An extended score-driven dynamic factor model:
Recovering composite indices from the pandemic

Mariia Artemova
Joint work with: **Dick van Dijk and Evgenii Vladimirov**

Erasmus University Rotterdam, Tinbergen Institute

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Motivation

- Dynamic factor models (DFMs) are workhorse models in macroeconomic and financial applications;
- Policy institutions rely on **Kalman filter**-based DFMs for:
  - constructing Coincident and Leading indices (**Philadelphia Fed**);
  - nowcasting GDP (**New York Fed, ECB, DNB**);
  - forecasting inflation (**ECB**);
  - financial market monitoring (**CISS index, ECB**).

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Ng (2021): "...covid has also created challenges for the modeling of economic time series . . . **Without any adjustment**, **the post-covid** observations will dominate to yield uninterpretable estimates, **messing up the pre-covid fit**."

Motivation

Philadelphia Fed

- for the state Coincident Indices:

  *"The observations from 2020 are treated as **missing data** to exclude the impacts of the extreme, idiosyncratic shock from the pandemic and to preserve states' historic business cycle characteristics."*

- has suspended the release of the state Leading Indices indefinitely:

  *"Given the sudden, extreme impact of the COVID-19 outbreak on initial unemployment claims in recent weeks, our researchers' **standard approach** for estimating the six-month change in coincident indices **is not appropriate**."*

Motivation

New York Fed has suspended its GDP nowcast releases based on the DFM methodology for two years because

"the COVID-19 pandemic generated considerable uncertainty and volatility with respect to macroeconomic data".

Updated New York Fed model features:

(i) **time-varying volatility**;

(ii) **non-Gaussian innovations**.

In this paper

1. We introduce an **Extended Score-Driven (ESD) DFM**:
    - accommodates **time-varying volatility** (GARCH-type) and **non-Gaussian innovations** (Student's t);
2. **Frequentist** ML estimation via prediction error decomposition;
3. The ESD-DFM **bridges** parameter-driven and score-driven DFMs:
    - generalizes existing **score-driven** factor models;
    - nests the classic **Gaussian (Kalman filter)** DFM as a special case;
4. **Empirical application:** recover the Philadelphia Fed and The Conference Board coincident and leading economic indices.

Illustrative example: A single-factor dynamic factor model

A single-factor Gaussian DFM

Observation equation:

$$\boldsymbol{y}_t = \boldsymbol{\lambda} f_t + \boldsymbol{\varepsilon}_t, \qquad \boldsymbol{\varepsilon}_t \sim \mathcal{N}(\mathbf{0}_N, \boldsymbol{\Sigma}), \tag{1}$$

- $\boldsymbol{y}_t = (y_{1t}, \ldots, y_{Nt})^\top$ with N fixed;
- f_t is a common factor;
- $\boldsymbol{\lambda}$ is a vector of loadings;
- $\boldsymbol{\varepsilon}_t$ are i.i.d. and $\boldsymbol{\Sigma}$ is diagonal;
- for simplicity and identification, $\frac{1}{N}\boldsymbol{\lambda}^\top \boldsymbol{\Sigma}^{-1} \boldsymbol{\lambda} = 1$.

Factor dynamics

Parameter-driven (PD) model (Durbin & Koopman 2012):

$$f_{t+1} = bf_t + \eta_{t+1}, \quad \eta_{t+1} \sim \mathcal{N}(0, \sigma_\eta^2). \tag{2}$$

The (steady-state) Kalman filter recursion:

$$f_{t+1|t} = bf_{t|t-1} + \frac{bp}{1+pN} \boldsymbol{\lambda}^\top \boldsymbol{\Sigma}^{-1}(\boldsymbol{y}_t - \boldsymbol{\lambda} f_{t|t-1}), \tag{3}$$

where $f_{t+1|t} := \mathbb{E}[f_{t+1}|\mathcal{F}_t]$ and $p > 0$ is the steady-state predictive variance.

Key limitation: ✗ analytical likelihood in non-Gaussian and nonlinear settings.

