Business Cycles with Pricing Cascades*

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Abstract

How do *large* aggregate and sector-specific shocks affect the macroeconomy? To answer this question, we develop a non-linear dynamic general equilibrium model featuring a disaggregated production economy with networks and *optimal* decisions on the timing and size of price adjustments. The interaction of our model ingredients creates equilibrium *cascades*: large movements in aggregates trigger additional price adjustment decisions at the extensive margin. Crucially, networks may dampen or amplify cascades, depending on the *type of shock* driving the business cycle. When faced with large *demand* shocks, such as monetary interventions, networks *dampen* cascades, thus slowing down price adjustment decisions and giving central banks substantial power to stimulate the real economy with limited inflationary consequences. In contrast, under aggregate or sector-specific *supply* shocks, networks *amplify* cascades, leading to fast increases in the frequency of repricing and large inflationary swings. Applied to Euro Area data, we show that it is the novel interaction of networks with pricing cascades that allows us to quantitatively match the surges in inflation and the repricing frequency in the post-Covid era.

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1 Introduction

The dynamics of aggregate prices and quantities over the business cycle has long been a central theme in economics. Recent events, such as the Covid pandemic and the Russian invasion of Ukraine, have brought revived attention to the topic, along with new evidence on the cyclical properties of the key macro variables. First, following a prolonged period of stability, we have witnessed the possibility of *large* inflationary swings in advanced economies, marked by persistent double-digit rates of price growth in the US, the UK and the Euro Area. All while the movements in aggregate activity have been milder and transitory. Second, we have learned that much the inflationary surge has come from rising *frequency* at which firms adjust their prices (Montag and Villar, 2023; Cavallo et al., 2024). Third, large granular shocks, hitting specific *sectors* such as energy and agriculture, have caused sizable consequences for the rest of the economy, despite their relatively small share in aggregate activity. While informative, such evidence cannot be analyzed through the lens of existing, and immensely influential, theoretical frameworks, featuring linearized, single-sector setups with a constant frequency of adjustment (Woodford, 2004; Galí, 2015). Such a discrepancy calls for a new framework to analyze aggregate prices and quantities over the business cycle – and this paper develops one.

Our novel general equilibrium framework features a previously unexplored combination of three crucial ingredients. First, a multi-sector structure with a fully unrestricted input-output architecture, allowing to capture empirically-realistic production networks. Second, firms' making pricing decisions in an optimal state-dependent manner, so that both the extensive and the intensive margins of adjustment are endogenous. Third, a fully non-linear solution strategy, tracing out the exact response of the economy to arbitrarily large shocks, either aggregate or sector-specific. The *interaction* of our three ingredients delivers a novel theoretical mechanism, namely pricing *cascades*: large movements in aggregates triggering possibly self-reinforcing adjustment decisions at the extensive margin. Crucially, networks can either dampen or amplify cascades, depending on the *type of shock* hitting the economy. For demand shocks, such as monetary interventions, networks dampen cascades, leading to muted price responses with a near-constant frequency of adjustment in equilibrium. In contrast, networks amplify cascades following either aggregate or sectoral supply shocks, with strong price responses led by extensive margin. As a result, the novel mechanism of pricing cascades allows our framework to produce realistic monetary non-neutrality, while simultaneously generating substantial inflationary surges following reasonably-sized and structurally-interpretable shocks. When estimated to Euro Area data, the interaction of networks with cascades allows our model to jointly match the surges in inflation and the repricing frequency in the post-Covid era.

For demand shocks, such as central bank monetary interventions, production networks shrink the magnitudes of desired price changes at the firm level, which in turn compresses the sizes of price gaps for all firms. As a result, the presence of networks *dampens* pricing cascades, lowering the response of the aggregate repricing frequency, *ceteris paribus*. Consequently, much larger monetary stimuli are feasible in equilibrium, as much larger shocks are required to reach the limit where all firms choose to adjust prices and the economy becomes money-neutral. Quantitatively, in a model estimated to match sectoral pricing moments in the Euro Area, under production networks a one-time monetary intervention can stimulate aggregate GDP by a maximum of 5%; meanwhile, removing input-output linkages shrinks the maximum possible stimulus to just over 2%.

In contrast, large aggregate and sector-specific *supply* shocks interact with the production network in a manner that is completely opposite to that under demand shocks. In particular, production networks amplify the firm-level desired price changes following supply shocks, hence expanding the price gaps. As a result, they *amplify* cascades in pricing, making the decision to adjust more likely *ceteris paribus*. As a result, large negative TFP shocks can lead to very fast increases in the aggregate repricing frequency, leading to substantial inflationary spikes. Quantitatively, following an aggregate TFP shock of -10%, production networks double the equilibrium fraction of adjusting firms and more than triple the impact response of inflation (from 5% to 17% monthly). Moreover, large TFP shocks to sectors with a high degree of network centrality, corresponding to important suppliers to other producers in the economy, can also drive up the aggregate repricing frequency, and thus create large aggregate inflationary responses, evolving non-linearly in the size of the shock.

As a further quantification of the role played by the interaction of networks with pricing cascades, we subject our model to the key structural shock series experienced by the Euro Area economy in the (post-)Covid years (2020-2024), and compare the model-implied dynamics of aggregate inflation and repricing frequency to that observed in the data. In particular, we feed in four shock series, corresponding to aggregate nominal demand, aggregate labor wedge, as well as the dynamics of energy and food prices. We find that the model successfully matches the five percentage point increase in the aggregate repricing frequency, as well as the aggregate inflation surge up to 11%

at the peak. In contrast, an otherwise identical model without networks generates at most a one percentage point increase in aggregate repricing frequency, as well as an aggregate inflation surge to only 5% at the peak. These results highlight the quantitative importance of our novel theoretical channel – the interaction of networks with pricing cascades – for explaining aggregate business cycle dynamics.

Literature review Our paper contributes to at least two broad strands of the literature. First, we add to the vast literature on state-dependent pricing in macroeconomics; see Costain and Nakov (2024) for a recent survey.[1] Under state-dependent pricing, the probability of a price change is affected by idiosyncratic and aggregate shocks, in contrast with time-dependent models such as Taylor (1979) or Calvo (1983). Our main contribution is to the literature on general equilibrium implication of state-dependent pricing, marked by the works of Golosov and Lucas (2007), Gertler and Leahy (2008) and Midrigan (2011) in the context of a single-sector model with small aggregate shocks and fixed menu costs.[2] This framework has been further explored analytically by Alvarez and Lippi (2022) and others, whose results provide model-based sufficient statistics linking the dynamics of macro aggregates to moments that can be measured in firm-level data. Subsequent work also considers one-sector models with state-dependent pricing subjected to large aggregate shocks, such as the papers of Karadi and Reiff (2019), Cavallo et al. (2024) and Blanco et al. (2024a). As for multi-sector models with state-dependent pricing, the seminal work by Nakamura and Steinsson (2010) studies the transmission of monetary shocks in a setup with heterogeneous pricing and roundabout production. More recent work by Carvalho and Kryvtsov (2021) studies a multi-sector framework with heterogeneous state-dependent pricing, but without networks, to rationalize aggregate price adjustment facts.

Relative to the aforementioned papers, we contribute by developing a general equilibrium model with an unrestricted input-output structure that we solve fully non-linearly for any aggregate or sector-specific shocks, either demand- or supply-side. We also introduce a novel theoretical channel that comes from the *interaction* of networks with pricing *cascades*.

Second, our paper is related to the growing literature on production networks and aggregate

[1] See also Nakamura and Steinsson (2013) and Klenow and Malin (2010) for earlier surveys.

[2] Several papers have developed "second generation" SDP models in which the price change probability is a *smoothly* increasing function of the gain from adjustment, e.g. Caballero and Engel (1992), Nakamura and Steinsson (2008), Costain and Nakov (2011), instead of the *step function* it is in the fixed menu cost model.

fluctuations. The seminal work by Acemoglu et al. (2012) considers a flexible-price setup and shows how production networks can amplify sector- or firm-specific shocks to create aggregate fluctuations. Subsequent work by Baqaee and Farhi (2020) studies aggregation properties in inefficient economies with networks. A separate strand of this literature analyzes linearized models with production networks and time-dependent pricing, both positively (Pasten et al., 2020; Ghassibe, 2021; Afrouzi and Bhattarai, 2023) and normatively (Rubbo, 2023; La'O and Tahbaz-Salehi, 2022).

We contribute to this literature by studying non-linear aggregate fluctuations in a setup with state-dependent pricing and arbitrarily large aggregate or sector-specific shocks.

Roadmap The remainder of the paper is structured as follows. Section 2 outlines the optimization problem faced by each type of agent in the economy and the numerical strategy to solve the equilibrium dynamics. Section 3 explains the key model mechanisms in a simplified version of our setup. Section 4 outlines our procedure for estimating the structural parameters of the model to match key sectoral micro-pricing moments for the Euro Area. Section 5 shows our quantitative results for monetary shocks. Section 6 turns to quantitative results for aggregate and sector-specific TFP shocks. Section 7 considers extensions to our baseline results. Section 8 describes our quantification exercise, where we assess the ability of our model to explain the aggregate dynamics of inflation and repricing frequency in the Euro Area. Section 9 concludes.

2 Model

We begin by introducing our theoretical model, which presents a novel combination of three key ingredients. First, it features a number of sectors populated by firms interconnected by production networks, which facilitate trade in intermediate inputs, both within and across sectors. Second, firms make optimal pricing decisions subject to menu costs. Third, we allow for both aggregate, sector-specific and firm-level shocks, and present a numerical strategy that allows to compute the economy-wide equilibrium dynamic response to an arbitrarily large disturbance of any origin.

2.1 Overview

Time is discrete and indexed by $t \in \{0, 1, 2, ...\}$. The economy is populated by three (types of) agents: households, firms and the government. There is a continuum of identical households, each consuming output and supplying labor. Firms are subdivided into N sectors, indexed by $i \in \{1, 2, ..., N\}$, each sector containing a continuum of monopolistically competitive firms of measure one; we use Φ_i to denote the set of all firms in sector i. The government consists of the central bank, which conducts policy by setting money supply, and the fiscal authority, which collects taxes from firms and rebates them to households in a lump-sum fashion.

2.2 Households

The representative households chooses a sequence of consumption, labor supply, and one-period nominal bond holdings to maximize expected lifetime utility:

$$\max_{\{C_t, L_t, B_t\}_{t \geq 0}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(C_t, L_t), \tag{1}$$

subject to the period-by-period budget constraint

$$P_t^C C_t + \mathbb{E}_t\{\Lambda_{t,t+1} B_{t+1}\} \quad \leq \quad B_t + W_t L_t + \sum_{i=1}^{N} \int_0^1 D_{i,t}(j) dj + T_t, \tag{2}$$

where C_t is consumption, L_t is labor supply, B_t is the level of nominal bond holdings, T_t is the level of lump-sum transfers from the government, $D_{i,t}(j)$ are the dividends received lump-sum from firm j in sector i at time t, $\Pi_t^C = \left(P_t^C / P_{t-1}^C\right)$ is the gross CPI inflation rate, W_t is the nominal wage and $\Lambda_{t,t+1}$ is the nominal stochastic discount factor of the household.

Total final consumption C_t is given by an aggregator over sector-specific varieties:

$$C_t = \mathcal{C}(C_{1,t}, ..., C_{N,t}) \tag{3}$$

where $\mathcal{C}(\cdot)$ is homogeneous of degree one and non-decreasing in each of the arguments. The household chooses consumption of each of the sector-specific varieties to minimize total expenditure $\sum_i P_{i,t} C_{i,t}$, subject to the aggregator in (3). The minimal cost of assembling such a basket of sectoral varieties aggregating to $C_t = 1$ pins down the consumption price index as $P_t^C = \mathcal{P}^C(P_{1,t}, ..., P_{N,t})$, where $\mathcal{P}^C$ is homogeneous of degree one and non-decreasing in each of the arguments.

Sectoral final consumption $C_{i,t}$ is in turn given by the following aggregator over firm-specific varieties:

$$C_{i,t} = \left\{ \int_0^1 \left[\zeta_{i,t}(j) C_{i,t}(j)\right]^{\frac{\epsilon-1}{\epsilon}} dj \right\}^{\frac{\epsilon}{\epsilon-1}}, \tag{4}$$

where $\epsilon > 1$ is the within-sector elasticity of substitution, $C_{i,t}(j)$ is the final demand for the output of firm $j \in [0,1]$ in sector i at time t, and $\zeta_{i,t}(j)$ is a firm-specific idiosyncratic *quality* process. The quality process follows a random walk in logs:

$$\log \zeta_{i,t}(j) = \log \zeta_{i,t-1}(j) + \sigma_i \varepsilon_{i,t}(j), \tag{5}$$

where $\varepsilon_{i,t}(j)$ is an *i.i.d.* Gaussian innovation with mean zero and standard deviation of one. The final demand for firm j in sector i is given by:

$$C_{i,t}(j) = \zeta_{i,t}(j)^{\epsilon-1} \left(\frac{P_{i,t}(j)}{P_{i,t}}\right)^{-\epsilon} C_{i,t}, \tag{6}$$

and the sectoral price index of sector i is given by:

$$P_{i,t} = \left[\int_0^1 \left(\frac{P_{i,t}(j)}{\zeta_{i,t}(j)}\right)^{1-\epsilon} dj\right]^{\frac{1}{1-\epsilon}}. \tag{7}$$

The representative household is also subject to a cash-in-advance constraint, which requires that the nominal money holdings are sufficient to cover the aggregate nominal final demand:

$$P_t^C C_t \leq M_t. \tag{8}$$

The aggregate money supply process $\{M_t\}_{t\geq 0}$ is set by the central bank, and agents treat this process as exogenous. An alternative is to consider a central bank which conducts monetary policy by setting the nominal interest rate according to a Taylor rule; we consider such an extension in Section 7.1.

We now specify the functional forms for household preferences. First, for household preferences over aggregate consumption and labor supply, we use the log-linear preferences of Golosov and Lucas (2007):

Assumption 1 (Golosov-Lucas preferences)**.** *The utility function over consumption and labor sup-*

ply is log-linear: $u(C_t, L_t) = \log C_t - L_t$.

Under such preferences, we obtain the following intra-temporal labor supply condition: $\frac{W_t}{P_t^C} = C_t$. When combined with the cash-in-advance constraint (8), it implies that the nominal wage equals money supply in every period: $W_t = M_t$. In addition, the nominal stochastic discount factor satisfies: $\Lambda_{t,t+1} = \beta \frac{P_t^C C_t}{P_{t+1}^C C_{t+1}} = \beta \frac{M_t}{M_{t+1}}$.

As for aggregation across final consumption of sectoral varieties, in the baseline model we assume it to take the Cobb-Douglas form:

Assumption 2 (Consumption aggregation)**.** *The consumption aggregator* $\mathcal{C}(\cdot)$ *is given by:*

$$\mathcal{C}(C_{1,t}, ..., C_{N,t}) = \iota^C \prod_{i=1}^{N} C_i^{\varpi_{i,t}^C}, \tag{9}$$

where $\iota^C \equiv \prod_{i=1}^N \varpi_i^{C - \varpi_i^C}$ *is a normalization term and* $\sum_i \varpi_i^c = 1$, $\varpi_i^c \geq 0, \forall i$.

Under this assumption, the equilibrium sectoral final consumption shares are constant over time: $\omega_{i,t}^C \equiv \frac{P_{i,t} C_{i,t}}{P_t^C C_t} = \varpi_i^C$. In Section 7.3 we consider a more general CES aggregator over sectoral consumption varieties.

2.3 Firms: production

The production function of firm j in sector i is given by:

$$Y_{i,t}(j) = \frac{1}{\zeta_{i,t}(j)} \times A_{i,t} \times \mathcal{F}_i \left[L_{i,t}(j), X_{i,1,t}(j), ..., X_{i,N,t}(j) \right], \tag{10}$$

where $\mathcal{F}_i(\cdot)$ is homogeneous of degree one and non-decreasing in inputs; $L_{i,t}(j)$ is the labor used by firm j in sector i at time t, $X_{i,k,t}(j)$ is intermediate inputs bought by firm j in sector i from sector k at time t. In addition, $A_{i,t}$ is an exogenous sector-specific total factor productivity process, while $\zeta_{i,t}(j)$ is the firm-level idiosyncratic quality process introduced in (5).

The intermediates demand $X_{i,k,t}(j)$ is in turn an aggregator over intermediates bought from each firm in sector k:

$$X_{i,k,t}(j) = \left\{ \int_0^1 \left[\zeta_{k,t}(j') X_{i,k,t}(j, j') \right]^{\frac{\epsilon - 1}{\epsilon}} dj' \right\}^{\frac{\epsilon}{\epsilon - 1}}, \tag{11}$$

where $X_{i,k,t}(j,j')$ is intermediates bought by firm j in sector i from firm j' in sector k, which satisfies the following demand condition in equilibrium: $X_{i,k,t}(j,j') = \zeta_{k,t}(j')^{\epsilon-1}\left(\frac{P_{k,t}(j')}{P_{k,t}}\right)^{-\epsilon} X_{i,k,t}(j)$.

Each firm chooses its labor and intermediate inputs in order to minimize the total cost of production, subject to the production technology in (10). The latter delivers the following marginal cost function for firm j in sector i at time t:

$$MC_{i,t}(j) = \zeta_{i,t}(j) \times \mathcal{Q}_i(W_t, P_{1,t}, ..., P_{N,t}; A_{i,t}) \tag{12}$$

where $\mathcal{Q}_i(\cdot)$ is the common component of the marginal cost index for all firms within a sector, which strictly falls in $A_{i,t}$ and is homogeneous of degree one and non-decreasing in the prices of all inputs.

In our baseline model, we assume that production technology takes a Cobb-Douglas form for all firms in all sectors:

Assumption 3 (Production technology)**.** *The production technology $\mathcal{F}_i(\cdot)$ for a firm j in sector i is given by:*

$$\mathcal{F}_i[L_{i,t}(j), X_{i,1,t}(j), ..., X_{i,N,t}(j)] = \iota_i L_{i,t}(j)^{\overline{\alpha}_i} \prod_{k=1}^{N} X_{i,k,t}(j)^{\overline{\omega}_{ik}}, \tag{13}$$

where $\iota_i \equiv \overline{\alpha}_i^{-\overline{\alpha}_i} \prod \overline{\omega}_{ik}^{-\overline{\omega}_{ik}}$ is a normalization term and $\overline{\alpha}_i + \sum_i \overline{\omega}_{ik} = 1$, $\overline{\alpha}_i$, $\overline{\omega}_{ik} \geq 0, \forall i$.

Under this assumption, the equilibrium labor cost shares and the input-output cost shares are constant over time and the same for all firms within a sector: $\alpha_{i,t} \equiv \frac{W_t L_{i,t}(j)}{MC_{i,t}(j) Y_{i,t}(j)} = \overline{\alpha}$, $\quad \omega_{i,k,t} \equiv \frac{P_{k,t} X_{i,k,t}(j)}{MC_{i,t} Y_{i,t}(j)} = \overline{\omega}_{ik}$. As with household preferences, in Section 7.3 we relax the Cobb-Douglas assumption and consider a more general CES production function.

2.4 Firms: equilibrium size

The goods market clearing condition for firm j in sector i is given by:

$$Y_{i,t}(j) = C_{i,t}(j) + \sum_{k=1}^{N} \int_0^1 X_{k,i,t}(j', j) dj'. \tag{14}$$

Aggregating up to the level of sectors, multiplying both sides by P_i and dividing by aggregate final nominal demand $P_t^C C_t$, one can express the sectoral sales share (Domar weight) $\lambda_i \equiv \frac{P_{i,t} Y_{i,t}}{P_t^C C_t}$ as:

$$\lambda_{i,t} = \omega_{i,t}^C + \sum_{k=1}^{N} \omega_{ki,t} \lambda_{k,t} \times \mu_{k,t}^{-1}, \tag{15}$$

where μ_k^{-1} is the sales-weighted harmonic average of firm-level markups in a sector k : $\mu_{k,t}^{-1} = \int_0^1 \frac{1}{\mu_{k,t}(j')} \times \frac{P_{k,t}(j) Y_{k,t}(j)}{P_{k,t} Y_{k,t}} dj$. Using the downward sloping demand condition for each firm, one can rewrite $\mu_{k,t}^{-1}$ as:

$$\mu_{k,t}^{-1} = \frac{\Delta_{k,t}}{\mathcal{M}_{k,t}}, \qquad \Delta_{k,t} \equiv (P_{k,t}/M_t)^{\epsilon} \int_0^1 \left(\frac{P_{k,t}(j')}{\zeta_{k,t}(j') M_t} \right)^{-\epsilon} dj', \qquad \mathcal{M}_{k,t} \equiv \frac{P_{k,t}}{\mathcal{Q}_{k,t}}, \tag{16}$$

where $\Delta_{k,t}$ is a measure of price dispersion within the sector and $\mathcal{M}_{k,t}$ is a measure of sectoral markup. Stacking the equation for sales shares across sectors, we can write it as:

$$\boldsymbol{\lambda}_t = \boldsymbol{\omega}_{\boldsymbol{C},t} + \tilde{\Omega}_t^T \boldsymbol{\lambda}_t \quad \Longrightarrow \quad \boldsymbol{\lambda}_t = (I - \tilde{\Omega}_t^T)^{-1} \boldsymbol{\omega}_{\boldsymbol{C},t} \tag{17}$$

where $\tilde{\Omega}_t$ is a $N \times N$ matrix whose $[i,j]$ entry is given by $[\tilde{\Omega}_t]_{i,j} = \omega_{ij,t} \left\{ \frac{\Delta_{i,t}}{\mathcal{M}_{i,t}} \right\}$. Having calculated the sectoral sales shares, one obtains the sectoral total output as $Y_{i,t} = \lambda_{i,t} \times M_t / P_{i,t}$ and then the size of an individual firm as $Y_{i,t}(j) = \zeta_{i,t}(j)^{\epsilon-1} \left(\frac{P_{i,t}(j)}{P_{i,t}} \right)^{-\epsilon} Y_{i,t}$.

