

Inflation, Inequality, and the Business Cycle

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Abstract

We introduce a nonlinear, state-dependent Phillips curve into a standard Heterogeneous Agent New Keynesian (HANK) model. We show that this nonlinearity is crucial for *jointly* matching the empirical properties of inflation and inequality. In our model, inflation and income inequality respond asymmetrically to business cycle fluctuations, increasing more sharply than declining. As a result, the model accounts for the observed positively skewed distributions of U.S. inflation rates and income inequality. In contrast, a version with a constant Phillips curve slope fails to replicate these empirical patterns, underscoring the importance of the nonlinear Phillips curve.

JEL Classification: E31, E32, E52

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1 Introduction

The inflation surge following the Covid-19 crisis caught many economists and central banks by surprise. Inflation in the U.S. and many other economies rose to levels not observed in decades. A similar surprise occurred after the Global Financial Crisis and Great Recession, when inflation rates fell less than predicted at the time. Recent research has shown that standard linearized New Keynesian models have difficulties to explain inflation developments during deep crises, mirroring the surprises by economists and central banks mentioned above. Harding, Lindé, and Trabandt (2022, 2023) have shown that a nonlinear New Keynesian model with a nonlinear Phillips curve, in which the slope is state-dependent, accounts much better for inflation dynamics in deep crises than the linearized model. We contribute to this literature by introducing household heterogeneity into a model with a nonlinear, state-dependent Phillips curve, allowing us to study how these observed inflation dynamics interact with the income distribution.

We include a state-dependent slope of the Phillips curve similar to e.g. Erceg, Jakab, and Lindé (2021) into an otherwise standard nonlinear HANK model similar to e.g. Auclert, Rognlie, and Straub (2024b). This model framework allows us to analyze the implications of a state-dependent Phillips curve slope on the propagation of demand and supply shocks in a heterogeneous agent environment. Importantly, it enables us to study the two-way interaction between inflation and inequality, both during inflation surges and in periods of persistently low inflation. Furthermore, we introduce countercyclical labor and profit income inequality and risk into our model to ensure that it accounts for the observed volatility in labor and aggregate income inequality in the U.S.

Our results suggest that - due to the state-dependency of the Phillips curve slope - inflationary pressures are amplified, while deflationary pressures are dampened, allowing the model to replicate the volatility and skewness of post-war U.S. inflation, GDP growth, and income inequality. The HANK environment allows us to study how supply and demand shocks propagate into inflation and affect the distribution of income. In particular, we find that inflationary cost-push shocks and contractionary demand shocks raise income inequality by reducing output. Due to countercyclical labor and profit income inequality and income risk, households face a greater risk of becoming less productive or receiving lower profit income during recessions. The resulting increase in inequality is amplified by the state-dependent Phillips curve slope. Conversely, reductions in inequality in response to cost-pull or expansionary demand shocks are more muted, as the positive output effects of these shocks are dampened relative to a model with a constant Phillips curve slope. Over the business cycle these results imply that inequality is positively skewed. Using U.S. data on the standard deviation of log household income - a standard measure of inequality - we show that this positive skewness in inequality in the model accords well with the data. This implies that inequality increases more strongly in recessions than

it falls in booms. A HANK model without a state-dependent Phillips curve slope and Gaussian shocks is not able to replicate these features observed in the data.

A growing strand of literature studies the relationship between inequality, inflation, and monetary policy.¹ Auclert (2019), Bilbiie (2018), and Kaplan, Moll, and Violante (2018) show that inequality affects the transmission of monetary policy in a HANK framework. Auclert et al. (2023) analyze the effects of an inflationary energy price shock and the implications for monetary and fiscal policies in a HANK model. Another strand of literature analyze the nexus between inequality and optimal monetary policy, see e.g. Acharya, Challe, and Dogra (2023), Bhandari et al. (2021), and McKay and Wolf (2022)). Bayer, Born, and Luetticke (2024) estimate a HANK model and show that business cycle fluctuations have implications for inequality. Their findings highlight how business cycle shocks propagate through heterogeneous household portfolios, amplifying inequality dynamics.

We contribute to the above cited literature by explicitly allowing for a nonlinear Phillips curve in a HANK model. Importantly we focus on the ability of our model to account for the skewness observed in both inflation and income inequality in U.S. data.

Our results are also related to recent empirical evidence on the impact of inflation and monetary policy on inequality. Coibion et al. (2017) find that income and consumption inequality in the U.S. increases in response to contractionary monetary policy shocks. Furceri, Loungani, and Zdzienicka (2018) find empirical evidence that monetary easing and tightening has asymmetric effects on income inequality – similar to our model-implied results. Empirical evidence by Pallotti et al. (2023) suggests that the recent inflation surge has affected households heterogeneously. Evidence in Del Canto et al. (2023) suggests that inflationary cost-push shocks widen the welfare distribution, while expansionary monetary policy shocks tighten it.

The remainder of the paper is organized as follows. In section 2 we introduce our model and the state-dependent Phillips curve. Section 3 describes our model calibration. In section 4 we present our results. Section 5 concludes.

¹More broadly our paper is related to a by now huge literature on Heterogenous Agent New Keynesian (HANK) models. It beyond the scope of this section to survey or list all contributions in this literature. For a partial body of work,, see the following papers and references therein: Acharya and Dogra (2020),Alves et al. (2020),Auclert and Rognlie (2018),Auclert et al. (2021) and Auclert, Rognlie, and Straub (2020, 2024a),Bayer et al. (2019),Bilbiie, Primiceri, and Tambalotti (2023), Bilbiie (2019, 2020, 2024), Bilbiie, Känzig, and Surico (2022), and Bilbiie, Monacelli, and Perotti (2024),Broer et al. (2020),Debortoli and Galí (2024),Fernández-Villaverde et al. (2023),Hagedorn et al. (2019),Kaplan and Violante (2018),McKay, Nakamura, and Steinsson (2016) and McKay and Reis (2016), Moll (2014),Oh and Reis (2012).

2 Model

This section develops our nonlinear HANK model. Our model setup is based on the nonlinear version of the canonical HANK model presented by Auclert, Rognlie, and Straub (2024b) and Auclert et al. (2021). It features a nonlinear Phillips curve with a state-dependent slope, Rotemberg nominal rigidities in wage setting, and a central bank that set the nominal policy rate.

