathbb{E}_t\hat{c}_{t+1}^w - \frac{1}{\sigma}\mathbb{E}_t r_{t+1} + \delta_1 \sum_{j=1}^{\infty} \beta^j \left[\frac{\sigma-1}{\sigma}(1-\delta_2)^{\frac{j}{\sigma}} - (1-\mu)^j\right] \mathbb{E}_t r_{t+1+j} \quad (23)$$

This encompasses several special cases in the literature. For $\delta_1 = 0$, we obtain the standard RANK Euler equation. If $\sigma = 1$ and $\delta_1 > 0$, we have a formulation that is equivalent to putting assets directly into the utility function. Finally, with $\sigma = 1$, $\delta_1 > 0$, and $\mu = 1$, we have a discounted Euler equation. Note that if $\sigma \leq 1$ ($EIS \geq 1$), then interest rates *at all future horizons* enter with a negative sign. Interest rate policy then always works in the conventional way. Moreover, the more a rate decrease (increase) is viewed as being persistent, the more it will stimulate (contract) demand.

In contrast, when $\sigma > 1$ ($EIS < 1$), monetary policy can affect the economy very differently depending on whether it only affects short-term rates, or if interest rates further out in the term structure are impacted. Going forward, we focus on the case where $EIS < 1$ (which, according to studies like Yogo (2005), Best et al. (2020), Ring (2024), and Crawley (2025), is the most empirically plausible case).

The first aspect to note from (23) is that a hike in the short-term rate r_{t+1} will always lower consumption (and vice versa for a cut). However, the effects of future rates on $\hat{c}_t^w$ will depend on the sign of $\left[\frac{\sigma-1}{\sigma}(1-\delta_2)^{\frac{j}{\sigma}} - (1-\mu)^j\right]$. This term captures the competition between valuation effects resulting from rate changes, versus the induced effects on asset demand (i.e., the desire to save for retirement).[15] Holding $\mathbb{E}_t\hat{c}_{t+1}^w$ constant, (23) shows that when σ is sufficiently high and/or the interest rate considered is sufficiently far into the future, a higher rate favors *more* current consumption. In other words, equation (23) shows that the partial effect of increasing interest rates on current consumption (holding $\mathbb{E}_t\hat{c}_{t+1}^w$ constant) will tend to change sign, from negative to positive, as one looks further in the future.[16] This arises as valuation effects only affect long-term assets, and these diminish further out in the future when $\mu > 0$ (which implies that the duration in assets is finite). Importantly, such sign-switching cannot arise under a discounted Euler equation formulation (more on this around Proposition 2 below).

However, (23) only provides a partial analysis (it is holding $\mathbb{E}_t\hat{c}_{t+1}^w$ constant and ignores retiree consumption). Before deriving explicit expressions for the impact of future rates on current activity, we need to ensure that the equilibrium of the system (17)-(21) is well

[15]Note that asset duration is governed by $(1-\mu)$. The duration of pension-related liabilities is increasing in $(1-\delta_2)$, as the expected duration of the retirement state is decreasing in the death probability δ_2.

[16]Because $\beta^j[\frac{\sigma-1}{\sigma}(1-\delta_2)^{\frac{j}{\sigma}} - (1-\mu)^j]$ will be positive for high enough j as long as $\sigma > 1$ and $(1-\mu) < 1$, meaning not all bonds are consols.

defined, i.e. stable and unique. Recall that monetary policy is governed by $(1+\phi)$, which expresses the degree to which expected interest rates are increased in response to expected inflation. The conventional Taylor principle suggests that we need $\phi > 0$. However, in our setup, the model maintains determinacy even if $\phi = 0$:

Proposition 1. *With $\theta > 0$ (sticky prices), a constant real rate policy ($\phi = 0$) is sufficient to deliver determinacy.*

Proofs of all propositions are in Appendix C. In light of Proposition 1, the rest of the paper will set $\phi = 0$ to ensure determinacy while simultaneously allowing us to discuss the effects of different real rate paths on activity (and see Appendix D for a visual representation of the model's determinacy region). Once we solve (18) and (19) forward, the impact of future rates on current activity and inflation can be expressed as:

$$\hat{y}_t = -\sum_{j=0}^{\infty} \psi_j \mathbb{E}_t \hat{r}_{t+1+j} \tag{24}$$

$$\hat{\pi}_t = -\sum_{j=0}^{\infty} \psi_j^{\pi} \mathbb{E}_t \hat{r}_{t+1+j} \tag{25}$$

with $\psi_0 = \frac{1}{\sigma}$, $\psi_0^{\pi} = \frac{\kappa}{\sigma}$,

$$\psi_j = (1-\delta_1)\psi_{j-1} - \xi_j^{\psi},$$
$$\psi_j^{\pi} = \beta\psi_{j-1}^{\pi} + \kappa\psi_j,$$

and

$$\xi_j^{\psi} \equiv \frac{\sigma-1}{\sigma}\left[\delta_1 - \gamma(1-\delta_1)\frac{1-\beta(1-\delta_2)^{\frac{1}{\sigma}}}{\beta(1-\delta_2)^{\frac{1}{\sigma}}}\right]\beta^j(1-\delta_2)^{\frac{j}{\sigma}} - \left[\delta_1 - \gamma(1-\delta_1)\frac{1-\beta(1-\mu)}{\beta(1-\mu)}\right]\beta^j(1-\mu)^j.$$

Here, each coefficient ψ_j represents the isolated impact that the real rate at horizon j has on output in the present (with ψ_j^{π} representing the same concept for inflation). It is always true that an increase in the "current" rate $\mathbb{E}_t\hat{r}_{t+1}$ depresses activity, as this is driven solely by intertemporal substitution ($\psi_0 = \frac{1}{\sigma}$). However, the effect of future interest rates becomes ambiguous as three forces are at play: intertemporal substitution, valuation effects, and effects on asset demand. Before exploring the ψ_j's when $\delta_1 > 0$ (i.e., when life-cycle forces are present), it is worth recalling that our model collapses to

RANK for $\delta_1 = 0$. In that case, $\psi_j = \frac{1}{\sigma}$ and $\psi_j^\pi = \kappa \frac{1-\beta^{j+1}}{1-\beta} \psi_j$ for all $j \geq 0$. This implies that near-term interest rate hikes always have the exact same effect on output as rates further out into the term structure (with this effect always equal to $-\frac{1}{\sigma}$).

In contrast, when $\delta_1 > 0$, the sign of ψ_j becomes dependent on the $EIS = 1/\sigma$. If the EIS is sufficiently large, rates at all horizons will have conventionally-signed effects as intertemporal substitution remains dominant. But when the EIS is sufficiently small, interest rates further out into the future will obtain the *unconventional* sign, since asset demand effects (driven by the interest income effect) will dominate:

Proposition 2. *For $\delta_1 > 0$ (i.e., when introducing retirement risk, giving rise to our "FLANK" model), we have that:*

(a) The ability of interest rates to affect activity and inflation in the conventional direction (i.e., with contractionary shocks lowering activity and inflation, and vice versa) is weakened relative to RANK: $\psi_j < \frac{1}{\sigma}$ and $\psi_j^\pi < \frac{\kappa}{\sigma} \frac{1-\beta^{j+1}}{1-\beta}$, for all $j \geq 1$;

(b) In the limit, taking the horizon j to infinity, $\mathbb{E}_t \hat{r}_{t+1+j}$ ceases to affect activity and inflation in the present: $\lim_{j\to\infty} \psi_j = 0$ and $\lim_{j\to\infty} \psi_j^\pi = 0$;

(c) At every horizon $j \geq 1$, ψ_j and ψ_j^π are decreasing in σ; they eventually become negative as σ is increased;

(d) The ability of interest rate policy to affect activity and inflation in the conventional direction is increasing in retirees' death probability (δ_2) and increasing in the duration of available assets (i.e., decreasing in μ) for all $j \geq 1$.

The key takeaway from Proposition 2 is that, with life-cycle forces, the effect that interest rates have on activity can vary along the yield curve – both quantitatively and qualitatively. In our FLANK setup, higher near-term rates can be conventional, i.e., contractionary ($\psi_j > 0$ for $j < \widetilde{j}$), whereas simultaneously higher rates further out into the term structure can be expansionary ($\psi_j < 0$ for $j > \widetilde{j}$). Figure 3 illustrates this by plotting how ψ_j evolves with the horizon j. As the dashed lines show, $\psi_j = 1/\sigma \; \forall j$ in RANK, whereas the solid lines convey the more involved forces present in FLANK.

Parts (a) and (b) of Proposition 2 are shared by models with a discounted Euler equation (McKay et al., 2017). Parts (c) and (d) are specific to FLANK. Part (c) implies that, in FLANK, interest rates further out in the yield curve may have *opposite* effects to that of near-term rates, with higher long-term rates *boosting* activity. This is something that can neither arise in RANK, nor under a discounted Euler equation. As discussed

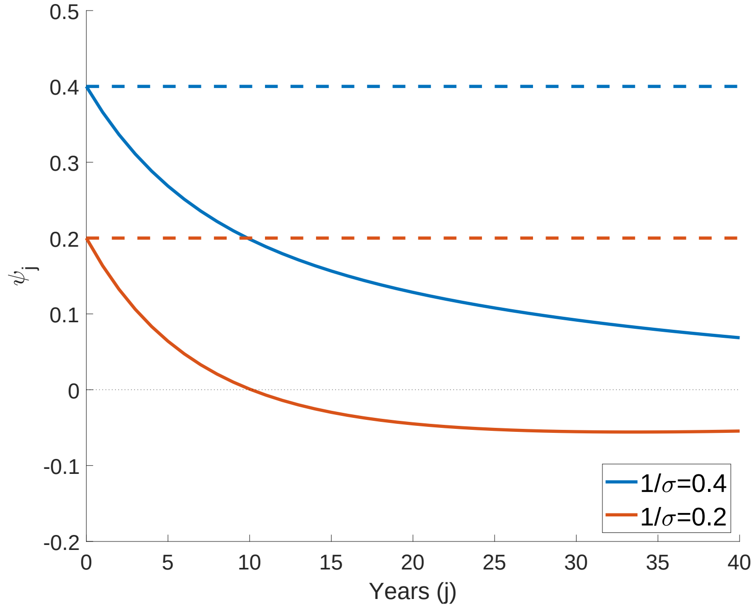

Figure 3: Evolution of ψ_j-coefficients along the yield curve in FLANK (solid) and RANK (dashed).

in Appendix E, this prediction of FLANK is consistent with the empirical observation that an inverted yield curve is often followed by an economic slowdown – with our model suggesting a causal link.

Part (d) of Proposition 2 provides additional insight on the ψ_j coefficients. It shows that interest rate policy loses potency in the conventional direction as households' expected time spent in retirement increases (lower δ_2). This increases the duration of household liabilities – with them having to finance a longer consumption stream in retirement, where households rely on asset income – meaning that low future rates (which are normally expansionary) incite more savings by working households and slower asset depletion by retirees.

Part (d) also implies that interest rate policy loses potency in the conventional direction when asset duration decreases (higher μ). The reason is that this weakens the asset valuation effect, which works in the usual direction (with lower rates being expansionary). This is relevant in considering how QE might affect the monetary transmission mechanism. Since it acts like an asset swap, with the central bank replacing high-duration assets (long-term bonds) with overnight central bank reserves of zero duration, QE can be seen as the central bank pushing up μ (lowering the share of long-term bonds held by the public[17]). This makes the economy less interest rate sensitive – rendering conventional

[17] Note that our Blanchard-Yaari-Gertler setup implies that Barro's (1974) Ricardian Equivalence does

monetary policy (conducted via the interest rate) less potent.[18]

It important to emphasize that Part (c) of Proposition 2 is central to our key results which are to follow, as it implies that persistent rate changes may have *qualitatively different* effects compared to more temporary ones.

5.3 Effect of interest rate persistence on potency and direction

We can now discuss how the potency of monetary policy shocks changes with their persistence. Consider a shock ε_t^i to the interest rate rule that follows an AR(1) process with autocorrelation parameter ρ_i (as specified in (15)). This implies that the policy shock induces a time path for the real rate given by $\mathbb{E}_t \hat{r}_{t+1+j} = \mathbb{E}_t \varepsilon_{t+j}^i = (\rho_i)^j \epsilon_t^i$. The impact responses to such a monetary policy shock are:

$$\hat{y}_t = -\Psi(\rho_i)\epsilon_t^i, \tag{26}$$

$$\hat{\pi}_t = -\frac{\kappa}{1-\rho_i\beta}\Psi(\rho_i)\epsilon_t^i, \tag{27}$$

where

$$\begin{aligned}\Psi(\rho_i) &\equiv \sum_{j=0}^{\infty} \psi_j \rho_i^j \\ &= \frac{1}{\sigma}\frac{(1-\gamma)(1-\delta_1)}{1-\rho_i(1-\delta_1)} - \left[\gamma + \frac{\delta_1(1-\gamma)}{1-\rho_i(1-\delta_1)}\right]\left[\frac{\frac{\sigma-1}{\sigma}}{1-\rho_i\beta(1-\delta_2)^{\frac{1}{\sigma}}} - \frac{1}{1-\rho_i\beta(1-\mu)}\right]\end{aligned} \tag{28}$$

captures the effect of a monetary policy shock ϵ_t^i with persistence ρ_i on current output. Since our model features no state variables, we have that $\hat{y}_t = \rho_i^t \hat{y}_0$ and $\hat{\pi}_t = \rho_i^t \hat{\pi}_0$ – implying that results continue to apply at all horizons $t \geq 0$.

Equations (26) and (27) have several interesting implications for how a shock's persistence affects potency. If either $\delta_1 = 0$ (no retirement preoccupations) or $\sigma \leq 1$, then more

not hold; because of this breakdown, the maturity structure of assets held by the public starts to matter. For $\delta_1 = 0$, Ricardian Equivalence holds and μ no longer matters for (24) and (25).

[18] Concerns related to this aspect of our model have recently come to the fore. As noted in Bloomberg (2023): "UK households are on aggregate about £10 billion ($12.7 billion) a year better off as a result of a jump in interest rates [...] At current rates, savers collectively are earning £24 billion more a year than in November 2021 [...] Respondents to GfK's June consumer confidence barometer said their personal finance situation had improved sharply last month, despite the surge in mortgage rates [...] The data suggests interest rates may not be as effective a monetary policy tool as they were in 2008".

persistent monetary policy shocks always have greater potency than temporary ones. In particular, when persistence ρ_i goes to 1, the potency of monetary shocks becomes very large, and goes to infinity if $\delta_1 = 0$ (i.e., in RANK). It is because of this potency that it is generally thought that monetary policy cannot keep real rates away from their flexible-price counterpart r^* for long periods without having major effects on inflation. However, in the presence of a retirement savings motive ($\delta_1 > 0$) and if $\sigma > 1$, the link between the persistence of monetary shocks and their effect on the economy becomes more involved.

While (26) and (27) show that the link between the persistence of monetary shocks and their effects on the economy depends on many parameters, Proposition 4 emphasizes the role played by the EIS $(1/\sigma)$. In particular, it emphasizes the existence of two threshold levels for σ where things qualitatively:

Proposition 3. *For* $\delta_1 = 0$, $\Psi(\rho_i) > 0$ *for all* $\rho_i \in [0, 1]$, $\partial\Psi(\rho_i)/\partial\rho_i > 0$, *and* $\lim_{\rho_i \to 1} \Psi(\rho_i) = \infty$.