2.5 Firms: pricing

The nominal profit of firm j in sector i at time t is given by:

$$D_{i,t}(j) = [(1 - \tau_{i,t}) P_{i,t}(j) - MC_{i,t}(j)] \times Y_{i,t}(j), \tag{18}$$

where $\tau_{i,t}$ is an exogenous sector-specific and time-varying sales tax levied by the government.[3] Denoting by $\tilde{P}_{i,t}(j) \equiv \frac{P_{i,t}(j)}{\zeta_{i,t}(j) M_t}$ the firm's quality-adjusted *real* price and by $\tilde{P}_{i,t} \equiv \frac{P_{i,t}}{M_t}$ the sectoral

[3] The proceeds of these taxes are then rebated to households as a lump-sum transfer $T_t = \sum_{i=1}^{N} \tau_{i,t} \int_0^1 P_{i,t}(j) Y_{i,t}(j) dj$.

real price index, we can write the firm-level *real* profits $\tilde{D}_{i,t}(j) \equiv \frac{D_{i,t}(j)}{M_t}$ as:

$$\tilde{D}_{i,t}(j) = \left(\frac{P_{i,t}}{M_t}\right)^{\epsilon-1} \times \left[(1-\tau_{i,t})\frac{P_{i,t}(j)}{\zeta_{i,t}(j)M_t} - \frac{\mathcal{Q}_{i,t}}{M_t}\right] \times \left(\frac{P_{i,t}(j)}{\zeta_{i,t}(j)M_t}\right)^{-\epsilon} \times \lambda_{i,t}$$

$$= \tilde{D}\left(\tilde{P}_{i,t}(j), \tau_{i,t}, \left\{\tilde{P}_{k,t}, \Delta_{k,t}, A_{k,t}\right\}_{k=1}^{N}\right). \tag{19}$$

Note that keeping track of the firm-level real profits requires knowing the firm's real quality-adjusted price, the own sectoral sales tax, as well as the real sectoral prices, price dispersions and productivities of all sectors in the economy.

Resetting the *nominal* price $P_{i,t}(j)$ involves the firm paying a sector-specific and possibly time-varying menu cost $\kappa_{i,t}$ measured in units of labor. The optimal reset price maximizes the firm's value, taking into account that this new price may not change for some period of time. In particular, when the nominal price does not change, the log of quality-adjusted real price $p_{i,t}(j) \equiv \log \tilde{P}_{i,t}(j)$ evolves according to

$$p_{i,t}(j) = p_{i,t-1}(j) + \log\left(\frac{P_{i,t-1}(i)}{\zeta_{i,t}(j)M_t}\right) - \log\left(\frac{P_{i,t-1}(j)}{\zeta_{i,t-1}(j)M_{t-1}}\right) = p_{i,t-1}(j) - \sigma_i\varepsilon_{i,t} - m_t, \tag{20}$$

where $m_t \equiv \Delta \log M_t$.

Without loss of generality, let $\eta_{i,t}(p)$ denote the probability that a firm in sector i with a quality adjusted log relative price p resets its price at t . Consider a firm with a real quality adjusted price p at the end of period t, and let $p_+ \equiv (p - \sigma_i\varepsilon_{i,t+1}(j) - m_{t+1})$. Then this firm's real value at the end of period t is given by the following Bellman equation:

$$V_{i,t}(p) = \tilde{D}_{i,t}(p) + \beta\mathbb{E}_t\left[\{1-\eta_{i,t+1}(p_+)\} V_{i,t+1}(p_+) + \eta_{i,t+1}(p_+)\left(\max_{p'} V_{i,t+1}(p') - \kappa_{i,t+1}\right)\right], \tag{21}$$

which consists of the current period real profits $\tilde{D}_{i,t}(p)$, as well as the discounted expected continuation value. The latter is computed taking into account that at time $t+1$ the nominal price does not change with probability $1-\eta_{i,t+1}(\cdot)$, whereas with probability $\eta_{i,t+1}(\cdot)$ the firm pays the menu cost and optimally resets the nominal price.

Our formulation of the pricing problem covers a wide range of existing models of price setting, corresponding to the different functional forms of $\eta_{i,t}(\cdot)$. In the baseline setup of our model, we consider a specific functional form for the probability of adjustment function $\eta_{i,t}(\cdot)$. In particular, following Golosov and Lucas (2007), we assume that a firm adjusts if and only if the value gain from adjustment in a given period exceeds the menu cost:

Assumption 4 (*Ss* pricing)**.** *Consider a firm in sector i with the quality adjusted log relative price p at time t. Then the probability that this firm adjusts its nominal price is given by:*

$$\eta_{i,t}(p) = \mathbf{1}(L_{i,t}(p) > 0) \tag{22}$$

where $\mathbf{1}(\cdot)$ is the indicator function, and

$$L_{i,t}(p) = \max_{p'} V_{i,t}(p') - V_{i,t}(p) - \kappa_{i,t} \tag{23}$$

is the gain from adjustment (or loss from inaction), net of the menu cost.

Note that although here we specify a problem of *price* setting under nominal rigidities, our setup can automatically handle rigidities in nominal *wage* setting as well by appropriately parameterizing the input-output structure. In particular, consider a setup with a sector called the labor union (LU), such that it only uses labor in production ($\overline{\alpha}_{LU} = 1$) and moreover it is the only sector purchasing labor directly from households ($\overline{\alpha}_{-LU} = 0$). Instead, other sectors purchase labor indirectly from the labor union as an intermediate input, such that $\overline{\omega}_{i,LU}$ represents the empirical cost share of labor for sector i. Then any rigidities in the price setting of the labor union sector are isomorphic to nominal wage rigidities. Moreover, the gap between the nominal wage W_t and the price index of the labor union sector $P_{LU,t}$ has a natural interpretation as the aggregate *labor wedge*.[4]

2.6 Equilibrium definition and solution method

In addition to the goods market clearing condition in (14), the equilibrium in our economy is also characterized by clearing of the labor market:

$$L_t = \sum_{i=1}^{N} \int_0^1 L_{i,t}(j)dj + \sum_{i=1}^{N} \kappa_{i,t} \int_0^1 \eta_{i,t}(p_{i,t}(j))dj, \tag{24}$$

[4]More formally, the labor wedge is the gap between the nominal marginal rate of substitution across consumption and labor $P_t^C \times MRS_t^{CL} = W_t$ and the nominal cost of labor faced by firms $P_{LU,t}$.

as well as by clearing in the market for bonds, which are in zero net supply: $B_t = 0$.

Having specified the optimality and market clearing conditions, we can now formally define the decentralized equilibrium in our economy:

Definition 1 (Equilibrium). *The equilibrium is a collection of prices $\{P_{i,t}(j)|j \in \Phi_i\}_{i=1}^N$, allocations $\left\{Y_{i,t}(j), L_{i,t}(j), C_{i,t}(j), \{X_{i,r,t}(j,j')|j' \in \Phi_r\}_{r=1}^N |j \in \Phi_i\right\}_{i=1}^N$, wage W_t and bond holdings B_t, which given the realizations of firm-level quality process $\{\zeta_{i,t}(j)|j \in \Phi_i\}_{i=1}^N$, sectoral productivities $\{A_{i,t}\}_{i=1}^N$, sectoral sales tax rates $\{\tau_{i,t}\}_{i=1}^N$ and money supply M_t satisfy agent optimization and market clearing conditions in every period.*

We now briefly outline our solution strategy, which we use to compute equilibrium prices and quantities given the realizations of exogenous processes. Full details of the numerical strategy are given in Appendix C.

As a first step, we compute the steady-state of our economy, defined as the equilibrium evaluated at the point where money supply growth and sectoral TFPs are at their unconditional mean values, and the firm-level prices are in their stationary distribution. In particular, for each sector we numerically solve the stationary Bellman equation and firms' price distributions on an evenly spaced grid of log quality adjusted real prices with step size Δp, $p_j \in \left[\underline{p}, \underline{p} + \Delta p, ..., \overline{p}\right]$, $j = 1, .., J$ grid points, so that $V_j = V(p_j)$. In the algorithm, introduced in Appendix C, we jointly search across firm-level prices in each sector and sector-specific sales taxes $\{\overline{\tau}_i\}_{i=1}^N$, so that we satisfy the equilibrium conditions and obtain steady-state real sectoral price indices equal to one.

Next, we compute the non-linear responses to a sequence of monetary and TFP shocks. We operate under the assumption of perfect foresight over aggregate and sectoral exogenous shocks, while maintaining uncertainty over the idiosyncratic innovations. To compute the responses, we first assume that there exists a finite period T, at which the economy is back in steady state. Then, starting from a guess for the sequences of sectoral and aggregate variables, we iterate *backward* from $t = T$ to $t = 0$ to solve for the micro value functions. Having obtained the micro value functions, we iterate *forward* from $t = 0$ to $t = T$, and numerically aggregate to obtain sectoral and aggregate variables. We repeat this backward-forward iteration until convergence. Appendix C formally details the algorithm to perform the backward-forward iteration.

3 Pricing cascades and networks: formal results

We now use a simplified version of our model in order to formally introduce the notion of pricing *cascades*: large movements in aggregates creating possibly self-reinforcing price adjustment decisions at the extensive margin. Moreover, we present analytical results regarding the novel interaction of pricing cascades with networks. In particular, we formally show that networks *dampen* cascades whenever the aggregate cycle is driven by demand shocks, whereas they *amplify* cascades driven by supply shocks. We also present several examples with particular network arrangements in order to solidify the intuition behind our novel theoretical results.

3.1 Static economy

In order to obtain intuition regarding the transmission of large shocks in our model, we consider a simplified setup obtained under two additional assumptions. First, we assume the economy to be static in the sense that agents fully discount the future:

Assumption 5 (Myopia)**.** *Agents fully discount the future in their objective function, so that* $\beta = 0$*.*

In particular, this setting implies that any firm's value function is simply given by contemporaneous profits, and hence the optimal quality-adjusted real reset price for any firm in a sector i is given by:

$$\tilde{P}^{*}_{i,t} = \frac{1}{1-\tau_{i,t}} \frac{\epsilon}{\epsilon - 1} \times \frac{1}{A_i} \prod_{k=1}^{N} \tilde{P}_{i,t}^{\overline{\omega}_{ik}} = \Gamma_{i,t} \times \tilde{\mathcal{Q}}_{i,t}. \tag{25}$$

where $\Gamma_{i,t} \equiv \frac{1}{1-\tau_{i,t}} \frac{\epsilon}{\epsilon-1}$ is the (exogenous) desired markup, whose variation across time and sectors is pinned down by the movements in the sectoral tax rates $\tau_{i,t}$.

Second, we assume a specific form of time-variation of the sector-specific menu cost $\kappa_{i,t}$:

Assumption 6 (Sectoral menu costs)**.** *The sector-specific menu cost follows the following process:* $\kappa_{i,t} = \overline{\kappa}_i (1-\tau_{i,t}) [\tilde{P}_{i,t}/\tilde{P}^{*}_{i,t}]^{\epsilon-1} \lambda_{i,t}$*, where* $\overline{\kappa}_i$ *is a sector-specific constant.*

The above two assumptions allow us to derive closed-form results regarding the interaction between networks, price adjustment decisions at the extensive margin, and the type of shocks hitting the economy.

The decision to change prices is based on whether the value gain from adjustment exceeds the menu cost. In the static setup, we can obtain a tractable approximation for the gain from adjustment as a function of the *price gap*, or the difference between the current and the optimal reset price:

Lemma 1 (Adjustment gains). *Suppose Assumptions 1-5 hold. Let $\tilde{p}_{i,t}(j) \equiv \log \tilde{P}_{i,t}(j) - \log \tilde{P}^*_{i,t}$ be the price gap for a firm j in sector i at time t. Then the profit gain from price adjustment satisfies:*

$$\tilde{D}^*_{i,t}(j) - \tilde{D}_{i,t}(j) \quad = \quad \frac{1}{2}(\epsilon - 1)(1 - \tau_{i,t})[\tilde{P}_{i,t}/\tilde{P}^*_{i,t}]^{\epsilon-1}\lambda_{i,t} \times [\tilde{p}_{i,t}(j)]^2 \quad + \quad \mathcal{O}[\tilde{p}_{i,t}(j)]^3 \tag{26}$$

where $\tilde{D}^*_{i,t}(j)$ is profits at the optimal reset price, $P_{i,t}$ is the real sectoral price index and $\lambda_{i,t}$ is the sectoral sales share (Domar weight).

To illustrate the interaction between networks and price adjustment decisions, consider the initial period ($t = 0$) in our economy. If the firm chooses to not adjust its nominal price, then the quality-adjusted real price in the initial period is given by:

$$\log \tilde{P}_{i,0}(j) = p_{i,-1}(j) - \sigma_i \varepsilon_{i,0}(j) - m_0 \tag{27}$$

where $p_{i,-1}(j)$ is the initial (exogenous) quality-adjusted real price of firm j in sector i, $\varepsilon_{i,0}(j)$ is the realization of the firm-level quality shock in period $t = 0$, and $m_0 \equiv \log(M_0/M_{-1})$ is the realization of money growth at $t = 0$. Given the expression for the optimal reset price in (25), we can write the firm-level price gap in the initial period as:

$$\tilde{p}_{i,0}(j) = -\sigma_i \varepsilon_{i,0}(j) - m_0 - \gamma_{i,0} + a_{i,0} - \sum_{k=1}^{N} \overline{\omega}_{ik} \log \tilde{P}_{k,0} + (p_{i,-1}(j) - \overline{\gamma}_i). \tag{28}$$

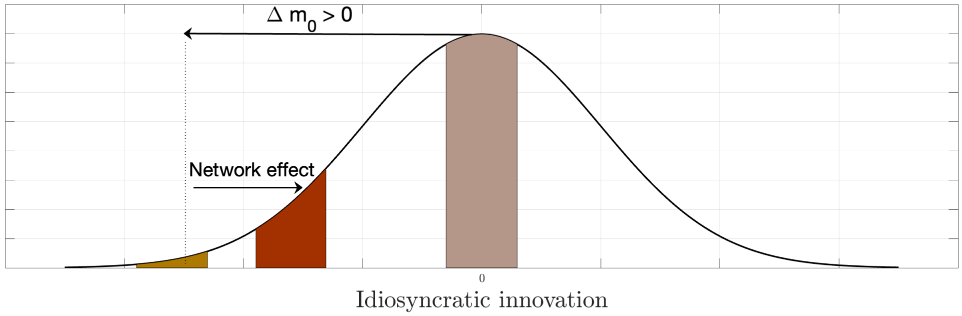
where $\overline{\gamma}_i \equiv \log \frac{\epsilon}{\epsilon-1}\frac{1}{1-\overline{\tau}_i}$, $a_{i,0} \equiv \log A_{i,0}$ and $\gamma_{i,0} \equiv \log \Gamma_{i,0} - \overline{\gamma}_i$.

Without loss of generality, normalize $p_{i,-1}(j) = \log \frac{\epsilon}{\epsilon-1}$; then given the realizations of aggregate and sectoral variables, the magnitude of the price gap of the specific firm j is pinned down by the realization of its idiosyncratic quality innovation $\varepsilon_{i,0}(j)$. We can use the approximate profit gain in Lemma 1 to determine the sector-specific *inaction regions*, defining the ranges for idiosyncratic innovations under which the firm will choose not to adjust:

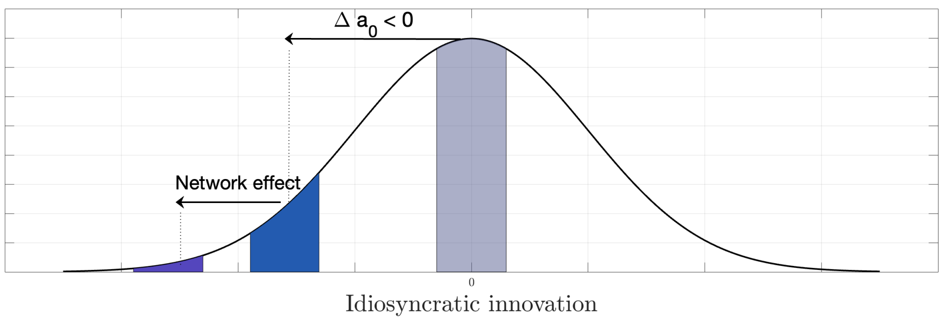
Lemma 2 (Inaction region). *Suppose Assumptions 1-6 hold. Given the realizations of aggregate and sectoral variables and normalizing $p_{i,-1}(j) = \log \frac{\epsilon}{\epsilon-1}\frac{1}{1-\overline{\tau}_i}$, let $\underline{\varepsilon}_{i,0}$ and $\overline{\varepsilon}_{i,0}$ be thresholds such*

Figure 1: **Networks and inaction regions**

(a) Dampening under monetary shocks



(b) Cascades under TFP/markup shocks



Notes: *the figure shows the contribution of production networks to the movements in the inaction region follows monetary and aggregate TFP shocks, respectively.*

that a firm in sector i will not adjust the price if it draws an innovation in $[\underline{\varepsilon}_{i,0}, \overline{\varepsilon}_{i,0}]$. Then,

$$[\sigma_i \underline{\varepsilon}_{i,0}, \quad \sigma_i \overline{\varepsilon}_{i,0}] \quad = \quad -m_0 - \gamma_{i,0} + a_{i,0} - \sum_{k=1}^{N} \overline{\omega}_{ik} \log \tilde{P}_{k,0} \quad \pm \quad \sqrt{\frac{2\overline{\kappa}_i}{\epsilon - 1}}, \tag{29}$$

where $\quad m_0 \equiv \log(M_0/M_{-1}), \quad \gamma_{i,0} \equiv \log \Gamma_{i,0} - \overline{\gamma}_i, \quad a_{i,0} \equiv \log A_{i,0} \quad$ *and* $\quad \overline{\gamma}_i \equiv \log \frac{\epsilon}{\epsilon-1} \frac{1}{1-\overline{\tau}_i}$.

Given the realizations of monetary, productivity, and desired markup shocks, which are independent of the presence of input-output linkages, we can now derive the effect of removing input-output linkages ($\overline{\omega}_{i,k} = 0, \forall i, k$) on the firm-level decision to adjust its price.

Consider an increase in the money supply $m_0 > 0$. According to Lemma 2, this increase in money supply, *ceteris paribus*, implies a leftward shift of the inaction region. In other words, more extreme (negative) realizations of idiosyncratic innovations are needed to prevent adjustment. At the same time, Lemma 2 also implies that as long as the pass-through of the money supply to sectoral prices is incomplete ($\log \tilde{P}_{k,0} < 0, \forall k$), the presence of networks attenuates the leftward shift of the inaction region for all firms that have a non-zero cost share of intermediate inputs. As a result, this weakly lowers the probability of price adjustment for any firm, creating *dampening* in price changes. Panel (a) of Figure 1 provides a graphical illustration of this mechanism. In the following proposition, we formalize the notion that, all else fixed, networks decrease the firm-level probability of adjustment following a monetary shock, thus *dampening* cascades:

Proposition 1 (Cascades and demand shocks)**.** *Suppose Assumptions 1-6 hold. Consider an increase in the money supply $m_0 > 0$. Then, as long as the pass-through of the money supply to sectoral prices is incomplete ($\log \tilde{P}_{k,0} < 0, \forall k$), production networks (weakly) lower the probability of adjustment for any firm following the monetary shock.*

In contrast, consider a sectoral productivity deterioration $a_{i,0} < 0$. According to Lemma 2, this productivity change creates a leftward shift of the inaction region for all firms in sector i. Moreover, as long as this productivity decline leads to a rise in price indices of other sectors ($\log \tilde{P}_{k,0} > 0, \forall k$), then Lemma 2 also implies that networks further amplify the leftward shift in the inaction region for all firms in sector i, as long as the cost share of intermediates in that sector is non-zero. In other words, even more extreme (negative) realizations of idiosyncratic innovations are needed to justify non-adjustment. As a result, contrary to the case of monetary shocks, the presence of networks weakly raises the probability of price adjustment for any firm in sector i, thus *amplifying* pricing cascades. Panel (b) of Figure 1 illustrates this mechanism graphically.

