Principled Identification of Structural Dynamic Models

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Preview of Results

- A **new perspective** on structural rotations in dynamic models
  - Reinterpret restrictions as first-order conditions
- **Cholesky** solves a sequence of constrained max-correlation problems
  - Different orderings impose different constraints on the same underlying objective
- **OASIS** gives the unconstrained, order- and scale-invariant benchmark rotation.
- Applied to **Proxy VARs** External instruments identification. OASIS estimates the most aligned rotation.
- **Empirical** Revisit 22+ SVAR studies

Notation: Structural and Reduced-Form

- Vector Autoregression (reduced-form)

$$X_t = \Phi_1 X_{t-1} + \cdots + \Phi_p X_{t-p} + \varepsilon_t.$$

ε_t is the **reduced-form shock** vector, $\text{var}(\varepsilon_t) = \Sigma \in \mathbb{R}^{n \times n}$, $\det \Sigma > 0$.

- Structural Vector Autoregression (SVAR)

$$A' X_t = A' \Phi_1 X_{t-1} + \cdots + A' \Phi_p X_{t-p} + \underbrace{A' \varepsilon_t}_{= u_t},$$

$u_t = A' \varepsilon_t$ is the vector of **structural shocks**. $\text{var}(u_t) = I_n$.

Identification Problem

(Drop subscript-t)

- Vector of structural shocks, $u = A'\varepsilon$,

$$\text{var}(u) = \text{var}(A'\varepsilon) = A'\Sigma A = I_n.$$

- Many A-matrices to choose from:

$$\mathcal{A} = \{A \in \mathbb{R}^{n\times n} : A'\Sigma A = I_n\}.$$

Same Problem: Square-Root of a Matrix

- There are many solutions to
$$\Sigma = BB'.$$
    - If B is a solution, then so is BR, for any orthogonal R ($R'R = I$).
- Popular choice: Lower triangular **Cholesky** decomposition: B
- Relation to $A'\Sigma A = I_n$:
$$A' = B^{-1} \quad \Rightarrow \quad A \in \mathcal{A}.$$

SVAR from Leeper, Sims, Zha (1996)

- Estimate VAR(4) with quarterly U.S. data 1959:Q1 to 2018:Q4.
  LSZ96. Shorter sample, interpolated monthly data.
- Reduced-form variance GDP Deflator = (Nominal GDP / Real GDP) x 100

$$\begin{bmatrix} \text{GDP} \\ \text{DEF} \\ \text{FFR} \\ \text{M2} \end{bmatrix} \qquad \Sigma = \begin{bmatrix} 0.5329 & 0.0031 & 0.0395 & 0.0141 \\ 0.0031 & 0.0602 & 0.0671 & -0.0165 \\ 0.0395 & 0.0671 & 1.0937 & -0.2470 \\ 0.0141 & -0.0165 & -0.2470 & 0.4472 \end{bmatrix}$$

- Cholesky: $A_c' = L^{-1}$ and $B_c = L$ where $\Sigma = LL'$

$$\begin{bmatrix} 1.370 & 0.000 & 0.000 & 0.000 \\ -0.023 & 4.076 & 0.000 & 0.000 \\ -0.067 & -1.102 & 0.992 & 0.000 \\ -0.069 & -0.039 & 0.362 & 1.601 \end{bmatrix} \qquad \begin{bmatrix} 0.730 & 0.000 & 0.000 & 0.000 \\ 0.004 & 0.245 & 0.000 & 0.000 \\ 0.054 & 0.272 & 1.008 & 0.000 \\ 0.019 & -0.068 & -0.228 & 0.625 \end{bmatrix}.$$

- Structural shocks $u_t = A_c' \varepsilon_t = B_c^{-1} \varepsilon_t$.

Cholesky Identification and its Implicit Criterion

Identification with Cholesky

- Lower triangular Cholesky decomposition

$$BB' = \Sigma, \qquad \text{with} \qquad B = \begin{bmatrix} * & 0 & \cdots & 0 \\ * & * & \ddots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ * & * & \cdots & * \end{bmatrix}.$$

- Are these $n(n-1)/2$ zero restrictions = FOCs to some optimization problem?

