Principled Identification of Structural Dynamic Models

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Abstract

We take a new perspective on identification in structural dynamic models: rather than imposing restrictions, we optimize an objective. This provides new theoretical insights into traditional Cholesky identification. A correlation-maximizing objective yields an Order- and Scale-Invariant Identification Scheme (OASIS) that selects the orthogonal rotation that best aligns structural shocks with their reduced-form innovations. We revisit a large number of SVAR studies and find, across 22 published SVARs, that the correlations between structural and reduced-form shocks are generally high.

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1 Introduction

Dynamic models in macroeconomics, such as structural vector autoregressions (SVARs) and local projections (LPs), aim to identify the latent economic shocks that drive macroeconomic fluctuations; Sims (1980), Jordà (2005), and Plagborg-Møller and Wolf (2021). A central assumption is that these structural shocks are mutually uncorrelated, unlike the contemporaneously correlated residuals of a reduced-form VAR. To recover structural shocks, researchers impose identification restrictions to pin down a rotation of the reduced-form innovations. Given a covariance matrix $\Sigma \in \mathbb{R}^{n\times n}$, the identification problem arises from the multiple solutions to $\Sigma = BB'$. The matrix B has n^2 elements, while the symmetric Σ contains only $n(n+1)/2$ unique elements. Full identification therefore requires $n(n-1)/2$ restrictions. A standard approach is to impose a triangular structure on B, in which case B is the Cholesky decomposition of Σ, see Sims (1980), Bernanke (1986), Blanchard and Watson (1986), long-run restrictions, see Blanchard and Quah (1989), Galí (1999), Francis and Ramey (2005), sign restrictions, see Canova and De Nicoló (2002), Uhlig (2005), Rubio-Ramírez et al. (2010). Each of these identification schemes has its own appeal and weaknesses, see e.g. Cooley and LeRoy (1985), Faust and Leeper (1997), Fry and Pagan (2011), Baumeister and Hamilton (2015), Ramey (2016), Stock and Watson (2016).

In this paper, we propose a principled approach to identification: formulate identification as an explicit optimization. Under standard regularity conditions, the corresponding first-order conditions play the same role as identifying assumptions, and many familiar assumption sets can be read as the first-order conditions of some optimization problem. This reframes the question from "Are the identifying restrictions plausible?" to "Is the objective sound for the economic question at hand?" In short, the discourse shifts from restrictions to objective soundness.

Our leading example is a maximum-correlation objective and variations thereof. We choose the orthogonal rotation that maximizes the average correlation between each structural shock and its corresponding reduced-form innovation, subject only to orthogonality of the structural shocks. We label the resulting identification scheme OASIS because it is order- and scale-invariant. The economic motivation is simple: each structural shock is typically associated with a particular variable, such that a strong association is to be expected. For example, a monetary policy shock is naturally associated with the federal funds rate (FFR), so we expect the structural monetary shock to be highly correlated with the reduced-form innovation to the FFR. Reduced-form shocks may be influenced by other structural shocks, which is the source of non-trivial correlations between reduced-form innovations.

Viewing identification from the perspective of an optimization problem also yields new insight into Cholesky identification. In particular, we show that the maximum-correlation objective is closely related to Cholesky identification, which can be seen as solving a constrained version of this optimization problem. Specifically, a Cholesky decomposition can be interpreted as the solution to sequential optimization problems, where in each step the objective is to maximize the correlation between a structural shock and the corresponding reduced-form innovation, subject to an increasing number of orthogonality constraints. Although Cholesky identification depends on the ordering of variables, all Cholesky decompositions share a common objective and differ only in the constraints imposed during the optimization process. This can partly explain why different Cholesky schemes often lead to similar results.

A high correlation between structural shocks and the corresponding reduced-form shocks is not an explicitly stated objective in SVAR modeling. However, it is nevertheless a characteristic of structural shocks in the empirical literature. In sixteen of the twenty-two SVAR studies we examine, the average correlation between reduced-form shocks and Cholesky-identified structural shocks exceeds 90%. The smallest average correlation is 78.5% and the largest is 99.8%.

Order sensitivity induced by triangular factorizations arises in other ways in VARs. For instance, the connectedness model by Diebold and Yilmaz (2009) used Cholesky to define variable specific shocks, before adopting the generalized impulse-response/variance-decomposition framework of Koop et al. (1996) and Pesaran and Shin (1998). The "generalized" shocks are order- and scale-invariant, but are not uncorrelated. Chan et al. (2024) address a related yet different aspect related to order-invariance and obtain an *order-invariant likelihood/posterior* for large Bayesian VARs by abandoning triangularity and exploiting stochastic volatility for identification (unique up to signs/permutations). Their decomposition is the (time-varying) reduced-form covariance matrix. This is different from OASIS, which is invariant to both permutations and rescalings.

Let $\bar{\rho}_*$ and $\bar{\rho}_{\mathrm{c}}$ denote the average correlation between structural shocks and reduced-form innovations under OASIS and Cholesky, respectively. Because OASIS maximizes the average correlation, we necessarily have $\bar{\rho}_* \geq \bar{\rho}_{\mathrm{c}}$. Moreover, we show, theoretically and empirically, that the average correlation achieved by OASIS is roughly twice as close to perfect correlation ($\bar{\rho} = 1$) as that obtained under Cholesky identification.

A second theoretical result concerns conventional Cholesky identification: although the Cholesky decomposition and the resulting impulse responses depend on the variable ordering, any Cholesky decomposition produces very similar values of $\bar{\rho}_{\mathrm{c}}$. Hence, the ordering primarily affects how the average correlation is distributed across the system's dimensions, rather than the overall level of average correlation.

We show that the average correlations, $\bar{\rho}_*$ and $\bar{\rho}_{\rm c}$, decrease as reduced-form shocks become more correlated. In the many empirical VAR studies we revisit, we find that the correlation matrix of the reduced-form errors, C, is often close to the identity matrix, and this explains that their average correlations are large. These and other empirical findings are fully consistent with our theoretical results.

The paper is organized as follows. We present OASIS in Section 2 and establish some of its theoretical properties. There we also establish theoretical results for identification with a Cholesky decomposition. Section 3 revisits some empirical studies and we compare the results for OASIS with those obtained with Cholesky. Concluding remarks are provided in Section 5. Proofs are presented in Appendix A.

2 Identification Schemes

Identification is central to empirical macroeconomics, from structural VARs to local projections (LPs) and IRF matching. To establish a common framework for discussing different identification schemes, we first set out the notation and assumptions that underlie these models.

We begin by formalizing the basic environment shared by these methods, and we largely follow the notation used in Hamilton (1994). Let $\varepsilon \in \mathbb{R}^n$ denote a vector of reduced-form shocks with a general nonsingular covariance matrix, $\Sigma \equiv \mathrm{var}(\varepsilon)$, and let $u = A'\varepsilon$ denote a vector of structural shocks for some matrix $A \in \mathbb{R}^{n\times n}$. These are time series, but to simplify the exposition, we suppress the subscript-t notation in this subsection.

The structural shocks, $u = A'\varepsilon$, are assumed to be uncorrelated and normalized; $\mathrm{var}(u) = I_n$, hence the requirement is $A'\Sigma A = I_n$. The set of A-matrices satisfying this requirement is denoted

$$\mathcal{A} \equiv \{A \in \mathbb{R}^{n\times n} : A'\Sigma A = I_n\},$$

and is uncountable for any nonsingular covariance matrix, Σ. The (lower triangular) Cholesky decomposition of the covariance matrix, Σ, is a common choice, which is given by $A_{\rm c} = L^{-1}$, where L is the lower triangular matrix satisfying $LL' = \Sigma$. This particular choice for A can be characterized as a sequential optimization problem.

Proposition 1 (Cholesky characterization)**.** *Consider the sequence of optimization problems,*

$$a_j \equiv \arg \max_{a:a'\Sigma a=1} \mathrm{corr}(a'\varepsilon, \varepsilon_j), \quad \textit{subject to} \quad a'\Sigma a_i = 0,\ \forall i < j, \tag{1}$$

for $j = 1, \ldots, n$. Then a_j is the j-th column of $A_{\rm c}$.

It follows that identification based on the Cholesky decomposition ensures a relatively high average correlation between the structural shocks $u_1, \ldots, u_n$ and their corresponding reduced-form shocks $\varepsilon_1, \ldots, \varepsilon_n$. This seems reasonable, because the structural shock to the j-th variable would naturally be embodied in ε_j. Naturally, ε_j is also contaminated with other shocks for $j \geq 2$, and this is responsible for Σ being a non-diagonal covariance matrix.

The Cholesky representation depends on the ordering of variables, and the chosen ordering is sometimes criticized for being (partly) arbitrary. The first structural shock will always be perfectly correlated with the corresponding reduced-form shock and, as j increases, the correlation between structural shocks and the corresponding reduced-form shocks tends to decrease because the number of constraints in (1) increases with j. It can therefore be said that Cholesky prioritizes high correlation for shocks associated with variables that appear first in the system.

An important point of this paper is that the basic principle that underlies Cholesky identification does not require a variable ordering. It is possible to treat all dimensions equally and maximize the total (or average) correlation. This maximization can be done simultaneously over all dimensions, rather than the constrained sequential maximization implicit in Cholesky. This leads to the OASIS identification scheme that we will introduce next.