Note that an identical mechanism of cascades also applies in the case of shocks to desired markups. Following an increase in desired markups, $\gamma_{i,0} > 0$, there is a leftward shift in the inaction region, which is further moved to the left as long as the markup shock is inflationary in the aggregate ($\log \tilde{P}_{k,0} > 0, \forall k$). In the following, we formalize the notion that networks create *cascades* in price adjustment decisions after TFP and markup shocks:

Proposition 2 (Cascades and supply shocks)**.** *Suppose Assumptions 1-6 hold. Consider a decrease in sectoral TFP $a_{i,0} < 0$ or an increase in sectoral desired markup $\gamma_{i,0} > 0$. Then, as long as such shocks lead to a rise in price indices of other sectors ($\log \tilde{P}_{k,0} > 0, \forall k$), production networks (weakly) increase the probability of adjustment for any firm in any other sector.*

In addition, note that a productivity or desired markup shock in a sector i can, in principle, increase the probability of price adjustment for firms in any other sector i'. This is true as long as the price indices of sectors used as suppliers by sector i' ($k : \overline{\omega}_{i'k} > 0$) rises following the productivity deterioration or markup increase in sector i.

3.2 Simple examples

We now solidify the intuition behind the formal results on cascades with the aid of several examples. In particular, we return to the dynamic version of our model, but consider concrete network arrangements in order highlight the key mechanisms. To facilitate further comparability between monetary and TFP shocks, for the remainder of this subsection we assume that both follow AR(1) in levels with persistence $\rho \in (0, 1)$.

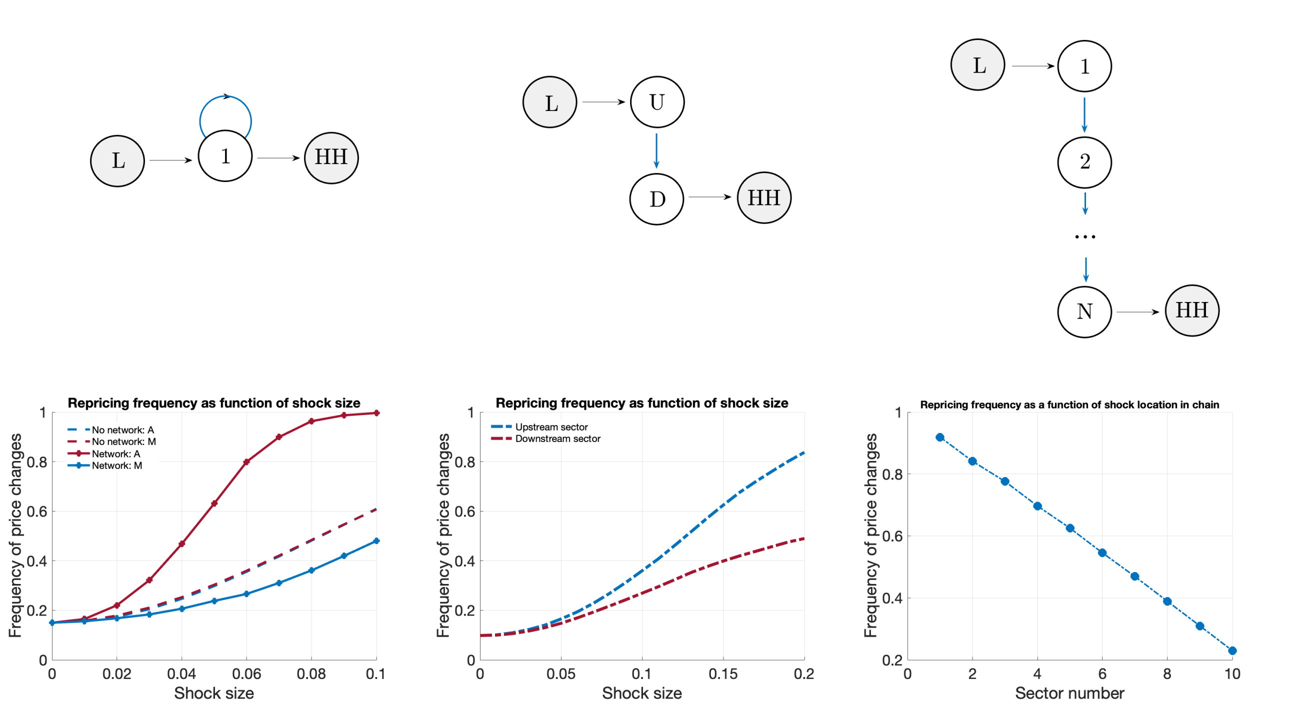
***Example 1*: roundabout production economy**

First, we consider a one-sector ($N = 1$) roundabout economy, where firms trade intermediate inputs with other firms in the same sector, as in the work of Basu (1995). Figure 2(a) illustrates such an arrangement graphically. Naturally, in the limit where we set the cost share of labor to one ($\overline{\alpha}_1 = 1$), the one-sector economy collapses to that of Golosov and Lucas (2007), where firms only use labor in production.

We use this simple example to illustrate how the presence of the network affects the response of the aggregate fraction of adjusting firms to monetary and productivity shocks of different sizes. As can be seen in the bottom panel of Figure 2(a), when there are no networks ($\overline{\alpha}_1 = 1$), monetary and productivity shocks are isomorphic in their effect on aggregate frequency. However, as soon as we add the roundabout production structure ($\overline{\alpha}_1 < 1$), the aggregate adjustment frequency responds much faster to productivity shocks relative to monetary shocks. This is because under monetary shocks, the network structure shrinks the desired price changes and the price gaps, thus *dampening* cascades, which leads to slower increases in the aggregate fraction of adjusters. In contrast, under TFP shocks, the presence of networks expands movements in desired price changes and hence the price gaps, thus *amplifying* cascades at the firm-level, leading to faster increases in the aggregate fraction of adjusters.

Figure 2: **Three example economies**

(a) Roundabout production economy (b) Two-sector vertical chain economy (c) N-sector vertical chain economy



***Notes**: the figure shows three example economies, as well as the responses of aggregate frequency of adjustment to monetary and TFP shocks.*

***Example 2*: two-sector vertical chain economy**

For our second example, we consider a two-sector economy ($N = 2$), which illustrates how the position of a sector in the network affects the transmission of sectoral productivity shocks to aggregate frequency. The top panel of Figure 2(b) presents the arrangement graphically: the upstream sector (U) only uses labor in production and supplies its output as an intermediate input to the downstream sector (D). Importantly, the two sectors have the same size in steady-state equilibrium, in the sense of having identical (cost-based) Domar weights. Moreover, their pricing moments are also the same in steady-state, hence their only ex ante difference comes from the position in the network.

We now consider sector-specific productivity shocks of different sizes and record their effect on the aggregate adjustment frequency. In the bottom panel of Figure 2(b) one can see that large shocks to the upstream sector deliver faster increases in the aggregate frequency, relative to equally sized shocks to the downstream sector. This is another way in which networks amplify cascades: shocks to the upstream sector affect the marginal cost, the optimal reset price, and hence the price gaps, of the downstream sector. As a result, shocks to the upstream sector trigger extensive margin price adjustment decisions both in the upstream *and* in the downstream sector. The opposite, however, is not true: shocks to the downstream sector only affect price gaps in the downstream sector itself and do not affect price adjustment decisions in the upstream sector.

This simple example illustrates an important point: when it comes to the effect of a sector-specific shock on aggregates, the position of the shocked sector in the network can matter over and above its size. In particular, here shocks to the upstream sector have a stronger effect on aggregate adjustment frequency, even though it is as large as the downstream sector in steady state. This runs contrary to a number of established network-irrelevance results, where the presence of networks make no difference over and above its effect on equilibrium size (Hulten, 1978; Baqaee and Farhi, 2020).

***Example 3*: N-sector vertical chain economy**

With our third example, we would like to illustrate how the interaction between the network position and pricing cascades extends beyond the two-sector arrangement. In particular, as depicted in the top panel of Figure 2(c), we consider an N-sector vertical chain economy. In such a setup, Sector 1 is the most upstream sector, which uses labor to produce a good that is supplied as an intermediate to Sector 2, which then supplies intermediates to Sector 3 and so on. Sector N is the least upstream sector, as it sells everything it produces as a final good to households. As before, all the N sectors have the same steady-state pricing moments and are equally big in the sense of having identical (cost-based) Domar weights. The only relevant dimension of heterogeneity is their position in the network.

In the bottom panel of Figure 2(c), we set $N = 10$ and plot the aggregate frequency response to large (-20%) sector-specific productivity shock to each sector. One can see that the shock to the most upstream Sector 1 delivers the largest increase in aggregate frequency. Moreover, the aggregate frequency response falls monotonically as we move down the supply chain and consider increasingly less upstream sectors. As before, this represents the interaction of networks with pricing cascades:

shocks to more upstream sectors affect, directly or indirectly, marginal costs and hence price gaps in a larger number of sectors, thus triggering a bigger increase in aggregate adjustment frequency.

4 Full model with Euro Area data

We now move to the quantitative analysis of our full dynamic model. In this section we outline the strategy to bring our model to the Euro Area data. In particular, we discipline the structural parameters of the model in order to make it consistent with the Euro Area economy disaggregated to 38 sectors. The household preferences and firms' production function parameters are estimated to match the observed consumption and input-output shares in the World Input-Output Tables. As for the sector-specific menu costs and variances of idiosyncratic shocks, those are estimated to fully match the observed sectoral frequencies and standard deviations of price changes in the PRISMA dataset for the Euro Area.

4.1 Parameterization and Calibration

We discipline the structural parameters of our model to the Euro Area data at monthly frequency. Table 1 summarizes our calibration.

For the aggregate parameters, the households' discount factor is set to $\beta = 0.96^{1/12}$ as in Golosov and Lucas (2007). The within-sector elasticity of substitution across varieties is $\epsilon = 3$ as in Midrigan (2011). We assume that aggregate money supply follows a random walk with drift:

$$\log M_t = \overline{\pi} + \log M_{t-1} + \varepsilon_t^M, \tag{30}$$

where $\overline{\pi}$ is the trend growth rate for money supply, which is also the equilibrium level of trend inflation; ε_t^M is an i.i.d. mean zero money growth innovation. The steady-state money growth rate is $\pi = 2\%$ per year, in line with the inflation target of the European Central Bank (ECB). As for the sectoral total factor productivities, we assume those to follow an AR(1) process:

$$\log A_{i,t} = \rho \log A_{i,t-1} + \varepsilon_{i,t}^A, \tag{31}$$

where $\rho \in (0,1)$ is the persistence parameter and $\varepsilon_{i,t}^A$ is an i.i.d. mean zero sector-specific productivity innovation. We set the persistence of TFP processes equal to $\rho = 0.9$.

We calibrate our economy to 38 production sectors of the Euro Area economy, following the classification in the World Input-Output Database (WIOD). The final consumption shares $\{\overline{\omega}_i^C\}_{i=1}^N$ and the input-output cost shares $\{\overline{\omega}_{ik}\}_{i,k=1}^N$ are taken from the 2014 input-output tables for the Euro Area based on WIOD.[5] Regarding the sectoral cost shares of labor $\{\overline{\alpha}_i\}_{i=1}^N$, they are taken from the 2014 National Income Accounts for the Euro Area, published by the EU KLEMS database.[6] In order to capture the possibility that wages are also sticky, we introduce an auxiliary labor union sector. In particular, we assume that the labor union sector is the only one that directly purchases labor from households and then sells it to the rest of the sectors as an intermediate input. In general, we work with $N = 39$ sectors: 38 production sectors and the auxiliary labor union sector.

Unlike in Section 3.1 above, we do not allow time variation in the sectoral menu costs. Instead, we consider the more conventional fixed menu cost setup (Golosov and Lucas, 2007), allowing the menu costs to vary in the cross section only:

Assumption 6′ (Fixed menu costs)**.** *The sector-specific menu cost follows the following process:* $\kappa_{i,t} = \overline{\kappa}_i$, *where* $\overline{\kappa}_i$ *is a sector-specific constant.*

This leaves us with two parameters per sector to estimate: the menu cost $\overline{\kappa}_i$, and the standard deviation of firm-level shocks σ_i. In line with evidence in Gautier et al. (2023), we assume that the sectors "Coke and Petroleum Products" and "Mining and Quarrying" have fully flexible prices at monthly frequency. We calibrate the price setting parameters in the labor union sector to match the frequency and standard deviation of nominal wage changes in Costain et al. (2022). For the remaining 36 sectors, we estimate the parameters $\{\overline{\kappa}_i\}_{i=1}^N$ and $\{\sigma_i\}_{i=1}^N$ to match the frequency and standard deviation of price changes in each sector in the Euro area, taken from Gautier et al. (2024), in steady state.

We also parameterize two auxiliary economies, for the purpose of benchmarking them against our baseline setup. First, we estimate the firm-level pricing parameters in a counterfactual economy without input-output linkages, for the same set of sector-specific frequencies and standard

[5] We make use of the EMuSe Calibration Toolkit developed by Hinterlang et al. (2023), which constructs the Euro Area input-output table by combining accounts of individual countries in the WIOD.

[6] The database can be accessed at `https://economy-finance.ec.europa.eu/economic-research-and-databases/economic-databases/eu-klems-capital-labour-energy-materials-and-service_en`.

Table 1: Parameter values (Euro Area, monthly)

		Aggregate parameters	
β	$0.96^{1/12}$	Discount factor (monthly)	Golosov and Lucas (2007)
ϵ	3	Goods elasticity of substitution	Midrigan (2011)
$\overline{\pi}$	0.02/12	Trend inflation (monthly)	ECB target
ρ	0.90	Persistence of the TFP shock	Half-life of seven months
		Sectoral parameters	
N	39	Number of sectors	Data from Gautier et al. (2024)
$\{\overline{\omega}_i^C\}_{i=1}^N$		Sector consumption weights	World IO Tables
$\{\overline{\omega}_{ik}\}_{i,k=1}^N$		Sector input-output matrix	World IO Tables
$\{\overline{\alpha}_i\}_{i=1}^N$		Sector labor weights	World IO Tables
		Firm-level pricing parameters	
$\{\overline{\kappa}_i\}_{i=1}^N$		Menu costs	Estimated to fit frequency, std dev.
$\{\sigma_i\}_{i=1}^N$		Std. dev. of firm-level shocks	of Δp from Gautier et al. (2024)

deviations of price changes in steady state. Such an economy features no linkages across the 38 production sectors, which are only linked to the labor union sector instead ($\overline{\omega}_{i,LU} = 1, \forall i \neq LU$). Second, we consider an economy with input-output linkages, but featuring time-dependent price setting as in Calvo (1983). The latter setup corresponds to having constant sector-specific pricing hazards ($\eta_i(p) = \overline{\eta}_i, \forall i$) and zero menu costs ($\overline{\kappa}_i = 0, \forall i$). We therefore estimate sector-specific constant hazards and variances of idiosyncratic shocks to match the same sectoral frequencies and standard deviations of price changes as in the steady state of our baseline setup.

4.2 Sectoral characteristics

In order to better understand the cross-sectional properties of the sectors we consider in our quantitative setup, we introduce two different measures of sectoral *centrality*. First, in order to capture the full degree to which a sector is important as a buyer of intermediate inputs from the rest of the economy, we use the following *customer centrality* metric:

$$\text{Customer Centrality}_i \quad \equiv \quad \sum_{j=1}^{N} (I - \overline{\Omega})_{ij}^{-1} - 1 \tag{32}$$

where $[\overline{\Omega}]_{ij} = \overline{\omega}_{ij}$ is the matrix of input-output cost shares. Intuitively, the customer centrality measure captures the total reliance of a sector on intermediate inputs, both direct and indirect. Naturally, if a sector only uses labor in production, its customer-centrality measure collapses to zero. Table B.1 in the Appendix reports the customer centrality measure for each of the 38 production sectors. The two sectors with the largest customer centrality are "Coke and petroleum products" (4.35) and "Chemicals and chemical products" (4.25), followed by "Paper and paper products" (3.97) and "Food and beverages" (3.94), while the smallest customer centrality is in "Education" (1.55).

Second, in order to capture the degree to which a sector is important as a provider of intermediate inputs to the rest of the economy, we introduce the following *supplier centrality* metric:

$$\text{Supplier Centrality}_i \quad \equiv \quad \sum_{j=1}^{N} (I - \overline{\Omega}^T)^{-1}_{ij} - 1 \tag{33}$$

The supplier centrality measure captures the total importance of a sector as a seller, either directly or indirectly, of intermediate inputs to the rest of the economy. The value of supplier centrality for each of the 38 production sectors is reported in Table B.1. The distribution of supplier centrality features a heavy right tail, with three sectors having a disproportionally larger measure than the rest: those are "Administration and support" (7.59), "Legal, accounting, management" (6.51) and "Chemicals and chemical products" (6.18). The sector with the smallest supplier centrality is "Fishing and aquaculture" (0.11).

5 Quantitative results: monetary shocks

For our first set of quantitative results, we present the general equilibrium dynamics of our economy following monetary shocks of different sizes. First, we show that the aggregate repricing frequency response to large monetary shocks is substantially attenuated by the presence of networks, so that the effect of cascades dampening is quantitatively sizable . As a result, the economy with networks features much stronger monetary non-neutrality, which manifests in a substantial flattening of the fully non-linear Phillips Curve. Second, we study sectoral frequency and price responses, and show that, *ceteris paribus*, sectors with a larger customer centrality exhibit larger movements in the fraction of adjusting firms and feature more size-dependence in their sectoral price responses.

5.1 Aggregate dynamics

Figure 3(a) shows the scaled (per % shock) responses of aggregate CPI inflation, aggregate GDP, as well as the unscaled fraction of adjusting firms to monetary shocks of two different magnitudes: 1% and 10%. Two key features are apparent. First, the scaled response of inflation increases in the size of the monetary shock, which represents a strong *size effect*. As can be seen in the frequency panel, this happens as the fraction of adjusting firms increases rapidly with larger shocks, reaching almost 30% for the 10% monetary shock.

Second, as shown in Figure 3(b), the contribution of production networks to the magnitudes of responses differs markedly between shocks of different sizes. For the small 1% shock, networks dampen the response of inflation and, as a result, amplify the response of aggregate GDP. This is the effect of production networks known from the prior literature, which employs linearized models with time-dependent pricing: input-output linkages create pricing complementarities, dampening inflation and amplifying the consumption response. At the same time, for the large 10% shock, the amplification of the aggregate GDP response due to networks is much greater. Importantly, this is because the 10% monetary shock delivers a markedly smaller increase in the repricing frequency relative to the economy without networks, which is the cascades *dampening* effect introduced earlier.

In Figure 4(a), we further investigate the interaction between networks and the response of repricing frequency by looking at a wide range of shock sizes and signs. One can see that networks consistently dampen the response of aggregate repricing frequency to monetary shocks of all sizes that we consider. For example, following a 10% monetary expansion, the aggregate frequency rises close to 45% in the multi-sector economy without networks, but increases only to 27% in an otherwise identical economy with input-output linkages.