2.1 Households

Households derive utility from consumption $c_{i,t}$ and disutility from supplying labor $n_{i,t}$. They earn income from three sources: labor income $z_{i,t}$, revenues from holding risk-free assets $r_t a_{i,t-1}$, and profit income $d_{i,t}$. We define capital income as the sum of asset and profit income, as we interpret profit income as dividends paid to households, similar to the definition in Debortoli and Galí (2024). Household i solves the following utility maximization problem:

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{c_{i,t}^{1-\sigma}}{1-\sigma} - \varphi \frac{n_{i,t}^{1+\nu}}{1+\nu} \right) \quad (1)$$

$$\text{s.t.} \quad c_{i,t} + a_{i,t} \leq (1 + r_t) a_{i,t-1} + z_{i,t} + d_{i,t} \quad (2)$$

$$a_{i,t} \geq \underline{a}. \quad (3)$$

Here, β denotes the discount factor, ν is the inverse Frisch elasticity, σ is the inverse intertemporal elasticity of substitution, and φ scales the disutility from labor. Households can buy assets subject to a borrowing constraint given by equation 3. The asset stock is idiosyncratic and type-specific. The households' after tax labor income is given by:

$$z_{i,t} = (1 - \tau_t) y_{i,t} \quad (4)$$

where $y_{i,t}$ and τ_t denote pre-tax labor income and labor income taxes, respectively. Pre-tax labor income is given by:

$$y_{i,t} = w_t n_{i,t} e_{i,t}. \quad (5)$$

Here, w_t is the real wage, $n_{i,t}$ denotes hours worked and $e_{i,t}$ is idiosyncratic, type-specific productivity. There exist n_e idiosyncratic productivity states. Following Auclert and Rognlie (2018) we allow for countercyclical inequality and income risk by introducing the following labor allocation rule:

$$n_{i,t} = n_t \frac{e_{i,t}^{\zeta_n \ln(n_t/n)}}{\mathbb{E} \left[e_i^{1+\zeta_n \ln(n_t/n)} \right]} \quad (6)$$

where $\zeta_n < 0$. Essentially, this rule generates countercyclical inequality since in a boom, low productivity households work more than in the steady state and high productivity households work less than in the steady state - thus implying that inequality declines in a boom. Also, with $\zeta_n < 0$, labor income risk is countercyclical, meaning in a boom, labor income risk falls. Note that if $\zeta_n = 0$, following Auclert, Rognlie, and Straub (2024b), we assume that all households are employed by a union, working the same amount of hours, which implies $n_{i,t} = n_t$. Note too that $\zeta_n = 0$ implies acyclical inequality and income risk.

Following Debortoli and Galí (2024), we assume that profit income is distributed among households in proportion to each household's productivity $e_{i,t}$. We also allow for countercyclical inequality in profit income and risk as above by assuming that aggregate dividends are allocated to households as follows:

$$d_{i,t} = d_t \frac{e_{i,t}^{1+\zeta_d \ln(d_t/d)}}{\mathbb{E} \left[e_i^{1+\zeta_d \ln(d_t/d)} \right]} \quad (7)$$

where $\zeta_d < 0$ and d_t denotes aggregate firm profits.

2.2 Phillips curve

Following Auclert, Rognlie, and Straub (2024b) we assume sticky wages but flexible prices in our model. In models with heterogeneous agents and nominal rigidities this assumption avoids countercyclical profits and thus large undesirable redistribution effects. Labor unions select the wage rate to maximize household utility and face quadratic nominal wage adjustment costs à la Rotemberg, governed by an adjustment cost parameter ϕ . In Appendix A we provide a step-by-step derivation of the following wage Phillips curve:

$$\pi_t^w (1 + \pi_t^w) = \kappa_t n_t \left(\varphi n_t^\nu - \frac{1}{\mu_w} w_t c_t^{-\sigma} \right) + \beta \mathbb{E}_t [\pi_{t+1}^w (1 + \pi_{t+1}^w)] + \epsilon_t, \quad (8)$$

with ϵ_t being a cost-push shock, $\mu_w = \frac{\varepsilon}{\varepsilon-1}$ as the steady state wage markup, and κ_t the state-dependent slope parameter of the Phillips curve.

Recent evidence suggests that especially in high and low inflation episodes a nonlinear, 'banana-shaped', Phillips curve is helpful to explain the inflation dynamics in the Great Recession and inflation surges. Harding, Lindé, and Trabandt (2022, 2023) use a Kimball aggregator in a nonlinear New Keynesian model to explain the missing deflation puzzle after the financial crisis and the post-covid inflation surge. Following Erceg, Jakab, and Lindé (2021) we introduce state-dependency of the Phillips curve slope by assuming the slope parameter κ_t to take the following functional form:

$$\kappa_t = \kappa e^{\chi(y_t - \bar{y})} \quad (9)$$

with $\chi \geq 0$ specifying the curvature of the Phillips curve and $\kappa = \frac{\varepsilon}{\phi}$. We use this simple functional form to model curvature and state-dependence while keeping our HANK model as transparent and computationally tractable as possible.

The functional form can be interpreted as follows, when the output is at its steady state value $\bar{y}$, the slope parameter κ_t becomes time-invariant. When output rises above steady state output, κ_t increases, therefore the Phillips Curve becomes steeper, accounting for the fact that wages are adjusted more strongly when the output gap is positive. When output falls below the steady state value κ_t decreases, and the Phillips curve becomes flatter.

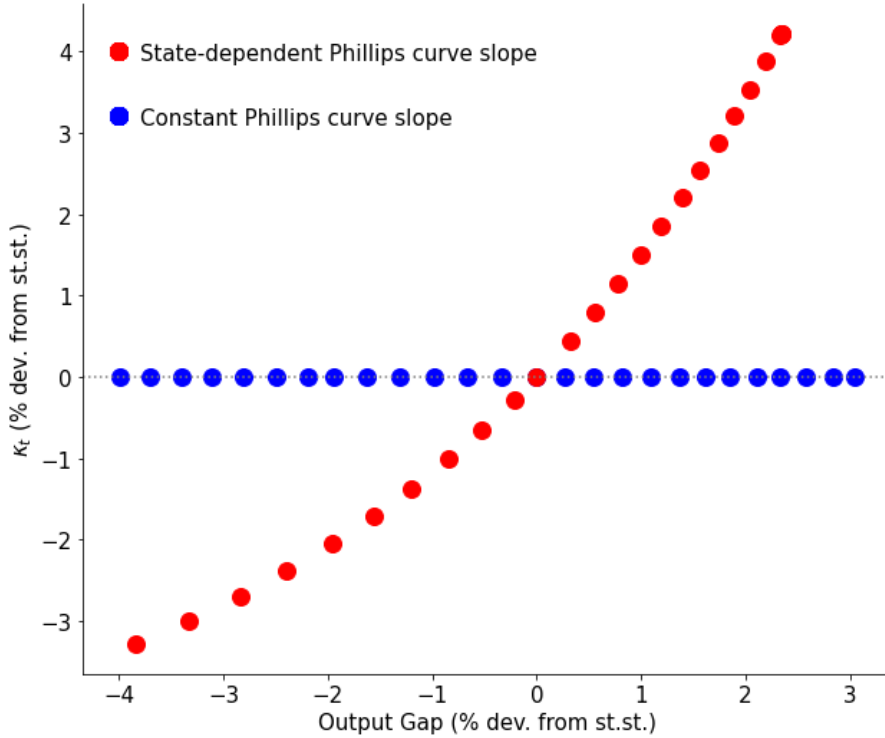


Figure 1: Relationship between output gap and the Phillips curve slope parameter κ_t .

The introduction of a state-dependent κ_t allows us analyze the impact of high and low inflation on inequality taking into account the results by Harding, Lindé, and Trabandt (2022) and Harding, Lindé, and Trabandt (2023).

