Optimal Debt Policy and Liquidity Taxation

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Abstract

U.S. public debt levels appear unsustainable, yet strong investor demand keeps interest rates on Treasuries low. Policymakers face a trade-off: Reduce long-run tax burdens through debt reduction or satiate safe asset demand through debt expansion. To quantify this trade-off, I solve for the optimal Ramsey policy of a government that issues safe assets (i.e., risk-free and liquid), raises distortionary taxes, and insures against aggregate risk. The calibrated model matches historical U.S. debt policy. Despite low interest rates, the model shows that recent crises were overly debt-financed and recommends reducing the debt to 65% of GDP over the next 10 years.

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1 Introduction

Public debt in the United States is elevated and growing. From 1980 to 2020 total public debt grew from 30% to 100% of GDP. Aggressive policy reactions to the Great Financial Crisis and the COVID-19 pandemic brought public debt to levels on par with World War II. Public debt expansion during this period pushed interest payments from 2.5% to 3.6% of GDP. Politicians, policymakers, investors, and academics alike worry that paying down the public debt requires substantial tax increases. Given these concerns does the U.S. have too much public debt? Did the U.S. excessively issue debt during recent crises?

Standard optimal debt models suggest U.S. public debt levels are well above the optimal level. Models with aggregate fiscal risk often find optimal debt levels at or below zero (Aiyagari et al. (2002) and Bhandari et al. (2017*a*)). Negative optimal debt levels emerge from the government's desire to smooth and lower future labor taxes. By amassing a "war-chest" of assets, i.e., negative debt, policymakers can fund all future expenditure and insure against future tax increases. Models with idiosyncratic risk (e.g., Aiyagari and McGrattan (1998) and Bayas-Erazo (2023)) find positive optimal debt levels but cannot reconcile the rise during crises.

The high public debt levels and low interest rates in the United States are inconsistent with these models of optimal debt issuance (Yared (2019)). Standard models predict that public debt expansions raise interest rates. Yet as public debt levels rose dramatically over the previous fifty years interest rates on Treasury bonds fell. Throughout this period, the real interest rate r generally remained below the growth rate g (Blanchard (2019)). Because tax-base growth outpaced interest payments on the debt, that is $r < g$, the U.S. avoided fiscal adjustment. In his provocative address Blanchard (2019) argues that additional U.S. debt may have "no fiscal cost." Despite recent interest rate hikes, this interest-growth rate differential will likely persist (CBO (2024) and Arslanalp and Eichengreen (2023)).

Introducing liquidity demand to standard debt models reconciles the observed high debt levels and low interest rates.[1] The United States sells public debt at low interest rates because it's debt is a safe asset (Caballero, Farhi and Gourinchas (2017); Barro et al. (2022)). Safe assets provide liquidity by uniquely facilitating trade – during crises (Woodford (1990); Holmström and Tirole (1998)), between borrowing-constrained agents (Aiyagari and McGrattan (1998); Brunnermeier, Merkel and Sannikov (2024)) and across generations (Diamond (1965); Blanchard (2019)). Holders of Treasury bonds are partially compensated by these liquidity services. Treasury bonds are more liquid than similarly safe private bonds (Krishnamurthy and Vissing-Jorgensen (2012)) and sovereign debt (Du, Im and Schreger (2018)). Recent work (e.g., Blanchard (2019), Mehrotra and

[1] In a quantitative model, Jiang et al. (2024) show that sufficiently high liquidity demand can explain the simultaneously high public debt and low interest rates in the United States.

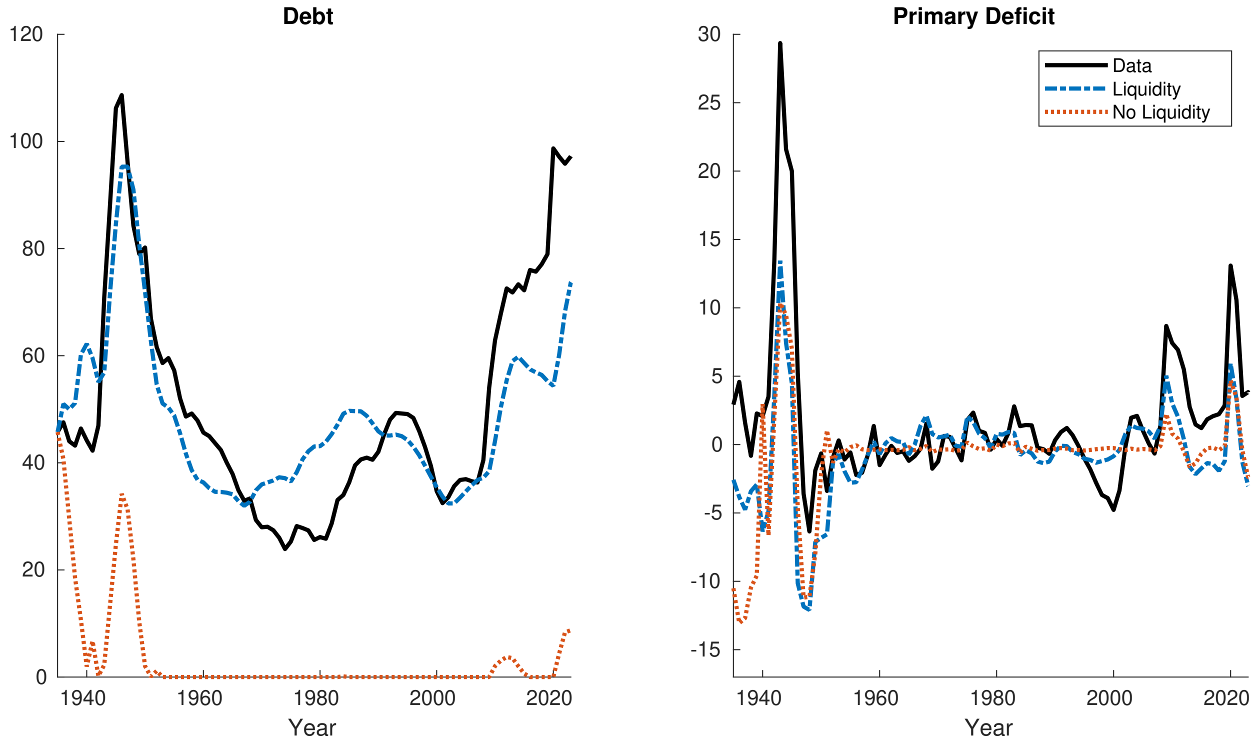

Figure 1: **Historical debt and primary deficit ratios.** This figure plots the actual and optimal debt and primary deficit ratios in the United States since 1935. Actual debt is the gross debt held by the public (FYGFDPUB in FRED). Optimal policy begins in 1935 with the same debt and expenditure level as the actual history, and debt is restricted to be positive. The blue dash-dot line is the optimal policy with liquidity demand, the orange dotted line is the optimal policy without liquidity demand.

Sergeyev (2021), Mian, Straub and Sufi (2025), Brunnermeier, Merkel and Sannikov (2024), and Kocherlakota (2023)) demonstrates that empirically plausible liquidity demand can sustain high public debt levels and low interest rates in the United States.

But does liquidity demand suggest that current debt levels are optimal? Just because policymakers can sustain high debt levels, does not mean they should. On the one hand, low interest rates on public debt present the opportunity to run primary deficits indefinitely at high debt levels.[2] In this scenario, policymakers can lower taxes and pay for government expenditure with inexpensive public debt. On the other hand, high public debt leaves the economy vulnerable to fiscal risk. A surprise increase in government spending — due to a financial crisis, epidemic, or war — would stress the U.S. budget. An unexpected rise in interest rates could necessitate sudden fiscal consolidation.

To quantify the impact of liquidity demand on optimal debt policy I consider a quantitative

[2]Since the value of government debt is above it's fundamental value there is a bubble on U.S. public debt (Brunnermeier, Merkel and Sannikov (2024) and Jiang et al. (2024)). Policymakers "mine" this bubble to fund tax cuts.

model with aggregate risk and liquidity demand. Specifically I solve for the optimal Ramsey policy in a quantitative model featuring liquidity demand, distortionary labor taxes, and fiscal risk. In my model the government levies labor taxes and issues safe — i.e., risk-free and liquid — public debt to cover surprise expenditure. The government raises revenue by either taxing labor or liquidity. Policymakers tax liquidity by under supplying public debt to suppress interest rates. Just as inflation is a tax on money's liquidity (Friedman (1971)), interest rate reduction is a tax on public debt's liquidity (Angeletos, Collard and Dellas (2023)). I introduce fiscal risk separately as surplus risk and interest rate risk. While surplus risk increases the cost of raising revenue, interest rate risks increases the cost of repayment. Following the literature (e.g., Lucas and Stokey (1983) and Bhandari et al. (2017*a*)) I introduce surplus risk through uninsurable exogenous shocks to government expenditure (see Section 3). I introduce interest rate risk with liquidity shocks (see Section 4) .

Calibrating my model to standard parameters and the government expenditure process in the United States, I find that liquidity is overtaxed during peacetime and debt is overused during crises. As a preview of the results, Figure 1 plots the actual and model implied optimal debt and primary deficits in the United States since 1935. The figure plots optimal debt policy with and without liquidity demand. In both models and the data, policymakers partially debt-finance wars and crises. However, the model without liquidity demand has a strong savings motive that leads to zero public debt in most years. According to the model with liquidity demand, U.S. policymakers relied too heavily on debt to fund recent crises. Debt reliance lead debt ratios to be 10-20 p.p. higher than optimal following the Great Financial Crisis and Covid Pandemic. The liquidity demand model also shows that policymakers overtax liquidity during peacetime. The optimal debt target exceeded actual midcentury policy. By under supplying debt, U.S. policymakers deprived markets of liquidity and kept interest rates artificially low.[3] Given current projections of government expenditure, the liquidity demand model recommends reducing the debt ratio to 65% over the next 10 years. This will require substantial primary surpluses, antithetical to current projections (e.g., CBO (2024)).

This quantitative debt model has a unique steady-state and produces ergodic distributions for optimal debt, labor taxes, and interest rates. I present analytical approximations of these distributions and illustrate them numerically. While the optimal debt target is 65% of GDP, there is always a potential debt-financed crisis. So, policymakers are unlikely to reach this target. The average optimal debt ratio across many stochastic expenditure realizations is 110%. This average debt ratio is high compared to the previous literature (0%-66%), similar to current levels (100%), and small compared to the theoretical debt limit (185%). Optimal labor taxes are higher and more volatile than earlier work and current levels. Optimal deficits are near zero during peacetime and 5-15%

[3] Choi, Kirpalani and Perez (Forthcoming) also find that the U.S. exploits it's role as a safe asset provider and under supplies safe assets.

during crises. Optimal interest rate suppression (67bp) is similar to the historical average.

Introducing household liquidity demand to optimal debt models overturns two standard findings: labor tax smoothing and negative optimal debt. First, even with complete markets perfect labor tax smoothing (i.e., labor tax volatility equaling zero) is suboptimal if households demand liquidity. Instead policymakers aim to simultaneously smooth labor *and* liquidity taxes. The liquidity tax smoothing incentive, absent from complete markets models (e.g., Lucas and Stokey (1983)), leads policymakers to partially distort labor taxes. The relative value of smoothing labor or liquidity taxes depends on the relative magnitudes of the Frisch elasticity and the liquidity demand parameters. Even with moderate liquidity demand, optimal policy deviates from perfect labor tax smoothing. Second, with incomplete debt markets positive debt is optimal. Normally in an incomplete markets model the policymaker amasses assets as a form of precautionary and buffer stock savings. Policymakers use this asset position to pay for government expenditure and keep labor taxes low. In the liquidity demand model, policymakers amass *debt* to supply households with liquidity. In contrast to other incomplete markets models (e.g., Aiyagari et al. (2002) and Bhandari et al. (2017*a*)), positive debt levels are optimal even with moderate liquidity demand. The relative value of saving assets or satiating liquidity depends on the relative magnitude of the intertemporal elasticity of substitution to the liquidity demand parameters.

Moving beyond surplus risk, I consider the optimal policy response to a liquidity crisis. The Great Depression, Volcker Disinflation, and Great Financial Crisis all featured a sudden rise in the demand for liquidity. In an extension of the baseline model, I solve for the optimal Ramsey policy given stochastic liquidity demand. I calibrate this quantitative model to match the historical fluctuations in Treasury's liquidity premium. The model recommends policymakers expand public debt in response to these kinds of liquidity crises. But debt expansion should only partially satiate surprise liquidity demand. The remaining liquidity demand should be channeled toward reducing interest rates and labor taxes. This policy echoes Sims (2025) findings in a perfect foresight model. Relatedly, Kocherlakota (2023) finds that liquidity risk can help sustain more public debt: the chance of a future demand spike reduces borrowing costs today. I find that U.S. policymakers can and *should* sustain more public debt to partially insure against future liquidity crises. Policymakers must carry enough public debt to ensure markets can access sufficient liquidity when needed. But policymakers should only partially satiate this demand and channel the rest toward lower interest rates and labor taxes.

Related Work This paper contributes to the debt management literature pioneered by Barro (1974, 1979, 1980). These papers were among the first to argue that taxes and debt should be smoothed across time. When markets are complete, Lucas and Stokey (1983) find that policymakers can perfectly insure households against labor tax fluctuations. I extend this literature by demonstrating that even with complete markets policymakers' desire to smooth liquidity taxes leads to volatile labor taxes. With incomplete markets and aggregate risk, Aiyagari et al. (2002) and Bhandari et al. (2017*a*) find policymakers cannot insure against tax fluctuations. Faced with this impossibility, policymakers build up a large asset position to insure against fiscal risk and reduce labor taxes. I attenuate this result, demonstrating that for reasonable liquidity demand a positive debt level is optimal. In the spirit of Barro (1989), I show that quantitative debt models with liquidity demand provide *positive* theories of how governments finance spending beyond *normative* theories of how they should.

This paper also contributes to the liquid debt literature. Private debt cannot provide the safety and liquidity of public debt. Public debt is uniquely valuable due to its singular ability to insure against aggregate and idiosyncratic shocks. While financial intermediaries can utilize credit lines to provide firms with insurance against idiosyncratic shocks, they are unable to insure against aggregate shocks like recessions, wars, and financial crises (Holmström and Tirole (1998) and Azzimonti and Yared (2019)). Because governments can borrow against the future by pledging future tax increases, public debt can uniquely insure against aggregate shocks. Public debt provides liquidity in heterogeneous agent models (e.g., Woodford (1990), Aiyagari and McGrattan (1998), Acikgoz et al. (2018), Brunnermeier, Merkel and Sannikov (2024)) by allowing households to partially insure themselves against uninsurable idiosyncratic risk through savings and risk-sharing. A more recent literature (e.g., Bhandari et al. (2021), Bayas-Erazo (2023), LeGrand and Ragot (2023), Auclert et al. (2024)) analyzes optimal debt policy with heterogeneous agents. Solving for optimal policy responses to aggregate shocks in these environments is computationally difficult, therefore this literature focuses on perturbations around a long-run steady-state.[4] This paper focuses on the globally optimal fiscal response to aggregate fiscal risk. I extend this literature by jointly considering both liquidity provision and fiscal risk. While public debt provides liquidity services which household's demand, debt sustainability concerns restrict policymakers from completely satiating this liquidity demand.

This paper additionally contributes to the low interest rate literature. Recent papers discussing the policy implications of low interest rates on public debt include Barro (2022), Mankiw (2022), Reis (2021), Kocherlakota (2023), Mian, Straub and Sufi (2025), Brunnermeier, Merkel and Sannikov (2024), Bayas-Erazo (2023), Choi et al. (2024), Mehrotra and Sergeyev (2021), Angeletos, Collard and Dellas (2023), Aparisi de Lannoy et al. (2025), and Sims (2025). The latter three are the most the most related to my paper. Angeletos, Collard and Dellas (2023) consider the optimal response of labor taxes and public debt to unexpected shocks in an economy with liquidity demand. They find that labor taxes should be front loaded to keep deficits initially lower and allow the government to earn "seigniorage" on the debt through liquidity taxation. My paper also finds that labor

[4]See Moll (2024) for a recent discussion of why these issues arise and computational methods to circumvent them.

taxes should initially absorb expenditure shocks, though the incentive to earn debt "seigniorage" is much weaker in my calibration. Moreover, I consider the optimal level of debt under uncertainty rather than with perfect foresight. Additionally, I calibrate a quantitative model to U.S. data while Angeletos, Collard and Dellas (2023) focus on the theoretical forces that dictate the optimal level of debt. Aparisi de Lannoy et al. (2025) consider the optimal maturity structure of public debt in an economy with liquidity demand. In contrast, I quantify the optimal level of debt rather than the maturity structure at a given level. Finally, Sims (2025) considers optimal liquidity provision and labor taxation in a model with perfect foresight and finds that a small liquidity tax is optimal. With quantitatively realistic fiscal risk, I find that a moderate liquidity tax during peacetime and a small liquidity tax during crises is optimal.

2 Model

In this section, I layout a dynamic stochastic general equilibrium model featuring liquidity demand, distortionary taxes, and fiscal risk. Before exploring optimal policy in Sections 3 and 4, I discuss how I calibrate this model and compute the equilibrium.

States of Nature Time is discrete and indexed by $t = 0, 1, 2, \ldots$. The state $s_t \in \mathbb{S}$ summarizes the exogenous stochastic disturbances in year t. The possible set of states $\mathbb{S}$ is finite with size S. I denote the history of states s_t up to and including year t by $s^t = s_0, s_1, \ldots, s_{t-1}, s_t \in \mathbb{S}^t$. The unconditional probability of history s^t at time t is $\Omega_t(s^t)$. These states s_t follow a Markov process which evolve according to transition probability matrix $\Omega(s'|s) = Pr[s_{t+1} = s'|s_t = s]$. For ease of notation, let Z_t denote the realization of an arbitrary variable Z in time t for history s^t; for example, consumption in year t given history s^t is $C_t = C_t(s^t)$. For most of the paper the state s^t is isomorphic to the government expenditure level $G_t = G(s^t)$. In an extension of the model in Section 4, the state s^t is isomorphic to the exogenous liquidity demand level $\nu_t = \nu(s^t)$. The economy consists of three agents: a representative firm, a representative household, and the government.

2.1 Firm

The firm employs labor N_t at real wage rate W_t and produces output Y_t using the production technology $F(N_t)$. Labor is the sole factor of production. Production is linear in labor supply, i.e., $F(N_t) = N_t$. Profit maximization by the firm implies that the wage rate is unity each year, i.e., $W_t = 1$. There are zero profits for all states s^t and times t.

2.2 Household

First, I explain how household liquidity demand is modeled. Second, I present households preferences and subsequent optimization conditions.

Liquidity Demand Liquidity demand is represented by bonds-in-the-utility function. Depicting the demand for government debt's liquidity as bonds-in-the-utility function echoes Sidrauski (1967)'s depiction of the demand for money's liquidity with money-in-the-utility. Both represent the excess demand for money and bonds beyond their function as a savings technology. Just as money-in-the-utility function allows tractable analysis of optimal monetary policy (e.g., Chari and Kehoe (1999)), bonds-in-the-utility function allows tractable analysis of optimal fiscal policy (e.g., Angeletos, Collard and Dellas (2023)). Specifically, optimal fiscal policy in an environment with aggregate risk. A voluminous literature, including Woodford (1990), Holmström and Tirole (1998), Aiyagari and McGrattan (1998), Blanchard (2019), Azzimonti and Yared (2019), and Brunnermeier, Merkel and Sannikov (2024), explores why firms and households have a particular demand for public debt over other kinds of private debt.[5] In Appendix A, I present an explicit microfoundation featuring precautionary and buffer stock savings that produces a bonds-in-the-utility function for the representative household.

The bonds-in-the-utility functional form,

$$w(B) = \begin{cases} (\nu+\phi)B - \phi B \sinh^{-1}(B) & B < B^{liq.sat.} \\ (\nu+\phi)B^{liq.sat.} - \phi B^{liq.sat.} \sinh^{-1}(B^{liq.sat.}) & B \geq B^{liq.sat.} \end{cases}. \tag{1}$$

closely matches empirical estimates of the liquidity premium on Treasuries. The utility of liquidity $w(B)$ is concave. The marginal utility of liquidity $w_B(B) = \nu - \phi \sinh^{-1}(B)$, for $B < B^{liq.sat.}$ is monotonically decreasing and equals zero when $B \geq B^{liq.sat.}$. Where $B^{liq.sat.}$ is the liquidity satiating debt ratio beyond which government debt provides no additional liquidity services. The liquidity satiating debt ratio is defined implicitly as the level at which the marginal utility of debt is zero $w_B(B^{liq.sat.}) = 0$ and explicitly as $B^{liq.sat.} = .5\exp\left(\frac{\nu}{\phi}\right) - .5\exp\left(-\frac{\nu}{\phi}\right)$. Define the debt-

[5] Money-in-the-utility function can be micro-founded with shopping-time models (e.g., McCallum (1983)) or cash-in-advance models (Bohn (1991)). Bonds-in-the-utility can similarly be microfounded by emphasizing public debt's role as a tradable asset (Woodford (1990)) or by emphasizing its role in relaxing collateral constraints (Holmström and Tirole (1998)). Aggregating the welfare of heterogeneous agents facing uninsurable idiosyncratic risk, as in Brunnermeier, Merkel and Sannikov (2024) and Angeletos, Collard and Dellas (2023), produces bonds-in-the-utility function in the representative welfare function. The addition of bonds-in-the-utility function to a representative agent concisely captures the liquidity services public debt provides to households facing uninsurable idiosyncratic risk. Adding bonds-in-the-utility function to a representative agent New Keynesian (RANK) model brings the model closer to a heterogeneous agent New Keynesian (HANK) model (Kaplan, Moll and Violante (2018)). Gaillard et al. (2023) finds that bonds-in-the-utility function is necessary in an Aiyagari (1994) model to match the joint distributions of consumption, wealth and capital income.

dependent liquidity risk-aversion as $\varepsilon(B) = -\frac{w_{BB}B}{w_B} = \frac{\phi}{\nu - \phi \sinh^{-1}(B)} \frac{B}{\sqrt{B^2+1}}$, where $\varepsilon(B) > 0$. The top panel of figure 2 plots the utility of liquidity $w(B)$, while the bottom panel plots the marginal utility of liquidity $w_B(B)$ in basis points.

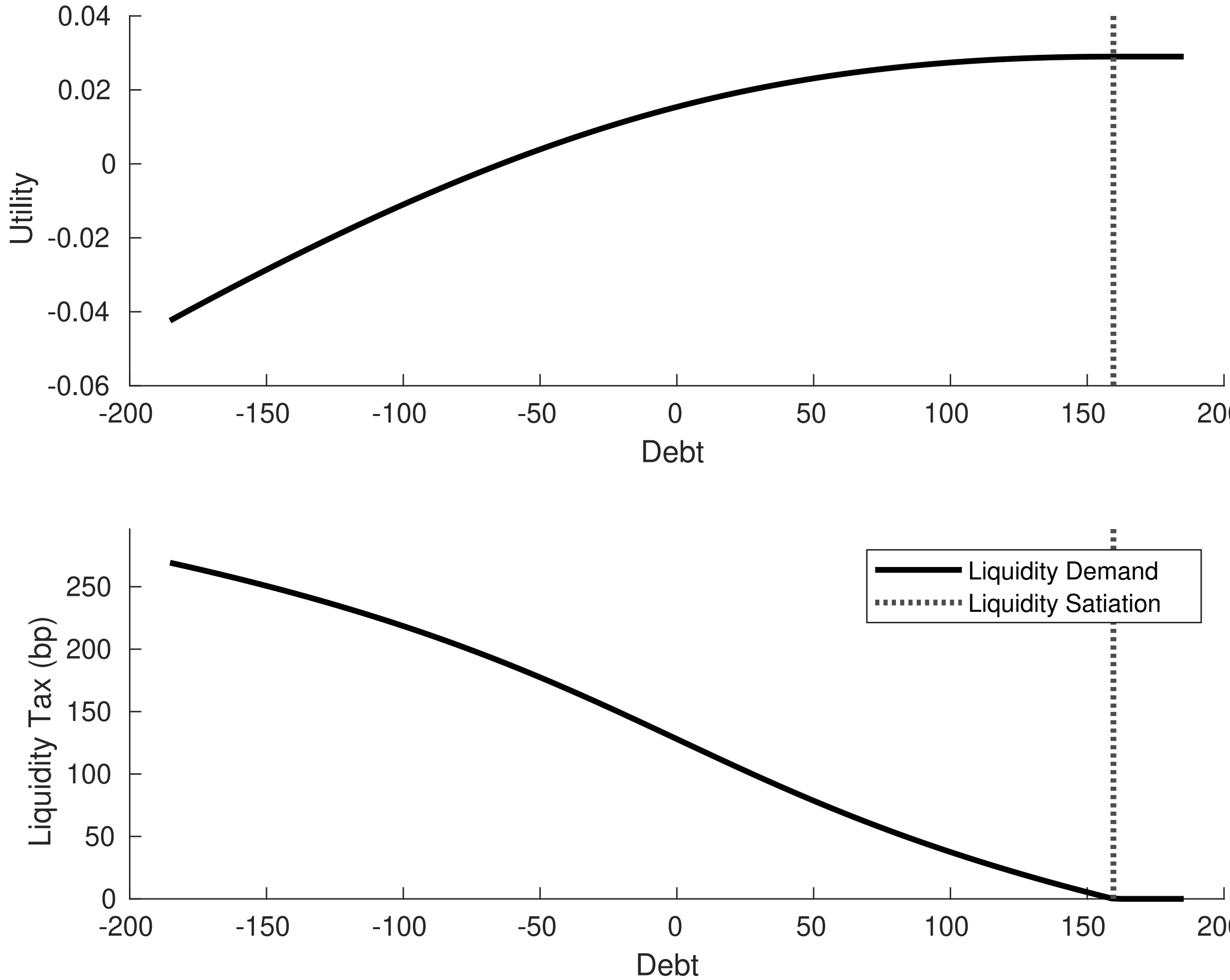

Figure 2: **Liquidity demand.** The top panel plots the utility over liquidity $w(B)$ and the bottom panel plots the marginal utility over liquidity $w_B(B)$. The x-axis denotes the the debt level. Marginal utility is reported in basis points.

