

Generative Economic Modeling

Hanno Kase¹, Matthias Rottner², **Fabio Stohler**³

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¹European Central Bank

²Bank for International Settlements, Deutsche Bundesbank

³University of Bonn

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Nonlinearly solving large economic models remains hard

⇒ limiting use of structural models for **risk assessment & forecasting**

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(Maliar, Maliar, and Winant, 2021; Han, Yang, and E, 2021; Azinovic, Gaegauf, and Scheidegger, 2022; Fernández-Villaverde, Hurtado, and Nuno, 2023; Kase, Melosi, and Rottner, 2024)

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Can we use **neural networks** with proven **conventional methods** we already know?

(cf. Heer & Maussner, 2024; Judd, 1998; Miranda & Fackler, 2004)

The Curse of Dimensionality Limits Economic Analysis

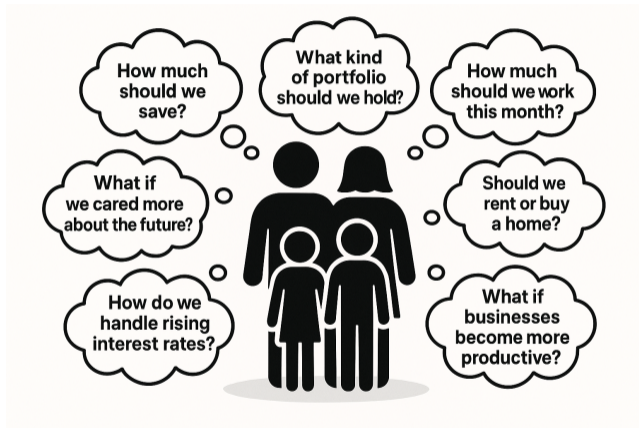


Figure 1: Full model

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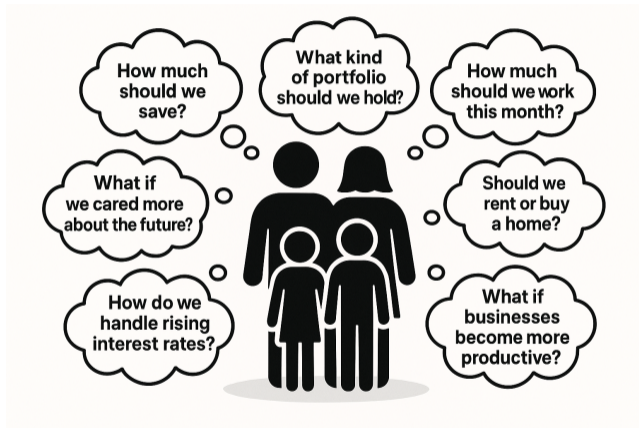


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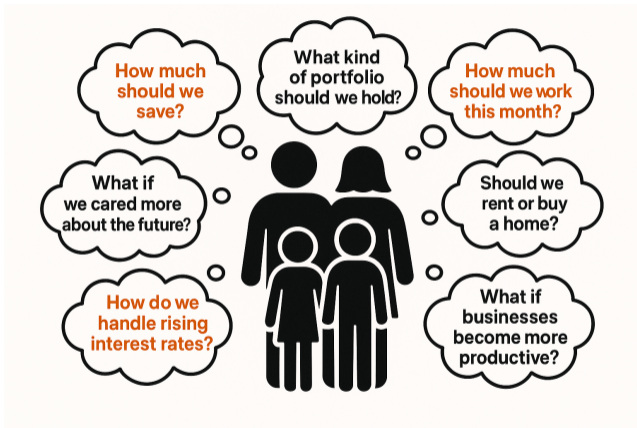


Figure 2: **Simplified** model

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- Forces researchers to focus on the key model components
- Conventional methods (VFI, PFI) work well for simplified models

