

Product turnover and endogenous price flexibility in uncertain times*

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Abstract

Price setting has become more flexible following several large adverse shocks (Covid-19, the Ukraine War). We argue that a shift to a high-uncertainty regime incentivizes firms to invest in their ability to adjust prices. We formalize this idea in a general equilibrium model with endogenous price flexibility and entry-exit. Under higher uncertainty, firms set prices more flexibly. This improves their resilience, reducing exit and output losses in response to negative supply shocks. Local projections on US data between 1993q1 and 2024q1 show that this predicted state-dependence of impulse responses is borne out in the data. Finally, higher monetary policy uncertainty can be welfare-improving when shocks are large.

Keywords: entry, exit, price flexibility, supply shocks, uncertainty.

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1 Introduction

Recent empirical evidence suggests that the frequency of price adjustment has risen in the wake of several large adverse shocks such as the Covid-19 pandemic or the Ukraine war, see Cavallo et al. (2024), Balleer et al. (2022), Montag and Villar (2022). At the same time, various measures of uncertainty have gone up, as noted by Altig et al. (2020) and Anayi et al. (2022), among others.¹

In this paper, we relate the increased flexibility in price setting to the rise in economic uncertainty that firms face. We argue that, in a regime of high uncertainty, more firms are likely to invest in their capability to adjust prices quickly.² This investment comes at a cost but makes firms more resilient. It enhances a firm’s chances to stay in the market when adverse shocks occur. Thus, endogenous price flexibility under uncertainty has important implications for firm dynamics and consequently for product turnover.

We formalize the idea that firms respond to increased uncertainty by investing in their ability to change prices in a general equilibrium model with endogenous price flexibility and firm/product entry-exit.³ We build on the model of endogenous entry and exit by Bilbiie and Melitz (2022). In that model, the output effects of a(n adverse) supply shock are amplified in a sticky-price model relative to a flexible-price model. This demand amplification is absent in the standard New Keynesian model with a constant number of producers. In their framework, it is more costly to change prices than it is for firms to enter or exit the market, or for multiproduct firms to stop producing certain goods.⁴ Here, we endogenize the degree of price flexibility following Devereux (2006). While that paper studies demand uncertainty, we focus on the supply side and allow for the volatility

¹ Notably, firms’ perception of uncertainty shifted. For instance, at the monthly frequency participants in the Bank of England’s Decision Makers Panel are asked about their expected prices and sales one year ahead. Respondents provide a point forecast as well as a distribution of expected outcomes. The survey shows a clear upward shift in firms’ perceived uncertainty after December 2019, i.e. at the start of the pandemic.

² In a similar spirit, Hall (2023) argues that a ‘seller in a more volatile environment will adopt policies that involve more frequent adjustments of the seller’s price, compared to one in a less volatile environment.’

³ We use the terms ‘firm’ and ‘product’ interchangeably throughout the paper.

⁴ Florin Bilbiie cites flight cancellations as an example of product exit: <https://twitter.com/FlorinBilbiie/status/1545367693880090624?>. Cavallo and Kryvtsov (2021) provide evidence of consumer product shortages during the pandemic.

of productivity to vary.

Our model predicts that, when productivity uncertainty is high, a larger share of firms change their prices each period, making price setting more flexible and firms more resilient to adverse supply shocks. Consequently, when such a shock hits, fewer firms exit the market than would under predetermined price flexibility. As a consequence, firm dynamics and the associated demand amplification play a much smaller role in the transmission of supply shocks compared to an environment where uncertainty is low.

This key prediction of our model, the dependence of macroeconomic responses to supply shocks on productivity uncertainty, is confirmed in the data. We identify regimes of TFP volatility in the US. We then show, using state-dependent local projection regressions, that producer prices respond more strongly, and net entry responds less strongly, to productivity shocks in times of high TFP volatility. This finding stands up to various robustness checks.

We also use our framework to study the consequences of monetary policy uncertainty. Introducing money supply volatility in our model with productivity shocks brings to the fore a hitherto unexplored channel of monetary policy. Similar to the case of productivity uncertainty, higher monetary policy uncertainty incentivizes firms to invest in price flexibility. This affects the transmission of supply shocks. When adverse productivity shocks hit, exit and output losses are smaller, and producer prices respond more strongly, under high monetary policy uncertainty. We show that higher monetary policy uncertainty can even be welfare-improving when productivity shocks are large.

Related literature. The pre-pandemic literature has found that greater volatility is associated with higher aggregate price flexibility. In micro data underlying the US consumer price index, Vavra (2013) shows that the standard deviation of price changes (i.e., price change dispersion) comoves strongly with the frequency of price adjustment. He proposes increases in firm-level volatility as a driving force that simultaneously increases both the frequency of adjustment and price change dispersion. Using German micro data, Bachmann et al. (2019) show that heightened firm-level volatility/uncertainty – measured as expectation errors from the Ifo Business Climate Survey – increases the probability of

a price change and leads to larger price changes.

A large theoretical literature considers the price setting decision of firms. According to the strand of the literature referred to as ‘state-dependent pricing models’ (Barro, 1972; Dotsey et al., 1999; Golosov and Lucas, 2007; Alvarez et al., 2011), a firm has a fixed cost of changing its price (‘menu cost’). When its current price differs from its desired price by a large amount, the firm finds it worthwhile to change its price. This implies that sufficiently large shocks will lead to price changes. These models are about *ex post* price flexibility, i.e. after a shock has occurred.

Instead, the model used here entertains the notion of *ex ante* price flexibility. Dutta et al. (2002) and Zbaracki et al. (2005) put forward the notion of ‘pricing capital’. They see pricing as a strategic capability, which requires investment in human resources, computer systems and organizational structures. The evidence in Zbaracki et al. (2004) furthermore suggests that the costs of determining and setting the right price for a product go beyond physical menu costs and encompass substantial managerial and customer costs as well.

Building pricing capability could take several forms. For instance, a firm may want to move into online sales. This requires investments in computer infrastructure and IT personnel. Gorodnichenko et al. (2018) and Rudolf and Seiler (2022) show that online prices change more frequently than offline prices. Cavallo (2018) documents that online competition has raised the frequency of price changes by US retailers between 2008 and 2017. Pricing decisions are increasingly aided by automated algorithms, while webscraping helps monitor competitors’ prices. Adams et al. (2024) document a tenfold rise in the adoption of AI pricing, which we might interpret as an investment in price flexibility, since 2010. A related example is supermarkets introducing electronic price displays, which allow for cheaper and more efficient price adjustment.⁵ Another type of investment in price flexibility are ‘price escalation clauses’. These clauses allow firms to increase a previously agreed-upon price in the event of unexpected cost rises, e.g. due to higher prices of raw materials or energy. Contracts with such clauses require the expertise of lawyers. The Bundesbank’s firm survey of August 2022 shows that the proportion of

⁵ For a quantification of the costs investing in an electronic shelf label system, see Levy et al. (1997).

firms using price escalation clauses has doubled since 2021, rising to 34%.⁶

We thus consider the firm’s decision to invest in price flexibility in response to a regime change, rather than its price setting decision following a one-time shock. As an implication, under a high volatility regime also a relatively small shock induces a large number of firms to change prices. This prediction finds support in Arndt and Enders (2023), who show that consumer prices respond more strongly to producer price shocks in high (inflation) volatility regimes, while no such state-dependence is observed with respect to the shock size.^{7,8} Our empirical evidence is complementary to Arndt and Enders (2023) as we find that productivity shocks lead to larger price responses when TFP volatility is high.

The following models share similarities with our framework. In Blanco et al. (2024), the fraction of firms’ price changes rises endogenously with inflation, creating an ‘inflation accelerator’ feedback loop that steepens the Phillips curve during high-inflation episodes. In Ball et al. (1988), firms choose the frequency at which they change prices. The equilibrium interval between price changes is shown to be decreasing in the variance of demand shocks. Werning (2022) adjusts the standard menu-cost model to allow for fixed costs of changing the pricing bands. He thereby entertains the notion that firms have to invest in price flexibility. Flynn et al. (2023) endogenize price flexibility by allowing firms to choose a supply function, i.e. the optimal adjustment in prices and quantities. However, they focus on demand uncertainty.⁹ None of those papers considers endogenous entry/exit.

Our paper most relates to Bilbiie and Melitz (2022), who show that supply shocks are amplified under entry and exit – relative to the New Keynesian model with a constant number of producers – *when prices are sticky*. Here, we endogenize price flexibility in a model with entry and exit for different levels of productivity uncertainty. Supply

⁶ See <https://www.bundesbank.de/en/bundesbank/research/survey-on-firms>.

⁷ In Benigno and Eggertsson (2023), the effect of supply shocks is also regime-dependent; in an exceptionally tight labor market, inflation responds more strongly.

⁸ Using US micro price data, Nakamura et al. (2018) find that the frequency of price adjustment rises with the inflation rate. Similar evidence is reported by Gagnon (2009) for Mexico and Alvarez et al. (2018) for Argentina. This supports the view that firms’ price setting behavior depends on the aggregate volatility that the firm is confronted with.

⁹ The model in Gasteiger and Grimaud (2023) also features an endogenous degree of price flexibility, which is, however, not micro-founded. Kurozumi (2016) analyses the determinacy properties of a Calvo (1983) model with endogenous price stickiness.

shocks are amplified under entry and exit – relative to the New Keynesian model – *when uncertainty is low*.

The remainder of the paper is structured as follows. Section 2 outlines the individual firm’s choice of price flexibility, followed by the general equilibrium model with entry and exit. In Section 3, we study the transmission of adverse supply shocks when price adjustment is endogenous to productivity uncertainty. Section 4 employs an empirical Markov switching model, combined with local projections, to estimate the effects of productivity shocks on prices and net entry in times of high and low TFP volatility in the US. In Section 5, we discuss monetary policy uncertainty and its welfare implications. Section 6 concludes.

2 Model

We develop a model in which firms face aggregate productivity uncertainty. Firms must decide *ex ante* whether or not to invest in price flexibility, subject to an idiosyncratic cost of doing so. Once productivity is realized, firms decide whether or not to produce. Production is subject to a fixed cost, which pins down profits and the number of producers in equilibrium. The two decisions, investment in price flexibility and the production choice, jointly determine the aggregate degree of price flexibility.

The timing in the model is as follows.

1. Potential producers learn their idiosyncratic cost of investing in price flexibility; they decide whether or not to become flexible price setters.
2. An aggregate productivity level is drawn.
3. Flexible-price firms reset their prices. Sticky-price firms keep prices at their predetermined level.
4. Firms decide whether or not to produce. Those firms with non-negative expected profits choose to produce and incur a fixed cost of production. This determines the equilibrium number of producers.

In Section 2.1, we show how price flexibility is determined as the solution to a particular investment problem facing the firm. Then in Section 2.2, we embed the firm's choice of price flexibility into a general equilibrium model with an endogenous production decision. Appendix A provides more details on the theoretical model.

2.1 Firm's choice of price flexibility

Firms produce differentiated intermediate goods indexed by $\omega \in [0, N]$ and compete under monopolistic competition, taking the nominal wage W as given. We assume a production function with labor as the only input,

$$Y(\omega) = Al(\omega), \quad (1)$$

where A is productivity and $l(\omega)$ is labor input that firm ω uses for production. The firm's operating cost is $Wl(\omega)$ or, from the production function (1), $W \frac{Y(\omega)}{A}$. The firm faces the following demand function:

$$Y(\omega) = \left(\frac{P(\omega)}{P} \right)^{-\theta} Y = P(\omega)^{-\theta} \hat{Y}, \quad (2)$$

where $\hat{Y} = P^\theta Y$ is market demand and $\theta > 1$ is the elasticity of substitution between goods varieties in the final goods firm's production function (see below). The intermediate goods firm chooses a price $P(\omega)$ to maximize expected discounted profits given by nominal revenues minus operating cost, $\mathbb{E}\Gamma\{P(\omega)Y(\omega) - WY(\omega)/A\}$, where Γ is the firm's discount factor. One can show that the latter equals $\Gamma = (PY)^{-1}$. Replacing firm output $Y(\omega)$ using the demand constraint (2), expected profits can be written as

$$\mathbb{E}\Gamma \left\{ \left[P(\omega) - \frac{W}{A} \right] \left(\frac{P(\omega)}{P} \right)^{-\theta} Y \right\}. \quad (3)$$

Then the price setting problem is to choose $P(\omega)$ in order to maximize (3).

Following Devereux (2006), we stipulate a firm-specific cost of price flexibility given by $\Phi(\omega)$, which is measured in terms of labor. If the firm invests in price flexibility, it

can choose its price after observing $\Theta = (P, Y, W, A)$.¹⁰ The first order condition to this problem is

$$\tilde{P} = \delta W/A, \quad (4)$$

where $\delta = \theta/(\theta - 1)$. The flexible-price firm's expected operating profits – excluding the costs of investing in price flexibility and fixed production costs – under the optimal price setting rule are

$$\tilde{V}(\Theta) = (\delta^{1-\theta} - \delta^{-\theta})\mathbb{E}\{\Gamma(W/A)^{1-\theta}\hat{Y}\}. \quad (5)$$

If the firm does *not* invest in price flexibility, the first order condition to its price setting problem is instead

$$\bar{P} = \delta \frac{\mathbb{E}\{\Gamma(W/A)\hat{Y}\}}{\mathbb{E}\{\Gamma\hat{Y}\}}. \quad (6)$$

Plugging the optimal price (6) into (3) and rearranging, the firm's expected operating profits are given by:

$$\bar{V}(\Theta) = (\delta^{1-\theta} - \delta^{-\theta})\mathbb{E}\{\Gamma(W/A)\hat{Y}\}^{1-\theta}\mathbb{E}\{\Gamma\hat{Y}\}^\theta. \quad (7)$$

The firm chooses to be a flexible price setter if the gain in expected discounted profits exceeds the expected discounted costs of investing in price flexibility, $\mathbb{E}\{\Gamma W\Phi(\omega)\}$. Because the cost of flexibility $\Phi(\omega)$ is known ex ante, we can take this term out of the expectations operator and write:

$$\Delta(\Theta) \equiv \frac{\tilde{V}(\Theta) - \bar{V}(\Theta)}{\mathbb{E}\{\Gamma W\}} \geq \Phi(\omega). \quad (8)$$

In (8), the term $\Delta(\Theta)$ captures the gains from price flexibility, while the cost of investing in price flexibility $\Phi(\omega)$ is an increasing and continuous function of $\omega \geq 0$. In Appendix A.1.1, we lay out in more detail that (8) describes the gains from price flexibility both in a general equilibrium model with product turnover (as specified in Section 2.2) and in a model without product turnover.

¹⁰ In Burstein (2006), a firm chooses a sequence of future prices conditional on current information. In contrast to our setup, a pricing plan cannot be made contingent on future aggregate variables.

To illustrate the role of uncertainty for the choice of price flexibility, we discuss how the gains from investing in price flexibility in (8) depend on the variances of the wage, productivity and market demand, and on the covariances between them. To this end, we derive the second-order approximation of the gain function (8) around the stochastic steady state; see Appendix A.1.2 for details.

Let's define $a \equiv \ln A - \mathbb{E} \ln A$, $w \equiv \ln W - \mathbb{E} \ln W$, and $\hat{y} \equiv \ln \hat{Y} - \mathbb{E} \ln \hat{Y}$. Thus, we use the lower-case letter x to denote the deviation of a variable (in logs), $\ln X$, from its stochastic mean $\mathbb{E} \ln X$. Defining $\mathbb{E} w^2 \equiv \sigma_w$, $\mathbb{E} a^2 \equiv \sigma_a$, etc., we approximate the gains from price flexibility (8) as follows,

$$\Delta(\Theta) \approx \frac{\Omega}{2}(\theta - 1)\theta[\sigma_w^2 + \sigma_a^2 - 2\sigma_{wa}], \quad (9)$$

where we define

$$\Omega \equiv \frac{V(\exp(\mathbb{E} \ln \Theta))}{\exp(\mathbb{E} \ln \Gamma + \mathbb{E} \ln W)} > 0, \quad (10)$$

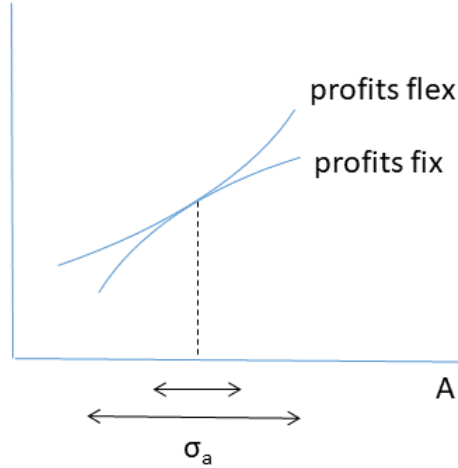
with $V(\exp(\mathbb{E} \ln \Theta))$ representing the operating profit function – of flexible-price firms or sticky-price firms – evaluated at the stochastic mean $\mathbb{E} \ln \Theta$. The gains from price flexibility depend positively on the variance of the wage and on the variance of productivity, and negatively on their covariance. The expression in square brackets in (9) is equal to $Var(\ln W - \ln A)$. Recall that $\theta > 1$ by assumption, and that $\Omega > 0$. Therefore, the gains from price flexibility are greater than zero, unless the wage is linear in productivity. In the latter case, the gains from price flexibility are exactly zero.

To understand the intuition why the gains from price flexibility are increasing in the size of productivity uncertainty, consider Figure 1. As shown in Appendix A.2, the flexible-price firm's profit function is convex in productivity A for $\theta > 2$, and the sticky-price firm's profit function is concave in A in this partial equilibrium exercise.¹¹ If productivity A is constant, there is no incentive to incur the cost of investing in price flexibility. The greater is the standard deviation of productivity σ_a , the greater is the

¹¹ The model feature that expected profits are larger for firms who have invested in price flexibility has support in Dutta et al. (2002), who note that ‘companies that don't have a well-developed pricing capability [...] will not extract as large a share of the value they create as competitors who have invested in pricing capabilities’.

divergence between expected profits of the flexible-price firm versus the sticky-price firm, and hence the greater are the gains from price flexibility. Therefore, volatility in productivity raises expected profits when prices are flexible relative to expected profits with preset prices.

Figure 1: Productivity uncertainty and gains from price flexibility: intuition



The flexible-price firms' profit function is convex, while the sticky-price firms' profit function is concave in productivity A . By Jensen's inequality, the gains from price flexibility are increasing in productivity uncertainty, measured as the standard deviation of productivity, σ_A . See also Bergin and Corsetti (2020). Notice that 'profits' in Figure 1 are operating profits excluding fixed costs and costs of investing in price flexibility.

Gorodnichenko and Weber (2016) study the stock returns of firms with different frequencies of price adjustment. They document that the returns of firms with stickier prices exhibit greater volatility after monetary shocks than do the returns of firms with more flexible prices, consistent with the menu cost model and the logic of Figure 1. Adams et al. (2024) show that stock returns of firms that adopted AI pricing are more sensitive to monetary policy shocks than non-adopters, in line with our notion of ex-ante price flexibility. We conjecture that this pattern carries over to other types of shocks, such as productivity shocks.

In the exposition so far we focused on the decision of an individual firm. Shocks to the productivity level A can be interpreted as idiosyncratic firm-level shocks as well as aggregate supply shocks. In any case, the gains from price flexibility rise in a firm's perception of productivity uncertainty. This interpretation is still valid in a general equilibrium model, where the price flexibility decision is undertaken by individual firms

that form expectations over the level of productivity (no matter whether the source of the shock is idiosyncratic or aggregate).

We now embed the price flexibility decision into such a general equilibrium model with entry and exit.

2.2 General equilibrium model with entry and exit

The household side of the model is standard. As for the firm side, the price setting decision and the entry-exit decision are intertwined, which makes the model more complex than the one in Bilbiie and Melitz (2022), who consider the two extreme cases where all firms are either flexible or sticky.

Households. Households choose consumption C , labor L and money holdings M to maximize utility given by

$$\ln C + \eta \ln \frac{M}{P} - \chi \frac{L^{1+\varphi}}{1+\varphi}, \quad (11)$$

where $\eta > 0$ is the velocity of money, $\chi > 0$ is the weight on labor disutility in household preferences, and $\varphi > 0$ is the inverse Frisch elasticity of labor supply, subject to the budget constraint, expressed in nominal terms as:

$$\Pi + (1 + \tau)WL + M_0 + \mathcal{M} \geq PC + T + M, \quad (12)$$

where Π are (total) firm profits, τ is a labor income subsidy from the government, W is the nominal wage, P is the price level, M_0 are initial money holdings, $\mathcal{M}$ is a transfer from the monetary authority, and T are lump-sum taxes.¹² The first order conditions to this problem yield the standard labor supply decision and money demand,

$$\chi L^\varphi PY = (1 + \tau)W, \quad (13)$$

$$M = \eta PY, \quad (14)$$

¹² We introduce a labor income subsidy to ensure that the flexible-price allocation is efficient. This will become relevant for the welfare analysis in Section 5.

where we have replaced consumption with aggregate output, using the market clearing condition $C = Y$.

Firms. Aggregate output is defined as in Dixit and Stiglitz (1977), except that we consider a continuum of producing firms $[0, N]$ rather than a discrete number of firms,

$$Y = \left(\int_0^N Y(\omega)^{\frac{\theta-1}{\theta}} d\omega \right)^{\frac{\theta}{\theta-1}}, \quad (15)$$

where $\theta > 1$ is the elasticity of substitution between goods varieties. The demand for good ω in (2) is the solution to the cost minimization problem whereby the firm chooses $Y(\omega)$ to minimize $\int_0^N P(\omega)Y(\omega)d\omega$, subject to the aggregator function (15), taking prices $P(\omega)$ as given. The price index is given by $P = (\int_0^N P(\omega)^{1-\theta} d\omega)^{1/(1-\theta)}$.

There are two types of firms. Firms on the interval $[0, z)$ set prices after the state of nature Θ has realized. The remaining firms on the interval $[z, N]$ set prices in advance. Flexible-price firms set a price $\tilde{P}$ and sticky-price firms set a price $\bar{P}$. This implies that the price index relevant for consumer welfare can be written as

$$P^{1-\theta} = z\tilde{P}^{1-\theta} + (N-z)\bar{P}^{1-\theta}. \quad (16)$$

We might define the producer price index as the price index that does not change with the number of firms or goods varieties, i.e. $p^{1-\theta} = N^{-1}[z\tilde{P}^{1-\theta} + (N-z)\bar{P}^{1-\theta}]$.