Cholesky = Sequential Max-Correlation

- Turns out: **Cholesky is equivalent to a sequence of optimization problems.** For $j = 1, \ldots, n$, let a_j be the solution to

$$\max_{a: a'\Sigma a = 1} \operatorname{corr}(a'\varepsilon, \varepsilon_j), \quad \text{s.t.} \quad a'\Sigma a_i = 0, \ \forall i = 1, \ldots, j-1.$$

  Then a_j is the j-th row of $A_c' = L^{-1}$
  (the reason sequential OLS can estimate Cholesky form).
  - **Objective identical** across all orderings.
  - Recursive ordering defines different (sequence of) constraints.

OASIS

OASIS: Baseline Case

- Drop constraints and **maximize average correlation**

$$\bar{\rho}(A) \equiv \frac{1}{n}\sum_{i=1}^{n} \mathrm{cor}(u_i, \varepsilon_i), \qquad u = A'\varepsilon.$$

- Simple solution based on correlation matrix $C = \mathrm{cor}(\varepsilon)$.
  - No additional constraint required.
  - Unique solution: Order- and Scale-Invariant.

Corollary 1

- **Solution**:
$$\bar{\rho}(A_*) = \frac{1}{n}\sum_{i=1}^{n} \text{cor}(u_i^*, \varepsilon_i) = \frac{1}{n}\sum_{i=1}^{n} \lambda_i^{1/2},$$
where $\lambda_1, \ldots, \lambda_n$ are the eigenvalues of $C = \mathrm{cor}(\varepsilon)$.
- $u^* = A_*'\varepsilon$ with
$$A_* = \Lambda_\sigma^{-1} C^{-1/2} \in \mathcal{A},$$
where $\Lambda_\sigma = \text{diag}(\sigma_1, \ldots, \sigma_n)$ is diagonal matrix with standard deviations, $\sigma_j^2 = \text{var}(\varepsilon_j)$.

More General: Weighted Maximum Correlation

- Objective is to maximize

$$\max_{A \in \mathcal{A}} \rho_w(A), \quad \text{where } \rho_w(A) \equiv \sum_{i=1}^{n} w_i \operatorname{cor}(u_i, \varepsilon_i),$$

for $w_i > 0$ and $u = A'\varepsilon$.

Theorem 1 (OASIS)

- Let $\lambda_{w,1}, \ldots, \lambda_{w,n}$ be eigenvalues of $\Lambda_w C \Lambda_w$ with

$$\Lambda_w \equiv \text{diag}(w_1, \ldots, w_n) \qquad \text{and} \qquad C = \text{cor}(\varepsilon).$$

- Then

$$\rho_w(A_*) = \sum_{i=1}^{n} \lambda_{w,i}^{1/2} \geq \rho_w(A) \quad \text{for all } A \in \mathcal{A},$$

where

$$A_* = \Lambda_\sigma^{-1} \Lambda_w (\Lambda_w C \Lambda_w)^{-1/2}$$

is the unique maximizer (i.e. $u^* = A_*' \varepsilon$).

Leeper, Sims, Zha (1996)

- $$\begin{bmatrix} \text{GDP} \\ \text{DEF} \\ \text{FFR} \\ \text{M2} \end{bmatrix} \qquad \Sigma = \begin{bmatrix} 0.5329 & 0.0031 & 0.0395 & 0.0141 \\ 0.0031 & 0.0602 & 0.0671 & -0.0165 \\ 0.0395 & 0.0671 & 1.0937 & -0.2470 \\ 0.0141 & -0.0165 & -0.2470 & 0.4472 \end{bmatrix}$$