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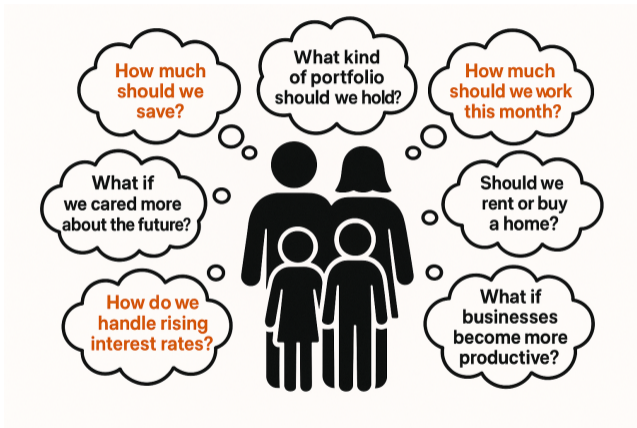


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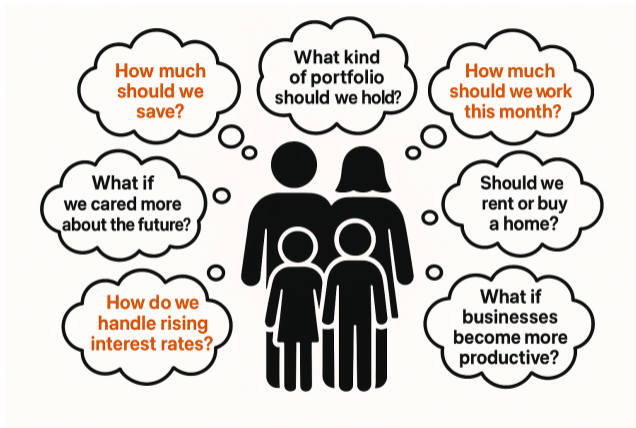


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⇒ **Generative Economic Modeling:**
Conventional Methods + Deep Learning

Recasting model solution as simulation and learning

1. **Solving Simplified Models**

- Solve simplified models nested within the full model
- Solve each with standard numerical methods

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3. Neural Network Approximation

- Train a neural network on the combined dataset
- The trained network generalizes to the full model

Results and Roadmap for the Talk

- **Practical demonstrations** across Asset Pricing and RBC model variants
 - Globally solving the full model vs. solving with generative economic modeling
 - Prediction errors comparable to training NN on the full model
 - Euler equation errors comparable to other methods
- **Application** to a Heterogeneous Agent New Keynesian (**HANK**) model
 - Understand transmission of financial shocks with heterogeneous agents
 - Interaction of multiple aggregate shocks with precautionary behavior

Generative Economic Modeling

- **Goal:** Solve model with variables \mathbb{X}_t , shocks ν_t , and parameters Θ

$$\mathbb{X}_t = f(\mathbb{X}_{t-1}, \nu_t \mid \Theta)$$

Challenge: numerical solution for $f(\cdot)$ is subject to the curse of dimensionality

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- **Solution:** Use a **simplified model** \tilde{f}^i :

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- **Generative Economic Modeling:** Generate full model from simplified models
 - Solve various simplified models \tilde{f}^i with $i \in \{a, b, c, \dots\}$
 - Simulate data $\bar{\mathbb{D}}$ from collection of simplified models
 - Train \bar{f}_{DNN} to approximate f through generative ability of neural network

$$f(\cdot) \approx \bar{f}_{DNN} = \arg \min_{\bar{W}} \mathbb{L}(\bar{W} \mid \bar{\mathbb{D}})$$

Proof of Concept

Analytical asset pricing model as a benchmark

Analytical tractable asset pricing model of Canzoneri, Cumby, and Diba (2007):

$$q_t = \beta \mathbb{E}_t \left[\exp \left(- \pi_{t+1} - \gamma \Delta c_{t+1} \right) \right]$$

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VAR(1) for variables $y_t = [\pi_t, \Delta c_t]'$ with shocks $\epsilon_t = [\epsilon_t^a, \epsilon_t^\zeta, \epsilon_t^\mu]'$:

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Analytical solution as expression of conditional mean and variance:

$$q_t = \beta \exp \left(- (a_{11} + \gamma a_{21}) \pi_t - (a_{12} + \gamma a_{22}) \Delta c_t \right. \\ \left. + \frac{1}{2} \left[\underbrace{(\eta_{11} + \gamma \eta_{21})^2}_{\epsilon_t^a} + \underbrace{(\eta_{12} + \gamma \eta_{22})^2}_{\epsilon_t^\zeta} + \underbrace{(\eta_{13} + \gamma \eta_{23})^2}_{\epsilon_t^\mu} \right] \right)$$