The liquidity demand parameters $\{\nu, \phi\}$ map to parameters estimated in other empirical work. $\nu \in \mathbb{R}$ is the debt-invariant spread on real interest rate, and $\phi \in \mathbb{R}_{++}$ is the semi-elasticity of the real interest rate spread with respect to debt. Increasing ν raises the marginal utility of liquidity at all levels of debt, while increasing ϕ makes liquidity demand more elastic. ν dictates the level of liquidity demand and ϕ controls the demand elasticity.

Recent papers use variations of this functional form. These papers (e.g., Krishnamurthy and Vissing-Jorgensen (2012), Angeletos, Collard and Dellas (2023), Mehrotra and Sergeyev (2021), Kekre and Lenel (2021), Mian, Straub and Sufi (2025)) typically parameterize the marginal value of liquidity with a linear-log specification (e.g., $w_B(B) = \nu - \phi \log(B)$), whereas I use a linear-inverse hyperbolic sine specification. My specification allows for negative debt (i.e., $B < 0$) which the linear-log specification mechanically rules out. The inverse hyperbolic sine function permits negative debt while still fitting the shape of the Treasury demand function. Krishnamurthy and Vissing-Jorgensen (2012) and Greenwood and Vayanos (2010) find that the linear-log specification describes the demand for public debt's liquidity, I find the inverse hyperbolic sine function fits equally well.

Preferences and Optimization Given initial bonds B_{-1}, the household consumes goods C_t, supplies labor N_t, and buys one-period real bonds B_t. Households preferences,

$$\mathbb{E}_0 \sum_t \beta^t \sum_{s^t} \Omega_t [u(C_t) - v(N_t) + w(B_{t-1})], \tag{2}$$

are time-separable in consumption utility $u(C_t) = \frac{C_t^{1-\sigma} - 1}{1-\sigma}$, labor disutility $v(N_t) = \Xi \frac{N_t^{1+\eta}}{1+\eta}$, and liquidity demand $w(B_t)$. Where $\beta \in (0,1)$ is the discount rate, Ω_t is the probability of state s^t in time t, $\sigma \geq 0$ is the inverse of the intertemporal elasticity of substitution (IES), $\eta > 1$ is the inverse of the Frisch elasticity, $\Xi > 0$ is the value of nonmarket time, $\nu \in \mathbb{R}$ is the debt-invariant spread on real interest rate, and $\phi \geq 0$ is the semi-elasticity of the real interest rate spread with respect to bonds.

The household takes labor taxation τ_t, interest rate on bonds R_t, and government transfers T_t as given. While the household owns the firm, there are no profits. For a given history s^t the household chooses consumption C_t, labor supply N_t, and their real bond holdings B_t to maximize their utility function (equation 2) given their time t household budget constraint,

$$C_t + \frac{B_t}{R_t} = (1 - \tau_t) N_t + B_{t-1} + T_t. \tag{3}$$

Optimization by the household leads to two first order conditions; the first optimality conditions is the labor-leisure decision,

$$(1 - \tau_t) = \frac{v_{N,t}}{u_{C,t}}. \tag{4}$$

The labor-leisure decision (equation 4) represents the tradeoff between consuming an additional unit this period (i.e., $u_{C,t}$) and working an extra unit (i.e., $v_{N,t}$). Holding consumption fixed, higher taxes reduce labor supply. Holding labor supply fixed, higher taxes reduce consumption . The second optimality condition is the liquidity-adjusted Euler equation,

$$\frac{1}{R_t} = \beta \underbrace{\sum_{s^{t+1}} \Omega_{t+1|t} [\frac{u_{C,t+1}}{u_{C,t}}}_{\text{Consumption-Savings}} + \underbrace{\frac{w_{B,t}}{u_{C,t}}}_{\text{Liquidity-Demand}}]. \tag{5}$$

The first term on the right hand side of the liquidity-adjusted Euler equation (5) captures household's demand for savings, while the second captures the demand for liquidity. Standard Euler equations solely capture the consumption-savings decision of the household; the tradeoff between consuming an additional unit this year $u_{C,t}$ or saving this additional unit and consuming more next year $u_{C,t+1}$. Beyond their role as a savings device, bonds B_t provide liquidity service. Liquidity demand produces a downward sloping demand for bonds B_t for a given interest rate R_t.

Household demand for liquidity produces a convenience yield on public debt. Let R_t^{rf} be the *risk-free* interest rate, i.e., the prevailing interest rate of a risk-free but illiquid asset,

$$R_t^{rf} = \beta^{-1} \frac{u_{C,t}}{\sum_{s_{t+1}} \Omega(s_{t+1}|s^t) u_{C,t+1}}. \tag{6}$$

For debt levels, B_t, *above* the liquidity-satiation level, $B^{liq.sat.}$ (dotted vertical line), households demand bonds solely for consumption smoothing. If the debt level lies *below* the liquidity-satiation level, households demand bonds for both consumption smoothing and liquidity provision. Excess demand for bonds beyond their role as a savings device produces a spread between the interest rate on *safe* bonds R_t and the risk-free interest rate R_t^{rf}. The convenience yield on public debt is the difference between the safe rate R_t and the risk-free rate R_t^{rf}. This spread, $\beta w_{B,t}/u_{C,t}$, is the convenience yield on public debt. Since there is no growth in this model, the interest rate spread $R_t - R_t^{rf}$ is the model equivalent of the $r-g$ spread discussed by Blanchard (2019) and Mehrotra and Sergeyev (2021).

2.3 Market Clearing

There is market clearing in the goods market and the bond market. Goods market clearing requires that household consumption C_t and government expenditure G_t must equal output $Y_t = F(N_t) = N_t$ each period. That is,

$$C_t + G_t = N_t. \tag{7}$$

Goods market clearing (equation 7) describes the demand for labor, while the labor-leisure decision (equation 4) describes the supply of labor. Together they pin down the equilibrium output N_t for a given tax rate τ_t and expenditure level G_t. [6]

[6]Implicitly, $N(\tau,G)^{\eta}(N(\tau,G)-G)^{\sigma} = (1-\tau)$.

Bond market clearings requires that household bond holdings B_t must equal the government's supply of public debt. The liquidity-adjusted Euler equation (5) together with the household budget equation (3) pins down the real interest rate R_t for a given debt level B_t. Households do not internalize market clearing restrictions.

2.4 Government

Policymakers affect the decisions of households and firms through government policy $p_t = \{\tau_t, T_t, B_t\}$. Where τ_t is labor taxation, $T_t \geq 0$ is lump-sum transfers to the household, and B_t is real debt issued. The government linearly taxes labor and provides lump-sum transfers. Heathcote, Storesletten and Violante (2017) argue that this taxation scheme approximates the U.S. tax system. [7]

The government faces surplus risk in the form of exogenous shocks to government spending G_t. When elevated I refer to these shocks as "wars" or "crises." The government is constained by an intertemporal budget constraint,

$$\frac{B_t}{R_t} = B_{t-1} + \overbrace{G_t - \tau_t N_t + T_t}^{\text{Primary Deficit at } t}. \tag{8}$$

The government uses three tools to pay for outstanding debt and government expenditure : labor taxation, debt rollover, and liquidity taxation. Let $\omega_t = \beta \frac{w_{B,t}}{u_{C,t}}$ be the tax rate on liquidity. Restricting the government to only issuing risk-free real debt rules out the possibility of using inflation to pay for outstanding debt B_t.[8] Labor taxation and debt rollover are standard in optimal debt models, taxing bond liquidity is novel. Liquidity taxation is implemented by undersupplying public debt. To illustrate, I decomposes the value of debt issuance $\frac{B_t}{R_t}$ into it's savings and liquidity value:

$$\frac{B_t}{R_t} = \overbrace{\frac{B_t}{R_t^{rf}}}^{\text{Savings value}} + \underbrace{\omega_t B_t}_{\text{Liquidity value}}. \tag{9}$$

Savings value of debt is the present discounted value of future primary surpluses. Liquidity value of debt is the present discounted value of future liquidity services.[9] The liquidity tax is zero if liquidity demand is satiated $B_t \geq B^{Liq.Sat.}$. Below the satiating level, $B_t < B^{Liq.Sat.}$, the government effectively taxes liquidity by suppressing the interest rate, $R_t < R_t^{rf}$. Just as inflation is a tax

[7] Lump-sum taxes are ruled out due to unmodeled wealth heterogeneity.

[8] While inflation is not explicitly defined in this economy, state-contingent real debt is equivalent to risk-free *nominal debt* subject to costless state-dependent inflation (Bohn (1990)) Explicitly, $B_t(s^t) = B_{t-1}^{nom}(s^{t-1})/P_t(s^t)$ where $P_t(s^t)$ is the state-dependent price level. Online Appendix E explores optimal debt policy with state-contingent debt.

[9] Brunnermeier, Merkel and Sannikov (2020) performs a similar decomposition and refers to this term as a bubble.

on money's liquidity services (Phelps (1973), Kimbrough (1986) and, Leeper and Zhou (2021)), interest rate suppression represents a tax on government bond's liquidity services. Others, e.g., Blanchard (2019), Mankiw (2022), and Barro (2022), refer to this suppression as "debt seigniorage." I describe this interest rate suppression as a liquidity tax to avoid confusion with the monetary definition of seigniorage and to reiterate that interest-rate suppression is welfare decreasing.

First-best First-best policy calls for eliminating labor and liquidity taxes, i.e., $\tau_t, \omega_t = 0, \forall t$. Setting these taxes to zero removes the policy-induced wedges in the labor-leisure decision (equation 4) and the liquidity-adjusted Euler equation (5). Eliminating liquidity taxes requires the government to issue enough debt to satiate liquidity demand. Policymakers with access to lump-sum taxes and transfers can implement first-best policy.[10] Alternatively, with access to a second risk-free but illiquid asset, policymakers could implement the first-best policy by longing risk-free assets and shorting safe asset. The short position in safe assets would satiate markets demand for liquidity and the long position in risk-free assets would fund future streams of government expenditure.[11] Ruling out lump-sum taxation and public asset ownership forces the government to resort to distortionary labor and liquidity taxation. Government policy is further restricted by labor and liquidity Laffer curves, natural debt and asset limits, and the state-contingency of debt.

Laffer Curves Laffer curves for both labor and liquidity taxation limit policymakers ability to raise tax revenues. The labor Laffer curve constrains taxation policy and the liquidity Laffer curve constrains debt policy. The former produces a maximum labor tax rate and the latter produces a maximum liquidity tax.

The labor Laffer curve emerges from the direct and indirect effect of increased labor taxes τ_t on labor tax revenues $\tau_t N(\tau_t, G_t)$. Directly, labor taxation τ_t raises the tax revenue per unit of labor supply N_t. Indirectly, labor taxes τ_t reduce labor supply, i.e., $N_\tau(\tau_t, G_t) < 0$, by disincentivizing households to work. Together these forces generate an inverted-U shaped Laffer curve for labor tax revenues with a single peak. The direct effect dominates to the left of the peak while the indirect effect dominates to the right. The peak of the Laffer curve corresponds to a state-dependent upper bound on labor taxation $\tau_t \leq \bar{\tau}(G_t)$, beyond which tax revenues decline. Elevated government expenditure G_t increases labor demand and consequently raises the peak of the Laffer curve: $G_t > G_t' \Rightarrow \bar{\tau}(G_t) > \bar{\tau}(G_t')$. Policymakers should never set labor taxes to the right of the labor Laffer curve peak as there is a lower labor tax that produces the same revenue[12]. The maximal labor tax rate resembles those estimated by Trabandt and Uhlig (2011).

[10] Specifically, policymakers would set lump-sum taxes to $T_t = G_t + \frac{R_t - 1}{R_t} B^{liq.sat.}$, where $R_t = R(G_t)$.

[11] Japan runs a similar sovereign wealth fund strategy, albeit investing in risky assets (Chien, Du and Lustig (2025)).

[12] Without commitment, taxation beyond the labor Laffer curve can be optimal (Debortoli, Nunes and Yared (2021)).

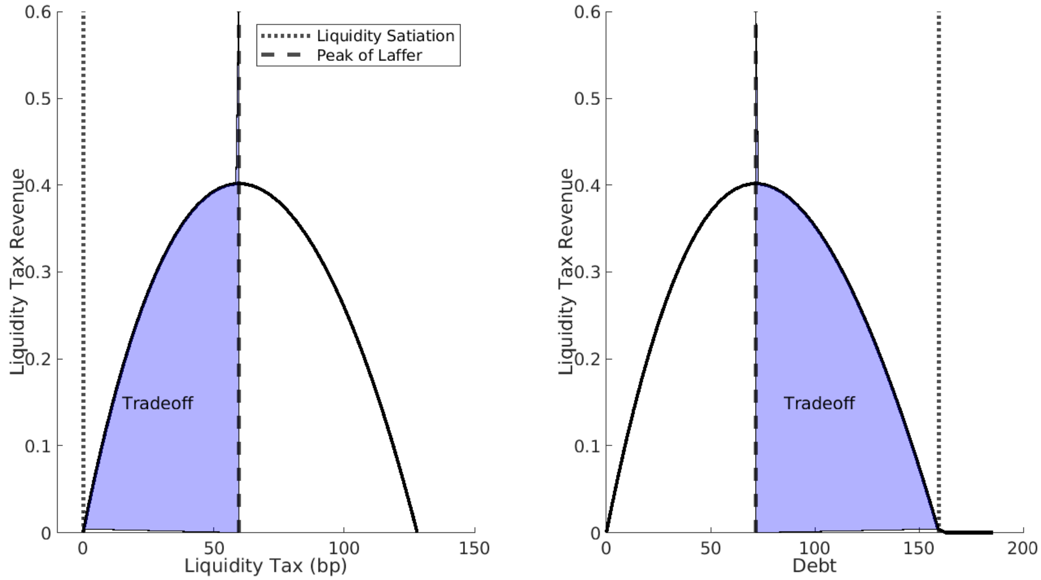

Figure 3: **Liquidity and debt Laffer curves.** This figure plots the tax revenue from the liquidity tax. The left panel plots the liquidity Laffer curve: liquidity tax against the liquidity tax revenue. The right panel plots the debt Laffer curve: debt ratio against the liquidity tax revenue.

The same intuition from the labor Laffer curve applies to the liquidity Laffer curve. Directly, liquidity taxes raise tax revenue per unit of public debt. Indirectly, liquidity taxes reduce households demand for public debt. The interaction between the direct and indirect effects generates the inverted U-shape common to Laffer Curves. The left panel of figure 3 plots the liquidity Laffer curve as the liquidity tax revenue $\omega_t B(\omega_t)$ for a given liquidity tax ω_t, holding consumption constant. In the public finance literature, Laffer curves are typically drawn like the liquidity Laffer curve with the tax rate on the x-axis. The peak of the liquidity Laffer curve is the liquidity tax that maximizes liquidity tax revenue, $\omega^{Laffer} = \arg\max_{\omega} \omega B(\omega)$.

A recent literature in macroeconomics (e.g., Brunnermeier, Merkel and Sannikov (2024), Mian, Straub and Sufi (2025)) presents these Laffer curves in terms of debt rather than liquidity taxation. However, the intuition from the liquidity Laffer curve is flipped for the debt Laffer curve. The right panel of figure 3 plots the debt Laffer curve as the liquidity tax revenue $\omega(B_t)B_t$ for a given debt level B_t. Directly, reducing debt reduces liquidity tax revenues. Indirectly, debt reduction increases the marginal value of liquidity, $w_{B,t}$, suppresses interest rates R_t and raises the liquidity tax $\omega(B_t)$. The peak of the debt Laffer curve is the debt ratio that maximizes liquidity tax revenue, i.e., $B^{Laffer} = \arg\max_{B} \omega(B)B$. The peak of the debt Laffer curve B^{Laffer} maps directly to the peak of the liquidity Laffer curve, i.e., $\omega^{Laffer} = \omega(B^{Laffer})$. For the particular function form of liquidity demand outlined in equation 1, the peak of this debt Laffer curve is *below* the debt satiating level.

The liquidity and debt Laffer curves help visualize optimal taxation and debt policy. First-best policy is to set labor and liquidity taxes to zero. Graphically, the first-best policy corresponds to the *left* corner of the liquidity Laffer curves and the *right* corner of the debt Laffer curve. Absent risk, optimal policy should never set the liquidity tax to the *right* of the liquidity Laffer curve peak or to the *left* of the debt Laffer curve peak. For any liquidity tax rate to the right of the peak there exists a tax rate to the left of the peak that raises the same revenue while imposing a lower tax rate on households. Absent risk, optimal liquidity taxes will be set in the tradeoff region between the minimum and maximum tax. On the liquidity Laffer curve this tradeoff region is to the *left* of the peak, while on the debt Laffer curve this region is to the *right* of the peak. Mian, Straub and Sufi (2025) refer to the area left of the debt Laffer curve peak as the "free-lunch" region. This "free-lunch" region maps to the right region of the liquidity Laffer curve. In this region the government can increase debt and raise liquidity tax revenues. Absent shocks, optimal debt policy should set the debt ratio somewhere in the tradeoff region between the peak of the debt Laffer curve and the liquidity satiating debt ratio. With shocks, the optimal debt level can be in this "free-lunch" region.

The liquidity satiation point is the bond equivalent of the Friedman (1971) rule for money. At this point, policymakers flood the market with liquid bonds, setting liquidity taxes to zero. While the peak of the debt Laffer curve is the bond equivalent of the Phelps (1973) rule for money. At this point, policymakers extract maximal liquidity taxes from bond holders by sapping the market of liquidity.

Natural Debt and Asset Limits Typically, liquidity demand does not raise the government's debt limit. Government's debt is limited by the natural debt limit – the maximum level of indebtedness at which the government debt can almost surely be repaid (Aiyagari (1994)). In order to find this debt limit, I consider the worst case scenario in which the government is stuck in a perpetual war while heavily indebted. In this scenario government expenditure G_t remains at its highest level $\bar{G}$ for all periods t and states s^t, public debt has reached its peak $\bar{B}$ and labor taxes must be set at the peak of their Laffer curve $\bar{\tau} = \bar{\tau}(\bar{G})$ to pay for both the war and the interest payments on the debt. Continually high taxes generate continually low output $\underline{\mathrm{N}}$. In this scenario, the real interest rate, $R(\bar{B}, \bar{\tau}) = \beta^{-1} \frac{u_C(\bar{\tau})}{u_C(\bar{\tau}) + w_B(\bar{B})}$, depends on the convenience yield of public debt $w_B(\bar{B})$ at the natural debt limit $\bar{B}$. However, for all reasonable calibrations the natural debt limit lies above the debt satiating level $B^{liq.sat.} < \bar{B}$. At the natural debt limit liquidity demand will be satiated, the convenience yield will be zero, and the natural debt limit is unaffected by liquidity demand.

Therefore the natural debt limit is $\bar{B} = \frac{\bar{\tau}\underline{\mathbf{N}} - \bar{G}}{1-\beta}$. [13]

While the convenience yield on public debt has little to no impact on the natural debt limit, it raises the government's asset limit. The natural asset limit is defined as the maximum asset level (i.e., negative debt level) at which all future government expenditure can be covered without labor taxes. In this scenario government expenditure G_t remains at its highest level $\bar{G}$ for all periods t and states s^t, public assets are at their peak $\underline{\mathbf{B}}$ and labor taxes are uniformly zero $\tau = 0$. At this limit, interest on assets covers all government expenditure. In this scenario, the real interest rate, $R(\underline{\mathbf{B}}, \bar{N}) = \beta^{-1}\frac{u_C(\bar{N})}{u_C(\bar{N}) + w_B(\underline{\mathbf{B}})}$, is suppressed by the convenience yield of public debt $w_B(\bar{B})$. The natural asset limit is implicitly defined by $\underline{\mathbf{B}} = -\frac{R(\underline{\mathbf{B}}, \bar{N})}{R(\underline{\mathbf{B}}, \bar{N}) - 1}\bar{G}$. Since the government is earning less interest on its assets, it must acquire more assets to pay for government expenditure $\bar{G}$ each year.

Contingent and Noncontingent Debt The value of state-contingent debt $B_{t+1}(s^{t+1})$ issued at year t after observing history s^t is contingent on history $s^{t+1} = (s^t, s_{t+1})$. With access to state-contingent debt, the government can vary payoffs at year $t+1$ based on the ex-post government expenditure level G_{t+1}. Therefore markets are complete in the sense that public debt can insure against future shocks. The value of noncontingent debt $B_t(s^t)$ issued at year t after observing history s^t is only contingent on the observable history of shocks s^t. Payoffs are not contingent on the ex-post government expenditure level G_{t+1}. Therefore markets are incomplete in the sense that public debt cannot insure against future shocks. Conversely, from the household's point-of-view noncontingent debt is risk-free since its payoffs are known at the time of purchase. The state-contingency of public debt radically alters optimal debt policy. Sections 3 and 4 consider optimal policy with noncontingent, that is risk-free debt. Online Appendix E considers optimal debt policy with contingent debt.[14]

2.5 Equilibrium

A *feasible government policy* is a government policy $p_t = \{\tau_t, T_t, B_t\}$ which respects (i) the Labor laffer curve, (ii) the natural debt and asset limits, and (iii) the lower bound on lump-sum transfers $T_t \geq 0$. For a given feasible government policy, a *competitive equilibrium* is a sequence $\{C_t, N_t, R_t\}$

[13] When liquidity demand is large enough that $\bar{B} < B^{liq.sat.}$, the natural debt limit is given implicitly $\bar{B} = \frac{R(\bar{B},\bar{\tau})}{R(\bar{B},\bar{\tau})-1}[\bar{\tau}\underline{\mathrm{N}} - \bar{G}]$. Even in this case, the liquidity demand has little impact on the debt limits since $\frac{R(\bar{B},\bar{\tau})}{R(\bar{B},\bar{\tau})-1} \approx (1-\beta)^{-1}$.

[14] Although antithetical to modern practice in the United States there exist a multitude of microfoundations and historical precedent for state-contingent debt. Ex-post devaluation of nominally risk-free debt through costless inflation can replicate state-contingent debt (Bohn (1990), Chari, Christiano and Kehoe (1991),Siu (2004), and Bhandari et al. (2017*b*)). Partially defaulting on real debt can also replicates state-contingent debt (Grossman and Van Huyck (1985), Adam and Grill (2017), and Arellano, Mateos-Planas and Ríos-Rull (2023)). Issuing a suite of risk-free real bonds of differing maturities can replicate state-contingent (Angeletos (2002), Buera and Nicolini (2004), and Aparisi de Lannoy et al. (2025)). State-contingency resembles the post-Civil War and Great Depression debt management of the United States.

that satisfies (i) optimality for households, (ii) optimality for firms, (iii) market clearing, and (iv) the government's intertemporal budget. The *Ramsey problem* is to maximize household welfare (equation 2) over all competitive equilibrium. Formal definitions for the italicized terms (e.g., *feasible government policy*) are found in Appendix B.

The Ramsey problem features unique ergodic distributions for debt, taxes, and interest rates. Sections 3.2 and 4.3 analytically derive the steady-state optimal debt, tax, and interest rates as well as a first-order approximation of the distribution. Although the policy problem is inherently non-convex, the Ramsey problem satisfies the Skiba (1978) conditions laid out by Angeletos, Collard and Dellas (2023) (Sec 4.C) which guarantee a unique ergodic distribution of debt.

2.6 Computation

In Sections 3 and 4, I solve for the global solution to the Ramsey problem using standard numerical methods (Section 20.3 of Ljungqvist and Sargent (2018)). The central difference between my model and the incomplete markets model in Ljungqvist and Sargent (2018), is the definition of the state variable. In the textbook model the value of government debt does not include it's liquidity services. Computational details for the optimal Ramsey policy are found in in Online Appendix C. Alternatively, the recursive contracts method of Marcet and Marimon (2019) could be used to find the solution.

2.7 Calibration

Table 1 summarizes my calibration. The model period is one year. I set the discount factor β to the standard value of .95, the intertemporal elasticity of substitution to the standard value of 1, the Frisch elasticity to .5 to match the estimates of Chetty et al. (2011), and the labor disutility Ξ to match Prescott (2004). I consider alternative calibrations in Section 3.5.

Parameter	Description	Value	Target
σ^{-1}	Intertemporal elasticity of substitution	1	Standard
η^{-1}	Frisch Elasticity	1/2	Chetty et al. (2011)
Ξ	Labor disutility	1.5	Prescott (2004)
β	Discount factor	.95	Standard
ν	Interest rate spread on liquid assets	.0192 (128 bp)	Convenience yield on public debt
ϕ	Semi-elasticity of interest rate spread with public debt	.0154 (103 bp)	Convenience yield on public debt

Table 1: **Baseline calibration.**

Liquidity Demand By log-linearizing the liquidity-adjusted Euler equation (5) around steady state, I can write the convenience yield $r_t^{rf} - r_t$ in terms of the structural liquidity demand parameters $\{\nu, \phi\}$ and the debt ratio b_t, i.e., $\frac{1}{C}(r_t^{rf} - r_t) = \nu - \phi \sinh^{-1}(b_t)$.[15] I use the spread between AAA corporate bonds r_t^{AAA} and long-maturity Treasury bonds $r_t^{Tres.}$ as a proxy for the convenience yield $r_t^{rf} - r_t$. AAA corporate bonds are similarly safe to Treasury bonds but do not offer the same liquidity services.