2.1 A Maximum Correlation Objective

Motivated by the implicit objective of Cholesky identification, we can simply maximize the average correlation between the elements of $u = A'\varepsilon$ and the corresponding elements of ε. In other words, we can drop the constraint imposed by the recursive ordering used in a Cholesky decomposition and simply maximize $\rho_w(A) = \sum_{i=1}^{n} \mathrm{corr}(u_i, \varepsilon_i)$. This defines an order- and scale-invariant identification scheme, which we label as OASIS.

More generally, we can maximize the weighted-correlation criterion

$$\rho_w(A) = \sum_{i=1}^{n} w_i \, \mathrm{corr}(u_i, \varepsilon_i), \qquad \text{for } A \in \mathcal{A},$$

for positive weights, $w_i > 0$, for $i = 1, \ldots, n$, where $u = A'\varepsilon$.

We introduce the following notation. Let $C \equiv \mathrm{corr}(\varepsilon)$ denote the correlation matrix of the reduced-form shocks. Then

$$C = \Lambda_\sigma^{-1} \Sigma \Lambda_\sigma^{-1},$$

where $\Lambda_\sigma \equiv \mathrm{diag}(\sigma_1, \ldots, \sigma_n)$ is the diagonal matrix of standard deviations, such that $\sigma_i^2 = \mathrm{var}(\varepsilon_i)$, $i = 1, \ldots, n$ are the diagonal elements of Σ. For a symmetric matrix, S, we let

$S^{1/2}$ denote the symmetric square root of S, and if, in addition, S is positive definite, then $S^{-1/2}$ is defined by the inverse of $S^{1/2}$.[1] For the correlation matrix with eigendecomposition, $C = Q\Lambda_\lambda Q'$, where $Q'Q = I_n$ and $\Lambda_\lambda \equiv \mathrm{diag}(\lambda_1, \ldots, \lambda_n)$ is the diagonal matrix with the eigenvalues of C, we have $C^{1/2} = Q\Lambda_\lambda^{1/2}Q'$ and $C^{-1/2} = Q\Lambda_\lambda^{-1/2}Q'$.

Theorem 1 (Weighted OASIS)**.** *Suppose* $\det(\Sigma) > 0$. *Let* $\Lambda_w = \mathrm{diag}(w_1, \ldots, w_n)$*, where* $w_i > 0$ $i = 1, \ldots, n$*. Then*

$$\rho_w(A_*) = \sum_{i=1}^{n} \kappa_i^{1/2} \geq \rho_w(A), \qquad \textit{for all } A \in \mathcal{A},$$

where $A_* = \Lambda_\sigma^{-1}\Lambda_w(\Lambda_w C \Lambda_w)^{-1/2}$ *is the unique maximizer and* $\kappa_1, \ldots, \kappa_n$ *are the eigenvalues of* $\Lambda_w C \Lambda_w$*. This identification scheme is order- and scale-invariant.*

Note $A_*'\Sigma A_* = I_n$ by construction, since $(\Lambda_w C\Lambda_w)^{-1/2}$ is the symmetric inverse square-root of $\Lambda_w C\Lambda_w$.

The weighted maximum correlation criterion allows the researcher to tilt identification toward more reliable or the most policy-relevant shocks by assigning variable-specific weights $w_i > 0$ in the objective $\tilde{\rho}(A) = \sum_i w_i \operatorname{corr}(u_i, \varepsilon_i)$. In practice, one may downweight dimensions believed to suffer greater measurement error (e.g., set w_i proportional to a reliability metric such as $1/\sigma_i$, where $\sigma_i^2 = \mathrm{var}(\varepsilon_i)$), or upweight variables whose shocks are of primary interest. The resulting solution is order- and scale-invariant but concentrates correlation where the signal is stronger or where the research question warrants greater emphasis.

2.1.1 Equal Weights

For the special case with equal weights we can,[2] without loss of generality, set $w_1 = \cdots = w_n = 1/n$ and define $\bar{\rho}(A) \equiv \frac{1}{n}\sum_{i=1}^{n} \operatorname{corr}(u_i, \varepsilon_i)$.

Corollary 1 (OASIS)**.** *Suppose that* $\det(\Sigma) > 0$. *Then*

$$\bar{\rho}(A_*) = \frac{1}{n}\sum_{i=1}^{n} \lambda_i^{1/2} \geq \bar{\rho}(A) \qquad \textit{for all } A \in \mathcal{A},$$

[1]The symmetric square root is given from the eigendecomposition. Note that $(\Lambda_w C\Lambda_w)^{-1/2} \neq \Lambda_w^{-1/2}C^{-1/2}\Lambda_w^{-1/2}$, unless Λ_w and C commute.

[2]This is the problem considered in Hansen and Tong (2024, 2025), where a vector of correlated standardized returns, $z = Bu$, is represented as a linear transformation of a vector of uncorrelated standardized variables, u. For interpretability in these models, it is desirable that each component z_i be highly correlated with its corresponding u_i, and the symmetric square root of the correlation matrix, $B = C^{1/2}$, solves this problem.

where $A_* = \Lambda_\sigma^{-1} C^{-1/2}$ is the unique solution to $\max_{A \in \mathcal{A}} \bar{\rho}(A)$. So, $u^* = A_*' \varepsilon$ attains the maximum average correlation, $\frac{1}{n} \sum_{i=1}^n \mathrm{corr}(u_i^*, \varepsilon_i) = \frac{1}{n} \sum_{i=1}^n \lambda_i^{1/2}$.

Unlike the eigendecomposition of Σ, which is not scale-invariant, OASIS uses the eigendecomposition of the correlation matrix $C = \Lambda_\sigma^{-1} \Sigma \Lambda_\sigma^{-1}$, yielding $A^* = \Lambda_\sigma^{-1} C^{-1/2}$ that is invariant to both scale and order.

A key property, which we used in the proofs, is that any $A \in \mathcal{A}$ can be expressed as $A = A_* R$, where R is orthonormal (a rotation matrix). That R is orthonormal follows by $I_n = A' \Sigma A = R' A_*' \Sigma A_* R = R' R$, and we have

$$R = A_*^{-1} A = (A^{-1} A_*)', \tag{2}$$

where the last identity follows by $A = A_* R \Leftrightarrow AR' = A_* R R' \Leftrightarrow AR' = A^{-1} A_*$. Moreover, for two identification schemes, A_1 and A_2 say, the rotation matrix, $R_{1,2} = A_1^{-1} A_2$, characterizes how $u_1 = A_1' \varepsilon$ can be rotated into $u_2 = A_2' \varepsilon = R' A_1' \varepsilon = R' u_1$. This helps explain differences in impulse response functions for different identification schemes. We will make use of this in our empirical analysis in Section 4.

2.2 OASIS for Proxy VARs

Proxy VARs rely on narrative/external measures to identify structural shocks, and the maximum correlation criterion is well suited for this framework. Let $z \in \mathbb{R}^r$ be (narrative/external) instrumental variables, where we suppress the dependence on time. As in an SVAR, the structural shocks are given by $A' \varepsilon \in \mathbb{R}^n$, but these are partitioned into $(u', v')' = A' \varepsilon$, where the elements of $u = \boldsymbol{a}' \varepsilon \in \mathbb{R}^r$ are those identified from the elements in z, and v is a vector of auxiliary shocks. Here $\boldsymbol{a}$ is the choice parameter that plays the same role as A did earlier, and the objective is to maximize

$$g(\boldsymbol{a}) = \sum_{j=1}^r w_j \,\mathrm{corr}(u_j, z_j), \qquad \text{for} \quad \boldsymbol{a} \in \mathcal{A}_r = \{\boldsymbol{a} \in \mathbb{R}^{n \times r} : \boldsymbol{a}' \Sigma \boldsymbol{a} = I_r\}.$$

The implicit identifying assumption is that z_j is correlated with the j-th structural shock, u_j, $j = 1, \ldots, r$.[3] The objective is therefore to determine a vector of uncorrelated structural shocks, u, where each element is highly correlated with the corresponding element of z.

For this problem, we introduce $C_{\varepsilon\varepsilon} = \mathrm{corr}(\varepsilon)$, $C_{\varepsilon z} = \mathrm{corr}(\varepsilon, z)$, and $\Lambda_w = \mathrm{diag}(w_1, \ldots, w_r)$, and a key quantity is

$$\Xi = C_{\varepsilon\varepsilon}^{-1/2} C_{\varepsilon z} \Lambda_w \in \mathbb{R}^{n \times r},$$

[3]Some require z_i to be uncorrelated with v, but this is generally incompatible with $\mathrm{var}(u) = I_r$.

and its singular value decomposition (SVD), $\Xi = U\Lambda_\xi V'$, for which $U'U = V'V = I_r$ and $\Lambda_\xi = \mathrm{diag}(\xi_1, \ldots, \xi_r)$.

Theorem 2. *Suppose* $\det(\Sigma) > 0$. *Then*

$$g(\boldsymbol{a}) \leq g(\boldsymbol{a}_*) = \sum_{i=1}^{r} \xi_i, \quad \forall \boldsymbol{a} \in \mathcal{A}_r,$$

where $\boldsymbol{a}_* \equiv \Lambda_\sigma^{-1} C_{\varepsilon\varepsilon}^{-1/2} UV'$. *If* $C_{\varepsilon z} \in \mathbb{R}^{n\times r}$ *has full column rank* r*, then* $\boldsymbol{a}_*$ *is the unique maximum of* g *over* $\mathcal{A}_r$.