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Assume the model would be **too complicated to solve**

Solving satellite versions of the model instead of the full model

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- **coefficients** on π_t and Δc_t learned from each partial dataset

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- **coefficients** on π_t and Δc_t learned from each partial dataset
- **coefficients** from shocks learned as shift from partial datasets with shock

Solving **Asset-Pricing** model:

1. Solving (sub)models

$$q_t^a = \dots$$

$$q_t^\zeta = \dots$$

$$q_t^\mu = \dots$$

Solving **Asset-Pricing** model:

1. Solving (sub)models
2. Simulating data

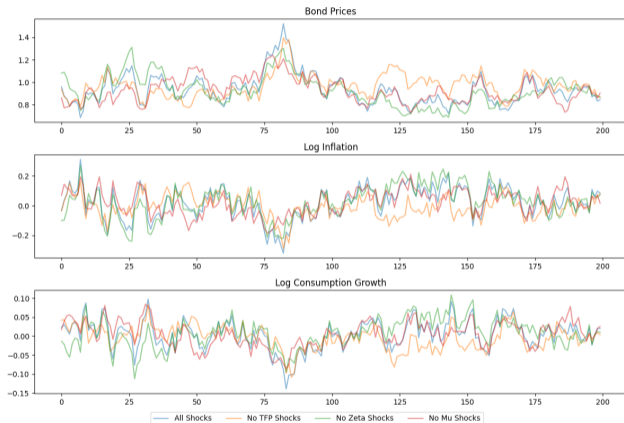


Figure 4: Simulation of economies with different combinations of aggregate shocks

Solving **Asset-Pricing** model:

1. Solving (sub)models
2. Simulating data and combining into one dataset

| q_t | π_t | Δc_t | ϵ_t^a | ϵ_t^ζ | ϵ_t^μ |
|----------|---------|--------------|----------------|--------------------|------------------|
| 0.99 | -0.02 | -0.01 | -0.12 | 1.04 | 0.0 |
| 1.06 | -0.04 | 0.02 | 1.63 | 1.85 | 0.0 |
| \vdots | | \vdots | | \vdots | |
| 1.01 | -0.08 | 0.02 | -0.12 | 0.0 | -1.73 |
| 0.99 | -0.03 | -0.01 | 1.63 | 0.0 | 0.45 |
| \vdots | | \vdots | | \vdots | |
| 1.06 | -0.11 | 0.01 | 0.0 | -1.0 | -1.72 |
| 0.94 | -0.01 | -0.07 | 0.0 | -0.39 | -0.86 |

Table 1: Simulation snapshots from combined dataset for q_t , π_t , Δc_t , ϵ_t^a , ϵ_t^ζ , and ϵ_t^μ

Solving **Asset-Pricing** model:

1. Solving (sub)models
2. Simulating data and combining into one dataset
3. Training the neural network

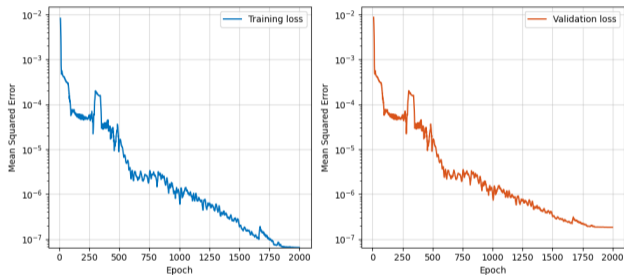


Figure 5: Loss function from training the NN

Solving **Asset-Pricing** model:

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Evaluate against **NNs** trained on data **generated by true model**

NN Specification

Evaluation Criteria

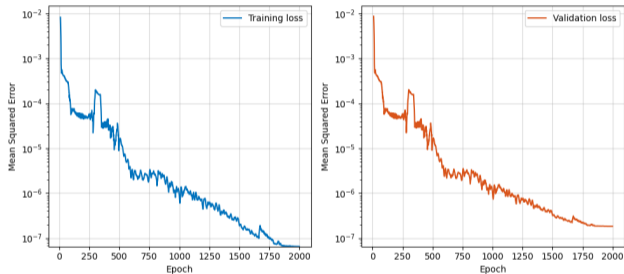


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Evaluating Generative Economic Modeling: Predicted values vs. actual values