I calibrate the liquidity demand parameters $\{\nu, \phi\}$ to match the convenience yield on public debt. Extending Krishnamurthy and Vissing-Jorgensen (2012)'s data, I calculate the interest rate spread $r_t^{AAA} - r_t^{Tres.}$ annually from 1919 to 2022. Similarly, I calculate the debt held by the public as a percent of GDP b_t over the same period. Since 1945, consumption-to-output, i.e., C has been relatively stable at .66. I regress the scaled interest rate spread $\frac{1}{C}(r_t^{AAA} - r_t^{Tres.})$ on a constant and hyperbolic sine of the debt ratio $\ln\left(b_t + \sqrt{b_t^2 + 1}\right)$ to recover the estimated parameters $\{\hat{\nu}, \hat{\phi}\}$.[16] These parameter values imply that the liquidity satiating level of the debt ratio is 160% and the peak of the debt Laffer curve is 72%. Repeating this estimation procedure with Krishnamurthy and Vissing-Jorgensen (2012)'s linear-log specification yields similar parameters, debt satiation levels, and debt Laffer peaks.

Government Expenditure Process Table 2 describes the estimated government expenditure process. To generate the stochastic government expenditure process G_t, I fit a two-state Markov-switching dynamic regression (MS-DR) model to the annual government expenditure to output ratio from 1929 to 2022. The stationary distribution of government expenditure is the sum of two independent normal distributions , $G_t \sim \pi_p N(G_p, \sigma_g) + \pi_w N(G_w, \sigma_g)$, where $\pi = [\pi_p, \pi_w]$ is the stationary distribution of of the Markov switching process P between peace and war states; G_p, G_w are the average government expenditure in peace and war time; and σ_g is the standard errors in peace and war time. The stationary distribution of government expenditure is $G \sim N(G^*, \sigma^*)$ where $G^* = \pi_p G_p + \pi_w G_w$ and $\sigma_g^* = \sigma_g \sqrt{\pi_p^2 + \pi_w^2}$.

I define the government expenditure to output ratio as the ratio of nominal government consumption and investment (GCEA in FRED) to the nominal gross domestic product (GDPA in FRED). This measure of government expenditure includes total national defense, income security, education and general government spending. It does not include transfers. Fitting the model reveals that the low government expenditure state is $\hat{G}_p = .2$, high government expenditure state is $\hat{G}_w = .45$, low state persistence is $\hat{\rho}_{low} = .99$, and high state persistence is $\hat{\rho}_{high} = .75$. Since estimating the probability of a crises is difficult with only one episode, I set the low state persistence to .95 to better match the historical experience of the United States prior to 1945, when wars were more frequent. In this calibration, crises occur every 20 years and last for 4 years; the probability

[15] Let $\frac{1}{R_t^{AAA}} = \mathbb{E}_t \beta \frac{u_{C,t+1}}{u_{C,t}}$, then by the liquidity-adjusted Euler equation (5) $\frac{1}{R_t} = \frac{1}{R_t^{AAA}} + \mathbb{E}_t[\frac{w_{B,t}}{u_{C,t}}]$. Let consumption be constant, i.e., $C_t = C$. Log-linearization of each side reveals $r_t^{AAA} - r_t^{Tres.} = C w_{B,t} = C[\nu - \phi \sinh^{-1}(B_t)]$.

[16] To think about these parameter estimates in basis points multiply each by $C \cdot 10^4$ =6,666.

Parameter	Description	Value
G_p	Peacetime government expenditure level	.20
G_w	Wartime government expenditure level	.45
ρ_p	Peacetime government expenditure state persistence	.95
ρ_w	Wartime government expenditure state persistence	.75
σ_g	Standard deviation	.030
G^*	Stationary expenditure	.24
σ_g^*	Stationary standard deviation	.025

Table 2: **Government expenditure process calibration.** All values calibrated to match the the level and change in annual government expenditure (GCEA in FRED) to output (GDPA in FRED) ratio from 1929 to 2022. The top panel is calibrated using a two-state Markov-switching dynamic regression model, the bottom panel is the stationary model.

of being in crisis in any given year is 17%, i.e., $\pi_p = .83$ and $\pi_w = .17$. This shock process in conjunction with the parameter values in Table 1 imply that the natural asset limit is -2,100% of GDP and the natural debt limit is 185% of GDP.

3 Optimal Policy with Stochastic Government Expenditure

This section characterizes the optimal Ramsey policy with incomplete bond markets. At the baseline calibration of the liquidity demand parameters, the satiation motive dominates the savings motive and the optimal debt target is positive and large. This large debt position is funded with high labor taxes, debt rollovers, and low liquidity taxes. The inclusion of liquidity demand helps rationalize historical U.S. public debt levels. These results are robust to alternative calibrations.

3.1 Ramsey Problem

Let $\mathscr{B} = B\sum_s \Omega_s(G)u_{c,s} + w_B B$ be the marginal value of debt. This definition includes the liquidity value of debt. The Ramsey planners optimal value function for $t \geq 1$ satisfies the Bellman equation:

$$\begin{aligned} V(\mathscr{B},G) = &\max_{(B,\{N_s,\mathscr{B}_s\})\in\Phi(\mathscr{B},G)} \sum_s \Omega_s(G)[u(N_s-G_s)-v(N_s)+w(B)+\beta V(\mathscr{B}_s,G_s)] \\ &\text{where } \Phi(\mathscr{B},G) = \{(B,\{N_s,\mathscr{B}_s\}) \in CE(\mathscr{B},G): \\ &\frac{u_{C,s}}{w_B+\sum_{s'}\Omega_{s'}u_{C,s'}}\mathscr{B} \leq u_{C,s}C_s - v_{N,s}N_s + \beta\mathscr{B}_s, \forall s\}. \end{aligned} \tag{10}$$

Where $\Omega_s(G)$ is the probability of state G_s conditional on state G and $CE(\mathscr{B},G)$ is the competitive equilibrium defined in section 2.5. Proposition 1 characterizes the interior Ramsey policy which solves problem 10.

Proposition 1 (Optimal Ramsey Policy)**.** *The optimal Ramsey policy is characterized by:*

(i) optimal debt policy: $w_B[(1-\varepsilon)\Phi - 1] = \sum_s \Omega_s(G)(\Phi_s - \Phi)u_{C,s}$

(ii) optimal tax policy: $\tau_s[(1+\eta)\Phi_s - 1] = (\sigma+\eta)\Phi_s + (\Phi_s - \Phi)B\frac{u_{CC,s}}{u_{C,s}}, \forall s$

(iii) government budget constraint: $\frac{u_{C,s}}{w_B + \sum_{s'}\Omega_{s'}u_{C,s'}}\mathscr{B} = u_{C,s}C_s - v_{N,s}N_s + \beta\mathscr{B}_s, \forall s$

(iv) resource constraint: $N_s = C_s + G_s, \forall s$

where $\Phi_s = V_{\mathscr{B},s}$ *is the tightness of the government budget constraints in state s and* $\varepsilon(B) = -\frac{w_{BB}B}{w_B}$ *is the liquidity risk aversion parameter.*

Without liquidity demand, the Ramsey problem (10) is identical to Aiyagari et al. (2002) with risk-aversion or Bhandari et al. (2017*a*) without price shocks. The construction of this Ramsey problem along with the time 0 Ramsey problem can be found in Online Appendix C.

Theory of Saving and Satiating Optimal debt policies critically depend on the *savings* motive of the government. Unable to engage in ex-post debt devaluation, the government assembles a war-chest of assets. Policymakers, worried about a future crisis, engage in precautionary savings to guard against future labor tax increases. Precautionary savings partially cover surprise government expenditure. Policymakers, worried about hitting the natural debt limit, engage in buffer stock savings. At the natural debt limit, labor taxes must cover all government expenditure shocks. To avoid this scenario the policymaker builds up a buffer stock. These governmental savings motive resemble the household savings motive in Bewley-Hugget-Aiyagri incomplete markets model.[17]

The size of this war chest depends on the degree of fiscal risk and the ability of the policymaker to engage in interest rate manipulation. Fiscal risk increases with the size and frequency of wars (i.e., G_w and $1-\rho_p$). At the extreme, linear consumption utility rules out interest rate manipulation. So policymakers save enough to hit the natural asset limit. At the natural asset limit, the policymaker pays for all government expenditure with the interest on their assets and keep labor taxes fixed at zero (Aiyagari et al. (2002)). Concave consumption utility allows the policymaker to manipulate interest rates. Policymakers save enough to insure against fiscal risk with a combination of labor taxes and interest payments (Bhandari et al. (2017*a*)). This savings level is less than the natural asset limit but still quite large.

In a model with liquidity demand, optimal debt policies also depend on the government's desire to *satiate* liquidity. With lump-sum taxes, the government issues debt until the liquidity tax is zero. This policy has echoes in macroeconomics and public finance. Namely the Friedman rule in macroeconomics and the zero-capital tax policy from public finance. Optimal policy in classical

[17] Shin (2006) contrasts the savings and satiating motive of the government in a two-agent Bewley model.

monetary models (e.g., Chari and Kehoe (1999)) follows a Friedman-rule: flood the economy with enough money to satiate liquidity demand (Chari, Christiano and Kehoe (1996)). If government debt, rather than money, provides liquidity services first-best policy follows a similar rule: flood the economy with enough debt to satiate liquidity demand (Sims (2025)). Optimal policy in neoclassical taxation models recommends a zero capital tax (Atkinson and Stiglitz (1976) and Chari and Kehoe (1999)) for different reasons. Since taxing capital effectively taxes future consumption it violates the uniform commodity taxation rule of Atkinson and Stiglitz (1976). Taxing government bonds by depressing interest rates also taxes future consumptions and is distortionary.

The optimal debt level is dictated by the relative power of the savings and satiating motives. Their relative power depends on the parameters of the model: a larger Frisch elasticity or intertemporal elasticity of substitution amplifies the savings motive while larger liquidity demand amplifies the satiation motive. At the baseline parameters, the satiation motive trumps the savings motive and the optimal debt target is .65 (i.e., 65% of GDP). In the presence of aggregate shocks, the average optimal debt level in the ergodic distribution is .1095 (i.e., 110% of GDP).

3.2 Optimal Policy

Optimal Debt Target The Ramsey problem outlined in proposition 1 has a unique steady-state equilibrium. Proposition 2 characterizes this equilibrium implicitly. At this equilibrium, policies can run primary deficits (i.e., $G^* > \tau^* N(\tau^*, G^*)$) as long as the gross real interest rate is low (i.e., $R^* < 1$). This holds if the marginal demand for liquidity is sufficiently larger than the marginal consumption utility (i.e., $w_B^* > \frac{1-\beta}{\beta} u_C^*$). Put differently, if the marginal value of *satiating* is sufficiently larger than the marginal value of *saving*, optimal policy is characterized by low interest rates, primary deficits, and a positive debt level. The baseline calibration features strong enough liquidity demand to meet this criteria.

Proposition 2 (Optimal policy targets)**.** *At the steady-state equilibrium, the optimal tax rate τ^*, gross interest rate R^*, and debt level B^* are implicitly defined as:*

$$\tau^*(B^*) = \frac{\sigma+\eta}{\varepsilon(B^*)+\eta}$$

$$R^*(G^*, B^*) = \frac{1}{\beta} \frac{u_C(\tau^*, B^*)}{u_C(\tau^*, B^*) + w_B(B^*)}$$

$$B^*(G^*) = \frac{R^*}{R^*-1}[\tau^* N(\tau^*, G^*) - G^*]$$

where $\varepsilon(B) = -\frac{w_{BB} B}{w_B}$ is the liquidity risk aversion parameter.

Figure 4 illustrates the unique steady-state equilibrium of the model. This figure plots the optimal path of debt, taxes, and interest rates starting from 100% debt to GDP after the government expenditure remains at peacetime levels (i.e., .2) for 75 years. Although no aggregate shocks occur during this period the policymaker must allow for fiscal space in case a shock occurs. This experiment illustrates what policymakers should do during periods of peace: optimal policy calls for reducing the debt level and labor taxes. The optimal debt target is substantially higher, and optimal interest rate is lower, when households demand liquidity.

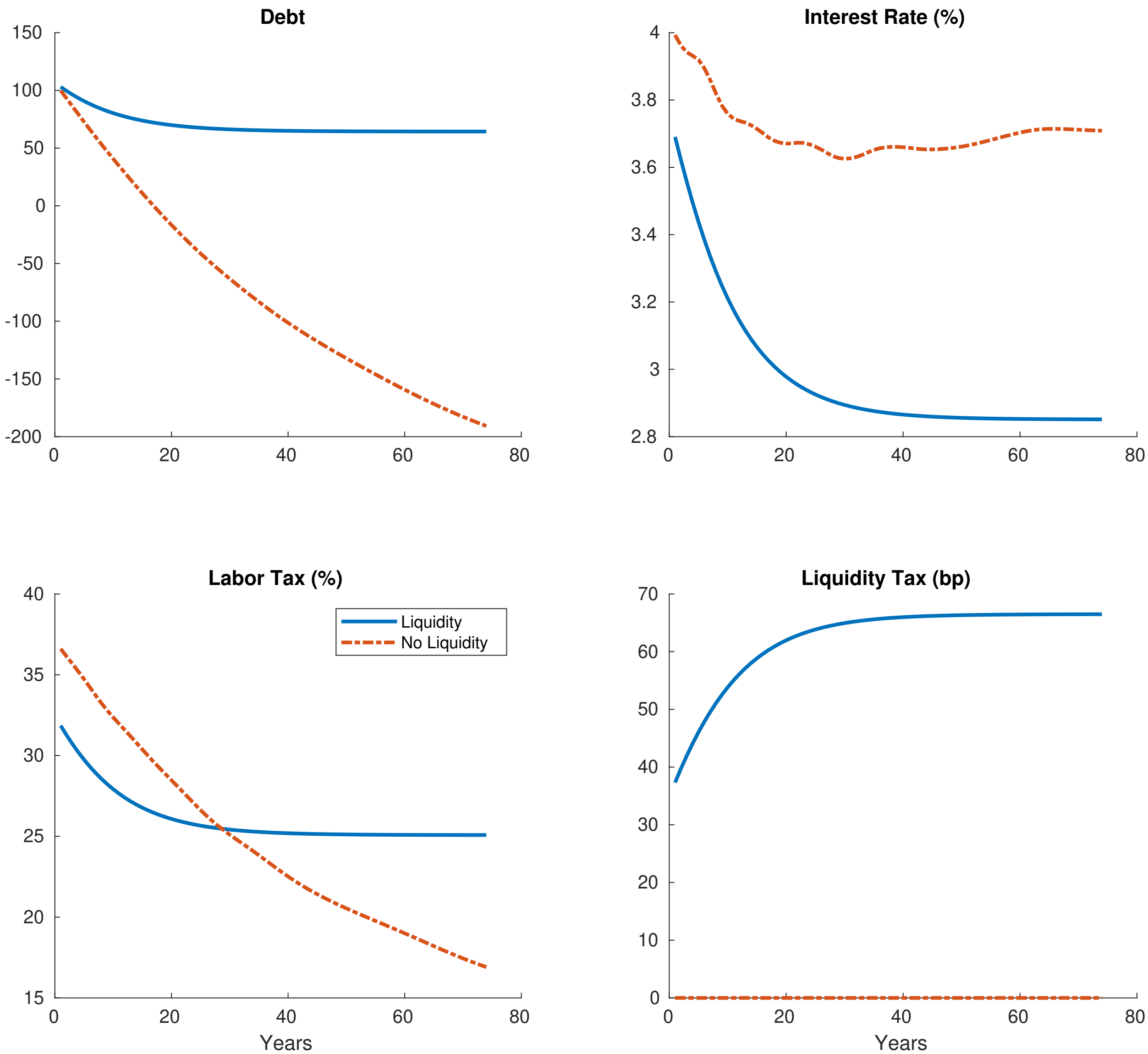

Figure 4: **Optimal policy targets.** This figure plots the optimal path of debt, taxes, and interest rates after a series of low expenditure shocks. In the baseline liquidity and no liquidity model the demand parameters equal $\{\nu, \phi\}$ and $\{0, 0\}$. Debt-to-output begins at 1 (i.e., 100% of GDP) for each.

Absent liquidity demand the policymaker quickly reduces the debt and eventually builds up a large asset position. This large asset position acts as precautionary savings against future aggregate shocks. With liquidity demand the policy maker slowly reduces the debt to approximately 64%, labor taxes to 25%, and interest rates to 2.85%. The incentive to satiate liquidity trumps the incentive to save assets, i.e., $w_B^* > \frac{1-\beta}{\beta} u_C^*$. Liquidity demand raises the optimal debt target and reduces the speed of convergence.

The optimal labor tax $\tau^* = \frac{\sigma+\eta}{\varepsilon^*+\eta}$ depends on the degree of risk aversion, the Frisch elasticity, and the degree of liquidity risk aversion. Greater risk aversion σ amplifies the savings motive leading to higher taxes. Greater Frisch elasticity η^{-1} indicates large labor supply responses to taxation and a lower optimal tax rate; greater liquidity risk aversion ε^* reduces the equilibrium interest rate and allows policymakers to reduce labor taxes.

The optimal liquidity tax $\omega^* = \beta \frac{w_B^*}{u_C^*}$ at the optimal debt target is approximately 67 basis points. Given households demand for liquidity, policymakers should target a higher labor taxes during peacetime. This 67bp liquidity tax keeps the gross interest rate $R^* = \beta^{-1}/(1+\omega^*)$ below the inverse of the discount factor. This interest rate suppression allows the government to reduce labor taxes while still running a balanced budget. Though the government must cover a larger debt position, interest rate suppression is large enough to keep labor taxes relatively low.

	$G_p = .17$	$G_p = .20$	$G_p = .24$
Debt	60.03	64.35	67.53
Labor Tax	20.07	25.08	30.13
Interest Rate	2.48	2.85	3.20
Liquidity Tax	74.22	66.50	60.08

Table 3: **Optimal policy targets.** This table describes the optimal policy targets of the liquidity demand model across different steady-state expenditure. The table presents the debt level, labor tax rate, interest rate, and liquidity tax (in basis points) for different peacetime expenditure levels. I simulate a model at each expenditure level for 200 years.

Policy targets depend on the equilibrium expenditure level. Table 3 displays the optimal policy targets for different peacetime expenditure levels. As suggested by proposition 2, debt levels, labor taxes, and interest rates rise with the peacetime expenditure level. Labor tax distortions are amplified by government expenditure. The more output is lost to government expenditure, the more valuable labor becomes. Policymakers partially debt-finance elevated expenditures. This debt financing reduces the optimal liquidity tax and raises the optimal labor tax.

This optimal debt target is greater than in a model without quantitatively realistic liquidity demand (e.g., Bhandari et al. (2017*a*)) and similar to a model with liquidity demand (e.g., Aiyagari and McGrattan (1998)). In a quantitative two-agent model with uninsurable aggregate risk and hand-to-mouth agents, Bhandari et al. (2017*a*) find that the optimal debt target is approximately 0% of output. The aggregate surplus risk and lack of complete markets leads the savings motive to dominate In a quantitative heterogeneous agent model with uninsurable idiosyncratic risk, Aiyagari and McGrattan (1998) find that the satiating motive dominates and the optimal debt target is approximately 66% of output.

Optimal Policy Response Proposition 3 describes the optimal policy responses of debt, taxes, and interest rates to a persistent government expenditure shock. All three positively comove with a government expenditure shock. More persistent shocks, lower Frisch elasticity, and greater liquidity risk aversion amplify the debt response and dampen the tax response. Greater liquidity tax, debt levels, and marginal liquidity risk aversion amplify the tax response and dampen the debt response. Since $\varepsilon^* > 0$, liquidity demand amplifies the interest rate response because greater debt reduces the liquidity premium. Risk aversion amplifies the debt, tax, and interest rate response.

Proposition 3 (Optimal policy response)**.** *The first-order responses of labor taxes and real interest rates to a government expenditure shock with persistence ρ are:*

$$\frac{dB}{dG}|^* \approx \frac{(1+\eta)}{(1-\varepsilon^*)}\frac{(1-\tau^*)}{\omega^*}\frac{\mathrm{d}N_s}{\mathrm{d}G_s}|^* + \sigma\frac{B^*/C^*}{(1-\varepsilon^*)\omega^*}(-\frac{\mathrm{d}C_s}{\mathrm{d}G_s}|^*)$$

$$\frac{d\tau_s}{dG_s}|^* \approx \frac{1-\rho}{\rho}\frac{(1+\eta)(1-\tau^*)+\sigma\varepsilon_B^* B^*/C^*}{\varepsilon^*+\eta}\omega^* R^* \frac{\mathrm{d}B}{\mathrm{d}G}|^*$$

$$\frac{dR_s}{dG_s}|^* = R^*[\sigma C^*(-\frac{\mathrm{d}C_s}{\mathrm{d}G_s}|^*) + \frac{\varepsilon^*}{\rho}\frac{\omega^* R^*}{B^*}\frac{\mathrm{d}B}{\mathrm{d}G}|^*]$$

where $\omega^* = \beta w_B^*/u_C^*$ is the equilibrium liquidity tax and $\frac{\mathrm{d}N_s}{\mathrm{d}G_s}|^* > 0, \frac{\mathrm{d}C_s}{\mathrm{d}G_s}|^* < 0$ are defined implicitly in Online Appendix C.5.

Figure 5 plots the optimal policy response from steady-state to a surprise expenditure shock. The figure illustrates optimal policy responses with and without liquidity demand. The government expenditure shock starts in period 20 and lasts for 10 years. Prior to the shock there is a long period of low expenditure. So policy tools begin the simulation at their optimal targets, visualized in figure 4 and described in table 3. Labor taxes, interest rates and debt levels rise while liquidity taxes fall. Therefore optimal labor and liquidity taxes are negatively correlated; policymakers should tax liquidity during peacetime and labor during crises.

Liquidity demand amplifies the rise in labor taxes during wars. Irrespective of the degree of liquidity demand, labor taxes fall during peace times and rise during wars. Liquidity demand dictates the magnitude of wartime rise and the speed of peacetime convergence. By raising the debt burden being serviced, liquidity demand amplifies the necessary increase in wartime labor taxes. Mechanically, liquidity taxes steadily fall as the government builds up debt.

Regardless of liquidity demand, wars are partially debt financed. The onset of a war triggers a gradual debt increase. Aiming to minimize future tax increases the policymaker opts for a partially debt and tax-financed war. Despite differences in liquidity demand all three economies experience similar increases in debt during the war; albeit from different starting debt levels (see Figure 4). Following the war's conclusion the policymaker steadily reduces the debt. Liquidity demand increases the speed of convergence to the steady-state level.

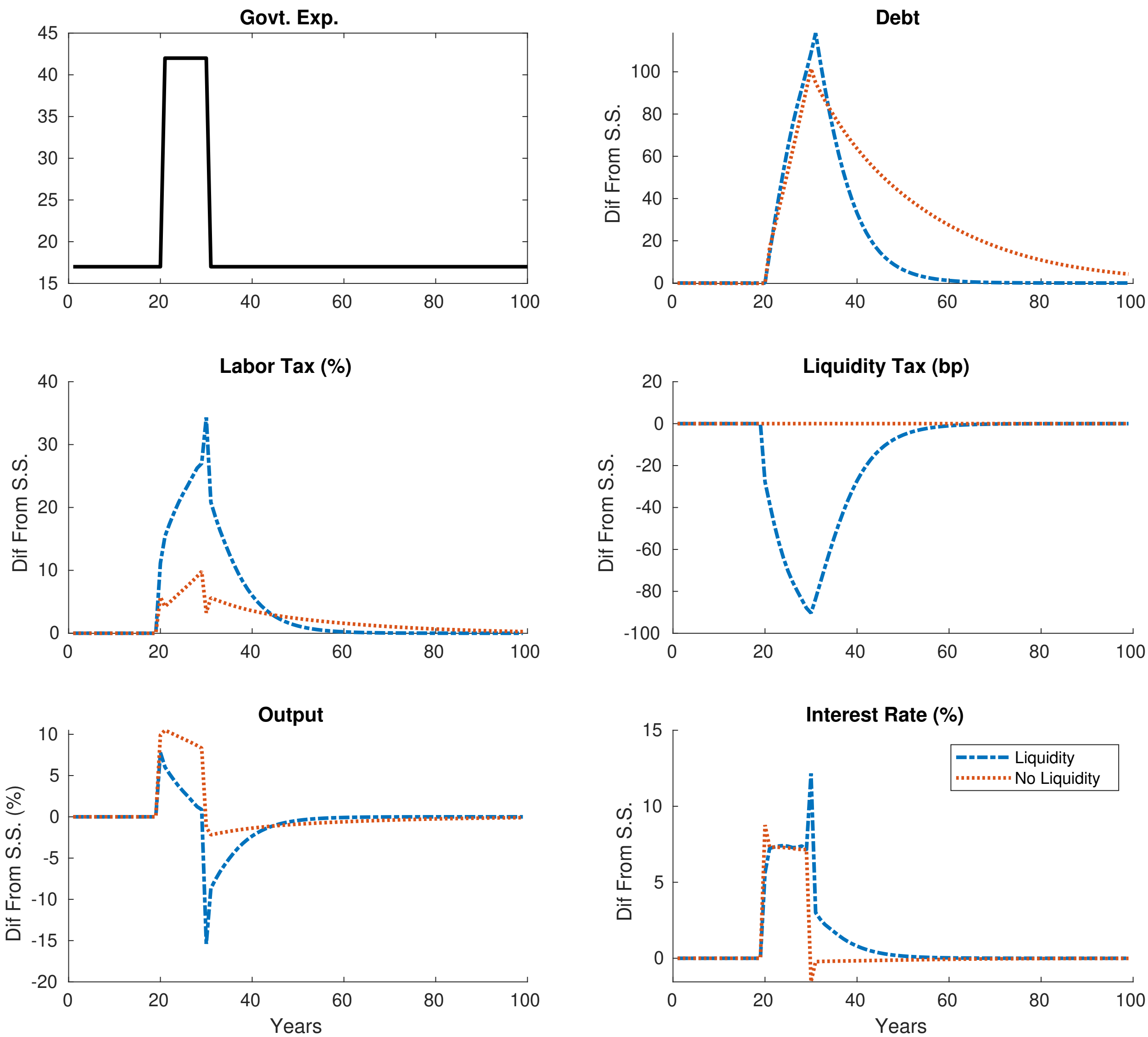

Figure 5: **Optimal policy response.** This figure plots the optimal policy response to an unforeseen government expenditure shock in the incomplete markets model. In the baseline liquidity and no liquidity model the demand parameters equal $\{\nu, \phi\}$ and $\{0,0\}$. The government expenditure shock starts in period 20 and lasts for 10 years. Prior to the shock there is a long period of low expenditure. The top left panel plots a single government expenditure shock. The remaining panels plot the differences from steady-state for each variable. Output is plotted as the percent difference while the remaining variables are plotted as level differences.