Interestingly, in the special case with equal weights, $\Lambda_w = I_r$, the singular values $\xi_1, \ldots, \xi_r$ of $\Xi = C_{\varepsilon\varepsilon}^{-1/2} C_{\varepsilon z}$ measure how informative the external instruments z are for the corresponding structural shocks. Each $\xi_i \in [0, 1]$ measures the maximal attainable correlation between a normalized linear combination of reduced-form innovations and the aligned instrument; $\xi_i = 0$ indicates no identifying content for that shock, whereas ξ_i near one indicates a very strong instrument. Because the optimal value satisfies $g(\boldsymbol{a}_*) = \sum_{i=1}^{r} \xi_i$, weak identification for any component (small ξ_i) directly lowers the objective and indicates fragility in that dimension. In empirical applications, it would be informative to report the ξ_i and/or their squares as canonical correlations, along with robust confidence intervals for these.

The result in Theorem 2 for proxy VARs can be adapted to impulse response function (IRF) matching. IRF matching is used to evaluate and estimate DSGE models by aligning their impulse responses with those derived from SVARs or local projections. The focus is typically on a smaller number of shocks, r, than the dimension of the system, n. For example, Basu and Bundick (2017) simulates data for seven ($n = 7$) observables using a DSGE model with $r = 3$ structural shocks. In this context, we can apply OASIS to the subset of shocks that are used to identify the model, $\tilde{\varepsilon}$ say, for which $\mathrm{var}(\tilde{\varepsilon})$ is nonsingular. Then, using Theorem 2 with $z = \tilde{\varepsilon}$, we can extract the r elements of ε.

2.3 Applications to SVARs and Local Projections

Let $\{X_t\}$ be a time series and let $\mathcal{F}_t$ be a filtration to which X_t is adapted. We are interested in how a shock to an element of X_t propagates to future values of X_t. In SVARs and LPs, the reduced-form shocks are typically defined by a linear projection:

$$\varepsilon_t = X_t - \Gamma Z_{t-1},$$

where $Z_t \in \mathcal{F}_t$. In a vector autoregression of order p, $\Phi(L)X_t = \mu + \varepsilon_t$, we have $\Gamma = (\mu, \Phi_1, \ldots, \Phi_p)$ and $Z_{t-1} = (1, X_{t-1}, \ldots, X_{t-p})'$. More generally, Z_{t-1} can include lagged values of other variables.

The impulse response function is defined by:

$$\text{IRF}(h) = \text{cov}(X_{t+h}, u_t),$$

where the (i, j)-th element of $\text{IRF}(h)$ measures the linear impact of $u_{j,t}$ on $X_{i,t+h}$.

In an SVAR, we have $\varepsilon_t = Bu_t$ with $B' = A^{-1}$, and if the underlying VAR is invertible, then $\text{IRF}(h) = \Psi_h B$, where Ψ_h is the h-th coefficient matrix in the moving-average (MA) representation:

$$X_t = \Phi(1)\mu + \sum_{h=0}^{\infty} \Psi_h \varepsilon_{t-h}, \tag{3}$$

because $\Psi_h \varepsilon_{t-h} = \Psi_h (A')^{-1} A' \varepsilon_{t-h} = \Psi_h B u_{t-h}$.

A local projection does not recover the IRF by deducing the MA coefficients from the VAR. Instead, the IRF is obtained from regressions, such as $X_{t+h} = \mu_h + \Theta_h \varepsilon_t + e_{t,t+h}$ for $h = 0, 1, 2, \ldots$, and the IRF (in matrix form) is given by $\text{IRF}(h) = \Theta_h A^{-1\prime} = \Theta_h B$. Alternatively, we can regress X_{t+h} on $u_t = A' \varepsilon_t$ and a constant, in which case the IRF is simply the coefficient matrix on u_t.

It is simple to compute the IRF for one identification scheme from those of another identification scheme. The relation between any IRF and that of OASIS is the following.

Proposition 2. *Let* $\text{IRF}(h)$ *denote the IRF resulting from identification with* $A \in \mathcal{A}$*. Then*

$$\text{IRF}(h) = \text{IRF}^*(h) R',$$

where $\text{IRF}^*(h)$ *is the IRF for* A^* *(OASIS) and* $R = A_*^{-1} A$ *is a rotation (orthonormal) matrix.*

Discrepancies in the IRFs can be attributed to the rotation matrix, R, that shows how structural shocks for one identification scheme can be expressed as a rotation (orthonormal linear combination) of the structural shocks of the other identification scheme, which may explain differences between IRFs across identification schemes. It is worth noting that two IRFs can be different even if their underlying structural shocks are highly correlated with each other and with the reduced-form shocks.

2.4 OASIS: Twice the Proximity to Perfect Correlation

Early in our empirical analysis, we noticed that

$$(1 - \bar{\rho}_{\text{c}}) \approx 2(1 - \bar{\rho}_*),$$

where $\bar{\rho}_{\rm c} \equiv \bar{\rho}(A^{\rm c})$ and $\bar{\rho}_* \equiv \bar{\rho}(A^*)$. The empirical ratio $(1-\bar{\rho}_{\rm c})/(1-\bar{\rho}_*)$ ranged from 1.83 to 2.21 across all studies, such that the average correlation under OASIS is about half as far from unity as that achieved by Cholesky. This is no coincidence, as the following Theorem shows.

Theorem 3. *Let* $d(C) = \frac{1}{n}\sum_{i\neq j} C_{ij}^2 = \frac{1}{n}\left\|C - I_n\right\|_F^2$. *Then*

$$\bar{\rho}_* = 1 - \tfrac{1}{8}d(C) + O(\mathrm{tr}\{E^3\}), \tag{4}$$

$$\bar{\rho}_{\rm c} = 1 - \tfrac{1}{4}d(C) + O(\mathrm{tr}\{E^3\}), \tag{5}$$

and $(1-\bar{\rho}_{\rm c})/(1-\bar{\rho}_*) = 2(1 + O(\|E\|_F)) = 2\left(1 + O(\sqrt{d(C)})\right)$ *where* $E = C - I_n$.

For the special case where C is an equicorrelation matrix, $C_{ij} = \rho$ for all $i \neq j$, we have $d(C) = \frac{1}{n}\sum_{i\neq j}\rho^2 = (n-1)\rho^2$. Thus, we should expect the average correlation between structural shocks and reduced-form shocks to decrease with the dimension, n. The exact result for the equicorrelation case is:

Corollary 2. *Suppose that* C *is an equicorrelation matrix, such that* $C_{ij} = \rho \in (-\frac{1}{n-1}, 1)$ *for all* $i \neq j$. *Then*

$$\bar{\rho}_* = \tfrac{1}{n}\sqrt{1 + (n-1)\rho} + \sqrt{1-\rho}\left(1 - \tfrac{1}{n}\right), \tag{6}$$

$$\bar{\rho}_{\rm c} = \tfrac{1}{n}\sum_{k=1}^{n}\sqrt{1 - \tfrac{(k-1)\rho^2}{(k-2)\rho+1}}. \tag{7}$$

Theorem 3 shows that OASIS (relative to Cholesky) reduces the distance between structural shocks and reduced-form shocks by a factor of two. This adds another testable implication that we will explore in the empirical section.

Note that $d(C)$ is $(n-1)$ times the average squared correlation coefficient, $\frac{1}{n(n-1)}\sum_{i\neq j} C_{ij}^2$, from which it follows that $d(C) = \frac{1}{n}\left\|C - I_n\right\|_F^2$, where $\|\cdot\|_F$ is the Frobenius norm.

Another significant consequence of Theorem 3 concerns the role of variable ordering in Cholesky identification.

Corollary 3. *The average correlation between structural shocks and reduced-form shocks, in a Cholesky decomposition, satisfies* $\bar{\rho}_{\rm c} = 1 - d(C)/4 + O(\mathrm{tr}\{E^3\})$, *regardless of the chosen ordering of the variables.*

Thus, the choice of variable ordering in Cholesky identification has little effect on the average correlation between structural and reduced-form shocks, especially when the reduced-form shocks are only moderately or weakly correlated. Low correlation between reduced-form shocks is found in nearly all of the empirical studies we revisit. This result can, in part, explain why many Cholesky-based empirical findings are reported to be robust to the ordering of variables.

3 Revisiting Empirical SVAR Studies

We conduct an extensive review of existing studies that employ a Cholesky decomposition in VARs to identify structural shocks. The studies are listed in Table 1, organized by topic (Monetary, Fiscal, Uncertainty, Financial, Oil, and Sectoral). We include a brief summary of these articles that predominantly use impact timing restrictions to reveal impulse responses.

3.1 OASIS and Cholesky Results

Table 2 provides detailed information about each of the empirical studies. We report the values of $\bar{\rho}_*$ and $\bar{\rho}_{\mathrm{c}}$ for each of the studies, where we have used the same ordering of variables as in the original articles. We also report the range of $\bar{\rho}_{\mathrm{c}}$ over all Cholesky orderings and descriptive statistics, such as the dimension of the VAR, n, the correlation between the two types of structural shocks (u^* and u), $\bar{\rho}_{*,\mathrm{c}}$, the average absolute correlation between reduced-form shocks, $\|C\|_1^\star = \frac{1}{n(n-1)}\sum_{i\neq j}|C_{ij}|$, the ratio, $\frac{1-\bar{\rho}_{\mathrm{c}}}{1-\bar{\rho}_*}$, (which should be about two according to Theorem 3), and $d(C) = \frac{1}{n}\sum_{i\neq j} C_{ij}^2$.