- Cholesky: $A_c' = L^{-1}$ and $B_c = L$ where $\Sigma = LL'$

$$\begin{bmatrix} 1.370 & 0.000 & 0.000 & 0.000 \\ -0.023 & 4.076 & 0.000 & 0.000 \\ -0.067 & -1.102 & 0.992 & 0.000 \\ -0.069 & -0.039 & 0.362 & 1.601 \end{bmatrix} \qquad \begin{bmatrix} 0.730 & 0.000 & 0.000 & 0.000 \\ 0.004 & 0.245 & 0.000 & 0.000 \\ 0.054 & 0.272 & 1.008 & 0.000 \\ 0.019 & -0.068 & -0.228 & 0.625 \end{bmatrix}.$$

- OASIS $A_*' = C^{-1/2}\Lambda_\sigma^{-1}$ and $B_* = \Lambda_\sigma C^{1/2}$

$$\begin{bmatrix} 1.372 & -0.016 & -0.030 & -0.035 \\ -0.006 & 4.186 & -0.127 & 0.026 \\ -0.044 & -0.542 & 1.032 & 0.285 \\ -0.032 & 0.070 & 0.182 & 1.574 \end{bmatrix} \qquad \begin{bmatrix} 0.730 & 0.005 & 0.020 & 0.013 \\ 0.002 & 0.243 & 0.032 & -0.010 \\ 0.029 & 0.135 & 1.020 & -0.186 \\ 0.011 & -0.026 & -0.119 & 0.657 \end{bmatrix}$$

Empirical Literature
18 papers with 22 VARs

Monetary Shocks

[B86]	Bernanke (1986b). VAR using monetary aggregates and interest rates as policy indicators.
[S95]	Strongin (1995). Identifies monetary shocks via Fed's component of nonborrowed reserves.
[LSZ96]	Leeper et al. (1996). SVAR analyzing monetary policy; emphasizes sensitivity to identification assumptions and robustness to ordering.
[CEE99]	Christiano et al. (1999). SVAR using federal funds rate as monetary policy instrument.
[CEE05]	Christiano et al. (2005). SVAR using interest rate shocks and impulse response matching to validate DSGE models.
[BL09]	Bjørnland and Leitemo (2009). SVAR identifying monetary shocks via interest rates and stock prices.

Fiscal Shocks

[BP02]	Blanchard and Perotti (2002). SVAR using government spending and net taxes with timing restrictions.
[RZ11]	Rossi and Zubairy (2011). SVAR using government spending, tax revenue, and interest rates.
[FG16]	Forni and Gambetti (2016). Uses forecast revisions (SPF) and government spending as shock proxy.

Uncertainty Shocks

[B09]	Bloom (2009). SVAR using stock market volatility (VXO) as proxy for uncertainty shocks.
[CCG14]	Caggiano et al. (2014). VAR using VIX and forecast dispersion to measure uncertainty.
[BB17]	Basu and Bundick (2017). SVAR identified via stock market volatility (VXO).
[BO23]	Bonciani and Oh (2023). VAR with uncertainty shocks proxied by macro uncertainty measures estimated by Jurado et al. (2015); compared to DSGE.

Financial Shocks

[GZ12]	Gilchrist and Zakrajšek (2012). VAR using credit spreads and bond premia as financial shock indicators.
[FGMS24]	Forni et al. (2024). VAR with nonlinear financial shock identification using a Vector Moving Average (VMA) model.

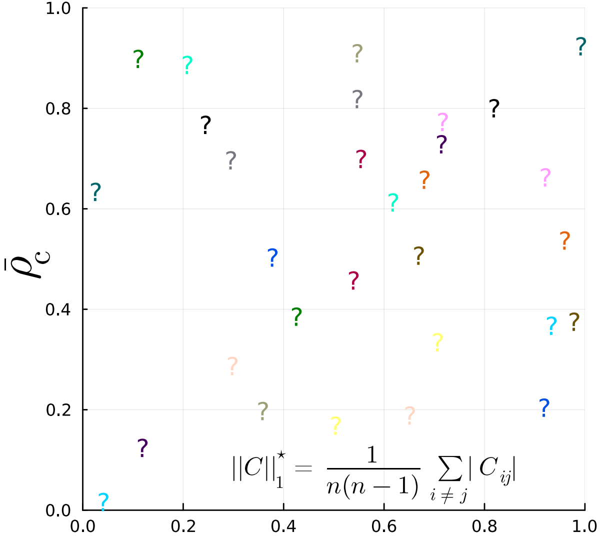
Oil Price Shocks

[LS04]	Leduc and Sill (2004). SVAR using real oil price as shock variable.
[LP18]	Lorusso and Pieroni (2018). SVAR using global oil prices to assess UK macro responses.