Factor dynamics

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Score-driven (SD) model (Artemova 2025):

$$f_{t+1} = bf_t + as_t \tag{4}$$

$$= bf_t + a \underbrace{\frac{1}{N} \boldsymbol{\lambda}^\top \boldsymbol{\Sigma}^{-1} (\boldsymbol{y}_t - \boldsymbol{\lambda} f_t)}_{=s_t := S_t \nabla_t}. \tag{5}$$

Predictive mean and variance of the data

Parameter-driven model

$$\begin{aligned} \mathbb{E}[\boldsymbol{y}_{t+1}|\mathcal{F}_t] &= \boldsymbol{\lambda} f_{t+1|t}, \\ \mathbb{V}[\boldsymbol{y}_{t+1}|\mathcal{F}_t] &= p\boldsymbol{\lambda}\boldsymbol{\lambda}^\top + \boldsymbol{\Sigma} \neq \textbf{diag}. \end{aligned} \tag{6}$$

Score-driven model

$$\begin{aligned} \mathbb{E}[\boldsymbol{y}_{t+1}|\mathcal{F}_t] &= \boldsymbol{\lambda} f_{t+1}, \\ \mathbb{V}[\boldsymbol{y}_{t+1}|\mathcal{F}_t] &= \boldsymbol{0} + \boldsymbol{\Sigma} = \textbf{diag}. \end{aligned} \tag{7}$$

Extended score-driven model

$$f_{t+1} = bf_t + as_t + cs_{t+1} \tag{8}$$
$$= bf_t + a\frac{1}{N}\boldsymbol{\lambda}^\top \boldsymbol{\Sigma}^{-1} (\boldsymbol{y}_t - \boldsymbol{\lambda} f_t) + c\frac{1}{N}\boldsymbol{\lambda}^\top \boldsymbol{\Sigma}^{-1} (\boldsymbol{y}_{t+1} - \boldsymbol{\lambda} f_{t+1}). \tag{9}$$

The predictive mean and variance of the data are

$$\mathbb{E}[\boldsymbol{y}_{t+1}|\mathcal{F}_t] = \boldsymbol{\lambda} f_{t+1|t},$$
$$\mathbb{V}[\boldsymbol{y}_{t+1}|\mathcal{F}_t] = \frac{c^2 + 2c}{N}\boldsymbol{\lambda}\boldsymbol{\lambda}^\top + \boldsymbol{\Sigma} \neq \textbf{diag}. \tag{10}$$

Extended score-driven (ESD) model

$$f_{t+1} = bf_t + as_t + cs_{t+1}$$

- factors f_{t+1} are not predetermined at time t, but predetermined at $t+1$;
- ESD-DFM bridges the gap between PD and SD:
  - ESD nests the PD DFM;
  - ESD reduces to SD DFM when $c = 0$;
- the idea of including innovations at time $t+1$ is similar to the real-time GARCH model (Smetanina 2017):
  - the real-time GARCH model can be thought of as a link between (observation-driven) GARCH and (parameter-driven) stochastic volatility models.

An extended score-driven dynamic factor model

General ESD model: observation equation

$$\boldsymbol{y}_t = \boldsymbol{\Lambda}(L)\boldsymbol{f}_t + \boldsymbol{\varepsilon}_t, \tag{11}$$

$$\boldsymbol{P}(L)\boldsymbol{\varepsilon}_t = \boldsymbol{u}_t, \qquad \boldsymbol{u}_t|\mathcal{F}_{t-1} \sim p_{\boldsymbol{u}}(\boldsymbol{u}_t; \boldsymbol{\Sigma}_t, \nu), \tag{12}$$

- $\boldsymbol{f}_t$ is an $r \times 1$ vector of common factors;
- $\boldsymbol{\Lambda}(L) = \boldsymbol{\Lambda}_0 + \boldsymbol{\Lambda}_1 L + \ldots + \boldsymbol{\Lambda}_m L^m$;
- $\boldsymbol{P}(L) = \boldsymbol{I} - \boldsymbol{P}_1 L - \ldots - \boldsymbol{P}_p L^p$, where $\boldsymbol{P}_j$ is diagonal;
- $\boldsymbol{u}_t$ is i.i.d. with conditional density $p_{\boldsymbol{u}}(\boldsymbol{u}_t; \boldsymbol{\Sigma}_t, \nu)$ from elliptical class;
- $\boldsymbol{\Sigma}_t = h_t^2 \boldsymbol{\Sigma}$ and $\boldsymbol{\Sigma}$ is diagonal;
- h_t is a common **volatility factor** shared across all series.