Many studies have a correlation matrix that is relatively close to the identity matrix. For instance, those of Bjørnland and Leitemo (2009) and Lorusso and Pieroni (2018) have an average absolute correlation of $\|C\|_1^* < 0.10$. This explains that any Cholesky decomposition in these studies will result in a nearly perfect correlation, $\bar{\rho} > 0.99$, between Cholesky-based structural shocks and reduced-form shocks. It is interesting that the correlations between the reduced-form shocks tend to be small. This is not a feature of least-squares estimation of vector autoregressions (VARs), because separate equation-by-equation estimation produces identical estimates and residuals. Rather, it suggests that reduced-form shocks in these models tend to be weakly correlated. This is helpful for identifying structural shocks because the identifying assumptions are close to being satisfied by the reduced-form shocks. We observe that $d(C)$ is particularly small in the studies of oil price shocks, which could stem from the exogeneity of oil price movements.

As predicted by Theorem 3, studies with large $d(C)$, such as Christiano et al. (2005), Blanchard and Perotti (2002), and Rossi and Zubairy (2011), have the largest differences between OASIS and Cholesky.

Table 1: Selected Structural VAR Studies with Causal-Ordering Restrictions by Topic

Topic/[Abbreviation]/Reference/Notes	
	Monetary Shocks
[B86]	Bernanke (1986). VAR using monetary aggregates and interest rates as policy indicators.
[S95]	Strongin (1995). Identifies monetary shocks via Fed's component of nonborrowed reserves.
[LSZ96]	Leeper et al. (1996). SVAR analyzing monetary policy; emphasizes sensitivity to identification assumptions and robustness to ordering.
[CEE99]	Christiano et al. (1999). SVAR using federal funds rate as monetary policy instrument.
[CEE05]	Christiano et al. (2005). SVAR using interest rate shocks and impulse response matching to validate DSGE models.
[BL09]	Bjørnland and Leitemo (2009). SVAR identifying monetary shocks via interest rates and stock prices.
	Fiscal Shocks
[BP02]	Blanchard and Perotti (2002). SVAR using government spending and net taxes with timing restrictions.
[RZ11]	Rossi and Zubairy (2011). SVAR using government spending, tax revenue, and interest rates.
[FG16]	Forni and Gambetti (2016). Uses forecast revisions (SPF) and government spending as shock proxy.
	Uncertainty Shocks
[B09]	Bloom (2009). SVAR using stock market volatility (VXO) as proxy for uncertainty shocks.
[CCG14]	Caggiano et al. (2014). VAR using VIX and forecast dispersion to measure uncertainty.
[BB17]	Basu and Bundick (2017). SVAR identified via stock market volatility (VXO).
[BO23]	Bonciani and Oh (2023). VAR with uncertainty shocks proxied by macro uncertainty measures estimated by Jurado et al. (2015); compared to DSGE.
	Financial Shocks
[GZ12]	Gilchrist and Zakrajšek (2012). VAR using credit spreads and bond premia as financial shock indicators.
[FGMS24]	Forni et al. (2024). VAR with nonlinear financial shock identification using a Vector Moving Average (VMA) model.
	Oil Price Shocks
[LS04]	Leduc and Sill (2004). SVAR using real oil price as shock variable.
[LP18]	Lorusso and Pieroni (2018). SVAR using global oil prices to assess UK macro responses.
	Sectoral
[FGKV25]	Fry-Mckibbin et al. (2025). SVAR for Australia, controlling for import penetration from China, in the sectoral output context.

Notes: Empirical studies that employed VARs or SVARs to identify and trace the effects of macroeconomic shocks. Aside from the first two studies, the results were reported to be robust to Cholesky reordering.

Table 2: OASIS and Cholesky Identification in SVARs

Study	n	OASIS $\bar{\rho}_*$	$\bar{\rho}_{\rm c}$	Cholesky min	max	$\bar{\rho}_{*,\rm c}$	$\lVert C \rVert_1^\star$	$\frac{1-\bar{\rho}_{\rm c}}{1-\bar{\rho}_*}$	$d(C)$
[B86]	6	0.965	0.935	0.933	0.936	0.970	0.17	1.88	0.29
—	6	0.962	0.930	0.928	0.931	0.968	0.18	1.87	0.32
[S95]	5	0.988	0.977	0.976	0.977	0.988	0.13	1.99	0.09
[LSZ96]	4	0.987	0.975	0.975	0.975	0.987	0.14	1.97	0.10
[CEE99]	7	0.967	0.933	0.932	0.936	0.963	0.13	2.02	0.24
[CEE05]	9	0.925	0.860	0.857	0.869	0.935	0.24	1.88	0.70
[BL09]	5	0.995	0.991	0.991	0.991	0.996	0.09	1.95	0.04
[BP02]	3	0.982	0.966	0.965	0.966	0.983	0.23	1.95	0.14
—	7	0.903	0.785	0.757	0.827	0.831	0.26	2.22	0.63
[RZ11]	8	0.915	0.834	0.815	0.849	0.908	0.22	1.95	0.61
[FG16]	7	0.969	0.935	0.934	0.939	0.963	0.15	2.09	0.22
—	8	0.968	0.935	0.934	0.939	0.965	0.14	2.04	0.24
[B09]	8	0.965	0.936	0.932	0.936	0.970	0.12	1.83	0.28
[CCG14]	4	0.988	0.976	0.975	0.976	0.988	0.13	1.98	0.09
[BB17]	8	0.935	0.871	0.869	0.881	0.933	0.22	1.99	0.53
[BO23]	9	0.945	0.897	0.896	0.904	0.952	0.22	1.88	0.54
[GZ12]	8	0.944	0.895	0.894	0.897	0.951	0.23	1.87	0.51
[FGMS24]	6	0.976	0.954	0.954	0.955	0.978	0.13	1.92	0.20
[LS04]	5	0.987	0.975	0.975	0.975	0.988	0.12	1.95	0.10
[LP18]	3	0.999	0.998	0.998	0.998	0.999	0.06	1.99	0.01
—	3	0.997	0.994	0.994	0.994	0.997	0.08	1.99	0.02
[FGKV25]	14	0.959	0.920	0.916	0.922	0.958	0.12	1.98	0.32

Notes: Abbreviations for each study appear in the first column. n is the dimension of the VAR, $\bar{\rho}_*$ is the average correlation between structural shocks and reduced-form shocks for OASIS. The corresponding correlation for Cholesky is denoted $\bar{\rho}_{\rm c}$ and min and max give the range of correlations for all variable permutations. $\bar{\rho}_{*,\rm c}$ is the average correlation between structural shocks identified by OASIS and Cholesky. $\lVert C \rVert_1^\star$ is the average absolute correlation in C.

Figure 1 is an illustration of some of the results in Table 2. We plot the values of $\bar{\rho}_*$ and $\bar{\rho}_c$ for each of the studies and use bars to indicate the range of Cholesky outcomes. It

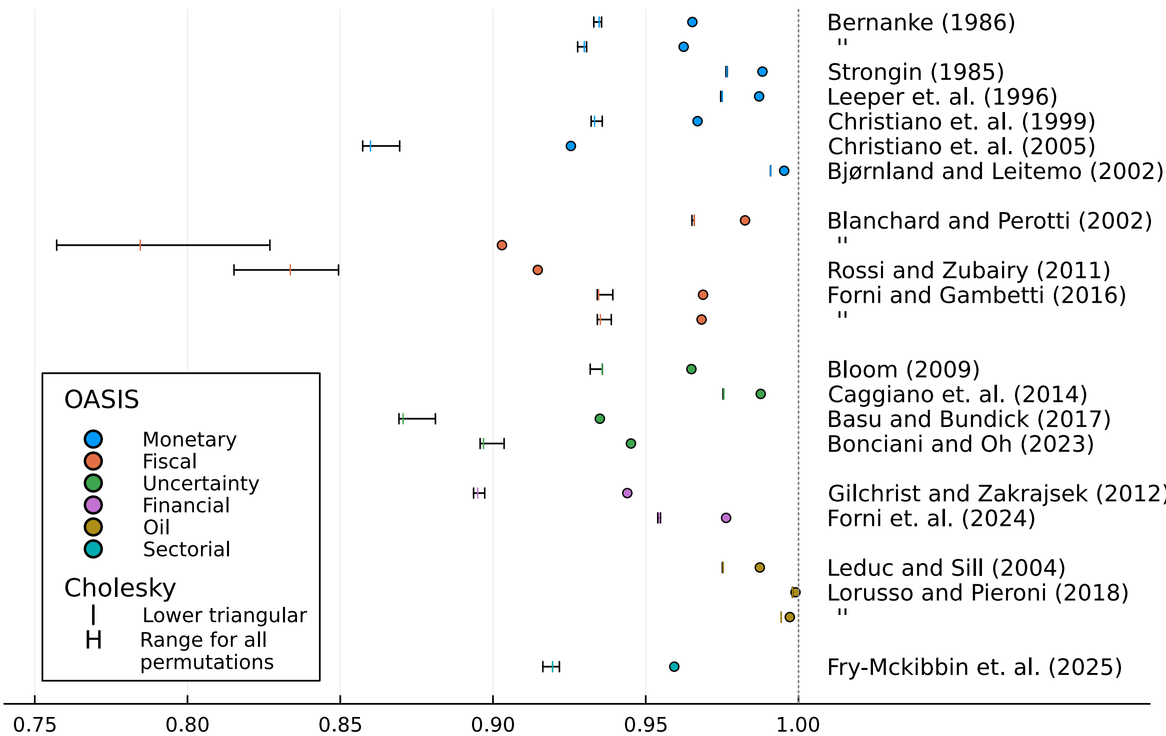

Figure 1: Selected studies. OASIS is shown for all; some Cholesky values lie below the range shown. Correlations are remarkably high, with a narrow range across Cholesky orderings.

is striking how large the correlations between structural shocks and reduced-form shocks are across all studies. All OASIS correlations are above 90% and 15 of the 21 studies have OASIS correlations above 95%. The same fifteen studies have Cholesky correlations above 90%. The range of Cholesky correlations, by considering all possible permutations of the n variables, is also fairly narrow, and for the nine studies where the Cholesky correlation is above 95%, all permutations have nearly identical average correlations, as predicted by Theorem 3. Only two studies, the second study with $n = 7$ in Blanchard and Perotti (2002) and Rossi and Zubairy (2011), have somewhat modest values of $\bar{\rho}_c$, and these are also the only two studies where the choice of Cholesky ordering can affect the average correlation to some extent. This does not reflect a weakness of these studies but is simply a consequence of the covariance structure of the variables under investigation. Even the largest VAR with $n = 14$ has a relatively narrow range, from 91.63% to 92.17%, for the average correlation between structural shocks and reduced-form shocks. This is remarkable because there are more than 87 billion different Cholesky orderings to consider, yet all of them have nearly the same value of $\bar{\rho}_c$.