Accounting for fixed production costs f , total profits (in real terms) of flexible-price firms and sticky-price firms, are

$$\tilde{\Pi} = \int_0^z \left[\frac{\tilde{P}(\omega)\tilde{Y}(\omega)}{P} - \frac{W}{P} (\tilde{l}(\omega) + f) \right] d\omega,$$

$$\bar{\Pi} = \int_z^N \left[\frac{\bar{P}(\omega)\bar{Y}(\omega)}{P} - \frac{W}{P} (\bar{l}(\omega) + f) \right] d\omega,$$

respectively. Fixed costs f are specified in terms of labor units. Under symmetry, all flexible-price firms that produce are alike and all sticky-price firms that produce are alike, i.e. they set the same price and produce the same output. Goods market clear-

ing, for flexible-price firms and for sticky-price firms, is given by $\tilde{Y} = (\tilde{P}/P)^{-\theta}Y$ and $\bar{Y} = (\bar{P}/P)^{-\theta}Y$, respectively. The two types of firms' demand for production labor is, respectively, $\tilde{l} = \tilde{Y}/A$ and $\bar{l} = \bar{Y}/A$. Total profits of flexible- and sticky-price firms become

$$\tilde{\Pi} = z \left\{ \left(\frac{\tilde{P}}{\bar{P}} \right)^{1-\theta} Y - \frac{W}{PA} \left[\left(\frac{\tilde{P}}{\bar{P}} \right)^{-\theta} Y \right] - \frac{W}{P} f \right\}, \quad (17)$$

$$\bar{\Pi} = (N - z) \left\{ \left(\frac{\bar{P}}{\bar{P}} \right)^{1-\theta} Y - \frac{W}{PA} \left[\left(\frac{\bar{P}}{\bar{P}} \right)^{-\theta} Y \right] - \frac{W}{P} f \right\}. \quad (18)$$

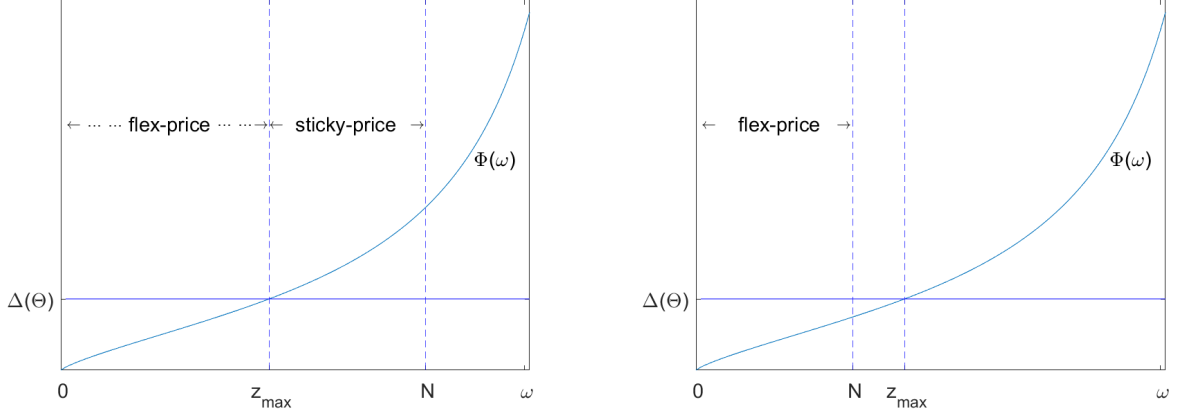
Production decision and aggregate price flexibility. The specification laid out above implies that the following relation determines the maximum number of flexible-price firms z_{max} ,

$$\Delta(\Theta) = \Phi(z_{max}). \quad (19)$$

In Devereux (2006), all firms – those that have invested in price flexibility and those that have not – produce. The number of producers is fixed and there is no product turnover. Consequently, the equilibrium degree of price flexibility z coincides with z_{max} in (19). We label the model without product turnover ‘New Keynesian’ model. Here, in contrast, the equilibrium z can be below z_{max} if some sticky-price firms choose not to produce once uncertainty about productivity is resolved.

More specifically, after the realization of productivity A , firms choose to produce if operating profits exceed fixed costs Wf . At the same time, we stipulate that due to entry and exit, profits of sticky-price firms are always zero in equilibrium, such that $\bar{V}(\Theta) = Wf$. The reasoning behind this is, first, that – by assumption – a new entrant has not had the time to invest in price flexibility and must therefore adopt the predetermined price. Second, sticky-price firms are the first to exit in the event of low productivity realizations. Thus, ex post price flexibility z is determined by the production decision, i.e. entry-exit after the productivity shock has occurred. This is illustrated in Figure 2. Consider the left hand plot. In the event of a sufficiently high productivity draw, many firms choose to produce and $N > z_{max}$. In that case, the ex-post price flexibility coincides with ex-ante maximum price flexibility, i.e. $z = z_{max}$, and there will be sticky-price firms

Figure 2: Determination of aggregate ex-post price flexibility



Determination of ex-post price flexibility in the aggregate: high productivity draw leading to interior solution (left hand side), low productivity draw leading to corner solution (right hand side).

with mass $N - z$. If instead productivity is very low, a small number of firms choose to produce and $N \leq z_{max}$. All firms that remain in the market are flexible price setters, such that $z = N$. See the right hand plot in Figure 2. Formally,

$$z = z_{max} \quad \text{if } N > z_{max}, \quad (20)$$

$$z = N \quad \text{if } N \leq z_{max}. \quad (21)$$

Overall, when $N > z_{max}$ the number of firms is determined by the requirement that total profits of sticky-price firms equal zero, i.e. $\bar{\Pi} = 0$, or using (18),

$$\left(\frac{\bar{P}}{P}\right)^{1-\theta} Y - \frac{W}{PA} \left[\left(\frac{\bar{P}}{P}\right)^{-\theta} Y \right] - \frac{W}{P} f = 0. \quad (22)$$

With $N \leq z_{max}$, the number of producers N is determined by the condition that total profits of flexible-price firms equal zero, $\tilde{\Pi} = 0$, or using (17),

$$\left(\frac{\tilde{P}}{P}\right)^{1-\theta} Y - \frac{W}{PA} \left[\left(\frac{\tilde{P}}{P}\right)^{-\theta} Y \right] - \frac{W}{P} f = 0. \quad (23)$$

Market clearing. Firms demand labor to produce, to cover fixed costs and to invest in price flexibility. Labor market clearing therefore requires

$$L = z \left(\frac{\tilde{P}}{\bar{P}} \right)^{-\theta} \frac{Y}{A} + (N - z) \left(\frac{\bar{P}}{\bar{P}} \right)^{-\theta} \frac{Y}{A} + Nf + \int_0^{z_{max}} \Phi(\omega) d\omega. \quad (24)$$

The last term on the right hand side captures the fixed cost incurred by the mass z_{max} of firms that choose price flexibility. Following Devereux (2006), we stipulate the following cost function for investing in price flexibility, $\Phi(\omega) = \Phi\omega$.

Table 1: Model equilibrium conditions

Max. price flexibility	$\Delta(\Theta) = \Phi(z_{max})$
Price flexibility	$z = z_{max} \quad \text{for } N > z_{max}$ $z = N \quad \text{for } N \leq z_{max}$
Flexible price	$\tilde{P} = \delta W/A$
Sticky price	$\bar{P} = \delta \mathbb{E}\{\Gamma(W/A)\hat{Y}\} / \mathbb{E}\{\Gamma\hat{Y}\}$
Price index	$P^{1-\theta} = z\tilde{P}^{1-\theta} + (N - z)\bar{P}^{1-\theta}$
Labor market	$L = [z(\tilde{P}/P)^{-\theta} + (N - z)(\bar{P}/P)^{-\theta}]Y/A + Nf + \frac{\Phi}{2}z_{max}^2$
Labor supply	$(1 + \tau)W = \chi L^\varphi Y P$
Money market	$Y = M/(\eta P)$
Entry condition	$(\bar{P}/P)^{1-\theta}Y - (W/(PA))(\bar{P}/P)^{-\theta}Y - Wf/P = 0, \quad \text{for } N > z_{max}$ $(\tilde{P}/P)^{1-\theta}Y - (W/(PA))(\tilde{P}/P)^{-\theta}Y - Wf/P = 0, \quad \text{for } N \leq z_{max}$

The model has nine endogenous variables z_{max} , z , $\tilde{P}$, $\bar{P}$, P , L , W , Y , and N , which are determined by the nine equilibrium conditions above.

A decentralized equilibrium is defined below, and Table 1 summarizes the model's equilibrium conditions.

Definition 1. *A decentralized equilibrium in the entry-exit model with an endogenous degree of price flexibility is a set $\{\tilde{P}, \bar{P}, z_{max}, W, Y, P, z, N, L\}$ that satisfies the two types of firms' price setting conditions (4) and (6), the firms' optimal investment in price flexibility (19), the household's first order conditions for labor (13) and money holdings (14), the price index (16), the production decision (20) or (21), the zero-profit condition (22) or (23), and labor market clearing (24), given an exogenous process for productivity A .*

We use numerical methods to obtain the equilibrium solution. It is crucial for the de-

termination of equilibrium price flexibility that firms make their decision to invest in price flexibility in the knowledge that productivity is volatile, rather than in an equilibrium with no uncertainty, where there is no role for risk arising from shocks.

2.3 Solution approach

The solution method has two steps. For a given productivity uncertainty, we first solve the firm's problem to choose price flexibility (from the first two lines of Table 1) under the constraints imposed by the general equilibrium model (remaining lines of Table 1). In a second step, we treat the equilibrium number of flexible-price firms z_{max} and the predetermined price $\bar{P}$ (both obtained in the first step) as given to simulate an exogenous shock to productivity A . This numerical approach is in line with our theoretical formulation of the price flexibility investment problem as firms decide on price flexibility before the actual state of nature can be observed.

In the following, we go into more detail regarding the solution algorithm. For a given uncertainty regime, we consider five exogenous states that span the set of possible productivity outcomes $\mathcal{S}_A = \{A_0, A_1, A_2, A_3, A_4\}$, where $A_i \in \mathbb{R}^+$ for $i = 0, 1, 2, 3, 4$. All states are equally likely. The degree of uncertainty is determined by the size of the range $[A_0, A_4]$. In a regime with low uncertainty, that range is smaller compared to a regime with high uncertainty. In each uncertainty regime, the preset price $\bar{P}$ and the number of flexible-price firms z_{max} are predetermined, i.e. set before productivity shocks realize (before A is drawn). The decision to invest in price flexibility also requires firms to form expectations on the total number of firms, $\mathbb{E}(N)$, and to learn their position on the $\Phi(\omega)$ -curve. To solve the model, we conduct a grid search on combinations of $\bar{P}$, z_{max} , and $\mathbb{E}(z)/\mathbb{E}(N)$ that are consistent with the expected (average) outcomes of the model in the $[A_0, A_4]$ range. The solution algorithm consists of the following steps:

Step 1: We make initial guesses for $\bar{P}$, z_{max} , and $\mathbb{E}(z)/\mathbb{E}(N)$. Then, we define a number of K grid point combinations of initial values around these guesses. We allow for different grid step sizes for the three corresponding grid vectors.

Step 2: For each $k = 1, 2, \dots, K$ and each possible productivity outcome $A_j \in \mathcal{S}_A$, we solve the model summarized in Table 1.

Step 3: For each grid point k , we then average the solutions across all productivity outcomes $A_j \in \mathcal{S}_A$ to compute the preset price $\bar{P}$ from (6) and $\mathbb{E}(z)/\mathbb{E}(N)$. Similarly, we compute expected gains from price flexibility from (8) and consequently the optimal number of flexible-price firms in equation (19).

Step 4: The algorithm converges if, for a grid point k , the solutions for $\bar{P}$, z_{max} , and $\mathbb{E}(z)/\mathbb{E}(N)$ in Step 3 deviate less than 10^{-4} from the initial values in Step 1. If there is no convergence, we redo Step 1 to Step 3 but take the solutions for $\bar{P}$, z_{max} , and $\mathbb{E}(z)/\mathbb{E}(N)$ in Step 3 as initial guesses in Step 1.

2.4 Calibration

In our baseline calibration, we follow Bergin and Corsetti (2008) and set the elasticity of substitution between varieties to $\theta = 3.8$. We set the inverse Frisch elasticity to $\varphi = 2$, a value which according to Keane and Rogerson (2012) is compatible with elasticities estimated using micro data. The cost of investing in price flexibility Φ is set such that, if in a deterministic solution all firms invested in price flexibility, the share of output spent on this type of investment would be equal to 1% of GDP (which is equivalent to revenue in our model). In the intermediate case where not all firms set prices flexibly, the corresponding fraction in our model is comparable to other papers. For instance, in the baseline analysis of Midrigan (2011), total resources to change prices are 0.34% of revenue. Levy et al. (1997) reports a fraction of 0.7% and Zbaracki et al. (2004) a larger number of 1.2%. The weight on labor in utility is normalized to $\chi = 1$ and the fixed cost of entry f is determined endogenously through the numerical computation of the steady state. In Section 3.4 below, we discuss the implications of assuming a higher elasticity of substitution between varieties ($\theta = 6$).

Table 2: Baseline parameter values

Parameter name	Value	Target / Reference
Substitution elasticity between goods	$\theta = 3.8$	Bergin and Corsetti (2008)
Inverse labor supply elasticity	$\varphi = 2$	Keane and Rogerson (2012)
Costs of investing in price flexibility	$\Phi = 0.0068$	1% of output for deterministic $z/N = 1$
Labor income subsidy	$\tau = 1/(\theta - 1)$	see Appendix B
Weight on labor in utility	$\chi = 1$	normalization
Velocity of money	$\eta = 0.99$	discount factor $\Gamma = (PY)^{-1} = \eta/M = 0.99$
Per-period production fixed cost	$f = 0.1607$	determined endogenously

3 Adverse supply shocks when price flexibility can adjust

In the following, we first show how the degree of price flexibility depends on the volatility of productivity. Second, we discuss the transmission of supply shocks under endogenous price flexibility. Third, we compare and contrast our model with entry and exit to the New Keynesian model with a fixed number of producers. Finally, we re-calibrate the model and show the effects of a drop in productivity for alternative assumptions regarding the elasticity of substitution between goods.

3.1 Productivity uncertainty and endogenous price flexibility

The gains from price flexibility (9) introduced in partial equilibrium in Section 2.1 can be approximated in the general equilibrium model with entry and exit in terms of the variance of productivity only,

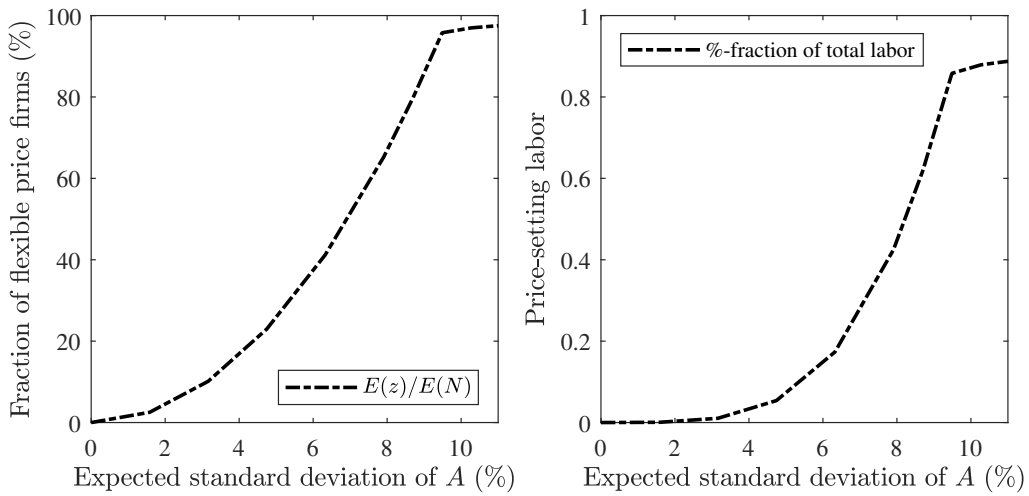
$$\Delta(\Theta) \approx \frac{\Omega}{2}(\theta - 1)\theta v^2 \sigma_a^2, \quad (25)$$

where the constant terms $\Omega > 0$ and v are derived in Appendix A. As this approximation mainly serves illustrative purposes, we have only considered the more relevant case $N > z_{max}$, i.e. the extensive margin only adjusts with entry and exit of sticky-price firms. We see from (25) that the gains from price flexibility are strictly increasing in σ_a^2 . Thus, a

larger shock variance implies a higher willingness of firms to invest in price flexibility.

Equation (25) cannot be solved analytically as Ω and v are evaluated at the stochastic mean of the model. We therefore continue with studying numerical simulations of the model under the baseline calibration outlined in Table 2. In this exercise, we consider a range of volatilities of the productivity shock σ_a that result – as an equilibrium outcome – in a varying fraction of flexible-price firms $\mathbb{E}(z)/\mathbb{E}(N)$. To solve the general equilibrium model specified in Table 1, we adopt the solution approach described in Section 2.3.

Figure 3: Endogenous price flexibility and shock volatility



Left panel: Ex ante expected fraction of flexible price firms for different *ex ante* volatilities of productivity A . Right panel: Labor devoted to investment in price setting capabilities as a fraction of total labor, for different *ex ante* volatilities of productivity A . The figure in the right panel simulates the model with five productivity states in the range of $[0.87, 1.13]$, consistent with a standard deviation of roughly 10%.

Figure 3 (left panel) shows how the fraction of flexible-price firms $\mathbb{E}(z)/\mathbb{E}(N)$ endogenously varies when we increase the standard deviation of the productivity process σ_a . In particular, if productivity is more volatile, the ex ante profits of a firm are larger under flexible prices than under sticky prices (as Figure 1 suggested). This implies that a larger fraction of firms choose to invest in price flexibility, and accordingly, z increases. With a very high productivity uncertainty, it becomes likely that flexible-price firms face negative profits and are forced to exit. This flattens the slope of the curve in Figure 3. Under our baseline calibration, a standard deviation σ_a of just around 9% is sufficient to make almost all firms choose price flexibility, such that $\mathbb{E}(z)/\mathbb{E}(N) \approx 1$ in equilibrium.

As outlined in the introduction, the last few years experienced a sizable upward shift

in (perceived) uncertainty. According to the Bank of England’s Decision Maker Panel, subjective firm-level uncertainty increased in the span March 2020 to December 2021 (compared to 2019) by around 50% for sales growth and 30% for price growth. In the model, a rise in uncertainty of that magnitude can – depending on its initial level – lead to a substantial increase in the fraction of flexible-price firms. For instance, at an initial level of uncertainty of around 5% that corresponds to a fraction of flexible price setters of 30%, a shift in uncertainty to 7.5% would result in roughly 50% flexible price setters (see Figure 3).

In our model, increased price flexibility under higher uncertainty comes at a cost in terms of labor that is not able to produce output. The right panel of Figure 3 shows the amount of labor that is devoted to setting prices as a fraction of aggregate labor in the economy. To illustrate, the figure simulates positive and negative productivity shocks with a standard deviation of roughly 10%. A similar figure would arise when assuming a lower standard deviation. When uncertainty rises, more labor is allocated to setting prices, reaching a bit less than 1% of the total labor force.

3.2 Shock transmission and uncertainty in the model

The mechanism highlighted in Figure 3 implies that in a regime with larger shock volatility, producer prices respond more strongly to supply shocks. To illustrate this, we mimic the case of a large shock comparable to the Covid-19 recession in early 2020. More specifically, we simulate a productivity drop by 10%, relative to equilibrium with the median technology state of $A = 1$, that results in an output decline of around 12%. This characterizes a realized shock that makes demand amplification through entry and exit under sticky prices powerful in Bilbiie and Melitz (2022). Their ‘entry-exit multiplier’ is nonlinear; it is very important for large productivity drops of around 10-15% and it is more muted for smaller productivity declines below 10%.¹³ Our mechanism, in contrast, is also important for small shocks, as they are transmitted with larger producer price hikes under high uncertainty. We discuss this case below.

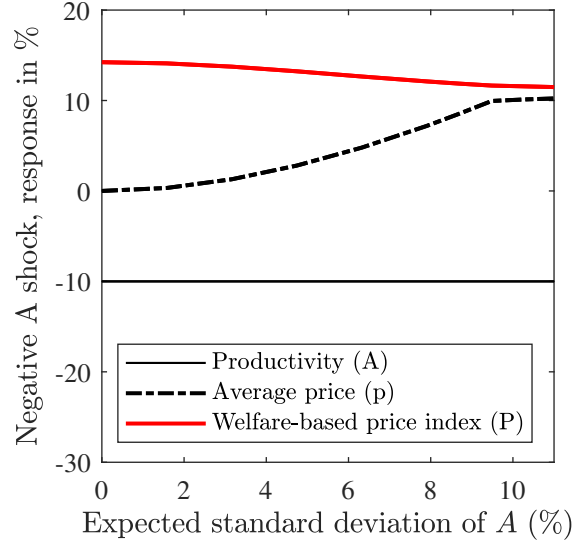
¹³ See Figure 1 in Bilbiie and Melitz (2022) for an illustration.

To avoid confusion, note that the realized productivity shock is not necessarily included in the range of possible productivity outcomes that the firm expect *ex ante* to choose if they invest in price flexibility. We characterize productivity uncertainty by the expected standard deviation of the range of possible productivity outcomes A (expressed in % around the median outcome that is normalized to 1). For instance – as an extreme example – it could be that firms expect that productivity tomorrow is the same as productivity today (i.e. the expected standard deviation of A is 0). Then there is zero uncertainty and no firm decides to invest in price flexibility. The productivity shock (that occurs *after* firms have invested in price flexibility) can still have any realization.

Consumer and producer price index. Figure 4 shows the impact response of the producer and consumer price index to an adverse supply shock, for different degrees of productivity uncertainty. The shock is modelled as a 10% drop in productivity A . Producer prices barely respond to the shock in the case where the volatility of A is low, i.e. expected supply shocks are on average small. In contrast, in a regime of high uncertainty, an adverse supply shock results in a strong response of firms' prices. This is a direct consequence of the greater investment in price flexibility, as discussed in Section 3.1.

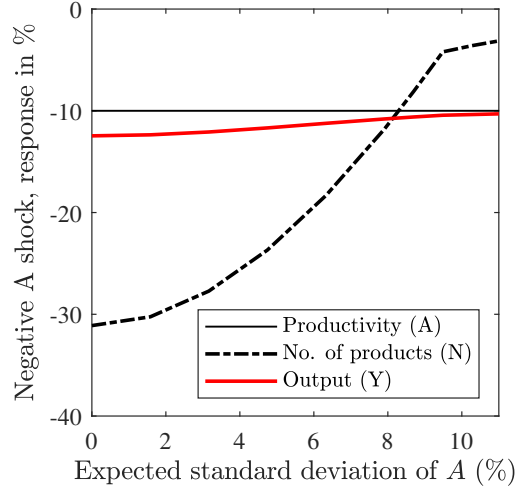
Notably, the pattern is different for the welfare-based (consumer) price index. This is due to the fact that, in a model with entry and exit, P declines in the number of available varieties, i.e. in the number of firms. In a regime of low supply shock volatility, the fraction of flexible-price firms is low (cf. Figure 3). Then, as we discuss below, there is more exit after an adverse supply shock. This implies a stronger response of the welfare-based price index, as we see in Figure 4. In a regime of high volatility, the effect of a negative supply shock on exit is markedly less pronounced, and thus the welfare-based price index rises by less. As emphasized by Bilbiie et al. (2007) and other contributions in the entry-exit literature, official CPI data provided by statistical agencies does not adjust (adequately) for new products. For this reason, the empirical CPI is closer to the average producer price, p , than it is to the welfare-based price index, P .