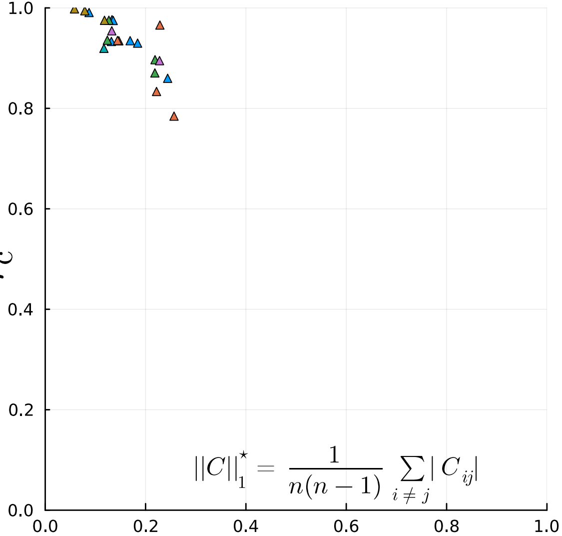
Sectoral

[FGKV25]	Fry-Mckibbin et al. (2025). SVAR for Australia, controlling for import penetration from China, in the sectoral output context.

Empirical literature: What are typical correlations?



Correlations in these 22 Studies



$\bar{\rho}_c$

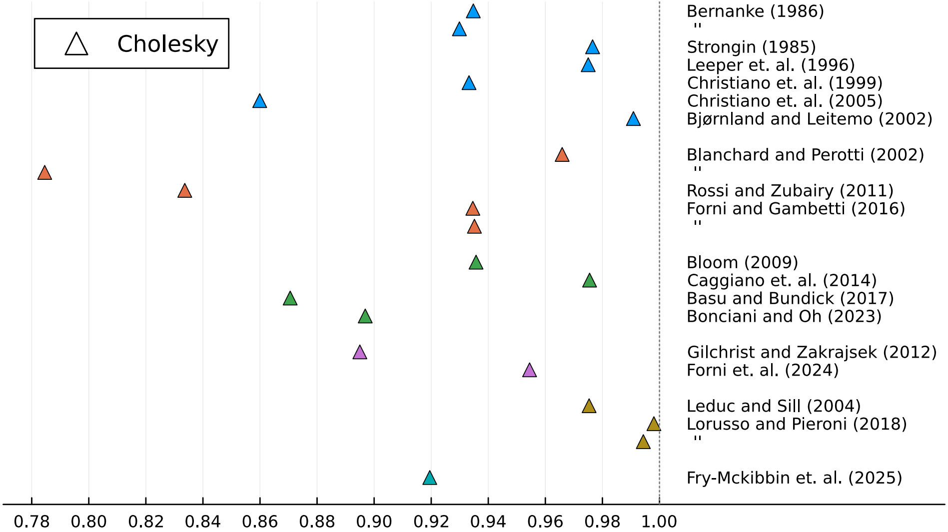
$$||C||_1^\star = \frac{1}{n(n-1)} \sum_{i \neq j} |C_{ij}|$$