Despite the average correlation being very similar for all Cholesky orderings, the resulting impulse responses need not be similar. An IRF for one identification scheme will be a mixture of several IRFs for a different identification scheme, where the linear combination is defined by the rotation matrix, R, that defines how structural shocks of one kind relate to structural shocks defined in a different way. Even a small rotation can lead to different conclusions about the impact of structural shocks.

3.2 Proximity to Perfect Correlation

The scatterplot in Figure 2 maps the proximity to perfect correlation, $-\log(1-\bar{\rho})$, against the residual dependence in the reduced-form shocks, $\log d(C)$, for OASIS and Cholesky across all empirical applications. The dashed reference lines are the relationships predicted by (4) and (5) of Theorem 3: $\bar{\rho}_* = 1 - \frac{1}{8}d(C) + O(\mathrm{tr}\{E^3\})$ and $\bar{\rho}_c = 1 - \frac{1}{4}d(C) + O(\mathrm{tr}\{E^3\})$, respectively, with $E = C - I_n$. Abstracting from the $O(\mathrm{tr}\{E^3\})$ terms, these become straight lines in a $(\log d(C), -\log(1-\bar{\rho}))$ plot with slope -1 and vertical intercepts $\log 8$ for OASIS and $\log 4$ for Cholesky.

Empirically, the points fall tightly around these lines: OASIS observations align near

$$-\log(1-\bar{\rho}) \approx -\log d(C) + \log 8,$$

while Cholesky observations align near

$$-\log(1-\bar{\rho}) \approx -\log d(C) + \log 4.$$

This visualization makes the "factor-of-two" result immediate: the vertical separation $\log 2$ is equivalent to $(1-\bar{\rho}_c) \approx 2(1-\bar{\rho}_*)$ in levels.

The interpretation is straightforward. The horizontal axis ($d(C) = \frac{1}{n}\|C - I\|_F^2$) measures how far the residuals are from being uncorrelated, such that a larger $d(C)$ means stronger contemporaneous comovement in reduced-form shocks. The vertical axis measures how close the identified structural shocks are to their corresponding reduced-form innovations. The nearly linear log–log relationship, with common slope -1, shows that both schemes degrade at the same *rate* as residual dependence rises, but OASIS is uniformly closer to perfect correlation as implied by theory.

Deviations from the dashed lines are modest and attributable to the higher-order remainder terms $O(\mathrm{tr}\{E^3\})$ in the expansions. Importantly, the choice of variable ordering influences this term, but the first-order expression $\bar{\rho}_c = 1 - \frac{1}{4}d(C) + O(\mathrm{tr}\{E^3\})$ holds regardless, and any choice of variable ordering will land near the same location in this figure. We

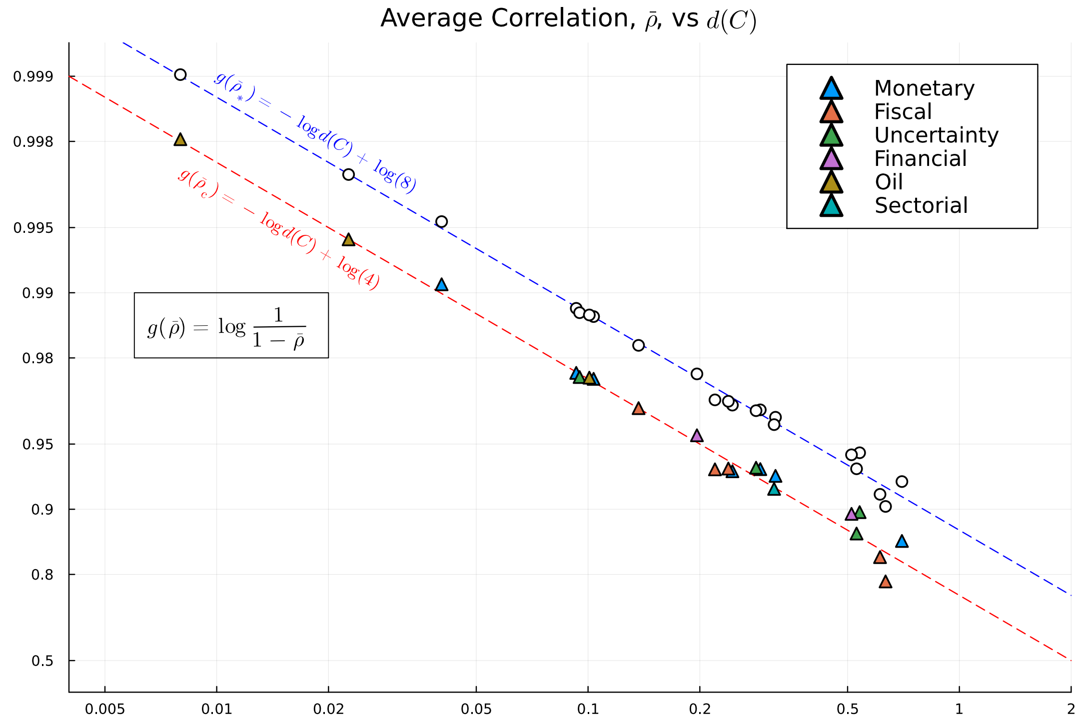

Figure 2: Scatterplot of $-\log(1-\bar{\rho})$ against $\log d(C)$. The values shown along the axes are $\bar{\rho}$ and $d(C)$. OASIS points lie near the line $y = -x + \log 8$, whereas Cholesky points lie near $y = -x + \log 4$.

observed that the values of $\bar{\rho}_c$ do not systematically lie above or below the dashed line for Cholesky, so it does not appear that the orderings in the empirical literature were selected with this in mind. By construction, OASIS is order-invariant, so each study contributes a single OASIS point, with no additional variation arising from the choice of variable ordering.

4 Detailed Result for a Monetary Policy VAR

In this section, we pursue a more detailed analysis of the SVAR used in Leeper et al. (1996) to study monetary shocks. We estimate a VAR(4) using quarterly U.S. data for the sample period 1959:Q1-2018:Q4. Leeper et al. (1996) used monthly data, which required them to interpolate quarterly GDP data to a monthly frequency.[4] The four variables are real GDP, the GDP deflator (DEF), the federal funds rate (FFR), and the money stock (M2). All variables are in log differences except the FFR, which is in levels.

The ordering of variables is (GDP, DEF, FFR, M2), which is the same as in Leeper et al. (1996). We estimate the model with OASIS and the conventional lower-triangular Cholesky decomposition, as well as the upper-triangular Cholesky decomposition, which is identical to

[4]In the appendix, we also estimate the SVAR with interpolated monthly data, along with additional robustness checks.

reversing the ordering of the variables. Results for an alternative ordering and for a different price index are reported in the appendix.

The three identification schemes, OASIS and lower/upper Cholesky, provide three sets of IRFs. Here we focus on the IRFs for a monetary shock.

Table 3: OASIS and Cholesky applied to Leeper, Sims, and Zha (1996)

	OASIS	Cholesky				
		Lower	Upper	min $\bar{\rho}_{\mathrm{c}}$	max $\bar{\rho}_{\mathrm{c}}$	
	$\mathrm{corr}(\varepsilon, u^*)$	$\mathrm{corr}(\varepsilon, u^{\mathrm{c}})$		Variable ordering		$\mathrm{corr}(u^*, u^{\mathrm{c}})$
GDP	0.9995	1.0000	0.9974	3	1	0.9995
DEF	0.9908	0.9999	0.9652	4	2	0.9908
FFR	0.9751	0.9641	0.9356	2	4	0.9752
M2	0.9831	0.9343	1.0000	1	3	0.9836
$\bar{\rho}$	0.9871	0.9745	0.9745	0.9745	0.9750	0.9879

Table 3 provides an example of how OASIS and Cholesky distribute the correlations across pairs of reduced-form shocks and structural shocks. The correlations are generally large. The individual correlations for OASIS are all above 97.5% with an average of 98.71%, whereas those for Cholesky range from 93.43% to 100% with an average value of 97.45%. Table 3 also reports the largest and smallest average correlation with Cholesky, and the corresponding variable orderings. There are twenty-four different ways to order the variables in this system. As predicted by Corollary 3, the average correlation is very similar across all variable orderings. In this application $\bar{\rho}_{\mathrm{c}}$ is within the narrow band between 97.45% and 97.50% for all Cholesky orderings. Note that the highest average Cholesky correlation is obtained by switching the order of the last two variables, FFR and M2. Also observe that the correlations for the individual pairs $\mathrm{corr}(\varepsilon_j, u_j^{\mathrm{c}})$ vary substantially with the chosen ordering of the variables.