Figure 4: Response of average prices and welfare-based price index to adverse supply shock



Impact response of productivity, the welfare-based price index and average producer prices to an adverse productivity shock for different *ex ante* volatilities of productivity A .

Figure 5: Response of number of firms and output to adverse supply shock



Impact response of the number of firms and of output to an adverse productivity shock for different *ex ante* volatilities of productivity A .

Firm dynamics and output. We have seen that, when productivity uncertainty (the standard deviation of A) is high, endogenous price flexibility as measured by $\mathbb{E}(z)/\mathbb{E}(N)$ is greater. This has important consequences for firm dynamics. The black dashed line in Figure 5 plots the impact response of the number of firms for different supply shock volatilities. Under a smaller supply shock volatility, i.e. in a situation where producers decide to invest little in price flexibility, the adjustment of the extensive margin is sizable.

In contrast, in an environment where supply shocks are on average large, i.e. under high productivity uncertainty, exit is substantially reduced. Thus, the level of productivity uncertainty is key for firm dynamics.¹⁴

The underlying intuition for this result is the following. If firms expect no large shocks to happen, i.e. in a low uncertainty environment, relatively few of them are willing to invest in their capacity to set prices flexibly. Then if – against the firms’ expectations – a large shock does materialize, most firms cannot reset prices. Some of them have to exit as the preset price is too low and would result in negative profits.¹⁵ Instead, if firms account for the possibility of large shocks, i.e. in a high uncertainty environment, more will invest in price flexibility. As a result, they are able to increase prices, and there is less exit.

This has implications for the output responses. With low productivity uncertainty, the adjustment in the extensive margin is sizable and the output response is relatively large. We note from the equilibrium condition for real money balances (see Table 1) – and since nominal money supply is fixed – that output is only affected by the current welfare-based price index. Thus, the decline in output in (red line in Figure 5) mirrors the increase in consumer prices (red line in Figure 4). Entry-exit amplifies supply shocks more under low productivity uncertainty – and therefore relatively sticky prices – in the sense that the output response is more pronounced when uncertainty is low.

Large upward shifts in productivity uncertainty, as for instance during the Covid-19 pandemic, markedly increase the number of firms that invest in price flexibility. As a consequence, the demand amplification mechanism through entry-exit highlighted in Bilbiie and Melitz (2022) is reduced when uncertainty is higher.

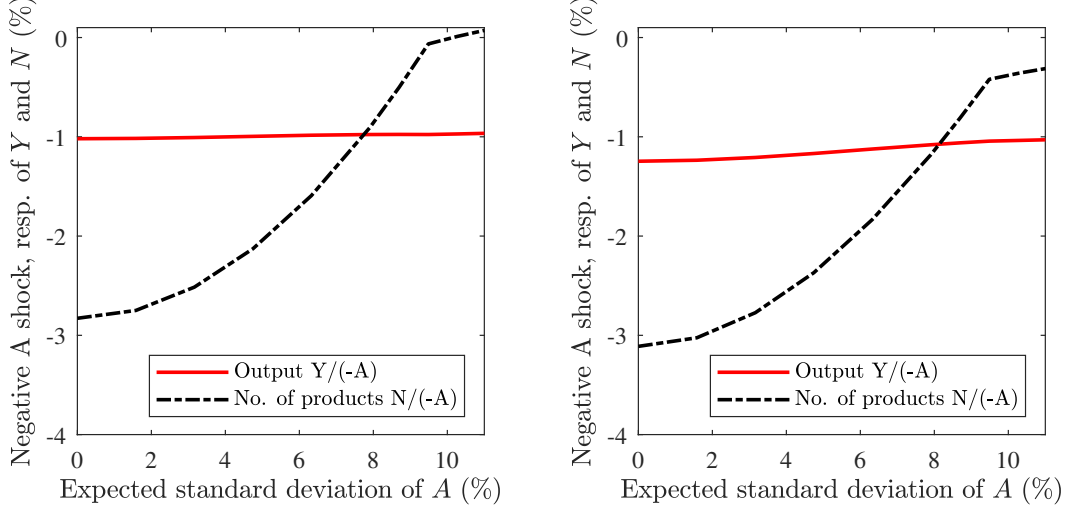
Notably, the potential for firms to shield themselves from expected large shocks by investing in price flexibility under high uncertainty is beneficial for consumers despite a larger pass-through of productivity declines to consumer prices at the product level. The

¹⁴ In Section 5, we discuss that this is also the case for monetary policy uncertainty.

¹⁵ As discussed in Section 2.2, in response to adverse supply shocks sticky-price firms exit first. Thus, the fraction of flexible-price firms in the market after shock realization z/N is higher than the ex ante fraction of flexible-price firms in the market $\mathbb{E}(z)/\mathbb{E}(N)$. This is demonstrated in Figure C.1 in the Appendix.

reason is that there is less exit and more goods varieties than in the case where firms have not invested in price flexibility before shocks happen. Section 5 discusses welfare implications of investing in price flexibility.

Figure 6: Response of number of firms and output: small vs. large shocks



Left panel shows impact response to a small adverse productivity shock (-1pp); right panel shows impact response to a large adverse productivity shock (-10pp). The impact of output and number of firms is normalized by the (negative) impact of productivity.

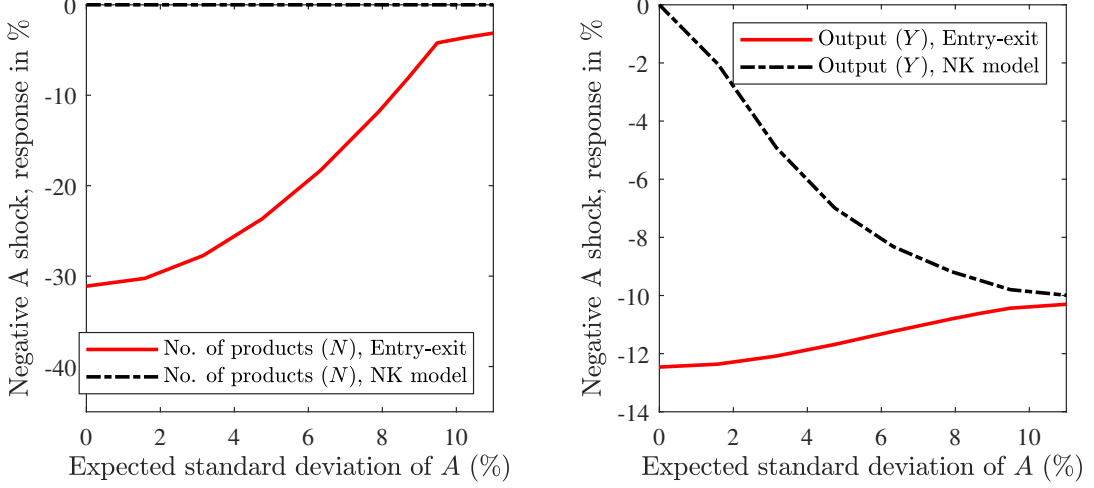
Large versus small shocks. In the baseline calibration, we assumed a rather large productivity disturbance of -10pp. Figure 6 compares a 10pp drop in productivity with a 1pp drop. The effects are more pronounced for the larger shock. Nevertheless, in an environment with high uncertainty also a relatively small shock induces a comparatively large number of firms to change prices. The degree of demand amplification – i.e. by how much entry/exit amplifies the productivity shock – does not, however, differ much across uncertainty regimes.

3.3 Entry-exit model versus New Keynesian model

To illustrate that firm dynamics are less pronounced in the transmission of productivity shocks under high uncertainty (as opposed to low uncertainty), we now compare our entry-exit model with a model featuring a constant number of firms. The latter is basically a New Keynesian (NK) model with productivity shocks; differently from the canonical NK model, however, price flexibility is endogenous. We implement the alternative model by

setting N equal to its value in the deterministic equilibrium throughout, effectively dropping the last equation in Table 1, which is the zero-profit condition. Figure 7 compares impulse responses of the two models across productivity uncertainty regimes.

Figure 7: NK model versus entry-exit model under endogenous price flexibility

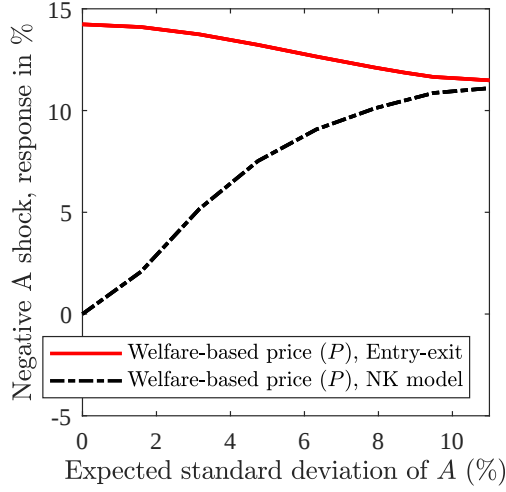


Impact response to a negative productivity shock under endogenous price flexibility, as a function of productivity uncertainty: New Keynesian versus entry-exit model.

We start by illustrating the role of the extensive margin across different productivity uncertainty regimes. The left panel of Figure 7 shows the impact response of N as a function of productivity uncertainty under endogenous price flexibility, in our baseline model with entry-exit and in the New Keynesian model with a constant number of producers (NK model). By construction, the response of N in the latter model is zero for all values of productivity uncertainty. In the entry-exit model, the impact response of N is decreasing in the degree of productivity uncertainty. The lower productivity uncertainty, the more rigid are prices, and the greater is the adjustment along the extensive margin.

This has implications for the output response in the two models, as can be seen in the right panel of Figure 7. We see that the impact response of Y increases in productivity uncertainty in the NK model. In the entry-exit model, it instead decreases. Consider the impact response of the welfare-based price index to an adverse productivity shock shown in Figure 8. In the NK model, the welfare-based price index – which is in this model equivalent to the average producer price – rises following the shock. The price index increases by more, the greater is productivity uncertainty. If uncertainty is very

Figure 8: Consumer price response: NK model versus entry-exit model



Impact response to an adverse supply shock under endogenous price flexibility, as a function of productivity uncertainty: New Keynesian versus entry-exit model.

low, prices barely change on impact. In the entry-exit model, however, the welfare-based price index depends not only on product prices, but also on the number of products. An adverse productivity shock leads to exit – more so when ex ante productivity uncertainty is low – reducing the number of products N and, through the variety effect, increasing the price index P . Thus for low productivity uncertainty, the change in the welfare-based price index reflects mostly the extensive margin. As the degree of productivity uncertainty increases, the variety effect is diminished, so that the response of the welfare-based price index is determined to a greater extent by the direct price effect.

Our key finding is that firm dynamics strongly amplify the effects of supply shocks in an environment of low productivity uncertainty. This result relates to Bilbiie and Melitz (2022), who show that, under sticky prices, aggregate supply disturbances are amplified through firm entry and exit. The New Keynesian model without entry and exit instead predicts that output responds little to adverse supply shocks when prices are sticky (under exogenous price flexibility) or when uncertainty is low (under endogenous price flexibility).

We now briefly discuss the differences in the labor market response in the model with entry and exit compared to the NK model (for more details, see Appendix C). In general, adverse supply shocks raise aggregate labor demand, as firms want to compensate

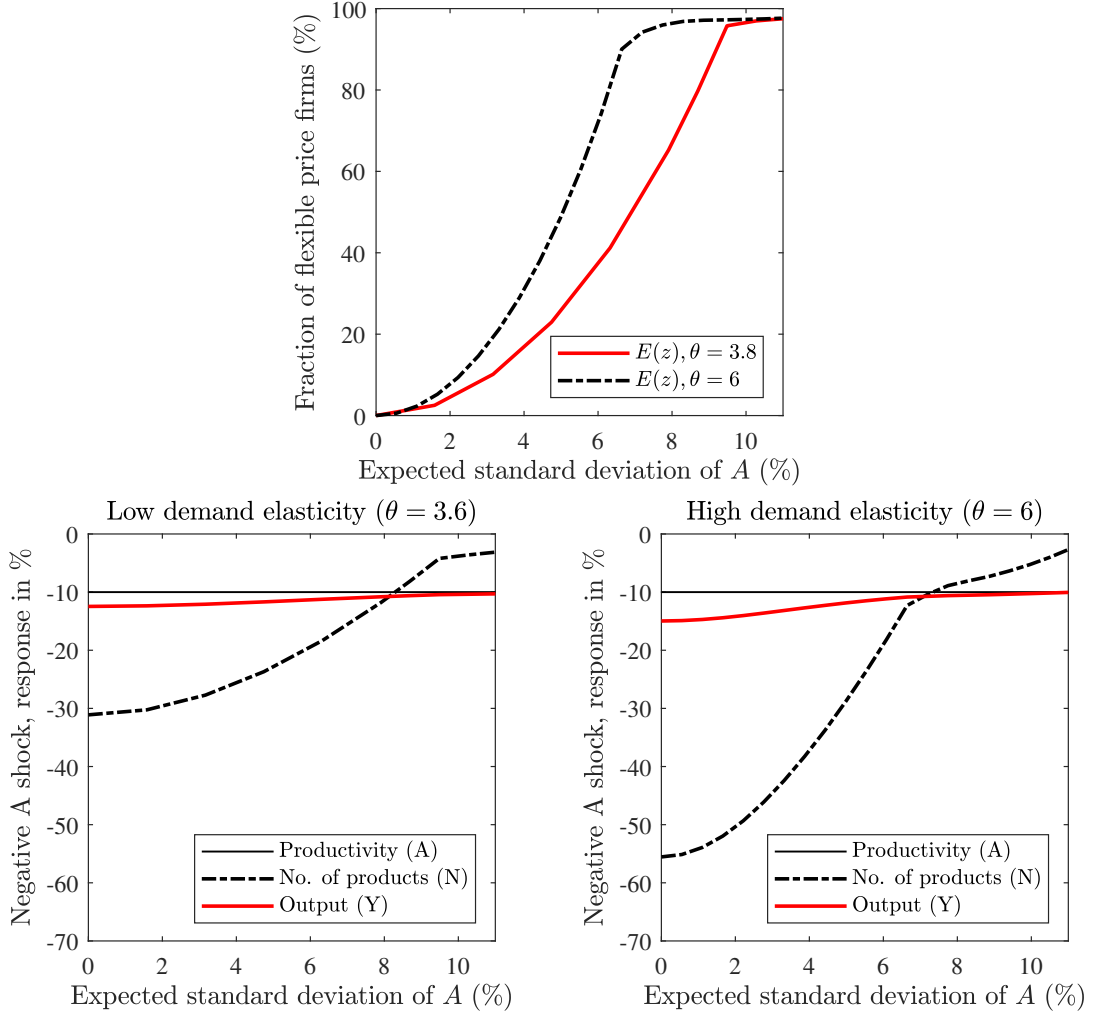
lower productivity with more labor input. This is more pronounced when uncertainty is lower and fewer firms have invested in price flexibility. The channel is key for the equilibrium labor outcome in the model without product turnover (NK model). In the NK model, nominal wages rise in response to higher aggregate labor demand. In the model with entry/exit, this channel is offset by the fact that in response to a large negative productivity shock many firms have to exit to avoid paying production fixed costs – more so under lower uncertainty when adverse productivity shocks cannot as easily be passed on to consumers via higher prices. This lowers the demand for labor, and therefore the wage. Thus, economy-wide production fixed costs, which consist of labor, decline. Overall, in the model with entry and exit, labor and nominal wages remain virtually unchanged (see Appendix, Figure C.2).

3.4 The role of market power

We now demonstrate graphically how productivity uncertainty affects equilibrium price flexibility and the response of the number of firms to an adverse supply shock, for different demand elasticities as measured by θ . The other parameters remain unchanged at their baseline values (see Table 2), except production fixed cost f and costs of investing in price flexibility Φ , which are calibrated to match the calibration targets.

We compare the baseline calibration where the demand elasticity is rather low $\theta = 3.8$, as in Bergin and Corsetti (2008), with a calibration $\theta = 6$, as in Bilbiie and Melitz (2022). The demand elasticity affects both price flexibility, which is an *ex ante* decision before shocks happen, and firm entry after shocks are realized. The upper panel of Figure 9 shows that, for a given level of productivity uncertainty (i.e. supply shock volatility), the fraction of flexible-price firms is higher when θ is larger. To see this, note from equation (8) that the gains from price flexibility $\Delta(\Theta)$ are greater for larger values of θ . Intuitively, when demand is fairly inelastic to the price (θ is low), for a firm price changes are not as effective to shift relative demand, which is necessary to avoid losses. Thus, for firm ω it is less advantageous to invest in price flexibility for a given investment cost $\Phi(\omega)$. To summarize, a lower θ makes prices less responsive to shocks; or in the words of Flynn

Figure 9: Price flexibility and firm dynamics: high vs. low demand elasticity



Upper panel: Fraction of flexible price firms for high and low demand elasticity θ . Lower panel: Impact response of number of firms and output to adverse supply shock for high and low demand elasticity θ .

et al. (2023), ‘market power flattens the AS curve’.

The lower panel of Figure 9 demonstrates how the entry-exit response to adverse supply shocks varies with the demand elasticity θ . Let us first consider the case of zero productivity uncertainty that imposes fully rigid prices as firms do not invest in price flexibility before shocks realize. Suppose that under such circumstances, productivity A falls and sticky-price firms are stuck with a price that is too low. As we recall from Figure 5, firms exit in great numbers in this case. In their model with fully rigid prices, which corresponds roughly to our case with no uncertainty regarding A , Bilbiie and Melitz (2022) show that, the higher the demand elasticity θ , the greater is the adjustment along the extensive margin, i.e. firm exit. Now, when productivity uncertainty rises, price

flexibility increases and more so under a higher θ , as indicated in Figure 9 and discussed above. This reduces the response of exit to the shock. Overall, with rising productivity uncertainty, the response of exit is reduced more strongly – i.e. the curve on the right panel in Figure 9 is steeper – under a higher θ .

4 Shock transmission and uncertainty in the data

Our model predicts that the transmission of productivity shocks depends on (firms’ expectations about) the volatility of those shocks. We wish to bring this prediction to the data and test the hypothesis that entry-exit responses to supply shocks are smaller, while price responses are larger, when the expected standard deviation of the productivity process is large.

To this end, we follow a two-step procedure. First, we identify productivity volatility regimes. We assume that firms expectations about the standard deviation of future productivity shocks are based on the current volatility regime. Second, we estimate separately the effects of productivity shocks on net entry and on prices via local projections. Details and additional robustness results are provided in Appendix D.

4.1 Identifying the state of productivity uncertainty

To gauge the state of productivity uncertainty, our baseline approach builds on the identification of TFP volatility regimes in Foerster and Matthes (2022). In particular, we estimate a Markov switching model with time-varying volatility in total factor productivity (TFP). We use the utilization-adjusted quarterly TFP series for the US from Fernald (2012) for the period 1948-2022. The approach yields estimates for the probability of being in a high-volatility regime.

In a robustness analysis, we compute TFP volatility by estimating an AR(1) model on the first difference of utilization-adjusted TFP and then constructing the 12-quarter moving average of the residual standard deviation (see Figure D.6 in the Appendix).

Our second-step results are robust to using either of these two approaches to identify

the state of TFP volatility. In the baseline, we use the Markov-switching approach. To save space, the second-step results building on identification via moving averages are relegated to Appendix D, Figures D.12 to D.14.

4.2 Regime-dependent effects of productivity shocks

We use the local projection method à la Jordà (2005) to separately estimate the average response of TFP shocks on (a) net entry, and (b) prices. In particular, we estimate state-dependent local projections at quarterly horizon $h \in \{0, \dots, 20\}$:

$$x_{t+h} - x_{t-1} = \text{Regime}_{t-1} \times \left(\delta^{h,r} + \beta^{h,r} TFP_t^{shock} + \sum_{j=1}^T \gamma_j^{h,r} Y_{t-j} \right) + \\ + (1 - \text{Regime}_{t-1}) \times \left(\delta^{h,nr} + \beta^{h,nr} TFP_t^{shock} + \sum_{j=1}^T \gamma_j^{h,nr} Y_{t-j} \right) + \epsilon_{t+h}, \quad (26)$$

where x_t is log-transformed outcome variable and ϵ is an error term. Y is a vector of control variables capturing lags of the TFP shocks as well as lags of first differences of x . We follow Montiel Olea and Plagborg-Møller (2021) in using lag augmentation and heteroskedasticity-robust standard errors to estimate standard errors that are robust to bias from serial correlation. We set the lag length T to 3 in the regression of net entry and to 2 to study price responses. With the longer lag length in the net entry regression we aim to capture time to build in firm dynamics. However, also with fewer lags the results remain qualitatively unchanged.

To be consistent with the theoretical model, we estimate state-dependent local projections where the state is defined by the volatility regime in place before the shock hits (Regime_{t-1}). The binary state indicator Regime_t captures a higher-than-average probability of being in a high-volatility regime at time t , where the average probability is computed for the sample period of regression (26). More specifically, Regime_t equals 1 when the four-quarter moving average probability of being in the high-volatility state exceeds the sample mean, and 0 otherwise.¹⁶

¹⁶ We follow a similar approach when we use moving averages to identify states of TFP volatility. Appendix D.3 plots the variable Regime_t for both approaches.

To proxy net entry, we rely on the BLS’s Business Employment Dynamics statistics, which provides establishment births and deaths from 1993Q2. The net entry index is computed as the cumulative sum of the difference between births and deaths, with base period 1993Q1. The total number of establishments in 1992 is from the Census Bureau’s Business Dynamics Statistics: <https://www.census.gov/data/tables/time-series/econ/bds/bds-tables.html>.¹⁷

As our measure of prices, we use the producer price index for final demand finished goods excluding foods and energy provided by the BLS. We restrict our sample to observations after 1993Q1 to align with the period in which establishment net entry data is available. In a robustness exercise, we use the BLS’s consumer price index for goods excluding food and energy. See the Appendix, Figure D.11.

In our baseline estimation of local projections regression (26), the productivity shock TFP_t^{shock} is the residual from an estimated AR(1) model of the first difference of utilization-adjusted TFP.^{18,19} To assess the plausibility of our estimated state-dependent impulse responses, we estimate unconditional local projection regressions, i.e. regressions without an interaction term $Regime_{t-1}$. The estimated unconditional responses indicate that – in line with standard models – following an adverse TFP shock net entry is significantly reduced, while producer prices increase, see Figure D.9 in Appendix D.