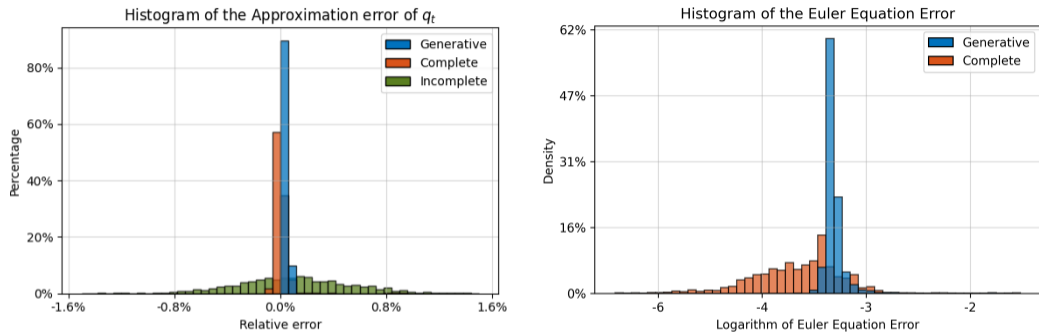


Figure 6: Prediction error distribution and Euler Equation Errors

- **Generative:** extrapolates with a NN trained on simulated data from submodels
- **Complete:** Train a NN on simulated data from true model

Accuracy of the prediction

| Model | $\log_{10}(\text{MSE})$ | $\log_{10}(\text{MAE})$ | $\log_{10}(\text{EEE})$ |
|----------------|-------------------------|-------------------------|-------------------------|
| Asset Price | -4.85 | -3.22 | -3.15 |
| Analytical RBC | | | |
| Nonlinear RBC | | | |
| Krusell-Smith | | | |
| Global HANK | | | |

Table 2: Mean Squared Error (MSE), Mean Absolute Error (MAE), and Euler Equation Error (EEE) of different models

RBC Model variants with increasing complexity

1. Baseline: tractable, analytically solvable RBC variant
2. Medium-scale: includes nonlinearities in capital adjustment cost friction
3. Heterogeneous agents: partial insurance against income risk

Evaluating Generative Economic Modeling: Heterogeneous Agent Model

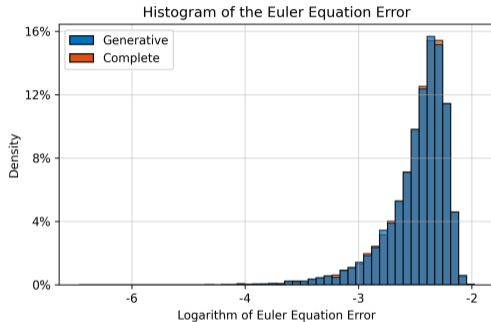
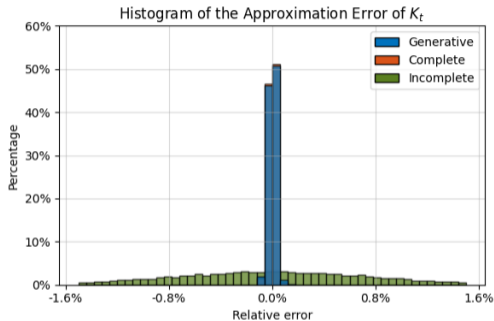


Figure 8: Graphical evaluation of model fit:
Prediction error distribution and Euler Equation Errors