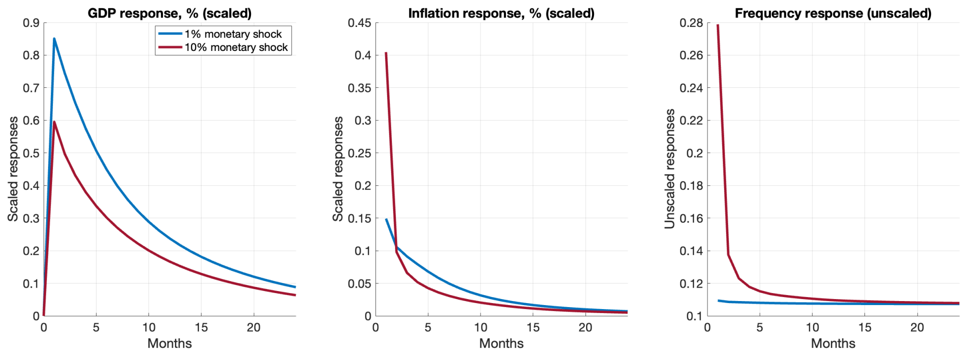
In order to further quantify the contribution of shock *dampening* to aggregate inflation responses, we decompose the effect of the money shock on inflation into three channels, following Blanco et al. (2024a) and Costain and Nakov (2011). Note that, up to a first-order approximation, inflation in the absence of the shock is equal to

$$\pi = \int \tilde{p}\eta(\tilde{p})\mathrm{d}g(\tilde{p}) \tag{34}$$

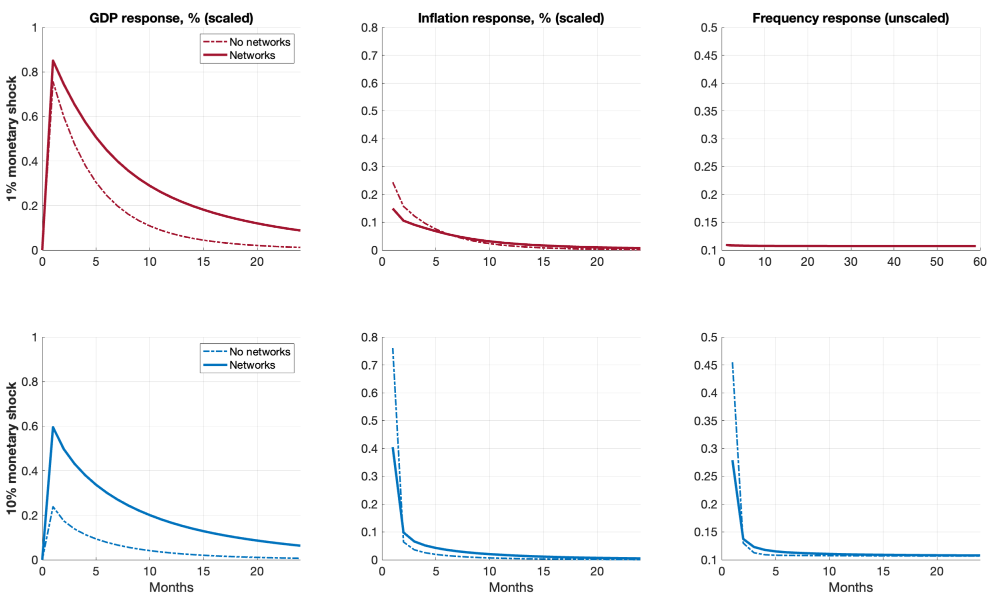
where $\tilde{p}$ is the desired log price change, $\eta(\tilde{p})$ is the adjustment hazard, and $g(\tilde{p})$ is the ergodic

Figure 3: **Effect of networks on aggregate responses**

(a) Responses of aggregates to monetary shocks



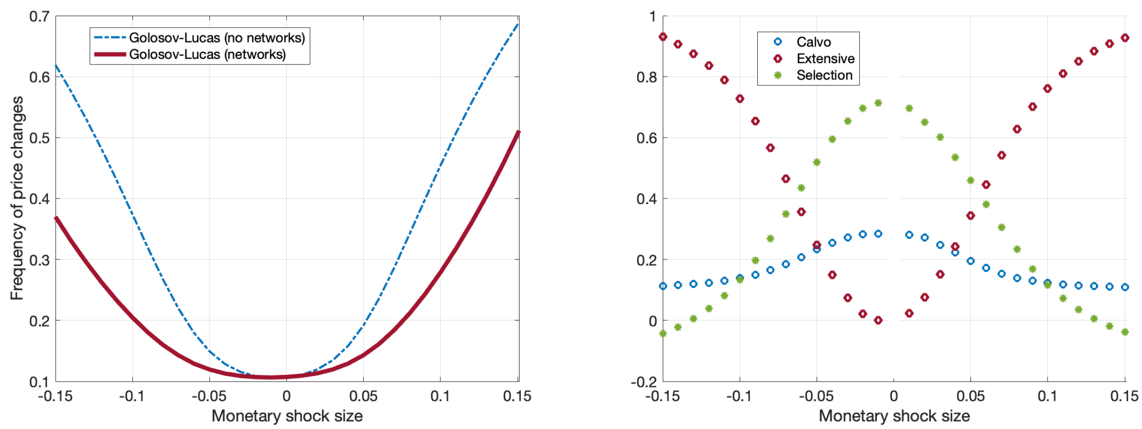
(b) Network contribution to aggregates' responses



***Notes**: the figure shows the responses of aggregate GDP, inflation and frequency of adjustment in response to monetary shocks of different sizes.*

Figure 4: **Aggregate frequency of adjustment after monetary shocks**

(a) Aggregate frequency response (b) Contribution of the extensive margin



***Notes**: the figure shows the impact responses of aggregate frequency of adjustment to monetary shocks of different sizes, with and without networks.*

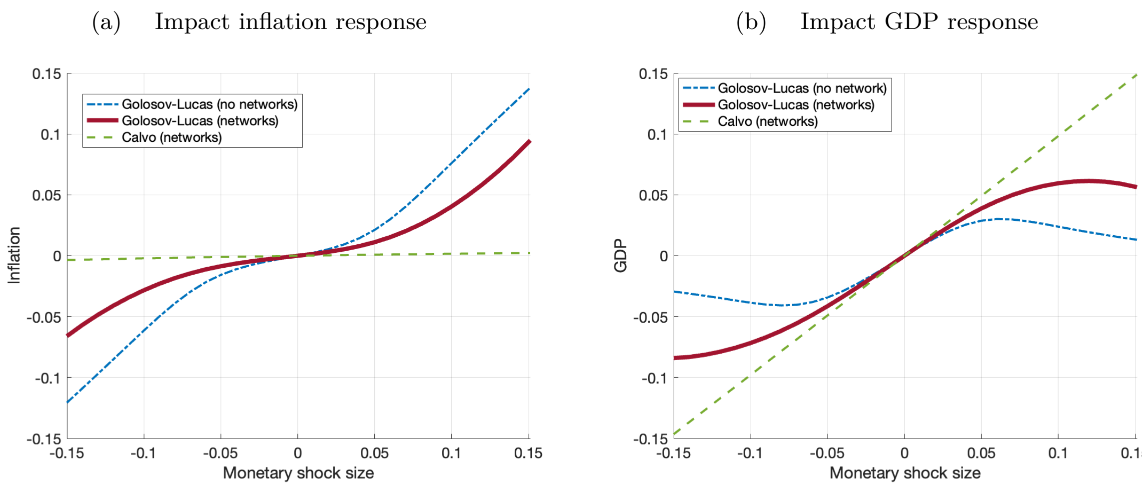
distribution of desired price changes across firms and sectors. The money shock increases all firms' desired price changes to $\tilde{p}+\alpha$, where $\alpha = p_1^* - p^* + \Delta m$ and where p_1^* is the average across sectors of the log reset price in the first period after the money shock and p^* is the average across sectors of the log reset price in the absence of the shock. The money shock changes the inflation rate to $\pi_1 = \int(\tilde{p}+\alpha)\eta_1(\tilde{p})\mathrm{d}g(\tilde{p})$ where $\eta_1(\tilde{p})$ is the new adjustment hazard after the shock. Following Blanco et al. (2024a), the change in inflation can then be decomposed into

$$\Delta\pi = \underbrace{\alpha\int\eta(\tilde{p})\mathrm{d}g(\tilde{p})}_{\text{Calvo}} + \underbrace{\alpha\int(\eta_1(\tilde{p})-\eta(\tilde{p}))\mathrm{d}g(\tilde{p})}_{\text{extensive}} + \underbrace{\int\tilde{p}(\eta_1(\tilde{p})-\eta(\tilde{p}))\mathrm{d}g(\tilde{p})}_{\text{selection}}. \tag{35}$$

In Figure 4b we show the resulting decomposition of the differential inflation responses in our model with networks compared to that without networks. The difference between the network and no-network cases is explained mainly by the selection effect for smaller shocks, and by the extensive margin component for shocks greater than 5 percent in absolute value. Therefore, for large shocks, most of the network contribution to the slowing down of the inflation response works through the *dampening* in pricing.

The dampening effect of networks on the repricing frequency has important implications for the responses of CPI inflation and aggregate consumption to large monetary interventions. In Figure

Figure 5: **Inflation and GDP responses to monetary shocks**

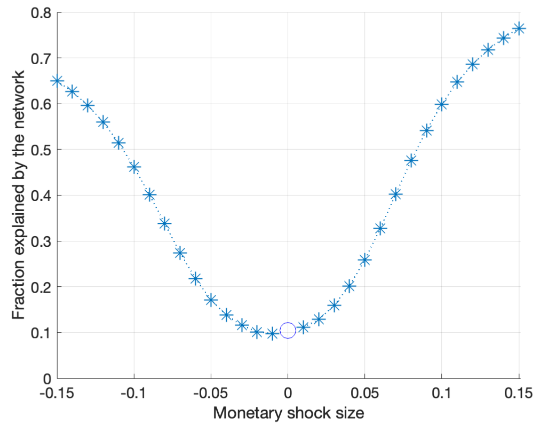


***Notes**: the figure shows the impact responses of inflation and GDP to monetary shocks of different sizes, in different configurations of the model.*

5 (a), we show that as the size of the monetary shocks increases, inflation in our baseline economy rises in a non-linear fashion: a 5% shock delivers 2% inflation on impact, whereas tripling the shock to 15% delivers a five-fold increase of inflation to 10%. At the same time, the figure also shows that an otherwise identical economy without networks features inflation rising even faster with larger monetary shocks. The fact that inflation rises relatively more slowly in the economy with networks reflects mainly the slower response of the fraction of adjusters, as documented in Figure 4. In order to quantify the importance of nonlinearity and state-dependent pricing, in Figure 5 (a) we also consider a version of our model with time-dependent pricing (Calvo, 1983), calibrated to match the sectoral frequencies of adjustment in steady state. Under such a time-dependent setup, even when solved fully non-linearly, inflation is rising more slowly as the monetary shock gets larger. The latter reflects the contribution of both the selection effect (for smaller shocks) and the extensive margin effect (for larger shocks) in delivering faster pricing increases in the state-dependent pricing model.

Figure 5 (b) shows that in the baseline economy with networks, the aggregate consumption response is hump-shaped in the size of monetary shocks and is maximized following a 12% monetary expansion, delivering an increase of almost 6%. At the same time, the equivalent economy without networks has its consumption response maximized following a 5% monetary shock, corresponding

Figure 6: **Output amplification**

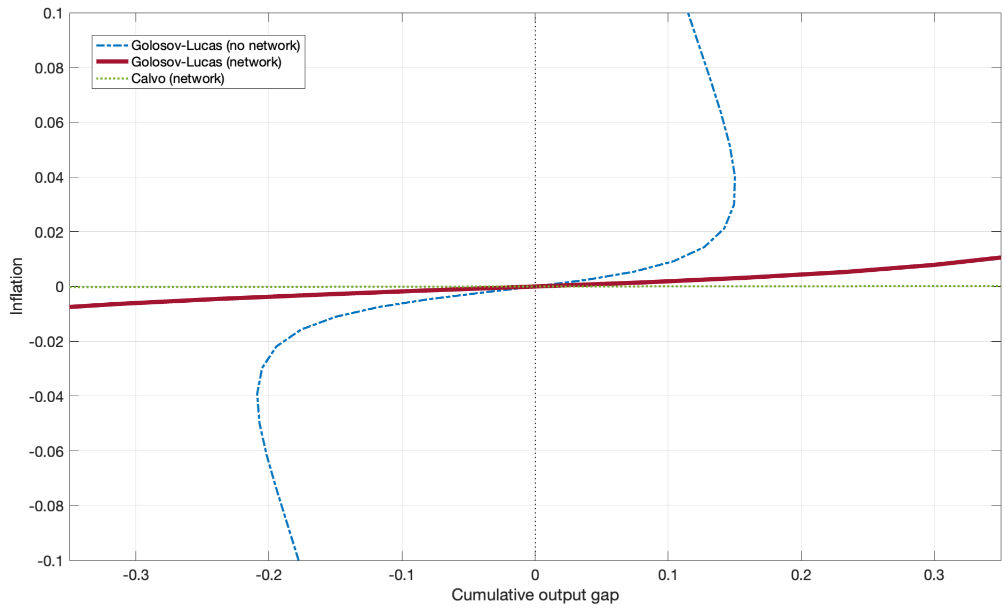


Notes*: the figure shows the contribution of networks to the impact responses of GDP to monetary shocks of different sizes.*

to a smaller increase of just over 3%. The higher maximal response of consumption under networks, as well as the fact that it occurs following a larger monetary shock, reflect the slower response of the fraction of adjusters, as documented in Figure 4. Figure 5 (b) (a) also shows the responses in the alternative setup with time-dependent Calvo (1983) pricing. With time-dependent pricing, one can see that even for very large shocks and a non-linear solution, the time-dependent setup has aggregate consumption rise quasi-linearly in the size of the monetary shock. Moreover, the non-linear time-dependent pricing results deviate substantially from the non-linear state-dependent solutions.

In order to formally measure the contribution of networks to the output response, in Figure 6 we construct, for each size of the monetary shock, the difference between the output response with and without networks, as a fraction of the former. One can see that for small monetary shocks, the contribution is in the neighborhood of 10-20%. Such magnitudes are consistent with prior estimates of network contributions in linearized time-dependent setups: for example, Ghassibe (2021) estimates the contribution to be just below 30% in such a setup. At the same time, one can also appreciate that as the size of the shock increases, the contribution of the network increases dramatically, reaching almost 80% for a 15% monetary expansion. This reflects the fact that for such large shocks, the fraction of adjusters rises much less in the economy with networks, delivering a much larger aggregate pass-through to inflation and hence a larger consumption response.

Figure 7: **Phillips curves**



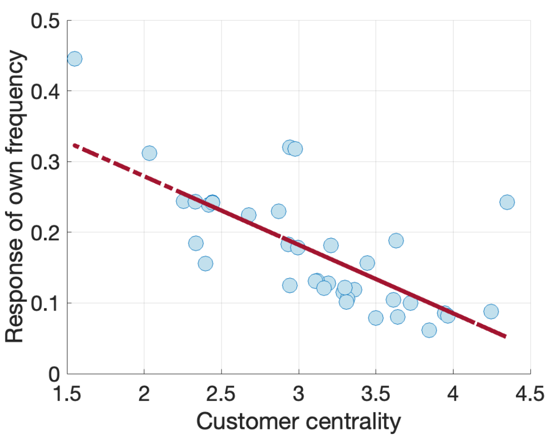
Notes: *the figure shows the fully non-linear Phillips curves in the various configurations of the model.*

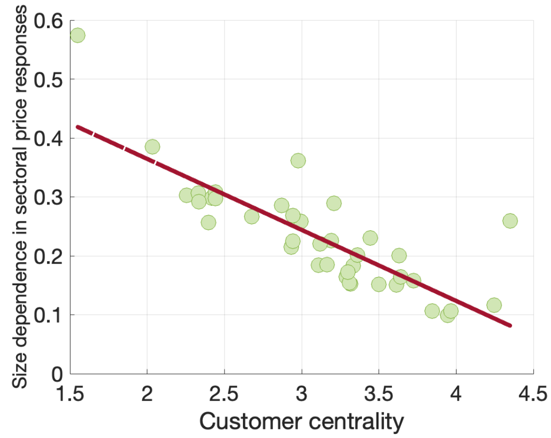
Figure 7 illustrates the trade-off between output stimulus and inflation under monetary interventions of different sizes. In particular, the figure traces a non-linear "Phillips curve" in the cumulative output gap–CPI inflation space, under different model configurations. In the network-based baseline economy, a cumulative output stimulus up to 5% or so can be achieved with little inflationary response, reflecting a locally flat Phillips curve. However, in a counterfactual economy without networks, the Phillips curve is steeper for small shocks and low output gap values. This suggests "flattening" of the Phillips curve due to networks, documented in previous studies under time-dependent pricing. Moreover, once the shocks are sufficiently large, the Phillips curve without networks becomes backward bending, with a maximum possible cumulative output stimulus of around 15%. This happens because, under very large shocks, the fraction of adjusters increases much faster in the economy without networks, as documented in Figure 4.

5.2 Disaggregated dynamics

Having analyzed the behavior of macroeconomic aggregates, we now move to studying sector-level behavior following monetary shocks of different sizes.

Figure 8: **Sectoral responses vs total intermediates exposure**





***Notes**: the figures plots the responses of frequencies and sectoral price indices to a large monetary shock.*

To better understand the role production networks play in dampening frequency responses, in Figure 8 (a) we plot a linear relationship between the sectoral frequency responses and a measure of *customer centrality*, given by the sum of rows of the Leontief inverse matrix, representing the total (direct and indirect) exposure of a sector to purchases of intermediate inputs from other sectors. As can be seen, a higher centrality of customers is associated with a smaller increase in frequency after the 10% monetary shock.

Similarly, we also investigate the association between customer centrality and the degree of size dependence in sectoral price responses to small vs. large monetary shocks. In Figure 8 (b) we plot a measure of size dependence in sectoral price responses, given by the difference in *normalized* responses to 10% and 0.1% monetary shocks, against the measure of customer centrality. Consistently with our mechanism, a higher total exposure to intermediate inputs lowers size dependence in the sectoral price response.

6 Quantitative results: TFP shocks

For our second set of quantitative results, we turn to the general equilibrium dynamics following aggregate and sector-specific total factor productivity (TFP shocks). First, we show that following large aggregate TFP shocks, the economy with networks features much stronger response of the repricing frequency, implying that the cascades amplification channel is indeed quantitatively important. The amplification of cascades in turn generates much stronger response of aggregate for a given shock, relative to the otherwise identical economy with time-dependent pricing. We also show that sectoral with a larger customer centrality exhibit stronger responses of the fraction of adjusters and more size dependence in sectoral price responses following large aggregate TFP shocks. Second, our results suggest that TFP shocks specific to sectoral with a large supplier centrality lead to more sizeable movements in the aggregate repricing frequency.

6.1 Aggregate TFP shocks

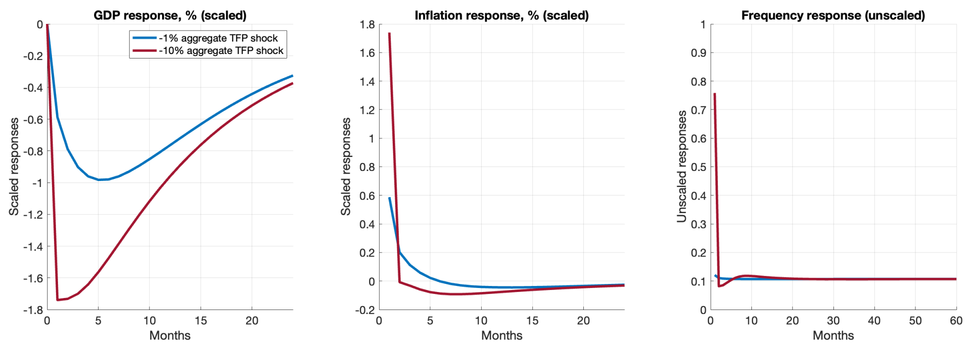
6.1.1 Aggregate dynamics

In Figure 9 (a), we report normalized responses of aggregate GDP and CPI inflation, as well as the aggregate fraction of adjusters in response to two negative aggregate TFP shocks: -1% and -10%. Just as with monetary shocks in the previous section, there is substantial *size dependence*: for the -1% shock the normalized response of GDP is -1%, whereas it is just under -2% for the -10% shock. At the same time, the normalized response of CPI inflation increases in the magnitude of the aggregate TFP shock, implying that the aggregate price changes rise more than proportionally in the size of the innovation. Quantitatively, the -1% shock generates a normalized impact response of CPI inflation of 0.6%, whereas the -10% corresponds to a normalized response of almost 1.7% on impact. Key to the observed size dependence is the endogenous response of the fraction of adjusters: for the -1% shock it remains unchanged, whereas the larger -10% shock brings the fraction of adjusters to almost 80%.