Proposition 4. *If* $\delta_1 > 0$, *then* $\lim_{\rho_i \to 1} \Psi(\rho_i)$ *is finite and* $\exists \sigma^*, \sigma^{**}$ *with* $\sigma^{**} > \sigma^*$, *such that for very persistent monetary policy shocks (*ρ_i *close to 1):*

(a) If $\sigma < \sigma^*$, *then* $\Psi(\rho_i) > 0$ *and* $\partial\Psi(\rho_i)/\partial\rho_i > 0$, *meaning that more persistent shocks have a stronger effect on activity in the conventional direction (i.e., with rate-increasing shocks lowering activity and vice versa);*

(b) If $\sigma > \sigma^*$, *then* $\partial\Psi(\rho_i)/\partial\rho_i < 0$, *meaning that increases in shock persistence* DECREASE *the shock's effect on activity in the conventional direction;*

(c) If $\sigma > \sigma^{**}$, *then* $\Psi(\rho_i) < 0$, *meaning that sufficiently persistent monetary policy shocks affect activity in the unconventional direction.*

The main aspect to note from Proposition 4(b) is that, when σ is high enough in FLANK, a more persistent monetary shock will be *less* potent than a more temporary one – giving a stark contrast with RANK (covered by Proposition 3). This "persistence-potency trade-off" arises because the effects of monetary shocks on consumption are not just driven by intertemporal substitution in FLANK. Instead, they are also shaped by how the rate change affects the desire to accumulate, and hold on to, assets (to ensure consumption in retirement). The latter depends on whether the lower (higher) rates are incentivizing households to hold more (less) wealth and whether valuation effects are sufficiently large to offset any changes in their desire to save. Proposition 4 shows that as σ increases, intertemporal substitution becomes less relevant and the impact on asset demand will eventually dominate the valuation effect. This then causes more persistent shocks to have less of an effect on activity than more temporary changes.[19] Interestingly, such a pattern has been observed by various empirical studies, including Uribe (2022), McKay and Wolf (2023, their Appendix C.2), Miescu (2023), Swanson (2024), and Braun et al. (2025); our Appendix E provides further evidence.

In fact, the operation of monetary policy can even flip sign, as implied by part (c) of the proposition. To visualize this, Figure 4 plots $\Psi(\rho_i)$ as ρ_i varies between 0.5 and 1.[20] The figure shows that, for rather transitory shocks, life-cycle forces do not affect the monetary transmission mechanism much (i.e., FLANK behaves much like RANK for low ρ_i). But as ρ_i increases, the two models diverge: whereas RANK implies that very persistent shocks are incredibly potent in the conventional direction (with this potency going to infinity in the limit), FLANK suggests the opposite may arise – with $\Psi(1)$ close to zero (or slightly negative) being a plausible outcome.

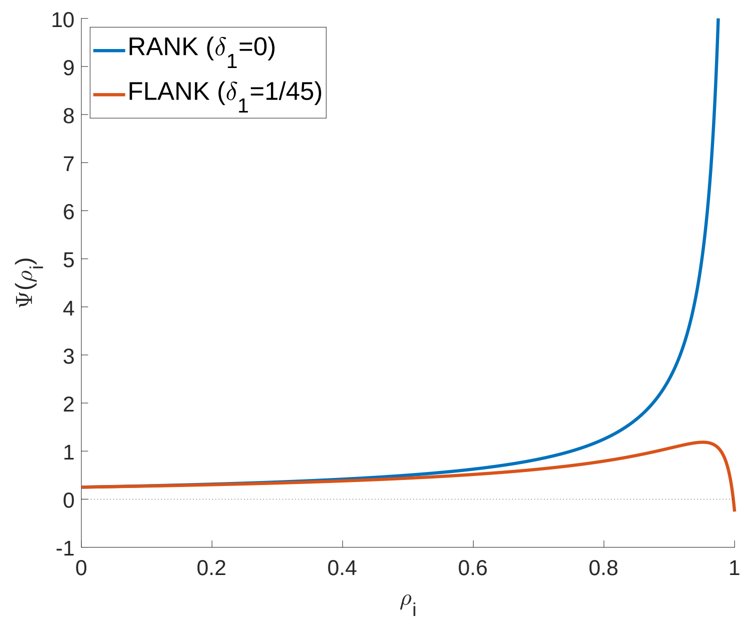

Figure 4: $\Psi^y(\rho_i)$ in RANK and FLANK. Other parameters calibrated as in footnote 20.

[19]This result is reminiscent of Lucas and Rapping (1969), who show that the response of labor supply may vary with the persistence of the wage impulse. When the latter is rather transitory, the substitution effect is likely dominant – making labor supply increase with the wage rate. But when the wage changes in a rather persistent manner, the income effect gains importance – potentially causing labor supply to fall with wages.

[20]This figure was generated using the following calibration at the annual frequency: $\sigma = 4$, $\beta = 0.96$, $\delta_1 = 1/45$ (an expected working life of 45 years), $\delta_2 = 1/20$ (an expected retired life of 20 years), and $\mu = 0.132$ (average bond maturity of 6 years).

5.4 The effect of (near-)permanent monetary policy shocks

At this stage, one might wonder: what does FLANK imply for a central bank's ability to keep its policy rate away from the natural rate r^* on a lasting basis? This is like asking whether the central bank can conduct monetary policy via an interest rate rule with an intercept (the long-run average rate the central bank is aiming for) different to the true r^*, without creating massive inflation or deflation.

The consequences of keeping r permanently away from r^* can be understood by considering the effects of a permanent monetary policy shock.[21] While our log-linearized model is not formally equipped to analyze truly permanent ($\rho_i = 1$) shocks, it is insightful to consider the analytical expression for Ψ^y that results in the limit where $\rho_i \to 1$ in equation (28). One then obtains:

$$\Psi(1) = \sum_{j=0}^{\infty} \psi_j^y = \underbrace{\frac{1}{\sigma}\frac{(1-\gamma)(1-\delta_1)}{\delta_1}}_{intertemporal\ substitution} - \underbrace{\frac{\sigma-1}{\sigma}\frac{1}{1-\beta(1-\delta_2)^{\frac{1}{\sigma}}}}_{asset\ demand} + \underbrace{\frac{1}{1-\beta(1-\mu)}}_{asset\ valuation}. \tag{29}$$

The first term captures intertemporal substitution. It is always positive and goes to zero as $EIS = \frac{1}{\sigma} \to 0$. The second term captures the asset demand effect. This is primarily driven by the duration of household liabilities, as governed by the death probability δ_2, which determines the expected length of retirement (where the household still wants to consume, but only enjoys interest income).[22] When $\sigma > 1$ this term works in the *unconventional* direction. The third term captures the asset valuation effect, driven by μ (asset duration). Whenever the sum of the last two terms is negative, meaning that the duration in the household's asset portfolio is not enough to compensate for its negative duration gap stemming from the need to finance consumption in retirement, the total effect $\Psi(1)$ could be close to zero (as the first term in (29) is positive). In the special case where $\frac{1}{\sigma} \to 0$ and $\mu \to 0$, we have $\Psi(1)$ *exactly* equal to 0 (as consumption then equals the flow value of wealth, $(r-1)a$, while the value of wealth is proportional to $\frac{1}{r-1}$).

Equation (29) also allows for an insightful reinterpretation, featuring just two forces. Combining the effects relating to intertemporal substitution and asset demand, one obtains a term that captures the effect of a permanent rate increase on the economy's average

[21] Agents in our model only care about the interest rate path, not about the decomposition between systematic monetary policy (including the intercept) and shocks to that rule (McKay and Wolf, 2023).

[22] The asset demand effect is maximized for $1/\sigma \to 0$. There, the household is infinitely risk averse, meaning that it only consumes its interest income – never daring to touch the principal (including capital gains) for fear of outliving assets. This actually seems a reasonable approximation to the observed behavior of retirees, who do not dissave much in retirement; recall footnote 3.

MPC out of financial wealth ($MPCoW$).[23] $\Psi(1)$ then reads:

$$\Psi(1) = \underbrace{\frac{1}{\sigma}\frac{(1-\gamma)(1-\delta_1)}{\delta_1} - \frac{\sigma-1}{\sigma}\frac{1}{1-\beta(1-\delta_2)^{\frac{1}{\sigma}}}}_{average\ MPC\ out\ of\ financial\ wealth\ (MPCoW)} + \underbrace{\frac{1}{1-\beta(1-\mu)}}_{asset\ valuation}. \tag{30}$$

Since the asset valuation effect always works in the conventional direction (higher rates depress demand), equation (30) implies that $\Psi(1) \approx 0$ is possible if the $MPCoW$ rises sufficiently in response to a permanent rate increase. In FLANK, this can easily happen because agents have less need to hold assets, to maintain a given retirement consumption stream, when assets generate a higher return.[24]

Ultimately, the above discussion is a quantitative question. For the calibration used in Figure 4, $\Psi(1)$ is indeed close to zero. But since there is uncertainty over the appropriate calibration, Figure 5 presents a heatmap.[25] It conveys the different values taken on by $\Psi(1)$ for different values of the $EIS(=\frac{1}{\sigma})$ and bond duration, as governed by μ. For the other parameters, of which there are only three, we fix $\delta_1 = 1/45$ (an expected working life of 45 years), $\delta_2 = 1/20$ (an expected retirement span of 20 years), and set $\beta = 0.96$.

Our biggest challenge relates to the plausible range for the EIS. Here, we draw from Best et al. (2020) which uses a frontier empirical strategy. Their preferred EIS estimate lies close to zero (at around 0.1). At the other end, they report values up to 0.3 (see their Table 3B, pooled estimate), so we go up to 0.35 to be inclusive of higher values (which are also consistent with Havránek's (2015) meta-analysis, reporting estimates around 0.3-0.4). For average bond maturities, we consider values above 5 years (i.e., $\mu \leq 0.2$) to capture a set of interpretations for assets held. Lower durations (higher μ) are appropriate when only thinking of government bonds (which, in the U.S., have an average duration of just over 5 years); higher durations (lower μ) are reasonable when thinking of a combination of bonds, equity, and real estate.[26] We aim to be quite inclusive in the range of parameters explored, to give a sense of the possible outcomes in FLANK.

[23] Averaging is over workers and retirees. The effect on the $MPCoW$ for retirees is given by $\frac{\sigma-1}{\sigma}\frac{1}{1-\beta(1-\delta_2)^{\frac{1}{\sigma}}}$, while that on working households equals $\frac{\sigma-1}{\sigma}\frac{1}{1-\beta(1-\delta_2)^{\frac{1}{\sigma}}} - \frac{1}{\sigma}\frac{1-\delta_1}{\delta_1}$. Since $\frac{1}{\sigma}\frac{1-\delta_1}{\delta_1} > 0$, the effect of r on the $MPCoW$ of working households is less positive (or more negative) than that on retirees – the reason being that interest income is less important to the former group.

[24] $\partial MPCoW/\partial r > 0$ also relates to Lettau and Ludvigson (2001), who present evidence from asset prices suggesting that household consumption increases with (expected) returns.

[25] Since our model is log-linearized, it is not formally equipped to handle fully permanent shocks, which is why Figure 5 is generated with $\rho_i = 0.99$. The same applies to Figure 6.

[26] Weber (2018) puts the duration of the S&P 500 at around 20 years. The duration of housing is estimated to be around 8 years (Burgert et al., 2024). In the Fed's FRB/US model, a highly persistent unit monetary policy shock changes household aggregate wealth holdings by approximately 10%, which corresponds to $\mu = 0.1$ in our FLANK model.

In Figure 5, blue areas represent positive values for $\Psi(1)$ (this is the "conventional" region); red areas represent *negative* values (where permanently lower rates *depress* activity). White areas indicate that $\Psi(1)$ is close to zero, with black lines marking iso-$\Psi(1)$ curves. An iso-$\Psi(1)$ curve of $\pm 1\%$ implies that a permanent real rate increase by 1 percentage point relative to r^*, would cause an output gap of 1%. With a standard Euler equation (including the discounted variant), the whole area would be dark blue. In particular, under RANK the entire surface would be valued at $+\infty$. In contrast, Figure 5 shows that negative values for $\Psi(1)$ are about as plausible as positive ones. FLANK thus gives little reason to believe that persistently low (high) interest rates are more likely to stimulate (depress) the economy, than to depress (stimulate) it. This, by itself, is an important take-away.[27]

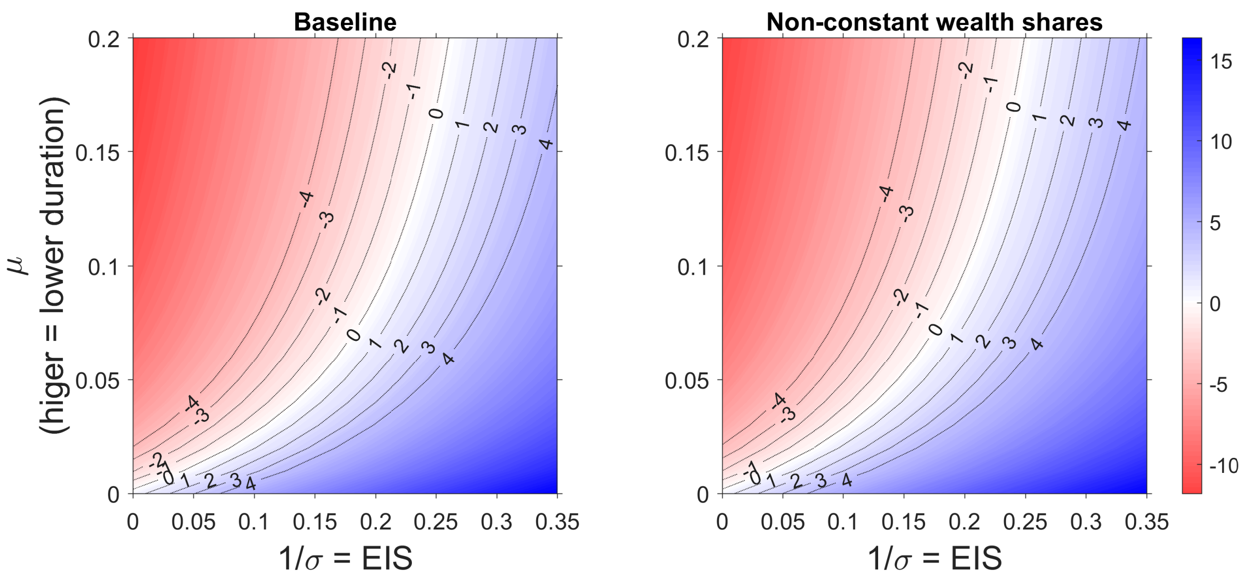

Figure 5: $\Psi(1)$ as a function of σ and μ in FLANK. The left panel shows this for the baseline model (featuring a transfer scheme to keep the wealth shares constant at their steady state), while the right panel does not impose this simplifying assumption. Other parameters are calibrated as in footnote 20.