Figure 1 illustrates the relationship between κ_t and the output gap in our model. We use the following parameters $\kappa = 0.05$ and $\chi = 25$ – see the calibration section below for details how these parameters are obtained. According to the figure, a positive output gap increases the slope of the Phillips curve. Conversely, a negative output gap reduces the slope of the Phillips curve. Thus, the model features a state-dependent slope of the Phillips curve.

2.3 Firms

The representative firm produces a continuum of intermediate goods $y_{j,t}$ using labor $n_{j,t}$ according to the following linear production function:

$$y_{j,t} = n_{j,t}. \quad (10)$$

The firm solves the following optimal flexible price setting problem under monopolistic competition:

$$\max_{P_{j,t}} D_{j,t} = P_{j,t} y_{j,t} - W_t n_{j,t} \quad (11)$$

$$\text{s.t. } y_{j,t} = n_{j,t} \quad (12)$$

$$y_{j,t} = \left(\frac{P_{j,t}}{P_t} \right)^{-\theta} y_t, \quad (13)$$

where $D_{j,t}$ are nominal profits, W_t is the nominal wage rate, $P_{j,t}$ is the price of good j and $\theta > 1$ is the substitution elasticity between intermediate goods. Equation 24 is the demand for good j . In equilibrium all firms charge the same price, i.e. $P_{j,t} = P_t$ so that the optimal price setting equation for firms can be expressed as:

$$P_t = \frac{\theta}{\theta - 1} W_t. \quad (14)$$

Thus, firms charge a constant mark-up $\mu_p = \frac{\theta}{\theta - 1}$ over their marginal cost (nominal wages). The real wage is therefore given by $w_t = \frac{1}{\mu_p}$. Real profits are given by

$$d_t = y_t - w_t y_t = \left(1 - \frac{1}{\mu_p} \right) y_t. \quad (15)$$

Note that real profits are procyclical in our model.

Finally, goods inflation π_t and wage inflation π_t^w are related as follows:

$$1 + \pi_t = (1 + \pi_t^w) \left(\frac{w_{t-1}}{w_t} \right). \quad (16)$$

2.4 Monetary and fiscal policy

We assume that the central bank follows a standard Taylor rule to set the nominal interest rate i_t :

$$1 + i_t = ((1 + r)(1 + \pi)) \left(\frac{1 + \pi_t}{1 + \bar{\pi}} \right)^{\phi_\pi} \left(\frac{y_t}{\tilde{y}_t} \right)^{\phi_y} e^{\gamma_t}, \quad (17)$$

where $\tilde{y}_t$ is potential output, which takes the value of steady-state output $\bar{y}$ in our model. r and π are the steady-state real interest rate and inflation rate, respectively. ϕ_π and

ϕ_y denote the Taylor rule parameters on inflation and the output gap. γ_t denotes an exogenous monetary policy shock which – following a large body of work in the HANK literature – we take as a stand-in for a demand shock affecting the economy.

The government budget constraint is given by:

$$\tau_t y_t + b_t = (1 + r_t)b_{t-1} + g_t, \quad (18)$$

where b_t is government debt and g_t is government consumption spending. We assume that government consumption spending is constant, i.e. $g_t = g$.

2.5 Aggregation

The aggregate resource constraint and asset market clearing condition are given by, respectively:

$$y_t = c_t + g_t \quad (19)$$

$$b_t = \int_0^1 a_{i,t} di. \quad (20)$$

2.6 Shocks

For the monetary policy shock γ_t and the cost-push shock ε_t , we specify the following AR(1) processes:

$$\gamma_t = \rho_\gamma \gamma_{t-1} + \eta_t^\gamma, \quad (21)$$

$$\varepsilon_t = \rho_\varepsilon \varepsilon_{t-1} + \eta_t^\varepsilon, \quad (22)$$

where $\eta_t^\gamma \sim \mathcal{N}(0, \sigma_\gamma^2)$ and $\eta_t^\varepsilon \sim \mathcal{N}(0, \sigma_\varepsilon^2)$ are exogenous shocks.

3 Calibration

We calibrate our HANK model to the following thirteen moments of key macroeconomic U.S. time series: the standard deviation, skewness and autocorrelation of inflation and GDP growth, and the correlation between inflation and GDP growth, and the mean, standard deviation and skewness of labor income inequality and labor income plus capital income inequality. We set model parameters so that the model-implied moments are

close to the mean data moments and within 95% data confidence intervals.²

3.1 Data

To match our model to U.S. macro data, we use U.S. data on annualized PCE inflation excluding food and energy and annualized GDP growth from 1967Q1 to 2019Q4. Figure 2 illustrates the evolution of quarterly inflation and GDP growth over the period considered. The inflation dynamics show that inflation surges are larger than inflation declines, resulting in positive skewness.

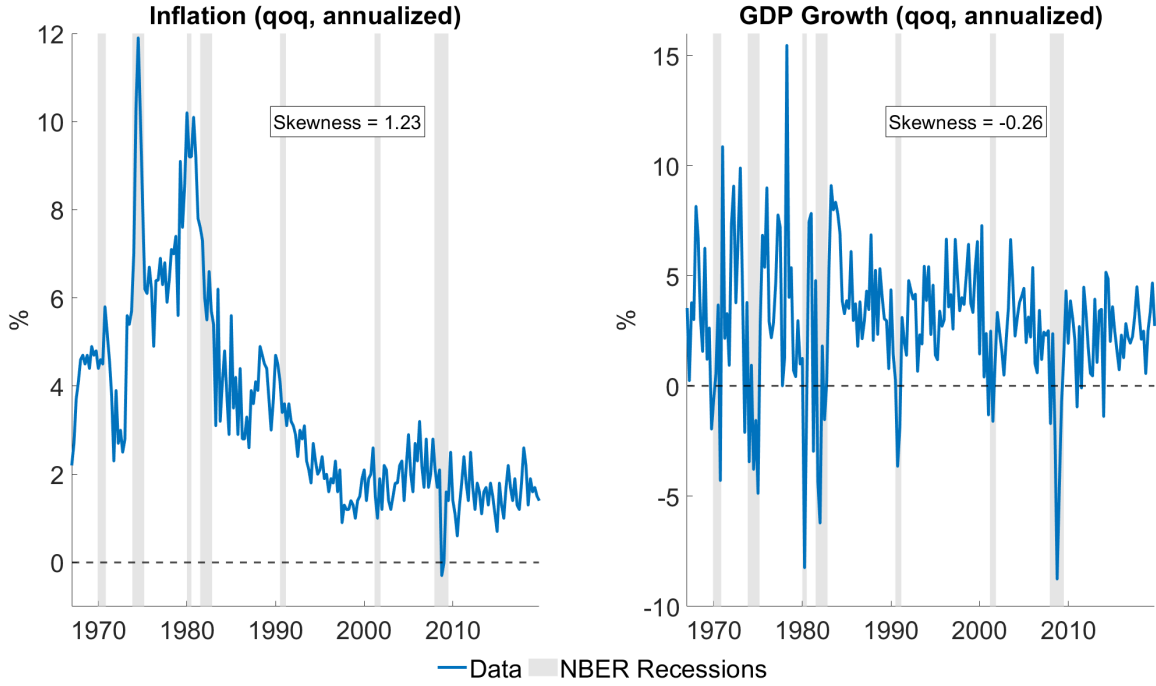


Figure 2: Quarterly U.S. inflation and GDP growth time series 1967-2019.