Seemingly paradoxically, liquidity demand smooths wartime labor taxes. Persistent expenditure shocks produce positive and negative spikes in labor taxes and interest rates at the beginning and end of wars.[18] These spikes mimic lump-sum taxes and transfers at the start and conclusion

[18]Optimal taxation policy in a model with capital taxes produces similar spikes (Farhi (2010)).

of wars. The positive spike in labor taxes at the war's onset spurs a negative spike in consumption facilitating a spike in interest rates. As the government is a net creditor, this positive interest rate spike mimics a lump-sum tax on households. The corresponding negative labor tax spike at the end of the war mimics a lump-sum transfer. Therefore these spikes resemble the inflation tax spikes in the complete markets model (see Online Appendix E). Liquidity demand dampens these spikes. Manipulating the interest rate to cover expenditure shocks is less efficient in the presence of liquidity demand.

Optimal Policy Distribution Optimal debt, tax, and interest rate policy have unique ergodic distributions. The opposing saving and satiating forces generate this distribution. I present both a numerical and analytical approximation of these distributions. We can approximate these ergodic distribution using a first-order Taylor expansion around the steady-state. Since government expenditure is a fat-tailed distribution, the policy instruments also inherent this fat-tailed distribution. 4 describes these distributions in detail.

Proposition 4 (Optimal policy distributions)**.** *To first-order, optimal debt, taxes, and interest rates are characterized by distributions:*

$B \sim N(B^*, \frac{dB}{dG}|^* \sigma_g^*)$

$\tau \sim N(\tau^*, \frac{d\tau_s}{dG_s}|^* \sigma_g^*)$

$R \sim N(R^*, \frac{dR_s}{dG_s}|^* \sigma_g^*)$

Where government expenditure is characterized by the stationary distribution $G_t \sim N(G^*, \sigma_g^*)$.

Figure 6 plots the ergodic distribution of labor taxes and debt levels. Liquidity demand markedly shifts the distribution of debt upwards and reduces debt fluctuations. Absent liquidity demand, the debt converges to a distribution centered at -300% of GDP and tends to fluctuate between -500 and -200%. While in a model with liquidity demand, the distribution of debt-to-output converges to a distribution centered at 109% and tends to fluctuate between 50 and 200%. In this case, debt fluctuations are typically buttressed by the debt Laffer curve peak and the debt satiating level. Though large and persistent crises can push the debt past the debt satiating level and long periods of peace can push it past the debt Laffer curve.

Liquidity demand markedly increases the level and volatility of labor taxes. Absent liquidity demand and risk-aversion, the policymaker builds a war chest of assets and reduces labor taxes to zero to replicate a complete markets outcome (Aiyagari et al. (2002)). Risk aversion leads the policymaker to accumulate less assets and minimize fiscal risk (Bhandari et al. (2017*a*)) by varying both the interest rate and labor taxes. Rather than paying for government expenditure with interest on assets and depriving households of liquidity the policymaker opts to tax labor and provide the household liquidity. Though liquidity demand had little impact on averages labor taxes in a

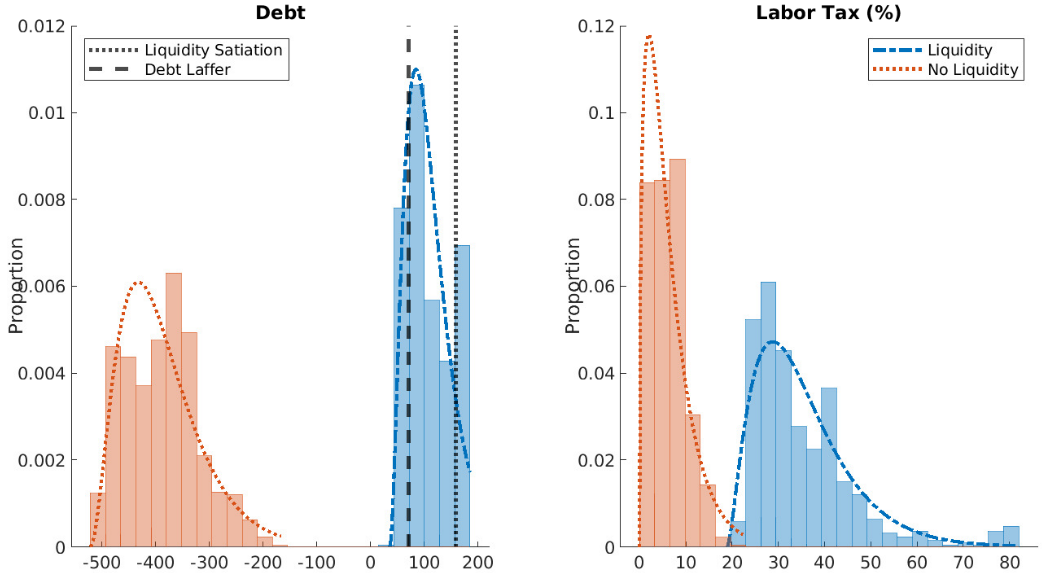

Figure 6: **Optimal policy distributions.** This figure plots the distribution of labor taxes and the debt-to-output ratio in the incomplete markets model. I overlay a kernel distribution on each distribution. In the baseline liquidity and no liquidity model the demand parameters equal $\{\nu, \phi\}$ and $\{0,0\}$. The left and right panels plots the distribution of labor taxes and debt-to-output ratios along with a Gamma distribution approximation. The debt satiation and debt Laffer curve levels plotted in the right panel are only relevant to the model with liquidity demand. There is a 1000 year burnin period prior to the 5000 year simulation.

complete markets model (see Online Appendix E), liquidity demand notably elevates the entire distribution of labor taxes in an incomplete markets model.

The policymaker utilizes liquidity taxes, albeit at a moderate level. Optimal policy prescribes "mining the bubble" to reduce labor taxation (Brunnermeier, Merkel and Sannikov (2024)). Though optimal liquidity taxation is very low averaging less than 30 basis points. Quantitatively, interest rate suppression of 30 basis points is far below the average historical convenience yield of 70 basis points (Krishnamurthy and Vissing-Jorgensen (2012)). During wars, the aggregate insurance motive of risk-free debt (Aiyagari et al. (2002)) outweighs the desire to tax liquidity. Debt financing the war causes the debt to cross the liquidity satiation point.

3.3 Taxation Decomposition

Each year, the policymaker must pay for exogenous government expenditure. There are three tools at the policymaker's disposal: (1) labor taxes (2) liquidity taxes and (3) debt rollover. The policymaker can tax labor income, tax government debt's liquidity services, or issue new debt. Since debt is noncontingent, there is no ex-post devaluation and thus no inflation taxes.

$$G_t = \overbrace{\tau_t N_t - T_t}^{\text{Labor Tax Rev.}} + \underbrace{\omega_t B_t}_{\text{Liquidity Tax Rev.}} + \overbrace{(\frac{B_t}{R_t^{rf}} - B_{t-1})}^{\text{Debt Rollover}} \tag{11}$$

Rearrange the government budget equation (8) to decompose the government expenditure across these three tools (equation 11): the net labor tax revenue $\tau_t N_t - T_t$, the liquidity tax revenue $\omega_t B_t$, and the debt rollover $B_t / R_t^{rf} - B_{t-1}$ as the value of debt rollover. Again, I use the risk-free interest rate R_t^{rf} rather than the prevailing interest rate R_t in defining the debt rollover to differentiate between liquidity taxation and debt rollover.[19]

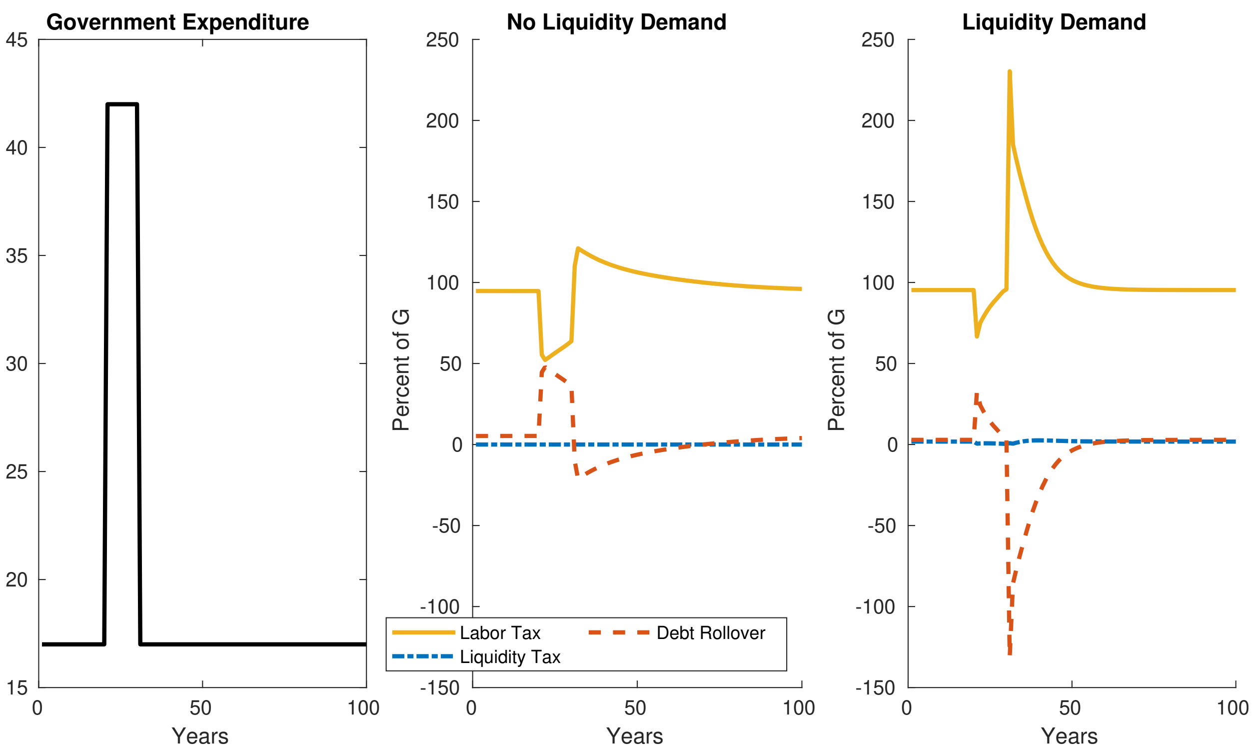

Figure 7: **Taxation decomposition.** This figure plots the decomposition of taxes for a incomplete markets model with liquidity demand and without liquidity demand. The left panel plots a one time government expenditure shock, the center panel plots the tax decomposition without liquidity demand, and the right panel plots the tax decomposition with liquidity demand. Prior to the shock there is a long period of low expenditure. Percentages are defined in terms of the government expenditure level G_t.

This decomposition, visualized in Figure 7, allows comparison of each tools importance across states of the world. The left panel plots the government expenditure shock, and the middle and right panels plots the magnitude of each tool as a percentage of government expenditure for economies

[19]In Online Appendix E, I consider optimal policy with state-contingent debt, which can be understood as a model with costless inflation. Unsurprisingly, inflation tax is the primary fiscal instrument in that environment.

with no liquidity demand and baseline liquidity demand. Prior to the shock there is a long period of low expenditure.

In both economies, labor taxes are the predominate tool to pay for government expenditure during peace time. Since labor taxes also pay for debt issued last year they can exceed 100% of government expenditure. At the start of a crisis, the policymaker rolls over the debt. Consistent with Barro (1979), the policymakers desire to smooth taxes leads debt and labor taxes to persistently increase following a government expenditure shock. Prolonged crises make continuing debt rollovers ever more costly and push the policymaker to resort to increasingly more labor taxation. The conclusion of the crisis causes the policymaker to steadily reduce labor taxes and the debt level.

Liquidity demand does not substantially alter this pattern of taxation, though it introduces a tertiary taxation tool. During peace time, liquidity taxes are utilized to reduce labor taxes. Therefore optimal policy calls for "eating a free lunch" (Mian, Straub and Sufi (2025)) or "mining the bubble" (Brunnermeier, Merkel and Sannikov (2024)) by taxing liquidity and *reducing* labor taxes during peacetime.

3.4 Historical Debt Policy

Optimal debt levels closely resemble actual debt levels in advanced countries. According to the IMF, the average debt level in advanced economies has risen from 50% in the 1980s to 115% in the 2010s. As of 2023, public debt to GDP sits at 93% in the European Union, 100% in the United Kingdom, 120% in the United States, and 260% in Japan. Only two advanced economies maintain a negative net debt position - Norway and Luxembourg. The 109% average optimal debt ratio in the model is similar in magnitude to these actual debt levels.

Although the average optimal debt ratio resembles the current debt ratio, recent debt policy was overly reliant on debt financing. Figure 8 plots the actual and optimal U.S. debt and primary deficit ratios since 2000. The optimal policy begins in the year 2000 with the same debt and expenditure levels as the United States. The quantitative model used for this exercise is nearly identical to the baseline model with noncontingent debt, except that this model rules out government asset accumulation (i.e., $B_t \geq 0,\ \forall t$). Fiscal stress from the Great Financial Crisis and the Covid Pandemic expanded the debt. Optimal policy also called for debt expansion in both cases. However, in both instances policymakers relied too heavily on debt-financing. As a percentage of GDP, primary deficit ratios were 5% too high during the former and 7% too high during the latter. Between crises policymakers failed to reduce the debt. By 2023, this over-reliance on debt-financing pushed debt ratios 20 p.p. too high.

Although over-reliant on debt, recent policy had moderate welfare impacts. To gauge the wel-

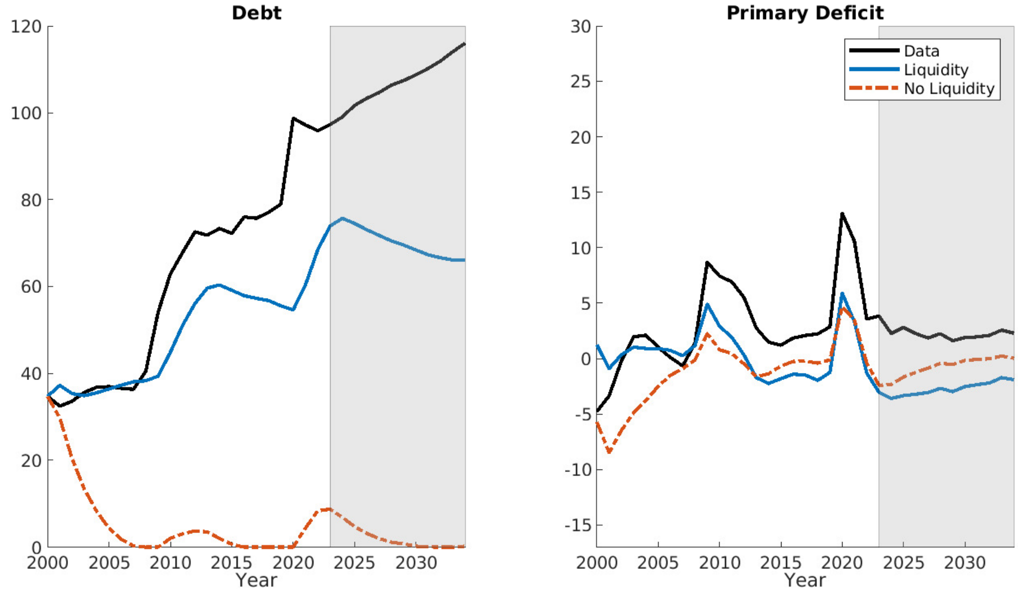

Figure 8: **Recent and projected debt policy.** This figure plots the actual and optimal debt and primary deficit ratios in the United States since 2000. Actual debt is the gross debt held by the public (FYGFDPUB in FRED). Optimal policy begins in 2000 with the same debt ratio and expenditure level as the actual history.

fare impacts, I compare the continuation value (i.e., $V(\cdot, G_{2023})$) of the actual debt (i.e., $\mathscr{B}^{act}_{2023}$) to the optimal debt (i.e., $\mathscr{B}^{opt}_{2023}$) at 2023 expenditure levels given the history of expenditure since 2000. Reducing debt to the optimal level increases welfare, i.e., $V(\mathscr{B}^{opt}_{2023}, G_{2023}) > V(\mathscr{B}^{act}_{2023}, G_{2023})$. To have a sense of scale, I compare this difference to the difference between optimal and the worst case: $V(\mathscr{B}^{opt}_{2023}, G_{2023}) - \min_{\mathscr{B}} V(\mathscr{B}, G_{2023})$. The welfare cost of the overreliance on debt during recent crises, was 5% of this worst case welfare loss.

According to the model, observers should worry about the projected rise in public debt. While recent debt policy resembles optimal policy, debt ratios are well above the optimal target. Moreover, debt ratios are projected to increase, not decrease, over the next 10-20 years (CBO (2020)). Policymakers should be running 2% surpluses not 2% deficits as projected. The United States was hit with two large fiscal crisis over the last 15 years. The Global Financial Crisis and Covid Pandemic put substantial stress on the fiscal budget and expanded the debt. According to the model, increasing the debt during crises is optimal; maintaining this debt during peacetime is not. Therefore, given the expected path of government expenditure, policymakers should aim to reduce debt to 70% of GDP over the next 10 years. If debt continues on its projected path, there will be substantial welfare losses. I use the same formula for welfare comparisons in 2034 as I used for 2023. Failure to reduce the debt to optimal would reduce welfare by 12% of the maximum welfare difference.

Although elevated, current and projected debt levels do not threaten fiscal sustainability. The natural debt limit is 185% of GDP. Hitting the natural debt limit requires an enormous crisis. Following a decade long crisis, on par with World War II, the United States would still have fiscal space. To see this, suppose the government maintains a peacetime debt level of 100%. Figure 5 makes clear that a surprise decade long war would increase the debt ratio by 100% to 200%. At this point the government would run out of fiscal space.

3.5 Alternative Calibrations

Model	Mean	Median	Std. Dev.	Target	Debt Limit
No Liquidity	-371.35	-375.18	88.12	-327.37	185.27
Partial Liquidity ($\nu/2, \phi/2$)	85.88	70.74	49.71	29.47	185.27
Baseline	109.84	97.22	41.27	64.35	185.27
Mian, Straub and Sufi (2025)	145.83	142.40	24.06	121.08	185.90
Jones (2024)	144.97	138.48	23.49	129.44	187.35
Income Effects (IES = .8)	146.40	126.94	62.31	76.29	267.47
Substitution Effects (IES = 1.25)	63.32	57.18	23.41	40.78	112.70
More Elastic Labor (Frisch = 1)	64.02	60.22	23.35	35.63	104.64
Less Labor Disutility ($\Xi = 1$)	144.95	123.16	59.80	86.27	283.96
More Patient ($\beta = .98$)	160.39	130.72	84.48	83.25	463.17
Heightened Crisis Prob. ($\bar{p} = .75$)	91.35	83.37	62.33	-14.03	185.27
Big Crisis ($\bar{G} = .60$)	30.25	30.23	21.43	-3.54	68.54

Table 4: **Alternative calibrations.** This table describes the optimal debt policy for different parameter calibrations. The first block present alternative liquidity demand calibrations. The second block present alternative preference calibrations. And the third block present different shock processes. The columns represent the mean, median, and standard deviation of the simulated debt, the target debt level, and the debt limit.

The optimal debt levels and targets are sensitive to liquidity demand calibrations and robust to alternative preference calibrations. Table 4 presents key statistics of the optimal debt policy for alternative calibrations. The first block present alternative liquidity demand calibrations. The second block present alternative preference calibrations. And the third block present different shock processes. To calculate the first three statistics I simulate a long series of stochastic expenditure shocks.[20] To calculate the optimal debt target I simulate a long series of low government expenditure.[21]

Optimal debt policy critically depends on the demand for liquidity. Therefore I consider alterative liquidity demand calibrations. Mian, Straub and Sufi (2025) and Jones (2024) directly

[20] I simulate 2000 years of stochastic government expenditure with a 100 year burnin.

[21] I simulate 200 years of constant government expenditure at the peacetime levels $G = .2$.

estimate the demand for liquidity through carefully identified surprise debt issuance which can be mapped to parameters (ν,ϕ). Mian, Straub and Sufi (2025) use the 2022 Georgia Senate runoff as a natural experiment for surprise debt issuance. Democrats had a 50% chance of winning and controlling the senate. Their victory in Georgia facilitated a nearly $2 Trillion debt-financed stimulus. Jones (2024) uses high frequency identification of surprise debt issuance at weekly Treasury auctions. Both papers estimated significantly stronger demand for liquidity than used to calibrate the baseline model: High debt-invariant spread for Mian, Straub and Sufi (2025) (i.e., $(\nu,\phi) = (.053, .036)$) and a low semi-elasticity for Jones (2024) (i.e., $(\nu,\phi) = (.016, .006)$). Consequently, the average optimal debt levels and targets for those calibrations are significantly higher. The liquidity demand estimated by Jones (2024) is strong enough to raise the debt limit, albeit by a small amount. Therefore the optimal debt levels advocated by the baseline model is likely a conservative estimate.

Household preference parameters also affect optimal debt policy. If income effects dominate substitution effects, optimal debt levels should be higher. The opposite is true for substitution effects. Microeconomic empirical studies typically estimates the IES to be below 1 while macro studies estimate the IES to be above 1 (Havránek (2015)). More elastic labor supply leads to lower optimal debt targets and levels. Micro estimates typically estimate households' labor supply to be less elastic than macro studies (Chetty et al. (2011)). More patient households allow for more fiscal expansion. Patient households demand a lower interest, making debt issuance less costly. More patient households lead to larger optimal debt targets and levels.

As the likelihood and magnitude of wars, i.e., fiscal risk, increases, the optimal debt levels and targets decrease. Big crises and heightened crisis probabilities significantly reduce the optimal target debt. The government expenditure process in the baseline model is already a conservative estimate: a crisis arrives every 20 years.

4 Optimal Policy with Stochastic Liquidity Demand

I solve for the optimal Ramsey policy in an extension of the baseline model with stochastic liquidity demand. Fluctuating liquidity demand introduces interest rate risk. I calibrate these fluctuations to closely match the historical fluctuations in Treasury bonds' liquidity premia. Following a rise in liquidity demand, optimal policy calls for issuing debt while reducing labor taxes and real interest rates.

4.1 A Model of Stochastic Liquidity Demand

The model in this sections builds on the baseline model in Section 2. Broadly, this model sets aside surplus risk to focus on interest rate risk. I abstract from surplus risk by setting government expenditure to a constant peacetime level (i.e., $G_t = G_p$, $\forall t$) and introduce interest rate risk through liquidity shocks (i.e., $\nu_t(s^t)$). Let the stochastic liquidity demand be modeled as,

$$\nu_t = (1-\alpha)\nu^* + \alpha \nu_{t-1} + u_t, \tag{12}$$

where the liquidity demand shocks u_t are normally distributed $u_t \sim N(0, \sigma_\nu)$. This AR(1) process for liquidity demand closely matches the history of interest rates on Treasury bonds. Kekre and Lenel (2021) utilize a similar process to model safety shocks. Liquidity shocks alter the household savings decision, producing a *stochastic liquidity-adjusted Euler equation*,

$$\frac{1}{R_t} = \frac{1}{R_t^{rf}} + \beta \sum_{s^{t+1}} \Omega(s^{t+1}|s^t) \frac{w_{B,t+1}}{u_{C,t}}. \tag{13}$$

where $w_{B,t+1} = [\nu_{t+1} - \phi \sinh^{-1}(B_t)]$. Although the stochastic liquidity-adjusted Euler equation (13) resembles the liquidity-adjusted Euler equation (5) in the baseline model. For a fixed debt level B_t, fluctuations in the liquidity demand ν_t lead to a stochastic convenience yield $R_t^{rf} - R_t$. Details of the model are found in Online Appendix D.