4.3 Estimation results

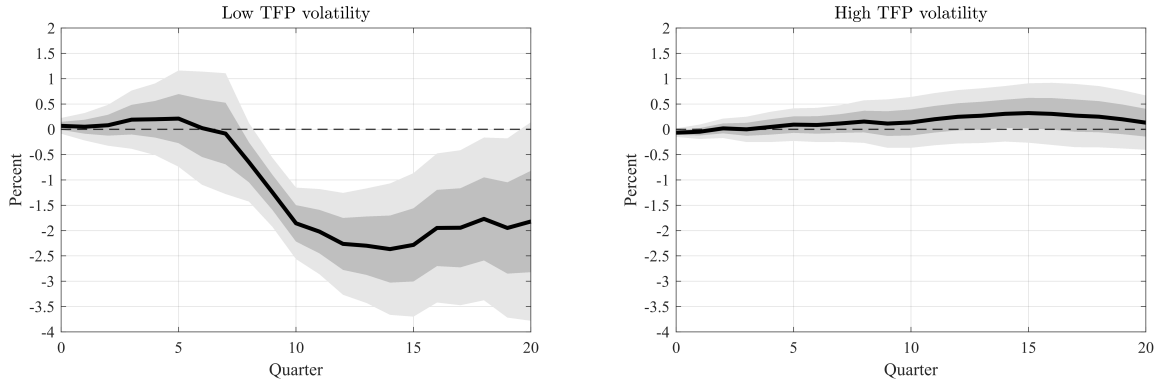
Figure 10 plots the response of net entry to an adverse shock that lowers TFP by one percent. The left hand plot shows that when productivity uncertainty was low before the shock hit, there is a sizable and rather persistent decline in net entry following a productivity shock, which becomes significant after around 8-10 quarters. In contrast, when TFP uncertainty was high before the shock hit, there is a muted and barely statistically significant response of net entry.

¹⁷ In a robustness analysis, we proxy net entry for the period prior to 1992 using net business formation from the BEA’s Survey of Current Business, which is available at the monthly frequency, starting in January 1948. The results remain robust. See Figure D.10 in Appendix D.

¹⁸ Appendix D.3 plots the shock series.

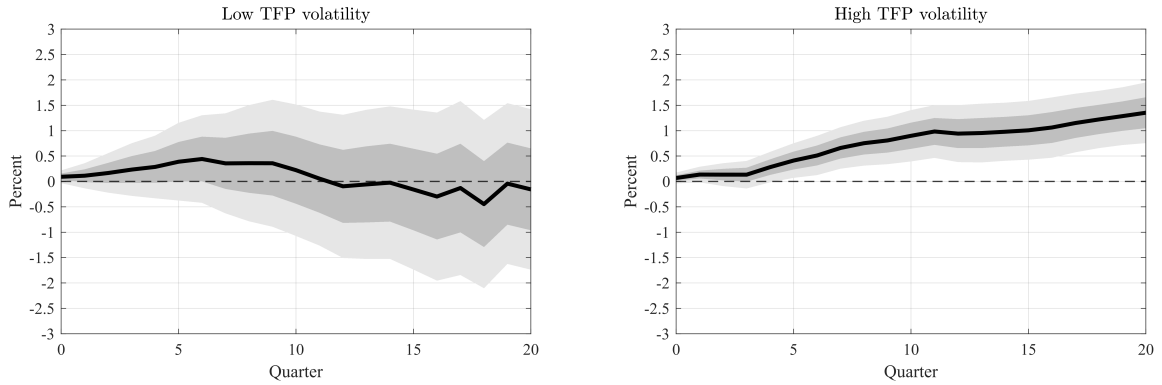
¹⁹ The results are very similar if we use instead the residuals from the Markov-switching model presented in Appendix D.1.

Figure 10: Empirical responses of net entry to 1 percent fall in productivity



Notes. Sample period 1993Q2-2022Q4. TFP volatility regimes based on Markov-switching model. Productivity shock measured as residual from AR(1) model estimated on first difference of utilization-adjusted TFP. Net entry measured as establishment entry rate. Dark (light) gray shaded area are 68% (95%) confidence bands.

Figure 11: Empirical responses of prices to 1 percent fall in productivity



Notes. Sample period 1993Q2-2024Q1. TFP volatility regimes based on Markov-switching model. Productivity shock measured as residual from AR(1) model estimated on first difference of utilization-adjusted TFP. Prices measured as producer price index (core final goods). Dark (light) gray shaded areas are 68% (95%) confidence bands.

Importantly, these results accord well with the findings in our theoretical model: the extensive margin adjustment to productivity shocks crucially hinges on the state of TFP shock volatility.

In Figure 11 we test another prediction of our model, namely that average prices respond relatively little in states of low TFP volatility, while the effects of productivity shocks on prices are more pronounced in times of high TFP volatility. Indeed, the estimation reveals that prices increase far less in response to productivity shocks in a regime of low TFP volatility. The effects are not statistically significant in this case. When TFP volatility had been large prior to the shock, however, a negative productivity shock has

a statistically significant and long lasting positive effect on producer prices. When we repeat the analysis for consumer prices, our findings are confirmed (see Appendix D).

Overall, our finding of regime-dependent effects of productivity shocks on the price level are consistent with our theoretical predictions. When ex ante uncertainty about TFP – measured by the state of TFP volatility – is high, adverse TFP shocks lead to higher price pressure compared to a regime in which ex ante uncertainty about TFP is low. These empirical estimates are consistent with the mechanism of our theoretical model whereby price adjustment shields firms from adverse supply shocks, which prevents exit.²⁰

5 Monetary policy uncertainty, price flexibility, and welfare

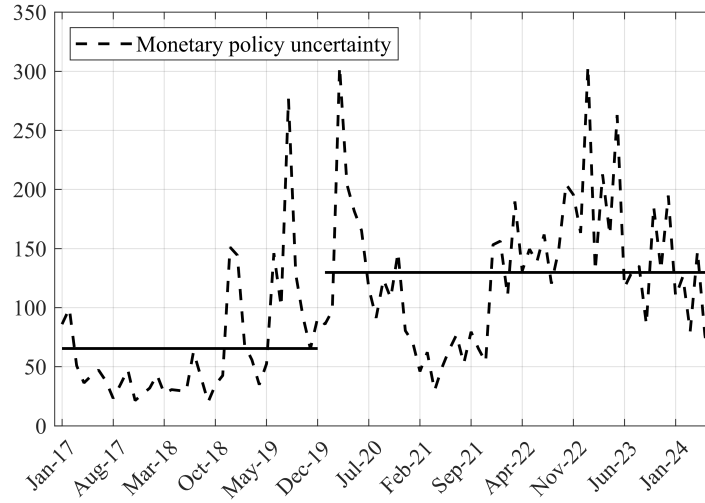
As is evident in US data, shifts in monetary policy uncertainty can be substantial. Since the beginning of the Covid-19 pandemic, the monetary policy uncertainty index of Baker et al. (2016) – a measure based on textual analysis of news articles – is on average markedly higher than in the pre-pandemic years, see Figure 12.²¹ In particular, from late 2021 onward perceived uncertainty about US monetary policy increased.

In our model in Section 3, we have kept the money stock M constant. We now relax this assumption. As for the transmission of monetary policy shocks, i.e. changes in the money stock, Bilbiie (2021) shows that, with free entry, monetary policy has no first-order effect on output under sticky prices – as long as product diversity is optimal under constant elasticity of substitution (CES) consumption preferences as in Dixit and Stiglitz (1977). In our model, we also assume standard CES preferences. Thus, a monetary policy shock leaves aggregate output unchanged regardless of the degree of nominal rigidity, while the number of firms responds more strongly to a monetary policy shock when more

²⁰ We also checked if our results are robust to controlling for the periods of the Great Financial Crisis and the Great Recession that followed, as well as the outbreak of the Covid-19 pandemic. For this purpose, we included period dummies in our regression. This robustness analysis yields results that are similar to the baseline regression. Details are available from the authors upon request.

²¹ The monetary policy uncertainty index of Husted et al. (2020) indicates a similar pattern.

Figure 12: Monetary policy uncertainty in the United States



Notes. Figure shows Monetary Policy Uncertainty index (normalized to 100 in 2010) based on textual analysis of news articles. Sources: Baker et al. (2016) and Haver Analytics. Monthly data, January 2017 to June 2024. Black solid line depicts pre- and post-December 2019 averages.

firms are sticky-price firms.²² This implies that, for any degree of price rigidity, the welfare-based consumer price index, which depends positively on producer prices and inversely on the number of varieties, responds roughly in the same way. This is because, when fluctuations in producer prices become larger (when price flexibility is higher), the response of the extensive margin of firms becomes smaller.²³

Importantly, in our framework the degree of price stickiness is endogenous. When monetary policy shocks are active, uncertainty about monetary policy itself affects the fraction of flexible-price firms in the economy. To study the role of monetary policy uncertainty, we simulate our baseline model assuming that firms expect monetary policy shocks to have a certain positive variance while, for the sake of clarity and without loss of generality, we switch off productivity uncertainty. Similar to the case of productivity uncertainty discussed in Section 3, the fraction of firms that set prices flexibly increases in the expected volatility of monetary policy shocks.²⁴ Under the presumption that central

²² As in Bilbiie (2021), a money supply shock has a negative second-order effect on output regardless of its sign.

²³ As emphasized by Bilbiie and Melitz (2022), in a standard model with entry and exit, there also exists an optimal money supply rule that can replicate the social planner allocation (see Appendix B for a derivation).

²⁴ See Figure C.4 in the online appendix. Devereux (2006) finds a similar result in a model without endogenous entry.

banks can control the volatility of their policy instrument at least to a certain extent, this result implies that they can affect inflation directly by affecting firms' choice of whether or not to invest in price flexibility.

Notice that a somewhat similar result can be found in Gray (1978), who studies optimal indexation of wages to the price level under money supply uncertainty. She shows that, when indexing is costly, the proportion of wage contracts that are indexed is an increasing function of the variance of monetary shocks. The intuition is that the gains from indexing – which are akin to the gains from investing in price flexibility in Devereux (2006) – increase in monetary variability.

5.1 Productivity shock transmission under monetary policy uncertainty

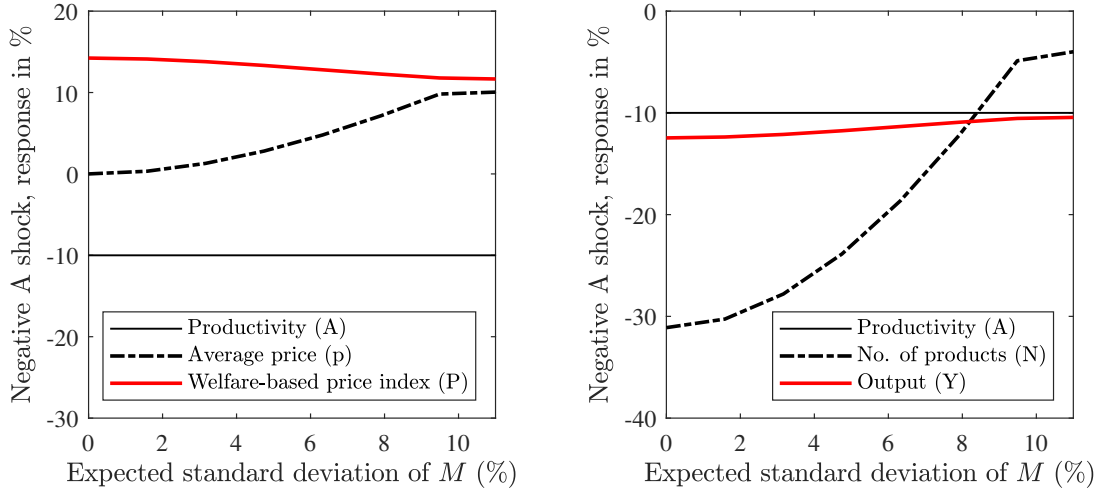
Introducing money supply uncertainty in a model with productivity shocks gives rise to a hitherto unexplored channel of monetary policy. In particular, we demonstrate that monetary policy uncertainty shapes the transmission of productivity shocks as a source of firm dynamics and the business cycle.

Firms' price setting decisions following a supply shock depend critically on the degree of monetary policy uncertainty. The black-dotted line in the left panel of Figure 13 shows the response of producer prices to a negative productivity shock, for different regimes regarding uncertainty about future monetary policy. When the expected (ex ante) money supply volatility is greater, there is higher producer price inflation in response to negative productivity shocks. Similar to the case of productivity uncertainty discussed in Section 3, exit is more pronounced when monetary policy uncertainty is low and prices are sticky, thus output and the welfare-based consumer price index respond more strongly.

5.2 Welfare consequences of monetary policy uncertainty

Central bank have (some) control over the level of monetary policy uncertainty, for instance through their communication. This raises the question how much monetary policy uncertainty is optimal when productivity shocks are present.

Figure 13: Responses to adverse supply shocks under different monetary policy uncertainty regimes



Notes. Responses to 10% drop in productivity under different monetary policy uncertainty regimes. In this exercise, there is no ex ante productivity uncertainty.

In our framework, welfare varies across uncertainty regimes that – as shown above – give rise to different degrees of price flexibility. This relates to previous literature that shows that price flexibility is key to studying welfare. A general result in (standard, no-entry) New Keynesian models is that of ‘divine coincidence’ (Blanchard and Galí, 2007). In the absence of real imperfections, the flexible-price allocation is dynamically efficient, and a policy that replicates this allocation maximizes welfare. For this result to hold, the policy maker needs to undo the monopolistic competition distortion with an appropriate labor or production subsidy. If such a subsidy is absent, labor and output in the flexible-price allocation are too low, see for instance Galí (2015).

Dixit and Stiglitz (1977) show that product diversity is optimal in the case where utility is a CES bundle over goods varieties. Accordingly, as argued in Bilbiie et al. (2007, 2019), in a New Keynesian model with endogenous entry-exit and a consumption aggregator of the CES-type, the flexible-price allocation remains efficient up to a static wedge that can be removed through a constant labor subsidy.

In our framework, uncertainty and the firms’ choice to invest in price flexibility add another dimension to welfare considerations. On the one hand, higher uncertainty induces firms to choose a higher degree of price flexibility, giving rise to an allocation closer to the optimal flexible-price allocation. On the other hand, greater price flexibility is costly

in that a greater part of the labor force is occupied with the ‘production’ of prices and, thus, is diverted away from producing consumption goods.

To compute welfare for different values of monetary policy uncertainty, we start with defining a social-planner benchmark. As we show in Appendix B, the social planner (or first best) allocation is given by a set $\{L, N, C\}$ satisfying

$$L = \left[\frac{\theta}{(\theta - 1)\chi} \right]^{1/(1+\varphi)}, \quad (27)$$

$$N = \frac{AL}{\theta f}, \quad (28)$$

$$C = (\theta - 1)fN^{\frac{\theta}{\theta-1}}. \quad (29)$$

The first best allocation arises in a world with flexible prices, no price flexibility investment costs, and an optimal labor income subsidy.

We now turn to a numerical exercise comparing welfare across different monetary policy uncertainty regimes. Let $\mathcal{U}$ denote expected utility, net of utility from real money balances, in the decentralized allocation

$$\mathcal{U} = \mathbb{E}\{U(C, L)\}. \quad (30)$$

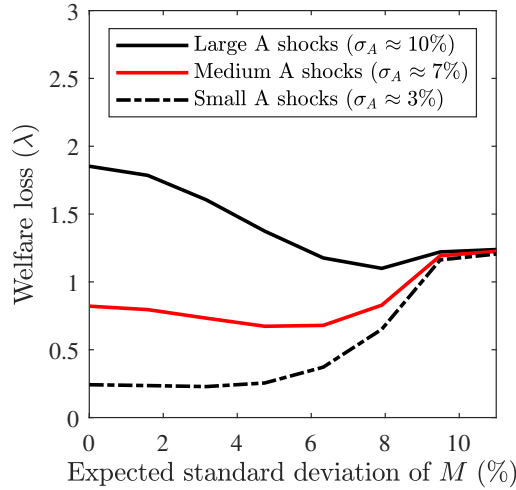
We define the welfare cost of price setting frictions in consumption equivalents, as do Schmitt-Grohé and Uribe (2007). More precisely, the welfare loss is the fraction λ of consumption in the first best allocation that households have to give up to attain the same utility as in the decentralized allocation. I.e., the welfare loss is defined implicitly as λ in

$$\mathcal{U} = \mathbb{E}\{U((1 - \lambda)C^{fb}, L^{fb})\}, \quad (31)$$

where C^{fb} and L^{fb} are consumption and labor in the first best allocation. Setting (30) equal to (31), using the utility function (11), and solving for the welfare loss, λ , we obtain

$$\lambda = 100 \left\{ 1 - \exp \left[(\mathbb{E}(U) - \mathbb{E}(U^{fb})) \right] \right\}. \quad (32)$$

Figure 14: Welfare loss (relative to first-best) under monetary policy uncertainty



Notes. Welfare loss λ , in percent of consumption in the first best allocation, see equation (32), across monetary policy uncertainty regimes.

We compute welfare for a range of monetary policy uncertainty regimes defined by different values of the money supply volatility σ_m . In each monetary policy uncertainty regime, the steady state is different, as the fraction of flexible-price firms and the cost of price setting flexibility varies. In the simulations, we assume that actual monetary policy shocks are absent; the economy is hit only by productivity shocks. This mainly serves illustrative purposes: we aim at characterizing the costs and benefits of monetary policy uncertainty that arise as productivity shocks are transmitted differently across regimes.

More specifically, we simulate productivity shocks with a certain standard deviation $\sigma_{a,simul}$. In this exercise, the standard deviation of the simulated shocks $\sigma_{a,simul}$ is assumed to be the same in each monetary policy uncertainty regime. To compute welfare, we simulate for a discrete range of technology draws. The set of possible states falls in a range with five elements and a standard deviation from the median state of 1 given by $\sigma_{a,simul}$.²⁵ We consider three possible cases of “small”, “medium” and “large” productivity shocks, with $\sigma_{a,simul}$ taking on the value 3%, 7% and 10%, respectively. Figure 14 shows the results.

When firms decide to invest in price flexibility as monetary policy becomes more

²⁵ To illustrate, we set ex ante productivity uncertainty to zero. The results may differ if we assumed positive levels of ex ante productivity uncertainty that exactly align with the actual shocks in the simulations. In this case, firms are prepared for large shocks and have invested in price flexibility. Thus, monetary policy uncertainty has little room to improve welfare. In practice, monetary policy uncertainty and productivity uncertainty are potentially correlated. We abstract from this complication here.

uncertain, some fraction of aggregate labor is allocated to price setting and is therefore not producing output. Welfare, which increases in consumption, is therefore smaller in regimes with very high productivity uncertainty. However, as discussed above, higher uncertainty results in more firms that set prices flexibly, which is beneficial for welfare. In the numerical example in Figure 14, this channel is dominant for a shift from a zero-uncertainty regime to a medium degree of uncertainty when the shocks hitting the economy are very large. In this case, a shift toward higher uncertainty can be beneficial in that firms are incentivized to invest more in price flexibility. In regimes with very high uncertainty, this benefit is offset as the costs of setting prices become large. When productivity shocks are small on average, there is not much benefit in higher monetary policy uncertainty. It is, on the contrary, optimal to keep monetary policy uncertainty at a rather low level.

A couple of other contributions study the consequences of ‘randomness’ in monetary policy; however, the mechanism is different from the one presented here. First, Dupor (2003) studies a model similar to ours (except for endogenous entry), with imperfect competition, predetermined prices and money-in-utility. Volatility in money growth on the one hand induces consumption volatility, which is detrimental for welfare. On the other, it leads firms to reduce their expected markup, thereby increasing expected sales.²⁶ If the latter effect dominates, a lower expected markup reduces the imperfect competition distortion, thereby raising welfare. Here, we abstract from the markup distortion by imposing an optimal labor supply subsidy.

Second, Ghironi and Ozhan (2020) put forward interest rate uncertainty as a policy tool, which they argue can be used to discourage inefficient capital inflows. In a two-country New Keynesian model, domestic interest rate volatility raises household savings at home through a precautionary savings motive. Moreover, as the home country asset return becomes more volatile, savers in both countries channel funds into foreign country bonds instead. Here, we abstract from financial market imperfections and focus on how money supply volatility can help to reduce the price setting distortion.

²⁶ This ‘precautionary sales’ motive is due to prudence, i.e. convex marginal utility.

6 Conclusion

We study the consequences of adverse supply shocks for output, prices and product diversity in a model where price flexibility is endogenous. More specifically, we consider a firm’s investment decision in a technology that allows the firm to change its price in response to shocks (Devereux, 2006). We show that the volatility of productivity shocks is critical for the equilibrium degree of price flexibility in an economy. When uncertainty rises and shocks are larger on average, more firms are willing to invest in price flexibility and this reduces product exit and output losses in the wake of negative supply shocks. In a local projections estimation on US data, we find evidence of this key model prediction. In regimes of high TFP volatility, prices respond more, and net entry responds less, to TFP shocks than in a regime of low TFP volatility. An additional result of our theory is that money supply volatility can be welfare-improving as long as the benefits from increased price flexibility exceed the costs of investing in price setting capabilities.

Our findings are highly relevant for policy. Greater uncertainty both at the firm level and at the aggregate level might prevail even once the effects of the pandemic and the Ukraine war eventually fade. Against the background of new risks arising from climate change and the green transition, as well as protectionism and deglobalisation, the European Central Bank’s executive board member Isabel Schnabel discusses the risk of entering a new era of “Great Volatility” (Schnabel, 2022). The consequences of higher volatility for the behavior of economic agents warrants further investigation.

A dynamic framework, in which firms may change their investment in price flexibility over time, would account for feedback effects of firm entry-exit on the degree of price flexibility, ex post being the ratio of flexible-price versus sticky-price firms in the market. We leave this challenge for future research.

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Online Appendix:

Product turnover and endogenous price flexibility in uncertain times

July 22, 2025

A Model and decentralized equilibrium

A.1 Firm's choice of price flexibility

The model derivation starts with the firm's problem to invest in price flexibility, which we borrow from Devereux (2006). We adapt his model in three dimensions. First, we introduce productivity A into the model. Second, we change the support on which differentiated goods are indexed from $[0, 1]$ to $[0, N]$. That is, we change the size of the mass of goods/firms from 1 to N (later, we will endogenize N by allowing for entry and exit). Third, we introduce a fixed startup cost in labor units f , which is needed to pin down profits and the number of producers in equilibrium once we introduce entry and exit.