General ESD model: time-varying parameters

$$\boldsymbol{f}_{t+1} = \boldsymbol{B}\boldsymbol{f}_t + \boldsymbol{A}\boldsymbol{s}_t + \boldsymbol{C}\tilde{\boldsymbol{s}}_{t+1}, \tag{13}$$

$$h_{t+1}^2 = w + \alpha s_t^h + \gamma h_t^2, \tag{14}$$

- $\boldsymbol{s}_t$ and s_t^h are functions of observations, i.e., the scaled (quasi) scores;
- $\tilde{\boldsymbol{s}}_t$ serves as an 'innovation term';
- $\tilde{\boldsymbol{s}}_t$ is always a Gaussian score in our framework.

Example: ESD-t DFM with GARCH

Consider $p = m = 0$ (no lags in $\boldsymbol{P}(L)$ or $\boldsymbol{\Lambda}(L)$) and $\boldsymbol{u}_t \sim t_\nu$.

$$\boldsymbol{f}_{t+1} = \boldsymbol{B}\boldsymbol{f}_t + \boldsymbol{A}\underbrace{\frac{1}{W_t}\frac{1}{N}\boldsymbol{\Lambda}_0^\top \boldsymbol{\Sigma}^{-1}\boldsymbol{u}_t}_{\text{score, downweighted}} + \boldsymbol{C}\underbrace{\frac{1}{N}\boldsymbol{\Lambda}_0^\top \boldsymbol{\Sigma}^{-1}\boldsymbol{u}_{t+1}}_{\text{innovation}},$$

$$h_{t+1}^2 = w + \alpha\frac{1}{N}\boldsymbol{v}_t^\top \boldsymbol{\Sigma}^{-1}\boldsymbol{v}_t + (\gamma - \alpha)h_t^2.$$

Key components:

- residuals $\boldsymbol{u}_t = \boldsymbol{y}_t - \boldsymbol{\Lambda}_0\boldsymbol{f}_t$;
- $W_t = \frac{\nu}{N+\nu+2}(1 + \boldsymbol{u}_t^\top \boldsymbol{\Sigma}_t^{-1}\boldsymbol{u}_t/\nu)$;
- W_t **automatically downweights** large $\|\boldsymbol{u}_t\|$;
- ν: degrees of freedom parameter. Gaussian limit as $\nu \to \infty$.

ESD filtering

$$\boldsymbol{f}_{t+1} = \boldsymbol{B}\boldsymbol{f}_t + \boldsymbol{A}\boldsymbol{s}_t + \boldsymbol{C}\underbrace{\frac{1}{N}\boldsymbol{\Lambda}_0^\top \boldsymbol{\Sigma}^{-1}\boldsymbol{P}(L)(\boldsymbol{y}_{t+1} - \boldsymbol{\Lambda}(L)\boldsymbol{f}_{t+1})}_{=\tilde{\boldsymbol{s}}_{t+1}}$$

- **Prediction step**: $\boldsymbol{f}_{t+1|t} := \mathbb{E}[\boldsymbol{f}_{t+1}|\mathcal{F}_t].$

$$\boldsymbol{f}_{t+1|t} = \boldsymbol{B}\boldsymbol{f}_t + \boldsymbol{A}\boldsymbol{s}_t, \tag{15}$$

since $\mathbb{E}\left[\boldsymbol{P}(L)(\boldsymbol{y}_{t+1} - \boldsymbol{\Lambda}(L)\boldsymbol{f}_{t+1})|\mathcal{F}_t\right] = 0.$

ESD filtering

$$\boldsymbol{f}_{t+1} = \boldsymbol{B}\boldsymbol{f}_t + \boldsymbol{A}\boldsymbol{s}_t + \boldsymbol{C}\underbrace{\tfrac{1}{N}\boldsymbol{\Lambda}_0^\top\boldsymbol{\Sigma}^{-1}\boldsymbol{P}(L)(\boldsymbol{y}_{t+1} - \boldsymbol{\Lambda}(L)\boldsymbol{f}_{t+1})}_{=\tilde{\boldsymbol{s}}_{t+1}}$$