To see how structural shocks from one identification scheme relate to those of a different identification scheme, we can compute the rotation matrix R defined in Proposition 2. The relationship between the Cholesky structural shocks, u^c, and the OASIS structural shocks, u^*, is given by $u^c = R'u^*$ and $u^* = Ru^c$, where $R = A_*^{-1}A$, and in this empirical application we have

$$R = \begin{bmatrix} 0.9995 & -0.0099 & -0.0227 & -0.0219 \\ 0.0071 & 0.9908 & -0.1341 & 0.0160 \\ 0.0273 & 0.1289 & 0.9752 & 0.1780 \\ 0.0172 & -0.0397 & -0.1748 & 0.9836 \end{bmatrix}$$

The monetary shock is the third variable in u^c and u^*, respectively, and the third row of the matrix R tells us how the monetary shock identified by OASIS is "relabeled" as different structural shocks when identified by Cholesky. For instance, the Cholesky structural shock to DEF is, in part, made up of the monetary shock as identified by OASIS, because $u_2^c = 0.1289u_3^* + \cdots$. Similarly, the monetary shock identified by Cholesky,

$$u_3^c = -0.0227u_1^* - 0.1341u_2^* + 0.9752u_3^* - 0.1748u_4^*,$$

has large components $-0.1341\,u_2^*$ and $-0.1748\,u_4^*$ that are interpreted as negative shocks to DEF and M2, respectively, by OASIS. This sheds light on discrepancies observed in the IRFs obtained with different identification schemes. A structural shock, identified with one identification scheme, is a convolution of multiple shocks identified with a different scheme. The exact relations between the two types of shocks are given from the R-matrix.

Cholesky's lower average correlation, $\bar{\rho}_c < \bar{\rho}_*$, reflects a systematic down-scaling of the eigenvalue contributions. From the proof of Corollary 1, the OASIS term is $\rho(A_*) = \sum_{i=1}^n \lambda_i^{1/2}$, whereas under Cholesky it becomes $\rho(A_c) = \sum_{i=1}^n M_{ii}\,\lambda_i^{1/2}$, with $\lambda_1, \ldots, \lambda_n$ the eigenvalues of C and $M = Q'R'Q$, where $C = Q\Lambda_\lambda Q'$. Since M is orthonormal, $|M_{ii}| \leq 1$ for all i, so the diagonal weights down-scale the eigenvalue square roots, yielding $\rho(A_c) \leq \rho(A_*)$. In this application we have

$$M = Q'R'Q = \begin{bmatrix} 0.9870 & -0.136 & 0.065\,30 & 0.0555 \\ 0.1256 & 0.9804 & 0.0153 & 0.1510 \\ -0.0625 & 0.0013 & 0.9964 & -0.0574 \\ -0.0785 & -0.1425 & 0.0520 & 0.9853 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} \sqrt{\lambda_1} \\ \sqrt{\lambda_2} \\ \sqrt{\lambda_3} \\ \sqrt{\lambda_4} \end{bmatrix} = \begin{bmatrix} 0.774 \\ 0.947 \\ 1.006 \\ 1.222 \end{bmatrix}.$$

The diagonal elements of M show how much the square roots of the eigenvalues of C get scaled down by Cholesky, which is the reason Cholesky has a smaller average correlation.

4.1 Empirical VARs with more reduced-form correlation

All the studies in Table 1 have reduced-form innovations with relatively low cross-correlations, as indicated by the small $\|C\|_1^\star$ and $d(C) = \frac{1}{n}\|C - I\|_F^2$ reported in Table 2. When $d(C)$ is small, OASIS and Cholesky both deliver structural shocks that align closely with their labeled reduced-form innovations.

To explore cases where the methods diverge more, we included three VARs with higher residual correlation: two drawn from existing work, and a third (a term-structure VAR) we constructed.

Table 4: Empirical VARs with More Correlations in Residuals

Topic/[Abbreviation]/Reference/Notes	
VARs with Correlated Residuals	
[EI95]	Engle and Issler (1995) VECM testing long-run relationship of U.S. sectoral output data
[DP05]	Dai and Philippon (2005) VAR identifying fiscal shocks via structural budget components; focuses on term structure.
[FHT25]	(This paper) A term structure VAR, see description in Section 4.1 for details.

Notes: See notes for Table 1

The first of these studies is Engle and Issler (1995). They did not use Cholesky, but did analyze productivity shocks, and their VAR has correlated residuals that, in part, are driven by input-output linkages across sectors. This VAR is therefore well suited for a comparison of OASIS and Cholesky in a setting with high residual correlations. Engle and Issler (1995) build on the Real Business Cycle model of Long and Plosser (1983) and show that cointegration among sectoral outputs implies cointegration among underlying productivity shocks, linking long-run comovement to technological fundamentals. Using a VECM and U.S. sectoral per-capita output data (1947–1989), they identify two cointegrating (long-run) vectors and six cofeature (shorter-run) vectors.

The second study is Dai and Philippon (2005) – an unpublished NBER working paper – that estimates an SVAR with quarterly data over the sample period 1970:1 to 2003:3. Their system has $n = 12$ variables (8 bonds and 4 observable macro variables), embedded within a no-arbitrage affine term structure model to identify the dynamic effects of fiscal policy shocks on interest rates. Using the Blanchard and Perotti (2002) identification scheme, the authors isolate structural shocks to deficits and decompose their impact on long-term yields into changes in expected short rates and risk premia. The authors find that fiscal deficits raise long-term interest rates significantly over time.

The third study is a term structure VAR, with seven yields spanning a wide range of maturities, from the overnight federal funds rate to the 10-year Treasury Bill (T-Bill) rate, as well as intermediate T-Bill rates at the 3-month, 6-month, 1-year, 3-year, and 5-year horizons. We estimate a VAR(12) using monthly data from September 1981 to January 2025.

The three VARs estimated in Table 5 have residuals with substantially more correlations than those in Table 2. This can be seen from the higher values of $\|C\|_1^\star$ and $d(C)$. Specifically, the highest values in Table 2 are $\|C\|_1^\star = 0.26$ and $d(C) = 0.70$, whereas the lowest values in Table 5 are $\|C\|_1^\star = 0.47$ and $d(C) = 2.00$. Consequently, the average correlations, $\bar{\rho}_*$ and $\bar{\rho}_{\mathrm{c}}$, are smaller and the difference between the two is larger, as predicted by Theorem 3. The

Table 5: OASIS and Cholesky in Highly Correlated VARs

Study	n	OASIS $\bar{\rho}_*$	Cholesky $\bar{\rho}_{\mathrm{c}}$	min	max	$\bar{\rho}_{*,\mathrm{c}}$	$\|C\|_1^\star$	$\frac{1-\bar{\rho}_{\mathrm{c}}}{1-\bar{\rho}_*}$	$d(C)$
[EI95]	8	0.821	0.704	0.695	0.718	0.867	0.47	1.66	2.00
[DP05]	8	0.621	0.345	0.301	0.407	0.477	0.64	1.73	3.29
[FHT25]	7	0.684	0.461	0.456	0.516	0.635	0.69	1.71	3.12

Notes: See notes for Table 2.

ratios, $\frac{1-\bar{\rho}_{\mathrm{c}}}{1-\bar{\rho}_*}$, deviate further from 2 because the third order term, $O(\mathrm{tr}\{(C-I)^3\})$, is larger for these three VARs.

5 Summary

In this paper, we have proposed a different perspective on identification in dynamic macroeconomic models. Instead of identification via restrictions, identification is achieved by solving an optimization problem. The maximum-correlation criterion maximizes the average correlation between structural shocks and the corresponding reduced-form innovations, and it leads to an order- and scale-invariant identification scheme (OASIS).

Reinterpreting the zero restrictions in a Cholesky decomposition as first-order conditions to an optimization problem shows that the maximum-correlation objective is closely related to the implicit optimization problem solved by Cholesky identification. It follows that a key characteristic of Cholesky identification is that it will generate a relatively large average correlation between structural shocks and the corresponding reduced-form shocks, regardless of the assumed causal ordering of variables. A relatively high average correlation between structural shocks and the corresponding reduced-form innovations is sensible from an economic perspective, which may help explain why Cholesky identification tends to produce sensible impulse response functions. We have also shown that the average correlation between structural shocks and reduced-form shocks is nearly identical for all variable orderings in Cholesky identification, such that the choice of causal ordering can be reinterpreted as a choice for distributing the average correlation over dimensions.

The OASIS identification maximizes the average correlation between structural shocks and reduced-form shocks. This provides a unique orthonormal rotation that avoids the ordering assumptions required by recursive Cholesky schemes. Both Cholesky and OASIS are scale-invariant, but only OASIS is order-invariant, which makes it especially well-suited for settings where the ordering of variables is arbitrary or where components are measured on different scales.

We established several theoretical results for OASIS and Cholesky, and the relationships between them. The patterns predicted by the analytical results are evident in empirical applications, including the many empirical SVARs we revisited.

Beyond conventional SVARs, OASIS is also applicable to local projections and the proxy VAR framework. OASIS can also be combined with other identification strategies, such as long-run and sign restrictions. While this is unlikely to have a simple tractable solution, it will avoid the problem of multiple admissible solutions, see Fry and Pagan (2011).