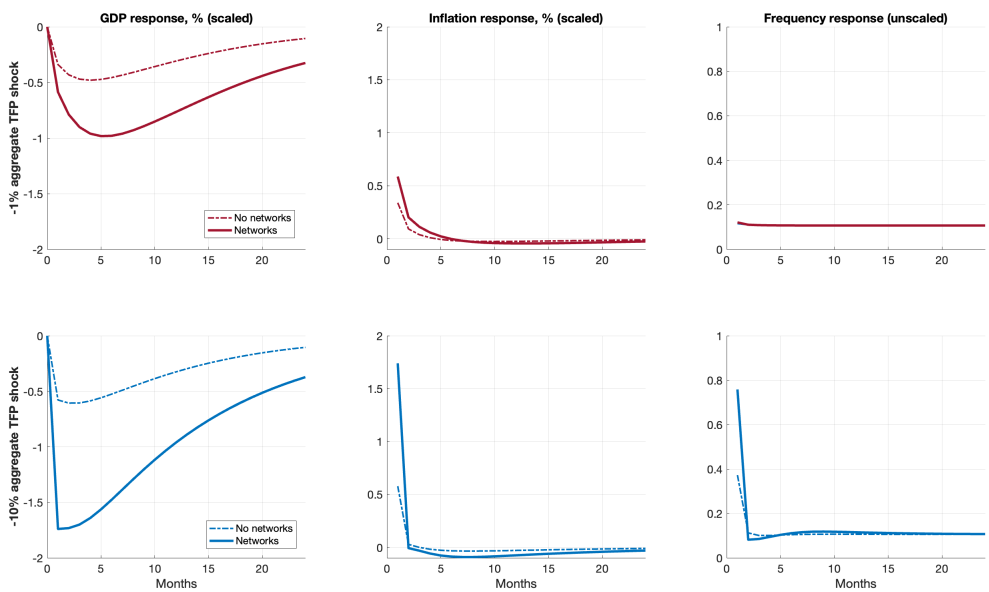
In order to understand the contribution of networks to the observed size dependence, in Figure 9 (b) we additionally document the responses to the same aggregate TFP in an otherwise identical economy without networks. For the -1% shock, networks amplify the response of aggregate GDP by a factor 2, while the normalized response of CPI inflation is nearly 0.3% without networks, compared to 0.6% under networks. Significantly, for the larger shocks of -10%, the network amplification of both aggregate consumption and CPI inflation is greater than under the small -1% shock. When it comes to the response of inflation, this is the opposite of what we observed under monetary shocks, where the amplification of inflation response was weakening as shocks became larger. To understand the difference, it is instructive to look at the response of adjustment frequencies. One can see that

Figure 9: **Responses to aggregate TFP shocks**

(a) Responses of aggregates to aggregate TFP shocks shocks



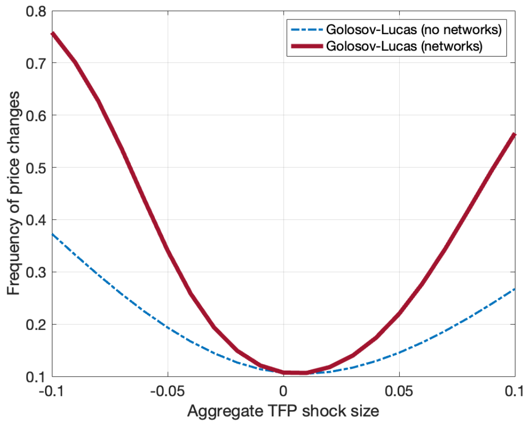
(b) Network contribution to aggregates' responses



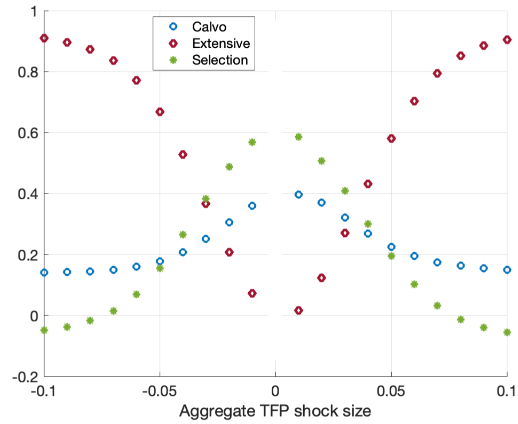
Notes: *the figure shows the responses of aggregate GDP, inflation and frequency of adjustment in response to aggregate TFP shocks of different sizes.*

Figure 10: **Aggregate frequency responses to aggregate TFP shocks**

(a) Aggregate frequency response



(b) Contribution of the extensive margin



***Notes**: the figure shows the impact responses of aggregate frequency of adjustment to aggregate TFP shocks of different sizes, with and without networks.*

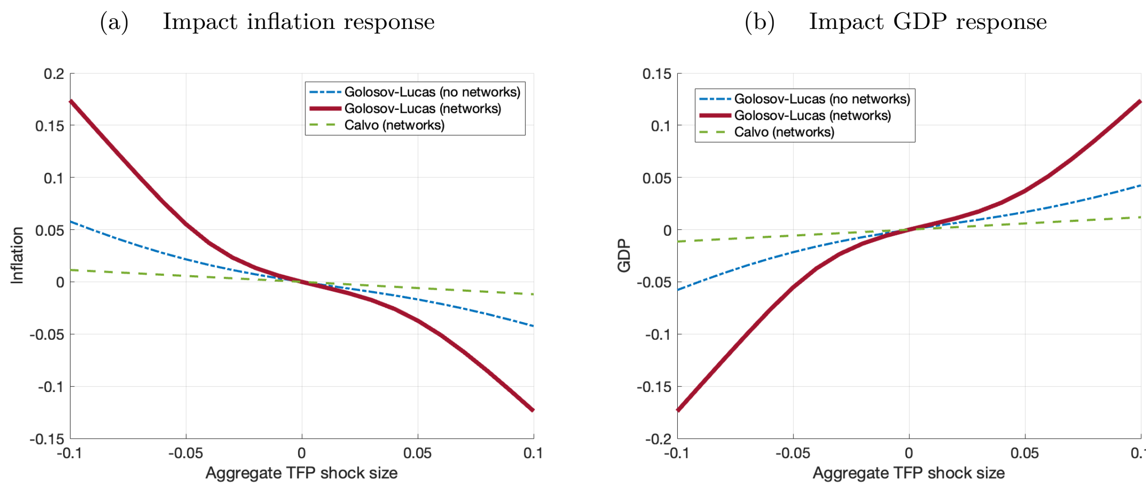
for the -10% shock, the fraction of adjusters increases substantially more in the economy with networks. This is the exact opposite of what is observed under monetary shocks, in which networks dampen the response of adjustment frequencies, and which represents the *cascades* effect.

In Figure 10(a), we further investigate the interaction between the aggregate repricing frequency and the size of the aggregate TFP shock. Contrary to what we established for monetary shocks, networks consistently and substantially amplify the response of the aggregate fraction of adjusters to aggregate TFP shocks. For example, following a -10% aggregate TFP shock, the economy with networks predicts a rise in the fraction of adjusters to 75%, while in an otherwise identical model without networks the corresponding increase in the aggregate adjustment frequency is up to just below 40%.

In Figure 10(b) we show the decomposition of the differential inflation responses in our model with networks compared to the one without networks. The difference between the network and no-network cases is explained mainly by the selection effect for smaller shocks, and by the frequency component for aggregate TFP shocks greater than 3 percent in absolute value.

The fact that networks amplify the response of adjustment frequency to aggregate TFP shocks also has notable implications for the dynamics of aggregate CPI inflation and GDP. In Figure 11 (a), we plot the impact response of aggregate CPI inflation to aggregate TFP shocks of different

Figure 11: **Inflation and output responses to aggregate TFP shocks**



***Notes**: the figure shows the responses of aggregate inflation and GDP to large aggregate TFP shocks under different configurations of the model.*

signs and sizes. The inflation response rises much faster in the economy with networks relative to the no-network benchmark, being almost 3.5 times higher after a -10% shock. We also report the inflation responses in an economy with networks, but with time-dependent (Calvo, 1983) pricing, matching the same sectoral frequencies of adjustment in steady state. One can see that the economy with time-dependent pricing predicts much smaller inflation responses.

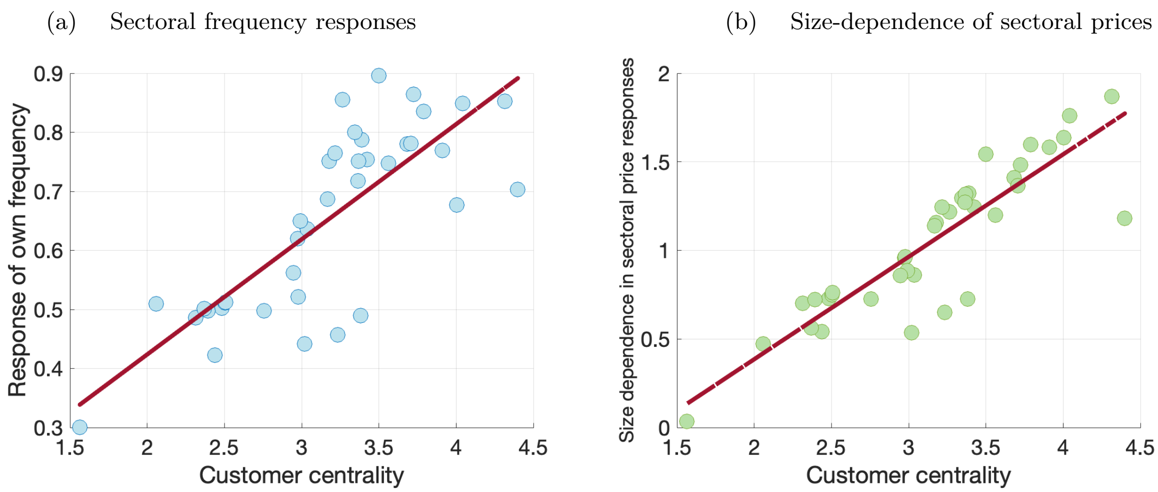
6.1.2 Disaggregated dynamics

We now turn to analyzing the responses of individual sectors to aggregate TFP shocks of different sizes.

In Figure 12(a), we plot a relationship between the sectoral frequency responses to the -10% aggregate TFP shock and the measure of customer centrality. There is a clear positive relationship between the two: *ceteris paribus*, increasing a sector's total exposure to intermediate inputs increases the fraction of firms in that sector that choose to adjust following the large aggregate TFP shock.

The established relationship between frequency response and exposure to intermediate inputs also has implications for the degree of size dependence in sectoral price dynamics. To see that, in Figure 12(b) we plot the linear relationship between a measure of sectoral price size dependence,

Figure 12: **Sectoral responses to aggregate TFP shock vs total intermediates exposure**



***Notes**: the figure shows the responses of sectoral frequency and sectoral price indices to large aggregate TFP shocks.*

given by the difference between normalized responses to -10% and -0.1% aggregate TFP shocks, and the sectoral customer centrality. The estimated relationship is positive, implying that a higher total exposure to intermediate inputs is associated with a greater degree of size dependence in the sectoral price response to a large aggregate TFP shock.

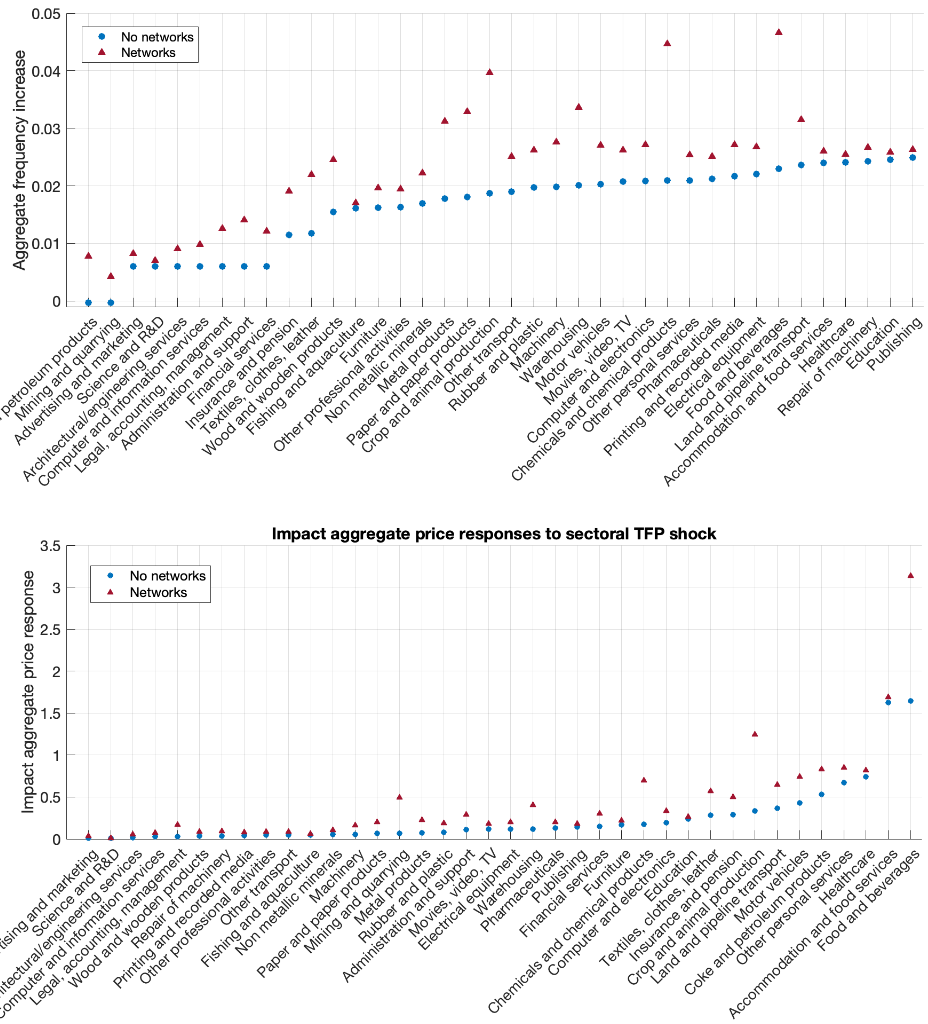
6.2 Sectoral TFP shocks

We now turn to the study of the transmission of sector-specific TFP shocks. We focus on the transmission of large contractionary sectoral shocks, modeled as a -20% reduction in sector-specific TFP.

In Figure 13(a), we show the responses of the aggregate fraction of adjusting firms to sector-specific TFP shocks. First, for all sectors, networks amplify the aggregate frequency response to sector-specific TFP shock. Second, for the majority of sectors, the effect of their own shock on aggregate frequency is relatively modest and not much bigger than in the otherwise identical economy without networks. Third, for a number of sectors, such as “Food and Beverages”, “Chemicals and Chemical Products” and “Warehousing”, networks deliver a substantial amplification of their own TFP shocks on aggregate adjustment frequency.

We also study the response of aggregate CPI inflation to sectoral TFP shocks. Specifically,

Figure 13: **Aggregate responses to sectoral TFP shocks**

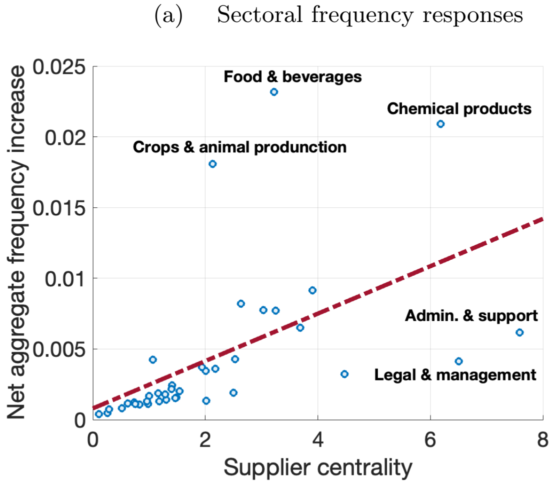


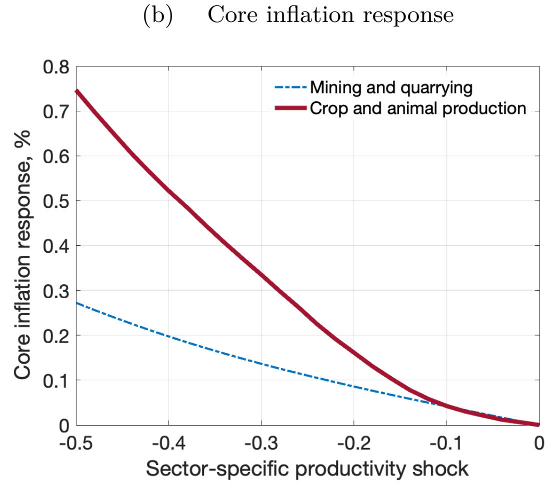
Notes*: the figure shows the responses of sectoral frequency and sectoral price indices to large aggregate TFP shocks.*

in Figure 13(b) we depict normalized impact responses of aggregate CPI inflation to large (-20%) negative sectoral TFP shocks. First, networks amplify the aggregate CPI inflation responses for all sectoral TFP shocks. Second, just as with aggregate frequencies, for the majority sectors, the network amplification is relatively modest. Third, some sectors are an exception: "Food and Beverages", "Mining and Quarrying" and "Chemicals and Chemical Products" have their own TFP shocks affect the aggregate CPI inflation response substantially.

In Figure 14, we investigate the relationship between a sector's *supplier centrality* and the aggregate consequences of that sector's TFP shocks. In Figure 14(a), one appreciates a very strong

Figure 14: **Aggregate responses to sectoral TFP shock vs sector centrality**





***Notes**: the figure shows the responses of sectoral frequency and sectoral price indices to large aggregate TFP shocks.*

positive relationship between the effect of a sectoral TFP shock on the aggregate frequency of adjustment and the centrality measure of that sector.

7 Extensions and robustness checks

In this section present three extensions to our baseline model. First, we consider a version of our economy in the cashless limit, where the central bank conducts monetary policy by setting the nominal interest rate, which endogenously responds to aggregate inflation and output according to a Taylor rule. Second, we relax the assumption of fixed menu costs, and consider a version of our economy with random free adjustment opportunities. Third, we extend our baseline model with a constant elasticity of substitution (CES) aggregation across sectors, which allows for the consumption and input-output shares to vary endogenously along the intensive margin.

7.1 Endogenous monetary policy

In our baseline results, the central bank conducts policy by setting an exogenous path of money supply. We now consider an extension that adds realism to the monetary policy conduct. In particular, we use the cashless limit setup of Woodford (2004) and Galí (2015), where the central bank conducts policy by setting the level of the nominal interest rate, which also responds endogenously to movements in macroeconomic aggregates. In particular, we assume that the nominal interest rate i follows a Taylor-type rule:

$$\log(1+i) = \log(\overline{\Pi}/\beta) + \phi_\pi \log(\Pi_t^C/\overline{\Pi}) + \varepsilon_t^i, \tag{36}$$

where $\overline{\Pi}$ is the steady-state level of CPI inflation, $\phi_\pi > 0$ pins down the strength of the central bank reaction to inflation deviations from target and ε_t^i is the monetary policy shock.

In Appendix D.2 we detail the full alternative version of our model in the cashless limit with the Taylor-type rule for the nominal interest rate. Here we present an overview of the key results. In Figure D.3 we report the responses of aggregate frequency and GDP to monetary shocks of different sizes. Just as in the cash economy, networks dampen cascades: the response of aggregate frequency is larger in the economy without networks. As before, this leads to stronger monetary non-neutrality. As for supply shocks, in Figure D.4 we report the responses of aggregate frequency and CPI inflation to aggregate TFP shocks of different sizes. Here we once again find that networks amplify cascades, since the frequency of adjustment responses stronger in the economy with networks, adding non-linearity to the inflation response.

7.2 Random menu costs

In our baseline results, we work under the assumption that nominal price re-setting is subject to a fixed sector-specific menu cost as in Golosov and Lucas (2007). In order to illustrate that our novel channel of interaction between networks and pricing cascades is not limited to the fixed menu cost setup, as an extension, we consider a random menu cost setup. More specifically, we use the CalvoPlus setup of Nakamura and Steinsson (2010), which assumes that each period a randomly selected fraction of firms within each sector draws a menu cost of zero, whereas the complementary fraction is still subject to the fixed menu cost.

Formally, the CalvoPlus setup corresponds to the following functional form of the probability of adjustment function $\eta_{i.t}(.)$:

Assumption 4′ (CalvoPlus pricing)**.** *Consider a firm in sector i with the quality adjusted log* *relative price p at time t. Then the probability that this firm adjusts its nominal price is given by:*

$$\eta_{i,t}(p) = \ell_i + (1 - \ell_i) \times \mathbf{1}(L_{i,t}(p) > 0) \tag{37}$$

where ℓ_i is the sectoral probability of drawing a zero menu cost, $\mathbf{1}(\cdot)$ is the indicator function, and

$$L_{i,t}(p) = \max_{p'} V_{i,t}(p') - V_{i,t}(p) - \overline{\kappa}_i \tag{38}$$

is the gain from adjustment (or loss from inaction), net of the menu cost.

Crucially, as the non-zero menu cost tends to infinity ($\overline{\kappa}_i \to \infty$), the pricing problem collapses to the time-dependent model of Calvo (1983), as only the randomly selected fraction ℓ_i in each sector gets to adjust. At the same time, setting the probability of drawing a zero menu cost to zero ($\ell_i = 0$) collapses the pricing problem in that sector to the fixed menu cost setup of Golosov and Lucas (2007).

In order to quantitatively discipline the probabilities of free adjustment, we estimate them so that, in steady state around 75% of all price adjustments are free in each sector, following Nakamura and Steinsson (2010) and Blanco et al. (2024b). As before, the non-zero menu costs and standard deviation of idiosyncratic shocks are estimated to jointly match the sector-specific frequencies and standard deviations of price changes in the Euro Area.

In Figure D.5 we study the responses of aggregate repricing frequency and GDP to monetary shocks of different sizes under CalvoPlus pricing. In panel (a) one can see that the response of aggregate repricing frequency, both with and without networks, is dampened relative to otherwise identical economies with fixed menu costs. This is because the presence of free adjustment opportunities implies that much larger shocks are needed for firms to get pushed out of their inaction region. At the same time, just like in the economy with fixed menu costs, the economy with networks features smaller frequency movements, which is the effect of dampening pricing cascades. As for the GDP responses in panel (b), the economy with networks and random menu costs features much stronger non-neutrality than an otherwise identical economy without networks.

As for the propagation of supply shocks, in Figure D.6 we report the responses of aggregate repricing frequency and CPI inflation to aggregate TFP shocks of different sizes. Panel (a) show that, as with monetary shocks, the introduction of random menu costs dampens the responses of frequency to aggregate TFP shocks, both with and without networks. At the same time, one can see that conditional on CalvoPlus pricing, the economy with networks features stronger movements in aggregate frequency, implying that networks amplify cascades, just as in the economy with fixed menu costs. The amplification of pricing cascades creates a strong nonlinearity in aggregate CPI dynamics, as can be seen in panel (b). For a -10% aggregate TFP shock, networks amplify the aggregate CPI response from 0.03 to 0.08 on impact.

7.3 Alternative elasticity of substitution across sectors

In our baseline analysis, we use Cobb-Douglas aggregation across sectors, as well as a Cobb-Douglas production technology. In this subsection we relax this assumption, and consider more general constant elasticity of substitution (CES) aggregation across sectoral consumptions, as well as across productive inputs.

First, we consider the following CES final consumption aggregator:

Assumption 2′ (CES consumption aggregation)**.** *The consumption aggregator $\mathcal{C}(\cdot)$ is given by:*

$$\mathcal{C}(C_{1,t}, ..., C_{N,t}) = \left(\sum_{i=1}^{N} \overline{\omega}_i^{C\,\frac{1}{\theta_c}} C_{i,t}^{\frac{\theta_c - 1}{\theta_c}}\right)^{\frac{\theta_c}{\theta_c - 1}}, \tag{39}$$

where $\theta_c > 0$ is the elasticity of substitution across sectoral varieties and $\sum_i \overline{\omega}_i^C = 1$, $\overline{\omega}_i^C \geq 0, \forall i$.