To measure income inequality in the U.S., we construct an inequality measure using data on household-level labor and capital income from the Current Population Survey (CPS). Following Heathcote et al. (2023) we rely on household data from the Annual Social and Economic (ASEC) supplement of the CPS from 1967 to 2019. We exclude households with zero or negative ASEC weight and households with no reference person or with no household member between 25 and 60 years of age. The labor income measure is constructed as the sum of wage earnings and income from self employment, divided by the number of adult equivalents in the household. The aggregate income measure additionally includes capital income like interest income or dividends.

²We adopt a calibration approach rather than a formal moment-based model estimation approach due to computational feasibility constraints. Specifically, for a given set of parameters, it takes about 2-3 seconds to solve our nonlinear HANK model for aggregate and distributional variables. To then compute model-implied moments, we need to simulate our model and it takes about 45 minutes for one simulation with sample size of 1000 quarters. Thus, it is unfortunately computationally infeasible to estimate model parameters.

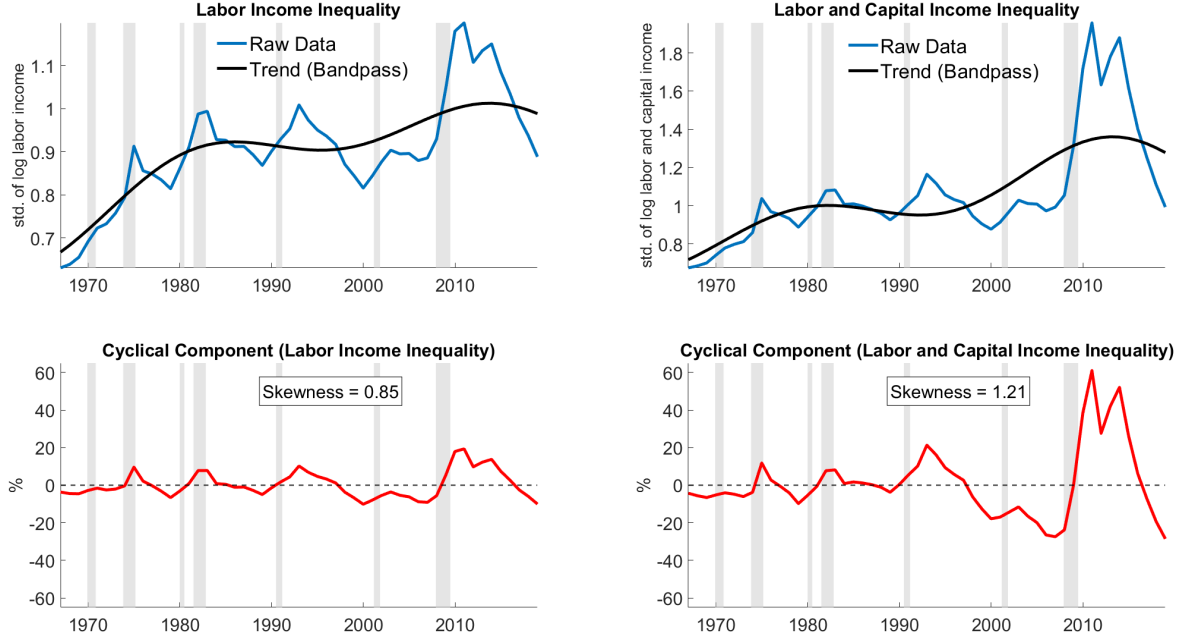


Figure 3: Cross-sectional standard deviation of log labor income and log labor + capital income (annual).

We use the standard deviation of log income to measure inequality in our sample. For each year in the sample, we discretize the income distribution using percentiles. We then take the logarithm of these percentiles and compute the weighted cross-sectional standard deviation in each year. Note that we exclude the lowest nine percentiles for labor income and the lowest seven percentiles for aggregate income, because income is equal to zero for them in some years. To still properly represent the bottom income percentiles, the tenth (eighth) percentile is weighed by the factor ten (eight), while all higher percentiles are weighed by the factor one. Note that the inequality data show a trend over time. Since we are interested to study business cycle dynamics in this paper, we detrend the inequality data. Following Bilbiie, Primiceri, and Tambalotti (2023) we use a band-pass filter that extracts fluctuations with periodicities lower or equal to 30 years. Figure 3 shows the raw data, trends, and business cycle components of the inequality data. Figure 3 shows that: i) inequality is countercyclical, i.e. increases in recessions, ii) inequality is highly volatile, and iii) inequality is skewed, i.e. over the business cycle, it rises more than it falls, on average. In Appendix B, we show that these empirical features of inequality data are robust to alternative detrending methods.

3.2 Parameters

The inverse elasticity of intertemporal substitution and the inverse Frisch elasticity are set to $\sigma = 1$ and $\nu = 2$, respectively. Further, the steady state gross wage mark-up is $\mu_w = 1.1$. We set the labor disutility parameter $\varphi = 1/\mu_w = 0.909$ such that steady

state labor and output are unity ($\bar{n} = \bar{y} = 1$, normalization). The Rotemberg wage adjustment cost parameter is set to $\phi = 208$. This parameter value is equivalent (in a linearized model version) to a Calvo wage stickiness parameter value that implies wage changes once every 1.5 years on average. The curvature parameter of the Phillips curve slope is set to $\chi = 25$ to match the positive skewness of inflation observed in the data. The steady-state gross price mark-up is $\mu_p = 1.2$. The Taylor rule parameters are set to $\phi_\pi = 1.5$ and $\phi_y = 0.2$. Steady-state government consumption spending is set to 20% of GDP. The steady-state net inflation rate is set to $\pi = 0$. The annual steady-state real interest rate is $\bar{r} = 0.01$. Annual steady-state government debt-to-GDP is set to 70%. With these parameter values, asset market clearing in steady state results in a value for the household discount factor β of roughly 0.97.

We simulate our model using exogenous AR(1) processes: γ_t (demand) and ε_t (cost-push). We set the persistence parameters for both AR(1) processes to $\rho_\gamma = 0.9$ and $\rho_\varepsilon = 0.9$. Stochastic shocks to both AR(1) processes are assumed to follow $\epsilon_t^\gamma \sim \mathcal{N}(0, \sigma_\gamma^2)$ and $\epsilon_t^\varepsilon \sim \mathcal{N}(0, \sigma_\varepsilon^2)$, where the standard deviations are set to $\sigma_\gamma = 0.000625$ and $\sigma_\varepsilon = 0.001$.