Small Reduced-Form Correlations

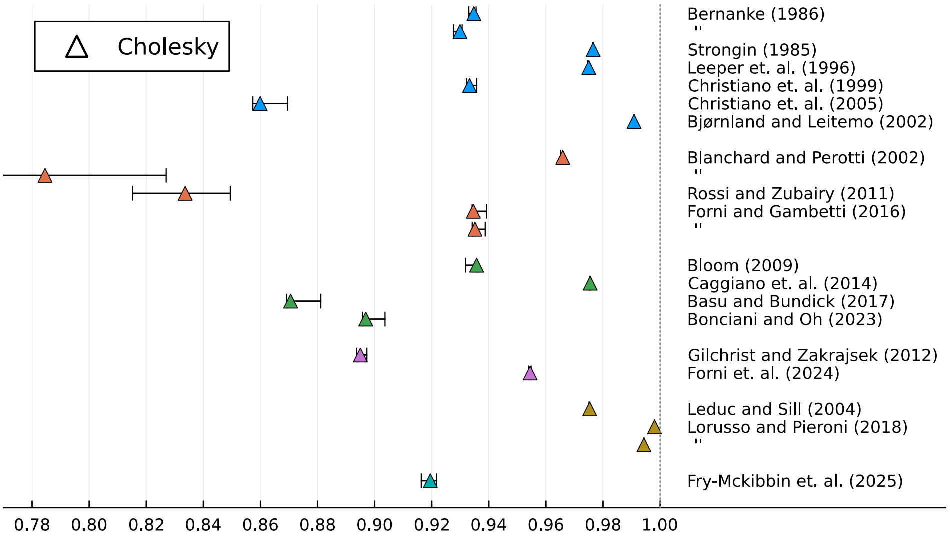
Study	n	OASIS $\bar{\rho}_*$	$\bar{\rho}_c$	Cholesky min	max	$\bar{\rho}_{*,c}$	$\Vert C\Vert_1^\star$	$\frac{1-\bar{\rho}_c}{1-\bar{\rho}_*}$	$d(C)$
[B86]	6		0.935				0.17		
—	6		0.930				0.18		
[S95]	5		0.977				0.13		
[LSZ96]	4		0.975				0.14		
[CEE99]	7		0.933				0.13		
[CEE05]	9		0.860				0.24		
[BL09]	5		0.991				0.09		
[BP02]	3		0.966				0.23		
—	7		0.785				0.26		
[RZ11]	8		0.834				0.22		
[FG16]	7		0.935				0.15		
—	8		0.935				0.14		
[B09]	8		0.936				0.12		
[CCG14]	4		0.976				0.13		
[BB17]	8		0.871				0.22		
[BO23]	9		0.897				0.22		
[GZ12]	8		0.895				0.23		
[FGMS24]	6		0.954				0.13		
[LS04]	5		0.975				0.12		
[LP18]	3		0.998				0.06		
—	3		0.994				0.08		
[FGKV25]	14		0.920				0.12		

- $C \simeq I +$ small-ish $\Rightarrow \mathrm{cor}(u_i, \varepsilon_i) = 1 - \mathrm{small}$.

Empirical Choleskys have large $\bar{\rho}_c$



All Choleskys have large and similar $\bar{\rho}_c$



...even 87 Billion Choleskys

Study	n	Cholesky $\bar{\rho}_c$	min	max	$\|C\|_1^\star$
[B86]	6	0.935	0.933	0.936	0.17
—	6	0.930	0.928	0.931	0.18
[S95]	5	0.977	0.976	0.977	0.13
[LSZ96]	4	0.975	0.975	0.975	0.14
[CEE99]	7	0.933	0.932	0.936	0.13
[CEE05]	9	0.860	0.857	0.869	0.24
[BL09]	5	0.991	0.991	0.991	0.09
[BP02]	3	0.966	0.965	0.966	0.23
—	7	0.785	0.757	0.827	0.26
[RZ11]	8	0.834	0.815	0.849	0.22
[FG16]	7	0.935	0.934	0.939	0.15
—	8	0.935	0.934	0.939	0.14
[B09]	8	0.936	0.932	0.936	0.12
[CCG14]	4	0.976	0.975	0.976	0.13
[BB17]	8	0.871	0.869	0.881	0.22
[BO23]	9	0.897	0.896	0.904	0.22
[GZ12]	8	0.895	0.894	0.897	0.23
[FGMS24]	6	0.954	0.954	0.955	0.13
[LS04]	5	0.975	0.975	0.975	0.12
[LP18]	3	0.998	0.998	0.998	0.06
—	3	0.994	0.994	0.994	0.08
[FGKV25]	14	0.920	0.916	0.922	0.12

- $14! = 87,178,291,200$ different orderings (Choleskys).