Under this assumption, the equilibrium final consumption shares are given by:

$$\omega_{i,t}^C \equiv \frac{P_{i,t}C_{i,t}}{P_t^C C_t} = \overline{\omega}_i^C \times \frac{\tilde{P}_{i,t}^{1-\theta_c}}{\sum_{k=1}^{N} \overline{\omega}_k^C \tilde{P}_{k,t}^{1-\theta_c}} \tag{40}$$

which is constant in the special case when the sectoral consumption aggregator is Cobb-Douglas ($\theta_c = 1$). It follows that the final consumption shares are time-varying and depend on relative movements in (real) sectoral price indices. Whenever final sectoral varieties are complements ($\theta_c \in (0, 1)$), a relative increase in a sectoral price index leads to a rise in that sector's final consumption share, and *vice versa* whenever the varieties are substitutes ($\theta_c > 1$).

Similarly, we also assume the following CES production technology:

Assumption 3′ (CES production technology)**.** *The production technology $\mathcal{F}_i(\cdot)$ for a firm j in sector i is given by:*

$$\mathcal{F}_i[L_{i,t}(j), X_{i,1,t}(j), ..., X_{i,N,t}(j)] = \frac{1}{\zeta_{i,t}(j)} \times A_{i,t} \times \left(\overline{\alpha}_i^{\frac{1}{\theta_i}} N_{i,t}^{\frac{\theta_i - 1}{\theta_i}}(j) + \sum_{k=1}^{N} \overline{\omega}_{ik}^{\frac{1}{\theta_i}} X_{i,k,t}^{\frac{\theta_i - 1}{\theta_i}}(j) \right)^{\frac{\theta_i}{\theta_i - 1}}, \quad (41)$$

where $\theta_i > 0$ is the elasticity of substitution across inputs and $\overline{\alpha}_i + \sum_i \overline{\omega}_{ik} = 1$, $\overline{\alpha}_i$, $\overline{\omega}_{ik} \geq 0, \forall i$.

Such a production technology delivers the following equilibrium cost shares of labor and intermediate inputs:

$$\alpha_{i,t} \equiv \frac{W_t N_{i,t}(j)}{MC_{i,t}(j) Y_{i,t}(j)} = \overline{\alpha}_i \times \frac{1}{\overline{\alpha}_i + \sum_{k'=1}^{N} \overline{\omega}_{ik'} \tilde{P}_{k',t}^{1-\theta_i}} \quad (42)$$

$$\omega_{ik,t} \equiv \frac{P_{k,t} X_{i,k,t}(j)}{MC_{i,t}(j) Y_{i,t}(j)} = \overline{\omega}_{ik} \times \frac{\tilde{P}_{k,t}^{1-\theta_i}}{\overline{\alpha}_i + \sum_{k'=1}^{N} \overline{\omega}_{ik'} \tilde{P}_{k',t}^{1-\theta_i}} \quad (43)$$

which are constant in the special case when the production function is Cobb-Douglas ($\theta_i = 1$). As with consumption aggregation, time variation in the input cost shares is pinned down by relative movements in (real) input prices. As before, whenever inputs are complements, a relative increase in the price of an input leads to an increase in the cost share of that input, and *vice versa* whenever inputs are substitutes.

We now revisit our key quantitative exercises in an economy with fixed menu costs and CES aggregation. We calibrate $\theta_c = \theta_i = 0.001, \forall i$, to consider an economy where goods are almost perfect complements, capturing the potential difficulty of substituting both consumption and production varieties. This may represent the supply chain disruptions that we observed during and after the Covid pandemic across the globe.

In Figure D.7, we study the propagation of monetary shocks in our economy with CES aggregation. First, once can see that, just like under Cobb-Douglas, networks dampen the response of frequency to monetary shocks. In other words, our key mechanism of interaction of networks and the extensive margin continues to hold under CES aggregation. Quantitatively, conditional on the presence of networks, moving from Cobb-Douglas to CES with $\theta_c = \theta_i = 0.001, \forall i$ delivers slightly larger frequency movements for expansions and slightly smaller frequency movements under monetary contractions. This is because under complements, sectors with rising prices see their input and consumption shares rise, thus creating a pro-inflation asymmetry.

As for supply disturbances, in Figure D.8 we study the propagation of aggregate TFP shocks. Just as in the economy with Cobb-Douglas, networks amplify the response of aggregate repricing frequency to aggregate TFP shocks. Therefore, our key mechanism that networks amplify pricing cascades continues to hold under CES aggregation. Also, as with monetary shocks, the fact that sectoral varieties are complements creates a pro-inflation asymmetry: conditional on networks, CES aggregation amplify frequency movements after negative TFP shocks, and dampens frequency movements following positive TFP shocks.

8 Application: post-COVID inflation in the Euro Area

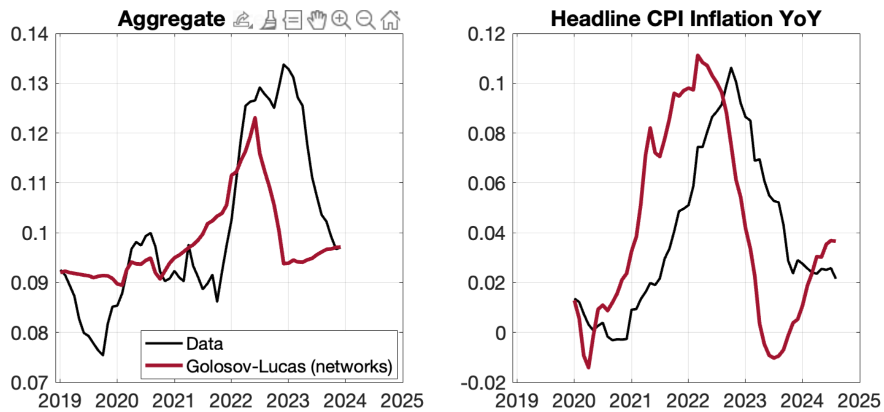
We would now like to assess whether the novel interaction between networks and pricing cascades is important for quantitatively explaining macroeconomic dynamics in the Euro Area in the (post-)Covid era. To do that, we feed four shock series into our model, corresponding to four major drivers of business cycles: money supply, energy price movements, food price movements and the dynamics of earnings in the labor market. We show that when subject to those four series, our model successfully captures the rise in the aggregate repricing frequency as well as the surge in consumer price inflation in the Euro Area. At the same time, removing either state-dependent pricing or networks dramatically worsens the quantitative performance of the model.

8.1 Matching selected time series

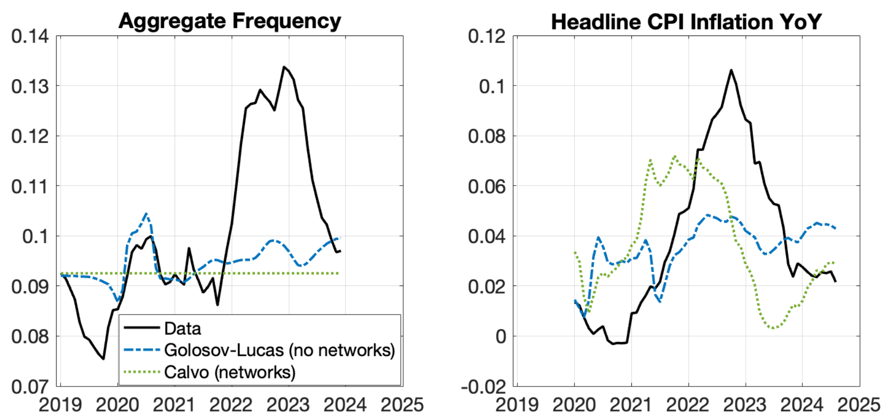
In our exercise, we consider four exogenous shock series. First, we feed in the Euro Area nominal GDP in order to approximate the aggregate money supply series $\{M_t\}_{2019:1}^{2024:6}$. Second, we are going to estimate an exogenous TFP process in the labor union sector $\{A_t^{\text{UNION}}\}_{2019:1}^{2024:6}$ so as to exactly match the Euro Area nominal hourly earnings series in equilibrium. Third, we fit an exogenous TFP process in the "Mining and Quarrying" sector $\{A_t^{\text{ENERGY}}\}_{2019:1}^{2024:6}$ in order to exactly match the real IMF Energy Price Index in equilibrium. Finally, we also feed in an exogeneous TFP series in the "Crop and Animal Production" sector $\{A_t^{\text{FOOD}}\}_{2019:1}^{2024:6}$ in order to exactly match the real IMF Food Price Index.

Figure 15: **Explaining the observed surge in frequency and inflation surge**

(a) Baseline model: menu costs and networks



(b) Alternative models



Notes: *the figure shows the model-implied changes in aggregate frequency of adjustment and aggregate CPI inflation versus the actual observed values in the Euro Area. Panel (a) considers the baseline model with production networks, whereas panel (b) considers an an otherwise identical economy without networks.*

8.2 Explaining the surge in frequency and inflation

Figure 15 shows the actual vs. model-implied changes in aggregate frequency of adjustment and aggregate CPI inflation in the Euro Area. In panel (a), one sees that the baseline non-linear model with state-dependent pricing and production networks successfully produces a surge in the aggregate frequency of adjustment of the magnitude observed in the Euro Area microdata. In addition, the baseline model can also generate an increase in aggregate CPI inflation of the same magnitude as observed in the actual data.

At the same time, as can be seen in panel (b) of Figure 15, an otherwise identical model without production networks fails to match the data, both when it comes to the frequency of adjustment and when it comes to CPI inflation. One sees that the aggregate frequency of adjustment remains essentially flat in the model without networks, apart from a minor uptick at the very beginning. At the same time, the model without networks generates an inflation increase only up to 4% annualized, while in the data it increased to as much as 10%.

9 Conclusions

In this paper, we develop and solve a novel quantitative dynamic general equilibrium model that allows us to study the transmission of large aggregate and sector-specific shocks in economies with realistic input-output linkages and fully optimal, state-dependent pricing decisions. Our framework makes predictions about the interactions of production networks with pricing decisions at the extensive margin, which is also quantitatively relevant for the dynamics of macroeconomic aggregates.

For aggregate nominal shocks, such as a change in the money supply, production networks shrink the magnitudes of desired price changes at the firm level, which in turn compresses the sizes of price gaps for all firms. As a result, the presence of networks slows the extensive margin of price adjustment decisions, lowering the response of the aggregate repricing frequency. At the same time, large aggregate and sector-specific total factor productivity (TFP) shocks interact with the production network in a manner opposite to that under monetary shocks. In particular, production networks amplify firm-level desired price changes following TFP shocks, expanding the price gaps, and making the decision to adjust more likely, *ceteris paribus*. As a result, large negative TFP shocks can lead to fast increases in the aggregate repricing frequency, and to rapid inflationary surges.

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Appendix

Business Cycles with Pricing Cascades

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A Proofs

Proof of Lemma 1. We want to find a second-order approximation of the firm-level profit function $\tilde{D}_{i,t}(j)$ in the log quality-adjusted real price of that firm $\log \tilde{P}_{i,t}(j)$ near the optimum $\log \tilde{P}^*_{i,t}$. By definition of the optimal reset price, $\frac{\partial \tilde{D}_{i,t}(j)}{\partial \log \tilde{P}_{i,t}(j)}|_{\tilde{P}_{i,t}(j)=\tilde{P}^*_{i,t}} = 0$. As for the second derivative, one can show that:

$$\frac{\partial^2 \tilde{D}_{i,t}(j)}{\partial \log \tilde{P}_{i,t}(j)^2} = \left[(1-\tau_{i,t})e^{(1-\epsilon)\log \tilde{P}_{i,t}(j)}(1-\epsilon)^2 - \epsilon^2 \tilde{\mathcal{Q}}_{i,t} e^{-\epsilon \log \tilde{P}_{i,t}(j)}\right] \times \tilde{P}^{\epsilon}_{i,t} Y_{i,t}. \tag{A.1}$$

Evaluating the second derivative at $\log \tilde{P}^*_{i,t}$, and after some algebra one obtains:

$$\frac{\partial^2 \tilde{D}_{i,t}(j)}{\partial \log \tilde{P}_{i,t}(j)^2}|_{\tilde{P}_{i,t}(j)=\tilde{P}^*_{i,t}(j)} = -(\epsilon-1)(1-\tau_{i,t})\left[\tilde{P}_{i,t}/P^*_{i,t}\right]^{\epsilon-1}\lambda_{i,t}. \tag{A.2}$$

Therefore, one can write the second-order approximation as:

$$\tilde{D}_{i,t} = \tilde{D}^*_{i,t} + \frac{1}{2}\frac{\partial^2 \tilde{D}_{i,t}(j)}{\partial \log \tilde{P}_{i,t}(j)^2}|_{\tilde{P}_{i,t}(j)=\tilde{P}^*_{i,t}(j)} \times [\tilde{p}_{i,t}(j)]^2 + \mathcal{O}[\tilde{p}_{i,t}(j)^3], \tag{A.3}$$

where $\tilde{p}_{i,t}(j) \equiv [\log \tilde{P}_{i,t}(j) - \log \tilde{P}^*_{i,t}]$ is the firm-level price gap. Inserting the expression for the second derivative, one obtains:

$$\tilde{D}^*_{i,t} - \tilde{D}_{i,t} = \frac{1}{2}(\epsilon-1)(1-\tau_{i,t})\left[\tilde{P}_{i,t}/\tilde{P}^*_{i,t}\right]^{\epsilon-1}\lambda_{i,t} \times [\tilde{p}_{i,t}(j)]^2 + \mathcal{O}[\tilde{p}_{i,t}(j)^3]. \tag{A.4}$$

□

Proof of Lemma 2. Focusing on period $t = 0$, a firm adjusts its price if the profit gain from adjustment exceeds the menu cost:

$$\tilde{D}_{i,0}(j)^* - \tilde{D}_{i,0}(j) \geq \kappa_{i,0} \tag{A.5}$$

Using the approximation for the profit gain in Lemma 1, as well as the menu cost form in Assumption

6, one can further rewrite the adjustment condition as:

$$\frac{1}{2}(\epsilon-1)(1-\tau_{i,0})\left[\tilde{P}_{i,0}/\tilde{P}^*_{i,0}\right]^{\epsilon-1}\lambda_{i,0}\times[\tilde{p}_{i,0}(j)]^2 \geq \overline{\kappa}_i(1-\tau_{i,0})[\tilde{P}_{i,0}/\tilde{P}^*_{i,0}]^{\epsilon-1}\lambda_{i,0}, \tag{A.6}$$

$$\implies \qquad [\tilde{p}_{i,0}(j)]^2 \geq \frac{2\overline{\kappa}_i}{\epsilon-1}. \tag{A.7}$$

Using the expression for the price gap in (28), as well as the normalization $p_{i,-1}(j) = \log\frac{\epsilon}{\epsilon-1}\frac{1}{1-\overline{\tau}_i}$, the adjustment condition becomes:

$$\left|-\sigma_i\varepsilon_{i,0}(j) - m_0 - \gamma_{i,0} + a_{i,0} - \sum_{k=1}^{N}\overline{\omega}_{ik}\log\tilde{P}_{k,0}\right| \geq \sqrt{\frac{2\overline{\kappa}_i}{\epsilon-1}}. \tag{A.8}$$

Therefore, the inaction region is given by:

$$[\sigma_i\underline{\varepsilon}_{i,0}, \quad \sigma_i\overline{\varepsilon}_{i,0}] \quad = \quad -m_0 - \gamma_{i,0} + a_{i,0} - \sum_{k=1}^{N}\overline{\omega}_{ik}\log\tilde{P}_{k,0} \quad \pm \quad \sqrt{\frac{2\overline{\kappa}_i}{\epsilon-1}}, \tag{A.9}$$

where $\quad m_0 \equiv \log(M_0/M_{-1}), \quad \gamma_{i,0} \equiv \log\Gamma_{i,0} - \overline{\gamma}_i, \quad a_{i,0} \equiv \log A_{i,0} \quad$ and $\quad \overline{\gamma}_i \equiv \log\frac{\epsilon}{\epsilon-1}\frac{1}{1-\overline{\tau}_i}$ □

Proof of Proposition 1. Before providing a proof for Proposition 1, it is useful to formally establish an auxiliary technical result:

Lemma A1. *Define $f_+(x;c) \equiv \Phi(c+x)-\Phi(-c+x)$ and $f_-(x;c) \equiv \Phi(c-x)-\Phi(-c-x)$, where $c>0$ is a parameter and $\Phi(.)$ is standard normal CDF. Then both $f_+(x;c)$ and $f_-(x;c)$ are decreasing in x for all $x>0$.*

Proof. First, consider $f_+(x;c)$. Notice that $f'_+(x) = \Phi'(c+x) - \Phi'(-c+x) = \phi(c+x) - \phi(-c+x)$, where $\phi(.)$ is standard normal PDF. Hence, $f'_+(0) = \phi(c) - \phi(-c) = 0$. As for any $x \in (0,c]$, one can deduce that $f'_+(x) = \underbrace{\phi(c+x)}_{<\phi(c)} - \underbrace{\phi(-c+x)}_{>\phi(-c)} < 0$. Further, for any $x > c$ it follows that $f'_+(x) = \phi(c+x) - \phi(-c+x) < 0$, since standard normal PDF is decreasing in positive inputs. All in all, we conclude that $f'_+(x) < 0$ for all $x > 0$.

Similarly, $f'_-(x) = -\Phi'(c-x) + \Phi'(-c-x) = -\phi(c-x) + \phi(-c-x)$. As before, $f'_-(0) = -\phi(c) + \phi(-c) = 0$. For any $x \in (0,c]$, $f'_-(x) = -\underbrace{\phi(c-x)}_{>\phi(c)} + \underbrace{\phi(-c-x)}_{<\phi(-c)} < 0$. As for any $x > c$, $f'_-(x) = -\phi(c-x) + \phi(-c-x) < 0$, since standard normal PDF is increasing in negative inputs. In total, we conclude that $f'_-(x) < 0$ for all $x > 0$. □

Armed with the additional result in Lemma A1, we are now ready to prove Proposition 1. Consider a monetary expansion $m_0 > 0$. The probability that a firm draws an idiosyncratic innovation that lies in the inaction region following the monetary expansion is given by:

$$\begin{aligned}
Pr(\underline{\varepsilon}_{i,0} \leq \varepsilon_{i,0}(j) \leq \overline{\varepsilon}_{i,0}) &= \Phi\left(\frac{1}{\sigma_i}\sqrt{\frac{2\overline{\kappa}_i}{\epsilon-1}} - \frac{1}{\sigma_i}\left\{m_0 + \sum_{k=1}^{N}\overline{\omega}_{ik}\log\overline{P}_{k,0}\right\}\right) \\
&\quad - \Phi\left(-\frac{1}{\sigma_i}\sqrt{\frac{2\overline{\kappa}_i}{\epsilon-1}} - \frac{1}{\sigma_i}\left\{m_0 + \sum_{k=1}^{N}\overline{\omega}_{ik}\log\overline{P}_{k,0}\right\}\right) \\
&= f_-\left(\frac{1}{\sigma_i}\left\{m_0 + \sum_{k=1}^{N}\overline{\omega}_{ik}\log\overline{P}_{k,0}\right\};\quad \frac{1}{\sigma_i}\sqrt{\frac{2\overline{\kappa}_i}{\epsilon-1}}\right), \qquad \text{(A.10)}
\end{aligned}$$

where $f_-(.)$ is defined in Lemma A1. Now, as long as the pass-through of the monetary expansion to sectoral prices is incomplete, $\log\tilde{P}_{k,0} < 0, \forall k$, it follows that $m_0 + \sum_{k=1}^{N}\overline{\omega}_{ik}\log\overline{P}_{k,0} < m_0$. Moreover, since $f_-(.)$ is falling in its positive inputs, it immediately follows that:

$$f_-\left(\frac{1}{\sigma_i}\left\{m_0 + \sum_{k=1}^{N}\overline{\omega}_{ik}\log\overline{P}_{k,0}\right\};\quad \frac{1}{\sigma_i}\sqrt{\frac{2\overline{\kappa}_i}{\epsilon-1}}\right) \quad > \quad f_-\left(\frac{1}{\sigma_i}m_0;\quad \frac{1}{\sigma_i}\sqrt{\frac{2\overline{\kappa}_i}{\epsilon-1}}\right). \qquad \text{(A.11)}$$

Hence, *ceteris paribus*, the probability of drawing a shock in the inaction region following a monetary expansion is higher in the economy with networks.