By shifting identification from hard restrictions to an objective function, we also shift the discourse: the question is no longer whether the restrictions are reasonable, but whether the objective is. Whether the maximum-correlation criterion is an appropriate way to define structural shocks is up for debate, and may depend on the application. One advantage is that it does rule out absurdly implausible identification schemes. A widely held view in the literature is that, without further identifying assumptions, any matrix B with $BB' = \Sigma$ yields an equally valid structural representation. Nevertheless, many such B-matrices are implausible, because they imply structural shocks that are essentially unrelated to the key variables with which they should be aligned. For example, labeling as "monetary" a structural shock that is uncorrelated with standard monetary variables, including the federal funds rate, would be difficult to defend. Maximum-correlation criteria and restricted versions thereof, such as OASIS and Cholesky decompositions, avoid these pathological B-matrices by explicitly seeking to relate structural shocks to target innovations.

Future research could explore extensions of OASIS to nonlinear and time-varying settings, and other areas where the order- and scale-invariant rotation of correlated shocks into uncorrelated shocks could be beneficial. In the presence of heteroskedasticity, one can apply OASIS period by period. From $\Sigma_t = \mathrm{var}(\varepsilon_t)$ one obtains $\Lambda_{\sigma,t}$ and C_t. The time-varying OASIS mapping is then $A_t^* = \Lambda_{\sigma,t}^{-1} C_t^{-1/2}$ and $u_t^* = (A_t^*)'\varepsilon_t$. OASIS remains order- and scale-invariant at each t. For instance, in ongoing research, we are applying these ideas to the Diebold–Yilmaz connectedness and network framework. Another possible application of OASIS is order-invariant stochastic volatility model estimation, see Chan et al. (2024), where OASIS can be applied to $\{\Sigma_t\}$, in a manner that retains order-neutral likelihoods while enforcing an order- and scale-invariant structural representation.

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A Appendix of Proofs

Proof of Theorem 1. From $\mathrm{cov}(u,\varepsilon) = A'\Sigma = A'\Lambda_\sigma C\Lambda_\sigma$ we have $\mathrm{corr}(u,\varepsilon) = A'\Lambda_\sigma C$, such that

$$\rho_w(A) = \mathrm{tr}\{A'\Lambda_\sigma C\Lambda_w\} = \mathrm{tr}\{\breve{A}'\Lambda_w C\Lambda_w\}, \qquad \breve{A}' \equiv A'\Lambda_\sigma\Lambda_w^{-1}.$$

From the eigendecomposition, $\Lambda_w C\Lambda_w = Q\Lambda_\kappa Q'$, we define $\breve{A}_*' = Q\Lambda_\kappa^{-1/2}Q'$ for which we have $\rho_w(A_*) = \mathrm{tr}(\Lambda_\kappa^{1/2}) = \sum_{i=1}^n \kappa_i^{1/2}$ for $A_* = Q\Lambda_\kappa^{-1/2}Q'\Lambda_w\Lambda_\sigma^{-1}$. It is straightforward to verify that $A_*\in\mathcal{A}$, and for any $A\in\mathcal{A}$, we have $A = A_*R$ for some orthonormal $R\in\mathbb{R}^{n\times n}$ (i.e., $R'R = I_n$), so that

$$\rho_w(A) = \mathrm{tr}\{R'A_*'\Lambda_\sigma C\Lambda_w\} = \mathrm{tr}\{R'Q\Lambda_\kappa^{1/2}Q'\} = \mathrm{tr}\{M\Lambda_\kappa^{1/2}\} = \sum_{i=1}^n M_{ii}\kappa_i^{1/2} \leq \sum_{i=1}^n \kappa_i^{1/2},$$

where $M = Q'R'Q$ is an orthonormal matrix, with $M_{ii}\leq 1$ for all $i,\ldots,n$. Finally, this identification scheme is order- and scale-invariant because the eigenvalues of $\Lambda_w C\Lambda_w$ do not depend on the ordering of variables nor their scale. □

Proof of Corollary 1. We begin by solving a simpler auxiliary problem, where the reduced-form shocks, denoted η_i, $i = 1,\ldots,n$, have unit variances such that $\mathrm{var}(\eta) = C$ is a correlation matrix. Here, a vector of structural shocks is given by $u = \tilde{A}'\eta$, where $\tilde{A}$ satisfies $\tilde{A}'C\tilde{A} = I_n$. The aggregate correlation between u_i and η_i is given by

$$\varrho(\tilde{A}) \equiv \sum_{i=1}^n \mathrm{corr}(u_i,\eta_i) = \mathrm{tr}\{\mathrm{cov}(\tilde{A}'\eta,\eta)\} = \mathrm{tr}\{\tilde{A}'C\}.$$

Let $C = Q\Lambda_\lambda Q'$ be the eigendecomposition of C. A particular choice for $\tilde{A}$ is $\tilde{A}_* = C^{-1/2} = Q\Lambda_\lambda^{-1/2}Q'$, which is symmetric and satisfies the requirement $\tilde{A}_*'C\tilde{A}_* = I_n$, and it follows that

$$\varrho(\tilde{A}_*) = \mathrm{tr}\{Q\Lambda_\lambda^{-1/2}Q'Q\Lambda_\lambda Q'\} = \mathrm{tr}\{\Lambda_\lambda^{1/2}\} = \sum_{i=1}^n \lambda_i^{1/2}.$$

The corresponding vector of structural shocks is given by $u^* = \tilde{A}_*'\eta$.

Let $\tilde{A}$ be an arbitrary matrix that satisfies $\tilde{A}'C\tilde{A} = I_n$, such that $\mathrm{var}(u) = I_n$ for $u = \tilde{A}'\eta$. Because $\tilde{A}$ and $\tilde{A}_*$ are both full-rank matrices, there exists a unique $R\in\mathbb{R}^{n\times n}$ for which $\tilde{A} = \tilde{A}_*R$, and it follows that $\mathrm{var}(u) = R'\,\mathrm{var}(u^*)R$. Moreover, since $\mathrm{var}(u) = \mathrm{var}(u^*) = I_n$ it follows that $RR' = I_n$, so that R is an orthonormal matrix, and we will sometimes refer to R as a rotation matrix. Next, define $M = Q'R'Q$, which is a product of orthonormal matrices, such that M is also an orthonormal matrix (this can also be verified directly with

$MM' = Q'R'QQ'RQ = I_n$). We now have the identity

$$\begin{aligned}\varrho(\tilde{A}) = \operatorname{tr}\{\tilde{A}'C\} &= \operatorname{tr}\{R'Q\Lambda_\lambda^{-1/2}Q'Q\Lambda_\lambda Q'\} \\ &= \operatorname{tr}\{Q'R'Q\Lambda_\lambda^{1/2}\} = \sum_{i=1}^{n} M_{ii}\lambda_i^{1/2} \leq \sum_{i=1}^{n} \lambda_i^{1/2} = \rho_\eta(\tilde{A}_*).\end{aligned}$$

The inequality follows from the fact that an orthonormal matrix satisfies $\max_{i,j}|M_{ij}| \leq 1$. This proves that $\tilde{A}_*$ leads to the maximal average correlation between the elements of $u^* = \tilde{A}_*'\eta$ and the corresponding elements of η.

To complete the proof, we observe that $\operatorname{corr}(u,\varepsilon) = \operatorname{cov}(u,\varepsilon)\Lambda_\sigma^{-1} = \operatorname{cov}(u,\Lambda_\sigma^{-1}\varepsilon) = \operatorname{cov}(u,\eta) = \operatorname{corr}(u,\eta)$, where $\eta = \Lambda_\sigma^{-1}\varepsilon$. Hence $A_* = \Lambda_\sigma^{-1}\tilde{A}_* = \Lambda_\sigma^{-1}Q\Lambda_\lambda^{-1/2}Q'$ maximizes the average correlation between u_i and ε_i. $\square$

Proof of Theorem 2. Let $\tilde{\boldsymbol{a}} \equiv C_{\varepsilon\varepsilon}^{1/2}\Lambda_\sigma\boldsymbol{a}$ so that $\tilde{\boldsymbol{a}}'\tilde{\boldsymbol{a}} = I_r$. Then

$$\operatorname{corr}(\boldsymbol{a}'\varepsilon, z)\Lambda_w = \boldsymbol{a}'\Lambda_\sigma C_{\varepsilon z}\Lambda_w = \boldsymbol{a}'\Lambda_\sigma C_{\varepsilon\varepsilon}^{1/2}C_{\varepsilon\varepsilon}^{-1/2}C_{\varepsilon z}\Lambda_w = \tilde{\boldsymbol{a}}'\Xi,$$

such that

$$g(\boldsymbol{a}_*) = \operatorname{tr}\{\boldsymbol{a}_*'\Lambda_\sigma C_{\varepsilon\varepsilon}^{1/2}\Xi\} = \operatorname{tr}\{VU'U\Lambda_\xi V'\} = \sum_{i=1}^{r}\xi_i,$$

and

$$g(\boldsymbol{a}) = \operatorname{tr}\{\tilde{\boldsymbol{a}}'\Xi\} = \operatorname{tr}\{\tilde{\boldsymbol{a}}'U\Lambda_\xi V'\} = \operatorname{tr}\{V'\tilde{\boldsymbol{a}}'U\Lambda_\xi\} = \sum_{i=1}^{r}M_{ii}\xi_i,$$

where M_{ii}, $i = 1,\ldots,r$ are the diagonal elements of $M \equiv V'\tilde{\boldsymbol{a}}'U$. Thus, $M_{ii} = v_i'\tilde{\boldsymbol{a}}'u_i$, where v_i and u_i are the i-th columns of V and U, respectively. Because $\tilde{\boldsymbol{a}}'\tilde{\boldsymbol{a}} = I_r$ and $\|v_i\| = 1$ we have $\|\tilde{\boldsymbol{a}}v_i\|^2 = v_i'(\tilde{\boldsymbol{a}}'\tilde{\boldsymbol{a}})v_i = v_i'v_i = 1$, such that

$$|M_{ii}| = |v_i'\tilde{\boldsymbol{a}}'u_i| \leq \|u_i\|\,\|\tilde{\boldsymbol{a}}v_i\| = 1,$$

by the Cauchy-Schwarz inequality and $\|u_i\| = 1$. Because $\xi_i \geq 0$ for all i (by the definition of the SVD) we have shown that $g(\boldsymbol{a}) \leq g(\boldsymbol{a}_*)$.