The heterogeneity in our model stems from heterogeneous idiosyncratic productivity states and heterogeneous asset holdings. There exist n_a idiosyncratic asset holding states, i.e. gridpoints on the asset grid, and n_e idiosyncratic productivity states. Following Auclert et al. (2021), the number of gridpoints is set to $n_a = 500$ for the asset distribution and $n_e = 11$ for the productivity grid. To match the moments of U.S. income inequality in the data, we set the cyclical labor income risk parameter to $\zeta_n = -4$ and the cyclical profit income risk parameter to $\zeta_d = -10$. This implies that both labor and profit income inequality and risk are countercyclical.

We solve the nonlinear HANK model using the Sequence-Space Jacobian software package developed by Auclert et al. (2021). The simulation results in Table 1 and 2 are generated by a long model simulation over 1000 quarters, where random unexpected demand and cost-push shocks are drawn every quarter. It takes about 45 minutes to simulate our nonlinear model to obtain the model-implied moments. Comparing the model-implied moments with those in the data reveals that overall, the model accounts reasonably well for the moments in the data. We discuss further details in the next section.

4 Results

In this section we report our results for the model simulation. First, we study how state-dependency in the Phillips curve slope affects the propagation of shocks in our model. Section 4.1 compares impulse responses to a demand and a cost-push shock in our model to a model with a constant Phillips curve slope. Section 4.2 presents the results from a model simulation of randomly drawn demand and supply shocks to assess

the ability of the model to match the characteristics from the data discussed in section 3.1.

4.1 Propagation of Shocks

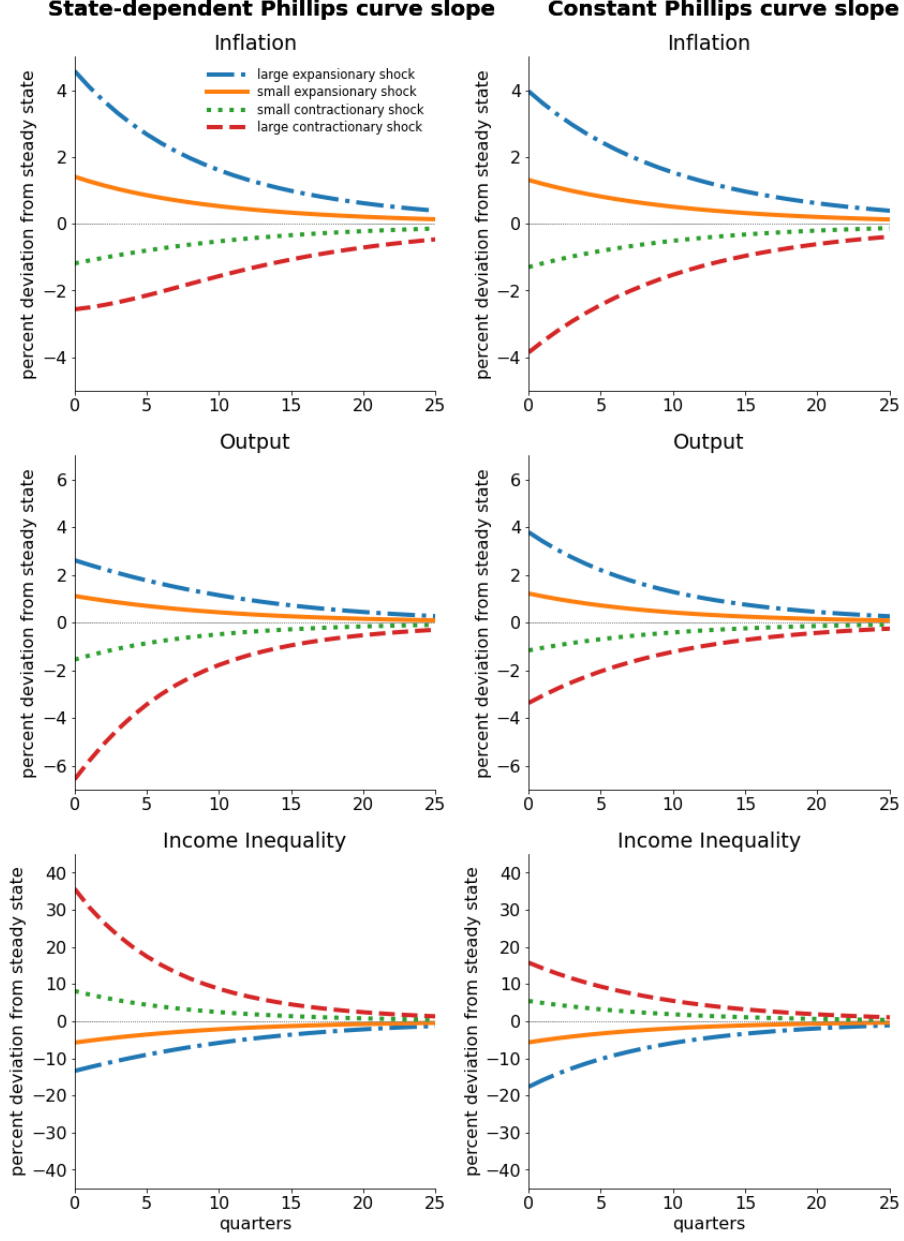


Figure 4: Impulse responses following a demand shock in our nonlinear HANK model.

The first column of Figure 4 shows the implications of a nonlinear, state-dependent Phillips curve slope for the propagation of demand shocks in the model. The Figure shows that introducing a state-dependent slope parameter κ_t into the Phillips curve has no significant impact for small demand shocks, represented by the green dotted and orange line. This is due to the approximate linearity of the Phillips curve close to the

steady state (see Figure 1). However, for larger shocks the responses following equal-sized positive vs. negative demand shocks become increasingly asymmetric.

The intuition for the asymmetries in the responses is that wages are adjusted more strongly when the economy is in a boom, i.e. when the output gap is positive, which leads to higher inflation. This dampens the upward adjustment in output following a positive demand shock. In a recession, i.e. when the output gap is negative, the drop in real wages is attenuated due to the nonlinearity of Phillips curve slope, which in turn dampens the drop in inflation. In this case the decrease in output is amplified as the central bank decreases the nominal interest rate less, as inflation falls less, and therefore the positive second-round effect of a lower real interest rate is smaller and output decreases more strongly.