Similarly, consider a monetary contraction $m_0 < 0$. The probability that a firm draws an idiosyncratic innovation that lies in the inaction region following the monetary contraction is given by:

$$\begin{aligned}
Pr(\underline{\varepsilon}_{i,0} \leq \varepsilon_{i,0}(j) \leq \overline{\varepsilon}_{i,0}) &= \Phi\left(\frac{1}{\sigma_i}\sqrt{\frac{2\overline{\kappa}_i}{\epsilon-1}} + \frac{1}{\sigma_i}\left\{-m_0 - \sum_{k=1}^{N}\overline{\omega}_{ik}\log\overline{P}_{k,0}\right\}\right) \\
&\quad - \Phi\left(-\frac{1}{\sigma_i}\sqrt{\frac{2\overline{\kappa}_i}{\epsilon-1}} + \frac{1}{\sigma_i}\left\{-m_0 - \sum_{k=1}^{N}\overline{\omega}_{ik}\log\overline{P}_{k,0}\right\}\right) \\
&= f_+\left(\frac{1}{\sigma_i}\left\{-m_0 - \sum_{k=1}^{N}\overline{\omega}_{ik}\log\overline{P}_{k,0}\right\};\quad \frac{1}{\sigma_i}\sqrt{\frac{2\overline{\kappa}_i}{\epsilon-1}}\right), \qquad \text{(A.12)}
\end{aligned}$$

where $f_+(.)$ is defined in Lemma A1. Now, as long as the pass-through of the monetary contraction to sectoral prices is incomplete, $\log\tilde{P}_{k,0} > 0, \forall k$, it follows that $-m_0 - \sum_{k=1}^{N}\overline{\omega}_{ik}\log\overline{P}_{k,0} < -m_0$. Moreover, since $f_+(.)$ is falling in its positive inputs, it immediately follows that:

$$f_+\left(-\frac{1}{\sigma_i}\left\{m_0 + \sum_{k=1}^{N}\overline{\omega}_{ik}\log\overline{P}_{k,0}\right\};\quad \frac{1}{\sigma_i}\sqrt{\frac{2\overline{\kappa}_i}{\epsilon-1}}\right) \quad > \quad f_+\left(-\frac{1}{\sigma_i}m_0;\quad \frac{1}{\sigma_i}\sqrt{\frac{2\overline{\kappa}_i}{\epsilon-1}}\right). \qquad \text{(A.13)}$$

Hence, *ceteris paribus*, the probability of drawing a shock in the inaction region following a monetary contraction is higher in the economy with networks. $\square$

Proof of Proposition 2. Consider a productivity deterioration $a_{i,0} < 0$ and/or a rise in desired markups $\gamma_{i,0} > 0$ in sector i. The probability that a firm in sector i' (which may or may not be the same as i) draws an idiosyncratic innovation that lies in the inaction region following productivity deterioration/markup increase in sector i is given by:

$$\begin{aligned} Pr(\underline{\varepsilon}_{i',0} \leq \varepsilon_{i',0}(j) \leq \overline{\varepsilon}_{i',0}) \quad &= \quad \Phi\left(\frac{1}{\sigma_{i'}}\sqrt{\frac{2\overline{\kappa}_{i'}}{\epsilon-1}} - \frac{1}{\sigma_{i'}}\left\{-a_{i',0} + \gamma_{i',0} + \sum_{k=1}^{N}\overline{\omega}_{i'k}\log\overline{P}_{k,0}\right\}\right) \\ &- \Phi\left(-\frac{1}{\sigma_{i'}}\sqrt{\frac{2\overline{\kappa}_{i'}}{\epsilon-1}} - \frac{1}{\sigma_{i'}}\left\{-a_{i',0} + \gamma_{i',0} + \sum_{k=1}^{N}\overline{\omega}_{i'k}\log\overline{P}_{k,0}\right\}\right) \\ &= \quad f_{-}\left(\frac{1}{\sigma_{i'}}\left\{-a_{i',0} + \gamma_{i',0} + \sum_{k=1}^{N}\overline{\omega}_{i'k}\log\overline{P}_{k,0}\right\}; \quad \frac{1}{\sigma_{i'}}\sqrt{\frac{2\overline{\kappa}_{i'}}{\epsilon-1}}\right), \end{aligned} \tag{A.14}$$

where $f_{-}(.)$ is defined in Lemma A1. Now, as long as the productivity deterioration/markup increase in sector i leads to a rise in sectoral prices of all other sectors, $\log\tilde{P}_{k,0} > 0, \forall k$, it follows that $-a_{i',0} + \gamma_{i',0} + \sum_{k=1}^{N}\overline{\omega}_{ik}\log\overline{P}_{k,0} > -a_{i',0} + \gamma_{i',0}$, $\forall i'$. Moreover, since $f_{-}(.)$ is falling in its positive inputs, it immediately follows that:

$$f_{-}\left(\frac{1}{\sigma_{i'}}\left\{-a_{i',0} + \gamma_{i',0} + \sum_{k=1}^{N}\overline{\omega}_{i'k}\log\overline{P}_{k,0}\right\}; \frac{1}{\sigma_{i'}}\sqrt{\frac{2\overline{\kappa}_{i'}}{\epsilon-1}}\right) < f_{-}\left(\frac{1}{\sigma_{i'}}\left\{-a_{i',0} + \gamma_{i',0}\right\}; \frac{1}{\sigma_{i'}}\sqrt{\frac{2\overline{\kappa}_{i'}}{\epsilon-1}}\right). \tag{A.15}$$

Hence, *ceteris paribus*, the probability that a firm in sector i' draws shock in the inaction region following a productivity deterioration/markup increase in sector i is lower in the economy with networks.

Similarly, consider a productivity improvement $a_{i,0} > 0$ and/or a fall in desired markups $\gamma_{i,0} < 0$ in sector i. The probability that a firm in sector i' (which may or may not be the same as i) draws an idiosyncratic innovation that lies in the inaction region following productivity improvement/markup decrease in sector i is given by:

$$
\begin{aligned}
Pr(\underline{\varepsilon}_{i',0} \leq \varepsilon_{i',0}(j) \leq \overline{\varepsilon}_{i',0}) \quad &= \quad \Phi\left(\frac{1}{\sigma_{i'}}\sqrt{\frac{2\overline{\kappa}_{i'}}{\epsilon-1}} + \frac{1}{\sigma_{i'}}\left\{a_{i',0} - \gamma_{i',0} - \sum_{k=1}^{N}\overline{\omega}_{i'k}\log\overline{P}_{k,0}\right\}\right) \\
&\quad - \Phi\left(-\frac{1}{\sigma_{i'}}\sqrt{\frac{2\overline{\kappa}_{i'}}{\epsilon-1}} + \frac{1}{\sigma_{i'}}\left\{a_{i',0} - \gamma_{i',0} - \sum_{k=1}^{N}\overline{\omega}_{i'k}\log\overline{P}_{k,0}\right\}\right) \\
&= \quad f_{+}\left(\frac{1}{\sigma_{i'}}\left\{a_{i',0} - \gamma_{i',0} - \sum_{k=1}^{N}\overline{\omega}_{i'k}\log\overline{P}_{k,0}\right\}; \quad \frac{1}{\sigma_{i'}}\sqrt{\frac{2\overline{\kappa}_{i'}}{\epsilon-1}}\right),
\end{aligned}
\tag{A.16}
$$

where $f_{+}(.)$ is defined in Lemma A1. Now, as long as the productivity improvement/markup decrease in sector i leads to a fall in sectoral prices of all other sectors, $\log \tilde{P}_{k,0} < 0, \forall k$, it follows that $-a_{i',0} + \gamma_{i',0} + \sum_{k=1}^{N}\overline{\omega}_{ik}\log\overline{P}_{k,0} < -a_{i',0} + \gamma_{i',0}$, $\forall i'$. Moreover, since $f_{+}(.)$ is falling in its positive inputs, it immediately follows that:

$$
f_{+}\left(-\frac{1}{\sigma_{i'}}\left\{a_{i',0} - \gamma_{i',0} - \sum_{k=1}^{N}\overline{\omega}_{i'k}\log\overline{P}_{k,0}\right\}; \frac{1}{\sigma_{i'}}\sqrt{\frac{2\overline{\kappa}_{i'}}{\epsilon-1}}\right) < f_{+}\left(-\frac{1}{\sigma_{i'}}\left\{a_{i',0} - \gamma_{i',0}\right\}; \frac{1}{\sigma_{i'}}\sqrt{\frac{2\overline{\kappa}_{i'}}{\epsilon-1}}\right). \tag{A.17}
$$

Hence, *ceteris paribus*, the probability that a firm in sector i' draws shock in the inaction region following a productivity deterioration/markup increase in sector i is lower in the economy with networks.

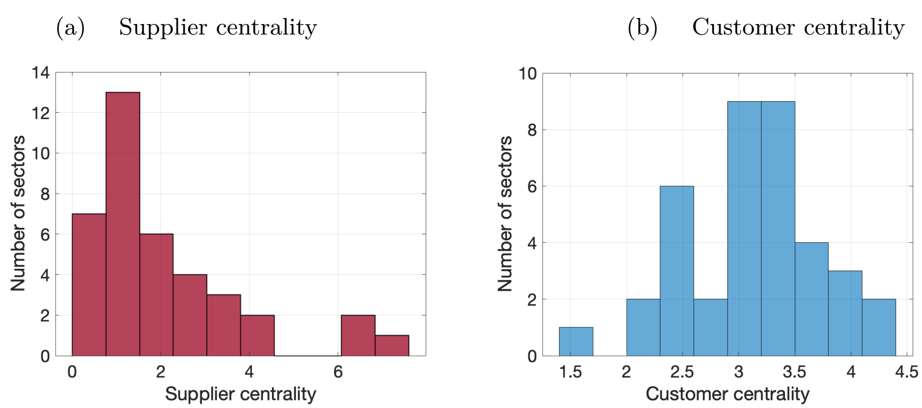
Note that while we provide a proof for sector-specific productivity/markup shocks, results are equivalent for aggregate productivity/markup shocks. This is the case since an aggregate productivity shock a is merely a combination of equally-sized sector-specific productivity shocks $a_i = a, \forall i$, and similarly for an aggregate markup shocks. $\square$

B Additional calibration details (Euro Area)

Table B.1: Sectors with Consumption Weights, Suppliers, and Customers Centrality

Sector Name	*Consumption Share*	*Supplier Centrality*	*Customer Centrality*
Crop and animal production	0.0290	2.1266	3.4452
Fishing and aquaculture	0.0039	0.1070	3.2103
Mining and quarrying	0.0051	2.5263	3.1919
Food and beverages	0.1430	3.2264	3.9448
Textiles, clothes, leather	0.0403	1.4829	3.6296
Wood and wooden products	0.0044	1.3072	3.6121
Paper and paper products	0.0079	2.6301	3.9664
Printing and recorded media	0.0035	1.0667	3.3331
Coke and petroleum products	0.0398	3.0308	4.3485
Chemicals and chemical products	0.0162	6.1838	4.2462
Pharmaceuticals	0.0140	0.7374	3.4977
Rubber and plastic	0.0088	2.0056	3.7245
Non metallic minerals	0.0066	0.9865	3.3615
Metal products	0.0081	3.2480	3.1201
Computer and electronics	0.0175	1.9379	3.2884
Electrical equipment	0.0115	1.4046	3.3162
Machinery	0.0066	2.1823	3.3086
Motor vehicles	0.0514	1.5453	3.8439
Other transport	0.0057	0.8266	3.6416
Furniture	0.0224	0.6181	3.1077
Repair of machinery	0.0030	1.1591	2.9322
Land and pipeline transport	0.0398	3.6878	2.9952
Warehousing	0.0125	3.9047	3.1653
Accommodation and food services	0.1475	1.0026	2.9424
Publishing	0.0138	0.7571	2.9781
Movies, video, TV	0.0131	1.2814	2.9446
Computer and information services	0.0069	2.5026	2.4193
Financial services	0.0391	4.4781	2.6784
Insurance and pension	0.0502	1.4619	3.2978
Legal, accounting, management	0.0079	6.5099	2.2538
Architectural/engineering services	0.0037	2.0129	2.3319
Science and R&D	0.0020	0.2637	2.4415
Advertising and marketing	0.0020	1.1808	2.8706
Other professional activities	0.0069	0.9650	2.3978
Administration and support	0.0293	7.5903	2.4434
Education	0.0261	0.5206	1.5508
Healthcare	0.0743	0.2930	2.0342
Other personal services	0.0765	1.3971	2.3334

Figure B.2: **Distributions of supplier and customer centrality (Euro Area, 38 sectors)**



Notes*: the figure shows the responses of sectoral frequency and sectoral price indices to large aggregate TFP shocks.*

C Details of the numerical algorithm

C.1 Steady state computation on a grid

For each sector, we solve the stationary Bellman equation and price distribution on an evenly spaced grid of log prices $\mathbf{\Gamma}$ with step size Δp, $p_j \in \left[\underline{p}, \underline{p} + \Delta p, ..., \overline{p}\right]$, $j = 1, .., J$ grid points, so that $V_j = V(p_j)$. The expectation $\mathbb{E}\left[V(p - \sigma\varepsilon_{t+1} - \pi) | p = p_j\right]$ is calculated as $\boldsymbol{TV}$ where we define transition matrix

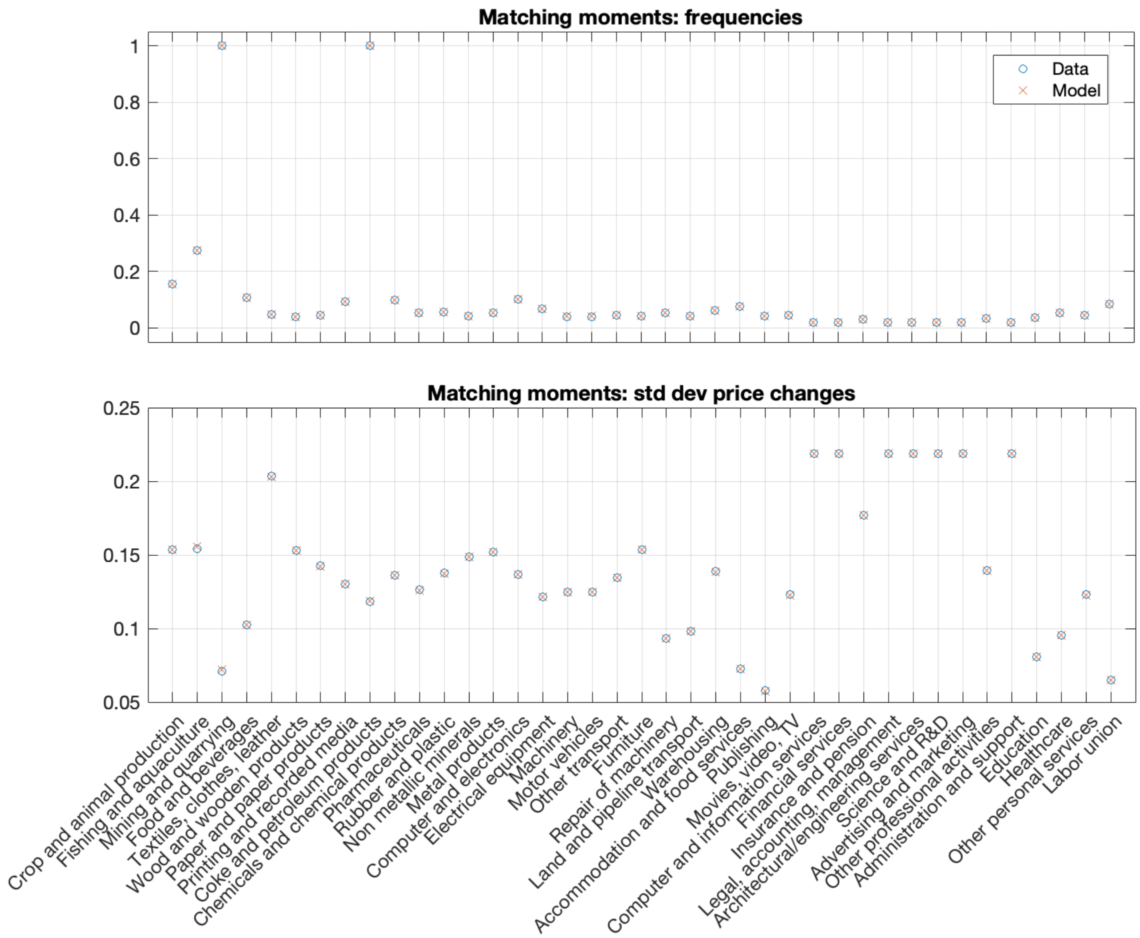
$$\boldsymbol{T} = \begin{bmatrix} \mathcal{T}_{1,1} & \mathcal{T}_{1,2} & \cdots & \mathcal{T}_{1,J} \\ \mathcal{T}_{2,1} & \mathcal{T}_{2,2} & \cdots & \mathcal{T}_{2,J} \\ \vdots & \vdots & \ddots & \vdots \\ \mathcal{T}_{J,1} & \mathcal{T}_{J,2} & \cdots & \mathcal{T}_{J,J} \end{bmatrix}.$$

with elements

$$\mathcal{T}_{j,k} = \int_{p=p_{k-1/2}}^{p_{k+1/2}} \psi\left(\frac{p - (p_j - \pi)}{\sigma}\right) dp = \Psi\left(\frac{p_{k+1/2} - (p_j - \pi)}{\sigma}\right) - \Psi\left(\frac{p_{k-1/2} - (p_j - \pi)}{\sigma}\right),$$

and where $p_{k-1/2} \equiv (p_{k-1} + p_k)/2$, $p_{k+1/2} \equiv (p_k + p_{k+1})/2$, $\psi(\cdot)$ is the standard normal probability density function, and $\Psi(\cdot)$ is the standard normal cumulative distribution function.

Figure B.3: Matching pricing moments for each sector



We also define the vectors

$$\boldsymbol{\phi} = \begin{bmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_J \end{bmatrix}, \ \eta = \begin{bmatrix} \eta_1 \\ \eta_2 \\ \vdots \\ \eta_J \end{bmatrix}, \ \boldsymbol{V} = \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_J \end{bmatrix}, \ \boldsymbol{D} = \begin{bmatrix} D_1 \\ D_2 \\ \vdots \\ D_J \end{bmatrix}$$

The Bellman equation in matrix notation is then given by

$$\boldsymbol{V} \quad = \quad \boldsymbol{D} + \beta \left[\boldsymbol{T} \left((1-\eta) \cdot \boldsymbol{V} \right) + \boldsymbol{T} \left(\eta \cdot \left(\boldsymbol{\phi}' \boldsymbol{V} - \kappa w \right) \right) \right]$$

where $\cdot$ denotes element-by-element multiplication. Vector $\boldsymbol{\phi}$ distributes unit probability mass to grid points adjacent to p^* according to the logit formula

$$\boldsymbol{\phi} = \frac{\exp(\boldsymbol{V}/\chi)}{\sum_{\Gamma} \exp(\boldsymbol{V}/\chi)}$$

with $\chi = 0.0005$. Note that $\boldsymbol{\phi}'\boldsymbol{V}$ performs smooth maximization as in eq.(21).

To solve the problem for N sectors with input-output linkages, we use the following algorithm. Start with a guess for the vector of steady-state price dispersions Δ_k and sectoral taxes τ_k, then:[7]

1. Given $\pi = \overline{\pi}$, compute the transition matrix $\boldsymbol{T}$
2. Using $\frac{W}{M} \equiv w = 1$, compute[8] $\omega_{ik} = \overline{\omega}_{ik} \times \frac{(P_k/M)^{1-\theta_i}}{\overline{\alpha}_i + \sum_{k'=1}^{N} \overline{\omega}_{ik'}(P_{k'}/M)^{1-\theta_i}} = \overline{\omega}_{ik}$
3. λ is given by eq.(15) and η by eq.(22)
4. With that, construct the profit matrix $\boldsymbol{D}$ as in eq.(19)
5. Iterate backward on the value function $\boldsymbol{V}$ above to convergence
6. To compute the distribution, iterate forward on

$$\boldsymbol{g} = (1-\eta) \cdot (\boldsymbol{T}'\boldsymbol{g}) + \boldsymbol{\phi}\eta' (\boldsymbol{T}'\boldsymbol{g}) . \tag{C.1}$$

   until convergence of $\boldsymbol{g}$.

7. Given the distribution, compute the residual vectors $resid_1$ and $resid_2$ as in

$$resid_1 = \Delta_k - (P_k/M)^{\epsilon} \int_0^1 \left(\frac{P_k(j')}{\zeta_k(j')M} \right)^{-\epsilon} dj', \tag{C.2}$$

$$resid_2 = P_k/M - \int_0^1 \left(\frac{P_k(j')}{\zeta_k(j')M} \right)^{1-\epsilon} dj' \tag{C.3}$$

8. Search for a vector of sectoral price dispersions and taxes such that $resid \to 10^{-14}$.

[7] We start with the guess $\Delta_k = 1$ and $\tau_k = -1/\epsilon$

[8] We are searching for taxes τ_k such that the steady state equilibrium is symmetric in sectoral prices, $P_k/M = 1$

C.2 Solving for impulse-responses in sequence space

We compute fully non-linearly the responses to an MIT shock in the space of sequences, iterating backward in time on the value function and forward in time on the law of motion of the distribution, under the assumption of perfect foresight. The steps are similar to those for computing the steady state; only this time we keep track of the sequences over time. We start by guessing sequences for time t from 1 to $T = 500$ months, for sectoral prices and price dispersions (our starting guess simply equals the steady-state value for these variables). The key assumption is that all stationary variables must return to steady state by period T. Given this initial guess, we compute the price of the final good and consumption over time using their definitions. Given that, we calculate λ_t as in eq.(15). We compute the profits D_t as in eq.(19). Iterating backward in time from $t = T$ to $t = 0$, we solve for the value function as in eq.(21). Given the value function, we can compute the gain from adjustment L_t and the adjustment hazard η_t. Once the backward iteration on the value function reaches period 0, we start from the steady-state distribution and iterate forward in time on the law of motion of the price distribution from period 1 until period T. Given the distribution, we can compute via eq.(7) the sectoral price indices, and by

$$\Delta_{k,t} \equiv (P_{k,t}/M_t)^{\epsilon} \int_0^1 \left(\frac{P_{k,t}(j')}{\zeta_{k,t}(j')M_t} \right)^{-\epsilon} dj'$$

the sectoral price dispersions. This provides us with an updated guess, with which we repeat the previous steps until the change in the sequences (of sectoral prices and price dispersions) becomes near zero.