Firms produce differentiated goods indexed by $\omega \in [0, N]$ and compete under monopolistic competition, taking the wage W as given. Following Devereux (2006), we assume a production function with labor as the only input,

$$Y(\omega) = Al(\omega)^\alpha, \tag{A.1}$$

where the parameter $\alpha \in (0, 1)$ captures the returns to labor in production, A is a productivity shock, $l(\omega)$ is total labor input that firm ω uses for production. The firm's operating cost is $Wl(\omega)$ or, from the production function (A.1),

$$W \frac{Y(\omega)^{1/\alpha}}{A}. \tag{A.2}$$

The firm faces the following demand function (to be derived below):

$$Y(\omega) = \left(\frac{P(\omega)}{P} \right)^{-\theta} Y = P(\omega)^{-\theta} \hat{Y}, \tag{A.3}$$

where $\hat{Y} = P^\theta Y$ is market demand and $\theta > 1$ is the elasticity of substitution between goods varieties in the final goods firm's production function (see below).

A.1.1 Price setting

The intermediate goods firm chooses a price $P(\omega)$ to maximize expected discounted operating profits given by nominal revenues minus operating cost (A.2), i.e. excluding fixed production costs (that do not depend on market demand or goods prices), $\mathbb{E}\Gamma\{P(\omega)Y(\omega) - \frac{W}{A}Y(\omega)^{1/\alpha}\}$, where Γ is the discount factor. Replacing firm output $Y(\omega)$ using the demand constraint (A.3), expected operating profits can be written as

$$\mathbb{E}\Gamma \left\{ P(\omega) \left(\frac{P(\omega)}{P} \right)^{-\theta} Y - \frac{W}{A} \left(\left(\frac{P(\omega)}{P} \right)^{-\theta} Y \right)^{1/\alpha} \right\}. \quad (\text{A.4})$$

Then the price setting problem is to choose $P(\omega)$ in order to maximize (A.4).

Price setting and expected profits with investment in price flexibility. If the firm invests in price flexibility, it can choose its price after observing $\Theta = \{\Gamma, W, \hat{Y}, A\}$. The first order condition to this problem yields the flexible-firms' optimal price

$$\tilde{P} = \delta \left[\left(\frac{W}{A} \right)^\alpha \hat{Y}^{1-\alpha} \right]^\zeta, \quad (\text{A.5})$$

where

$$\delta = \left(\frac{\theta}{\alpha(\theta - 1)} \right)^{\alpha\zeta}, \quad (\text{A.6})$$

and

$$\zeta = \frac{1}{\alpha + \theta(1 - \alpha)}. \quad (\text{A.7})$$

With a linear production function where $\alpha = 1$, we have $\delta = \frac{\theta}{\theta - 1}$ and $\zeta = 1$, such that the optimal price simplifies to:

$$\tilde{P} = \frac{\theta}{\theta - 1} \frac{W}{A}. \quad (\text{A.8})$$

Now, we derive the flexible-price firm's expected profits under the optimal price setting rule. Using the definition of market demand $\hat{Y} = P^\theta Y$ and plugging in the optimal price (A.5), we write the firm's expected discounted operating profits (A.4) as follows

$$\mathbb{E}\Gamma \left\{ (\delta^{1-\theta} - \delta^{-\theta/\alpha}) \left[\left(\frac{W}{A} \right)^{\alpha(1-\theta)} \hat{Y} \right]^\zeta \right\}.$$

Let's denote the firm's expected profits under investment in price flexibility as $\tilde{V}(\Theta)$, such that

$$\tilde{V}(\Theta) = \Psi \mathbb{E} \left\{ \Gamma \left[\left(\frac{W}{A} \right)^{\alpha(1-\theta)} \hat{Y} \right]^\zeta \right\}. \quad (\text{A.9})$$

where

$$\Psi = \delta^{1-\theta} - \delta^{-\theta/\alpha}. \quad (\text{A.10})$$

Price setting and expected profits without investment in price flexibility. If the firm does *not* invest in price flexibility, we can solve for the (common) sticky-firm's price as follows

$$\bar{P} = \delta \frac{\mathbb{E}\{\Gamma(W/A)\hat{Y}^{1/\alpha}\}^{\alpha\zeta}}{\mathbb{E}\{\Gamma\hat{Y}\}^{\alpha\zeta}}. \quad (\text{A.11})$$

Plugging the optimal price (A.11) into (A.4), the sticky-price firm's expected profits are given by

$$\bar{V}(\Theta) = \Psi \mathbb{E}\{\Gamma(W/A)\hat{Y}^{1/\alpha}\}^{(1-\theta)\alpha\zeta} \mathbb{E}\{\Gamma\hat{Y}\}^{\theta\zeta}. \quad (\text{A.12})$$

Realized profits with production fixed cost and entry/exit decisions. In equilibrium, firms set prices optimally and produce when operating profits are larger than production fixed costs (in terms of labor), Wf . Moreover, under entry and exit the equilibrium sticky-price firm profits are always zero. Thus, ex post profits of a firm ω are given by

$$\tilde{\Pi}(\omega) = \max(\tilde{V}(\Theta) - Wf, 0),$$

$$\bar{\Pi}(\omega) = \max(\bar{V}(\Theta) - Wf, 0) = 0.$$

The latter equation implies that $\bar{V} = Wf$. Moreover,

$$\tilde{\Pi}(\omega) = \max(\tilde{V}(\Theta) - \bar{V}(\Theta), 0) = \tilde{V}(\Theta) - \bar{V}(\Theta). \quad (\text{A.13})$$

Investment in price flexibility. The firm chooses to be a flex-price firm if the gain in expected discounted profits exceeds the expected discounted costs of investing in price flexibility. This can be formulated as

$$\max(\tilde{V}(\Theta) - Wf, 0) > \mathbb{E}\{\Gamma W \Phi(\omega)\}.$$

From (A.13), the gap between profits of flexible-price firms and fixed-price firms is equal to $\max(\tilde{V}(\Theta) - \bar{V}(\Theta), 0) = \tilde{V}(\Theta) - \bar{V}(\Theta)$. This implies that

$$\tilde{V}(\Theta) - \bar{V}(\Theta) > \mathbb{E}\{\Gamma W \Phi(\omega)\}.$$

Because the cost of price flexibility $\Phi(\omega)$ is known ex ante, we can take this term out of the expectations operator and write:

$$\Delta(\Theta) \equiv \frac{\tilde{V}(\Theta) - \bar{V}(\Theta)}{\mathbb{E}\{\Gamma W\}} \geq \Phi(\omega). \quad (\text{A.14})$$

In (A.14), the term $\Delta(\Theta)$ captures the gain from price flexibility. .

A.1.2 Approximation of the gain function (8)

Substitution of flexible-price firms' and sticky-price firms' expected profits, equations (A.9) and (A.12), into equation (A.14) gives

$$\Delta(\Theta) = \frac{\Psi}{\mathbb{E}\{\Gamma W\}} \left(\mathbb{E}\Gamma[(W/A)^{\alpha(1-\theta)}\hat{Y}]^\zeta - \mathbb{E}\{\Gamma(W/A)\hat{Y}^{1/\alpha}\}^{\alpha(1-\theta)\zeta} \mathbb{E}\{\Gamma\hat{Y}\}^{\theta\zeta} \right). \quad (\text{A.15})$$

To express the variables in logarithmic form, we transform this into

$$\Delta(\Theta) = \frac{\Psi}{\mathbb{E}\exp(\ln \Gamma + \ln W)} \left\{ \mathbb{E}\exp(\ln \Gamma + \alpha(1-\theta)\zeta(\ln W - \ln A) + \zeta \ln \hat{Y}) \right. \\ \left. - \left[\mathbb{E}\exp(\ln \Gamma + \ln W - \ln A + \frac{1}{\alpha} \ln \hat{Y}) \right]^{\alpha(1-\theta)\zeta} \left[\mathbb{E}\exp(\ln \Gamma + \ln \hat{Y}) \right]^{\theta\zeta} \right\}. \quad (\text{A.16})$$

Let's define $g \equiv \ln \Gamma - \mathbb{E} \ln \Gamma$, $a \equiv \ln A - \mathbb{E} \ln A$, $w \equiv \ln W - \mathbb{E} \ln W$, and $\hat{y} \equiv \ln \hat{Y} - \mathbb{E} \ln \hat{Y}$. Up to second order, the approximation of equation (A.16) around the stochastic mean $\mathbb{E} \ln \Theta$ is equal to

$$\begin{aligned} \Delta(\Theta) &\approx \Delta(\exp(\mathbb{E} \ln \Theta)) - \Delta(\exp(\mathbb{E} \ln \Theta))\mathbb{E}(g + w) \\ &+ \Delta(\exp(\mathbb{E} \ln \Theta))\mathbb{E}(g^2 + w^2 + 2gw) \\ &+ \Omega\{\mathbb{E}[g + \alpha(1-\theta)\zeta(w - a) + \zeta\hat{y}] - \mathbb{E}[(1-\theta)\alpha + \theta]\zeta g + \alpha(1-\theta)\zeta(w - a) + ((1-\theta)\zeta + \theta\zeta)\hat{y}\} \\ &+ \frac{1}{2}\mathbb{E}[g^2 + \alpha^2(1-\theta)^2\zeta^2(w^2 + a^2) + \zeta^2\hat{y}^2] \\ &+ \mathbb{E}[\alpha(1-\theta)\zeta gw + \zeta g\hat{y} + \zeta^2\alpha(1-\theta)w\hat{y} - \alpha(1-\theta)\zeta(ga + \alpha(1-\theta)\zeta wa + \zeta\hat{y}a)] \\ &- \frac{1}{2}[\alpha(1-\theta)\zeta\mathbb{E}(g^2 + w^2 + a^2 + \alpha^{-2}\hat{y}^2) + \theta\zeta\mathbb{E}(g^2 + \hat{y}^2 + 2g\hat{y})] \\ &- \alpha(1-\theta)\zeta\mathbb{E}(gw + \alpha^{-1}g\hat{y} + \alpha^{-1}w\hat{y} - ga - wa - \alpha^{-1}\hat{y}a)\}, \end{aligned} \quad (\text{A.17})$$

where we define $\Omega \equiv \frac{V(\exp(\mathbb{E} \ln \Theta))}{\exp(\mathbb{E} \ln \Gamma + \mathbb{E} \ln W - \mathbb{E} \ln A)}$. We know that $\Omega > 0$, as the numerator of expression (10), $V(\exp(\mathbb{E} \ln \Theta))$, represents the operating profit function – of flexible-price firms or sticky-price firms – evaluated at the stochastic mean $\mathbb{E} \ln \Theta$ and the denominator of (10) is an exponential function, which is always positive.

We can simplify (A.17) as follows. First, by definition, the gains from price flexibility evaluated at the stochastic mean are zero, i.e. $\Delta(\exp(\mathbb{E} \ln \Theta)) = 0$, as profits of flexible-price firms and profits of preset-price firms, evaluated at the constant mean value $\mathbb{E} \ln \Theta$, are equal. For this reason, the first three terms on the right hand side of equation (A.17) are zero. Further, this implies that $V(\exp(\mathbb{E} \ln \Theta)) = \tilde{V}(\exp(\mathbb{E} \ln \Theta)) = \bar{V}(\exp(\mathbb{E} \ln \Theta))$ in expression (10). Second, we have that $\mathbb{E}(g) = \mathbb{E}(w) = \mathbb{E}(\hat{y}) = \mathbb{E}(a) = 0$, so the fourth and fifth terms in the previous expression are zero. Simplifying equation (A.17) in this

way, we obtain

$$\begin{aligned}\Delta(\Theta) \approx & \Omega \left\{ \frac{1}{2} \mathbb{E}[g^2 + \alpha^2(1-\theta)^2 \zeta^2(w^2 + a^2) + \zeta^2 \hat{y}^2] \right. \\ & + \mathbb{E} [\alpha(1-\theta) \zeta g w + \zeta g \hat{y} + \zeta^2 \alpha(1-\theta) w \hat{y} - \alpha(1-\theta) \zeta (g a + \alpha(1-\theta) \zeta w a + \zeta \hat{y} a)] \\ & - \frac{1}{2} [\alpha(1-\theta) \zeta \mathbb{E}(g^2 + w^2 + a^2 + \alpha^{-2} \hat{y}^2) + \theta \zeta \mathbb{E}(g^2 + \hat{y}^2 + 2g\hat{y})] \\ & \left. - \alpha(1-\theta) \zeta \mathbb{E}(g w + \alpha^{-1} g \hat{y} + \alpha^{-1} w \hat{y} - g a - w a - \alpha^{-1} \hat{y} a) \right\}.\end{aligned}$$

Rearranging the equation by collecting all the second order terms in g , we obtain

$$\begin{aligned}\Delta(\Theta) \approx & \Omega \left\{ \frac{1}{2} \mathbb{E}[\alpha^2(1-\theta)^2 \zeta^2(w^2 + a^2) + \zeta^2 \hat{y}^2] \right. \\ & + \mathbb{E} [\zeta^2 \alpha(1-\theta) w \hat{y} - \alpha(1-\theta) \zeta (\alpha(1-\theta) \zeta w a + \zeta \hat{y} a)] \\ & - \frac{1}{2} [\alpha(1-\theta) \zeta \mathbb{E}(w^2 + a^2 + \alpha^{-2} \hat{y}^2) + \theta \zeta \mathbb{E}(\hat{y}^2)] \\ & \left. - \alpha(1-\theta) \zeta \mathbb{E}(\alpha^{-1} w \hat{y} - w a - \alpha^{-1} \hat{y} a) \right\} \\ & + \frac{1}{2} \Omega \mathbb{E}[1 - \alpha(1-\theta) \zeta - \theta \zeta] g^2 + \Omega \mathbb{E}[\alpha(1-\theta) \zeta - \alpha(1-\theta) \zeta] g w \\ & + \Omega \mathbb{E}[\zeta - \theta \zeta - (1-\theta) \zeta] g \hat{y} + \Omega \mathbb{E}[-\alpha(1-\theta) \zeta + \alpha(1-\theta) \zeta] g a.\end{aligned}$$

All the terms in g , the last four lines in the above equation, drop out. Collecting terms, simplifying, and rearranging, we end up with

$$\Delta(\Theta) \approx \Omega \alpha(1-\theta) \zeta^2 \theta \left\{ \frac{1}{2} \mathbb{E} \left[-w^2 - a^2 - \frac{(1-\alpha)^2}{\alpha^2} \hat{y}^2 - 2 \frac{1-\alpha}{\alpha} w \hat{y} + 2 w a + 2 \frac{1-\alpha}{\alpha} \hat{y} a \right] \right\}.$$

Defining $\mathbb{E}(w^2) \equiv \sigma_w$, $\mathbb{E}(a^2) \equiv \sigma_a$, etc., we can rewrite as

$$\Delta(\Theta) \approx \frac{\Omega}{2} (\theta - 1) \theta \alpha \zeta^2 \left[\sigma_w^2 + \frac{(1-\alpha)^2}{\alpha^2} \sigma_{\hat{y}}^2 + \sigma_a^2 + \frac{2(1-\alpha)}{\alpha} \sigma_{w\hat{y}} - \frac{2(1-\alpha)}{\alpha} \sigma_{a\hat{y}} - 2\sigma_{wa} \right]. \quad (\text{A.18})$$

The gains from price flexibility depend positively on the variance of the wage, the variance of market demand, the variance of productivity, and on the covariance between the wage and market demand. They further depend negatively on the covariance between productivity and the wage, and on the covariance between productivity and market demand. In fact, the expression in square brackets in (A.18) is equal to $\text{Var}(\ln W + \frac{1-\alpha}{\alpha} \ln \hat{Y} - \ln A)$. Up to second order, the gains from price flexibility do not depend on the properties of the stochastic discount factor Γ .

Special case 1: No productivity shocks. If productivity is constant, $A = \bar{A}$, we can set its variance (and all covariances) to zero, $\sigma_a^2 = 0$, such that equation (A.18) simplifies

to:

$$\Delta(\Theta) \approx \frac{\Omega}{2}(\theta - 1)\theta\alpha\zeta^2 \left[\sigma_w^2 + \frac{(1 - \alpha)^2}{\alpha^2}\sigma_{\hat{y}}^2 + \frac{2(1 - \alpha)}{\alpha}\sigma_{w\hat{y}} \right], \quad (\text{A.19})$$

which is the expression derived in Devereux (2006). The gains from price flexibility depend positively on the variance of the wage, the variance of market demand, and their covariance. We see from (A.19) that uncertainty in market demand plays a role only if $\alpha < 1$. If the production function is linear in labor, i.e. $\alpha = 1$, marginal costs are independent of output and a change in market demand does not induce a firm to change its price.

Special case 2: Linear production function. In the case of a linear production function, i.e. $\alpha = 1$, we have $\zeta = 1$ and equation (A.18) simplifies to:

$$\Delta(\Theta) \approx \frac{\Omega}{2}(\theta - 1)\theta [\sigma_w^2 + \sigma_a^2 - 2\sigma_{wa}]. \quad (\text{A.20})$$

The gains from price flexibility depend positively on the variance of the wage and on the variance of productivity, and negatively on their covariance. In fact, the expression in square brackets in (A.20) is equal to $\text{Var}(\ln W - \ln A)$. Recall that $\theta > 1$ by assumption, and that $\Omega > 0$. Therefore, the gains from price flexibility are greater than zero, unless the wage is linear in productivity. In the latter case, the gains from price flexibility are exactly zero.

A.2 Firm profits concave in productivity

In the following, we derive the curvature of the profit function for flexible-price and sticky-price firms. We show that the flexible-price firm's profit function is convex in A for $\theta > 2$, and the sticky-price firm's profit function is concave in A in this partial equilibrium exercise (conditional on the production function being linear, i.e. $\alpha = 1$).

Both types of firms have ex post (nominal) operating profits given by $\Pi(\omega) = P(\omega)Y(\omega) - WL(\omega)$. Plugging in labor $L(\omega) = Y(\omega)/A$ and demand for firm ω 's output $Y(\omega) = (P(\omega)/P)^{-\theta}Y$, this becomes $\Pi(\omega) = (P(\omega) - \frac{W}{A})P(\omega)^{-\theta}\hat{Y}$, where $\hat{Y} = P^\theta Y$ is market demand.

First, suppose firm ω is a **flexible-price** firm. Substituting the optimal flexible price $\tilde{P}(\omega) = \frac{\theta}{\theta-1} \frac{W}{A}$, we obtain the profit function $\tilde{\Pi}(\omega) = (\frac{\theta}{\theta-1} \frac{W}{A})^{1-\theta} \frac{P^\theta Y}{\theta}$. Notice that this is a partial equilibrium exercise; we do not consider how the wage or market demand change with productivity. Bearing this in mind, we differentiate the profit function twice with respect to A , taking the wage W and market demand $\hat{Y}$ as given, to obtain $\frac{\partial^2 \tilde{\Pi}(\omega)}{\partial A \partial A} = (\theta - 2)\delta^{-\theta} A^{\theta-3} W^{1-\theta} \hat{Y}$, where $\delta = \frac{\theta}{\theta-1}$. The profit function is convex in A , i.e. the second derivative is positive, if demand is sufficiently elastic such that $\theta > 2$.

Second, if firm ω is a **sticky-price** firm, it sets a price $\bar{P}(\omega) = \delta \frac{\mathbb{E}\{\Gamma(W/A)\hat{Y}\}}{\mathbb{E}\{\Gamma\hat{Y}\}}$. When

differentiating sticky-price profits $\bar{\Pi}(\omega)$ with respect to productivity A , we treat the preset price $\bar{P}(\omega)$ as a constant. The second derivative of the profit function is given by $\frac{\partial^2 \bar{\Pi}(\omega)}{\partial A \partial A} = -2 \frac{W}{A^3} \bar{P}(\omega)^{-\theta} \hat{Y} < 0$, which is negative for all values of θ . For a sticky-price firm, the profit function is concave in productivity A .

A.3 Endogenous entry-exit

We now consider the general equilibrium model with endogenous entry and exit.

Firms. Aggregate output is defined as in (15), where $\theta > 1$ is the elasticity of substitution between goods varieties. The demand for good ω is the solution to problem of choosing $Y(\omega)$ to minimize expenditure $\int_0^N P(\omega) Y(\omega) d\omega$, subject to the aggregator function (15), taking prices $P(\omega)$ as given. The solution is given by (2) for all $\omega \in (0, N)$. The price index is derived from the identity that the value of aggregate output is equal to total expenditure, i.e. $PY = \int_0^N P(\omega) Y(\omega) d\omega$, where P , the price of bundle Y , is interpreted as the price index. Substituting $Y(\omega)$ in the expenditure function using the demand equation (2), and rearranging, we find that the aggregate price index is given by $P = (\int_0^N P(\omega)^{1-\theta} d\omega)^{\frac{1}{1-\theta}}$. There are two types of firms. Firms on the interval $(0, z)$ set prices after the state of nature has realized. We call these firms flexible-price firms. The remaining firms on the interval (z, N) set prices in advance; we call these sticky-price firms. Flexible-price firms set a price $\tilde{P}(\omega)$ and sticky-price firms set a price $\bar{P}(\omega)$. This implies that the price index can be written as $P^{1-\theta} = \int_0^z \tilde{P}(\omega)^{1-\theta} d\omega + \int_z^N \bar{P}(\omega)^{1-\theta} d\omega$.

Symmetric equilibrium. We restrict attention to symmetric equilibria. The household's expenditure can be expressed as $PC = \int_0^z \tilde{P}(\omega) \tilde{Y}(\omega) d\omega + \int_z^N \bar{P}(\omega) \bar{Y}(\omega) d\omega$. Given that all flexible-price firms are alike, and all sticky-price firms are alike, we can write this as

$$PC = [z\tilde{P}\tilde{Y} + (N-z)\bar{P}\bar{Y}]. \quad (\text{A.21})$$

Also, the price index can be written as

$$P^{1-\theta} = [z\tilde{P}^{1-\theta} + (N-z)\bar{P}^{1-\theta}], \quad (\text{A.22})$$

with prices of flexible-price firms and sticky-price firms given by (A.5) and (A.11).