Why are Choleskys Similar?

- A "how far is C from I_n" metric.

$$d(C) = \frac{1}{n}\sum_{i\neq j} C_{ij}^2 = \frac{1}{n}\|C - I_n\|_F^2 = \frac{1}{n}\|E\|_F^2,$$

  with $E \equiv C - I_n$
- Theorem 2 (Part I). **Any Cholesky decomposition satisfies:**

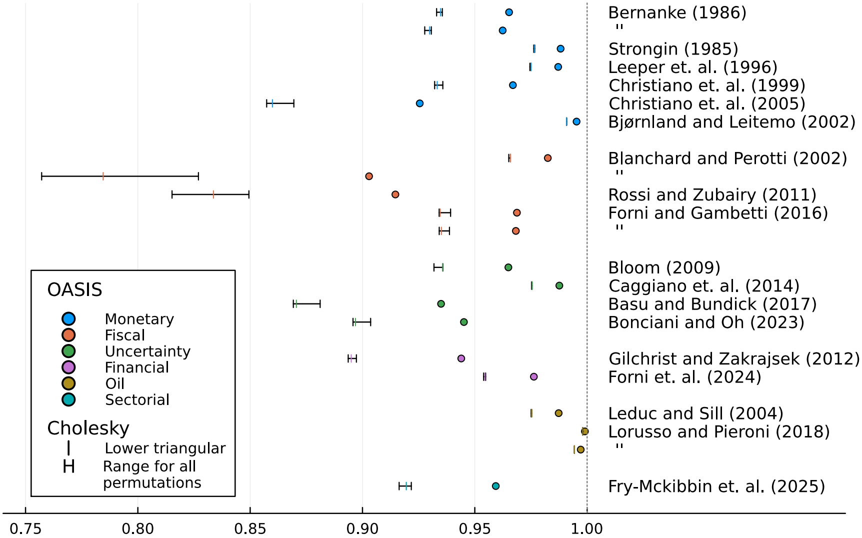
$$\bar{\rho}_{\mathrm{c}} = 1 - d(C)/4 + O(E^3).$$

  - $\bar{\rho}_{\mathrm{c}}$ is pinned down near $1 - d(C)/4$ for all orderings.
  - Ordering mainly redistributes $\sum_{i\neq j} C_{ij}^2$ across shocks:

$$\frac{1}{4}\sum_{i\neq j} C_{ij}^2 \approx \sum_j (1 - \mathrm{corr}(u_j, \varepsilon_j)).$$

Add OASIS to the Picture

Average Correlations: All Cholesky and OASIS



Twice as close to "perfect correlation"