- **Update step:** $\boldsymbol{f}_{t+1} = \mathbb{E}[\boldsymbol{f}_{t+1}|\mathcal{F}_{t+1}]$.

$$\boldsymbol{f}_{t+1} = \boldsymbol{f}_{t+1|t} + \boldsymbol{K}^\star \boldsymbol{e}_{t+1}, \tag{16}$$

where $\boldsymbol{e}_{t+1} := \boldsymbol{y}_{t+1} - \mathbb{E}[\boldsymbol{y}_{t+1}|\mathcal{F}_t]$ and $\boldsymbol{K}^\star := (\boldsymbol{I} + \boldsymbol{C})^{-1}\boldsymbol{C}\frac{1}{N}\boldsymbol{\Lambda}_0^\top\boldsymbol{\Sigma}^{-1}$.

ML Estimation

$$
\begin{aligned}
\hat{\boldsymbol{\theta}}_T &= \arg\max_{\boldsymbol{\theta}\in\Theta} L_T(\boldsymbol{\theta}) \\
&= \arg\max_{\boldsymbol{\theta}\in\Theta} \frac{1}{T}\sum_{t=2}^{T} -\frac{1}{2}\log|\boldsymbol{\Omega}_t(\boldsymbol{\theta})| + \log g\left(\boldsymbol{e}_t(\boldsymbol{\theta})^\top \boldsymbol{\Omega}_t^{-1}(\boldsymbol{\theta})\boldsymbol{e}_t(\boldsymbol{\theta})\right),
\end{aligned}
$$

where

- $\boldsymbol{e}_t = \boldsymbol{y}_t - \mathbb{E}[\boldsymbol{y}_t|\mathcal{F}_{t-1}]$ is the one-step-ahead prediction error;
- $\boldsymbol{\Omega}_t = \mathbb{V}(\boldsymbol{e}_t(\boldsymbol{\theta})|\mathcal{F}_{t-1}) = \mathbb{V}(\boldsymbol{y}_t|\mathcal{F}_{t-1})$ is the conditional covariance;
- $g(\cdot)$ is the density generator of the class of elliptical distributions associated with $p_{\boldsymbol{u}}(\cdot;\boldsymbol{\Sigma}_t,\boldsymbol{\nu})$.

Empirical application

Recovering economic indices: CEI

Philadelphia Fed produces the **Coincident Economic Index** (CEI), which captures the current state of the economy.

- PHI Fed uses Gaussian PD DFM
  - Nonfarm payroll employment, unemployment rate, average weekly hours (manufacturing), real personal income;
  - Sample: Jan 1959 – Dec 2025;
  - PHI Fed treats all **2020** observations as **missing** when estimating DFM.

CEI: Model fit comparison

		PD-$\mathcal{N}$	SD-$\mathcal{N}$	ESD-$\mathcal{N}$	SD-t	ESD-t
$p = m = 0$	Loglik	-3540.48	-4492.97	-3537.37	-2164.56	**-2081.75**
	BIC	7141.02	9046.01	7141.47	4395.86	**4236.90**
$p = m = 1$	Loglik	-3415.78	-4410.86	-3396.96	-2184.09	**-1958.27**
	BIC	6945.01	8935.16	6914.03	4488.30	**4043.33**

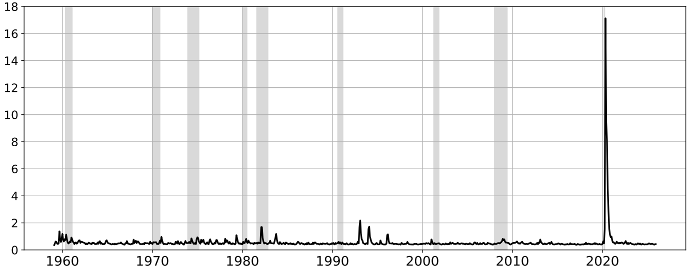
Table 1: We report the log-likelihood value (Loglik) and Bayesian information criterion (BIC) for PD, SD, and ESD models with Gaussian ($\mathcal{N}$) or Student's t (t) distributions. m is the number of lags of the common factors; p is the autoregressive order for the idiosyncratic components. All model specifications are without conditional heteroskedasticity.