Goods market clearing, for flexible-price firms and for sticky-price firms respectively, is as follows,

$$\tilde{Y} = (\tilde{P}/P)^{-\theta} Y, \quad (\text{A.23})$$

$$\bar{Y} = (\bar{P}/P)^{-\theta} Y. \quad (\text{A.24})$$

Rewriting the production function (A.1) as

$$l(\omega) = \frac{Y(\omega)^{1/\alpha}}{A}, \quad (\text{A.25})$$

the two types of firms' demand for production labor is, respectively, $\bar{l} = \frac{\bar{Y}^{1/\alpha}}{A}$ and $\tilde{l} = \frac{\tilde{Y}^{1/\alpha}}{A}$. Labor market clearing implies

$$L = \left[z \frac{\tilde{Y}^{1/\alpha}}{A} + (N - z) \frac{\bar{Y}^{1/\alpha}}{A} \right] + Nf + \int_0^{z_{max}} \Phi(\omega) d\omega. \quad (\text{A.26})$$

The last term on the right hand side captures the fixed cost incurred by the measure z_{max} of firms that choose to invest in price flexibility. The second-last term on the right hand side captures production fixed costs. We can derive labor by plugging flexible-price firm output $\tilde{Y}$ and sticky-price firm output $\bar{Y}$ into the labor market clearing condition (A.26),

$$L = \left[\frac{z}{A} \left(\left(\frac{\tilde{P}}{\bar{P}} \right)^{-\theta} Y \right)^{1/\alpha} + \frac{N - z}{A} \left(\left(\frac{\bar{P}}{\bar{P}} \right)^{-\theta} Y \right)^{1/\alpha} \right] + Nf + \int_0^{z_{max}} \Phi(\omega) d\omega. \quad (\text{A.27})$$

The wage equation can be simplified by substituting output Y from (14) to obtain $(1 + \tau)W = \chi L^\varphi M / \eta$. Total consumption output is found by aggregating over sticky-price and flexible-price firms,

$$Y = \frac{z\tilde{P}\tilde{Y} + (N - z)\bar{P}\bar{Y}}{P}. \quad (\text{A.28})$$

Entry and exit. Our timing assumption stipulates that, after the price flexibility decision and after the shock has happened, an individual flexible-price firm with $\omega \in (0, z)$ produces if

$$\frac{\tilde{P}(\omega)\tilde{Y}(\omega)}{P} - \frac{W}{P}\tilde{l}(\omega) \geq Wf, \quad (\text{A.29})$$

while an individual sticky-price firm with $\omega \in (z, N)$ produces if

$$\frac{\bar{P}(\omega)\bar{Y}(\omega)}{P} - \frac{W}{P}\bar{l}(\omega) \geq Wf. \quad (\text{A.30})$$

New entrants into the market have not invested in price flexibility and can therefore only compete with the (market-wide) pre-set price. They base their production decision on (A.30). Moreover, flexible-price firms are less likely to exit as price flexibility allows them to adjust prices in response to (adverse) supply shocks. This can be seen by comparing equations (A.29) and (A.30), which differ with respect to the individual firm price (that is either the flexible optimal price or the pre-set price).

A.4 Determination of optimal price flexibility with entry/exit

In Section 3, we assume a linear production function. Thus, we consider the special case of the model derived in (A.3) in which $\alpha = 1$. For a given z/N , the model is log-linear, except for the price index (16), labor market equilibrium (24), and the zero-profit condition (22). The log-linear model equations are

$$\ln \tilde{P} = \ln \delta - \ln A + \ln \chi + \varphi \ln L + \ln M - \ln \eta, \quad (\text{A.31})$$

$$\ln W = \ln \chi + \varphi \ln L + \ln M - \ln \eta, \quad (\text{A.32})$$

$$\ln Y = \ln M - \ln \eta - \ln P. \quad (\text{A.33})$$

Note that the price index, labor and the number of firms are not log-linear (due to Jensen's inequality) at the point of approximation $\mathbb{E} \ln P$, $\mathbb{E} \ln L$ and $\mathbb{E} \ln N$.

In the following, to simplify the notation in the approximated functions, we define $\tilde{N} = z$ and $\bar{N} = N - z$. Note that the function defining $\tilde{N}$ has kinks. Under positive shocks, or under negative shocks in a situation where there is a sufficient number of steady-state sticky-price firms $\bar{N}$ and the shocks are sufficiently small, $\tilde{N} = z$ remains constant at z_{max} . Only when the number of flexible-price firms is very large relative to the number of sticky-price firms might large adverse supply shocks lead to a situation where all sticky-price firms exit the market, which gives rise to extensive adjustments for flexible-price firms.

For simplicity, we focus on the cases in which $\tilde{N} = z$ remains constant at z_{max} , such that the extensive margin only adjusts with sticky-price firms. I.e., we only consider the case $N > z_{max}$. This is the most important case that only excludes very large adverse shocks.

Second-order approximation of the price index. We express the price index as

$$P = \{\exp(\ln \tilde{N}) \exp((1 - \theta) \ln \tilde{P}) + \exp(\ln \bar{N}) \exp((1 - \theta) \ln \bar{P})\}^{1/(1-\theta)}. \quad (\text{A.34})$$

Since we want to approximate $\ln P$ rather than P , we take logs of (A.34),

$$\ln P = \frac{1}{1 - \theta} \ln \left\{ \exp(\ln \tilde{N}) \exp((1 - \theta) \ln \tilde{P}) + \exp(\ln \bar{N}) \exp((1 - \theta) \ln \bar{P}) \right\}. \quad (\text{A.35})$$

The first derivative of (A.35) w.r.t. $\ln \tilde{P}$ is given by

$$\frac{\partial \ln P}{\partial \ln \tilde{P}} = \frac{1}{1 - \theta} \frac{(1 - \theta) \exp(\ln \tilde{N}) \exp((1 - \theta) \ln \tilde{P})}{\exp(\ln \tilde{N}) \exp((1 - \theta) \ln \tilde{P}) + \exp(\ln \bar{N}) \exp((1 - \theta) \ln \bar{P})}.$$

Cancelling terms, this simplifies to

$$\frac{\partial \ln P}{\partial \ln \tilde{P}} = \frac{\exp(\ln \tilde{N}) \exp((1 - \theta) \ln \tilde{P})}{\exp(\ln \tilde{N}) \exp((1 - \theta) \ln \tilde{P}) + \exp(\ln \bar{N}) \exp((1 - \theta) \ln \bar{P})}.$$

The first derivative of (A.35) w.r.t. $\ln \bar{N}$ is

$$\frac{\partial \ln P}{\partial \ln \bar{N}} = \frac{1}{1 - \theta} \frac{\exp(\ln \bar{N}) \exp((1 - \theta) \ln \bar{P})}{\exp(\ln \tilde{N}) \exp((1 - \theta) \ln \tilde{P}) + \exp(\ln \bar{N}) \exp((1 - \theta) \ln \bar{P})}.$$

Noting that $\bar{P}$ and $\ln \tilde{N}$ are constant, we obtain the second-order approximation of (A.35) around the stochastic mean as

$$\begin{aligned} \ln P \approx \ln \left(\{ \exp(\mathbb{E} \ln \tilde{N}) \exp((1 - \theta) \mathbb{E} \ln \tilde{P}) + \exp(\mathbb{E} \ln \bar{N}) \exp((1 - \theta) \ln \bar{P}) \}^{1/(1 - \theta)} \right) \\ + \varpi(\mathbb{E}z, \mathbb{E}N) \tilde{p} + \frac{1}{1 - \theta} (1 - \varpi(\mathbb{E}z, \mathbb{E}N)) \bar{n} + \text{2nd order terms}, \end{aligned} \quad (\text{A.36})$$

where $\tilde{p} \equiv \ln \tilde{P} - \mathbb{E} \ln \tilde{P}$, $\bar{n} \equiv \ln \bar{N} - \mathbb{E} \ln \bar{N}$, and

$$\varpi(\mathbb{E}z, \mathbb{E}N) = \frac{\mathbb{E}z \exp(\mathbb{E}(1 - \theta) \ln \tilde{P})}{\mathbb{E}z \exp(\mathbb{E}(1 - \theta) \ln \tilde{P}) + (\mathbb{E}N - \mathbb{E}z) \exp(\mathbb{E}(1 - \theta) \ln \bar{P})}.$$

Second-order approximation of labor market clearing. We start by expressing the variables on the right hand side of (A.27) in logarithmic form:

$$\begin{aligned} \ln L = \ln \left\{ \exp \left[\left(-\theta \ln \tilde{P} + \theta \ln P + \ln M - \ln \eta - \ln P \right) + \ln \tilde{N} - \ln A \right] + \right. \\ \left. + \exp \left[\left(-\theta \ln \bar{P} + \theta \ln P + \ln M - \ln \eta - \ln P \right) + \ln \bar{N} - \ln A \right] + \exp(\ln N + \ln f) + \int_0^{z_{max}} \Phi(i) di \right\}. \end{aligned} \quad (\text{A.37})$$

Approximating around the stochastic mean $l = \ln L - \mathbb{E} \ln L$, etc., and noting that $\bar{P}$ and $\tilde{N}$ are constant yields

$$\ln L \approx \ln \Lambda_0 + \varsigma'(m - \hat{\eta} - a - p) + [\varsigma'(1 - \vartheta(\mathbb{E}z, \mathbb{E}N)) + \varsigma''] \bar{n} + \varsigma' \theta (p - \vartheta(\mathbb{E}z, \mathbb{E}N) \tilde{p}) + \text{2nd order terms},$$

where the terms Λ_0 , ς' , ς'' , and $\vartheta(\mathbb{E}z, \mathbb{E}N)$ are given by:

$$\begin{aligned} \Lambda_0 = \left[\exp(\mathbb{E} \ln \tilde{N}) \exp \left(-\theta \mathbb{E} \ln \left(\frac{\tilde{P}}{\bar{P}} \right) \right) + \exp(\mathbb{E} \ln \bar{N}) \exp \left(-\theta \mathbb{E} \ln \left(\frac{\bar{P}}{\bar{P}} \right) \right) \right] \\ \times \exp \left(\mathbb{E} \ln \left(\frac{M}{\eta \bar{P}} \right) \right) \exp(-\mathbb{E} \ln A) + \exp(\mathbb{E} \ln N + \ln f) + \int_0^{z_{max}} \Phi(i) di, \end{aligned} \quad (\text{A.38})$$

$$\varsigma' = \frac{\Lambda_0 - \exp(\mathbb{E} \ln N + \ln f) - \int_0^{\tilde{N}_0} \Phi(i) di}{\Lambda_0}, \quad \varsigma'' = \frac{\exp(\mathbb{E} \ln \bar{N} + \ln f)}{\Lambda_0},$$

$$\vartheta(\mathbb{E}z, \mathbb{E}N) = \frac{\mathbb{E}z \exp(-\theta \mathbb{E} \ln \tilde{P})}{\mathbb{E}z \exp(-\theta \mathbb{E} \ln \tilde{P}) + (\mathbb{E}N - \mathbb{E}z) \exp(-\theta \mathbb{E} \ln \bar{P})} > 0.$$

Note that $\Lambda_0 > 0$ and $\varsigma \in (0, 1)$. Moreover, the function $\vartheta(\mathbb{E}z, \mathbb{E}N)$ has the properties $\vartheta(0, \mathbb{E}N) = 0$, and $\vartheta(\mathbb{E}z, \mathbb{E}N) = 1$ for $\mathbb{E}z = \mathbb{E}N$.

Second-order approximation of the zero-profit condition. From the free entry condition in (A.30), we have

$$(\bar{P}/P)^{1-\theta} Y = (W/(PA))[(\bar{P}/P)^{-\theta} Y + Af]. \quad (\text{A.39})$$

Combining with the money market condition $M = \eta PY$ and labor market clearing (A.27), we arrive at

$$(\bar{P}/P)^{1-\theta} \frac{M}{\eta W} (N - z_{\max}) = L - \frac{\Phi}{2} z_{\max}^2 - z_{\max} f - \frac{z_{\max}}{A} (\tilde{P}/P)^{-\theta} \frac{M}{P\eta}. \quad (\text{A.40})$$

We start by taking a Taylor approximation of the right hand side. $\bar{P}$ and $\tilde{N} = z_{\max}$ are fixed. Moreover, the right-hand-side expression (RHS) can be expressed as

$$\ln RHS = \ln \left\{ \exp(\ln L) - \frac{\Phi}{2} z_{\max}^2 - z_{\max} f - z_{\max} \frac{\exp(-\theta \ln \tilde{P})}{\exp((1-\theta) \ln P)} \frac{\exp(\ln M)}{\exp(\ln A) \exp(\ln \eta)} \right\}. \quad (\text{A.41})$$

The expression varies in $\ln A$, $\ln L$, $\ln \tilde{P}$, $\ln P$, and $\ln M$. We treat η as constant. The corresponding first-order derivatives are

$$\frac{\partial \ln RHS}{\partial \ln L} = \frac{\exp(\ln L)}{RHS}, \quad (\text{A.42})$$

$$\frac{\partial \ln RHS}{\partial \ln \tilde{P}} = \theta \frac{z_{\max} \exp(-\theta \ln \tilde{P}) \exp(\ln M)}{RHS \exp((1-\theta) \ln P) \exp(\ln A) \exp(\ln \eta)}, \quad (\text{A.43})$$

$$\frac{\partial \ln RHS}{\partial \ln P} = -(\theta - 1) \frac{z_{\max} \exp(-\theta \ln \tilde{P}) \exp(\ln M)}{RHS \exp((1-\theta) \ln P) \exp(\ln A) \exp(\ln \eta)}, \quad (\text{A.44})$$

$$\frac{\partial \ln RHS}{\partial \ln A} = -\frac{\partial \ln RHS}{\partial \ln M} = \frac{z_{\max} \exp(-\theta \ln \tilde{P}) \exp(\ln M)}{RHS \exp((1-\theta) \ln P) \exp(\ln A) \exp(\ln \eta)}. \quad (\text{A.45})$$

The left hand side of (A.40) has only multiplicative terms. Further, we have defined $\bar{N} = N - z$ and, as $z = z_{\max}$, $\bar{N} = N - z_{\max}$. Overall, we arrive at the following Taylor

approximation of the log free entry condition in (A.40)

$$(\theta - 1)p + m - w + \bar{n} \approx \frac{\exp(\ln L)}{RHS}l + \frac{z_{max} \exp(\ln \tilde{P}) \exp(\ln M)}{RHS \exp((1 - \theta) \ln P) \exp(\ln A) \exp(\ln \eta)} [\theta \tilde{p} - (\theta - 1)p - m + a]. \quad (\text{A.46})$$

Defining $O_1 = \frac{\exp(\ln L)}{RHS}$ and $O_2 = \frac{z_{max} \exp(\ln \tilde{P}) \exp(\ln M)}{RHS \exp((1 - \theta) \ln P) \exp(\ln A) \exp(\ln \eta)}$, and noting that $w = \varphi l + m$, we rearrange this as

$$(\theta - 1)p - \varphi l + \bar{n} \approx Const + O_1 l + O_2 [\theta \tilde{p} - (\theta - 1)p - m + a] \quad (\text{A.47})$$

$$\Leftrightarrow \bar{n} \approx Const - (\theta - 1)(O_2 + 1)p + O_2 \theta \tilde{p} - O_2 m + O_2 a + (\varphi + O_1)l. \quad (\text{A.48})$$

A.4.1 Determination of optimal price flexibility

In the following, to focus on the volatility of productivity, we assume – as in the benchmark specification – fixed money supply, i.e. $m = 0$. Remember that $\tilde{n} = 0$ and $\hat{\eta} = 0$.

We can rewrite the approximated aggregate price index as

$$p \approx Const_1 + \varpi(\mathbb{E}z, \mathbb{E}N)\tilde{p} + \frac{1}{1 - \theta}(1 - \varpi(\mathbb{E}z, \mathbb{E}N))\bar{n}. \quad (\text{A.49})$$

The labor market clearing condition is

$$l \approx Const_2 + \varsigma'(-a - p) + \varsigma'\theta(p - \vartheta(\mathbb{E}z, \mathbb{E}N)\tilde{p}) + [\varsigma'(1 - \vartheta(\mathbb{E}z, \mathbb{E}N)) + \varsigma'']\bar{n}, \quad (\text{A.50})$$

and the free entry condition (A.48)

$$\bar{n} \approx Const - (\theta - 1)(O_2 + 1)p + O_2 a + O_2 \theta \tilde{p} + (\varphi + O_1)l. \quad (\text{A.51})$$

Substitution of $\tilde{p}$ from (A.49) into (A.51) gives

$$\bar{n} \approx Const - (\theta - 1)(O_2 + 1)p + O_2 a + \frac{\theta O_2}{\varpi(\mathbb{E}z, \mathbb{E}N)} \left[p - \frac{1 - \varpi(\mathbb{E}z, \mathbb{E}N)}{1 - \theta} \bar{n} \right] + (\varphi + O_1)l.$$

Rearranging gives

$$\begin{aligned} \bar{n} \left[1 + \frac{\theta O_2}{\varpi(\mathbb{E}z, \mathbb{E}N)} \frac{1 - \varpi(\mathbb{E}z, \mathbb{E}N)}{1 - \theta} \right] &\approx \\ &\approx Const - (\theta - 1)(O_2 + 1)p + O_2 a + \frac{\theta O_2}{\varpi(\mathbb{E}z, \mathbb{E}N)} p + (\varphi + O_1)l. \end{aligned} \quad (\text{A.52})$$

Now we substitute $\tilde{p}$ from (A.49) into the labor market condition (A.50)

$$l \approx Const_2 + \varsigma'(-a-p) + \varsigma'\theta \left(p - \frac{\vartheta(\mathbb{E}z, \mathbb{E}N)}{\varpi(\mathbb{E}z, \mathbb{E}N)} \left[p - \frac{1 - \varpi(\mathbb{E}z, \mathbb{E}N)}{1 - \theta} \bar{n} \right] \right) + [\varsigma'(1 - \vartheta(\mathbb{E}z, \mathbb{E}N)) + \varsigma'']\bar{n}. \quad (\text{A.53})$$

Collecting terms and setting $\iota = \frac{\vartheta(\mathbb{E}z, \mathbb{E}N)}{\varpi(\mathbb{E}z, \mathbb{E}N)} > 0$, gives

$$l \approx Const_2 - \varsigma'a + [\varsigma'(\theta - 1 - \theta\iota)]p + [\varsigma'(\theta\iota \frac{1 - \varpi(\mathbb{E}z, \mathbb{E}N)}{1 - \theta} + 1 - \vartheta(\mathbb{E}z, \mathbb{E}N)) + \varsigma'']\bar{n}. \quad (\text{A.54})$$

We plug in from (A.52) and define $\tilde{\vartheta} = \frac{\theta}{\theta-1} \frac{\varpi(\mathbb{E}z, \mathbb{E}N)-1}{\varpi(\mathbb{E}z, \mathbb{E}N)}$ to get

$$l \approx Const - \varsigma'a + [\varsigma'(\theta - 1 - \theta\iota)]p + [\varsigma'(\iota\tilde{\vartheta}\varpi(\mathbb{E}z, \mathbb{E}N) + 1 - \vartheta(\mathbb{E}z, \mathbb{E}N)) + \varsigma''] \frac{1}{1 + O_2\tilde{\vartheta}} \times \left\{ O_2a - \left[(\theta - 1)(O_2 + 1) - \frac{\theta O_2}{\varpi(\mathbb{E}z, \mathbb{E}N)} \right] p + (\varphi + O_1)l \right\}. \quad (\text{A.55})$$

We define $O_3 = [\varsigma'(\iota\tilde{\vartheta}\varpi(\mathbb{E}z, \mathbb{E}N) + 1 - \vartheta(\mathbb{E}z, \mathbb{E}N)) + \varsigma''] \frac{1}{1 + O_2\tilde{\vartheta}}$, and $O_4 = \frac{1}{\varpi(\mathbb{E}z, \mathbb{E}N)}$. Then

$$l \approx Const + \frac{O_2O_3 - \varsigma'}{1 - (\varphi + O_1)O_3}a - \frac{O_3[(\theta - 1)(O_2 + 1) - \theta O_4O_2] - \varsigma'(\theta - 1 - \theta\iota)}{1 - (\varphi + O_1)O_3}p. \quad (\text{A.56})$$

Turning to the equation for flexible prices, we have

$$\tilde{p} \approx Const + \frac{1}{\varpi(\mathbb{E}z, \mathbb{E}N)}p - \frac{\tilde{\vartheta}}{\theta}\bar{n}. \quad (\text{A.57})$$

We use (A.52) to get

$$\tilde{p} \approx Const + O_4p - \frac{\tilde{\vartheta}}{\theta(1 + O_2\tilde{\vartheta})} \{ O_2a - ((\theta - 1)(O_2 + 1) - \theta O_4O_2)p + (\varphi + O_1)l \}. \quad (\text{A.58})$$

Ignoring constants and assuming that $m = \hat{\eta} = 0$, the log-linear approximation of the flexible price $\tilde{P}$ from (A.31) is given by

$$\tilde{p} = -a + \varphi l. \quad (\text{A.59})$$

Setting equal the price index (A.58) and the flexible price (A.59) to eliminate $\tilde{p}$, we get

$$O_4 p - \frac{\tilde{\vartheta}}{\theta(1 + O_2 \tilde{\vartheta})} \{O_2 a - [(\theta - 1)(O_2 + 1) - \theta O_4 O_2] p + (\varphi + O_1) l\} = -a + \varphi l.$$

Multiplying by $\theta(1 + O_2 \tilde{\vartheta})$ and collecting terms in p , l , and a yields

$$\begin{aligned} \{\theta(1 + O_2 \tilde{\vartheta}) O_4 - \tilde{\vartheta}[(\theta - 1)(O_2 + 1) - \theta O_4 O_2]\} p = \\ = [\tilde{\vartheta} O_2 - \theta(1 + O_2 \tilde{\vartheta})] a + [\tilde{\vartheta}(\varphi + O_1) + \theta(1 + O_2 \tilde{\vartheta}) \varphi] l. \end{aligned}$$

Plugging in (A.56) to eliminate l (while ignoring constants), rewriting, and solving for p , we obtain

$$p = Const + \frac{\kappa_1}{\kappa_2} a, \quad (\text{A.60})$$

with

$$\begin{aligned} \kappa_1 &= [\tilde{\vartheta} O_2 - \theta(1 + O_2 \tilde{\vartheta})] + [\tilde{\vartheta}(\varphi + O_1) + \theta(1 + O_2 \tilde{\vartheta}) \varphi] \frac{O_2 O_3 - \zeta'}{1 - (\varphi + O_1) O_3}, \\ \kappa_2 &= \theta(1 + O_2 \tilde{\vartheta}) O_4 - \tilde{\vartheta}[(\theta - 1)(O_2 + 1) - \theta O_4 O_2] \\ &\quad + [\tilde{\vartheta}(\varphi + O_1) + \theta(1 + O_2 \tilde{\vartheta}) \varphi] \times \frac{O_3[(\theta - 1)(O_2 + 1) - \theta O_4 O_2] - \zeta'(\theta - 1 - \iota)}{1 - (\varphi + O_1) O_3}. \end{aligned}$$

We then, again ignoring constants, substitute into the approximation of the labor equation

$$l \approx Const \left[-\frac{O_3[(\theta - 1)(O_2 + 1) - \theta O_4 O_2] - \zeta'(\theta - 1 - \iota)}{1 - (\varphi + O_1) O_3} \frac{\kappa_1}{\kappa_2} + \frac{O_2 O_3 - \zeta'}{1 - (\varphi + O_1) O_3} \right] a. \quad (\text{A.61})$$

Variances and covariances. Recall that in the gain function (A.18), with a linear production function ($\alpha = 1$), we have the variance of $\ln W - \ln A$. Plugging in equations (A.32) and (A.33) to replace $\ln W$ and $\ln Y$, we get $\ln W - \ln A = \ln \chi + \varphi \ln L + \ln M - \ln \eta - \ln A$. Subtracting the stochastic mean and taking the variance operator on both sides, while we still assume $m = \hat{\eta} = 0$, we have on the right hand side

$$Var(a) + \varphi^2 Var(l) - 2\varphi Cov(l, a). \quad (\text{A.62})$$

Notably, variations in the price level do not matter for the gains from price flexibility.