Errors for individual policy functions

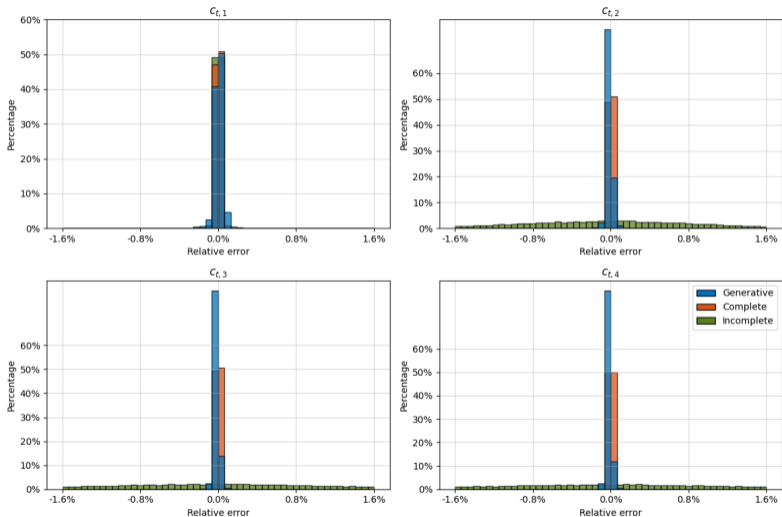


Figure 9: Comparison of error distributions for consumption function at individual grid points

Numerical Accuracy of GEM Approximations

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| Krusell-Smith | -7.10 | -4.00 | -2.09 |
| Global HANK | | | |

Table 3: Mean Squared Error (MSE), Mean Absolute Error (MAE), and Euler Equation Error (EEE) of different models

- Generative Economic Modeling yields accurate approximations across models
- Euler Equation Error (EEE) remains competitive with benchmark

Global solution of a HANK model with financial frictions

HANK Model with Financial Frictions - Nonlinearity Meets Amplification

- **Firms**
 - Face monopolistic competition and price adjustment costs
 - Produce using labor, subject to TFP shocks and a **cash-in-advance constraint**

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- **Government**
 - Issues bonds, consumes, and levies distortionary taxes on labor and profits
 - Sets the nominal interest rate via a Taylor rule
- **Why is this model interesting?**
 - Features **125 million state combinations** - global solution typically infeasible
 - **Amplification** of financial shocks **through heterogeneity**
 - Heterogeneity relevant for **optimal policy?**

Accuracy of Solution of Financial HANK model

| Model | $\log_{10}(\text{MSE})$ | $\log_{10}(\text{MAE})$ | $\log_{10}(\text{EEE})$ |
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| Global HANK | | | 0.55 |

Table 4: Mean Squared Error (MSE), Mean Absolute Error (MAE), absolute and relative to baseline; and Euler Equation Error (EEE)

- Accuracy in line with other models

Scenario Analysis: Financial Shocks Trigger Nonlinear Recessions

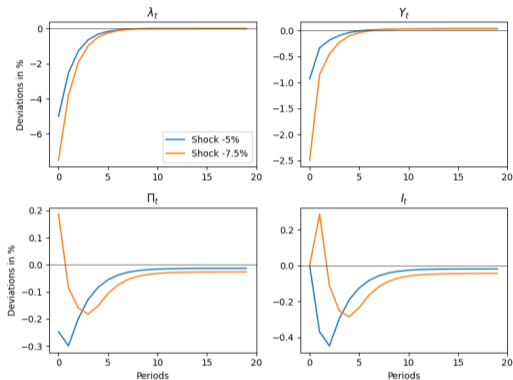


Figure 10: Generalized impulse responses to a financial shock of varying size

Acts as a supply shock:

- Financial shock limits firms' ability to finance inputs \Rightarrow output falls
- Inflation rises, pushing up nominal and real rates \Rightarrow demand contracts further

Global solution needed for the constraint:

- Economy's response is **nonlinear** in the size of the financial shock

Core Steps of GEM:







1. Solve a collection of **simplified models**
2. Simulate **synthetic data** from each specification
3. Train a **neural network** on the aggregated dataset





Takeaways for applied use:

- **Extends** existing solution methods – no new toolkit required
- Builds directly on established **conventional methods**
- Neural networks used **only for training** on simulated data

Thank you!