CEI: ESD-DFM fit comparison

		ESD-$\mathcal{N}$		ESD-t	
	h_t	const	GARCH	const	GARCH
$p = m = 0$	Loglik	-3537.37	-2548.63	-2081.75	**-2002.94**
	BIC	7141.47	5177.34	4236.90	**4092.64**
$p = m = 1$	Loglik	-3396.96	-2459.17	-1958.27	**-1869.74**
	BIC	6914.03	5051.80	4043.33	**3879.61**

Table 2: h_t indicates the conditional variance dynamics: constant (homoskedasticity) and linear GARCH-type (GARCH).

CEI: Predicted volatility factor



- COVID-19 appears as a shock to volatility.

CEI: Predicted factor

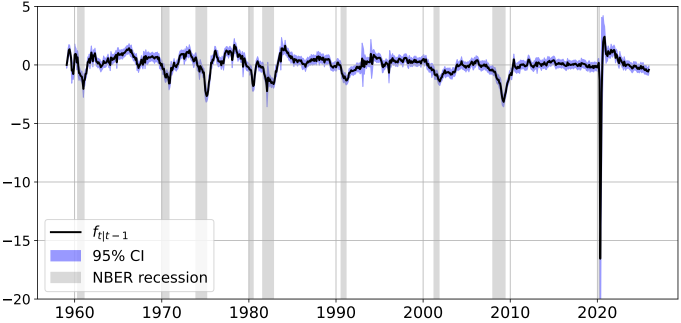


Figure 1: **Predicted factor $\hat{f}_{t+1|t}$ with 95% confidence bands.**

- The factor is statistically different from zero during all NBER recessions;
- Standard score-driven models cannot provide confidence bands.

PHI Fed Coincident Economic Index

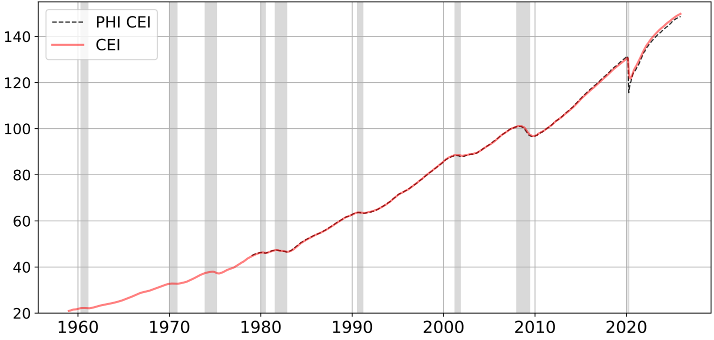


Figure 2: **Reconstructed CEI in levels vs. official PHI Fed CEI.**

- The ESD-constructed index resembles the PHI Fed CEI;
- The 2020 dip is **smaller** due to robustness – but no observations excluded.

Recovering economic indices: LEI

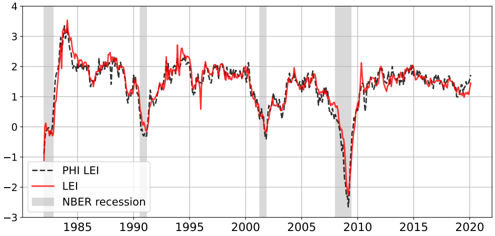
The PHI Fed **discontinued** the **Leading Economic Index** (LEI) after 2020 – we revive it with the ESD DFM.

- **LEI** predicts the six-month growth rate of CEI.
  - Leading variables: housing permits, initial unemployment claims, ISM delivery times, interest rate spread (June 1976 – Dec 2025);
  - VAR-based construction.
- **Our approach:** estimate common factor f_t^L from the leading variables via ESD-DFM, then model:

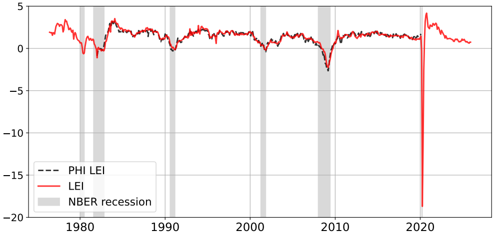
$$\Delta_6 \log \mathrm{CEI}_{t+6} = \beta_0 + \beta_c(L)\Delta \log \mathrm{CEI}_t + \beta_\ell(L) f_t^L + \varepsilon_t,$$

so that $\mathrm{LEI}_t = \mathbb{E}[\Delta_6 \log \mathrm{CEI}_{t+6} \mid \mathcal{F}_t]$.