Liquidity demand fluctuations also affect the government's problem. Fluctuating liquidity demand alters the natural asset limit and the debt Laffer curves. The natural asset limit is the maximal asset limit at which the government can pay for future expenditure at the minimum interest rate. Governments earning low interest rates on their savings require more assets. Since interest rates are stochastic even at a fixed asset position, the natural asset limit varies with the degree of liquidity demand (i.e., $\underline{\mathrm{B}}(\nu_t)$). If positive liquidity shocks push the interest rate above the minimum, the government can remain at the natural asset limit and rebate the excess return on its assets to the household via a transfer. The debt Laffer curve is now stochastic, meaning that the peak of the debt Laffer curve and the liquidity satiating point vary with the state of the world. Keeping liquidity completely satiated following a rise in liquidity demand requires debt issuance.

Since public debt is noncontingent, first-best debt policy aims to satiate *expected* liquidity demand. Too much debt causes excessive labor taxation in low liquidity demand states. Too little debt causes excessive liquidity taxation in high liquidity demand states. The persistence of liquidity demand shocks implies that positive shocks raise the expected level.

Calibrating Liquidity Shocks The liquidity demand spread parameter follows an AR(1) process, $\nu_t = (1-\alpha)\nu^* + \alpha\nu_{t-1} + u_t$. By log-linearizing the stochastic liquidity adjusted Euler equa-

Parameter	Description	Value	Basis Points	Target
ν^*	Average interest spread on liquid assets	.0192	128	Avg. residual convenience yield
α	Persistence of liquidity shocks	.685	N/A	Autocorrelation of residual convenience yield
σ_ν	Standard deviation of interest rate shocks	.0042	28	Std. dev. of residual convenience yield
ϕ	Semi-elasticity of interest rate spread with public debt	.0154	103	Convenience yield

Table 5: **Stochastic liquidity demand model calibration.**

tion (13) around steady state, I can write the convenience yield $r_t^{rf} - r_t$ in terms of the stochastic liquidity demand ν_t and the debt-to-output level B_t: $\frac{Y}{C}(r_t^{rf} - r_t) = \nu_t + \phi \sinh^{-1}(B_t)$. I use the spread between AAA corporate bonds r_t^{AAA} and long-maturity Treasury bonds $r_t^{Tres.}$ as a proxy for the convenience yield $r_t^{rf} - r_t$. AAA corporate bonds are similarly safe to Treasury bonds but do not offer the same liquidity services. I calibrate the stochastic properties of ν_t to match the AAA/Treasury spread given historical debt-to-GDP levels. Table 5 lists the calibrated parameters and corresponding targets.

Let the residual convenience yield be the convenience yield unexplained by the debt-to-GDP level. Conditional on the debt level, I set ν to .0192 to match the average residual convenience yield, α to .685 to match the autocorrelation of the residual convenience yield, and σ^ν to .2828 to match the standard deviation of the residual convenience yield. This parameterization approximates the historical convenience yield well ($R^2 = .60$). The average convenience yield in the data is 82 basis points in the model and the data, standard deviation is 45 basis points in the data and 35 basis points in the model, skewness is 68 basis points in the data and 89 in the model, and autocorrelation is .75 in the data and .31 in the model. Figure 5 in Online Appendix D.1 plots the historical calibrated liquidity demand shocks.

4.2 Ramsey Problem

Redefine the value of debt as $\mathscr{B} = B\sum_s \Omega_s u_{c,s} + B\sum_s \Omega_s w_{B,s}$. The Ramsey planner's optimal value function for $t \geq 1$ satisfies the Bellman equation:

$$\begin{aligned} V(\mathscr{B}, \nu) = \max_{(B,\{N_s,\mathscr{B}_s\})\in\Phi(\mathscr{B},\nu)} \sum_s \Omega_s [u(N_s - G) - v(N_s) + w(B, \nu_s) + \beta V(\mathscr{B}_s, \nu_s)] \\ \text{where } \Phi(\mathscr{B}, \nu) = \{(B, \{N_s, \mathscr{B}_s\}) \in CE(\mathscr{B}, \nu) : \\ \frac{u_{C,s}}{\sum_s \Omega_s [u_{C,s} + w_{B,s}]} \mathscr{B} \leq u_{C,s} C_s - v_{N,s} N_s + \beta \mathscr{B}_s, \forall s\}. \end{aligned} \tag{14}$$

Where $\Omega_s = \Omega(\nu_s|\nu)$ is the probability of state ν_s conditional on state ν and $CE(\mathscr{B}, \nu)$ is the competitive equilibrium defined in section 2.5. Proposition 5 characterizes the optimal interior Ramsey problem which solves problem (14).

Proposition 5 (Optimal Ramsey Policy with Liquidity Demand Shocks)**.** *The optimal interior Ramsey policy is characterized by:*

(i) optimal debt policy : $\sum_s \Omega_s w_{B,s}[(1-\varepsilon_s)\Phi - 1] = \sum_s \Omega_s(\Phi_s - \Phi)u_{C,s}$

(ii) optimal tax policy: $\tau_s[(1+\eta)\Phi_s - 1] = (\sigma+\eta)\Phi_s + (\Phi_s - \Phi)B\frac{u_{CC,s}}{u_{C,s}}$

(iii) government budget constraint: $\frac{u_{C,s}}{\sum_s \Omega_s[u_{C,s}+w_{B,s}]}\mathscr{B} = u_{C,s}C_s - v_{N,s}N_s + \beta\mathscr{B}_s$

(iv) resource constraint: $N_s = C_s + G$

where $\Phi = V_{\mathscr{B}}$ *and* $\Phi_s = V_{\mathscr{B},s}$ *are the tightness of the government budget constraints and* $\varepsilon_s = -w_{BB,s}B/w_{B,s}$ *is the state-dependent liquidity risk aversion parameter.*

The Ramsey problem (14) can be seen as a generalization of Sims (2025) (Section 5) without the assumption of perfect foresight. Alternatively, this Ramsey problem is a generalization of Kocherlakota (2023) with endogenous interest rates. The construction of this Ramsey problem along with the time 0 Ramsey problem can be found in Online Appendix D.2.

4.3 Optimal Policy

Optimal Policy Target The Ramsey problem outlined in proposition 5 has a unique steady-state equilibrium. Proposition 6 characterizes this equilibrium implicitly. This equilibrium is identical to the steady-state equilibrium in the environment with government expenditure shocks (characterized by proposition 2).

Proposition 6 (Optimal Steady-State Policy with Liquidity Demand Shocks)**.** *At the steady-state equilibrium, the optimal tax rate* τ^**, gross interest rate* R^**, and debt level* B^* *are implicitly defined as:*

$$\tau^*(\nu^*, B^*) = \frac{\sigma+\eta}{\varepsilon(\nu^*, B^*)+\eta}$$
$$R^*(\nu^*, B^*) = \frac{1}{\beta}\frac{u_C(\tau^*, B^*)}{u_C(\tau^*, B^*)+w_B(B^*, \nu^*)}$$
$$B^*(\nu^*) = \frac{R^*}{R^*-1}[\tau^* N(\tau^*, \nu^*) - G]$$

where $\varepsilon(\nu^*, B^*) = -\frac{w_{BB}(\nu^*)B^*}{w_B(\nu^*)}$ *is the liquidity risk aversion parameter.*

Optimal Policy Response Proposition 7 describes the optimal policy responses of debt, taxes, and interest rates to a persistent liquidity demand shock. Debt rises while the effect on taxes and interest rates is ambiguous. The effect on taxes depends on the persistence of the shock — as long as the liquidity demand shock is sufficiently persistent and the debt response is small, i.e., $\alpha > \frac{\varepsilon_B^*}{\varepsilon^*}\frac{\mathrm{d}B}{\mathrm{d}\nu}^*$, taxes fall. If the debt response is small enough, the rise in the liquidity premium ultimately leads interest rates to fall. Lower Frisch elasticity and greater liquidity risk aversion amplify the debt response and dampen the tax response. Greater liquidity tax, debt levels, and marginal liquidity risk aversion amplify the tax response and dampen the debt response. Risk aversion amplifies the debt, tax, and interest rate responses. Steady-state taxes dampen the debt and tax response, while steady-state interest rates amplify the tax and interest rate responses.

Proposition 7 (Policy Response to Liquidity Demand Shocks)**.** *The first-order responses of labor taxes and real interest rates to a liquidity demand shock with persistence α are:*

$$\frac{dB}{dv}|^* \approx [\frac{1+\eta}{1-\varepsilon^*}\frac{(1-\tau^*)}{\omega^*} - \frac{\sigma}{\omega^*}\frac{B^*}{C^*}]\frac{\mathrm{d}C_s}{\mathrm{d}v_s}|^*$$

$$\frac{d\tau_s}{dv_s}|^* \approx -\omega^* R^*[\frac{(1+\eta)(1-\tau^*)+\sigma(1-\alpha)/\alpha\ B^*/C^*}{\varepsilon^*+\eta}][\alpha\varepsilon^* - \varepsilon^*_B\frac{\mathrm{d}B}{\mathrm{d}v}|^*]$$

$$\frac{dR_s}{dv_s}|^* = R^*[\sigma C^*(-\frac{\mathrm{d}C_s}{\mathrm{d}G_s}|^*) + \frac{\varepsilon^*}{\alpha}\frac{\omega^* R^*}{B^*}\frac{\mathrm{d}B}{\mathrm{d}v}|^* - \frac{\beta R^*}{u^*_C}]$$

where $\omega^* = \beta w^*_B/u^*_C$ is the equilibrium liquidity tax and $\frac{\mathrm{d}C_s}{\mathrm{d}G_s}|^* > 0$ is defined implicitly in Online Appendix D.5.

Figure 9 plots the optimal policy response from steady-state to a surprise liquidity shock. The liquidity shock starts in period 20 and lasts for 10 years. Prior to the shock there is a long period of low liquidity demand. Each year the policymaker is uncertain if the liquidity crisis will continue another year. On impact labor taxes, interest rates, and the debt ratio fall while liquidity taxes rise. Throughout the crisis labor taxes, liquidity taxes, interest rates, and debt ratio rise while liquidity taxes fall.

Liquidity crises are a boon to the government budget; policymakers increase debt in response. During the liquidity crisis policymakers gradually issue debt to satiate liquidity demand. The debt level jumps on impact, then increases slowly, because policymakers hedge against the possibility of the liquidity crisis ending. After the liquidity crisis abates, policymakers slowly lower the debt level. Debt converges slowly outside the crisis so the policymaker can smooth labor taxes.

The policymaker takes advantage of the reduced financing costs to lower labor taxes. Greater liquidity demand induces lower interest payments. These low interest payments allow the government to reduce labor taxes. During the liquidity crisis labor taxes gradually rise as debt expands, but remain below the pre-crisis level. Labor tax cuts spur a jump in output. While policymakers partially debt-finance liquidity crises, they also resort to liquidity taxation. About half of the liquidity shock (50 bp) manifests as a rise in the liquidity tax (25 bp). Completely satiating liquidity would require a substantial increase in the debt. Policymakers forego this increase to reduce future labor taxes. On impact, liquidity shocks lower interest rates.

Optimal Policy Distribution Optimal debt, tax, and interest rate policy have unique ergodic distributions. The opposing saving and satiating forces generate this distribution. I present both a

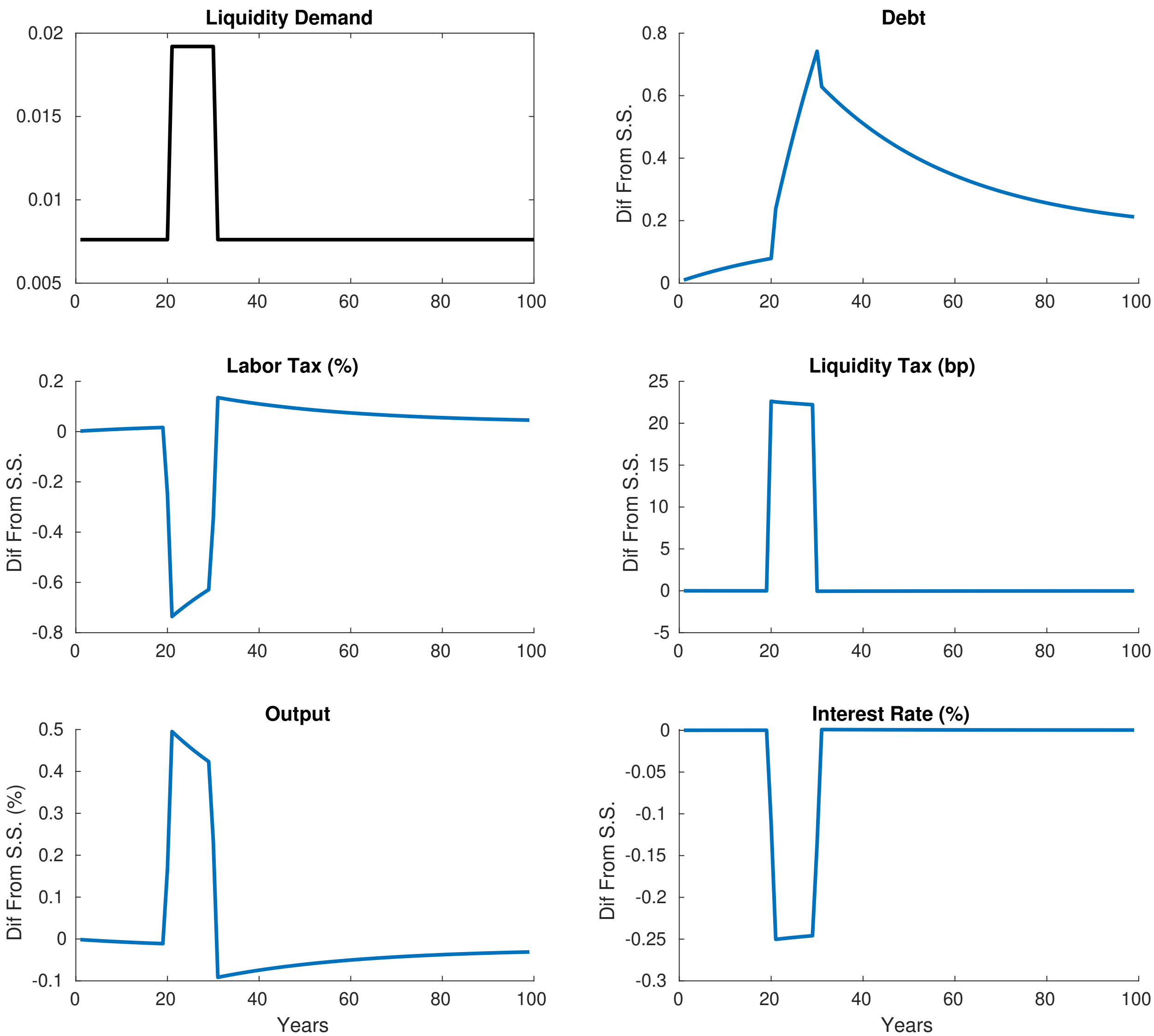

Figure 9: **Policy response to liquidity demand shocks.** This figure plots the optimal policy response to an unforeseen liquidity demand shock. The liquidity demand shock begins in year 20 and lasts for 10 years. Prior to the shock there is a long period of low liquidity demand. The top left panel plots a single liquidity demand shock. The remaining panels plot the differences from steady-state for each variable. Output is plotted as the percent difference while the remaining variables are plotted as level differences.

numerical and analytical approximation of these distributions. Using first-order Taylor expansion around the steady-state, I approximate these ergodic distribution. Policy instruments inherent the normal distribution of liquidity demand shocks. Proposition 8 describes these distributions in detail.

Proposition 8 (Stationary Distributions with Liquidity Demand Shocks)**.** *Optimal debt, taxes, and interest rates are characterized by normal distributions:*

$B \sim N(B^*, \frac{dB}{dv}|^* \frac{\sigma_v}{\sqrt{(1-\alpha^2)}})$
$\tau \sim N(\tau^*, \frac{d\tau_s}{dv_s}|^* \frac{\sigma_v}{\sqrt{(1-\alpha^2)}})$
$R \sim N(R^*, \frac{dR_s}{dv_s}|^* \frac{\sigma_v}{\sqrt{(1-\alpha^2)}})$
Where liquidity demand is characterized by an AR process $v_t = (1-\alpha)v^* + \alpha v_{t-1} + u_t$*, with stationary distribution* $v_t \sim N(v^*, \frac{\sigma_v}{\sqrt{(1-\alpha^2)}})$.

Figure 6 in Appendix D.5 plots the ergodic distribution of key policy variables. Debt levels are centered at 140% and fluctuate very little. (s.d. 2.5%). The liquidity demand process is not particularly persistent ($\alpha = .69$). Therefore liquidity shocks dissipate quickly. Since the policymaker satiates expected liquidity, not current liquidity, the optimal debt level is relatively constant. Moreover, because government expenditure is constant, total government expenditure remains relatively constant. This allows labor taxes to remain fixed. Labor taxes average 30% and fluctuate little (s.d. 1 %). However, regardless of liquidity crises, government expenditure and debt payments are labor tax-financed. Rather than impact the debt level or labor taxes, the 42 basis point jumps in liquidity demand manifest as liquidity tax jumps and interest rate drops. Interest rates average 4.9% and fluctuate moderately (s.d. 30 basis points). Liquidity taxes average 33 basis points and fluctuate substantially (s.d. 25 basis points). Their combined variation (55 basis points) is large enough to absorb the liquidity demand shocks.

5 Conclusion

In this paper, I argue that historically U.S. debt policy has been nearly optimal. To make this argument, I solve for the optimal Ramsey policy in an incomplete markets model featuring liquidity demand and surplus risk. I find that the optimal debt target is 65% of GDP and that debt should increase in response to surprise expenditure. Additionally, in extensions of the baseline model I demonstrate that high debt levels are optimal for policymakers facing liquidity risk.

Standard incomplete market models do not match the historical public debt levels or the convenience yield in the United States. This discordance between models and data stems from the precautionary savings motive that dominant standard incomplete markets models. By calibrating my model to also match the liquidity demand implied by the low interest rate on Treasury bonds, I find markedly higher optimal debt levels than other incomplete markets models. In contrast to recent papers that similarly match the low interest rates on public debt, I match also match the historical fluctuations in debt.

The stark results in this paper stem from the public's demand for U.S. Treasuries. The safety and liquidity of U.S. Treasuries makes them a more attractive savings vehicle than any other private bonds. Given this attractiveness, U.S. policymakers have two options: deprive the market of safe-assets to tax liquidity or satiate the markets desire for safe-assets by taxing labor. For various parameter values and shock processes this paper recommends the former policy during peacetime and latter policy during crises.

By focusing on the impact of liquidity demand on optimal debt levels, this paper sets aside important channels. Policymakers should take seriously the possibility of debt cliffs, self-fulfilling debt crises, and default more generally. Even if the U.S. can pay back its debt, bond vigilantes could force a fiscal consolidation if they get nervous. Policymakers should also consider who is buying the debt. If liquidity demand is mainly coming from emerging markets, policymakers should keep the debt closer to the peak of the debt Laffer curve, allow international bond holders to pay the liquidity tax, and channel these revenues to debt reduction.

A Microfoundation

There are two types of households: constrained and unconstrained. Constrained households are unable to borrow or lend money, while the unconstrained households can do both. A portion $\rho(b)$ of households are unconstrained while the rest $1-\rho(b)$ are constrained each period. This proportion depends on the amount of government debt b. This model resembles the OLG structure of Blanchard (1985) and Angeletos, Lian and Wolf (2024) but introduces a precautionary savings motive which is central to heterogeneous agent models like Aiyagari (1994) and Auclert et al. (2024).

A.1 Constrained Households

Constrained households fully consume their labor income and savings. They maximize

$$W(b,\lambda) = \max_{n,c} u(c) - v(n)$$
$$\text{s.t. } c = n + \lambda b$$

where c, n are consumption and labor supply, b is savings, and $\lambda \in [0,1]$ is the portion of constrained households savings that is liquid. We can think of households with a high λ as being wealthy hand-to-mouth and those with a low λ as being poor hand-to-mouth where λ is the realization of some distribution Λ. This distribution dictates the distribution of assets for the constrained households and the proportion of poor and wealthy hand-to-mouth households.

A.2 Unconstrained Households

Unconstrained households can borrow or lend money b' with no constraints. They take taxes τ and the price of bonds Q as given. Unconstrained households maximize

$$U(b) = \max_{n,c,b'} u(c) - v(n) + \beta\rho(b')U(b') + \beta(1-\rho(b'))E[W(b',\lambda)]$$
$$\text{s.t. } c + Qb' = (1-\tau)n + T + b$$

where $E[W(b',\lambda)] = \int W(b',\lambda)d\lambda$ is the expected value of being constrained and $\rho(b) = \frac{\rho}{1+\exp(-\gamma b)}$ is the probability of remaining unconstrained. This probability is logistic and depends on the asset position of unconstrained households. The more households save, the more likely they are to survive and not become hand-to-mouth. This mimics the precautionary savings motive in a standard Aiyagari (1994))model. Households that die with assets or liabilities pool carry them over to the state in which they are hand-to-mouth. This mimics the buffer-stock savings motive found in OLG models (e.g., Diamond (1965); Blanchard (2019)).

Taking first-order conditions we see that the standard labor-leisure condition

$$\frac{v_n}{u_c} = (1-\tau). \tag{15}$$

holds. Combining the envelope condition and the first-order condition for borrowing, we arrive at the Euler equation

$$Q = \beta E[\frac{u_{c'}}{u_c}] + \beta(1-\rho(b'))\frac{E[W_{b'}] - U_{b'}}{U_b} + \beta\rho'(b')\frac{U(b') - E[W(b',\lambda)]}{U_b}. \tag{16}$$

Where the first-term on the right-hand side is the risk-free rate, the second-term represents the demand for precautionary savings and the final term is the demand for buffer-stock savings. The precautionary savings term is the product of the probability of dying (i.e., $1-\rho(b')$) times the excess marginal value of assets when constrained (i.e., $E[W_{b'}] - U_{b'}$). The buffer-stock savings term is the product of the marginal increase in the chance of surviving (i.e., $\rho'(b')$) times the excess value from surviving $U(b') - E[W(b',\lambda)]$. Both terms are positive, and imply that the equilibrium nominal interest rate $i = Q^{-1} - 1$ in steady-state is low (i.e., $i < \beta^{-1} - 1$).

A.3 Representative Household

For a given debt level b the proportion of unconstrained households is $\rho(b)$. Therefore the expected welfare of the representative household is

$$V(b) = \rho(b)U(b) + (1-\rho(b))\int W(b,\lambda)d\lambda. \tag{17}$$

Rearranging this expression, and using the definition of the unconstrained households problem, we can see that

$$V(b) = \max_{n,c,b'} u(c) - v(n) + w(b) + \beta E[V(b')]$$
$$\text{s.t. } c + Qb' = (1-\tau)n + T + b$$

where $w(b) = [1-\rho(b)][U(b) - \int W(b,\lambda)d\lambda]$ is the value of liquidity: the product of the probability of being constrained and the excess value of being unconstrained.

B Equilibrium Definition

Before describing optimal policy, I define a feasible allocation, feasible government policy, competitive equilibrium, and the Ramsey problem. These definitions apply in the main text and appendices.

Definition B.1 (Feasible Allocation)**.** Given an initial debt level B_0 and the stochastic government expenditure process $G_t(s^t)$, a *feasible allocation* is a stochastic process $f_t(s^t) = \{C_t(s^t), N_t(s^t)\}$ for consumption and output which satisfies market clearing (equation 7) and whose time t elements are measurable with respect to the initial debt level B_0 and the history of government expenditure shocks $G_t(s^t)$.

Definition B.2 (Competitive Equilibrium)**.** Given an initial debt level B_0 and the stochastic government expenditure process $G_t(s^t)$, a *competitive equilibrium* is a feasible allocation, a feasible government policy, and a stochastic real interest rate process $R_t(s^t)$ which satisfies the household's labor-leisure decision (equation 4), the household's liquidity-adjusted Euler equation (5) and the government's intertemporal budget constraint (equation 8).

Definition B.3 (Feasible Government Policy)**.** Let $\mathbb{P}_{\mathbb{t}}$ be the space of feasible government policies. A *government policy* $p_t(s^t) = \{\tau_t(s^t), T_t(s^t), B_t(s^t)\} \in \mathbb{P}_{\mathbb{t}}$ is a stochastic process for labor taxation $\tau_t(s^t)$, transfers $T_t(s^t)$, the real debt level $B_t(s^t)$ whose time t elements are measurable with respect to the initial debt level B_0 and the history of government expenditure shocks $G_t(s^t)$. A *feasible government policy* is a policy $p_t(s^t) \in \mathbb{P}_t(s^t)$ which respects the labor Laffer curve (equation **??**), the restriction on transfers $T_t(s^t) \in \mathbb{R}, \forall s^t$, the natural debt limit (equation **??**) and the natural asset limit $B_t(s^t) \geq 0$.

Definition B.4 (Ramsey Problem)**.** The *Ramsey problem* is to maximize expected household utility (equation 2) over all competitive equilibria. I define the *continuation Ramsey problem* to be the Ramsey problem faced by the policymaker at time $t \geq 1$ after state s_t has been realized. I define the *initial Ramsey problem* to be the Ramsey problem faced by the policymaker at time $t = 0$.

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Online Appendix

C Optimal Policy with Government Expenditure Shocks

This section outlines the optimal Ramsey problem for the incomplete markets economy. I describes how to reach the solution using the recursive contracts method of Marcet and Marimon (2019). The methodology closely follows Ljungqvist and Sargent (2018) section 20.3.