The area where $\Psi(1)$ is *exactly* equal to zero, is of measure zero – making it not very relevant. Nonetheless, the figure shows that there is a considerable area where $\Psi(1)$ may be considered quite small. Recall that over the period from 1990 to 2019, the U.S. output gap varied by several percentage points without inflation moving much. This suggests that, when inflation expectations are well anchored, an output gap of a few percentage points

[27]While much of our discussion focuses on the idea that $\Psi^y(1)$ may be close to zero, the possibility of $\Psi^y(1)$ being negative (instead of positive) also suggests that low-for-long policies may have contributed to depressing the economy instead of stimulating it.

might not affect inflation by a lot. Accordingly, Figure 5 hosts a considerable region where a permanent departure of real rates from r^* could be consistent with inflation remaining close to target.

At this stage, one may recall that the above results were derived under the assumption that the relative wealth share of retired versus working households was held constant via a tax-transfer scheme. Does this affect the properties of $\Psi^y(1)$? Figure 5's right panel, which shows the same heatmap *without* the simplifying assumption (in which case we can still solve the model numerically), points to robustness: both the qualitative and quantitative features of $\Psi^y(1)$ are virtually unchanged.

5.5 Monetary versus demand shocks in FLANK

Another interesting feature of FLANK is that it breaks the equivalence (for example present in RANK) between monetary shocks and other types of demand shocks. Here, we illustrate this point by considering shocks to the discount rate (recall equation (16)), but the point is more general. To see this, observe that there exists an equivalent representation to equations (24)-(25), which were derived under $\varepsilon_t^\beta = 0$, when allowing for discount rate shocks. In particular, Appendix C shows that the effects of discount rate shocks "ε_t^β" (where $\varepsilon_t^\beta > 0$ represents an *increase* in patience, thus representing a negative demand shock) on output and inflation are given by:

$$\hat{y}_t = -\sum_{j=0}^{\infty} \omega_j \mathbb{E}_t \varepsilon_{t+j}^\beta \tag{31}$$

$$\hat{\pi} = -\sum_{j=0}^{\infty} \omega_j^\pi \mathbb{E}_t \varepsilon_{t+j}^\beta \tag{32}$$

with $\omega_0 = \frac{1}{\sigma}$, $\omega_0^\pi = \frac{\kappa}{\sigma}$,

$$\begin{aligned}
\omega_j &= (1-\delta_1)\omega_{j-1} - \xi_j^\omega, \\
\omega_j^\pi &= \beta\omega_{j-1}^\pi + \kappa\omega_j, \text{ and} \\
\xi_j^\omega &\equiv -\frac{1}{\sigma}\left[\delta_1 - \gamma(1-\delta_1)\frac{1-\beta(1-\delta_2)^{\frac{1}{\sigma}}}{\beta(1-\delta_2)^{\frac{1}{\sigma}}}\right]\beta^j(1-\delta_2)^{\frac{j}{\sigma}}.
\end{aligned}$$

Crucially, with $\delta_1 > 0$, the coefficients on the discount rate shock at each horizon $j > 0$ are *not* proportional to those for the monetary policy shock. For RANK ($\delta_1 = 0$)

the coefficients *are* proportional. In that case, a monetary policy shock induces the exact same dynamics as a discount rate shock – meaning that the former is extremely well-suited to offset the latter. However, in FLANK that is no longer true: while discount rate shocks continue to operate via intertemporal substitution, policy-induced shocks to the interest rate are "special" as they come with an offsetting effect (changes in interest income affecting asset demand) that give rise to a persistence-potency trade-off. Note that the time-t impact of a persistent AR(1) discount rate shock on output equals $\hat{y}_t = -\Omega(\rho_\beta)\varepsilon_t^\beta$, with:

$$\begin{aligned}\Omega(\rho_\beta) &\equiv \sum_{j=0}^{\infty} \omega_j \rho_\beta^j \\ &= \frac{1}{\sigma}\frac{(1-\gamma)(1-\delta_1)}{1-\rho_\beta(1-\delta_1)} + \frac{1}{\sigma}\frac{\gamma + \frac{\delta_1(1-\gamma)}{1-\rho_\beta(1-\delta_1)}}{1-\rho_\beta\beta(1-\delta_2)^{\frac{1}{\sigma}}}\end{aligned}$$

From this, it is easy to see that $\Omega(\rho_\beta) > 0$ for all $\rho_\beta \in [0,1]$, with $\partial\Omega(\rho_\beta)/\partial\rho_\beta > 0$ (meaning that more persistent shocks are more potent in the conventional direction).

These findings suggest that monetary policy is less well equipped to offset demand shocks in FLANK, especially when demand shocks are very persistent.

6 The (ir)relevance of r*

FLANK implies that the effects of interest rates on activity will vary along the yield curve, likely switching sign along the way. This section will show that this has important implications for the relevance of the natural rate (r^*) as a policy anchor, and for r^* estimation. In particular, our setup implies that precise knowledge of r^* may not be very important for inflation-targeting central banks – as the system may be very "forgiving" to the central bank working with a biased value for r^*. This indicates that central banks might still be able to fulfill their mandate in a satisfactory way, even if they are ill-informed about the true value of r^*.

Standard models suggest that the location of r^* is crucial for central banks to be aware of, since keeping rates away from that level for too long is bound to force inflation away from target.[28] In contrast, FLANK suggests that central banks may be much less constrained by r^*, potentially making r^* quasi-irrelevant and opening the door for monetary policy to influence longer-term real rates. To further clarify the extent to which monetary

[28] This notion also appears to be gaining popularity in practice, with the number of central bank speeches referring to the "natural/neutral interest rate" having risen sharply since 2015 (Borio, 2021).

policy is constrained by r^*, consider the class of models where activity $\hat{y}_t$ can be related to the future path of interest rates via:

$$\hat{y}_t = -\sum_{j=0}^{\infty} \psi_j \mathbb{E}_t(r_{t+1+j} - r^*).$$

As previously noted, this formulation (also shown in equation (24)) hosts the standard RANK model as well as our FLANK setup – with the models differing only with respect to implied coefficients for ψ_j.

Now consider a central bank which misperceives r^*, where (in line with Section 3) we denote the central bank's perception of r^* by $r^{*,CB}$ (which can be seen as the central bank's long-run target for r). Would this misperception be problematic? The determination of output is now given by:

$$\hat{y}_t = -\sum_{j=0}^{\infty} \psi_j \mathbb{E}_t(r_{t+1+j} - r^{*,CB}) - \Psi(1)(r^{*,CB} - r^*),$$

This shows that the relevance of r^* for $\hat{y}_t$ depends crucially on the value of $\Psi(1)$. When activity is determined by a standard representative agent Euler equation, $\Psi(1) = \infty$. In this case, making sure that $r^{*,CB}$ equals r^* is *absolutely crucial* for monetary authorities as deviations of $r^{*,CB}$ from r^* would have huge implications for activity and consequently inflation.[29]

However, in FLANK, $\Psi(1)$ may actually be close to *zero*. Deviations of $r^{*,CB}$ from r^* then do not affect activity much. And if the Phillips curve is not very steep, as for example argued by Hazell et al. (2022), an $(r^{*,CB} - r^*)$-gap could have only a small effect on inflation. Therefore, when $\Psi(1)$ is small, a central bank could potentially adopt a policy rule where its long-term anchor for real rates $r^{*,CB}$ is substantially different from the true r^* without causing any major economic disruption.

In this sense, knowing r^* becomes quasi-irrelevant for the conduct of monetary policy, as the system is very forgiving to the central bank working with a biased r^*-belief. In the special case where $\Psi(1)$ is *exactly* zero, r^* becomes indeterminate and the central bank can set its long-term goal $r^{*,CB}$ freely, without any direct implications for output and inflation. Still, the choice for $r^{*,CB}$ will affect asset prices.

[29] This logic captures why central banks are often thought to be heavily constrained by r^*, while it also explains why there is a Forward Guidance puzzle (Del Negro et al., 2013).

7 Discussion: assumptions and extensions

7.1 Assumptions

In this section we discuss various assumptions underlying our model. We will point out why our current assumptions could be easily relaxed and why they would not likely change our key insights. We also discuss how our results should be seen as "local", placing implicit bounds on how far interest rates could deviate from the true r^*.

Hand-to-mouth agents. Our model treats all households as intertemporal optimizers. This may seem inappropriate given the evidence supporting the presence of hand-to-mouth (HtM) consumers (Kaplan et al., 2014). Accordingly, the mechanisms in our model may appear relevant only for the financially well-off. We concur with this, but do not view it as a drawback for two reasons. First, the well-off account for much of total consumption demand – making them a natural focal point (in U.S. data, the wealthiest 20% account for nearly half of total consumption; Abbott and Brace, 2020). Second, one of the main insights from the HtM literature is that the dynamics of aggregate activity will primarily be driven by the behavior of optimizing households – even if the later are only a fraction of the total population (Werning, 2015). With HtM households, the decisions of optimizers are transmitted to wider economy via the non-optimizing households – potentially yielding amplification (Bilbiie, 2020; 2024), though recent micro-data from Norway fails to provide evidence for such amplification (Bilbiie et al., 2025). Either way, as long as the fraction of income going to HtM households is relatively stable, treating the economy as if solely driven by optimizers is a decent approximation. This is the interpretation we favor, recognizing that the modelled behavior may only reflect a subset of the population. While our model's structure is flexible enough to add HtM households, we choose not to – as this would complicate the setup without adding anything new.

Bequests. While our FLANK model does not include a bequest motive, we believe that its insights should carry through and may even be strengthened with such an extension. Bequest motives would likely accentuate the asset demand force present in FLANK. If parents not only care about the value of any assets they pass on, but also about the expected rate of return, a simple modelling approach is to think of bequests as consumption past death. A bequest motive can then be proxied by lowering δ_2. To gauge the impact of this on $\Psi(1)$ (i.e., the effect that a permanent increase in real rates has on consumption demand) the left panel of Figure 6 regenerates our heatmap after reducing δ_2 from $\frac{1}{20}$ to

$\frac{1}{30}$. As the figure shows, the range of parameter values where $\Psi(1)$ is close to zero (or positive) expands – centered around slightly higher values of the EIS. For example, with an EIS just below 0.3, there is now a large range for μ (governing asset duration) where $\Psi(1)$ is small.

Equity. Agents only hold government bonds in our model. This may seem restrictive, as it neglects equity. Adding an equity market is straightforward. In our baseline setup, working households own all firms. Alternatively, firm equity could be traded in a market featuring workers and retirees, with the equity price responding to interest rates as implied by standard arbitrage. We have explored this modification and have not found it to affect our main results – motivating our choice for the simpler setup. The reason for this robustness lies in the fact that interest rates affect equity and long-term bond prices in the same direction. So, while the introduction of equity would make the model's asset valuation channel more involved, it would not change its nature. There are nonetheless two aspects that would change with equity. The first concerns the strength of the valuation channel. With only long-term bonds, the strength of this channel is governed by bond duration. But with equity, the strength would also be governed by the equity risk premium.[30] This does not change the main mechanism, but influences how to calibrate the model (as discussed in footnote 26). The second aspect that would change with equity, is that it would open the door to exploring changes in risk premiums (Caramp and Silva (2021) offer an analysis along these lines), which is also related to the literature on safe asset demand (Caballero et al., 2016; 2017). At the moment, the route via which the central bank in our model can affect longer-term rates runs entirely through the expectations hypothesis. But a "low for long" policy might also trigger a search for yield, making investors move more into equity. We suspect that such a shift would compress the equity premium, making the risky rate of return fall by more than the safe rate – *adding* to the retirement savings challenge – but leave a formal analysis for future work.[31]

Housing. Along similar lines, the logic of the model would continue to hold if households were allowed to save in a housing asset. So, while our model contains a long-term bond as

[30]The steady-state value of equity would equal $\frac{d}{r+rp}$, where d is the dividend payment, r is the real rate, and rp is an equity risk premium. Recall that the steady-state bond price in the model is given by $\frac{1}{r+\mu}$, where $1/\mu$ governs bond duration. This illustrates that a lower equity premium implies that asset prices are more sensitive to real rate changes, which parallels the role played by bond duration.

[31]In reality, the risk premium appears to have *risen* since the 1980s (for reasons outside of our model), implying that the risky rate of return has fallen by less than the safe rate (Reis, 2022). This is no panacea for retirement savings though, as investing in riskier options implies that one ends up with a different (riskier) pension, to which one might again respond by wishing to hold more assets.

the asset through which saving takes place, the exact nature of the asset is of secondary importance. The more important issue is that this asset has positive duration, i.e., that its price "q" is inversely related to the interest rate.

Physical capital. A next extension in this line is to enrich the model with productive physical capital K. While one might think that the accompanying "investment channel" of monetary policy could overturn some of our findings, this turns out not to be so. This is shown by the right panel of Figure 6, which plots the heatmap after extending our model with capital and quadratic investment adjustment costs (details are in Appendix F). It looks similar to Figure 5, which abstracts from capital, suggesting that is a decent approximation for the question central to this paper. To understand why, it helps to think of r^* as the interest rate which sets long-run excess demand ("XD") to 0. The natural logarithm of this object can be defined as $\ln XD = \ln(C + I) - \ln F(K, L) = \ln(C + \nu K) - \ln F(K, L)$, where "$\nu$" is capital's depreciation rate. Differentiating with respect to $\ln r$, whilst holding consumption and labor supply constant, gives $\frac{\partial \ln XD}{\partial \ln r}\Big|_{C,L} = \left(\frac{I}{Y} - \frac{K \cdot \partial F/\partial K}{F(K,L)}\right) \frac{\partial \ln K}{\partial \ln r}$. Since the investment share $\frac{I}{Y}$ tends to be smaller than the capital share $\frac{K \cdot \partial F/\partial K}{F(K,L)}$ (in the U.S., the former is about 20% versus 30% for the latter), one can see that the partial effect of higher interest rates is to create excess *demand* (not excess supply) in the natural case where $\frac{\partial \ln K}{\partial \ln r} < 0$. The reason lies in the fact that investment does not just come with a demand aspect to it, but also affects future supply; on balance, investment tends to expand long-run supply by more than long-run investment demand in realistic calibrations. Hence, what is required for excess demand to be strongly negatively related to r, is that consumption is strongly negatively related to r. This explains our focus on the latter.

Local versus global. While we only offer a local analysis of our model in this paper, it is relevant to mention how results would change with a global analysis. In our local analysis, real rates can deviate from r^* for long periods of time without doing much to activity or inflation. However, if the deviation became very large, many of the local properties could change. As shown in Beaudry et al. (2024), the underlying asset demand function is C-shaped. This means that, at very high real rates, asset demand will eventually always become increasing in returns (even for $EIS \ll 1$). This implies that large deviations in interest rates away from r^* would not be possible without creating a large economic boom or contraction. Hence, from a global perspective, r^* should be viewed as remaining relevant, but knowing it with great precision might not be very important.