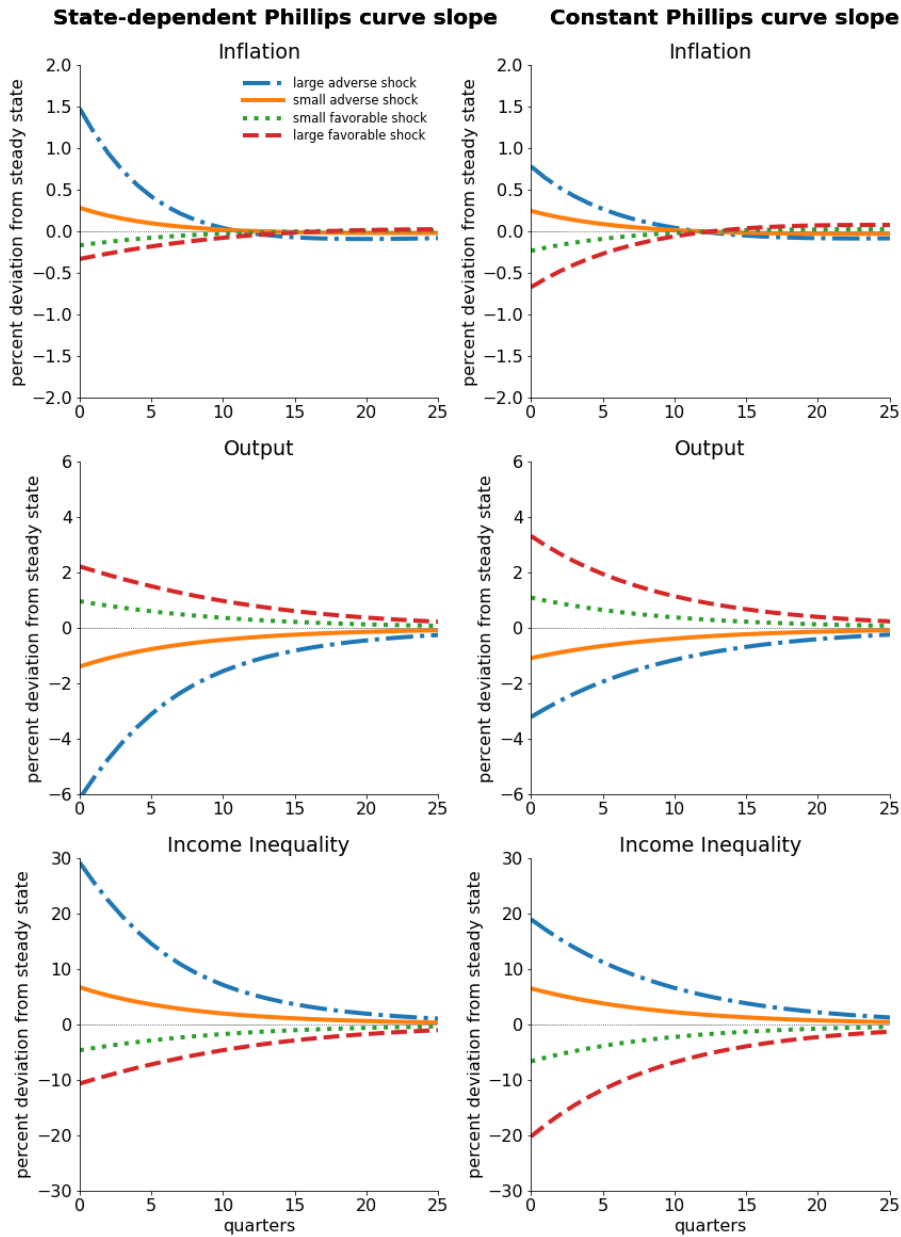


Figure 5: Impulse responses following a cost-push shock in our nonlinear HANK model.

The response of the standard deviation of log income shows that this asymmetric response to shocks also carries over to our measure for inequality. Following a negative demand shock, output and inflation decrease. This decrease in output leads to a stronger dispersion of labor productivity states, as agents face countercyclical inequality and income risk. A negative demand shock thus increases the risk of becoming less productive, which leads to an increase in inequality. Similarly, the fall in output increases the risk of having lower profit income due to countercyclical profit income inequality and risk, which increases capital income inequality. In the nonlinear model, the fall in output is amplified due to the state-dependency of the Phillips curve explained above. This in turn exacerbates the increase in income inequality. Conversely, following a positive demand shock, output and inflation increase. Therefore, income inequality decreases, as agents face lower risk of becoming less productive and having lower profit income. As the increase in output is dampened in our nonlinear model, the decrease in inequality is dampened as well. The second column of Figure 4 shows that using a constant Phillips curve slope, the impact of small and large demand shocks is symmetric, meaning that equal-sized positive and negative demand shocks cause equal-sized increases and decreases in e.g. inflation.

Figure 5 shows that the state-dependent Phillips curve generates similar asymmetries in the economy's response to a cost-push shock. In particular, the inflation increase following a positive supply shock is amplified, whereas the decline after an equal-sized negative supply shock is dampened. When a positive cost-push shock raises inflation, the stronger inflationary response induces the central bank to raise nominal interest rates more aggressively. This leads to a larger contraction in output compared to our model featuring a constant Phillips curve. As in the case of a contractionary demand shock, this amplified drop in output exacerbates income inequality, as agents face higher countercyclical labor and profit inequality and income risk. Conversely, following a negative cost-push shock (cost-pull shock), inflation falls, prompting the central bank to lower interest rates. However, since inflation declines less than in the model featuring a constant Phillips curve, the increase in output is also more muted. Consequently, the reduction in income inequality is less pronounced.

To analyze the propagation of shocks to income inequality in more detail, Figure 6 breaks down the response of income inequality to a demand and cost-push shock into its labor and capital income components. The shocks correspond to the large adverse shocks in Figure 4 and 5. Labor income inequality accounts for approximately 80 percent of the aggregate response to these shocks. The remaining increase in income inequality is driven by countercyclical profit income inequality and profit income risk. Following an inflationary cost-push shock or a contractionary demand shock, output and aggregate profits decline, increasing the risk of lower profit income for households.

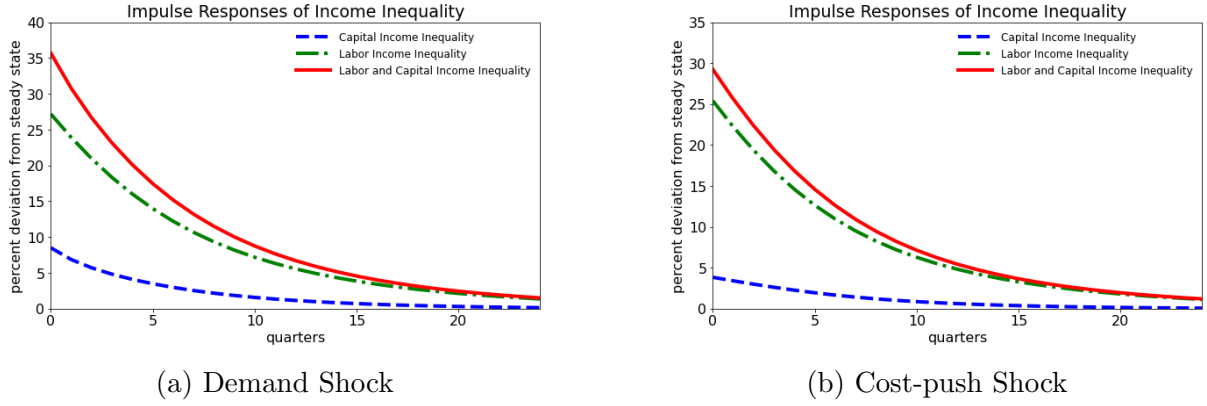


Figure 6: Impulse responses of income inequality components.