Gains from price flexibility. We remain in the special case where $\alpha = 1$ and use the first-order approximations l to derive the second-order moments. Applying the variance

operator to the approximated labor equation (A.61) yields

$$Var(l) = \left[-\frac{O_3[(\theta-1)(O_2+1) - \theta O_4 O_2] - \varsigma'(\theta-1-\theta_l) \kappa_1}{1 - (\varphi + O_1)O_3} \frac{\kappa_1}{\kappa_2} + \frac{O_2 O_3 - \varsigma'}{1 - (\varphi + O_1)O_3} \right]^2 Var(a),$$

$$Cov(l, a) = - \left[-\frac{O_3[(\theta-1)(O_2+1) - \theta O_4 O_2] - \varsigma'(\theta-1-\theta_l) \kappa_1}{1 - (\varphi + O_1)O_3} \frac{\kappa_1}{\kappa_2} + \frac{O_2 O_3 - \varsigma'}{1 - (\varphi + O_1)O_3} \right] Var(a).$$

Now we substitute these expressions into (A.62) to obtain:

$$Var(a) \left\{ 1 + \varphi^2 \left[-\frac{O_3[(\theta-1)(O_2+1) - \theta O_4 O_2] - \varsigma'(\theta-1-\theta_l) \kappa_1}{1 - (\varphi + O_1)O_3} \frac{\kappa_1}{\kappa_2} + \frac{O_2 O_3 - \varsigma'}{1 - (\varphi + O_1)O_3} \right]^2 \right. \\ \left. + 2\varphi \left[-\frac{O_3[(\theta-1)(O_2+1) - \theta O_4 O_2] - \varsigma'(\theta-1-\theta_l) \kappa_1}{1 - (\varphi + O_1)O_3} \frac{\kappa_1}{\kappa_2} + \frac{O_2 O_3 - \varsigma'}{1 - (\varphi + O_1)O_3} \right] \right\}.$$

Defining the constant $v = 1 + \varphi \left(-\frac{O_3[(\theta-1)(O_2+1) - \theta O_4 O_2] - \varsigma'(\theta-1-\theta_l) \kappa_1}{1 - (\varphi + O_1)O_3} \frac{\kappa_1}{\kappa_2} + \frac{O_2 O_3 - \varsigma'}{1 - (\varphi + O_1)O_3} \right)$, we can plug in the expression into the gain function

$$\Delta(\Theta) \approx \frac{\Omega}{2}(\theta-1)\theta v^2 \sigma_a^2. \quad (\text{A.63})$$

As $\frac{\Omega}{2}(\theta-1)\theta v^2 > 0$, the gains from price flexibility increase in the productivity shock volatility.

B Social planner allocation and optimal policy

A general result in the class of models considered here is that optimal monetary policy requires the central bank to set the money supply in such a way as to replicate the flexible-price allocation.²⁷ King and Wolman (1999) and Woodford (2003) show that under price stickiness, the flexible-price allocation can be implemented as a policy that stabilizes prices perfectly. If firms optimally choose not to change prices, the presence of price rigidity does not affect the equilibrium allocation.

B.1 Model with constant number of firms

In the model where the number of firms is constant, we can easily show this for a closed economy with productivity shocks, following the approach in Devereux (2006). If all prices are sticky, the price index is $P = \bar{P}$, which using (6) can be written as $P = \delta \frac{\mathbb{E}\{\Gamma(W/A)Y\}}{\mathbb{E}\{\Gamma Y\}}$. Replacing the household discount factor Γ with $(PY)^{-1}$ and noting that

²⁷ For this result to hold, we need to assume that the monopolistic competition distortion is undone with an appropriate labor or production subsidy. If such a subsidy is absent, the flexible-price allocation is inefficient and consequently, price stability is not optimal, as shown in Adão et al. (2003). The same authors also show that the monetary friction can be undone by following the Friedman rule, i.e. setting the nominal interest rate to zero.

$P = \bar{P}$, this simplifies to $P = \delta \mathbb{E}\{W/A\}$. Consumption is given by the money market clearing condition,

$$C = \frac{M}{\eta P}. \quad (\text{B.1})$$

Combining (B.1) first with the goods market clearing condition, $C = Y$, and second with the production function, $Y = AL$, we can write labor as

$$L = \frac{M}{\eta P A}. \quad (\text{B.2})$$

Combining the wage, $W = \chi L^\varphi P Y$, with the production function, $Y = AL$, we obtain $W/A = \chi L^{1+\varphi} P$. Thus the price index becomes $P = \delta \mathbb{E}\{\chi L^{1+\varphi} P\}$. Then, eliminating labor L using (B.2), we get $P = \delta \mathbb{E}\{\chi (M/(\eta P A))^{1+\varphi} P\}$. We rewrite this to obtain

$$P = (\delta \chi)^{1/(1+\varphi)} \left(\mathbb{E} \left\{ \left(\frac{M}{\eta A} \right)^{1+\varphi} \right\} \right)^{1/(1+\varphi)}. \quad (\text{B.3})$$

From (B.2) and (B.3) we can show that expected labor disutility is

$$\frac{\chi}{1+\varphi} \mathbb{E} L^{1+\varphi} = \frac{\chi}{1+\varphi} \frac{\mathbb{E}[M/(\eta A)]^{1+\varphi}}{P^{1+\varphi}} = \frac{\chi}{1+\varphi} \frac{\mathbb{E}[M/(\eta A)]^{1+\varphi}}{(\delta \chi)^{1/(1+\varphi)} \mathbb{E}[M/(\eta A)]^{1+\varphi}},$$

which is a constant. Therefore, expected utility depends only on the expected value of log consumption, and not on labor. Using the expression for consumption in (B.1) and ignoring constants, we can write expected utility as follows,

$$\mathbb{E} \ln C = \mathbb{E} \left\{ \ln \left(\frac{M}{\eta} \right) - \ln P \right\}. \quad (\text{B.4})$$

Assume that there is a finite number of states of nature Σ and let a state be denoted $\epsilon \in \Sigma$. We abstract from velocity shocks and treat η as constant. Let the money supply be state-contingent. The monetary policy problem is to choose $M(\epsilon)$ so as to maximize expected utility (B.4). Deriving and rearranging the first order condition, we obtain the optimal monetary policy rule

$$M(\epsilon) = \Xi A(\epsilon), \quad (\text{B.5})$$

where $\Xi = \eta (\mathbb{E}[M/(\eta A)]^{1+\varphi})^{1/(1+\varphi)}$ is a constant. According to the optimal rule (B.5), the money supply should move in proportion to productivity, as shown in Bilbie and Melitz (2022).

What is the intuition for this result? Suppose that a negative productivity shock hits, such that A falls. Under fully flexible prices, firms would raise prices in proportion to the rise in marginal costs. But in the presence of price stickiness, the prices of some (or all) firms are too low after the shock; those firms that cannot raise prices will end

up producing too much given the current level of aggregate demand: there is a positive output gap. The central bank must reign in demand by lowering the money supply.

Indeed, the optimal policy is to move the money supply in exact proportion to the shock in A . As we see from (B.3), this policy leads to a constant price level P ; all terms on the right hand side of the equation except M and A are constant, but since the money stock fluctuates in proportion to A , the ratio M/A is constant. This implies that the price level – in the model with a constant number of firms – does not change. Therefore, in the model without entry-exit, optimal monetary policy eliminates the need for ex post price adjustment completely.

B.2 Model with entry and exit

In the model with entry and exit, optimal monetary policy will also eliminate the need for price adjustment, but this does not imply that the welfare-based price index, denoted P in the model, is constant. This is because the welfare-based price index depends also on the number of firms.

Households. Households maximize utility given by

$$U = \ln C - \chi \frac{L^{1+\varphi}}{1+\varphi}, \quad (\text{B.6})$$

subject to the budget constraint, expressed in real terms as

$$dN + (1 + \tau)W^R L \geq C + T, \quad (\text{B.7})$$

where d are firm profits, N is the number of producers, W^R is the real wage, τ is a labor income subsidy, and T are lump-sum taxes. Combining the first order conditions for consumption C and labor L , we obtain the standard labor supply decision,

$$\chi L^\varphi = \frac{1}{C}(1 + \tau)W^R. \quad (\text{B.8})$$

Final output and demand for individual varieties. Aggregate output is defined as:

$$Y = \left(\int_0^N Y(\omega)^{\frac{\theta-1}{\theta}} d\omega \right)^{\frac{\theta}{\theta-1}}, \quad (\text{B.9})$$

where $\theta > 1$ is the elasticity of substitution between goods varieties. The demand for good ω is the solution to problem of choosing $Y(\omega)$ to minimize $\int_0^N P(\omega)Y(\omega)d\omega$, subject to the aggregator function (B.9), taking prices $P(\omega)$ as given. The solution is given by (2) above.

Price index. The price index is derived from the identity that the value of aggregate output is equal to the number of producers times output per firm, i.e.

$$PY = \int_0^N P(\omega)Y(\omega)d\omega. \quad (\text{B.10})$$

Substituting $Y(\omega)$ in the expenditure function using the demand equation (2) and rearranging, we find that the aggregate price index is given by

$$P = \left(\int_0^N P(\omega)^{1-\theta} d\omega \right)^{\frac{1}{1-\theta}}. \quad (\text{B.11})$$

Intermediate goods firms. Firms produce differentiated goods indexed by $\omega \in [0, N]$ and compete under monopolistic competition, taking the real wage W^R as given. Following Devereux (2006), we assume a linear production function with labor as the only input,

$$Y(\omega) = Al(\omega), \quad (\text{B.12})$$

where A is an aggregate technology shock and $l(\omega)$ is total labor input that firm ω uses for production. The firm's total production cost is $W^R(l(\omega) + f)$, where f is a fixed per-period cost or, from the production function (B.12),

$$W^R \left(\frac{Y(\omega)}{A} \right) + W^R f. \quad (\text{B.13})$$

The firm faces the demand function

$$Y(\omega) = \left(\frac{P(\omega)}{P} \right)^{-\theta} Y = P(\omega)^{-\theta} \hat{Y}, \quad \text{where } \hat{Y} = P^\theta Y. \quad (\text{B.14})$$

The firm chooses price $P(\omega)$ to maximize discounted expected profits given by revenues minus total cost (B.13), $\mathbb{E}\Gamma \{ P(\omega)Y(\omega)/P - W^R Y(\omega)/A - W^R f \}$, where Γ is the discount factor. Replacing firm output $Y(\omega)$ using the demand constraint (B.14), expected profits can be written as

$$\mathbb{E}\Gamma \left\{ P(\omega) \left(\frac{P(\omega)}{P} \right)^{-\theta} Y - W^R \left(\frac{P(\omega)}{P} \right)^{-\theta} \frac{Y}{A} - W^R f \right\}. \quad (\text{B.15})$$

Then the price setting problem is to choose $P(\omega)$ in order to maximize (B.15).

Symmetry. Under symmetry, the price index (B.11) simplifies to

$$P = N^{\frac{1}{1-\theta}} \mathbf{p}, \quad (\text{B.16})$$

where $\mathbf{p}$ is the symmetric firm price. This can be written as

$$\rho = N^{\frac{1}{\theta-1}}. \quad (\text{B.17})$$

Total consumption expenditure (B.10) becomes $PY = N\mathbf{p}\mathbf{y}$, which we can solve for (symmetric) firm output to obtain $\mathbf{y} = Y/(N\rho)$. Let μ denote the firms markup over marginal cost, that is,

$$\mu = \frac{\rho}{W^R/A}. \quad (\text{B.18})$$

Firm ω 's profit is

$$\mathbf{d} = \frac{\mathbf{p}}{P}\mathbf{y} - W^R(\ell + f), \quad (\text{B.19})$$

where $\mathbf{y}$ is the (symmetric) labor input. Total profits of all firms are, therefore, $\mathbf{d}N = (\mathbf{p}/P)\mathbf{y}N - W^R(\ell + f)N$. Using the relation $\ell = \mathbf{y}/A$ from the production function (B.12), and rearranging, this becomes

$$\mathbf{d}N + W^RfN = \left(\frac{\mathbf{p}}{P} - \frac{W^R}{A} \right) \mathbf{y}N. \quad (\text{B.20})$$

Using the household's budget constraint (B.7), holding with equality, we can replace the term $\mathbf{d}N$ in (B.20). Then, imposing the government budget balance $T = \tau W^RL$, we obtain

$$C + W^RfN = W^RL + \left(\frac{\mathbf{p}}{P} - \frac{W^R}{A} \right) \mathbf{y}N. \quad (\text{B.21})$$

Equation (B.21) can be seen as an aggregate accounting identity, where the second term on the left hand side is investment and the second term of the right hand side is profit income.

Aggregating goods production across firms, i.e. multiplying firm output in (B.12) by N , we can write $\mathbf{y}N = A(L - Nf)$, where we have used the relation $L = (\ell + f)N$. To obtain aggregate output, we must multiply the above equation by ρ , set $Y = \mathbf{y}\rho N$, and use the price index (B.16), such that

$$Y = N^{\frac{1}{\theta-1}} A(L - Nf). \quad (\text{B.22})$$

We now derive the equilibrium, considering the flexible-price case and the sticky-price case.

Equilibrium. The number of firms is determined by a zero-profit condition for aggregate profits every period. Setting $\mathbf{d} = 0$ and $\mathbf{y} = A\ell$ in the expression for firm profits (B.19) yields $(\mathbf{p}/P)A\ell - W^R(\ell + f) = 0$. Rearranging, and solving for ℓ we obtain firm-level labor demand, $\ell = \frac{W^R}{\rho A - W^R} f$. Rearranging once more, using the markup rule (B.18)

written as $\mu = \rho A / W^R$ and multiplying both sides by N , this becomes

$$L = \frac{\mu}{\mu - 1} f N, \quad (\text{B.23})$$

where we have used the relation $L = (\ell + f)N$. The zero-profit condition implies that the aggregate accounting relation (B.21) simplifies to

$$Y = C = W^R L. \quad (\text{B.24})$$

Combining (B.24) with the labor supply equation (13) and imposing $\Gamma = C^{-1}$, we see that labor is constant,

$$L = \left(\frac{1 + \tau}{\chi} \right)^{\frac{1}{1+\varphi}}. \quad (\text{B.25})$$

There are four equations (B.16), (B.18), (B.22), and (B.23), determining ρ , W^R , N , Y . Labor is constant at $L = [(1 + \tau)/\chi]^{\frac{1}{1+\varphi}}$ from (B.25).

Under **flexible prices**, the markup is constant at $\mu = \frac{\theta}{\theta-1}$ from (B.17). We can consolidate the resulting five equations to obtain a recursive three-equation system determining labor (B.25), the number of firms,

$$N = \frac{L}{\theta f}. \quad (\text{B.26})$$

and output (B.22). Combining (B.26) with the relation $L = (\ell + f)N$, we see that firm-level labor input is given by

$$\ell = (\theta - 1)f. \quad (\text{B.27})$$

Plugging (B.27) into the firm's production function (A.1), we see that firm output is

$$y = A(\theta - 1)f. \quad (\text{B.28})$$

Under **sticky prices**, individual prices are fixed, such that the price index (B.16) becomes

$$P = N^{-\frac{1}{\theta-1}} \bar{p}. \quad (\text{B.29})$$

The relative price (B.17) is

$$\rho = \frac{\bar{p}}{P} = N^{\frac{1}{\theta-1}}, \quad (\text{B.30})$$

and the markup μ is no longer constant, but instead determined by (B.18). The central bank sets the money supply M , which determines the price index through the money

demand equation combined with goods market clearing $C = Y$,

$$M = \eta PY. \quad (\text{B.31})$$

Then, given the money supply, output is determined by

$$Y = \frac{M}{\eta \bar{p}} N^{\frac{1}{\theta-1}}. \quad (\text{B.32})$$

Plugging the relative price (B.30) and the aggregate resource constraint (B.24) into the markup equation (B.18), we obtain $\mu = \frac{AL}{Y} N^{\frac{1}{\theta-1}}$. Combined with (B.32), this becomes

$$\mu = AL \frac{\eta \bar{p}}{M}. \quad (\text{B.33})$$

We rearrange the labor demand equation (B.23) as $N = \frac{\mu-1}{\mu} \frac{L}{f}$ and plug in the expression for the markup (B.33) to obtain

$$N = \left(L - \frac{M}{\eta \bar{p} A} \right) \frac{1}{f}. \quad (\text{B.34})$$

Social planner allocation. The following exposition follows Lewis (2013). The social planner sets consumption C , labor L and the number of firms N to maximize utility (B.6), $U = \ln C - \chi L^{1+\varphi}/(1+\varphi)$, subject to the resource constraint (B.22), rewritten as $L = CN^{-\frac{1}{\theta-1}}A + fN$. Consolidating the first order conditions, the social planner allocation satisfies,

$$\chi L^\varphi C = AN^{\frac{1}{\theta-1}}, \quad (\text{B.35})$$

$$f = \frac{1}{\theta-1} CN^{-\frac{1}{\theta-1}-1} A^{-1}. \quad (\text{B.36})$$

Upon inspection of the social planner conditions, we see that the intersectoral optimality condition (B.36) corresponds to equations (B.22) and (B.26) combined, which determine output as a function of the number of firms in the decentralized allocation with flexible prices, $C = (\theta-1)fN^{\frac{1}{\theta-1}+1}A$. Plugging in the intrasectoral condition (B.35) written as $N^{\frac{1}{\theta-1}} = \chi L^\varphi C/A$, and rearranging gives the number of firms as a function of labor,

$$N = \frac{L}{\theta f} \underbrace{\frac{\theta}{(\theta-1)\chi L^{1+\varphi}}}_{\text{product diversity wedge}}. \quad (\text{B.37})$$

The intrasectoral optimality condition (B.35) is consistent with the flexible-price allocation where labor is constant and equal to $L^{SP} = \left[\frac{\theta}{(\theta-1)\chi} \right]^{\frac{1}{1+\varphi}}$. This is the value of labor for which the product diversity wedge between the social planner allocation and the flexible-

price allocation is equal to 1, see (B.37). Recall that in the flexible-price allocation, labor is equal to $L^{flex} = \left[\frac{1+\tau}{\chi} \right]^{\frac{1}{1+\varphi}}$. Therefore, without a labor income subsidy, $\tau = 0$, labor supply in the flex-price allocation is too low,

$$L^{flex} \underbrace{\left[\frac{\theta}{\theta-1} \left(\frac{1}{1+\tau} \right) \right]^{\frac{1}{1+\varphi}}}_{\text{labor wedge}} = L^{SP}. \quad (\text{B.38})$$

As we can see from (B.38), the static wedge can be removed through a constant labor subsidy equal to $1 + \tau = \frac{\theta}{\theta-1}$. See also Bilbiie et al. (2019).

Optimal monetary policy under sticky prices. As argued in Bilbiie et al. (2007), with endogenous entry-exit, the flexible-price allocation remains efficient. Therefore, monetary policy needs to be set such that the flexible price $\tilde{P}$ is constant. From the price setting condition (4), this means that the nominal wage has to move in proportion to productivity A , such that W/A is constant.

Suppose now that all prices are *sticky*. Then there are $\bar{N}$ firms and the aggregate price index is given by $P = (\int_0^{\bar{N}} P(\omega)^{1-\theta} d\omega)^{1/(1-\theta)}$. In a symmetric equilibrium, all prices are identical, such that $P(\omega) = \bar{p}$. Thus, the price index becomes $P = \bar{N}^{\frac{1}{1-\theta}} \bar{p}$. With the goods price $\bar{p}$ given by (6), the price index is

$$P = \bar{N}^{\frac{1}{1-\theta}} \delta \frac{\mathbb{E}\{\Gamma(W/A)Y\}}{\mathbb{E}\{\Gamma Y\}}. \quad (\text{B.39})$$

Under the optimal policy, the number of firms in the sticky-price allocation is the same of the number of firms in the flexible-price allocation, such that we can set $\bar{N}$ equal to $\tilde{N}$ in (B.39). Moreover, the optimal policy imposes a constant ratio W/A . With these two properties, the price index simplifies to

$$P = \delta(W/A) \tilde{N}^{\frac{1}{1-\theta}}. \quad (\text{B.40})$$

Taking logs of (B.40), we can state that the optimal welfare-based price index moves inversely with the number of firms as follows:

$$\ln P = \ln[\delta(W/A)] - \frac{1}{\theta-1} \ln \tilde{N}.$$

Implementation. How does the central bank implement this policy of keeping average prices constant, such that the optimal welfare-based price index is given by (B.40)? The optimal policy consists in setting the money supply M such that the number of firms under sticky prices (B.34) is equal to the number of firms in the flexible-price allocation. Thus, setting (B.34) equal to (B.26), $\left(L - \frac{M}{\eta \bar{p} A} \right) \frac{1}{f} = \frac{L}{\theta f}$, and solving for M , we obtain

the optimal money supply rule,

$$M = \eta \bar{p} \frac{\theta - 1}{\theta} AL. \quad (\text{B.41})$$

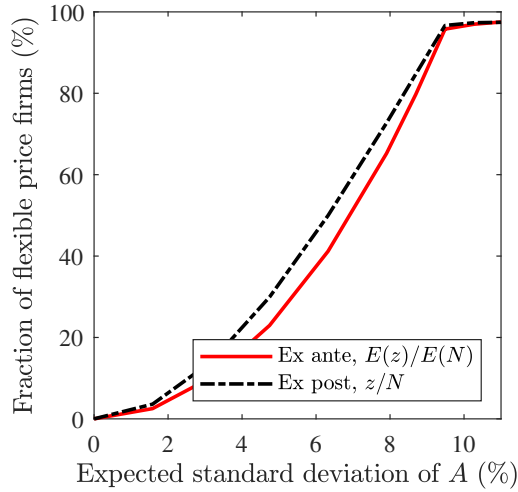
As explained in Bilbiie and Melitz (2022), the optimal rule prescribes that the money supply should move one-for-one with productivity A . So, when there is a negative supply shock and A falls, the central bank should respond in a countercyclical fashion, lowering the money supply. While this conclusion is the same as in the model without entry-exit, the intuition for this result is different: Here, in the model with sticky prices and flexible wages, the central bank increases entry by reducing the money supply.

Combining the aggregate accounting relation (B.24), $W = PY/L$, with money market clearing (B.31), $M = \eta PY$, we can derive the nominal wage as $W = \frac{M}{\eta L}$. Now plugging in the monetary policy rule (B.41), we obtain $W = \bar{p} \frac{\theta-1}{\theta} A$. This shows that under the monetary policy rule (B.41), the ratio W/A is indeed constant.

C Additional results: model

Ex post shifts in the fraction of flexible-price firms

Figure C.1: Ex post shifts in the fraction of flexible-price firms



Simulation of a 10 % productivity drop.