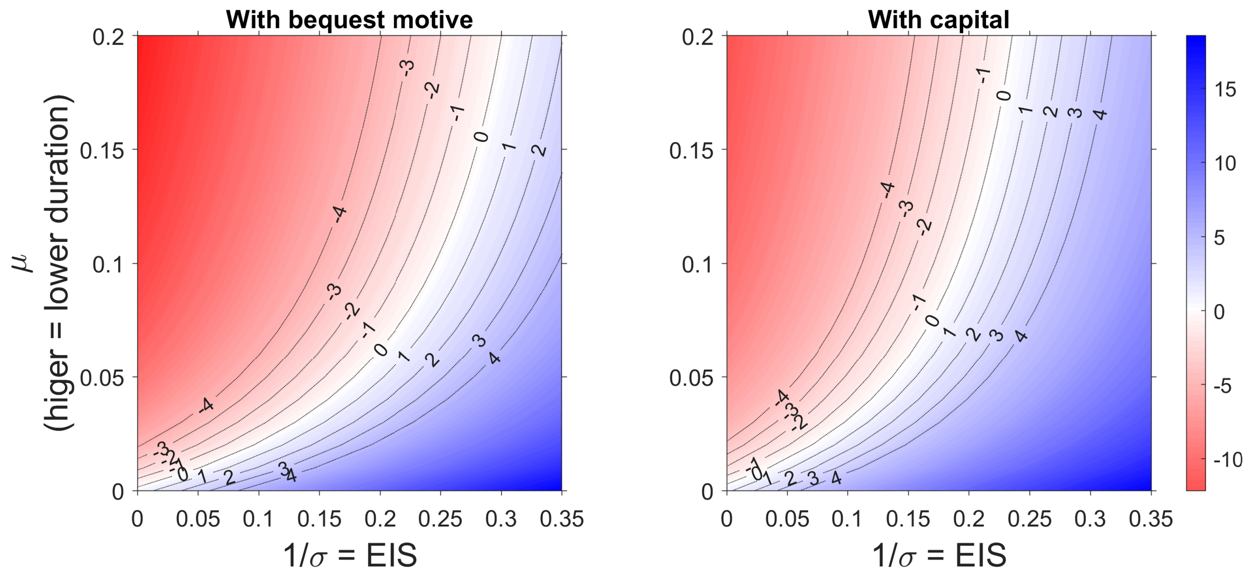

Figure 6: $\Psi(1)$ as a function of σ and μ in FLANK. The left panel shows this when proxying a bequest motive by setting $\delta_2 = 1/30$, while the right panel extends the model with capital. Other parameters are calibrated as in footnote 20.

7.2 Extensions for future work

By offering a highly tractable framework combining life-cycle forces and monetary policy, our work opens several avenues for future work. Our finding that conventional monetary policy may be less potent when retirement preoccupations are more prevalent (or when household assets are of shorter duration) suggests that central banks may need to move the interest rate *by more* to achieve a given effect on output and prices in an aging society (or a "post-QE world" where central banks hold large long-term bond portfolios). This may have adverse consequences for financial stability. We do not model these interactions here, but such an extension could be warranted.

Second, while FLANK is already heterogenous-agent in nature (featuring workers and retirees), other forms of heterogeneity would be interesting to consider. Heterogeneity in the MPC out of wealth would be a natural candidate. Empirical studies find that this object varies across the wealth distribution, with richer households having lower MPCs (Di Maggio et al., 2020; Chodorow-Reich et al., 2021). In that case, our model's logic suggests that greater inequality can weaken the monetary transmission mechanism – as the "asset valuation effect" is normally an important force working in the conventional direction. But when consumption demand of asset holders is not very sensitive to valuation effects, this channel loses potency. To analyze such questions, the model of Bardoczy and Velasquez-Giraldo (2024), which combines MPC-heterogeneity with life-cycle dynamics, seems to hold great potential.

When it comes to adding realism, countries typically do not solely rely on funded pension plans – also providing retirees with a basic retirement income via a pay-as-you-go (PAYG) system, financed by taxing workers. The generosity of such schemes however tends to be limited,[32] leaving an important role for the saving dynamics central to our paper – a role that would only gain importance if one were to explicitly model bequest motives (a PAYG pension cannot be bequeathed to one's offspring). Our model also clarifies that the importance of retirement preoccupations to the monetary transmission mechanism is greater when PAYG pensions are less generous. As demographic forces (higher old-age dependency ratios) are putting PAYG systems under pressure (OECD, 2021), our paper suggests that the importance of retirement preoccupations to monetary policy may rise over time.

Our model can also serve as a guide to empirical researchers in formulating the correct specification when trying to estimate the MPC out of wealth – with our model showing how and why to control for the accompanying *level* of interest rates. If wealth levels are high because of low discount rates, the MPC is likely to be low, as households would want to hold on to their assets to compensate for the lower flow return. This suggests that the MPC out of financial wealth not only varies with wealth holdings (with richer households having a lower MPC) but also with the prevailing level of long-term rates. Recent empirical findings in Di Maggio et al. (2020) and Fagereng et al. (2021) are indeed hinting in this direction, pointing towards a higher MPC out of dividend payouts relative to capital gains stemming from lower rates of interest.

It would also be interesting to characterize optimal policy in FLANK. Since the model suggests that very persistent rate changes might not affect demand by much, this implies that interest rate policy may be ill-equipped to offset persistent demand shocks. The latter may be better left to fiscal policy, with monetary policy instead focusing on stabilization in response to disturbances that are deemed more transient in nature.

Finally, to us, the region of the model's parameter space where $\Psi(1) \approx 0$ carries considerable appeal: not only can it explain why central banks appear to have significant control over longer-term real rates, but also why central banks have been quite successful in fulfilling their mandate despite being very imperfectly informed about the location of r^*. In this light, it is interesting to explore what can widen the range where $\Psi(1)$ is small.

[32]For example: 2023 U.S. Social Security payments were about $1,782 per month (see https://www.cbpp.org/sites/default/files/atoms/files/8-8-16socsec.pdf). Most young, working Americans are moreover pessimistic about their future Social Security benefits (Turner and Rajnes, 2021), increasing the importance of their own saving efforts.

Our initial explorations suggest that a bequest motive can do so (recall the discussion around Figure 6) but there may be other avenues. One possibility is to consider Epstein-Zin (1989) preferences, which allow the EIS and coefficient of relative risk aversion to be calibrated separately (rather than imposing they are each other's inverse).

8 Conclusion

Long-term real interest rates are usually seen as being driven by real forces, with monetary policy playing a transitory role – otherwise just *following* secular trends. This paper has argued that this view may be incomplete, with there actually existing a causal (though non-deliberate) link between the stance of monetary policy and long-term real rates.

We began by documenting that temporary demand shocks, to which the Federal Reserve responds systematically, appear to leave a highly persistent imprint on forward real rates and estimates of r^* – suggesting that monetary policy may have a longer reach over real rates than standard models imply.

We then proposed a mechanism through which such "real rate hysteresis" can arise. The key object is the sensitivity of aggregate demand to very persistent deviations of real interest rates from the natural rate, denoted $\Psi(1)$. In standard New Keynesian models, this sensitivity is large: keeping real rates away from r^* for long should generate substantial excess demand, pushing inflation away from target, and forcing the central bank to reverse course. In contrast, we showed that once life-cycle forces are introduced, $\Psi(1)$ can be much smaller – potentially even close to zero. The reason is that persistent changes in real rates affect not only intertemporal substitution and asset valuations, but also households' desired asset holdings for retirement. When the elasticity of intertemporal substitution is low, the asset demand effect can broadly offset the valuation channel.

While we do not claim to know with certainty that the net effects are in fact approximately zero – even though it is consistent with various empirical observations and calibrations offered in this paper – we do argue that such a possibility opens the door to a fundamentally different view regarding the powers of central banks. Especially, it offers an interpretation on the observed link between policy rates and long-term real rates that does not rely on central banks having private information. Instead, there may well exist a "persistence-potency trade-off", making the persistent component of monetary policy much less potent than commonly thought. Our model implies that if a central bank chooses to keep real rates low for a prolonged period, as many central banks did post-GFC, this may not boost the economy much (or even cause a slight contraction).

If the central bank processes this disappointing outcome through the lens of a standard model, it is likely to conclude that r^* has fallen, which perpetuates the loose stance of monetary policy. The main effect of such a low-for-long policy would then be to lower long-term rates and boost asset valuations.

As a result, if central banks set policy based on a misperceived estimate of r^*, they would receive very few signals suggesting they are mistaken. In this sense, the economy is rather forgiving to such misperceptions. Accordingly, actions by a central bank may end up driving real rates over long periods of time, without them realizing this. It can lead to cases where a rate cut, initially intended to be purely temporary, acquires additional persistence as it subsequently induces the central bank to erroneously lower its estimate of r^* (and vice versa for a rate hike). It then becomes rational for markets to view central bank decisions and statements as relevant for long-term rates, even if they do not think central banks have private information about r^*.

In sum, there may be a "long arm of monetary policy". Not because central banks deliberately choose long-run real interest rates, but because the economy provides too little feedback to prevent their beliefs about the associated natural rates from becoming self-sustaining.

References

Abadi, Joseph, Markus K. Brunnermeier, and Yann Koby (2023), “The Reversal Interest Rate”, *American Economic Review*, 113, pp. 2084-2120.

Abbott, Brant and Robin Brace (2020), “Has Consumption Inequality Mirrored Wealth Inequality in the Survey of Consumer Finances?”, *Economics Letters*, 193, 109289.

Ahmed, Rashad, Claudio Borio, Piti Disyatat, and Boris Hofmann (2024), “Losing Traction? The Real Effects of Monetary Policy when Interest Rates Are Low”, *Journal of International Money and Finance*, 141, 102999.

Aizenman, Joshua, Yin-Wong Cheung, and Hiro Ito (2019), “The Interest Rate Effect on Private Saving: Alternative Perspectives”, *Journal of International Commerce, Economics and Policy*, 10 (1), 1950002.

Ang, Andrew, Monika Piazzesi, and Min Wei (2006), “What Does the Yield Curve Tell Us About GDP Growth?”, *Journal of Financial Economics*, 131 (1-2), pp. 359-403.

Auclert, Adrien (2019), “Monetary Policy and the Redistribution Channel”, *American Economic Review*, 109 (6), pp. 2333-2367.

Auclert, Adrien, Hannes Malmberg, Frédéric Martenet, and Matthew Rognlie (2024), “Demographics, Wealth, and Global Imbalances in the Twenty-First Century”, mimeo, Stanford University.

Auclert, Adrien, Matthew Rognlie, and Ludwig Straub (2025), “Fiscal and Monetary Policy with Heterogeneous Agent”, *Annual Review of Economics*, forthcoming.

Bardóczy, Bence and Mateo Velásquez-Giraldo (2024), “HANK Comes of Age”, Finance and Economics Discussion Series No. 052, Washington: Board of Governors of the Federal Reserve System.

Barro, Robert J. (1974), “Are Government Bonds Net Wealth?”, *Journal of Political Economy*, 82 (6), pp. 1095-1117.

Beaudry, Paul, Katya, Kartashova, and Césaire Meh (2024), “Asset Demand and Real Interest Rates”, NBER Working Paper No. 32248.

Best, Michael Carlos, James S. Cloyne, Ethan Ilzetzki, and Henrik J. Kleven (2020), “Estimating the Elasticity of Intertemporal Substitution Using Mortgage Notches”, *Review of Economic Studies*, 87 (2), pp. 656-690.

Bianchi, Francesco, Martin Lettau, and Sydney C. Ludvigson (2022), “Monetary Policy and Asset Valuation”, *Journal of Finance*, 77 (2), pp. 967-1017.

Bielecki, Marcin, Michał Brzoza-Brzezina, and Marcin Kolasa (2022), “Intergenerational Redistributive Effects of Monetary Policy”, *Journal of the European Economic Association*, 20 (2), pp. 549-580.

Bilbiie, Florin O. (2008), “Limited Asset Markets Participation, Monetary Policy and (Inverted) Aggregate Demand Logic”, *Journal of Economic Theory*, 140 (1), pp. 162-196.

Bilbiie, Florin O. (2020), "The New Keynesian Cross", *Journal of Monetary Economics*, 114, pp. 90-108.

Bilbiie, Florin O. (2024), "Monetary Policy and Heterogeneity: An Analytical Framework", *Review of Economic Studies*, forthcoming.

Bilbiie, Florin O., Sigurd Mølster Galaasen, Refet S. Gürkaynak, Mathis Mæhlum, and Krisztina Molnar (2025), "HANKSSON", mimeo, Cambridge University.

Blanchard, Olivier J. (1985), "Debt, Deficits, and Finite Horizons", *Journal of Political Economy*, 93 (2), pp. 223-247.

Bloomberg (2023), *Savings Lift Helps Blunt UK Household Mortgage Pain*, www.bloomberg.com/news/articles/2023-07-04/uk-households-better-off-as-savings-lift-blunts-mortgage-pain.

Borella, Margherita, Mariacristina De Nardi, Fang Yang, Johanna P. Torres Chain (2025), "Why Do Households Save and Work?", NBER Working Paper No. 33874.

Borio, Claudio (2021), "Navigating by r*: safe or hazardous?", SUERF Policy Note, No. 255, September.

Braun, Robin, Silvia Miranda-Agrippino and Tuli Saha (2025), "Measuring Monetary Policy in the UK: The UK monetary Policy Event-Study Database", *Journal of Monetary Economics*, 149, 103645.

Brülhart, Marius, Jonathan Gruber, Matthias Krapf, and Kurt Schmidheiny (2022), "Behavioral Responses to Wealth Taxes: Evidence from Switzerland", *American Economic Journal: Economic Policy*, 14 (4), pp. 111–150.

Burgert, Matthias, Johannes Eugster, and Victoria Otten (2024), "The Interest Rate Sensitivity of House Prices: International Evidence on its State Dependence", SNB Working Papers No. 1/2024.

Caballero, Ricardo J., Emmanuel Farhi, and Pierre-Olivier Gourinchas (2016), "Safe Asset Scarcity and Aggregate Demand", *American Economic Review*, 106 (5), pp. 513-518.

Caballero, Ricardo J., Emmanuel Farhi, and Pierre-Olivier Gourinchas (2017), "The Safe Assets Shortage Conundrum", *Journal of Economic Perspectives*, 31 (3), pp. 29-46.

Caballero, Ricardo J. and Alp Simsek (2024), "Central Banks, Stock Markets, and the Real Economy", *Annual Review of Financial Economics*, 16, pp. 179-205.

Caballero, Ricardo J., Tomás E. Caravello, and Alp Simsek (2025), "FCI-star", mimeo, MIT.

Caramp, Nicolas and Dejanir H. Silva (2021), "Monetary Policy and Wealth Effects: The Role of Risk and Heterogeneity", mimeo, UC Davis.

Caramp, Nicolas and Dejanir H. Silva (2023), "Fiscal Policy and the Monetary Transmission Mechanism", *Review of Economic Dynamics*, 51, pp. 716-746.

Caravello, Tomás E., Alisdair McKay, and Christian K. Wolf (2024), "Evaluating Policy Counterfactuals: A VAR-Plus Approach", mimeo, MIT.

Cavallino, Paolo and Damiano Sandri (2023), "The Open-economy ELB: Contractionary Monetary Easing and the Trilemma", *Journal of International Economics*, 140, 103691.

Cesa-Bianchi, Ambrogio, Richard Harrison, and Rana Sajedi (2023), "Global R*", CEPR Discussion Paper No. 18518.

Chodorow-Reich, Gabriel, Plamen T. Nenov, and Alp Simsek (2021), "Stock Market Wealth and the Real Economy: A Local Labor Market Approach", *American Economic Review*, 111 (5), pp. 1613-1657.