4.2 Model vs. Data Comparison

Table 1 compares the results from our model simulation of randomly drawn demand and cost-push shocks over 1000 quarters to the moments observed in the data. The nonlinear model with the state-dependent Phillips curve slope matches the standard deviations, skewness and (auto-)correlations of inflation and GDP growth well. In contrast, the model with a constant Phillips curve matches the data considerably less well, especially the positive skewness of inflation.

Table 1: Model vs. Data Comparison

	Model			Data	
	State-dep. slope	Const. slope	Mean	95% CI	
Standard deviation π_t	2.50	1.82	2.29	2.00	2.55
Skewness π_t	1.08	-0.11	1.23	0.93	1.53
Autocorrelation π_t	0.90	0.92	0.92	0.89	0.94
Standard deviation Δy_t	3.40	2.64	3.14	2.71	3.58
Skewness Δy_t	0.13	0.03	-0.22	-0.93	0.49
Autocorrelation Δy_t	-0.01	-0.01	0.30	0.15	0.45
Correlation $\pi_t, \Delta y_t$	-0.21	-0.18	-0.06	-0.25	0.14

Figure 7 plots the simulation results of inflation, output, and inequality for a sample of 1000 demand and cost-push shocks in our model. The figure also shows the results using the same shocks in a version of our model with a constant Phillips curve slope. It shows that inflation surges are almost twice as large when the slope of the Phillips curve is state-dependent compared to the constant slope case. By contrast, deflationary pressures are dampened, at least to some extent. The asymmetry of inflation impulse responses is more pronounced for larger shocks. In addition, Figure 7 shows that over a long time period in which the economy is hit by supply and demand shocks, inequality tends to increase much more strongly than it decreases. The standard deviation of log household income

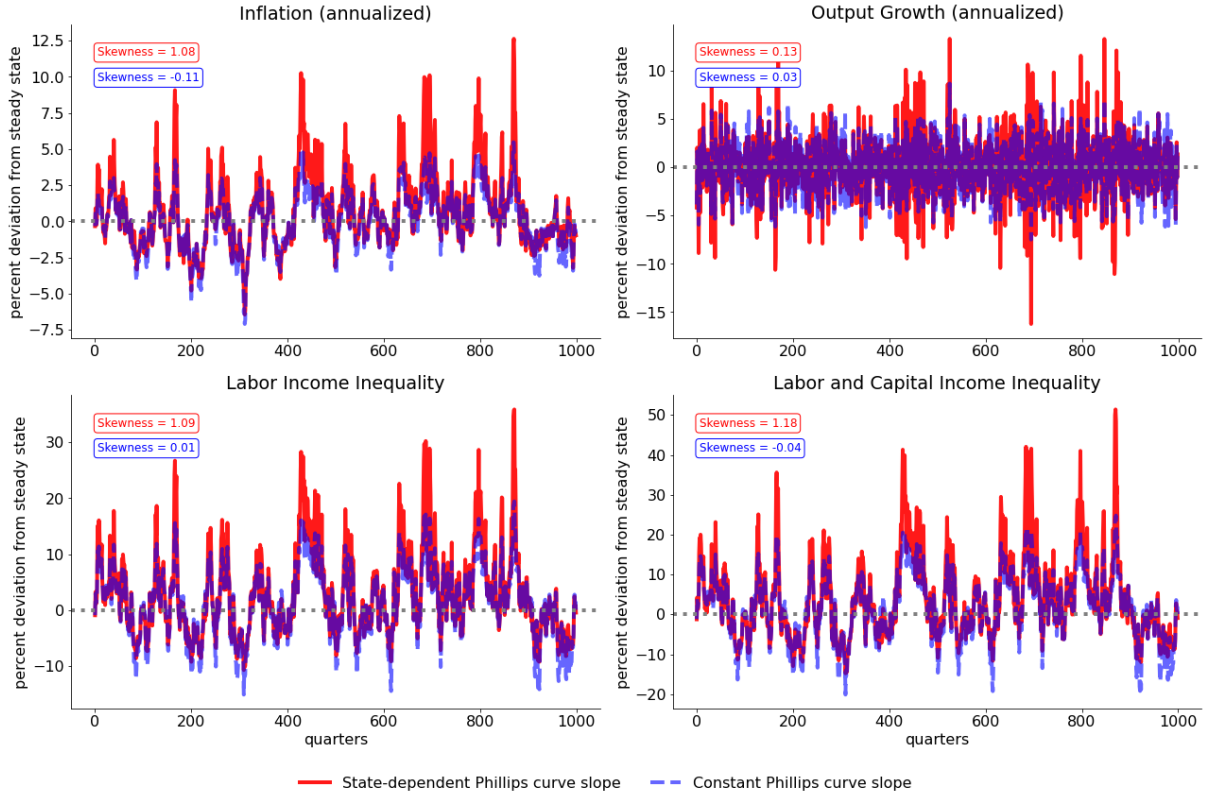


Figure 7: Long simulation in our nonlinear HANK model over 1000 quarters with demand and cost push shocks.

increases by almost twice as much in the state-dependent Phillips curve slope model compared to a model including a constant Phillips curve, while the reductions in boom periods are dampened. Also, labor income inequality in our model increases in recessions. The decrease in output induces a stronger dispersion in productivity states, which leads to an increase in income inequality. Finally, Figure 7 shows that the increase in aggregate income inequality is even more pronounced in recessions due to countercyclical profit income equality and risk.

Using our long simulation we can not only match the mean standard deviation of log household income of the U.S., but also the skewness of the standard deviation of log household labor income, as shown in Table 2. The inclusion of a state-dependent Phillips curve slope allows the model to reproduce the positive skewness of both labor and aggregate household income inequality observed in the data. This implies that income inequality increases more strongly in recessions than it falls in boom periods. A model with a constant Phillips curve slope predicts a skewness close to zero, which implies a symmetric response of inequality to equal-sized favorable and adverse shocks.

In our framework, countercyclical labor and profit income inequality and risk, governed by the parameters ζ_n and ζ_d , are crucial to account for the pronounced volatility of inequality observed in the data. Countercyclical labor income risk implies that households are more likely to experience declines in productivity during recessions. This feature

Table 2: Standard Deviation of Logs: Data vs. Model Comparison

	Model		Data		
	State-dep. slope	Const. slope	Mean	95% CI	
<i>Labor Income Inequality</i>					
Mean	0.92	0.92	0.92	0.88	0.95
Skewness	1.09	0.01	0.85	0.32	1.34
Standard Deviation	0.07	0.05	0.07	0.05	0.08
<i>Labor and Capital Income Inequality</i>					
Mean	0.93	0.93	1.07	0.99	1.15
Skewness	1.18	-0.04	1.15	0.40	1.77
Standard Deviation	0.10	0.07	0.18	0.13	0.23

is intended to capture the elevated risk of unemployment during economic downturns, which may be an important driver of inequality fluctuations over the business cycle (see Chang and Schorfheide (2023)). Similarly, countercyclical profit income inequality and risk serves as a proxy for a risky asset, allowing for greater volatility in capital income inequality without requiring a full two-asset structure.