Philadelphia Fed LEI



(a) Subsample: Jan 1982 – Feb 2020



(b) Full sample

Figure 3: **Estimated LEI vs. the official PHI Fed LEI.**

- The estimated LEI matches the official LEI pre-2020;
- We continue producing the LEI after COVID: large 2020 drop and positive growth after July 2020.

Recovering The Conference Board Indices

The Conference Board (TCB) produces the Coincident Economic Index (CEI) and Leading Economic Index (LEI).

- **Non-model-based approach** (indices are computed as a weighted average of the series);
- CEI and LEI are expected to move in the **same direction** most of the time.

TCB puzzle

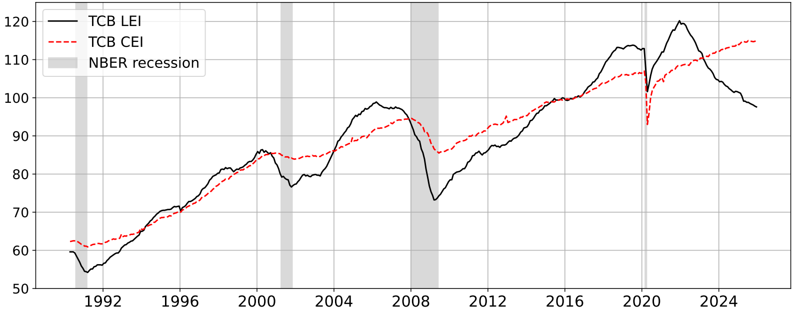


Figure 4: **TCB LEI and CEI.**

- Since December 2021, TCB CEI and LEI for the US have exhibited a **divergent pattern**: CEI has **trended upward**, while LEI has been **declining** steadily since December 2021.

TCB puzzle resolved

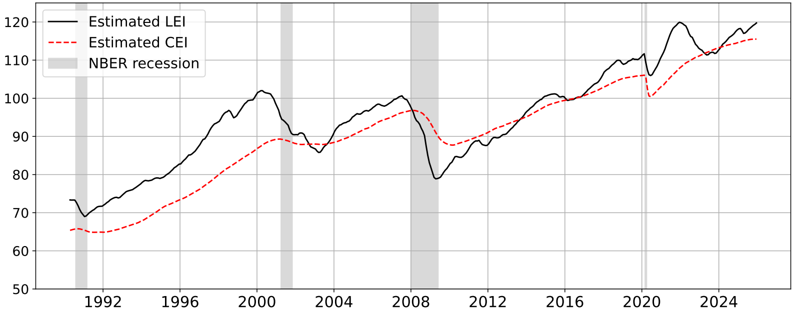


Figure 5: **Estimated LEI and CEI.**

Conclusion

- The ESD DFM bridges score-driven and parameter-driven models and allows for time-varying volatility and heavy tails;
- The data favor ESD-t models with conditional heteroskedasticity;
- Indices are 'recovered': Philadelphia Fed indices can be produced after COVID and TCB divergence puzzle is resolved.

Thank you!

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Appendix

Monte Carlo simulations: Measurement equation

$$\boldsymbol{y}_t = \boldsymbol{\lambda} f_t + \boldsymbol{u}_t,$$

- $N = 4$ and $T = 2 \times 500$;
- $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)^\top$ with $\lambda_i = 1 - 0.15 \times (i-1)$;
- $\boldsymbol{\Sigma} = \mathrm{diag}(\sigma_1^2, \sigma_2^2, \sigma_3^2, \sigma_4^2)$ with $\sigma_i^2 = 0.2 + 0.2 \times (i-1)$;
- $\boldsymbol{u}_t \sim \mathcal{N}(\mathbf{0}_N, \boldsymbol{\Sigma})$ or $\boldsymbol{u}_t \sim t_5(\mathbf{0}_N, \boldsymbol{\Sigma})$;
- Evaluate out-of-sample fit based on the log-score rule (LSR).