Cochrane, John H. and Monica Piazzesi (2002), "The Fed and Interest Rates: A High-Frequency Identification", *American Economic Review*, 92 (2), pp. 90-95.

Crawley, Edmund (2025), "Do Households Substitute Intertemporally? 10 Structural Shocks That Suggest Not", Finance and Economics Discussion Series No. 021, Washington: Board of Governors of the Federal Reserve System.

Daniel, Kent, Lorenzo Garlappi, and Kairong Xiao (2021), "Monetary Policy and Reaching for Income", *Journal of Finance*, 76 (3), pp. 1145-1193.

Debortoli, Davide and Jordi Galí (2024), "Heterogeneity and Aggregate Fluctuations: Insights from TANK Models", *NBER Macroeconomics Annual 39.*

De Nardi, Mariacristina, Eric French, and John Bailey Jones (2016), "Savings After Retirement: A Survey", *Annual Review of Economics*, 8 (1), pp. 177-204.

Del Negro, Marco, Marc Giannoni, and Christina Patterson (2013), "The Forward Guidance Puzzle", Federal Reserve Bank of New York Staff Report 574.

Den Haan, Wouter J., Pontus Rendahl, and Markus Riegler (2018), "Unemployment (Fears) and Deflationary Spirals", *Journal of the European Economic Association*, 16 (5), pp. 1281-1349.

Di Maggio, Marco, Amir Kermani, and Kaveh Majlesi (2020), "Stock Market Returns and Consumption", *Journal of Finance*, 75 (6), pp. 3175-3219.

Eggertsson, Gauti B., Neil R. Mehrotra, and Jacob A. Robbins (2019), "A Model of Secular Stagnation: Theory and Quantitative Evaluation", *American Economic Journal: Macroeconomics*, 11 (1), pp. 1-48.

Eggertsson, Gauti B., Ragnar Juelsrud, Lawrence H. Summers, and Ella G. Wold (2024), "Negative Nominal Interest Rates and the Bank Lending Channel", *Review of Economic Studies*, 91 (4), pp. 2201-2275.

Epstein, Larry G. and Stanley E. Zin (1989), "Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: a Theoretical Framework", *Econometrica*, 57 (4), pp. 937-996.

Fagereng, Andreas, Martin B. Holm, Benjamin Moll, and Gisle Natvik (2021), "Saving Behavior Across the Wealth Distribution: The Importance of Capital Gains", mimeo, LSE.

Fella, Giulio, Martin B. Holm, and Thomas Pugh (2024), "Saving After Retirement and Preferences for Residual Wealth", Bank of Canada Staff Working Paper No. 2024-21.

Fujiwara, Ippei and Yuki Teranishi (2008), "A Dynamic New Keynesian Life-Cycle Model: Societal Aging, Demographics, and Monetary Policy", *Journal of Economic Dynamics and Control*, 32 (8), pp. 2398-2427.

Gabaix, Xavier (2020), "A Behavioral New Keynesian Model", *American Economic Review*, 110 (8), pp. 2271–2327.

Galí, Jordi (2021), "Monetary Policy and Bubbles in a New Keynesian Model with Overlapping Generations", *American Economic Journal: Macroeconomics*, 13 (2), pp. 121-167.

Galí, Jordi, J. David López-Salido, and Javier Vallés (2007), "Understanding the Effects of Government Spending on Consumption", *Journal of the European Economic Association*, 5 (1), pp. 227-270.

Gertler, Mark (1999), "Government Debt and Social Security in a Life-Cycle Economy", *Carnegie-Rochester Conference Series on Public Policy*, 50, pp. 61-110.

Gertler, Mark and Peter Karadi (2015), "Monetary Policy Surprises, Credit Costs, and Economic Activity", *American Economic Journal: Macroeconomics*, 7 (1), pp. 44-76.

Gornemann, Nils, Keith Kuester, and Makoto Nakajima, (2014), "Doves for the Rich, Hawks for the Poor? Distributional Consequences of Systematic Monetary Policy", mimeo.

Greenwald, Daniel L., Matteo Leombroni, Hanno Lustig, and Stijn van Nieuwerburgh (2023), "Financial and Total Wealth Inequality with Declining Interest Rates", mimeo, NYU Stern.

Guerrieri, Veronica and Guido Lorenzoni (2017), "Credit Crises, Precautionary Savings, and the Liquidity Trap", *Quarterly Journal of Economics*, 132 (3), pp. 1427-1467.

Hansen, Stephen, Michael McMahon, and Matthew Tong (2019), "The Long-Run Information Effect of Central Bank Communication", *Journal of Monetary Economics*, 108, pp. 185-202.

Hanson, Samuel G. and Jeremy Stein (2015), "Monetary Policy and Long-Term Real Rates", *Journal of Financial Economics*, 115 (3), pp. 429-448.

Harvey, Campbell R. (1988), "The Real Term Structure and Consumption Growth", *Journal of Financial Economics*, 22 (2), pp. 305-333.

Hazell, Jonathon, Juan Herreño, Emi Nakamura, and Jón Steinsson (2022), "The Slope of the Phillips Curve: Evidence from U.S. States", *Quarterly Journal of Economics*, 137 (3), pp. 1299-1344.

Hillenbrand, Sebastian (2025), "The Fed and the Secular Decline in Interest Rates", *Review of Financial Studies*, 38 (4), pp. 981-1013.

Havránek, Tomáš (2015), "Measuring Intertemporal Substitution: The Importance of Method Choices and Selective Reporting", *Journal of the European Economic Association*, 13 (6), pp. 1180-1204.

Hofmann, Boris, Zehao Li, and Steve Pak Yeung Wu (2025), "Monetary Policy and the Secular Decline in Long-Term Interest Rates: A Global Perspective", BIS Working Paper No. 1252.

Holston, Kathryn, Thomas Laubach, and John C. Williams (2017), "Measuring the Natural Rate of Interest: International Trends and Determinants", *Journal of International Economics*, 108 (1), pp. S59-S75.

Jakobsen, Katrine, Kristian Jakobsen, Henrik Kleven, and Gabriel Zucman (2020), "Wealth

Taxation and Wealth Accumulation: Theory and Evidence From Denmark", *Quarterly Journal of Economics*, 135 (1), pp. 329-388.

Kaplan, Greg, Giovanni L. Violante, and Justin Weidner (2014), "The Wealthy Hand-to-Mouth", *Brookings Papers on Economic Activity,* 48 (Spring), pp. 77-153.

Kaplan, Greg, Benjamin Moll, and Giovanni L. Violante (2018), "Monetary Policy According to HANK", *American Economic Review*, 108 (3), pp. 697-743.

Lettau, Martin and Sydney Ludvigson (2001), "Consumption, Aggregate Wealth, and Expected Stock Returns", *Journal of Finance*, 56 (3), pp. 815-849.

Lucas, Robert E. and Leonard A. Rapping (1969), "Real Wages, Employment, and Inflation", *Journal of Political Economy*, 77 (5), pp. 721-754.

McKay, Alisdair, Emi Nakamura, and Jón Steinsson (2016), "The Power of Forward Guidance Revisited", *American Economic Review*, 106 (10), pp. 3133–3158.

McKay, Alisdair, Emi Nakamura, and Jón Steinsson (2017), "The Discounted Euler Equation: A Note", *Economica*, 84 (336), pp. 820-831.

McKay, Alisdair and Christian K. Wolf (2023), "What Can Time-Series Regressions Tell Us About Policy Counterfactuals?", *Econometrica*, 91 (5), pp. 1695-1725.

Mertens, Karel and Morten O. Ravn (2013), "The Dynamic Effects of Personal and Corporate Income Tax Changes in the United States", *American Economic Review*, 103 (4), pp. 1212-1247.

Mian, Atif, Ludwig Straub, and Amir Sufi (2021), "Indebted Demand", *Quarterly Journal of Economics*, 136 (4), pp. 2243-2307.

Miescu, Mirela S. (2023), "Forward Guidance and the Mitigation Hypothesis: An Empirical Assessment", mimeo, Lancaster University.

Moll, Benjamin (2020), "Comment on: Sources of U.S. Wealth Inequality: Past, Present, and Future", *NBER Macroeconomics Annual*, 35.

Nabar, Malhar (2011), "Targets, Interest Rates, and Household Saving in Urban China", IMF Working Paper No. 11/223.

Nakamura, Emi and Jón Steinsson (2018), "High-Frequency Identification of Monetary Non-Neutrality: The Information Effect", *Quarterly Journal of Economics*, 133 (3), pp. 1283-1330.

Obstfeld, Maurice (2023), "Natural and Neutral Real Interest Rates: Past and Future", NBER Working Paper No. 31949.

OECD (2021), *Pensions at a Glance*, Paris, OECD.

Oh, Hyunseung and Ricardo Reis (2012), "Targeted Transfers and the Fiscal Response to the Great Recession", *Journal of Monetary Economics*, 59, pp. S50-S64.

Pflueger, Carolin and Gianluca Rinaldi (2022), "Why Does the Fed Move Markets so Much? A Model of Monetary Policy and Time-Varying Risk Aversion", *Journal of Financial Economics*, 146 (1), pp. 71-89.

Piazzesi, Monika (2005), "Bond Yields and the Federal Reserve", *Journal of Political Economy*, 113 (2), pp. 311-344.

Plagborg-Møller, Mikkel and Christian K. Wolf (2021), "Local Projections and VARs Estimate the Same Impulse Responses", *Econometrica*, 89 (2), pp. 955-980.

Rachel, Łukasz and Lawrence H. Summers (2019), "On Falling Neutral Real Rates, Fiscal Policy,and the Risk of Secular Stagnation", *Brookings Papers on Economic Activity.*

Rajan, Raghuram (2013), "A Step in the Dark: Unconventional Monetary Policy after the Crisis", Andrew Crockett Memorial Lecture, BIS.

Ramey, Valerie A. (2016), "Macroeconomic Shocks and Their Propagation", *Handbook of Macroeconomics*, 2, pp. 71-162.

Ravn, Morten O. and Vincent Sterk (2017), "Job Uncertainty and Deep Recessions", *Journal of Monetary Economics*, 90, pp. 125-141.

Reis, Ricardo (2022), "Which R-star: Public Bonds or Private Investment? Measurement and Policy Implications", mimeo, LSE.

Rigon, Lorenzo (2022), "The Fed's Power over R-star", mimeo, Stanford University.

Ring, Marius A.K. (2024), "Wealth Taxation and Household Saving: Evidence from Assessment Discontinuities in Norway", *Review of Economic Studies*, forthcoming.

Roberts, John M. (1995), "New Keynesian Economics and the Phillips Curve", *Journal of Money, Credit, and Banking*, 27 (4), pp. 975-984.

Romer, Christina D. and David H. Romer (2004), "A New Measure of Monetary Shocks: Derivation and Implications", *American Economic Review*, 94 (4), pp. 1055-84.

Rotemberg, Julio J. (1982), "Monopolistic Price Adjustment and Aggregate Output", *Review of Economic Studies*, 49 (4), pp. 517-531.

Rungcharoenkitkul, Phurichai and Fabian Winkler (2023), "The natural rate of interest through a hall of mirrors", mimeo, Federal Reserve Board.

Shapiro, Adam Hale, Moritz Sudhof, and Daniel J. Wilson (2020), "Measuring News Sentiment", FRB San Francisco Working Paper 2017-01.

Skinner, Thomas J. and Jeromin Zettelmeyer (1995), "Long Rates and Monetary Policy: Is Europe Different?", mimeo, MIT.

Sterk, Vincent and Silvana Tenreyro (2018), "The Transmission of Monetary Policy Through Redistributions and Durable Purchases", *Journal of Monetary Economics*, 99, pp. 124-137.

Stock, James H. and Mark W. Watson (2012), "Disentangling the Channels of the 2007–09 Recession", *Brookings Papers on Economic Activity*, Spring, pp. 81-135.

Swanson, Eric T. (2024), "The Macroeconomic Effects of the Federal Reserve's Conventional and Unconventional Monetary Policies", *IMF Economic Review*, 72, pp. 1152-1184.

Turner, John A. and David Rajnes (2021), "Workers' Expectations About Their Future Social Security Benefits: How Realistic Are They?", *Social Security Bulletin*, 81 (4), pp. 1-17.

Uribe, Martin (2022), "The Neo-Fisher Effect: Econometric Evidence from Empirical and Optimizing Models", *American Economic Journal: Macroeconomics*, 14 (3), pp. 133-162.

Van den End, Jan Willem, Paul Konietschke, Anna Samarina, and Irina Stanga (2020), "Macroeconomic Reversal Rate: Evidence From a Nonlinear IS-curve", DNB Working Paper No. 684.

Weber, Michael (2018), "Cash Flow Duration and the Term Structure of Equity Returns", *Journal of Financial Economics*, 128 (3), pp. 486-503.

Werning, Ivan (2015), "Incomplete Markets and Aggregate Demand", NBER Working Paper No. 21448.

Wieland, Johannes F. and Mu-Jeung Yang (2020), "Financial Dampening", *Journal of Money, Credit and Banking*, 52 (1), pp. 79-113.

Woodford, Michael (2001), "Fiscal Requirements for Price Stability", *Journal of Money, Credit and Banking*, 33 (3), pp. 669-728.

Yaari, Menahem E. (1965), "Uncertain Lifetime, Life Insurance, and the Theory of the Consumer", *Review of Economic Studies*, 32 (2), pp. 137-150.

Yogo, Motohiro (2004), "Estimating the Elasticity of Intertemporal Substitution When Instruments Are Weak", *Review of Economics and Statistics*, 86 (3), pp. 797-810.

Appendix

A Additional robustness check for Figure 1

Here, we present an additional robustness check for Figure 1. In particular, we take a two-step approach. First, we orthogonalize the sentiment index with respect to the unemployment rate and the (logged) VIX index. Next, we use the residuals as an internal instrument in a VAR featuring our baseline variables (as visible in Figure 1). As shown in Figure A1, this produces very similar results to our baseline approach.

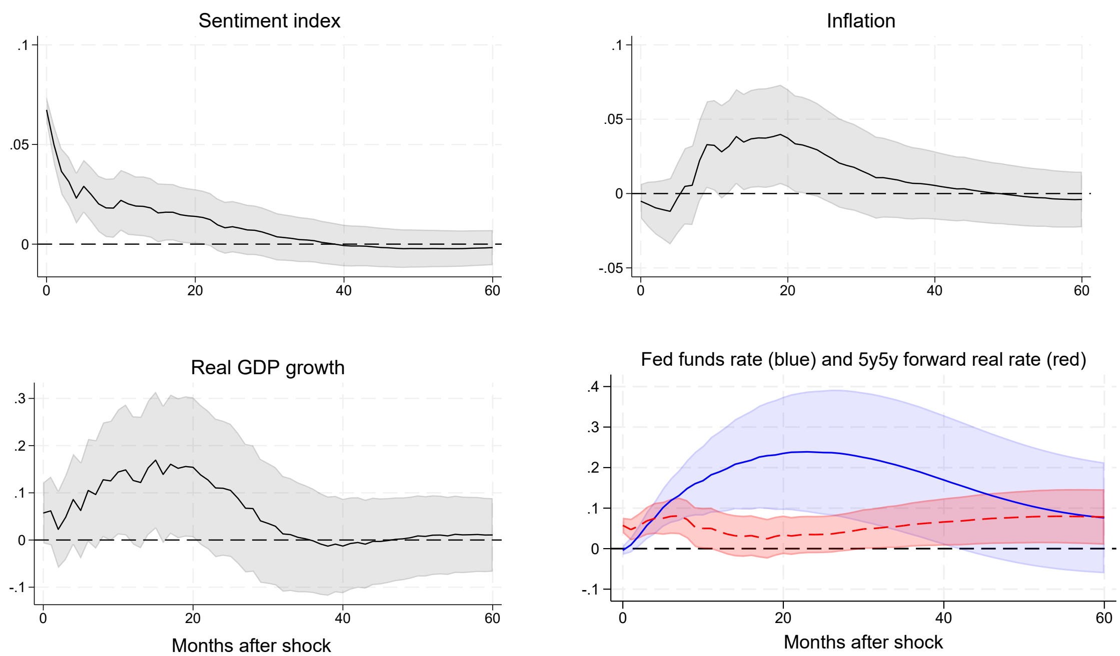

Figure A1: IRFs to a 1 standard deviation sentiment shock when first orthogonalizing the sentiment index with respect to the unemployment rate and VIX. Shaded areas represent 95% confidence bands.