C.1 Primal Method

Risk-free debt requires that the household and government choose tomorrow's debt level today. Therefore the household budget constraint,

$$C_t + \frac{B_t}{R_t} = (1-\tau_t)N_t + B_{t-1} + T_t,$$

now indexes next periods debt level B_t with time index t to indicate that this debt level is chosen after history s^t. After substituting out the labor tax $1-\tau_t$ with the labor leisure decision (equation 4), and the real interest rate with the liquidity-adjusted Euler equation (5), the household budget is written in primal form as

$$u_{C,t}B_{t-1} = u_{C,t}[C_t - T_t] - v_{N,t}N_t + \beta \sum_{s_{t+1}} \Omega(s_{t+1}|s^t)[u_{C,t+1}B_t + w_{B,t}B_t]. \tag{18}$$

Where debt-to-output tomorrow $b_{t+1} = \frac{B_t}{N_{t+1}}$ depends on the debt issued today and output tomorrow. Just as in the state-contingent debt case (Section 4), selecting the optimal debt level B_t is complicated by the future budget constraint. The debt issued by the government today B_t, will impact tomorrows budget. The welfare impacts of the debt issuance today depend on the marginal utility of consumption tomorrow $u_{C,t+1}$ and the marginal value of liquidity tomorrow $w_b(b_{t+1})$. Since these are uncertain, the *expected* value of public debt is $\mathscr{B}_t = \mathbb{E}_t[u_{C,t+1}]B_t + w_{B,t}B_t$. Let $\mathscr{R}_t = \frac{w_{B,t-1}+u_{C,t}}{w_{B,t-1}+\mathbb{E}_{t-1}[u_{C,t}]}$ be the marginal utility adjusted realized interest rate. Let $\mathscr{T}_t = [u_{C,t} - v_{N,t}]N_t$ be the marginal utility adjusted labor tax revenue. Let $\mathscr{Q}_t = w_{B,t-1}B_{t-1}$ be the liquidity tax revenue. Let $\mathscr{G}_t = u_{C,t}G_t$ be the marginal utility adjusted government expenditure level. The primal budget (equation 18) can be rewritten as,

$$\mathscr{B}_{t-1}\mathscr{R}_t + \mathscr{G}_t \leq \mathscr{T}_t + \mathscr{Q}_t + \beta\mathscr{B}_t. \tag{19}$$

Where the inequality emerges from the availability of lump-sum transfers. This version of the budget greatly simplifies analysis as it allows the budget to be written in terms of one state-variable, the expected value of the future debt $\mathscr{B}_t$. Writing the budget condition in this way makes clear that the expected value of the future debt $|debt_{t-1}$ after history s^{t-1} affects the budget constraint for all possible histories $s^t = (s^{t-1}, s_t)$. The configuration of the budget allows me to utilize the expectations parameterization method of Marcet and Marimon (2019) when solving for optimal policy.

C.2 Ramsey Problem

In this section I describe the Ramsey problem faced by the policymaker recursively. The policymaker, aiming to maximize expected household utility (equation 2) at time $t \geq 1$, solves the following Bellman Equation,

$$\begin{aligned} V(\mathscr{B}, G) = \max_{\{B, C_s, N_s, \mathscr{B}_s\}} & \sum_{s \in S} \Omega_s [u(C_s) - v(N_s) + w(B) + \beta V(\mathscr{B}_s, G_s)] \\ \text{s.t } & u_C(C_s)B \leq u_C(C_s)C_s - v_N(N_s)N_s + \beta \mathscr{B}_s \\ & C_s = N_s - G_s \\ & \mathscr{B} = B \sum_{s \in S} \Omega_s [u_C(C_s)] + w_B(B)B \\ & \underline{\mathrm{N}}(G_s) \leq N_s \leq \bar{N}(G_s) \\ & \underline{\mathrm{B}} \leq B \leq \bar{B}. \end{aligned} \tag{20}$$

In which utility maximization $V(\mathscr{B}, G)$ is subject to the intertemporal budget conditions (with corresponding shadow values $\Omega_s \Phi_s$), market clearings (with corresponding shadow values $\Omega_s \Theta_s$, government debt commitment (with corresponding shadow value Φ), and the government policy feasibility constraints on taxes (with corresponding shadow values $\underline{\rho}_{N_s}, \bar{\rho}_{N,s}$), transfers (with corresponding shadow value ρ_T), and debt issuance (with corresponding shadow value $\underline{\rho}_B, \bar{\rho}_B$). The probability of the next state $\Omega_s = \Omega(G_s|G)$ depends on the current state. The policymaker selects the debt level B before the state s is realized, but can condition consumption C_s, output N_s, transfers T_s and the expected value of the debt $\mathscr{B}_s$ on next period's state.

The intertemporal budget conditions Φ_s state that the expected value of the debt due today $\mathscr{B}$ must be paid for by a combination of labor taxation $u_C(C_s)C_s - v_N(N_s)N_s$, liquidity taxation $w_b(\frac{B}{N_s}, s)\frac{B}{N_s}$, and new expected debt issuance $\beta \mathscr{B}_s$. The shadow price of commitment Γ restricts today's policymaker to honor the expected value of government debt $\mathscr{B}$ promised by yesterday's planner. This constraint indicates that today's expected value of government debt $\mathscr{B}$ is the sum of the expected debt burden $\sum_{s \in S} \Omega(G_s|G)[u_C(C_s)N_s b_s]$ and the liquidity tax $w_B(B)B$ which suppresses interest payments on today's debt by taxing liquidity. The labor feasibility condition ρ_N restricts taxation to be below the peak of the labor Laffer curve . The debt issuance limits are the natural debt limit and the natural asset limit, $Nb > 0$. Government expenditure G_s and liquidity demand $w(b_s, s)$ are indexed by the state s to indicate that expenditure and the degree of liquidity demand are exogenous.

Optimal policy consists of decision rules $\{C_s(\mathscr{B},G), N_s(\mathscr{B},G), T_s(\mathscr{B},G), b(\mathscr{B},G), \mathscr{B}_s(\mathscr{B},G)\}$ for consumption, output, transfers, public debt and the expected value of public debt next period as well as a value function $V(\mathscr{B},G)$. Moreover, these decision rules produce shadow values $\{\Phi_s(\mathscr{B},G), \Theta_s(\mathscr{B},G), \Gamma(\mathscr{B},G)\}$ for the intertemporal budget condition, market clearing, and government debt commitment conditions. As in the previous section, the shadow value of the budget constraints $\Phi_s(\mathscr{B},G)$ will control labor taxes, while the shadow value of the government commitment condition $\Gamma(\mathscr{B},G)$ will control the liquidity tax. The relative values of each will dictate the relative level of labor and liquidity taxes. The following sections flesh out how and why.

At time $t = 0$, the policymaker solves the following Bellman Equation,

$$
\begin{aligned}
W(B_0, G_0) = \max_{\{C_0, N_0, \mathscr{B}_0\}} & u(C_0) - v(N_0) + w(B_0) + \beta V(\mathscr{B}_0, G_0) \\
\text{s.t } & u_{C,0} B_0 \leq u_{C,0} C_0 - v_{N,0} N_0 + \beta \mathscr{B}_0 \\
& C_0 = N_0 - G_0 \\
& \underline{\mathrm{N}}(G_0) \leq N_0 \leq \bar{N}(G_0).
\end{aligned} \tag{21}
$$

As in the previous section, I focus my analysis on the continuation Ramsey problem (equation 20) as the initial Ramsey problem (equation 21) is standard (Ljungqvist and Sargent (2018) Section 20.3).

The proceeding section presents analytical approximations of the stationary distribution of key policy variables. I approximate stochastic government expenditure as $G_t \sim N(G^*, \sigma^*)$ using a normal distribution. Alternatively, I could use an alternative model of government expenditure:

$$
\begin{aligned}
G_t &= (1-\rho)G^* + \rho G_t + \sigma_t z_t \\
\sigma_t^2 &= (1-\gamma-\delta)\sigma^2 + \gamma \sigma_{t-1}^2 + \delta \sigma_{t-1}^2 z_{t-1}^2 \\
z_t &\sim N(0,1)
\end{aligned}
$$

This AR-GARCH process has expenditure persistence ρ, GARCH coefficient γ, and ARCH coefficient δ, such that $\gamma + \delta < 1$. The stationary distribution of this model f is a fat-tailed distribution of mean G^* and variance $\frac{\sigma^2}{(1-\rho^2)}$. Although this alternative AR-GARCH model better explains the fat-tailed distrbutions key piolicy variables.

C.3 Optimality Conditions

The optimal Ramsey policy is found by taking first-order conditions of the continuation Ramsey problem:

$$
\begin{aligned}
\Phi &= V_{\mathscr{B}}(\mathscr{B}, G) \\
\Phi_s &= V_{\mathscr{B},s}(\mathscr{B}_s, G_s) \\
u_{C,s}[1-(1-\sigma)\Phi_s] - v_{N,s}[1-(1+\eta)\Phi_s] &= (\Phi - \Phi_s)Bu_{CC,s} + (\underline{\rho}_s^N - \bar{\rho}_s^N) \\
\sum_s \Omega_s(\Phi - \Phi_s)u_{C,s} &= w_B[1-(1-\varepsilon(B))\Phi] + (\underline{\rho}_B - \bar{\rho}_B)
\end{aligned}
$$

where $\varepsilon(B) = w_{BB}B + w_B$ is the liquidity aversion parameter. The shadow values of the Laffer curve constraints $(\underline{\rho}_s^N, \bar{\rho}_s^N)$ and the natural debt and asset constraints $(\underline{\rho}_B, \bar{\rho}_B)$ equal zero when the constraints don't bind. The first-two conditions show that the marginal value of the value of liquidity is equal to the budget tightness each period. The third equation describes the optimal tax policy. To see this remember that $\tau_s = 1 - v_{N,s}/u_{C,s}$. At a steady state interior to the Laffer curve, at which $\Phi = \Phi_s$, the optimal tax equation shows that $\tau_s(\Phi_s) = \frac{(\sigma+\eta)\Phi_s}{(1+\eta)\Phi_s - 1}$. The fourth equation describes the optimal debt policy, which is a tradeoff between debt reduction $\sum_s \Omega_s(\Phi - \Phi_s)u_{C,s}$ and liquidity provision $w_B[1-\varepsilon(B)\Phi]$.

The first-order conditions of the initial Ramsey policy are

$$
\begin{aligned}
V_{\mathscr{B}}(\mathscr{B}_0, G_0) &= \Phi_0 \\
u_{C,0}[1-(1-\sigma)\Phi_0] - v_{N,0}[1-(1+\eta)\Phi_0] &= -\Phi_0 Bu_{CC,0} + \underline{\rho}_0^N - \bar{\rho}_0^N \\
W_B(B_0, G_0) &= w_{B,0} + \Phi_0 u_{C,0}
\end{aligned}
$$

C.4 Steady-State

The steady-state of the optimal policy can be written analytically with a set of implicit functions. The interior steady-state is given by the optimal debt policy, the optimal tax policy, the intertemporal government budget constraint, the resource constraint, and the stochastic government expenditure process:

$$\Phi = 1/(1-\varepsilon(B))$$

$$u_C[1-(1-\sigma)\Phi] = v_N[1-(1+\eta)\Phi]$$

$$(1-\beta)\mathscr{B} = u_C C - v_N N + \beta w_B B$$

$$N = C + G$$

$$G = G^*$$

where we have assumed that taxes respect the labor Laffer curve $\underline{\mathrm{N}}(G) \leq N \leq \bar{N}(G)$ and the optimal debt level respects the natural debt and asset limits $\underline{\mathrm{B}} \leq B \leq \bar{B}$. In contrast to the model without liquidity demand, there is a unique steady-state that depends on the elasticity of liquidity risk aversion $\varepsilon(B)$. By substituting the budget tightness $\Phi(B)$ into the optimal tax policy condition, I can write the optimal tax level in terms of elasticities and debt

$$\tau(B) = \frac{\sigma+\eta}{\varepsilon(B)+\eta} \tag{22}$$

where σ is the degree of risk aversion and η^{-1} is the Frisch elasticity. An increase in risk aversion amplifies the precautionary savings motive, leading policymakers to raise the equilibrium asset level, and thus raise taxes. As liquidity risk aversion $\varepsilon(B)$ increases, so does the convenience yield on public debt. This lowers the interest rate and allows for lower taxes in steady-state. As policymakers increase the debt B they reduce this elasticity, tightening the budget and raising the optimal tax rate. Here is this simple equation we see the central trade-off in the model between labor and liquidity tax reduction.

Given the tax rate and the government expenditure level, the labor-leisure condition and the resource constraint pin down a unique labor supply $N(\tau,G)$ and consumption $C(\tau,G)$. Since the tax rate depends on the debt level, we can write an implicit function for the optimal debt level in steady state using the budget equation

$$B = \frac{R(B,G)}{R(B,G)-1}[\tau(B)N(B,G)-G] \tag{23}$$

where the equilibrium gross interest rate $R(B,G) = \frac{1}{\beta}\frac{u_C(G,B)}{u_C(G,B)+w_B(B)}$ depends on the steady-state debt and tax policy. From this expression we can see that if liquidity demand is strong enough (i.e. ν is big enough) that the gross interest rate is negative (i.e. $R-1<0$) the optimal steady-state policy can be characterized by permanent deficits $\tau(B)N(B,G)-G<0$ and positive debt $B>0$. Together the implicit functions for debt, taxes, and interest rates $B(G), \tau(B), r(B,G)$ define a unique steady-state $\{B^* = B(G^*), \tau^* = \tau(B^*), R^* = R(B^*,G^*)\}$ at the steady-state government expenditure level G^*

C.5 First-Order Approximation of Optimal Policy Distribution

Around this steady-state, government expenditure has first-order effects on optimal debt, tax, and interest rates. I approximate each using a first-order Taylor expansion

$$B \approx B^* + \frac{dB}{dG}|^* G_s$$
$$\tau \approx \tau^* + \frac{d\tau_s}{dG_s}|^* G_s$$
$$R \approx R^* + \frac{dR_s}{dG_s}|^* G_s$$

where $\frac{dB_s}{dG_s}|^*$ is the marginal effect of government expenditure on debt at steady-state. Optimal debt, taxes, and interest rates will, to first-approximation, follow the same normal distribution as government expenditure

$$B_s \sim N(B^*, \frac{dB}{dG}|^* \sigma^*) \tag{24}$$
$$\tau_s \sim N(\tau^*, \frac{d\tau_s}{dG_s}|^* \sigma^*) \tag{25}$$
$$R_s \sim N(R^*, \frac{dR_s}{dG_s}|^* \sigma^*) \tag{26}$$

In the last subsection, I gave an implicit description of the steady-state $\{B^*, \tau^*, r^*\}$ at the steady-state government expenditure level G^*. In order to full characterize the first-order approximation of optimal debt, taxes, and interest rates we need the first-order effects of government expenditure on each. I will again write implicit functions for these marginal effects $\{\frac{dB}{dG}|^*, \frac{d\tau_s}{dG_s}|^*, \frac{dR_s}{dG_s}|^*\}$.

To solve for these marginal effects, I take the derivative of the resource constraint, government budget, optimal tax policy, optimal debt policy, and the shock process with respect to last periods expenditure G. At steady-state,

$$\frac{\mathrm{d}G_s}{\mathrm{d}G}|^* = \rho$$
$$\frac{\mathrm{d}N_s}{\mathrm{d}G_s}|^* = \frac{\mathrm{d}C_s}{\mathrm{d}G_s}|^* + 1$$
$$[(\rho^{-1} - \beta)u_C^* - \beta(1-\varepsilon)w_B^*]\frac{\mathrm{d}B}{\mathrm{d}G}|^* = (1-\sigma)u_C^*\frac{\mathrm{d}C_s}{\mathrm{d}G_s}|^* - (1+\eta)v_N^*\frac{\mathrm{d}N_s}{\mathrm{d}G_s}|^* - B^*u_{CC}^*\frac{\mathrm{d}C_s}{\mathrm{d}G_s}|^*$$
$$\frac{u_{CC}^*}{u_C^*}\frac{\mathrm{d}C_s}{\mathrm{d}G_s}|^* - \frac{v_{NN}^*}{v_N^*}\frac{\mathrm{d}N_s}{\mathrm{d}G_s}|^* = \frac{(1-\sigma)u_C^* - (1+\eta)v_N^* + (1-\rho)/\rho B^* u_{CC}^*}{[1-(1+\eta)\Phi^*]v_N^*}\frac{\mathrm{d}\Phi_s}{\mathrm{d}G_s}^*$$
$$\varepsilon_B^* w_B^* \Phi^* \frac{\mathrm{d}B}{\mathrm{d}G}|^* = [u_C^* + (1-\varepsilon^*)w_B^*]\frac{\mathrm{d}\Phi_s}{\mathrm{d}G_s}|^*$$

where $*$ notation indicates the steady-state value of a variables. The first equation is the derivative with respect to the shock series, the second equation is with respect to the budget equation, the third is with respect to the optimal tax policy, and the last equation is with respect to the optimal debt policy. The final four are a system of linear equations in variables $\{\frac{\mathrm{d}C_s}{\mathrm{d}G_s}|^*, \frac{\mathrm{d}N_s}{\mathrm{d}G_s}|^*, \frac{\mathrm{d}\Phi_s}{\mathrm{d}G_s}|^*, \frac{\mathrm{d}B}{\mathrm{d}G}|^*\}$. Therefore, we can solve for each, however the solutions are quite complex. When $\sigma = 1$, and $\rho, \beta = 1$ then

$$\frac{\mathrm{d}B}{\mathrm{d}G}|^* \approx \frac{(1+\eta)}{(1-\varepsilon^*)}\frac{(1-\tau^*)}{\omega^*}\frac{\mathrm{d}N_s}{\mathrm{d}G_s}|^* + \sigma\frac{B^*/C^*}{(1-\varepsilon^*)\omega^*}(-\frac{\mathrm{d}C_s}{\mathrm{d}G_s}|^*) \tag{27}$$

Using these values we can solve for the marginal effects on taxes and interest rates

$$\frac{d\tau_s}{dG_s}|^* = \frac{(1-\sigma) - (1+\eta)(1-\tau^*) - \sigma(B^*/C^*)(1-\rho)/\rho}{\varepsilon^* + \eta}\frac{(-\varepsilon_B^*)w_B^*}{u_c^* + (1-\varepsilon^*)w_B^*}\frac{\mathrm{d}B}{\mathrm{d}G}|^*$$
$$\frac{dR_s}{dG_s}|^* = R^*[\frac{u_{CC}^*}{u_C^*}\frac{\mathrm{d}C_s}{\mathrm{d}G_s}^* + \varepsilon^*\frac{\beta}{\rho}\frac{w_B^*}{u_C^*}\frac{R^*}{B^*}\frac{\mathrm{d}B}{\mathrm{d}G}^*]$$

When risk aversion is close to unite, i.e. $\sigma \approx 1$, liquidity risk aversion is approximately zero, i.e. $\varepsilon^* \approx 0$, and we can approximate the marginal effect of government expenditure on optimal taxes as

$$\frac{d\tau_s}{dG_s}|^* \approx \frac{1-\rho}{\rho}\frac{(1+\eta)(1-\tau^*) + \sigma\varepsilon_B^* B^*/C^*}{\varepsilon^* + \eta}\omega^* R^* \frac{\mathrm{d}B}{\mathrm{d}G}|^* \tag{28}$$

where $\omega^* = \beta w_B^*/u_C^*$ is the equilibrium liquidity tax.

C.6 Time-Series

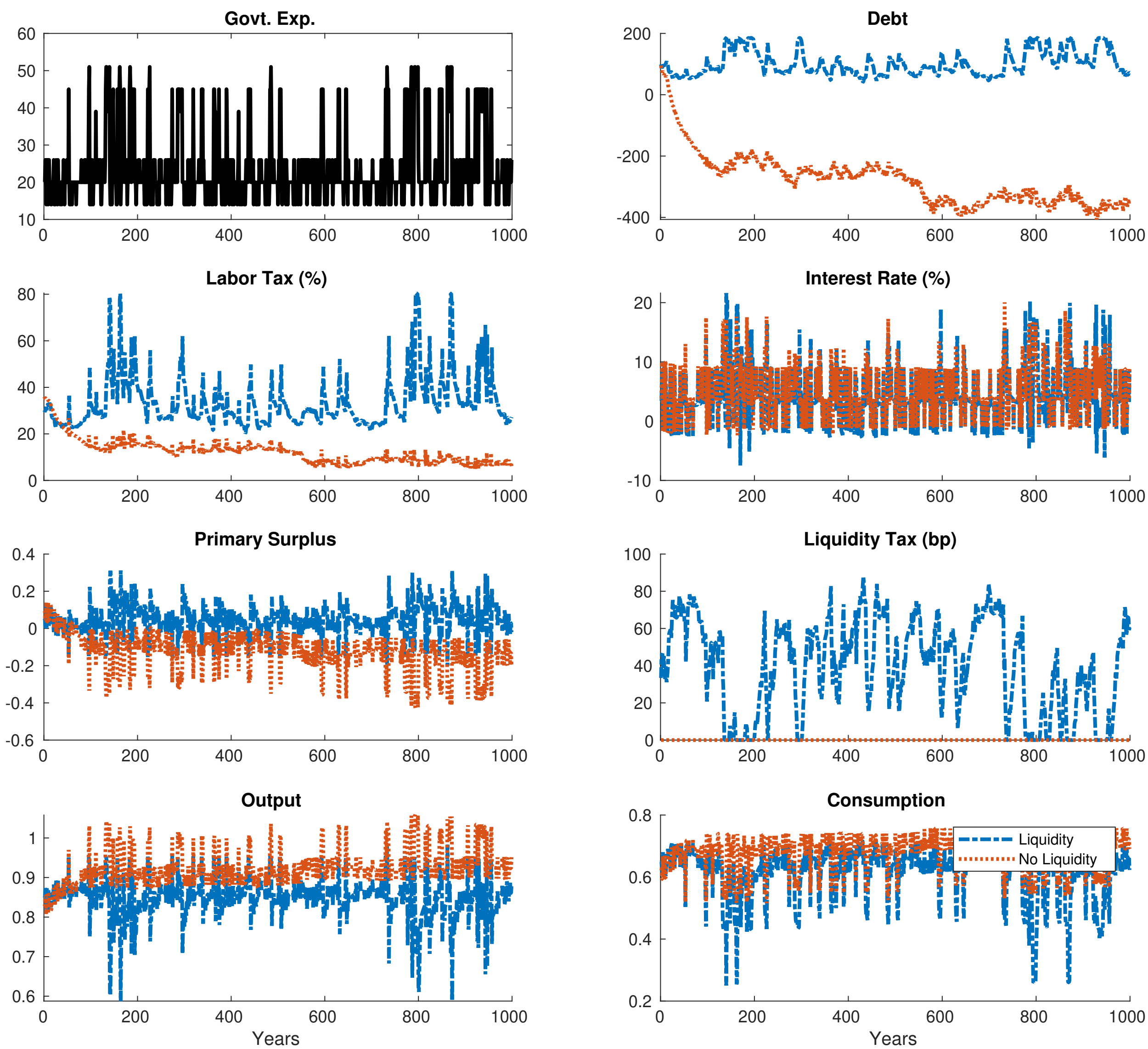

Figure 1: **Optimal policy time-series with government expenditure shocks**. This figure plots relevant variables over a 1000 year simulation starting at $B = 100$ in the incomplete markets model for different levels of liquidity demand. In the baseline liquidity and no liquidity model the demand parameters equal $\{\nu, \phi\}$ and $\{0, 0\}$.

Figure 1 plots the first 1000 years of the 10,000 year simulation used to calculate the ergodic distributions in figure 6.

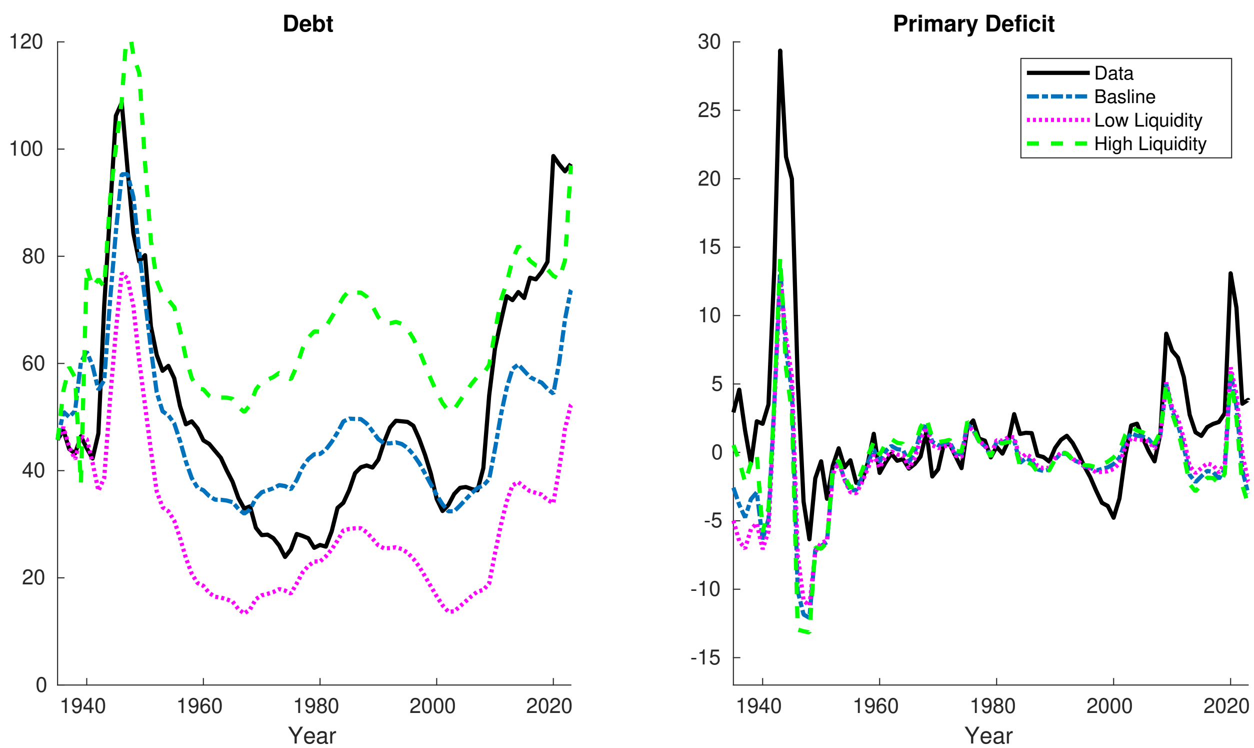

Figure 2: **Impact of liquidity demand on historical debt policy**. This figure plots the actual and optimal debt and primary deficit ratios in the United States since 1935. Actual debt is the gross debt held by the public (FYGFDPUB in FRED). Optimal policy begins in 1935 with the same debt and expenditure level as the actual history. I plot optimal policy for theoretical models with varying levels of liquidity demand: baseline, low (i.e., $(\nu/1.5, \phi/1.5)$), and high (i.e., $(1.5\nu, 1.5\phi)$).