5 Conclusion

We introduce a nonlinear Phillips curve with a state-dependent slope into a standard Heterogeneous Agent New Keynesian (HANK) model. This modification enables our model to replicate the positive skewness of inequality and inflation observed in post-war U.S. data, a feature that a model with a constant Phillips curve slope fails to capture. Our results indicate that output declines lead to rising income inequality due to countercyclical labor and profit income inequality and risk. Households face an increased likelihood of lower productivity, reducing both labor and profit income. The state-dependent Phillips curve amplifies these effects, exacerbating inequality increases in recessions while dampening its reduction during booms. This result holds for supply and demand shocks in our model.

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A Derivation of wage Phillips curve

In this section, we derive the nonlinear wage Phillips curve given by equation A. In our set-up, unions face quadratic nominal wage adjustment costs à la Rotemberg (1982). At time t , union j sets its wage $W_{j,t}$ to maximize the utility of its average worker as in Auclert, Rognlie, and Straub (2024b). The maximization problem is defined as follows:

$$\max_{W_{j,t}} \sum_{s=0}^{\infty} \mathbb{E}_t \left[\frac{c_t^{1-\sigma}}{1-\sigma} - \varphi \frac{n_t^{1+\nu}}{1+\nu} - \frac{\phi}{2} \left(\frac{W_{j,t+s}}{W_{j,t+s-1}} - 1 \right)^2 \right], \quad (23)$$

$$\text{s.t.} \quad n_{j,t} = \left(\frac{W_{j,t}}{W_t} \right)^{-\varepsilon} n_t. \quad (24)$$

Unions combine individual labor into tasks, which face demand given by (24).

Using (24), household real earnings are defined as follows:

$$z_t = \int_0^1 \frac{W_{j,t}}{P_t} n_{j,t} dj = \frac{1}{P_t} \int_0^1 W_{j,t} \left(\frac{W_{j,t}}{W_t} \right)^{-\varepsilon} n_t dj \quad (25)$$

We assume that all income from the union wage change is consumed immediately, which implies $\frac{\partial c_t}{\partial W_{j,t}} = \frac{\partial z_t}{\partial W_{j,t}}$ by the envelope theorem:

$$\frac{\partial c_t}{\partial W_{j,t}} = \frac{\partial z_t}{\partial W_{j,t}} = (1-\varepsilon) \frac{1}{P_t} \left(\frac{W_{j,t}}{W_t} \right)^{-\varepsilon} n_t = (1-\varepsilon) \frac{1}{P_t} n_{j,t} \quad (26)$$

The derivative of hours worked by household i (from (24)) with respect to wage $W_{j,t}$ is given by:

$$\frac{\partial n_{i,t}}{\partial W_{j,t}} = -\varepsilon \frac{n_{j,t}}{W_{j,t}} \quad (27)$$

Using (26) and (27), we obtain the following first-order condition of the union:

$$c_t^{-\sigma} (1-\varepsilon) \frac{1}{P_t} n_{j,t} + \varepsilon \varphi n_t^\nu \frac{n_{j,t}}{W_{j,t}} - \phi \frac{1}{W_{j,t-1}} \left(\frac{W_{j,t}}{W_{j,t-1}} - 1 \right) + \beta \phi \mathbb{E}_t \frac{W_{j,t+1}}{W_{j,t}^2} \left(\frac{W_{j,t+1}}{W_{j,t}} - 1 \right) = 0 \quad (28)$$

In equilibrium all unions set the same wage, which implies $W_{j,t} = W_t$ and $n_{j,t} = n_t$:

$$c_t^{-\sigma} (1-\varepsilon) \frac{1}{P_t} n_t + \varepsilon \varphi n_t^\nu \frac{n_t}{W_t} - \phi \frac{1}{W_{t-1}} \left(\frac{W_t}{W_{t-1}} - 1 \right) + \beta \phi \mathbb{E}_t \frac{W_{t+1}}{W_t^2} \left(\frac{W_{t+1}}{W_t} - 1 \right) = 0 \quad (29)$$

Define wage inflation such that $\pi_t^W = \frac{W_t}{W_{t-1}} - 1$:

$$c_t^{-\sigma}(1 - \varepsilon)\frac{1}{P_t}n_t + \varepsilon\varphi n_t^\nu \frac{n_t}{W_t} - \phi \frac{1}{W_{t-1}}\pi_t^W + \beta\phi\mathbb{E}_t \frac{1}{W_t}(\pi_{t+1}^W + 1)\pi_{t+1}^W = 0 \quad (30)$$

$$\Leftrightarrow c_t^{-\sigma}(1 - \varepsilon)w_t n_t + \varepsilon\varphi n_t^\nu n_t - \phi(\pi_t^W + 1)\pi_t^W + \beta\phi\mathbb{E}_t(\pi_{t+1}^W + 1)\pi_{t+1}^W = 0, \quad (31)$$

where $w_t = \frac{W_t}{P_t}$ is the real wage.

Finally, this can be rearranged such that we obtain our nonlinear wage Phillips curve:

$$\pi_t^w(1 + \pi_t^w) = \frac{\varepsilon}{\phi}n_t \left(\varphi n_t^\nu - \frac{\varepsilon - 1}{\varepsilon}w_t c_t^{-\sigma} \right) + \beta\mathbb{E}_t[\pi_{t+1}^w(1 + \pi_{t+1}^w)]. \quad (32)$$

In our model we then introduce the state-dependent slope parameter $\kappa_t = \frac{\varepsilon}{\phi}e^{\chi(y_t - \bar{y})}$ as described in section 2. Equation (32) represents the special case when output is at potential, i.e. $y_t = \bar{y}$.

B Robustness of de-trending method

Table 3: Data moments std of logs with different time series filters

	Band-pass 30y (baseline)	Band-pass 8y	HP filter	Hamilton
<i>Labor Income Inequality</i>				
Skewness	0.76	0.58	0.52	1.31
Standard Deviation	0.05	0.03	0.03	0.07
<i>Labor and Capital Income Inequality</i>				
Skewness	1.15	0.49	0.74	2.09
Standard Deviation	0.19	0.08	0.07	0.18

Table 3 presents the skewness and standard deviation of the standard deviation of log labor income (LI) and log labor plus capital income (LCI) in the dataset described in section 3.1 using four different de-trending methods to check the robustness of using the band-pass filter to extract fluctuations with periodicities lower than 30 years. First, we use a band-pass filter with an upper bound of 8 years, as often used for business cycle analysis. This yields a smaller, but still positive, skewness and a smaller standard deviation. Next, we use a two-sided Hodrick-Prescott (HP) filter, where we set the smoothing parameter to $\lambda = 6.25$, as is suggested for annual data by Ravn and Uhlig (2002). The results for the two-sided HP filter are close to our baseline results. Finally, we use a Hamilton filter (Hamilton (2018)), where we set the lead length to 2 and the lag length to 1, as recommended for annual data. The resulting cyclical component shows almost the same volatility as in the baseline analysis, but an even higher skewness. On average, all de-trending methods yield a skewness of 0.79 for labor income and 1.12 for labor and capital income.