Study	n	OASIS $\bar{\rho}_*$	Cholesky $\bar{\rho}_c$	Cholesky min	Cholesky max	$\bar{\rho}_{*,c}$	$\lVert C \rVert_1^\star$	$\frac{1-\bar{\rho}_c}{1-\bar{\rho}_*}$	$d(C)$
[B86]	6	0.965	0.935	0.933	0.936	0.970	0.17	1.88	0.29
—	6	0.962	0.930	0.928	0.931	0.968	0.18	1.87	0.32
[S95]	5	0.988	0.977	0.976	0.977	0.988	0.13	1.99	0.09
[LSZ96]	4	0.987	0.975	0.975	0.975	0.988	0.14	1.94	0.10
[CEE99]	7	0.967	0.933	0.932	0.936	0.963	0.13	2.02	0.24
[CEE05]	9	0.925	0.860	0.857	0.869	0.935	0.24	1.88	0.70
[BL09]	5	0.995	0.991	0.991	0.991	0.996	0.09	1.95	0.04
[BP02]	3	0.982	0.966	0.965	0.966	0.983	0.23	1.95	0.14
—	7	0.903	0.785	0.757	0.827	0.831	0.26	2.22	0.63
[RZ11]	8	0.915	0.834	0.815	0.849	0.908	0.22	1.95	0.61
[FG16]	7	0.969	0.935	0.934	0.939	0.963	0.15	2.09	0.22
—	8	0.968	0.935	0.934	0.939	0.965	0.14	2.04	0.24
[B09]	8	0.965	0.936	0.932	0.936	0.970	0.12	1.83	0.28
[CCG14]	4	0.988	0.976	0.975	0.976	0.988	0.13	1.98	0.09
[BB17]	8	0.935	0.871	0.869	0.881	0.933	0.22	1.99	0.53
[BO23]	9	0.945	0.897	0.896	0.904	0.952	0.22	1.88	0.54
[GZ12]	8	0.944	0.895	0.894	0.897	0.951	0.23	1.87	0.51
[FGMS24]	6	0.976	0.954	0.954	0.955	0.978	0.13	1.92	0.20
[LS04]	5	0.987	0.975	0.975	0.975	0.988	0.12	1.95	0.10
[LP18]	3	0.999	0.998	0.998	0.998	0.999	0.06	1.99	0.01
—	3	0.997	0.994	0.994	0.994	0.997	0.08	1.99	0.02
[FGKV25]	14	0.959	0.920	0.916	0.922	0.958	0.12	1.98	0.32

Theorem 2 (OASIS and Cholesky)

- Define, $E \equiv C - I_n$ (how far is ε from being uncorrelated).
- Recall $d(C) \equiv \frac{1}{n}\sum_{i \neq j} C_{ij}^2 = \frac{1}{n}\|E\|_F^2$. Then

$$\begin{aligned} \bar{\rho}_* &= 1 - \tfrac{1}{8} d(C) + O(\mathrm{tr}\{E^3\}), \\ \bar{\rho}_{\mathrm{c}} &= 1 - \tfrac{1}{4} d(C) + O(\mathrm{tr}\{E^3\}), \end{aligned}$$

and

$$(1 - \bar{\rho}_{\mathrm{c}})/(1 - \bar{\rho}_*) = 2(1 + O(\|E\|_F)) = 2\left(1 + O(\sqrt{d(C)})\right).$$

- **Results: OASIS is twice as close to "perfection"**.

Visualizing the Empirical Support

- Theorem 2: $\bar{\rho}_* \approx 1 - \frac{1}{8}d(C)$ such that

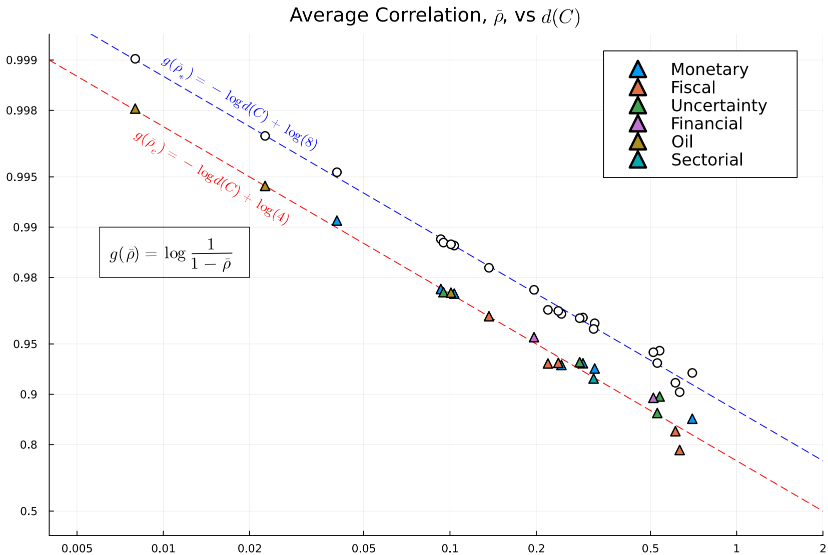
$$\begin{aligned} \log(1-\bar{\rho}_*) &\approx \log\tfrac{1}{8} + \log d(C) \\ -\log(1-\bar{\rho}_*) &\approx \log 8 - \log d(C) \end{aligned}$$