B Equilibrium and steady state

The equilibrium of the model is described by the following equations:

$$
\begin{aligned}
y_t &= \frac{\vartheta c_t^w + (1-\vartheta)\, c_t^r}{1 - \frac{\theta}{2}\left(\pi_t - \bar{\pi}\right)^2} \\
c_t^r &= a_t^r \left[(\Gamma_t)^{-1} - 1 \right]^{-1} \\
\left(c_t^w\right)^{-\sigma} &= \beta_t \left\{ (1-\delta_1)\, \mathbb{E}_t \left[\left(c_{t+1}^w\right)^{-\sigma} r_{t+1} \right] + \delta_1 \mathbb{E}_t \left[\left(a_t^w r_{t+1}^w + \tau_{t+1}^r \right)^{-\sigma} \left(\Gamma_{t+1}\right)^{-\sigma} r_{t+1} \right] \right\} \\
\left[(\Gamma_t)^{-1} - 1 \right]^{\sigma} &= (1-\delta_2)\, \beta_t \mathbb{E}_t \left[r_{t+1} \left(r_{t+1}^r \Gamma_{t+1} \right)^{-\sigma} \right] \\
\left(\pi_t - \bar{\pi}\right) \pi_t &= \lambda \left[\left(\frac{y_t}{\vartheta A} \right)^{1+\varphi} - 1 \right] + \mathbb{E}_t \left[\Lambda_{t,t+1}^w \left(\pi_{t+1} - \bar{\pi} \right) \pi_{t+1} \frac{y_{t+1}}{y_t} \right] \\
\Lambda_{t,t+1}^w &= \beta_t \frac{(1-\delta_1) \left(c_{t+1}^w\right)^{-\sigma} + \delta_1 \left(a_t^w r_{t+1}^w + \tau_{t+1}^r \right)^{-\sigma} \left(\Gamma_{t+1}\right)^{-\sigma}}{\left(c_t^w\right)^{-\sigma}} \\
\Lambda_{t,t+1}^r &= (1-\delta_2)\, \beta \frac{\left(r_{t+1}^r \Gamma_{t+1} \right)^{-\sigma}}{\left(\Gamma_t^{-1} - 1 \right)^{\sigma}} \\
q_t b^g &= \vartheta a_t^w + (1-\vartheta)\, a_t^r \\
0 &= \vartheta \left(1 - \alpha_t^w\right) a_t^w + (1-\vartheta)\left(1-\alpha_t^r\right) a_t^r \\
r_{t+1}^r &= r_{t+1} + \left[\frac{1 + (1-\mu)\, q_{t+1}}{q_t} - r_{t+1} \right] \alpha_t^r \\
r_{t+1}^w &= r_{t+1} + \left[\frac{1 + (1-\mu)\, q_{t+1}}{q_t} - r_{t+1} \right] \alpha_t^w \\
1 &= \mathbb{E}_t \left[\Lambda_{t,t+1}^r \frac{1 + (1-\mu)\, q_{t+1}}{q_t} \right] \\
1 &= \mathbb{E}_t \left[\Lambda_{t,t+1}^w \frac{1 + (1-\mu)\, q_{t+1}}{q_t} \right] \\
a_t^r &= \left[(1-\delta_2)\, a_{t-1}^r r_t^r + \delta_2 \left(a_{t-1}^w r_t^w + \tau_t^r \right) \right] (1 - \Gamma_t) \\
i_t &= r \bar{\pi} \left(\frac{\mathbb{E}_t \left[\pi_{t+1}\right]}{\bar{\pi}} \right)^{1+\phi} e^{\varepsilon_t^i} \\
r_{t+1} &= \frac{i_t}{\pi_{t+1}}
\end{aligned}
$$

For a zero inflation target ($\bar{\pi} = 1$) and $\tau^r = 0$, the steady-state real rate r solves:

$$
\frac{y}{r - \left[(1-\delta_2)\,\beta r\right]^{\frac{1}{\sigma}}} \frac{1 + \delta_1 \frac{\left[(1-\delta_2)\beta r\right]^{\frac{1}{\sigma}}}{1-(1-\delta_2)\left[(1-\delta_2)\beta r\right]^{\frac{1}{\sigma}}}}{\left[\frac{1-(1-\delta_1)\beta r}{\delta_1 \beta r} \right]^{\frac{1}{\sigma}} + \frac{\delta_1}{1-(1-\delta_2)\left[(1-\delta_2)\beta r\right]^{\frac{1}{\sigma}}}} = \frac{b^g}{r - 1 + \mu}
$$

The left-hand side of this equation represents the steady-state demand for savings, while the right-hand side captures the steady-state value of the assets supplied to the economy. Let $\gamma \equiv (1-\vartheta)\, c^r / y$ denote the share of steady-state output consumed by retirees, and $\varsigma \equiv a^r / a^w$

denote the steady-state financial wealth of retirees relative to workers. Their equations are

$$\varsigma = \frac{\delta_2 \left[(1-\delta_2)\beta r\right]^{\frac{1}{\sigma}}}{1-(1-\delta_2)\left[(1-\delta_2)\beta r\right]^{\frac{1}{\sigma}}}$$

$$\gamma = \frac{\delta_1}{\left[\frac{1-(1-\delta_1)\beta r}{\delta_1 \beta r}\right]^{\frac{1}{\sigma}} \left\{1-(1-\delta_2)\left[(1-\delta_2)\beta r\right]^{\frac{1}{\sigma}}\right\} + \delta_1}$$

Now assume that $a_t^r = \varsigma a_t^w$, $r = \beta^{-1}$ and τ_{t+1}^r is unexpected. The log-linearized equilibrium equations are then given by:

$$\begin{aligned}
\hat{y}_t &= (1-\gamma)\hat{c}_t^w + \gamma \hat{c}_t^r \\
\hat{c}_t^r &= \hat{q}_t + \left[\beta (1-\delta_2)^{\frac{1}{\sigma}}\right]^{-1} \hat{\Gamma}_t \\
\hat{c}_t^w &= (1-\delta_1)\left(\mathbb{E}_t \hat{c}_{t+1}^w - \frac{1}{\sigma}\mathbb{E}_t \hat{r}_{t+1}\right) + \delta_1 \left(\hat{q}_t + \left[\beta (1-\delta_2)^{\frac{1}{\sigma}}\right]^{-1} \hat{\Gamma}_t\right) - \frac{1-\delta_1}{\sigma}\varepsilon_t^\beta \\
\hat{\Gamma}_t &= \beta (1-\delta_2)^{\frac{1}{\sigma}} \left[\mathbb{E}_t \hat{\Gamma}_{t+1} + \frac{\sigma-1}{\sigma}\mathbb{E}_t \hat{r}_{t+1} - \frac{1}{\sigma}\varepsilon_t^\beta\right] \\
\hat{\pi}_t &= \lambda (1+\varphi)\hat{y}_t + \beta \mathbb{E}_t \hat{\pi}_{t+1} \\
\hat{q}_t &= \beta (1-\mu)\mathbb{E}_t \hat{q}_{t+1} - \mathbb{E}_t \hat{r}_{t+1} \\
\hat{r}_{t+1} &= \hat{i}_t - \mathbb{E}_t \hat{\pi}_{t+1} - \varrho \\
\hat{i}_t &= \varrho + (1+\phi)\mathbb{E}_t \hat{\pi}_{t+1} + \varepsilon_t^i
\end{aligned}$$

with $a_t^w = a_t^r = q_t$, $r_{t+1}^r = r_{t+1}^w = r_{t+1}$, and $\varrho \equiv \log r$.

C Proofs of Propositions

C.1 Proof of Proposition 1

When $\phi = 0$, the equilibrium dynamics are captured by:

$$\begin{bmatrix} \hat{c}_t^w \\ \hat{\Gamma}_t \\ \hat{\pi}_t \\ \hat{q}_t \end{bmatrix} = \begin{bmatrix} 1-\delta_1 & \delta_1 & 0 & \beta\delta_1(1-\mu) \\ 0 & \beta(1-\delta_2)^{\frac{1}{\sigma}} & 0 & 0 \\ \kappa(1-\gamma)(1-\delta_1) & \kappa(1-\gamma)\delta_1 + \kappa\gamma & \beta & \kappa\beta(1-\mu)\left[(1-\gamma)\delta_1+\gamma\right] \\ 0 & 0 & 0 & \beta(1-\mu) \end{bmatrix} \begin{bmatrix} \mathbb{E}_t \hat{c}_{t+1}^w \\ \mathbb{E}_t \hat{\Gamma}_{t+1} \\ \mathbb{E}_t \hat{\pi}_{t+1} \\ \mathbb{E}_t \hat{q}_{t+1} \end{bmatrix}$$

The four eigenvalues of this system are $\left\{\beta, \beta(1-\mu), 1-\delta_1, \beta(1-\delta_2)^{1/\sigma}\right\}$. Since $\beta, \mu, \delta_1, \delta_2 \in (0,1)$ and $\sigma > 0$ then all four eigenvalues are less than 1 in modulus and the system has a unique stable solution.

C.2 Proof of Proposition 2

We start by deriving the "yield curve representation" of $\hat{y}_t$ and $\hat{\pi}_t$, equations (24) and (25). Assume $\phi = 0$, such that $\hat{r}_{t+1} = \varepsilon_t^i$. Solving q and Γ forward yields

$$\begin{aligned}\hat{q}_t &= -\mathbb{E}_t\hat{r}_{t+1} - \sum_{j=1}^{\infty}\beta^j\left(1-\mu\right)^j\mathbb{E}_t\hat{r}_{t+1+j}\\ \hat{\Gamma}_t &= \frac{\sigma-1}{\sigma}\sum_{j=0}^{\infty}\left[\beta\left(1-\delta_2\right)^{\frac{1}{\sigma}}\right]^{j+1}\mathbb{E}_t\hat{r}_{t+1+j} - \frac{1}{\sigma}\sum_{j=0}^{\infty}\left[\beta\left(1-\delta_2\right)^{\frac{1}{\sigma}}\right]^{j+1}\varepsilon_{t+j}^{\beta}\end{aligned}$$

Plug these into the workers' Euler equation to obtain

$$\begin{aligned}\hat{c}_t^w = {} & \left(1-\delta_1\right)\mathbb{E}_t\hat{c}_{t+1}^w - \frac{1}{\sigma}\mathbb{E}_t r_{t+1} + \delta_1\sum_{j=1}^{\infty}\beta^j\left[\frac{\sigma-1}{\sigma}\left(1-\delta_2\right)^{\frac{j}{\sigma}} - \left(1-\mu\right)^j\right]\mathbb{E}_t r_{t+1+j}\\ & - \frac{1}{\sigma}\left[\delta_1\sum_{j=1}^{\infty}\beta^j\left(1-\delta_2\right)^{\frac{j}{\sigma}}\mathbb{E}_t\varepsilon_{t+j}^{\beta} + \varepsilon_t^{\beta}\right]\end{aligned}$$

Through repeated substitution, we can express $\hat{c}_t^w$ as a function of future real interest rates and demand shocks only, as follows:

$$\hat{c}_t^w = -\sum_{j=0}^{\infty}\tilde{\psi}_j\mathbb{E}_t\hat{r}_{t+1+j} - \sum_{j=0}^{\infty}\tilde{\omega}_j\mathbb{E}_t\varepsilon_{t+j}^{\beta}$$

where $\tilde{\psi}_0 = \tilde{\omega}_0 = \frac{1}{\sigma}$ and

$$\begin{aligned}\tilde{\psi}_j &= \tilde{\psi}_{j-1}\left(1-\delta_1\right) - \delta_1\beta^j\left[\frac{\sigma-1}{\sigma}\left(1-\delta_2\right)^{\frac{j}{\sigma}} - \left(1-\mu\right)^j\right]\\ &= \left(1-\delta_1\right)^j\tilde{\psi}_0 - \delta_1\sum_{i=1}^{j}\left(1-\delta_1\right)^{j-i}\beta^i\left[\frac{\sigma-1}{\sigma}\left(1-\delta_2\right)^{\frac{i}{\sigma}} - \left(1-\mu\right)^i\right]\end{aligned}$$

$$\begin{aligned}\tilde{\omega}_j &= \tilde{\omega}_{j-1}\left(1-\delta_1\right) + \frac{\delta_1}{\sigma}\beta^j\left(1-\delta_2\right)^{\frac{j}{\sigma}}\\ &= \left(1-\delta_1\right)^j\tilde{\omega}_0 + \frac{\delta_1}{\sigma}\sum_{i=1}^{j}\left(1-\delta_1\right)^{j-i}\beta^i\left(1-\delta_2\right)^{\frac{i}{\sigma}}\end{aligned}$$

Now, plug the equations derived above for q and Γ into the retirees' consumption function to obtain a similar representation for $\hat{c}_t^r$:

$$\hat{c}_t^r = -\sum_{j=0}^{\infty} \bar{\psi}_j \mathbb{E}_t \hat{r}_{t+1+j} - \sum_{j=0}^{\infty} \bar{\omega}_j \mathbb{E}_t \varepsilon_{t+j}^{\beta}$$

where $\bar{\psi}_0 = \bar{\omega}_0 = \frac{1}{\sigma}$ and

$$\begin{aligned} \bar{\psi}_j &= \beta^j \left[(1-\mu)^j - \frac{\sigma-1}{\sigma} (1-\delta_2)^{\frac{j}{\sigma}} \right] \\ \bar{\omega}_j &= \frac{1}{\sigma} \beta^j (1-\delta_2)^{\frac{j}{\sigma}} \end{aligned}$$