Monte Carlo simulations: Factor dynamics

- **DGP 1 (PD-AR(1)):** $f_{t+1} = 0.9 f_t + 0.5\,\eta_t,\ \eta_t \sim \mathcal{N}(0,1)$.
- **DGP 2 (ESD, $c = 2$):** $f_{t+1} = 0.9 f_t + 0.2 s_t + 2\,\tilde{s}_{t+1}$.
- **DGP 3 (ESD, $c = 0.2$):** $f_{t+1} = 0.9 f_t + 0.2 s_t + 0.2\,\tilde{s}_{t+1}$.

Gaussian DGP	PD-$\mathcal{N}$	PD$_{11}$-$\mathcal{N}$	SD-$\mathcal{N}$	ESD-$\mathcal{N}$
DGP 1	✓	✓	✗	✓
DGP 2	✗	✓	✗	✓
DGP 3	✗	✗	✗	✓

Table 3: **Estimated Gaussian models.**
✓ correctly specified ✗ misspecified.

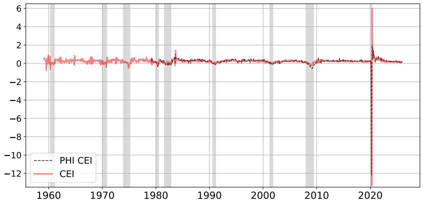
PD-$\mathcal{N}$ - parameter-driven DFM with AR(1) factors
PD$_{11}$-$\mathcal{N}$ - parameter-driven DFM with ARMA(1,1) factors

Additionally, we estimate SD-t and ESD-t models.

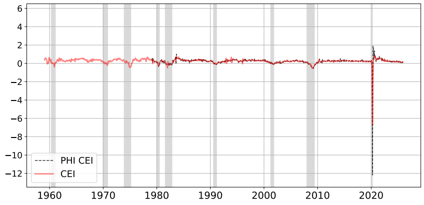
Out-of-sample Results: LSR

	PD-$\mathcal{N}$	PD$_{11}$-$\mathcal{N}$	SD-$\mathcal{N}$	ESD-$\mathcal{N}$	SD-t	ESD-t
$\boldsymbol{u}_t \sim \mathcal{N}(\mathbf{0}_N, \boldsymbol{\Sigma})$						
DGP 1: PD AR(1)	**4.6930**	4.6938	4.9523	4.6938	4.9471	4.6940
DGP 2: ESD, $c = 2$	5.1540	**5.1476**	5.9534	**5.1476**	5.9413	5.1479
DGP 3: ESD, $c = 0.2$	4.3370	4.2973	4.2552	**4.2345**	4.2556	4.2348
$\boldsymbol{u}_t \sim t_5(\mathbf{0}_N, \boldsymbol{\Sigma})$						
DGP 1: PD AR(1)	5.5734	5.5745	5.7266	5.5745	5.5678	**5.3771**
DGP 2: ESD, $c = 2$	6.2105	6.2107	7.0262	6.2107	6.8174	**5.9133**
DGP 3: ESD, $c = 0.2$	5.3684	5.3468	5.3652	5.3285	5.0069	**4.9827**

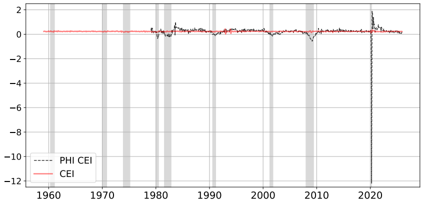
Table 4: The table reports the Monte Carlo average of the logarithmic scoring rule (LSR).



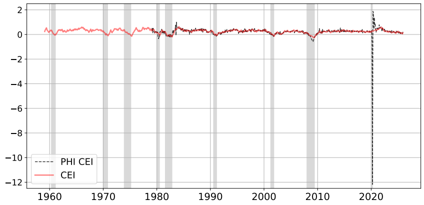
(a) ESD-$\mathcal{N}$, $\hat{b} = 0.34$



(b) ESD-$\mathcal{N}$ with GARCH, $\hat{b} = 0.9$



(c) PD-$\mathcal{N}$, $\hat{b} = 0.02$



(d) SD-t, $\hat{b} = 0.93$

Figure 6: **Estimated CEI together with the PHI CEI for different DFMs.**