Moreover, if $C_{\varepsilon z}$ has full column rank, then $\xi_i > 0$ for all i, and

$$g(\boldsymbol{a}) = g(\boldsymbol{a}_*) \Leftrightarrow M_{ii} = 1,\ \forall i \Leftrightarrow u_i = \tilde{\boldsymbol{a}}v_i,\ \forall i \Leftrightarrow U = \tilde{\boldsymbol{a}}V,$$

which implies $\tilde{\boldsymbol{a}} = UV' = \tilde{\boldsymbol{a}}_*$. This shows that $\boldsymbol{a}_*$ is the unique maximizer in this case. $\square$

Proof of Theorem 3. Let $\epsilon_1,\ldots,\epsilon_n$ be the eigenvalues of $E = C - I_n$. Then $\sum_{i=1}^n\epsilon_i = \operatorname{tr}(E) = 0$ and $d(C) = \frac{1}{n}\|E\|_F^2 = \frac{1}{n}\sum_{i=1}^n\epsilon_i^2$, where the last identity uses that E is symmetric.

Moreover, the eigenvalues of C are related to those of E, by $\lambda_i = 1 + \epsilon_i$, for $i = 1, \ldots, n$. For OASIS, $\tilde{A}_* = Q\,\Lambda_\lambda^{-1/2} Q'$, we have by Corollary 1 that $\mathrm{tr}(\tilde{A}_*' C) = \sum_{i=1}^n \lambda_i^{1/2}$. The Taylor series for an element in this sum is:

$$\lambda_i^{1/2} = (1 + \epsilon_i)^{1/2} = 1 + \tfrac{1}{2}\epsilon_i - \tfrac{1}{8}\epsilon_i^2 + \tfrac{1}{16}\epsilon_i^3 - \cdots, \tag{8}$$

and by adding these up, we find

$$\bar{\rho}_* = \tfrac{1}{n}\,\mathrm{tr}(\tilde{A}_*' C) = 1 - \tfrac{1}{8n}\sum_{i=1}^n \epsilon_i^2 + O\left(\tfrac{1}{n}\sum_{i=1}^n \epsilon_i^3\right) = 1 - \tfrac{1}{8}d(C) + \tfrac{1}{n}O(\mathrm{tr}\{E^3\}),$$

where we used $\sum_{i=1}^n \epsilon_i^3 = \mathrm{tr}(E^3)$.

Next, we consider $\tilde{A}_{\mathrm{c}}' = L^{-1}$ based on the Cholesky factorization, $C = L\,L'$, where L is lower triangular (with positive diagonal entries). It follows that $\tilde{A}_{\mathrm{c}}' C = L^{-1} L\,L' = L'$, so that $\mathrm{tr}(\tilde{A}_{\mathrm{c}}' C) = \mathrm{tr}(L) = \sum_{i=1}^n L_{ii}$. From the identity, $1 = C_{ii} = \sum_{j \le i} L_{ij}^2$, we have

$$L_{ii} = \sqrt{1 - \sum_{j<i} L_{ij}^2} = 1 - \tfrac{1}{2}\sum_{j<i} L_{ij}^2 - \tfrac{1}{8}\left(\sum_{j<i} L_{ij}^2\right)^2 - \cdots, \tag{9}$$

where we used the same Taylor expansion as in (8). Next, $L_{ij} = C_{ij} + O(E^2) = E_{ij} + O(E^2)$, such that summing over i in (9) yields

$$\mathrm{tr}(L) = n - \tfrac{1}{2}\sum_{i=1}^n \sum_{j<i} E_{ij}^2 + O\left(\mathrm{tr}\{E^3\}\right).$$

Since $d(C) = \frac{2}{n}\sum_{i>j} E_{ij}^2$, we can conclude that

$$\tfrac{1}{n}\,\mathrm{tr}(\tilde{A}_{\mathrm{c}}' C) = 1 - \tfrac{1}{4}d(C) + O\left(\mathrm{tr}\{E^3\}\right).$$

Combined, we have

$$\frac{1 - \bar{\rho}_{\mathrm{c}}}{1 - \bar{\rho}_*} = 2\frac{1 + O\left(\mathrm{tr}\{E^3\}/d(C)\right)}{1 + O\left(\mathrm{tr}\{E^3\}/d(C)\right)} = 2\left(1 + O\left(\|E\|_F\right)\right) = 2\left(1 + O\left(\sqrt{d(C)}\right)\right),$$

using $\mathrm{tr}\{E^3\} = O(\|E\|_F^3)$ and $d(C) = \|E\|_F^2/n = O(\|E\|_F^2)$. □

Proof of Corollary 2. From Archakov and Hansen (2024) we have that the r-th power of an equicorrelation matrix can be expressed as

$$C^r = (1 + (n-1)\rho)^r P_n + (1-\rho)^r P_n^{\perp}. \tag{10}$$

where $P_n = \iota_n(\iota_n'\iota_n)^{-1}\iota_n' = \frac{1}{n}\iota_n\iota_n'$ and $P_n^{\perp} = I - P_n$ are orthogonal projection matrices and $\iota_n \in \mathbb{R}^n$ is the vector of ones. So, (6) follows directly from (10) with $r = 1/2$,

$$\bar{\rho}_* = \tfrac{1}{n}\,\mathrm{tr}\{C^{1/2}\} = \tfrac{1}{n}\sqrt{1+(n-1)\rho} + \sqrt{1-\rho}\left(1-\tfrac{1}{n}\right).$$

For the Cholesky decomposition, $C = LL'$, we have $\bar{\rho}_{\mathrm{c}} = \frac{1}{n}\,\mathrm{tr}\{L\} = \frac{1}{n}\sum_{k=1}^{n} L_{kk}$. A diagonal element L_{kk} is the standard deviation of η_k after controlling for $\eta_1, \ldots, \eta_{k-1}$, where $\mathrm{var}(\eta) = C$. Standard projection arguments give us the linear regression, $\eta_k = \rho\iota_{k-1}'C_{k-1}^{-1}\eta_{1:k-1} + L_{kk}u_k$, where $\eta_{1:k-1} = (\eta_1, \ldots, \eta_{k-1})'$, and the error variance is given by

$$\begin{aligned}
L_{kk}^2 &= 1 - \rho^2\iota_{k-1}C_{k-1}^{-1}\iota_{k-1}' \\
&= 1 - \rho^2\iota_{k-1}\left[(1+(k-2)\rho)^{-1}P_{k-1} + (1-\rho)^{-1}\left(P_{k-1}^{\perp}\right)\right]\iota_{k-1}' \\
&= 1 - \rho^2(k-1)/(1+(k-2)\rho),
\end{aligned}$$

where we used (10) with $r = -1$. This proves (7). □

Proof of Corollary 3. Follows directly from Theorem 3. □

B Sample Periods in Empirical Analysis

We used the original data for most of the studies revisited in our empirical analysis. In four of the studies, we used larger sample periods as detailed in Table B.1.

Table B.1: Sample Periods of Structural VAR Studies by Shock Type

Study ID	**Original Sample Period**	**Estimation Sample Period**
	Monetary Shocks	
B86	1953:Q1 – 1984:Q4	1959:Q1 – 2024:Q4
S95	1959:M1 – 1992:M2	same
LSZ96	1960:M1 – 1996:M3	1959:Q1 – 2018:Q2
CEE99	1965:Q3 – 1995:Q2	same
CEE05	1965:Q3 – 1995:Q3	same
BL09	1983:M1 – 2002:M12	same
	Fiscal Shocks	
BP02	1960:Q1 – 1994:Q4	1947:Q1 – 2018:Q3
RZ11	1954:Q4 – 2006:Q4	same
FG16	1981:Q3 – 2013:Q3	same
	Uncertainty Shocks	
B09	1962:M6 – 2008:M6	same
CCG14	1962:Q3 – 2012:Q3	same
BB17	1986:Q1 – 2014:Q4	same
BO23	1960:Q2 – 2018:Q2	same
	Financial Shocks	
GZ12	1973:Q3 – 2010:Q3	same
FGMS24	1973:M1 – 2019:M12	same
	Oil Price Shocks	
LS04	1972:Q1 – 2000:Q4	same
LP18	1976:M1 – 2014:M12	same
	Sectoral	
FGKV25	1988:Q1 – 2019:Q4	same
	Highly Correlated VARs	
DP05	1970:Q1 – 2003:Q3	same
EI95	1947 – 1989	2005:Q1 – 2024:Q4
FHT25	NA	1981:M9 – 2025:M1

Notes: See notes for Table 1 for the abbreviation used for studies. We report the original sample period and use “same” if replication codes were available and the original sample period was used for estimation. Otherwise we give our estimation sample period. ”M” and ”Q” are used to indicate samples with monthly and quarterly data, respectively. The original sample in EI95 was based on Annual Data.