C.7 Historical Debt Policy

Figure 2 plots the actual and model implied optimal debt and primary deficits in the United States since 1935 for liquidity demand calibrations. As the degree of liquidity demand increases, the optimal debt levels increase uniformly for all years. Moreover, the optimal debt expansions during World War II, the Great Financial Crisis, and the Covid-19 Pandemic increase in magnitude.

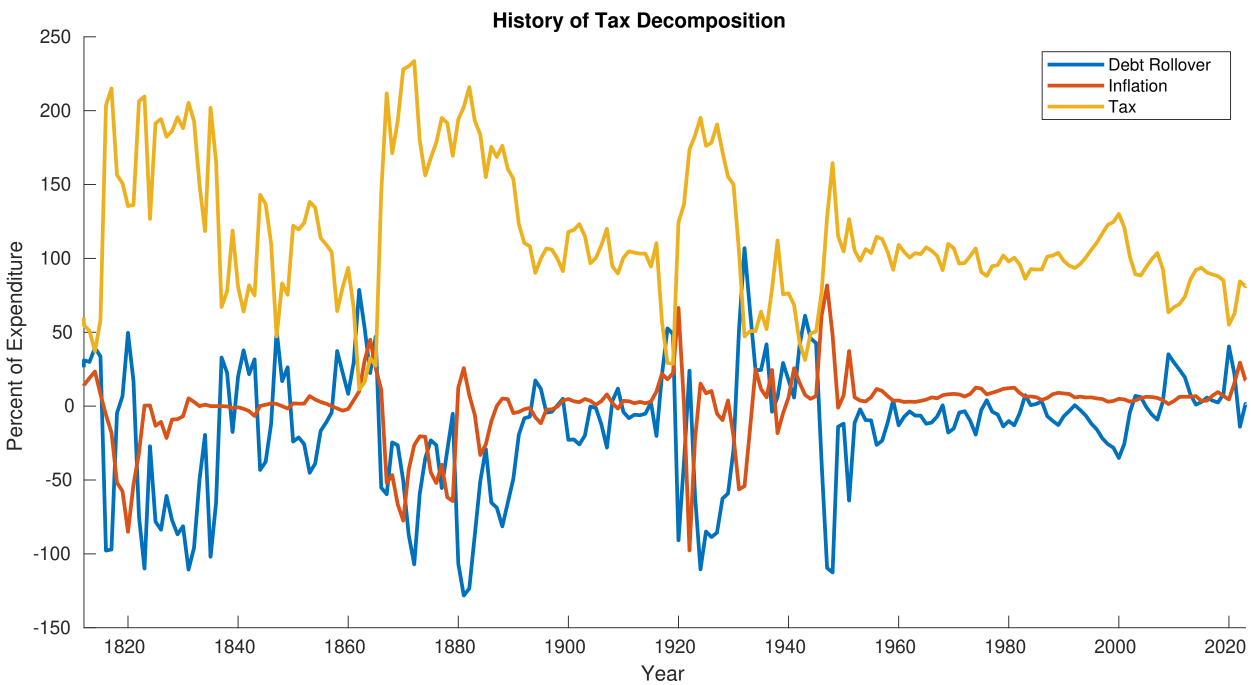

Figure 3: **Historical tax decomposition**. This figure decomposes historical government expenditure into tax, debt, and inflation financing from 1810 to 2023.

Figure 3 plots the decomposition of government expenditure into debt rollover, inflation, and tax revenues. To perform this decomposition I define the tax revenues as the total nominal tax revenues; the inflation tax revenue to be $\pi_t B_{t-1}$ where π_t is the inflation rate in year t; and debt rollover to be the remaining source of revenue needed to fund government expenditure G_t. This definition of debt rollover also includes liquidity tax revenue. The figure makes clear that tax revenues has been the dominant financing tool of policy makers since 1960. Inflation taxation was a dominant tool during WWI and WWII.

C.8 Time Inconsistency

Optimal policy is time-inconsistent. The "period-0" planner has no commitment plan, and thus can deviate from the continuation or timeless policy. However, this time-inconsistency problem does not lead to substantial variation in optimal policy. Figure 4 plots the historical debt and primary deficits of the U.S. against the model implied optimal policy, beginning the simulation in different years. For each year (i.e., 1940,1960,1980, and 2000) the simulation begins at the same debt and government expenditure level (e.g., B_{1940}, G_{1940}) as the U.S. While policy differ across the different start dates, the differences are not substantial. For instance the optimal policy after 2010 are nearly identifical across the different histories.

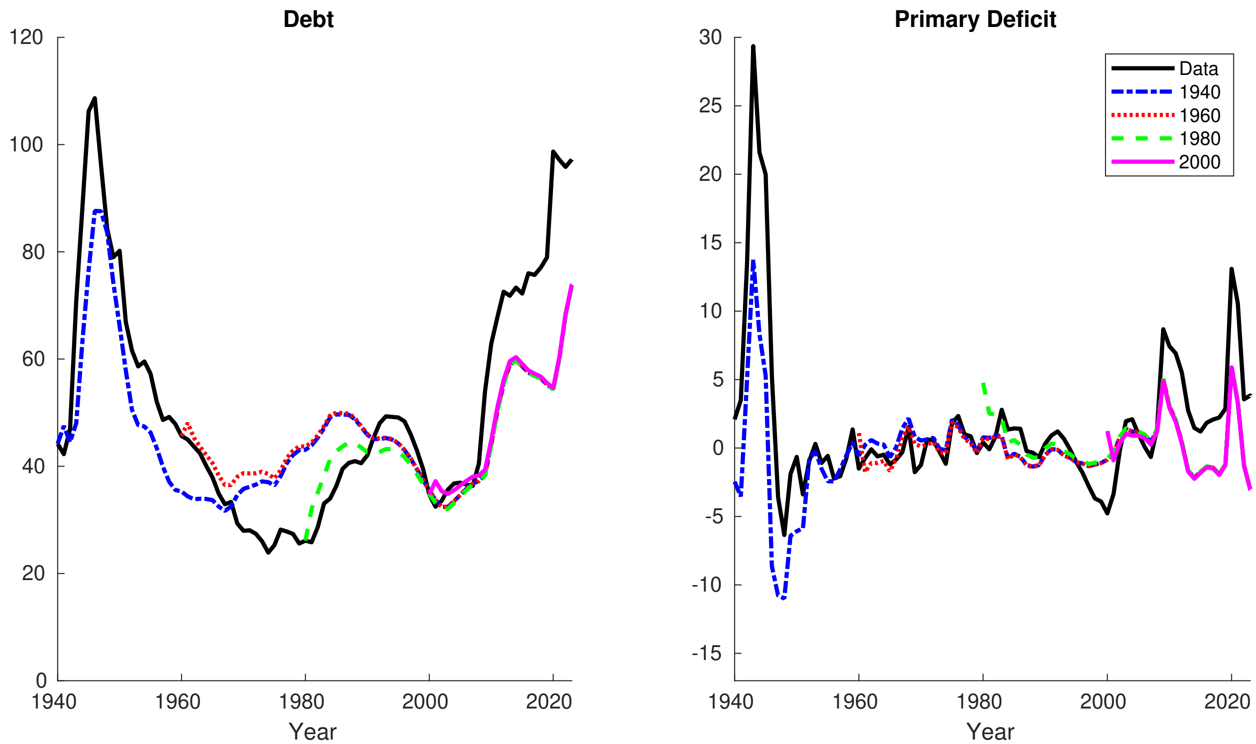

Figure 4: **Time inconsistency**. This figure plots the optimal policy path given different histories. Each line represents a different simulation beginning at the corresponding debt and government expenditure level of that year (e.g., B_{1940}, G_{1940}).

D Optimal Policy with Liquidity Demand Shocks

D.1 Calibrated Stochastic Liquidity Demand

Figure 5 illustrates these calibrated liquidity shocks. The top panel plots the historical debt-to-GDP level, the middle panel plots the estimated liquidity shocks, and the bottom panel plots the actual and estimated convenience yield. Decades of high debt, for instance the 1940s and 1950s, lead to low convenience yield (Krishnamurthy and Vissing-Jorgensen (2012)). However the debt levels only explains some of the movement in convenience yields. Stochastic liquidity demand appreciably improves the fit ($R^2 = .19$ vs. $R^2 = .60$) compared to a model with static liquidity demand. Unlike the static model, the stochastic liquidity demand model closely matches major liquidity demand episodes including the Great Depression, Volcker Disinflation, and Great Financial Crisis.

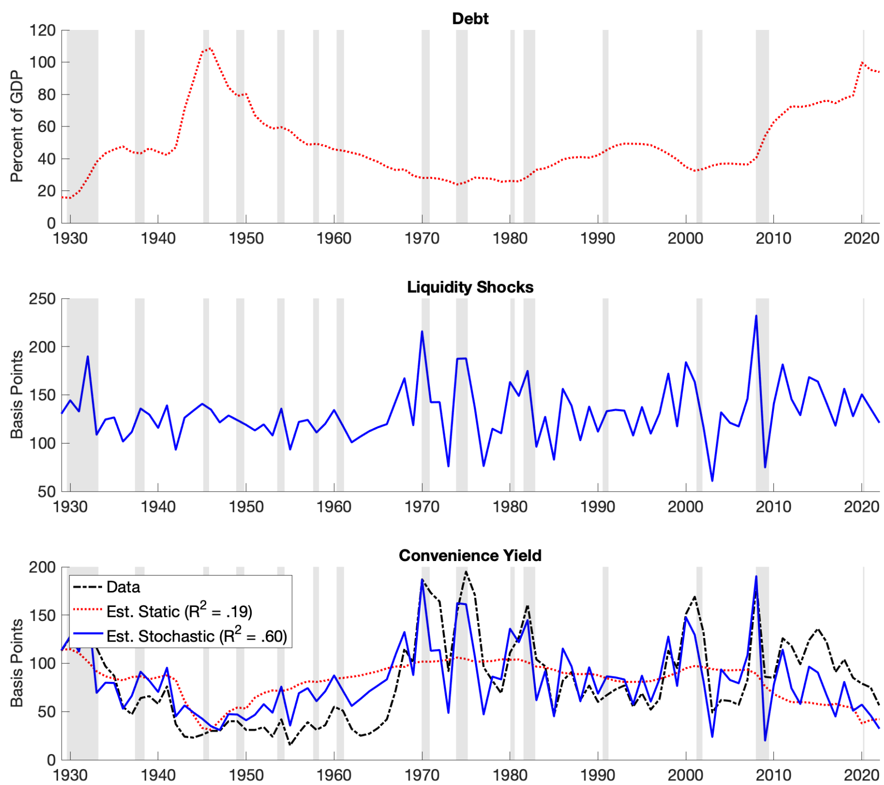

Figure 5: **Calibrated liquidity demand shocks.** This figure plots the estimated liquidity shocks and the estimated convenience yield. The top panel plots plots the historical debt-to-GDP level, the middle panel plots the estimated liquidity shocks in basis points, and the bottom panel plots the actual and estimated convenience yield in basis points. The static model features a constant spread parameter ν while the stochastic model allows for time-varying spread parameter ν_t.

D.2 Ramsey Problem

The stochastic liquidity demand model is nearly identical to the noncontingent debt model. The only difference being the source of aggregate risk. In this model policymakers aim to insure against *liquidity risk* rather than *surplus* risk. History s^t describes the history of liquidity demand shocks $\{\nu_s\}_{s=0}^t$. Transition probabilities are functions of liquidity demand $\Omega_{t+1}(s^{t+1}|s^t) = \Omega(\nu_{t+1}|\nu_t)$. The Ramsey problem is otherwise identical to equation 20.

$$\begin{aligned} V(\mathscr{B},\nu) = \max_{\{B,C_s,N_s,T_s,X_s\}} & \sum_{s\in S}\Omega(\nu_s|\nu)[u(C_s) - v(N_s) + w(\frac{B}{N_s},\nu_s) + \beta V(\mathscr{B}_s,\nu_s)] \\ \text{s.t } & u_C(C_s)B = u_C(C_s)[C_s - T_s] - v_N(N_s)N_s + \beta\mathscr{B}_s && (\Omega(\nu_s|\nu)\Phi_s) \\ & C_s = N_s - G_s && (\Omega(\nu_s|\nu)\Theta_s) \\ & \mathscr{B} = \sum_{s\in S}\Omega(\nu_s|\nu)[u_C(C_s)B + w_b(\frac{B}{N_s},\nu_s)\frac{B}{N_s}] && (\Gamma) \\ & \underline{\mathrm{N}}_s \le N_s \le \bar{N}_s && (\Omega(\nu_s|\nu)\rho_{N_s}) \\ & \underline{\mathrm{B}} \le B \le \bar{B} && (\rho_B\sum_{s\in S}\Omega(\nu_s|\nu)) \end{aligned} \tag{29}$$

At time $t = 0$, the policymaker solves the following Bellman Equation,

$$\begin{aligned} W(B_0,\nu_0) = \max_{\{C,N,\mathscr{B}\}} & u(C) - v(N) + w(B_0,\nu_0) + \beta V(\mathscr{B},\nu_0) \\ \text{s.t } & u_C B_0 \le u_C C - v_N N + \beta\mathscr{B} && (\Phi) \\ & C = N - G_0 && (\Theta) \\ & \underline{\mathrm{N}}(G_0) \le N \le \bar{N}(G_0) && (\rho_N). \end{aligned} \tag{30}$$

As in the previous section, I focus my analysis on the continuation Ramsey problem (equation 29) as the initial Ramsey problem (equation 30) is standard (Ljungqvist and Sargent (2018) Section 20.3).

D.3 Optimality Conditions

The optimal Ramsey policy is found by taking first-order conditions of the continuation Ramsey problem:

$$\Phi = V_{\mathscr{B}}(\mathscr{B}, G)$$
$$\Phi_s = V_{\mathscr{B},s}(\mathscr{B}_s, G_s)$$
$$u_{C,s}[1-(1-\sigma)\Phi_s] - v_{N,s}[1-(1+\eta)\Phi_s] = (\Phi - \Phi_s)Bu_{CC,s} + (\underline{\rho}_s^N - \bar{\rho}_s^N)$$
$$\sum_s \Omega_s(\Phi - \Phi_s)u_{C,s} = \sum_s \Omega_s w_{B,s}[1-(1-\varepsilon_s(B))\Phi] + (\underline{\rho}_B - \bar{\rho}_B)$$

where $\varepsilon(B) = w_{BB}B + w_B$ is the liquidity aversion parameter. The shadow values of the Laffer curve constraints $(\underline{\rho}_s^N, \bar{\rho}_s^N)$ and the natural debt and asset constraints $(\underline{\rho}_B, \bar{\rho}_B)$ equal zero when the constraints don't bind. The first-order conditions of the initial Ramsey policy are

$$V_{\mathscr{B}}(\mathscr{B}_0, G_0) = \Phi_0$$
$$u_{C,0}[1-(1-\sigma)\Phi_0] - v_{N,0}[1-(1+\eta)\Phi_0] = -\Phi_0 Bu_{CC,0} + \underline{\rho}_0^N - \bar{\rho}_0^N$$
$$W_B(B_0, G_0) = w_{B,0} + \Phi_0 u_{C,0}$$

D.4 Steady-State

The steady-state of the optimal policy can be written analytically with a set of implicit functions. The interior steady-state is given by the optimal debt policy, the optimal tax policy, the intertemporal government budget constraint, the resource constraint, and the stochastic government expenditure process:

$$\Phi = 1/(1-\varepsilon)$$
$$u_C[1-(1-\sigma)\Phi] = v_N[1-(1+\eta)\Phi]$$
$$(1-\beta)\mathscr{B} = u_C C - v_N N + \beta w_B B$$
$$N = C + G$$
$$\nu = \nu^*$$

where we have assumed that taxes respect the labor Laffer curve $\underline{N}(G) \leq N \leq \bar{N}(G)$ and the optimal debt level respects the natural debt and asset limits $\underline{B} \leq B \leq \bar{B}$. By substituting the budget equation into the optimal tax equation, I can write the steady-state as:

$$\tau^*(\nu^*,B^*) = \frac{\sigma+\eta}{\varepsilon(\nu^*,B^*)+\eta}$$

$$R^*(\nu^*,B^*) = \frac{1}{\beta}\frac{u_C(\tau^*,B^*)}{u_C(\tau^*,B^*)+w_B(B^*,\nu^*)}$$

$$B^*(\nu^*) = \frac{R^*}{R^*-1}[\tau^* N(\tau^*,\nu^*)-G]$$

D.5 First-Order Approximation of Optimal Policy

$$\frac{\mathrm{d}\nu_s}{\mathrm{d}\nu}|^* = \alpha$$

$$\frac{\mathrm{d}N_s}{\mathrm{d}\nu_s}|^* = \frac{\mathrm{d}C_s}{\mathrm{d}\nu_s}|^*$$

$$[(\alpha^{-1}-\beta)u_C^* - \beta(1-\varepsilon^*)w_B^*]\frac{\mathrm{d}B}{\mathrm{d}\nu}|^* = [(1-\sigma)u_C^* - (1+\eta)v_N^* - u_{CC}^*B]\frac{\mathrm{d}C_s}{\mathrm{d}\nu_s}|^* + \alpha\beta B^*$$

$$[\frac{u_{CC}^*}{u_C^*} - \frac{v_{NN}^*}{v_N^*}]\frac{\mathrm{d}C_s}{\mathrm{d}\nu_s}|^* = \frac{(1-\sigma)u_C^* - (1+\eta)v_N^* + (1-\alpha)/\alpha\; u_{CC}^*B^*}{[1-(1+\eta)\Phi^*]v_N^*}\frac{\mathrm{d}\Phi_s}{\mathrm{d}\nu_s}|^*$$

$$\Phi^*\varepsilon_B^* w_B^*\frac{\mathrm{d}B}{\mathrm{d}\nu}|^* = [(1-\alpha)u_C^* + (1-\varepsilon^*)w_B^*]\frac{\mathrm{d}\Phi_s}{\mathrm{d}\nu_s}|^* + \alpha\varepsilon^* w_B\Phi^*$$

Since risk aversion is unity. liquidity risk aversion is approximately zero, and $\alpha^{-1}-\beta\approx 0$, the partial derivative of debt is approximately

$$\frac{\mathrm{d}B}{\mathrm{d}\nu}|^* \approx [\frac{1+\eta}{1-\varepsilon^*}\frac{(1-\tau^*)}{\omega^*} - \frac{\sigma}{\omega^*}\frac{B^*}{C^*}]\frac{\mathrm{d}C_s}{\mathrm{d}\nu_s}|^* - \frac{\alpha}{(1-\varepsilon^*)u_C}B^*$$

Therefore the partial derivative of taxes is approximately

$$\frac{\mathrm{d}\tau_s}{\mathrm{d}\nu_s}|^* \approx -[\frac{(1+\eta)(1-\tau^*)+\sigma(1-\alpha)/\alpha\; B^*/C^*}{\varepsilon^*+\eta}][\alpha\varepsilon^* - \varepsilon_B^*\frac{\mathrm{d}B}{\mathrm{d}\nu}|^*]\omega^* R^*$$

while the partial derivative of interest rates is

$$\frac{dR_s}{d\nu_s}|^* = R^*[\sigma C^*(-\frac{\mathrm{d}C_s}{\mathrm{d}G_s}|^*) + \frac{\varepsilon^*}{\alpha}\frac{\omega^* R^*}{B^*}\frac{\mathrm{d}B}{\mathrm{d}\nu}|^* - \frac{\beta R^*}{u_C^*}]$$

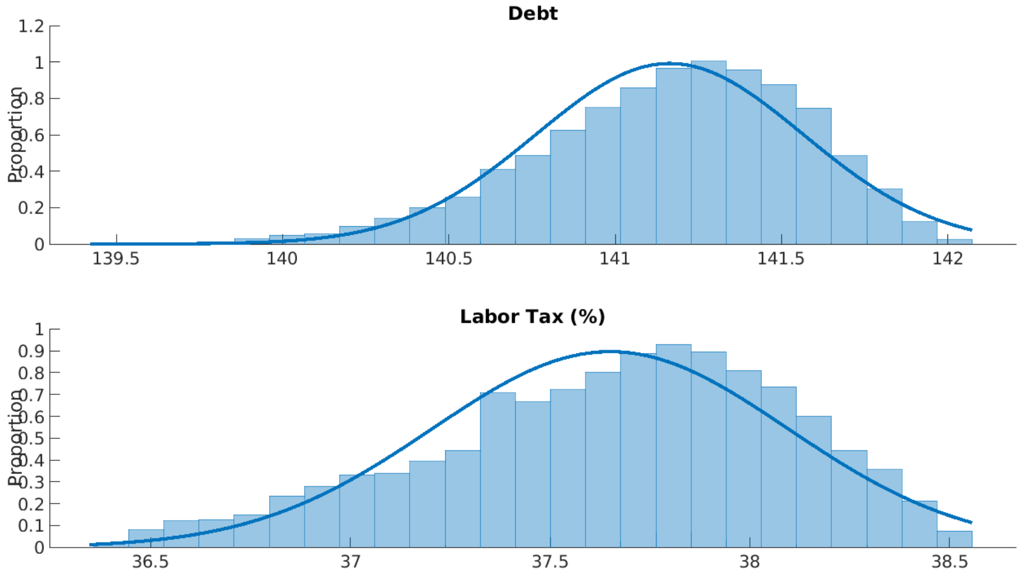

Figure 6: **Policy distributions with liquidity demand shocks.**. This figure plots the distribution of debt and labor taxes in the stochastic liquidity demand model. I overlay a normal kernel distribution on each distribution. There is a 100 year burnin period prior to the 10,000 year simulation.

Figure 6 plots the ergodic distribution of key policy variables. For the static baseline parameters, I plot liquidity satiation (dotted line) and debt Laffer curve peak (dashed line) in the debt-to-GDP panel. Mean liquidity demand is 130 basis points and fluctuates greatly (s.d. 42 basis points). At the extremes, positive and negative liquidity shocks shift between 50 to 200 basis points – an 150 basis point difference. Despite large swings in the demand for liquidity, the optimal debt level is high. Interest rates and liquidity taxes absorb most of the variation in liquidity demand. While the debt ratio and labor taxes remain relatively constant.

E Optimal Policy with Contingent Debt

This section characterizes the optimal Ramsey policy for an economy with complete markets. Compared to optimal policy without liquidity demand, labor taxes are more volatile, the public debt is less volatile, and the inflation tax is smaller. Optimal policy is characterized by a central tradeoff between reducing labor and liquidity tax volatility. The relative desirability of each depends on the degree of liquidity demand and the labor supply response to labor taxes. The former is dictated by the liquidity demand parameters and latter by the Frisch elasticity.

E.1 Primal Method

Rewriting the problem faced by policymaker in primal form (i.e. removing prices) simplifies analysis. To convert the problem to primal form, I rewrite the household budget constraint by substituting out the prices faced by households. By substituting the labor tax $1-\tau_t$ with the labor-leisure decision (equation 4) and the real interest rate R_t with the liquidity-adjusted Euler equation (5) the household budget is represented in primal form in equation 31.

$$u_{C,t}B_t = u_{C,t}[C_t - T_t] - v_{N,t}N_t + \beta \sum_{s_{t+1}} \Omega(s_{t+1}|s^t)[u_{C,t+1}B_{t+1} + w_b(b_{t+1})b_{t+1}] \tag{31}$$

Everything in the primal budget (equation 31) is priced by marginal utility in order to price debt and taxation in terms of utility. The left-hand side of primal budget is the value of the debt burden. The first two terms on the right-hand side of the primal budget are the total taxes paid to the government net of transfers and government expenditure, i.e. surpluses. The third term on the right-hand side of the primal budget is the expected value of government debt net of the expected liquidity tax $w_b(b_{t+1})b_{t+1}$.

Conditional on government expenditure G_t, the labor tax τ_t is isomorphic to the output level N_t. Therefore, for the policymaker selecting the optimal labor tax τ_t amounts to selecting the optimal output level N_t for a given expenditure level. Put differently, labor tax is a function of the output and expenditure$\tau_t = \tau(N_t, G_t)$.

Selecting the optimal state-contingent debt level $B_{t+1}(s_{t+1})$ is complicated by the future budget constraint. State-contingent debt levels chosen today will be debt obligations tomorrow. The welfare impact of these debt obligations crucially depends on the relative marginal utility of consumption today $u_{C,t}$ compared to tomorrow $u_{C,t+1}$. These marginal utilities price the relative cost taxation, today versus tomorrow, in terms of utility. Future debt also provides liquidity services, priced at the marginal value of liquidity $w_b(b_t, s_t)$. When liquidity is satiated, the marginal value of liquidity is zero, $w_b(b_t, s_t) = 0$ and the value of public debt exclusively depends on the marginal cost of taxation $u_{C,t+1}B_{t+1}$. More generally, the total value of public debt including the value of its liquidity services, in terms of utility is $u_{C,t+1}B_{t+1} + w_b(b_{t+1}, s_{t+1})b_{t+1}$.