Finally, we can use these representations for $\hat{c}_t^w$ and $\hat{c}_t^r$ to rewrite $\hat{y}_t$ as

$$\hat{y}_t = -\sum_{j=0}^{\infty} \psi_j \mathbb{E}_t \hat{r}_{t+1+j} - \sum_{j=0}^{\infty} \omega_j \mathbb{E}_t \varepsilon_{t+j}^{\beta}$$

where $\psi_j \equiv (1-\gamma)\tilde{\psi}_j - \gamma\bar{\psi}_j$ and $\omega_j \equiv (1-\gamma)\tilde{\omega}_j - \gamma\bar{\omega}_j$, which imply $\psi_0^y = \omega_0^y = \frac{1}{\sigma}$ and

$$\begin{aligned} \psi_j &= \frac{1}{\sigma}(1-\gamma)(1-\delta_1)^j - (1-\gamma)\delta_1 \sum_{i=1}^{j} (1-\delta_1)^{j-i} \beta^i \left[\frac{\sigma-1}{\sigma}(1-\delta_2)^{\frac{i}{\sigma}} - (1-\mu)^i \right] \\ &\quad - \gamma\beta^j \left[\frac{\sigma-1}{\sigma}(1-\delta_2)^{\frac{j}{\sigma}} - (1-\mu)^j \right] \\ &= (1-\delta_1)\psi_{j-1} - \frac{\sigma-1}{\sigma} \left[\delta_1 - \gamma(1-\delta_1) \frac{1-\beta(1-\delta_2)^{\frac{1}{\sigma}}}{\beta(1-\delta_2)^{\frac{1}{\sigma}}} \right] \beta^j (1-\delta_2)^{\frac{j}{\sigma}} \\ &\quad + \left[\delta_1 - \gamma(1-\delta_1) \frac{1-\beta(1-\mu)}{\beta(1-\mu)} \right] \beta^j (1-\mu)^j \\ \omega_j &= \frac{1}{\sigma}(1-\gamma)(1-\delta_1)^j - \frac{1}{\sigma}(1-\gamma)\delta_1 \sum_{i=1}^{j} (1-\delta_1)^{j-i} \beta^i (1-\delta_2)^{\frac{i}{\sigma}} + \gamma\frac{1}{\sigma}\beta^j (1-\delta_2)^{\frac{j}{\sigma}} \\ &= (1-\delta_1)\omega_{j-1} + \frac{1}{\sigma} \left[\delta_1 - \gamma(1-\delta_1) \frac{1-\beta(1-\delta_2)^{\frac{1}{\sigma}}}{\beta(1-\delta_2)^{\frac{1}{\sigma}}} \right] \beta^j (1-\delta_2)^{\frac{j}{\sigma}} \end{aligned}$$

Finally, solve $\hat{\pi}_t$ forward to obtain $\hat{\pi}_t = \kappa \sum_{j=0}^{\infty} \beta^j \mathbb{E}_t \hat{y}_{t+j}$. Then use the representation derived above to express $\hat{\pi}_t$as follows:

$$\hat{\pi}_t = -\sum_{j=0}^{\infty} \psi_j^{\pi} \mathbb{E}_t \hat{r}_{t+1+j} - \sum_{j=0}^{\infty} \omega_j^{\pi} \mathbb{E}_t \varepsilon_{t+j}^{\beta}$$

where $\psi_0^\pi = \kappa\psi_0$, $\omega_0^\pi = \kappa\omega_0$, and

$$\begin{aligned}\psi_j^\pi &= \beta\psi_{j-1}^\pi + \kappa\psi_j \\ \omega_j^\pi &= \beta\omega_{j-1}^\pi + \kappa\omega_j\end{aligned}$$

Proof of 2.a If $\delta_1 = 0$, then $\psi_j^y = \frac{1}{\sigma}$ and $\psi_j^\pi = \frac{\kappa}{\sigma}\frac{1-\beta^{j+1}}{1-\beta}$, for all $j \geq 0$. If $\delta_1 > 0$, then

$$\begin{aligned}\psi_1^y &= \frac{1}{\sigma} - \frac{1}{\sigma}\left[\delta_1 + \gamma\left(1-\delta_1\right)\right]\left[1 - \beta\left(1-\delta_2\right)^{\frac{1}{\sigma}}\right] - \left[\delta_1 + \gamma\left(1-\delta_1\right)\right]\left[\beta\left(1-\delta_2\right)^{\frac{1}{\sigma}} - \beta\left(1-\mu\right)\right] \\ \psi_2^y &= \frac{1}{\sigma} - \frac{1}{\sigma}\left\{\left[1-\delta_1+\beta\left(1-\delta_2\right)^{\frac{1}{\sigma}}\right]\left[\delta_1+\gamma\left(1-\delta_1\right)\right]+\delta_1\right\}\left[1-\beta\left(1-\delta_2\right)^{\frac{1}{\sigma}}\right] \\ &\quad - \left\{\left[\delta_1+\gamma\left(1-\delta_1\right)\right]\left[\beta\left(1-\delta_2\right)^{\frac{1}{\sigma}}+\beta\left(1-\mu\right)\right]+\delta_1\left(1-\gamma\right)\left(1-\delta_1\right)\right\}\left[\beta\left(1-\delta_2\right)^{\frac{1}{\sigma}}-\beta\left(1-\mu\right)\right] \\ \psi_3^y &= \ldots\end{aligned}$$

If $\delta_2 < \mu$, then they are all strictly smaller than $\frac{1}{\sigma}$. Since $\psi_j < \frac{1}{\sigma}$ for all $j \geq 1$ and $\psi_j^\pi = \kappa\sum_{i=0}^{j}\beta^{j-i}\psi_i$, then also $\psi_j^\pi < \frac{\kappa}{\sigma}\frac{1-\beta^{j+1}}{1-\beta}$ for all $j \geq 1$.

Proof of 2.b Solve ψ_j backward to obtain:

$$\psi_j = \left(1-\delta_1\right)^j\psi_0 - \sum_{i=1}^{j}\left(1-\delta_1\right)^{j-i}\xi_i^\psi$$

where $\xi_i^\psi \equiv \frac{\sigma-1}{\sigma}\left[\delta_1 - \frac{\gamma}{(1-\delta_1)^{-1}}\frac{1-\beta(1-\delta_2)^{\frac{1}{\sigma}}}{\beta(1-\delta_2)^{\frac{1}{\sigma}}}\right]\beta^j\left(1-\delta_2\right)^{\frac{i}{\sigma}} - \left[\delta_1 - \frac{\gamma}{(1-\delta_1)^{-1}}\frac{1-\beta(1-\mu)}{\beta(1-\mu)}\right]\beta^i\left(1-\mu\right)^i$. Now, since $\lim_{i\to\infty}\xi_i^\psi = 0$ then also $\lim_{j\to\infty}\psi_j = 0$, provided that $\delta_1 > 0$. Since $\psi_j^\pi = \kappa\sum_{i=0}^{j}\beta^{j-i}\psi_i$, then also $\lim_{j\to\infty}\psi_j^\pi = 0$.

Proof of 2.c The derivative of ψ_j with respect to σ is

$$\begin{aligned}\frac{\partial\psi_j}{\partial\sigma} &= -\frac{1}{\sigma^2}\left(1-\gamma\right)\left(1-\delta_1\right)^j - \gamma\beta^j\left[\frac{1}{\sigma^2}\left(1-\delta_2\right)^{\frac{j}{\sigma}} + \frac{\sigma-1}{\sigma}\left(1-\delta_2\right)^{\frac{j}{\sigma}}\left[-\ln\left(1-\delta_2\right)\right]\frac{j}{\sigma^2}\right] \\ &\quad - \left(1-\gamma\right)\delta_1\sum_{i=1}^{j}\left(1-\delta_1\right)^{j-i}\beta^i\left[\frac{1}{\sigma^2}\left(1-\delta_2\right)^{\frac{i}{\sigma}} + \frac{\sigma-1}{\sigma}\left(1-\delta_2\right)^{\frac{i}{\sigma}}\left[-\ln\left(1-\delta_2\right)\right]\frac{i}{\sigma^2}\right]\end{aligned}$$

Since all of the terms are negative (recall that $\delta_2 \in [0,1]$, therefore $-\ln\left(1-\delta_2\right) > 0$), then $\partial\psi_j/\partial\sigma < 0$. The derivative of ψ_j^π with respect to σ is $\partial\psi_j^\pi/\partial\sigma = \kappa\sum_{i=0}^{j}\beta^{j-i}\partial\psi_i/\partial\sigma$, which is therefore also negative. Finally, note that

$$\lim_{\sigma\to+\infty}\psi_j = -\left(1-\gamma\right)\delta_1\sum_{i=1}^{j}\left(1-\delta_1\right)^{j-i}\beta^i\left[1-\left(1-\mu\right)^i\right] - \gamma\beta^j\left[1-\left(1-\mu\right)^j\right] < 0$$

which is strictly negative, as $\mu \in (0,1]$. Since ψ_j is continuous in σ and its limit for $\sigma \to +\infty$ is negative, then $\exists\sigma < +\infty$ such that $\psi_j < 0$. Similarly, since $\psi_j^\pi = \kappa \sum_{i=0}^{j} \beta^{j-i}\psi_i$ is continuous in σ and $\lim_{\sigma\to+\infty} \psi_j^\pi = \kappa \sum_{i=0}^{j} \beta^{j-i} \lim_{\sigma\to+\infty} \psi_i < 0$, then $\exists\sigma < +\infty$ such that $\psi_j^\pi < 0$.

Proof of 2.d The derivatives of ψ_j with respect to δ_2 and μ are

$$\frac{\partial\psi_j}{\partial\delta_2} = \frac{1}{\sigma}\frac{\sigma-1}{\sigma}\frac{(1-\gamma)\,\delta_1 \sum_{i=1}^{j} (1-\delta_1)^{j-i}\,\beta^i i\,(1-\delta_2)^{\frac{i}{\sigma}} + \gamma\beta^j j\,(1-\delta_2)^{\frac{j}{\sigma}}}{1-\delta_2} > 0,$$
$$\frac{\partial\psi_j}{\partial\mu} = -\frac{(1-\gamma)\,\delta_1 \sum_{i=1}^{j} (1-\delta_1)^{j-i}\,\beta^i i\,(1-\mu)^i + \gamma\beta^j j\,(1-\mu)^j}{1-\mu} < 0.$$

Since $\psi_j^\pi = \kappa \sum_{i=0}^{j} \beta^{j-i}\psi_i$, the derivatives of ψ_j^π with respect to δ_2 and μ are

$$\frac{\partial\psi_j^\pi}{\partial\delta_2} = \kappa \sum_{i=0}^{j} \beta^{j-i}\frac{\partial\psi_i}{\partial\delta_2} > 0,$$
$$\frac{\partial\psi_j^\pi}{\partial\mu} = \kappa \sum_{i=0}^{j} \beta^{j-i}\frac{\partial\psi_i}{\partial\mu} < 0.$$

C.3 Proof of Proposition 3

We start by deriving equation (28). Assume $\phi = 0$ and $\mathbb{E}_t\varepsilon_{t+1}^i = \rho_i\varepsilon_t^i$. Then $\hat{y}_t = -\sum_{j=0}^{\infty} \psi_j \mathbb{E}_t \hat{r}_{t+1+j} = -\sum_{j=0}^{\infty} \psi_j\,(\rho_i)^j\,\varepsilon_t^i = -\Psi\,(\rho_i)\,\varepsilon_t^i$, where

$$\begin{aligned}
\Psi(\rho_i) &= \frac{1}{\sigma} + \sum_{j=1}^{\infty}(1-\delta_1)\psi_{j-1}\rho_i^j + \delta_1\left[1-\gamma\frac{1-\delta_1}{\delta_1}\frac{1-\beta(1-\mu)}{\beta(1-\mu)}\right]\sum_{j=1}^{\infty}(\beta\rho_i)^j(1-\mu)^j \\
&\quad - \delta_1\frac{\sigma-1}{\sigma}\left[1-\gamma\frac{1-\delta_1}{\delta_1}\frac{1-\beta(1-\delta_2)^{\frac{1}{\sigma}}}{\beta(1-\delta_2)^{\frac{1}{\sigma}}}\right]\sum_{j=1}^{\infty}(\beta\rho_i)^j(1-\delta_2)^{\frac{j}{\sigma}} \\
&= \frac{1}{\sigma} - (1-\delta_1)\rho_i\Psi(\rho_i) + \delta_1\left[1-\gamma\frac{1-\delta_1}{\delta_1}\frac{1-\beta(1-\mu)}{\beta(1-\mu)}\right]\frac{\beta\rho_i(1-\mu)}{1-\beta\rho_i(1-\mu)} \\
&\quad - \delta_1\frac{\sigma-1}{\sigma}\left[1-\gamma\frac{1-\delta_1}{\delta_1}\frac{1-\beta(1-\delta_2)^{\frac{1}{\sigma}}}{\beta(1-\delta_2)^{\frac{1}{\sigma}}}\right]\frac{\beta\rho_i(1-\delta_2)^{\frac{1}{\sigma}}}{1-\beta\rho_i(1-\delta_2)^{\frac{1}{\sigma}}} \\
&= \frac{1}{\sigma}\frac{(1-\gamma)(1-\delta_1)}{1-\rho_i(1-\delta_1)} - \left[\gamma + \frac{\delta_1(1-\gamma)}{1-\rho_i(1-\delta_1)}\right]\left[\frac{\sigma-1}{\sigma}\frac{1}{1-\beta\rho_i(1-\delta_2)^{\frac{1}{\sigma}}} - \frac{1}{1-\beta\rho_i(1-\mu)}\right]
\end{aligned}$$

Now, if $\delta_1 = 0$ then $\Psi\,(\rho_i) = \frac{1}{\sigma}\frac{1}{1-\rho_i}$. This expression is strictly positive, for all $\rho_i \in [0,1)$, and diverges to ∞ as $\rho_i \uparrow 1$.