fabio.stohler@uni-bonn.de

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Appendix

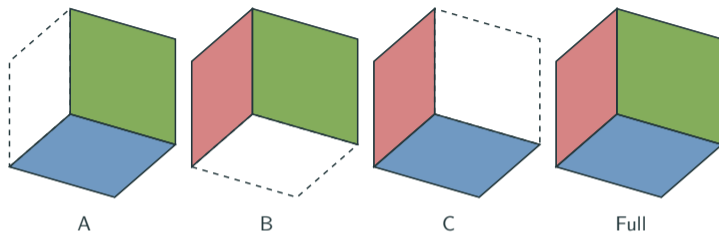


Figure 12: Graphical illustration for $n_\nu = 3$ and $n_{\tilde{\nu}} = 2$

- Model with shocks ν of length n_ν , simplified models with shocks $\tilde{\nu}$ of length $n_{\tilde{\nu}}$

$$\binom{n_\nu}{n_{\tilde{\nu}}} = \frac{n_\nu!}{n_{\tilde{\nu}}!(n_\nu - n_{\tilde{\nu}})!}$$

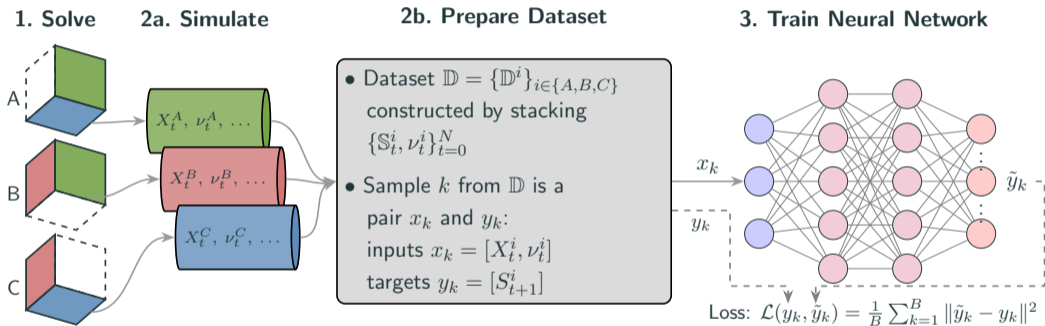


Figure 13: Flow chart of the generative economic modeling method

Train one NN on the full model and another on the combined simulated data.

- **Training Setup:**

- Network architecture: 5 hidden layers, each with 128 neurons
- Activation function: CELU
- Optimizer: AdamW, minimizing mean squared error (MSE)
- Learning rate: cosine annealing schedule, decaying from 10^{-3} to 10^{-7}

- **Evaluation Metric: Euler Equation Error (EEE)**

$$EEE = u'(\underbrace{\bar{f}_{DNN}^c(\mathbb{X}_t, \nu_t | \Theta)}_{c_t}) - \beta \mathbb{E}_t \left[\underbrace{\bar{f}_{DNN}^R(\mathbb{X}_{t+1}, \nu_{t+1} | \Theta)}_{R_{t+1}} u'(\underbrace{\bar{f}_{DNN}^c(\mathbb{X}_{t+1}, \nu_{t+1} | \Theta)}_{c_{t+1}}) \right]$$

Proposition

If all households are ex-ante and ex-post identical, depreciation is deterministic and full, $\delta(u_t) = 1$, capacity utilization is fixed at $u_t = 1$, there are no capital adjustment costs $\phi = 0$, the discount factor shock is inactive $\sigma_\zeta^2 = 0$, and per period felicity is of King, Plosser, and Rebelo (1988) (KPR)-form given by $u(C_t, N_t) = \ln C_t - \omega \frac{N_t^{1+\gamma}}{1+\gamma}$. Then the policy functions of the representative household are

$$N_t(\mu_t) = \left[\frac{\mu(1 - \tau^L)(1 - \alpha)}{\mu_t \omega (1 + \tau^C)(\mu - (1 - \tau^K)\alpha\beta)} \right]^{\frac{1}{1+\gamma}}, \quad (1)$$

$$C_t(K_t, A_t, Z_t, \mu_t) = \left(1 - \frac{\alpha\beta}{\mu_t} \right) Y_t(K_t, A_t, Z_t, N_t(\mu_t)), \quad (2)$$

$$\text{and } K_{t+1}(K_t, A_t, Z_t, \mu_t) = \frac{\alpha\beta}{\mu_t} Y_t(K_t, A_t, Z_t, N_t(\mu_t)), \quad (3)$$

where $Y_t(K_t, A_t, Z_t, N_t(\mu_t))$ denotes the Cobb-Douglas production function. Given the policy functions of the household, the prices in the economy can be determined by the first-order conditions of the firm.