- In a log-log scatterplot

$$g(\bar{\rho}) \approx \alpha + \beta \log d(C), \qquad g(\bar{\rho}) = -\log(1-\bar{\rho}),$$

  we should expect
  - $\alpha_* = +\log 8$ and $\alpha_{\mathrm{c}} = +\log 4$.
  - $\beta = -1$.

Visualizing Empirical Evidence (Log-log scatterplot)



LSZ96 Additional Details

- Average correlations and more.

Table 3: OASIS and Cholesky applied to Leeper, Sims, and Zha (1996)

	OASIS	Cholesky				
		Lower	Upper	min $\bar{\rho}_c$	max $\bar{\rho}_c$	
	$\text{corr}(\varepsilon, u^*)$	$\text{corr}(\varepsilon, u^c)$		Variable ordering		$\text{corr}(u^*, u^c)$
GDP	0.9995	1.0000	0.9974	3	1	0.9995
DEF	0.9908	0.9999	0.9652	4	2	0.9908
FFR	0.9751	0.9641	0.9356	2	4	0.9752
M2	0.9831	0.9343	1.0000	1	3	0.9836
$\bar{\rho}$	0.9871	0.9745	0.9745	0.9745	0.9750	0.9879

- Here $d(C) = 0.1035$ and $\|C\|_1^\star = 0.1355$ (average $|C_{ij}|$).

Is Max-Correlation Sensible?

- A structural shock often associated with a particular variable.
  - A monetary policy shock is **naturally associated** with the federal funds rate (FFR)
  - We would expect the structural monetary shock to be highly correlated with the FFR innovation.
- Can u_j be too correlated with ε_j?
  - Kilian, Plante, & Richter (2025) counter example.
  - Known measurement errors?
- Yet. 16/22 have $\bar{\rho}_c = \frac{1}{n}\sum_{j=1}^{n} \mathrm{corr}(u_j, \varepsilon_j) > 90\%$, all $\bar{\rho}_c > 78.5\%$.

Proxy VARs

Proxy VARs (IV-SVARs)

- External-instrument approach of Stock and Watson.
  Formalized/popularized in SVARs by Mertens and Ravn (2013).
- External instruments $z_1, \ldots, z_r$, provide the identifying content.
- $(u', v')' = A'\varepsilon$ with $u = \mathbf{a}'\varepsilon \in \mathbb{R}^r$ are the structural shocks and v is a vector of auxiliary shocks.
- OASIS **selects the rotation most aligned with the instruments**.
- **Theorem 3**: Objective:

$$\max_{\mathbf{a} \in \mathcal{A}_{n,r}} \sum_{i=1}^{r} w_i \operatorname{corr}(u_i, z_i).$$

- This is a principled way to operationalize instrument relevance.

OASIS for Proxy VARs

- Handles multiple instruments jointly.
- Gives an ordering-free rotation inside the instrument-identified shock space.
- Can be combined with additional sign/zero restrictions if desired. (At the expense of elegance).
- In applications: $\hat{\xi}_1, \ldots, \hat{\xi}_r$ from (singular-value decomposition) serve as diagnostic statistics for instrument strength.

Summary

Summary

- **New perspective** on structural rotations in dynamic models.
  - **Shifts discourse** from restrictions to objective
  - **New insight** about Cholesky.
- **OASIS**: Maximum-correlation objective
  - Benchmark rotation in **SVARs**.
  - Empirical studies have $C \approx I_n$ and large $\bar{\rho}_{\mathrm{c}}$
  - **Proxy VARs**: Most aligned with instruments rotation.
- Much more in paper
  - **Diebold-Yilmaz** Connectedness/Networks