C.4 Proof of Proposition 4

Notice that

$$\lim_{\rho_i \to 1} \Psi(\rho_i) = \frac{1}{\sigma} \frac{(1-\gamma)(1-\delta_1)}{\delta_1} - \frac{\sigma-1}{\sigma} \frac{1}{1-\beta(1-\delta_2)^{\frac{1}{\sigma}}} + \frac{1}{1-\beta(1-\mu)}$$

which is finite, since $\delta_1 > 0$, $\beta(1-\delta_2)^{\frac{1}{\sigma}} < 1$ and $\beta(1-\mu) < 1$

The derivative of Ψ with respect to ρ_i evaluated at $\rho_i = 1$ is

$$\begin{aligned} \left.\frac{\partial \Psi}{\partial \rho_i}\right|_{\rho_i=1} = & \frac{1-\gamma}{\sigma} \left(\frac{1-\delta_1}{\delta_1}\right)^2 - \frac{\sigma-1}{\sigma} \frac{\beta(1-\delta_2)^{\frac{1}{\sigma}}}{\left[1-\beta(1-\delta_2)^{\frac{1}{\sigma}}\right]^2} + \frac{\beta(1-\mu)}{[1-\beta(1-\mu)]^2} \\ & - (1-\gamma)\frac{(1-\delta_1)}{(\delta_1)^2}\left[\frac{\sigma-1}{\sigma}\frac{1}{1-\beta(1-\delta_2)^{\frac{1}{\sigma}}} - \frac{1}{1-\beta(1-\mu)}\right] \end{aligned}$$

By setting, $\left.\frac{\partial \Psi}{\partial \rho_i}\right|_{\rho_i=1} = 0$ we obtain an implicit expression for σ^*:

$$\sigma^* = 1 + \frac{\left[1-\beta(1-\delta_2)^{\frac{1}{\sigma^*}}\right][1-\beta(1-\mu)]}{\beta(1-\delta_2)^{\frac{1}{\sigma^*}} - \beta(1-\mu)} \frac{(1-\gamma)\frac{1-\delta_1}{\delta_1}\left[1-\delta_1 + \frac{1}{1-\beta(1-\mu)}\right] + \delta_1 \frac{\beta(1-\mu)}{[1-\beta(1-\mu)]^2}}{(1-\gamma)\frac{1-\delta_1}{\delta_1} + \delta_1 \frac{1-\beta(1-\delta_2)^{\frac{1}{\sigma^*}}\beta(1-\mu)}{\left[1-\beta(1-\delta_2)^{\frac{1}{\sigma^*}}\right][1-\beta(1-\mu)]}}$$

Therefore, $\left.\frac{\partial \Psi}{\partial \rho_i}\right|_{\rho_i=1} > 0$ iff $\sigma < \sigma^*$ and $\left.\frac{\partial \Psi}{\partial \rho_i}\right|_{\rho_i=1} < 0$ iff $\sigma > \sigma^*$. This proves 4.b and the second part of 4.a. To prove 4.c, and the first part of 4.a, we set $\Psi(1) = 0$ and solve for σ to obtain and implicit expression for σ^{**} :

$$\sigma^{**} = 1 + \frac{\left[1-\beta(1-\delta_2)^{\frac{1}{\sigma^{**}}}\right][1-\beta(1-\mu)]}{\beta(1-\delta_2)^{\frac{1}{\sigma^{**}}} - \beta(1-\mu)} \left[(1-\gamma)\frac{1-\delta_1}{\delta_1} + \frac{1}{1-\beta(1-\mu)}\right]$$

Therefore, $\Psi(1) > 0$ iff $\sigma < \sigma^{**}$ and $\Psi(1) < 0$ iff $\sigma > \sigma^{**}$. It's easy to show that $\sigma^{**} - \sigma^* > 0$. Since Ψ is continuous in ρ_i, then $\exists \epsilon > 0$ such that the claims just proved for $\Psi(1)$ also apply to $\Psi(\rho_i)$, for $\rho_i \in (1-\epsilon, 1]$.

D Region of determinacy

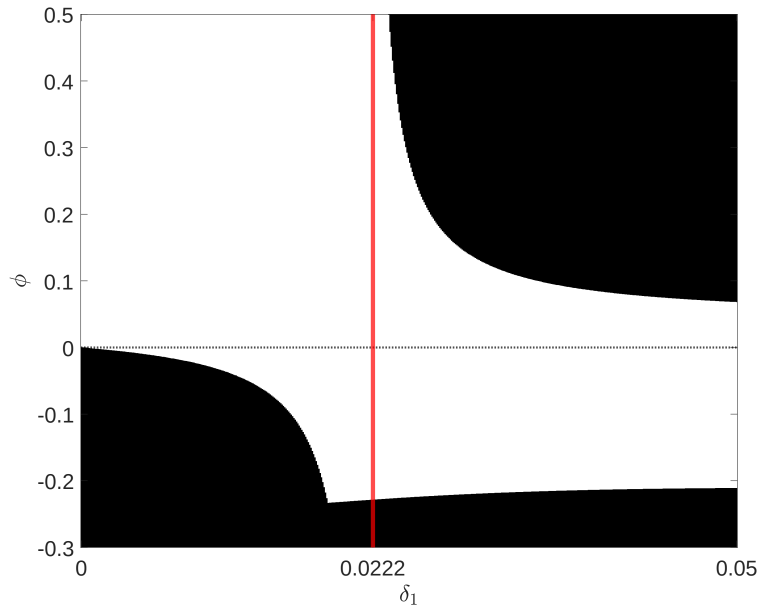

Figure D2: Visual representation of our model's region of determinacy (in white) as a function of ϕ and δ_1; red line represents our baseline choice for $\delta_1 = 1/45$. Other parameters calibrated as in footnote 20.

E Do the effects of monetary policy shocks vary with persistence?

An important feature of FLANK – distinguishing it from the standard New Keynesian model – is that rather transient monetary policy shocks do more to affect real activity in the conventional direction, than more persistent shocks (such as those associated with forward guidance). These contrasting predictions open the door to an empirical test, which is what we do here.

For the U.S., it has been observed (by, e.g., McKay and Wolf (2023)) that the monetary policy shock series by Romer and Romer (2004, "RR") rapidly leads to a short-lived peak in the Federal funds rate, while the shock of Gertler and Karadi (2015, "GK") captures a different dimension of monetary policy, more inclusive of "forward guidance", with the shock inducing a more delayed and persistent response in the policy rate.

To see whether these different shocks also yield different responses in activity, we generate IRFs by following Plagborg-Møller and Wolf (2021) in ordering the shock first in a recursive VAR (estimated at the monthly frequency) that also contains the Federal funds rate, the natural log of the CPI, the natural log of the commodity price index, and the natural log of Industrial

Production (our measure of real activity[33]). All data are taken from Ramey (2016), who – in turn – used the updated RR series of Wieland and Yang (2020).

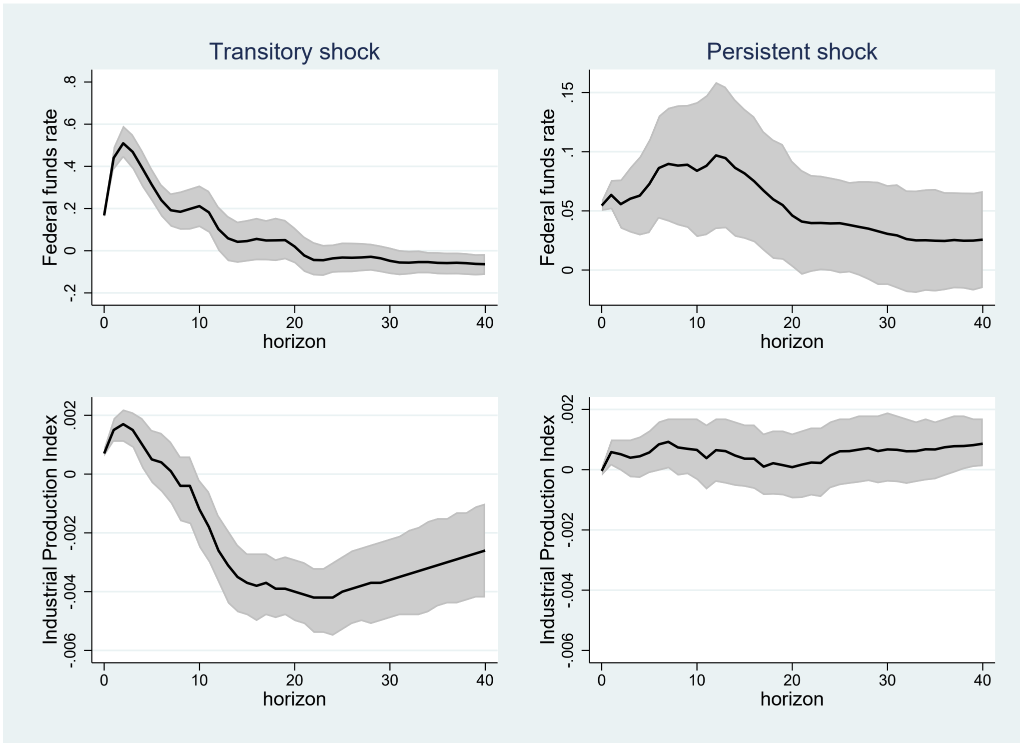

Figure E3: Response of Federal funds rate and industrial production index to monetary policy shocks of different persistence (transitory shock = RR; persistent shock = GK). VAR estimated at monthly frequency. Shaded areas represent 16th and 84th percentile confidence bands, obtained via bootstrapping.

As Figure D2 shows, the RR shock – which induces a more transient increase in the Federal funds rate compared to the GK shock – leads to a stronger contraction in real activity; using a different specification, McKay and Wolf (2023, Appendix C.2) obtain a similar finding, pointing towards some robustness of the bottomline conclusion.[34] While this is strongly at odds with the standard New Keynesian model (where the potency of monetary policy shocks is *increasing* in persistence – even under a discounted Euler equation), the apparent emergence of a "persistence-potency trade-off" is more in line with our FLANK model.

An alternative interpretation is to question the validity of, especially, the more persistent shock (which we simply took from Gertler and Karadi (2015)). It is however interesting to

[33]Looking at the response of the unemployment rate leads to the same conclusion.

[34]While McKay and Wolf (2023) find more evidence of the GK shock lowering activity, it is striking how – also in their specification – the RR shock is more potent on output, despite that impulse giving rise to a much smaller area under curve of the interest rate response. In the standard New Keynesian model, the strength of the activity response should be *increasing* in the area under the curve of the interest rate response (recall Proposition 3).

observe that other studies (using different shock series) have also found evidence to suggest that the potency of monetary policy shocks on activity decreases with persistence. Examples include Miescu (2023, for the U.S.), Swanson (2024, for the U.S.), and Braun et al. (2025, for the U.K.). A similar result is reported in Uribe (2022, for the U.S.), who takes a rather different approach to shock identification (not relying on high-frequency methods, but exploiting cointegrating relationships). FLANK is furthermore consistent with the observation that yield curve inversions tend to be followed by economic slowdowns (Harvey, 1988).[35] Our model suggests that such inversions might be more than "just" a recession signal, pointing to a potential causal link stemming from the notion that the combination of high short-term rates with lower long-term rates is contractionary on both ends of the curve.

Regardless of this, further empirical work aimed at identifying the causal impact of highly persistent monetary policy shocks would be desirable – also since it can help in the construction of policy counterfactuals (McKay and Wolf, 2023; Caravello et al., 2024).

F Extension with physical capital

In the extension with physical capital, good-producing firms operate the production function:

$$y_t = A\left(\ell_t\right)^{\eta}\left(k_{t-1}\right)^{1-\eta},$$

where $\eta \in (0,1)$. All capital is owned by households who rent it to good-producing firms and invest to produce new capital. Investment is subject to a quadratic adjustment cost, such that producing inv_t new units of capital costs $inv_t + \frac{\iota}{2}\left(\frac{inv_t}{k_{t-1}} - \nu\right)^2 k_{t-1}$ units of output, with $\iota > 0$. Existing capital depreciates at rate $\nu \in [0,1]$. Hence, its law of motion is $k_t = inv_t + (1-\nu)\, k_{t-1}$.

The optimal investment policy is:

$$inv_t = \left(\nu + \frac{q_t^k - 1}{\iota}\right) k_{t-1},$$

where q_t^k denotes the price of capital which is determined by the households' first order conditions:

$$1 = \mathbb{E}_t\left[\Lambda^r_{t,t+1}\frac{u_t + (1-\nu)\, q^k_{t+1}}{q^k_t}\right],$$
$$1 = \mathbb{E}_t\left[\Lambda^w_{t,t+1}\frac{u_t + (1-\nu)\, q^k_{t+1}}{q^k_t}\right],$$

[35] Also see Ang et al. (2006), who find that short-term rates have most predictive power when it comes to forecasting future GDP. This is again in line with our FLANK model, which implies that the short-term rate bears the least ambiguous relation to activity.

where $u_t = \frac{1-\eta}{\eta}\frac{\epsilon-1}{\epsilon}\chi\left(\frac{y_t}{\vartheta A}\right)^{1+\frac{1+\varphi}{\eta}}\left(\frac{1}{k_{t-1}}\right)^{1+(1-\eta)\frac{1+\varphi}{\eta}}$ is the rental rate of capital. The returns on the portfolios of assets held by retirees and workers are:

$$r_{t+1}^r = r_{t+1} + \left[\frac{1+(1-\mu)\,q_{t+1}}{q_t} - r_{t+1}\right]\alpha_t^r + \left[\frac{1+(1-\nu)\,q_{t+1}^k}{q_t^k} - r_{t+1}\right]\breve{\alpha}_t^r,$$
$$r_{t+1}^w = r_{t+1} + \left[\frac{1+(1-\mu)\,q_{t+1}}{q_t} - r_{t+1}\right]\alpha_t^w + \left[\frac{1+(1-\nu)\,q_{t+1}^k}{q_t^k} - r_{t+1}\right]\breve{\alpha}_t^w,$$

where α_t^j denotes the share of household-j wealth invested in long-term bonds and $\breve{\alpha}_t^j$ the share invested in capital. Market clearing in the asset markets requires:

$$q_t b^g = \vartheta\alpha_t^w a_t^w + (1-\vartheta)\,\alpha_t^r a_t^r,$$
$$q_t^k k_t = \vartheta\breve{\alpha}_t^w a_t^w + (1-\vartheta)\,\breve{\alpha}_t^r a_t^r,$$
$$0 = \vartheta\,(1-\alpha_t^w-\breve{\alpha}_t^w)\,a_t^w + (1-\vartheta)\,(1-\alpha_t^r-\breve{\alpha}_t^r)\,a_t^r,$$

while goods market clearing implies:

$$y_t = \frac{\vartheta c_t^w + (1-\vartheta)\,c_t^r + inv_t + \frac{\iota}{2}\left(\frac{inv_t}{k_{t-1}} - \nu\right)^2 k_{t-1}}{1-\frac{\theta}{2}\,(\pi_t-\bar{\pi})^2}.$$

Finally, the real marginal cost of production is $\frac{\chi}{\eta}\left(\frac{y_t}{\vartheta A}\right)^{\frac{1+\varphi}{\eta}}\left(\frac{1}{k_{t-1}}\right)^{(1-\eta)\frac{1+\varphi}{\eta}}$. Hence, the Phillips curve becomes:

$$(\pi_t-\bar{\pi})\,\pi_t = \lambda\left[\frac{\chi}{\eta}\left(\frac{y_t}{\vartheta A}\right)^{\frac{1+\varphi}{\eta}}\left(\frac{1}{k_{t-1}}\right)^{(1-\eta)\frac{1+\varphi}{\eta}} - 1\right] + \mathbb{E}_t\left[\Lambda_{t,t+1}^w\,(\pi_{t+1}-\bar{\pi})\,\pi_{t+1}\frac{y_{t+1}}{y_t}\right].$$

All other equations remain unchanged.

For a zero inflation target ($\bar{\pi}=1$) and $\tau^r=0$, the steady-state real rate r solves:

$$\frac{y-\nu k}{r-[(1-\delta_2)\,\beta r]^{\frac{1}{\sigma}}}\,\frac{1+\frac{\delta_1[(1-\delta_2)\beta r]^{\frac{1}{\sigma}}}{1-(1-\delta_2)[(1-\delta_2)\beta r]^{\frac{1}{\sigma}}}}{\left[\frac{1-\beta(1-\delta_1)r}{\beta\delta_1 r}\right]^{\frac{1}{\sigma}} + \frac{\delta_1}{1-(1-\delta_2)[(1-\delta_2)\beta r]^{\frac{1}{\sigma}}}} = \frac{b^g}{r-1+\mu} + k,$$

where:

$$k = (\eta)^{\frac{1}{1+\varphi}}\left(\frac{\epsilon-1}{\epsilon}\,\frac{1-\eta}{r-1+\nu}\right)^{\frac{1}{\eta}},$$
$$y = \vartheta A\,(\eta)^{\frac{1}{1+\varphi}}\left(\frac{\epsilon-1}{\epsilon}\,\frac{1-\eta}{r-1+\nu}\right)^{\frac{1-\eta}{\eta}}.$$