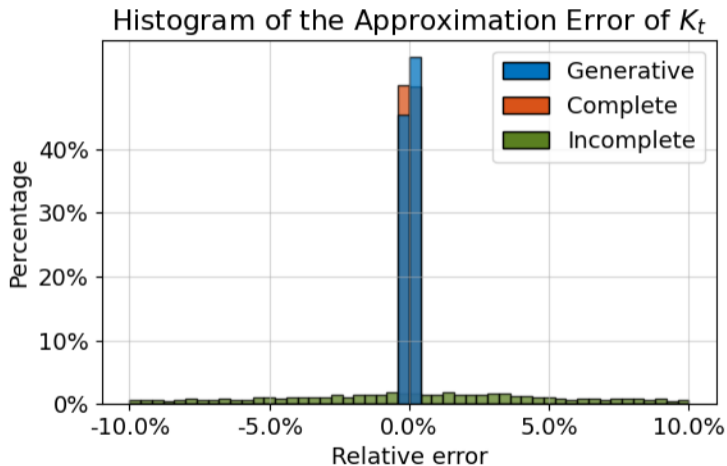


Figure 15: Error distribution of surrogate model for analytical model

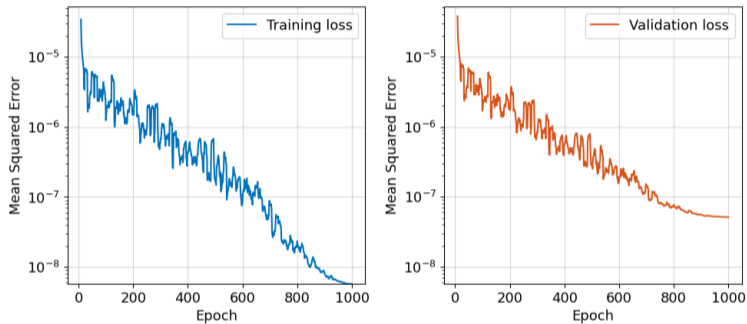


Figure 16: Loss in training and validation data over iterations

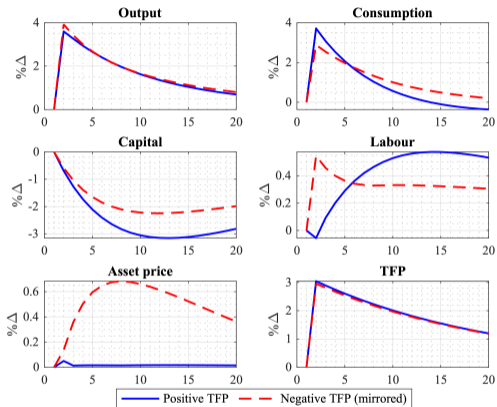


Figure 17: IRFs and Nonlinear Propagation of the TFP Shock

- Nonlinear medium-sized RBC with state-dependent capital adjustment costs
- Model is solved with global solution method (time iteration)
- Generalized IRFs illustrate nonlinear effect of capital adjustment costs

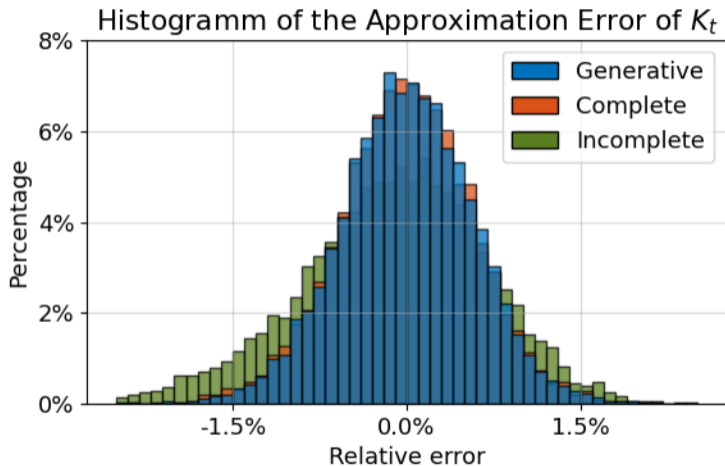


Figure 19: Error distribution of surrogate model for nonlinear Rep. Agent