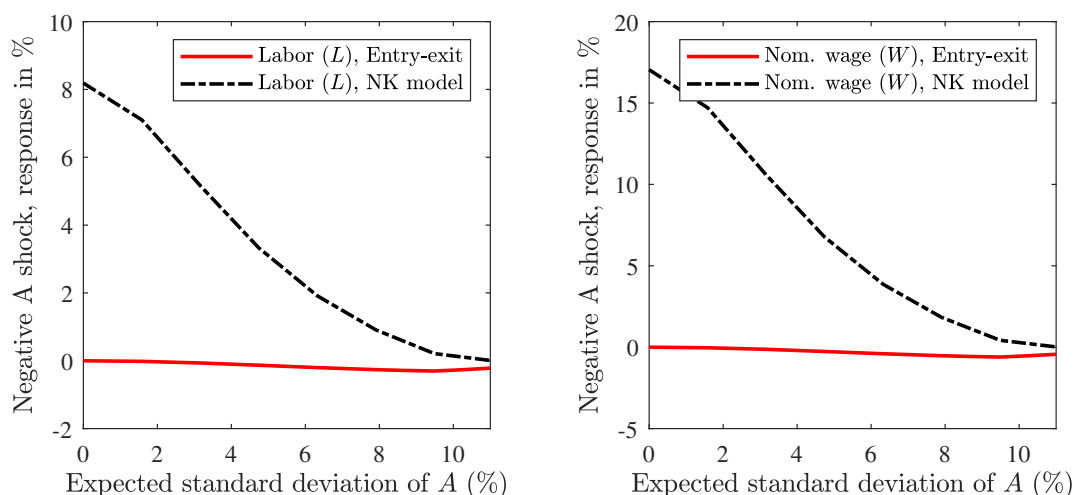
Labor and wage dynamics with and without product turnover

Investment in price flexibility under productivity uncertainty affects how labor markets respond once productivity shocks actually occur. In a regime of low productivity uncertainty, where only few firms have invested in price flexibility, adverse supply shocks raise labor demand as firms want to compensate lower productivity with more labor input.

Instead, in a regime of high productivity uncertainty where firms expect large shocks to happen, more of them shield themselves by investing in price flexibility. Then, when shocks actually occur, more firms can pass on productivity declines to consumers by raising their prices. Figure C.2 shows that this channel is key for the equilibrium labor outcome in the model without product turnover (NK model). In the NK model, nominal wages rise in response to higher aggregate labor demand.

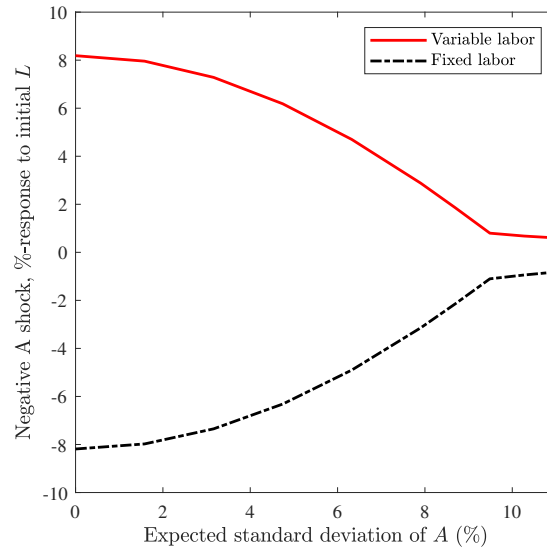
The channel is also present in the model with product turnover. It is, however, offset by the fact that with a large negative productivity shock, a large number of firms decide to exit to avoid paying production fixed costs – more so under lower productivity uncertainty. Thus, economy-wide production fixed costs, which consist of labor, decline (see Figure C.3). Overall, in the model with entry and exit, labor and nominal wages remain unchanged (see Figure C.2) despite some slight adjustments due to endogenous shifts in the composition of sticky- and flexible-price firms (see Figure C.1).

Figure C.2: Labor and wage response: NK model versus entry-exit model



Impact response to an adverse supply shock (10% productivity drop) as a function of productivity uncertainty: New Keynesian versus entry-exit model.

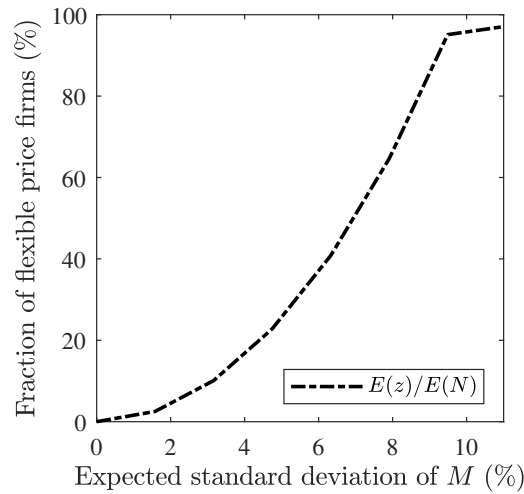
Figure C.3: Labor response under entry-exit: Variable versus fixed labor in production



Impact response to an adverse supply shock under endogenous price flexibility. Expressed in % change relative to the initial total aggregate labor L .

Monetary policy uncertainty and endogenous price flexibility

Figure C.4: Monetary policy uncertainty and endogenous price flexibility



Fraction of flexible-price firm across monetary policy uncertainty regimes.

D Additional results: empirics

D.1 Markov-switching model to detect TFP volatility regimes

In the first step, we use the utilization-adjusted quarterly TFP series for the US from Fernald (2012)²⁸ and estimate a Markov switching model with time-varying volatility in total factor productivity (TFP) as in Foerster and Matthes (2022):

$$\Delta \ln A_t = \mu + \sigma(s_t)\varepsilon_t, \quad (\text{D.1})$$

where μ and σ denote the average TFP growth rate and the volatility of TFP growth, respectively. Volatility is governed by a two-state Markov processes indexed by s_t , where s_t takes either a high or a low value and has transition matrix

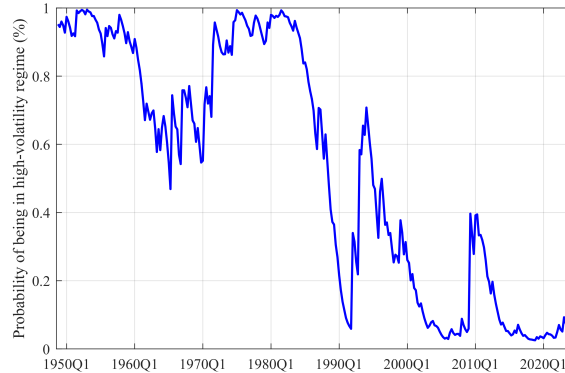
$$P = \begin{bmatrix} p_{LL} & p_{LH} \\ p_{HL} & p_{HH} \end{bmatrix} = \begin{bmatrix} p_{LL} & 1 - p_{LL} \\ 1 - p_{HH} & p_{HH} \end{bmatrix}. \quad (\text{D.2})$$

Estimating the five parameters of the TFP growth process in (D.1), μ , σ^L , σ^H , p_{LL} , p_{HH} , with maximum likelihood, we obtain the results displayed in Table D.1.

Table D.1: Parameter estimates for TFP growth process

	μ	$\log(\sigma^L)$	$\log(\sigma^H)$	p_{LL}	p_{HH}	$\log L$
Estimate	1.103	1.043	1.351	0.9939	0.9941	-807.379
(s.e.)	(0.190)	(0.062)	(0.068)	(0.010)	(0.010)	

Figure D.5: States of TFP volatility: Markov-switching model

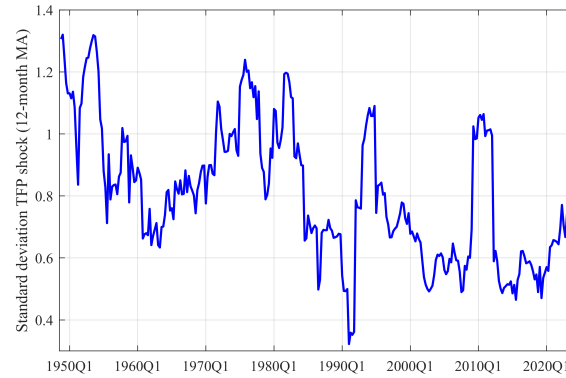


Notes. Sample period 1948Q4-2022Q4. Based on quarterly utilization-adjusted TFP from Fernald (2012). Filtered estimates of being in a high-volatility regime based on a Markov-switching model (see also Foerster and Matthes (2022)).

²⁸ The latest data set is available at http://www.frbsf.org/economic-research/files/quarterly_tfp.xlsx.

D.2 Alternative identification of TFP volatility states

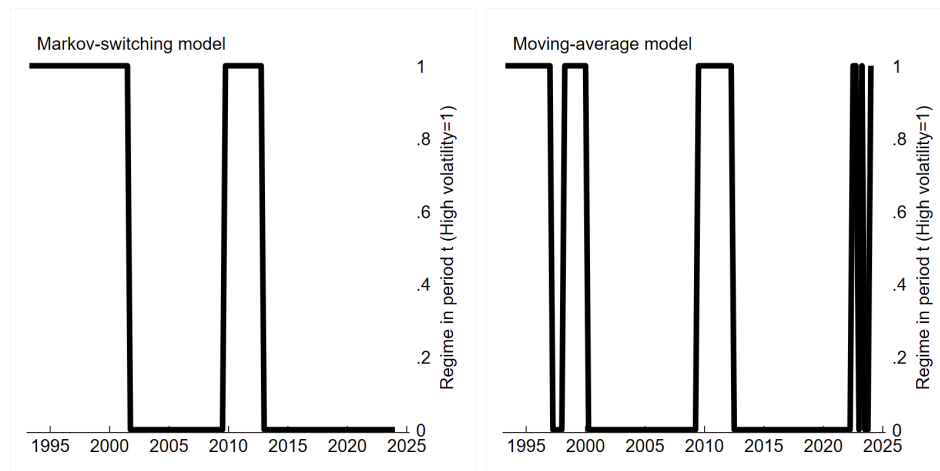
Figure D.6: States of TFP volatility: moving average model



Notes. Sample period 1993Q2-2024Q1. Shocks are computed from an AR (1) model of quarterly utilization-adjusted TFP from Fernald (2012) for the time period 1950Q1-2024Q1.

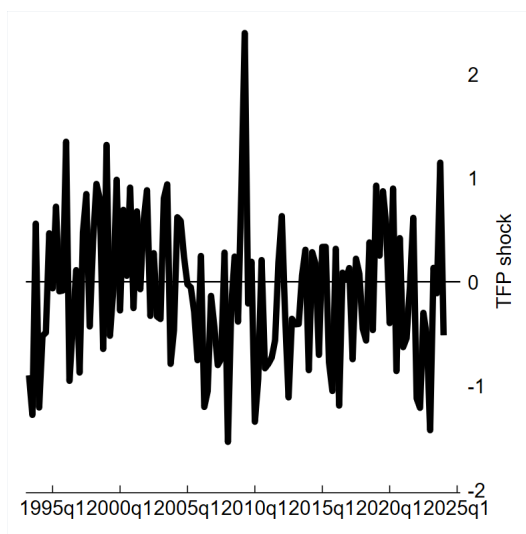
D.3 Uncertainty regimes and shock series 1993Q2-2024Q1

Figure D.7: Uncertainty regimes 1993Q2-2024Q1: Markov-switching model and moving average model



Notes. Identified uncertainty regimes. See description below regression (26).

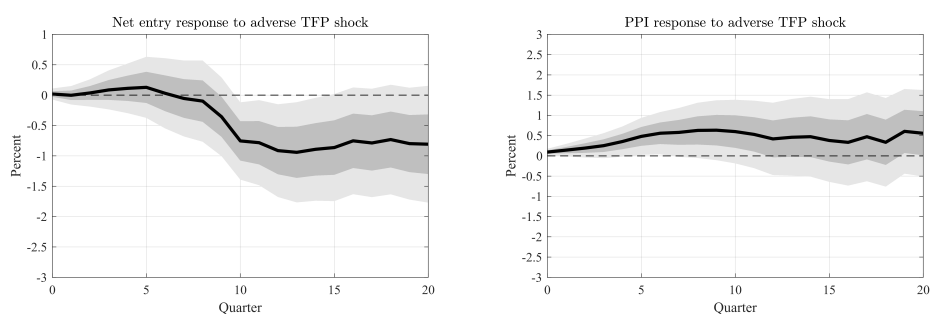
Figure D.8: TFP shock series



Notes. Shock series computed as residual from an AR(1) regression of utilization-adjusted TFP from Fernald (2012) for 1950Q1-2024Q1.

D.4 Unconditional responses to productivity shocks

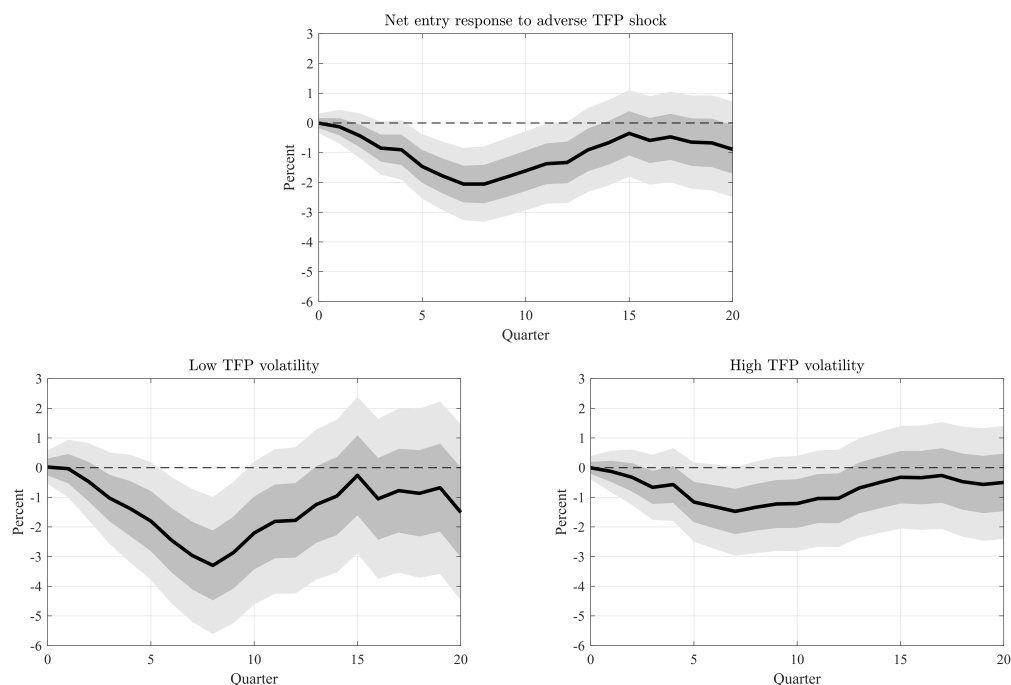
Figure D.9: Empirical response of net entry and prices to 1 percent fall in productivity



Notes. Dark (light) gray shaded areas are 68% (95%) confidence bands. Left panel: Net entry measured as net establishment entry rate. Sample period 1993Q2-2022Q4. Right panel: Prices measures as producer price index (core). Sample period 1974Q1-2024Q1.

D.5 Net entry measured as net business formation (prior to 1992)

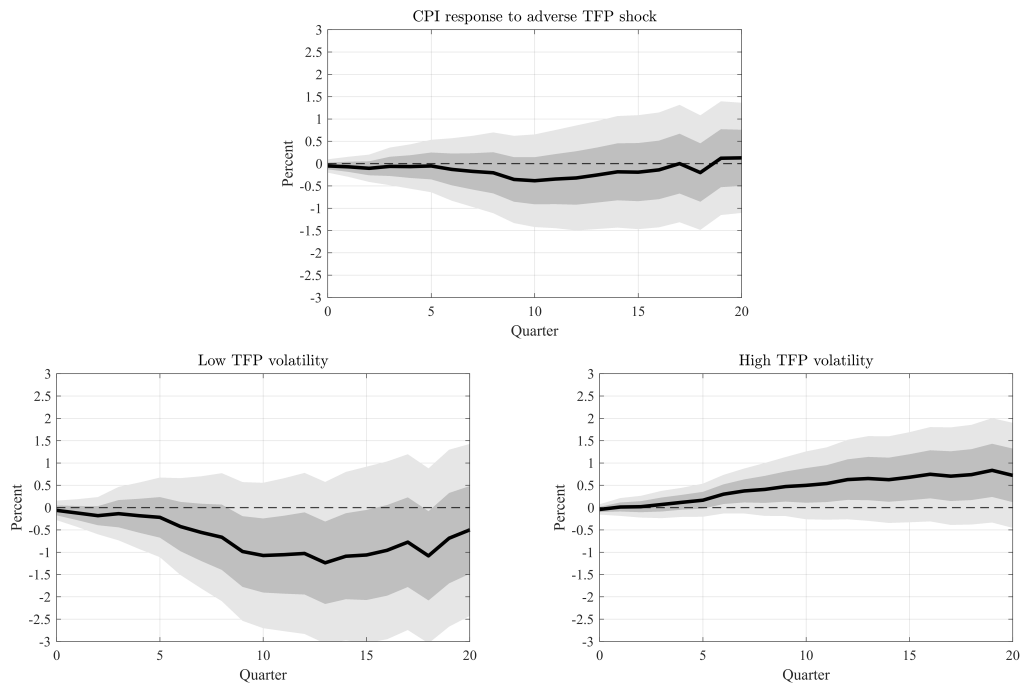
Figure D.10: Empirical response of net entry to 1 percent fall in productivity



Notes. Net entry measured as net business formation, a discontinued monthly index reported in BEA's Survey of Current Business, see page C-29 of this file: <https://apps.bea.gov/scb/pdf/NATIONAL/BUSCYCLE/1994/1194cpgs.pdf>. Upper panel: Unconditional net entry response to adverse TFP shock. Lower panel: Net entry response to adverse TFP shock conditional on being in regime of relatively high or low TFP volatility. Sample period 1948Q1-1992Q1. Dark (light) gray shaded areas are 68% (95%) confidence bands.

D.6 Prices measured as consumer price index

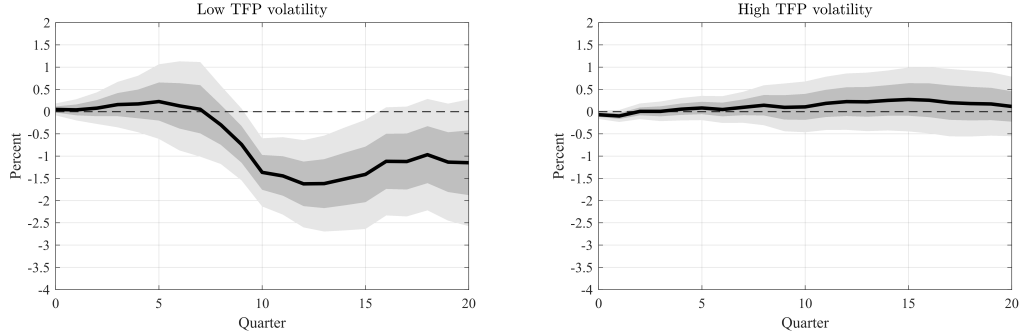
Figure D.11: Empirical response of prices to 1 percent fall in productivity



Notes. Prices measured as CPI for goods excluding foods and energy. Upper panel: Unconditional response to adverse TFP shock. Lower panel: TFP volatility regimes based on Markov-switching model. Response to adverse TFP shock conditional on being in regime of high or low TFP volatility. Sample period 1957Q1-2024Q1. Dark (light) gray shaded areas are 68% (95%) confidence bands.

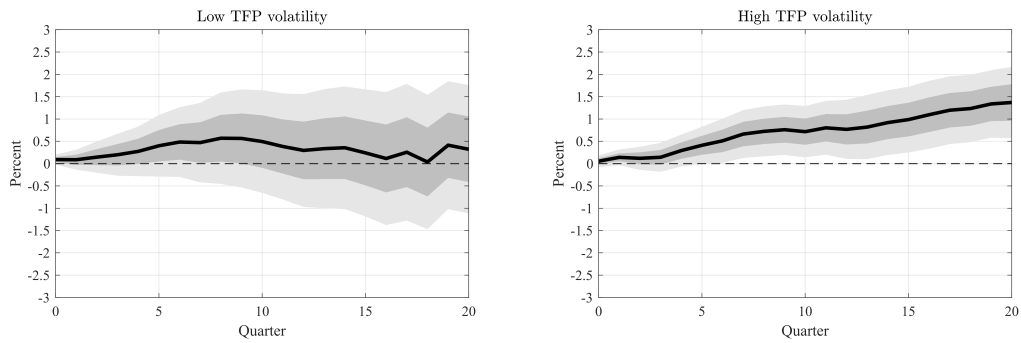
D.7 Alternative identification of TFP volatility states: Local projections

Figure D.12: Empirical response of net entry to 1 percent fall in productivity



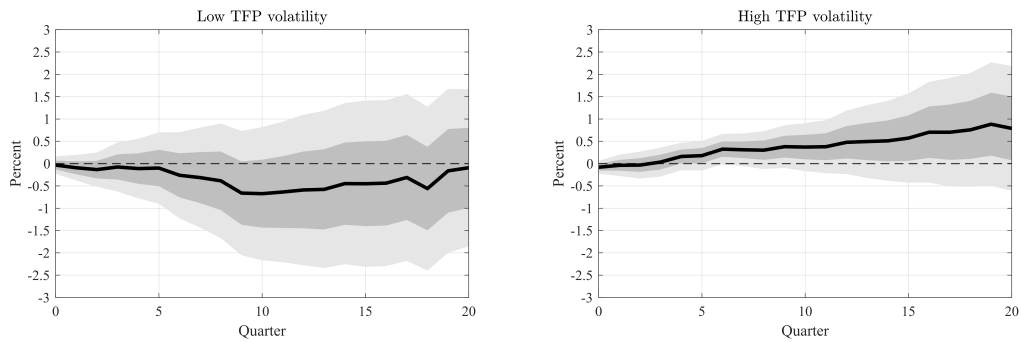
Notes. TFP volatility regimes based on moving average of TFP shock standard deviation. Net entry measured as net establishment entry rate. Sample period 1993Q2-2022Q4. Dark (light) gray shaded areas are 68% (95%) confidence bands.

Figure D.13: Empirical response of producer prices to 1 percent fall in productivity



Notes. Volatility regimes based on moving average of TFP shock standard deviation. Prices measured as PPI (core final goods). Sample period 1993Q2-2024Q1. Dark (light) gray shaded areas are 68% (95%) confidence bands.

Figure D.14: Empirical response of consumer prices to 1 percent fall in productivity



Notes. Volatility regimes based on moving average of TFP shock standard deviation. Prices measured as CPI (core). Sample period 1993Q2-2024Q1. Dark (light) gray shaded areas are 68% (95%) confidence bands.

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