E.2 Ramsey Problem

This section outlines the optimal Ramsey problem for the complete markets economy. I describes how to reach the solution using the recursive contracts method of Marcet and Marimon (2019). The methodology closely follows Ljungqvist and Sargent (2018) section 20.2.

Let $\mathscr{B}_t = u_{C,t}B_t + w_b(b_t)b_t$, $\mathscr{T}_t = u_{C,t}[\tau(N_t, G_t)N_t]$, $\mathscr{Q}_t = w_b(b_t, s_t)b_t$, $\mathscr{G}_t = u_{C,t}G_t$ and $\mathscr{R} = \frac{1}{\beta}$ be the marginal utility adjusted debt, labor tax revenues, liquidity tax revenue, government expenditure, and interest rate. The primal budget (equation 31) can be rewritten as,

$$\mathscr{B}_t + \mathscr{G}_t \leq \mathscr{T}_t + \mathscr{Q}_t + \frac{1}{\mathscr{R}} \sum_{s_{t+1}} \Omega(s_{t+1}|s^t)\mathscr{B}_{t+1}. \tag{32}$$

Where the inequality emerges from the availability of lump-sum transfers. This version of the budget equation greatly simplifies analysis as it allows the budget to be written in terms of one state-variable, the value of the future debt $\mathscr{B}_{t+1}$, rather than two, the future debt level B_{t+1} and future promised consumption C_{t+1}.

Throughout this section, and the proceeding sections, I describe the Ramsey problem faced by the policymaker recursively. The policymaker, aiming to maximize expected household utility (equation 2) at time $t \geq 1$ solves the following Bellman equation,

$$\begin{aligned}
V(\mathscr{B}, G) = &\max_{\{B,C,N,\mathscr{B}_s\}} u(C) - v(N) + w(b,s) + \beta \sum_{s \in S} \Omega(G_s|G)V(\mathscr{B}_s, G_s) \\
\text{s.t } & u_C(C)B \leq u_C(C)C - v_N(N)N + \beta \sum_{s \in S} \Omega(s'|s)\mathscr{B}_s \\
& C = N - G \\
& \mathscr{B} = u_C(C)B + w_b(\frac{B}{N})(\frac{B}{N}) \\
& \underline{\mathrm{N}}_s \leq N \leq \bar{N}_s \\
& \underline{\mathrm{B}} \leq B \leq \bar{B}
\end{aligned} \tag{33}$$

In which utility maximization $V(\mathscr{B}, G)$ is subject to the intertemporal budget condition (with corresponding shadow value Φ), market clearing (with corresponding shadow value Θ, government debt commitments (with corresponding shadow value Γ), and the government policy feasibility constraints on taxes (with corresponding shadow value ρ_N), transfers (which leads to the inequality in the budget), and debt issuance (with corresponding shadow value ρ_B). The intertemporal budget condition Φ states that the value of the debt due today X must be paid for by a combination of labor taxation $u_C(C)C - v_N(N)N$, liquidity taxation $w_b(b)b$, and new debt issuance

$\beta \sum_{s' \in S} \Omega(G_s|G)\mathscr{B}_s$. The shadow price of commitment Γ restricts today's policymaker to honor the real value of government debt X promised by yesterday's planner. This constraint indicates that today's real value of government debt X is the sum of the debt burden $u_C(C)Nb$ and the liquidity tax $w_b(b)b$ which suppresses interest payments on today's debt. The labor feasibility condition ρ_N restricts taxation to be below the peak of the labor Laffer curve (equation **??**). The debt issuance limits are the natural debt limit and the natural asset limit, $Nb > 0$.

Optimal policy consists of decision rules $\{C(\mathscr{B},G), N(\mathscr{B},G), T(\mathscr{B},G), b(\mathscr{B},G), \mathscr{B}_s(\mathscr{B},G;G_s)\}$ for consumption, output, transfers, public debt and the state-contingent value of public debt next period as well as a value function $V(\mathscr{B},G)$. Moreover, these decision rules produce state-dependent shadow values $\{\Phi(\mathscr{B},G), \Theta(\mathscr{B},G), \Gamma(\mathscr{B},G)\}$ for the intertemporal budget condition, market clearing, and government debt commitment conditions. The shadow value of the budget constraint $\Phi(\mathscr{B},G)$ will control labor taxes. while the shadow value of the government commitment condition $\Gamma(\mathscr{B},G)$ will control the liquidity tax. The relative values of each will dictate the relative smoothing of labor taxes versus liquidity. The following sections flesh out how and why.

At time $t = 0$, the policymaker solves the following Bellman Equation,

$$
\begin{aligned}
W(B_0, G_0) = \max_{\{C,N,\mathscr{B}_s\}} & \; u(C) - v(N) + w\Big(\frac{B_0}{N}\Big) + \beta \sum_{s \in S} \Omega(G_s|G) V(\mathscr{B}_s, G_s) \\
\text{s.t } & \; u_C B_0 \le u_C C - v_N N + \beta \sum_{s \in S} \Omega(G_s|G)\mathscr{B}_s && (\Phi) \\
& C = N - G_0 && (\Theta) \\
& \underline{\mathrm{N}}(G_0) \le N \le \bar{N}(G_0) && (\rho_N).
\end{aligned}
\tag{34}
$$

I restrict my analysis in the main text to the *continuation* Ramsey problem (equation 33), as the *initial* Ramsey problem (equation 34) is standard (Ljungqvist and Sargent (2018) Section 20.2). While the solution to the initial Ramsey problem depends on the degree of liquidity-demand, the initial planner does not pick the initial debt level. Therefore the influence of liquidity demand on the optimal debt policy is primarily mediated through the continuation Ramsey problem (equation 33).

Theory of Labor and Liquidity Tax Smoothing In standard models with contingent debt, optimal debt policies critically depend on the labor tax-smoothing motive of policymakers. Labor taxes reduce welfare by distorting household's labor-leisure decision (equation 4). Ideally, the policymaker would set labor taxes to zero in all periods to eliminate these distortions. Given that lump-sum taxes are unavailable, labor taxation becomes necessary to cover expenditure shocks. Second-best policy smoothes these distortion across periods (Barro (1979)).

In my model, optimal debt policies also critically depend on the *liquidity* tax-smoothing motive of policymakers. Liquidity taxes distort the household's savings decision (equation 5). First-best policy sets liquidity taxes to zero. Similar to the Friedman (1971) policy prescription for money, this policy outcome is achieved by flooding the market with public debt. Optimal debt policy and optimal liquidity tax policy are two-sides of the same coin. Second-best policy smoothes the savings decision distortions across periods by keeping the debt level constant.

With state-contingent debt, the policymaker can perfectly smooth labor taxes. Smoothing labor taxes across periods of high and low government expenditure requires ex-post debt revaluations. When expenditure is high the value of debt holdings declines to keep total government expenditure – government expenditure and interest payments – constant across states. Policymakers keep labor taxes smooth by allowing debt to fluctuate.

Absent liquidity demand these debt revaluations have no direct welfare impact on households. If the household demands liquidity, debt reductions deprive the economy of liquidity (Angeletos, Collard and Dellas (2023) and Sims (2025)). Debt fluctuation raise liquidity tax volatility. Second-best labor and liquidity tax policy are at odds. Since simultaneously smoothing labor taxes and debt is impossible the policymaker faces a tradeoff between tax instruments. Without liquidity demand the policymaker perfectly smooths labor taxes. With strong enough liquidity demand the policymaker perfectly smooths liquidity taxes.

E.3 Steady-State

Figure 7 plots the long-run targets of key policy variables including debt, labor taxes, interest rates, and liquidity taxes. The liquidity demand model features higher debt levels and tax rates, along with a lower interest rate. Substantially higher debt is partially funded by labor taxes and liquidity taxes. With state-contingent bonds and no liquidity demand, policymaker smooths all future labor taxes at a level commensurate with the initial debt level. In this case, the debt target is an increasing function of the initial debt. With liquidity demand, there is a unique average debt ratio independent of the initial debt.

E.4 Policy Response

Figure 8 plots the optimal policy response from steady-state to a surprise expenditure shock. The figure illustrates optimal policy responses for three models. Specifically, "full liquidity" in which the liquidity demand parameters (ν, ϕ) are set to the baseline calibration in table 1 and "no liquidity demand" which sets both liquidity demand parameters to zero. The government expenditure shock starts in period 20 and lasts for 10 years. Prior to the shock there is a long period of low expenditure. Each year the policymaker is uncertain if the crisis will persistent another year. Labor taxes, liquidity taxes, and interest rates rise while debt to output falls.

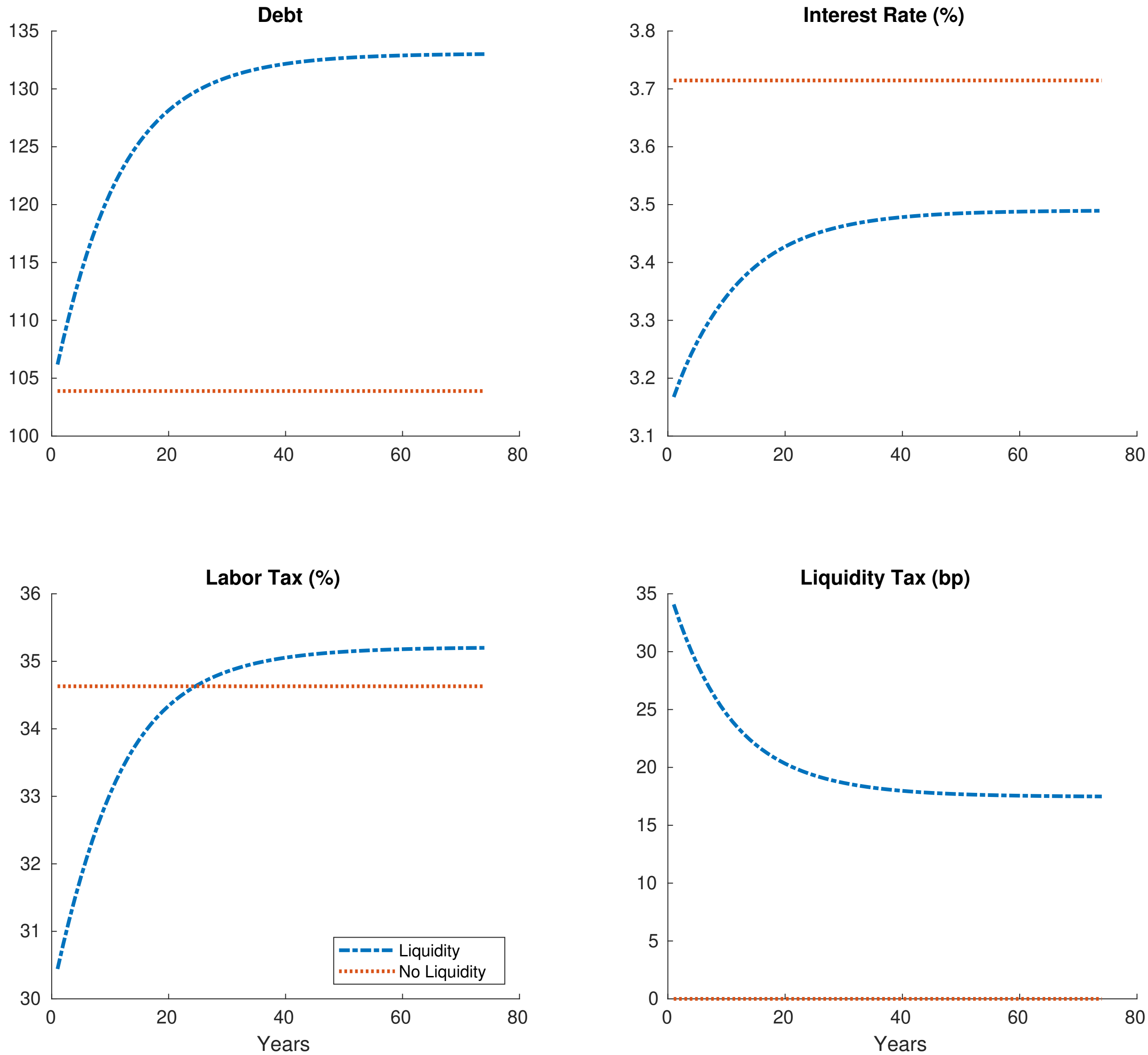

Figure 7: **Optimal policy level with complete markets**. This figure plots relevant variables over a 75 year simulation starting at $B = 100$ in the complete markets model for different levels of liquidity demand. In the baseline liquidity and no liquidity model the demand parameters equal $\{\nu, \phi\}$ and $\{0, 0\}$. The top left panel plots the distribution of government expenditure shock.

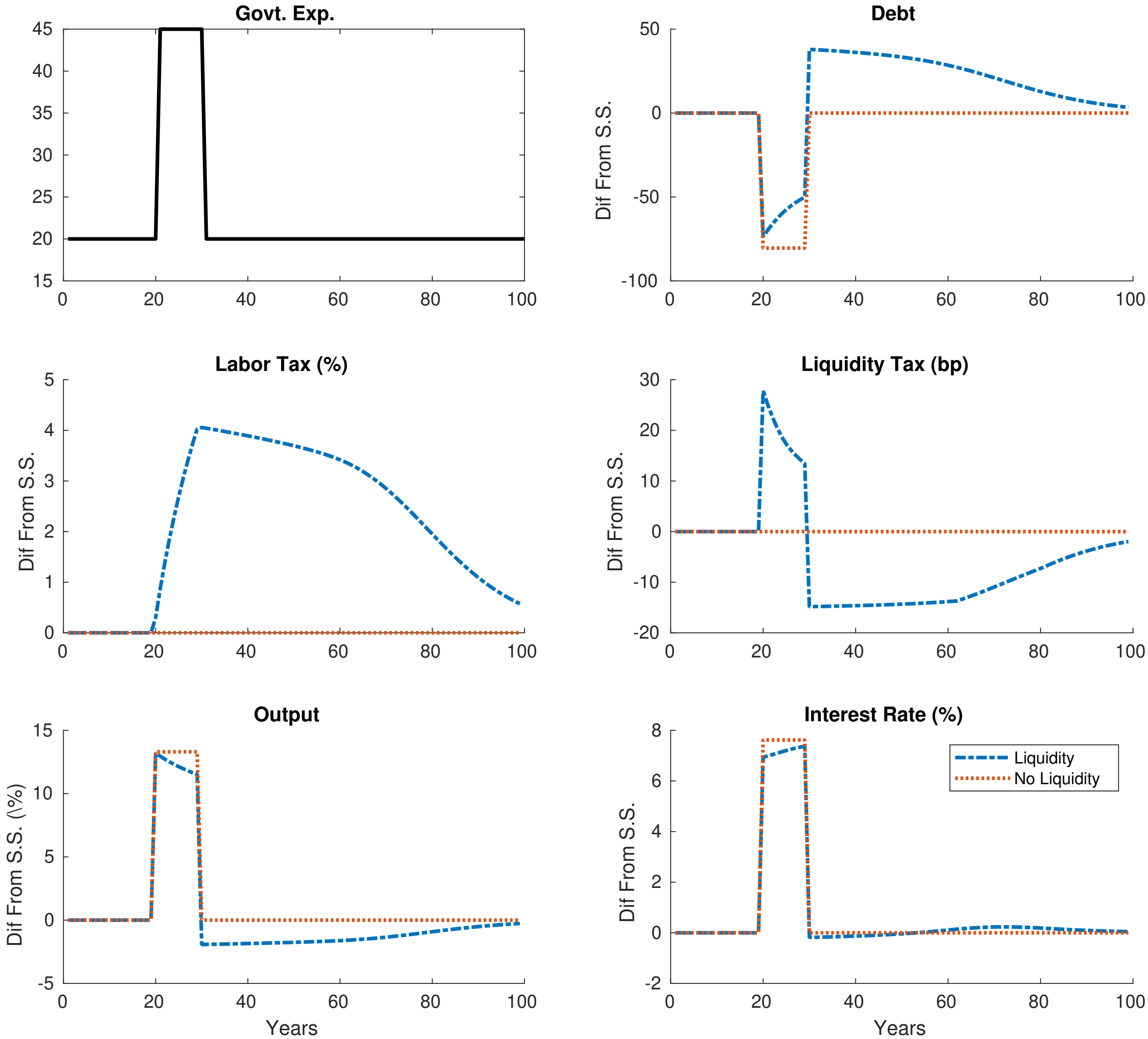

Figure 8: **Policy response with complete markets.** This figure plots the optimal policy response to an unforeseen government expenditure shock in the complete markets model. In the baseline liquidity and no liquidity model the demand parameters equal $\{\nu, \phi\}$ and $\{0,0\}$. The government expenditure shock starts in period 20 and lasts for 10 years. Prior to the shock there is a long period of low expenditure. The top left panel plots a single government expenditure shock. The remaining panels plot the differences from steady-state for each variable. Output is plotted as the percent difference while the remaining variables are plotted as level differences.

The magnitude of the labor tax increase during the crisis is proportional to the degree of liquidity demand. Without liquidity demand, labor taxes do not change with liquidity demand labor taxes increase by 4 p.p. from steady state. Persistent crises lead to larger increases in labor taxes than transitive episodes. For instance, a 1 p.p. labor tax increase for a one year crisis and a 10 p.p.

labor tax increase for a twenty year crisis.

In the absence of liquidity demand, the policymaker devalues the debt by 60 p.p. at the start of the war, reducing the debt ratio by 105 p.p.. This devaluation is sufficiently large to keep the policymaker from raising labor taxes. With full liquidity demand, the policymaker still devalue debt at the start of the war, but this devaluation only reduces the debt ratio by 77 p.p.. This devaluation is insufficiently large to cover the surprise expenditure, forcing the policymaker to make up the budget shortfall by raising labor taxes.

As the degree of liquidity demand increases the volatility of liquidity taxes increases while the volatility of the normalized liquidity tax decreases. Liquidity demand mechanically increases the magnitude and volatility of liquidity taxes. Moving from an economy with some liquidity demand to the baseline calibration, a tenfold increase in the degree of liquidity demand, increases the magnitude of liquidity taxes by 1.2 basis points and the volatility by 1.8 basis points. Higher degree of liquidity demand leads to smoother normalized liquidity taxes. With some liquidity demand, normalized liquidity taxes jump .6 on impact while for full liquidity they jump .5. With complete markets, optimal policy calls for taxing liquidity during crises.

The policymaker aims to simultaneously smooth labor and liquidity taxes. Absent implausibly large output and interest rate responses, perfectly smoothing both taxes is infeasible[22]. Therefore the policymaker faces a tradeoff: smooth labor taxes by ex-post revaluing the debt or smooth liquidity taxes by limiting debt revaluations. Higher liquidity demand leads the policymaker to favor liquidity tax smoothing. More responsive labor supply leads the policymaker to favor labor tax smoothing. This tradeoff is discussed in more detail in Appendix **??**.

E.5 Taxation Decomposition

Each year, the policymaker must cover exogenous government expenditure. There are four tools at the policymaker's disposal: (1) labor taxes (2) liquidity taxes (3) inflation taxes and (4) debt rollover. The policymaker can tax labor income, tax government debt's liquidity services, reduce the value of last year's debt through inflation, or issue new debt. By rearranging the government budget equation (8), I decompose the government expenditure across these four tools:

$$G_t = \overbrace{\tau_t N_t - T_t}^{\text{Labor Tax Rev.}} + \underbrace{\mathbb{E}_t[\omega_{t+1} b_{t+1}]}_{\text{Liquidity Tax Rev.}} + \overbrace{(\mathbb{E}_{t-1}[B_t] - B_t)}^{\text{Inflation Tax Rev.}} + \underbrace{\left(\frac{B_{t+1}}{R_t^{CS}} - \mathbb{E}_{t-1}[B_t]\right)}_{\text{Debt Rollover}}. \tag{35}$$

I define the net labor tax income $\tau_t N_t - T_t$ as the labor tax revenue, the average expected liquidity tax income $\mathbb{E}_t[\omega_{t+1} b_{t+1}]$ as the liquidity tax revenue, the difference between expected and

[22] Simultaneously insuring tax payers and government bond holders is impossible (Jiang et al. (2026)).

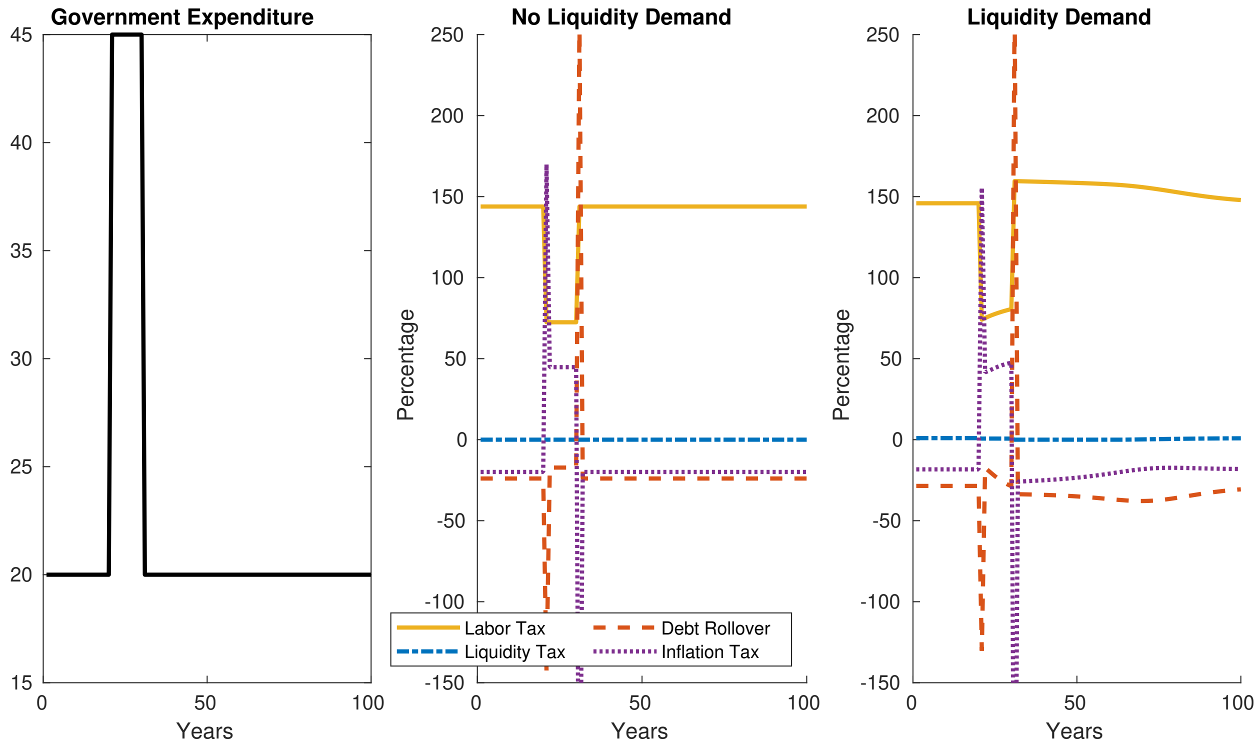

Figure 9: **Tax decomposition with complete markets.** This figure plots the decomposition of taxes for a complete markets model with liquidity demand and without liquidity demand. The left panel plots a one time government expenditure shock, the center panel plots the tax decomposition without liquidity demand, and the right panel plots the tax decomposition with liquidity demand. Prior to the shock there is a long period of low expenditure. Percentages are defined in terms of the government expenditure level G_t.

actual real debt $\mathbb{E}_{t-1}[B_t] - B_t$ as the inflation tax revenue, and the difference between the savings's value of newly issued and expected real debt $\frac{B_t}{R_t^{CS}} - \mathbb{E}_{t-1}[B_t]$ as the value of debt rollover. I use the consumption smoothing interest rate R_t^{CS} rather than the prevailing interest rate R_t to differentiate between liquidity taxation and debt rollover.

This decomposition, visualized in Figure 9, allows comparison of each tools importance across states of the world. The left panel plots the government expenditure shock, and the middle and right panels plots the magnitude of each tool as a percentage of government expenditure for economies with no liquidity demand and baseline liquidity demand. Prior to the shock there is a long period of low expenditure.

In both economies, labor taxes are the predominate tool to pay for government expenditure during peace time. Labor taxes, as a percentage of government expenditure, exceed 100% because they are also pay for outstanding debt obligations. At the start of a war, the policymaker inflates away much of the debt. During the war a mixture of inflation and labor taxes pay for government expenditure. In order to incentivize households to hold this inflationary debt, the policymaker promises a bonanza at the conclusion of the war. This bonanza is facilitated by a spike in deflation. These jumps in inflation and deflation mimic the lump-sum taxes and transfers of first-best policy.

Liquidity demand does not substantially alter this pattern of taxation. The spike in inflation at the start of wars and the spike in deflation at the end are both smaller. Labor tax income, as a percentage of government expenditure, asymptotes downward during peace time and asymptotes upward during war times. Liquidity tax income is near zero for all states of the world. Therefore optimal policy is not to "eat a free lunch" (Mian, Straub and Sufi (2025)) or "mine the bubble" (Brunnermeier, Merkel and Sannikov (2024)) by taxing liquidity and *reducing* labor taxes, but rather to satiate liquidity demand (Blanchard (2019) and Sims (2025)) by *raising* labor taxes.