D Additional results, extensions and robustness

D.1 Additional figures for main text

In Figure D.1 (a) we report the impact responses of sectoral fractions of adjusters to a 10% monetary shock. First, for all sectors, the fraction of adjusters increases by more in the economy without networks, relative to the baseline economy with input-output linkages. For some sectors, such as "publishing", the percentage of adjusters increases to almost 100%.

Beyond frequency responses, in Figure D.1 (b) we additionally document impact responses of sectoral price indices, normalized by the size of the monetary shock (10%). Like with the frequency responses, the baseline economy with networks features a smaller response relative to an otherwise identical economy without networks.

In Figure D.2(a) we report responses of sector-specific fractions of adjusting firms to a -10% aggregate TFP shock. First, for all sectors the fraction of adjusters is larger in the baseline economy with networks, relative to the otherwise identical economy without networks. Second, for a number of sectors, such as "Warehousing" and "Land and Pipeline Transport" the fraction rises almost to 100% following the shock. At the same time, for some other sectors, such as "Computer and Information Services", the frequency rises relatively modestly.

In Figure D.2(b) we additionally report the impact responses of sector-specific prices to a -10% aggregate TFP shock. Importantly, for all sectors, the overall price response is larger in an economy with networks, relative to the otherwise identical economy without networks.

D.2 Cashless limit

The representative household chooses a sequence of consumption, labor supply and one-period nominal bond holdings to maximize expected lifetime utility:

$$\max_{\{C_t,L_t,B_t\}_{t\geq 0}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(C_t, L_t), \tag{D.1}$$

subject to the period-by-period budget constraint

$$P_t^C C_t + B_t \quad = \quad (1+i_{t-1})B_{t-1} + W_t L_t + \sum_{i=1}^{N} \int_0^1 D_{i,t}(j)dj + T_t, \tag{D.2}$$

where C_t is consumption, L_t is labor supply, B_t is the level of nominal bond holdings, T_t is the level of lump-sum transfers from the government, $D_{i,t}(j)$ are the dividends received lump-sum from firm j in sector i at time t, $\Pi_t^C = \left(P_t^C/P_{t-1}^C\right)$ is the gross CPI inflation rate, W_t is the nominal wage and i_t is the nominal interest rate set by the central bank.

The nominal interest rate is set by the central bank according to the following Taylor rule:

$$\log(1+i_t) = \log(\overline{\Pi}/\beta) + \phi_\pi \log(\Pi_t^C/\overline{\Pi}) + \varepsilon_t^i, \tag{D.3}$$

where $\overline{\Pi}$ is the steady-state level of CPI inflation, $\phi_\pi > 1$ pins down the strength of the central bank's reaction to inflation deviations from target and ε_t^i is the monetary policy shock.

We assume the following form of households' preferences:

$$u(C_t, L_t) = \frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{L_t^{1+\varphi}}{1+\varphi}. \tag{D.4}$$

Note that the Golosov-Lucas log-linear preferences which we use in the main text arise as a special case when $\sigma \to 1$ and $\varphi = 0$.

Given the presence of possibly non-zero steady-state inflation and the non-stationarity of the quality processes, we appropriately normalize our variables. Unlike in the main text, where we normalize by money supply, in the current cashless setting, we instead normalize by the aggregate CPI price level P_{t-1}^C. In particular, we let $\tilde{P_{i,t}}(j) \equiv \frac{P_{i,t}(j)}{\zeta_{i,t}(j) P_{t-1}^C}$ be the quality-adjusted real price, $\tilde{P}_{i,t} \equiv \frac{P_{i,t}}{P_{t-1}^C}$ be the real sectoral price, and $\tilde{W}_t \equiv \frac{W_t}{P_{t-1}^C}$ be the real wage. Then the equilibrium conditions for the aggregate real variables are given by:

$$C_t^{-\sigma} = \beta \mathbb{E}_t \left[\frac{1+i_t}{\Pi_{t+1}^C} C_{t+1}^{-\sigma} \right] \tag{D.5}$$

$$C_t^{\sigma} L_t^{\varphi} = \tilde{W}_t / \Pi_t^C \tag{D.6}$$

$$L_t = \Pi_t^C \frac{C_t}{\tilde{W}_t} \left[1 - \sum_{i=1}^N \lambda_{i,t} \left(1 - \frac{\Delta_{i,t}}{\mathcal{M}_{i,t}} \right) \right] + \sum_{i=1}^N \kappa_{i,t} \int_0^1 \eta_{i,t}(j) dj. \tag{D.7}$$

where $\lambda_{i,t}$ is the sectoral Domar weight (sales) share, $\Delta_{i,t}$ is the within-sector dispersion of real prices and $\mathcal{M}_{i,t}$ is the sectoral markup, which are given by:

$$\lambda_{i,t} = \omega_{i,t}^C + \sum_{k=1}^N \omega_{k,i,t} \lambda_{k,t} \frac{\Delta_{i,t}}{\mathcal{M}_{i,t}}, \qquad \Delta_{i,t} \equiv \tilde{P}_{i,t}^{\epsilon} \int_0^1 \tilde{P}_{i,t}(j)^{-\epsilon} dj, \qquad \mathcal{M}_{i,t} \equiv \frac{\tilde{P}_{i,t}}{\tilde{\mathcal{Q}}_{i,t}}. \tag{D.8}$$

The real sectoral price indices and marginal costs in turn satisfy:

$$\tilde{P}_{i,t}^{1-\epsilon} = \int_0^1 \tilde{P}_{i,t}(j)^{1-\epsilon} dj, \qquad \tilde{\mathcal{Q}}_{i,t} = \mathcal{Q}_i \left[\tilde{W}_t, \tilde{P}_{1,t}, ..., \tilde{P}_{N,t}; A_{i,t} \right], \qquad \Pi_t^C = \mathcal{P}^{\mathcal{C}} \left[\tilde{P}_{1,t}, ..., \tilde{P}_{N,t} \right]. \tag{D.9}$$

If the nominal price is not adjusted, then the quality-adjusted real price evolves according to:

$$p_{i,t}(j) = p_{i,t-1}(j) - \sigma_i \varepsilon_{i,t}(j) - \pi^C_{t-1}, \tag{D.10}$$

where $\pi^C_{t-1} \equiv \log \Pi^C_{t-1}$.

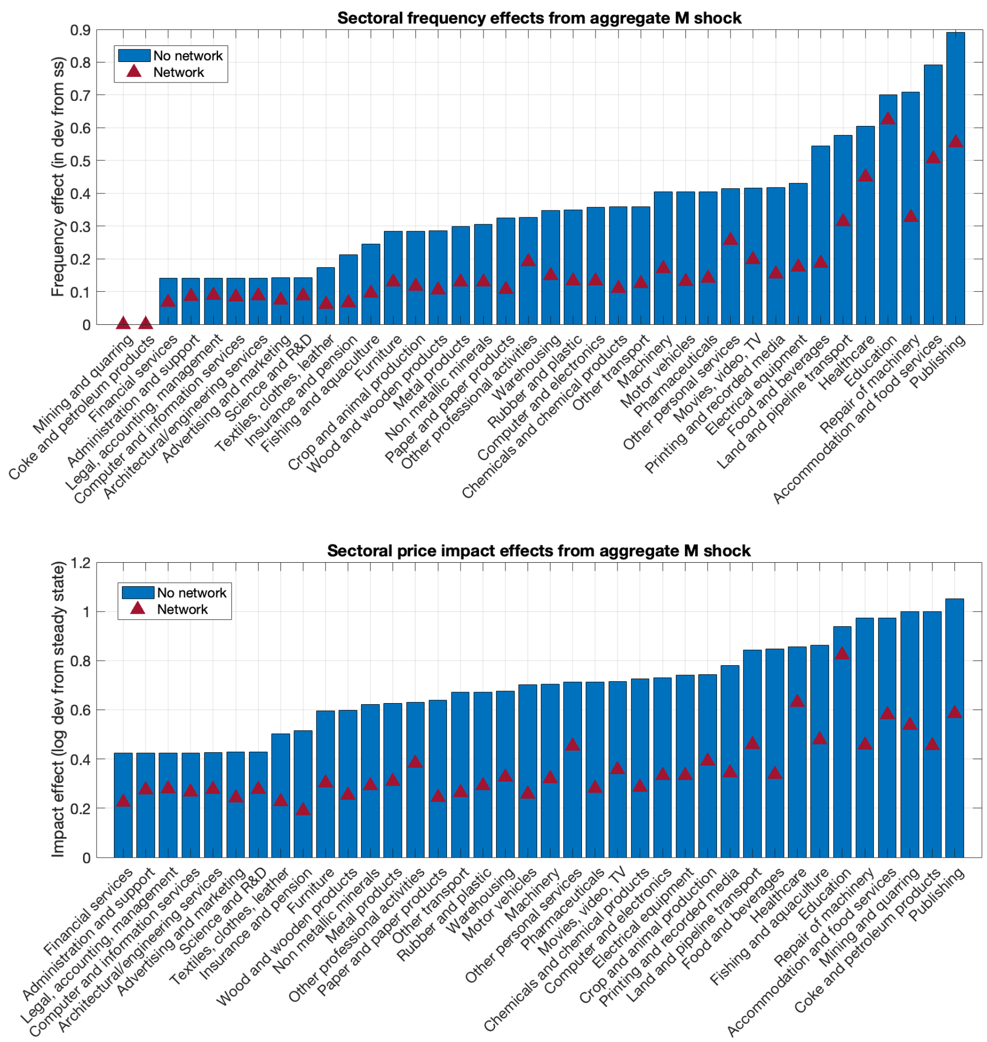
The per-period real profits of a firm are given by:

$$\tilde{D}_{i,t}(j) = \tilde{P}^{\epsilon-1}_{i,t} \left[(1-\tau_{i,t})\tilde{P}_{i,t}(j) - \tilde{\mathcal{Q}}_{i,t} \right] \tilde{P}_{i,t}(j)^{-\epsilon} \times \lambda_{i,t} \times C_t \times \Pi^C_t. \tag{D.11}$$

Finally, consider a firm with real quality-adjusted price p at the end of period t, and let $p_+ \equiv (p - \sigma_i \varepsilon_{i,t+1}(j) - \pi^C_t)$, where $\pi^C_t \equiv \log \Pi^C_t$. Then this firm's real value at the end of period t is given by the following Bellman equation:

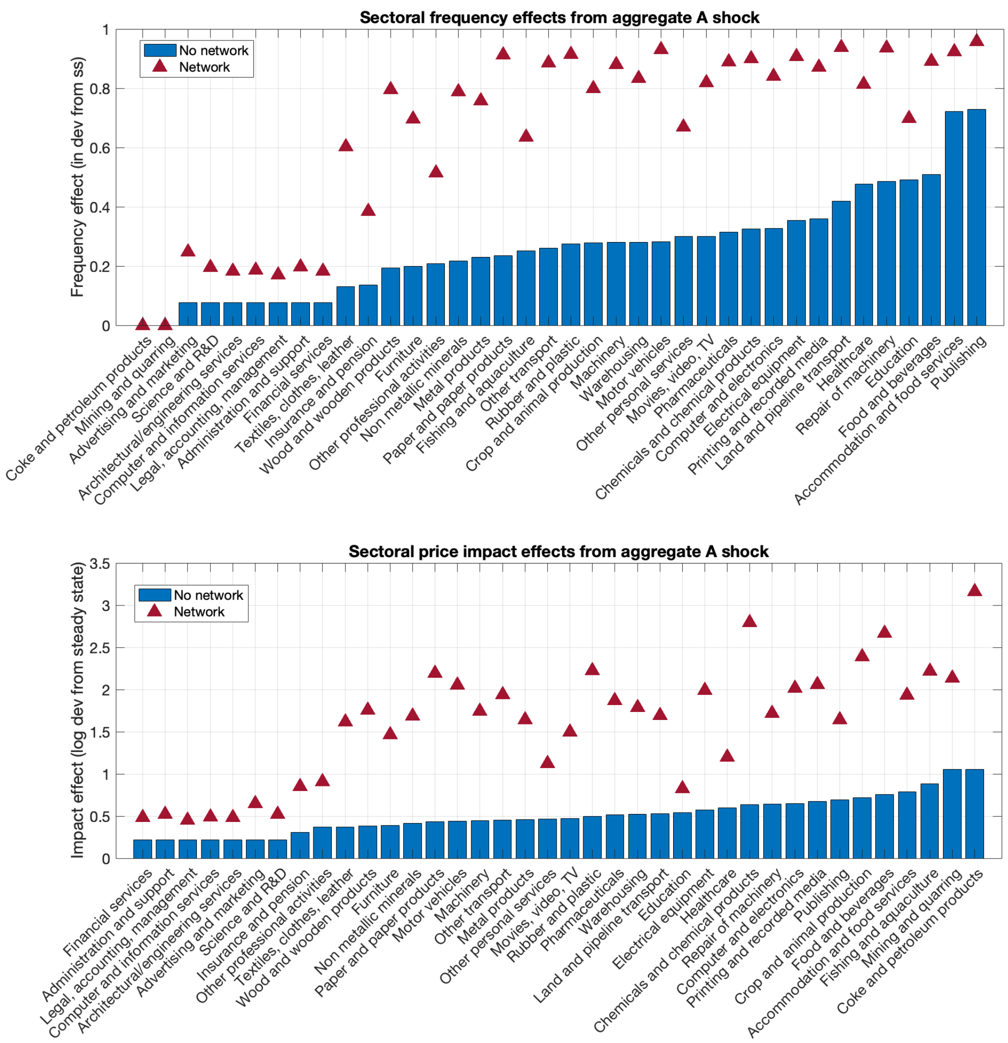
$$\begin{aligned} V_{i,t}(p) \quad = \quad & \tilde{D}_{i,t}(p) \quad + \\ & + \beta \mathbb{E}_t \left[\{1 - \eta_{i,t+1}(p_+)\} \frac{C^{-\sigma}_{t+1}}{C^{-\sigma}_t} \frac{\Pi^C_t}{\Pi^C_{t+1}} V_{i,t+1}(p_+) \right] + \\ & + \beta \mathbb{E}_t \left[\eta_{i,t+1}(p_+) \frac{C^{-\sigma}_{t+1}}{C^{-\sigma}_t} \frac{\Pi^C_t}{\Pi^C_{t+1}} \left(\max_{p'} V_{i,t+1}(p') - \kappa_{i,t+1} \tilde{W}_{t+1} \right) \right]. \end{aligned}$$

Figure D.1: **Sectoral responses to a monetary shock**



Notes: *the figure shows the responses of sectoral frequency and sectoral price indices to large aggregate TFP shocks.*

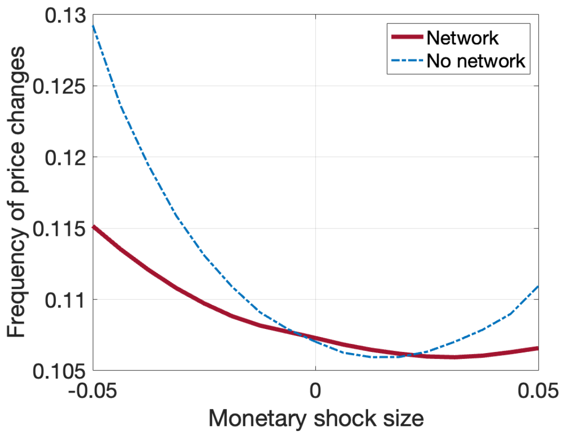
Figure D.2: **Sectoral responses to a aggregate TFP shock**



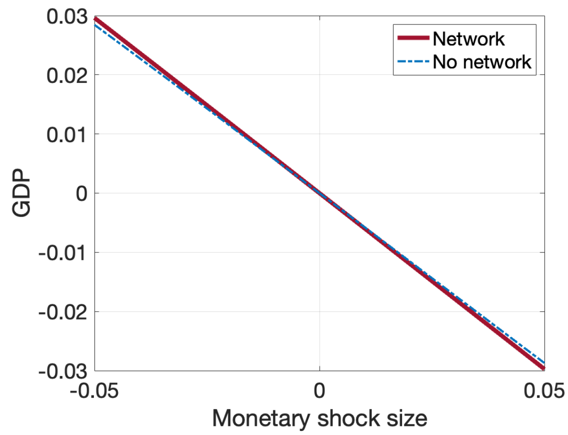
Notes: *the figure shows the responses of sectoral frequency and sectoral price indices to large aggregate TFP shocks.*

Figure D.3: **Frequency and GDP responses to monetary shocks: Taylor rule**

(a) Aggregate frequency response



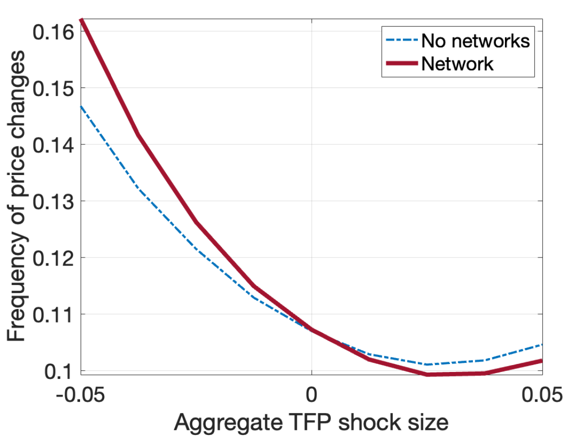
(b) Impact GDP response



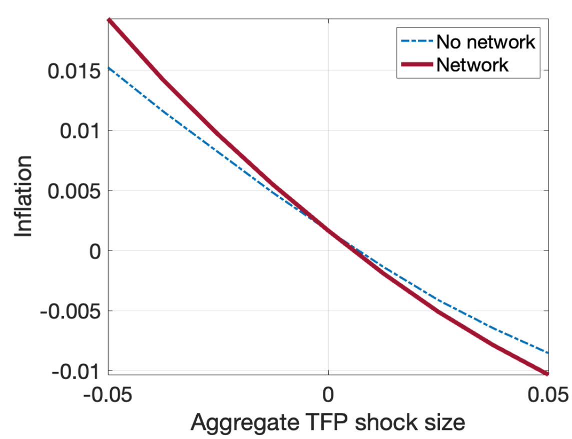
***Notes**: the figure shows the impact responses of aggregate frequency of adjustment to monetary shocks of different sizes, with and without networks.*

Figure D.4: **Frequency and inflation responses to agg. TFP shocks: Taylor rule**

(a) Aggregate frequency response



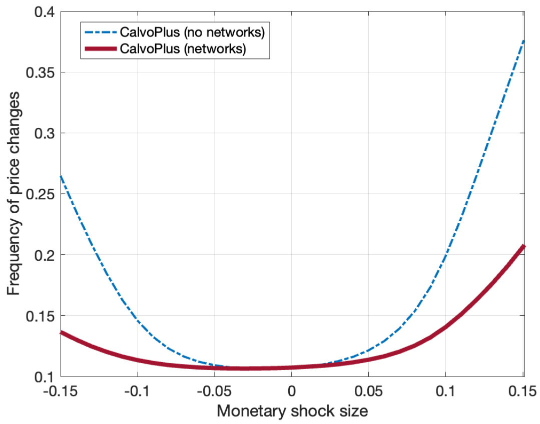
(b) Impact inflation response



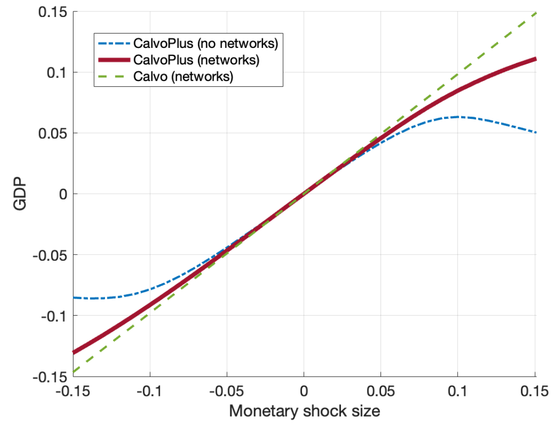
***Notes**: the figure shows the impact responses of aggregate frequency of adjustment to monetary shocks of different sizes, with and without networks.*

Figure D.5: **Frequency and GDP responses to monetary shocks: CalvoPlus**

(a) Aggregate frequency response



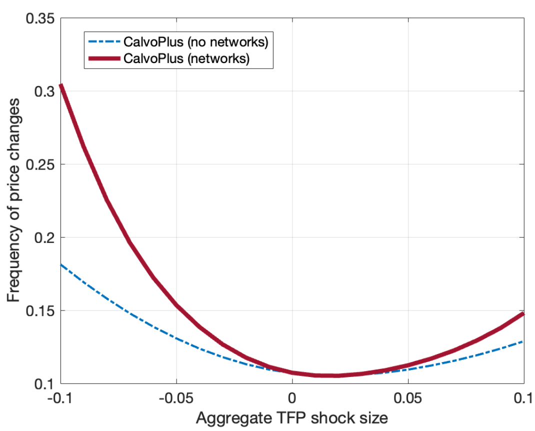
(b) Impact GDP response



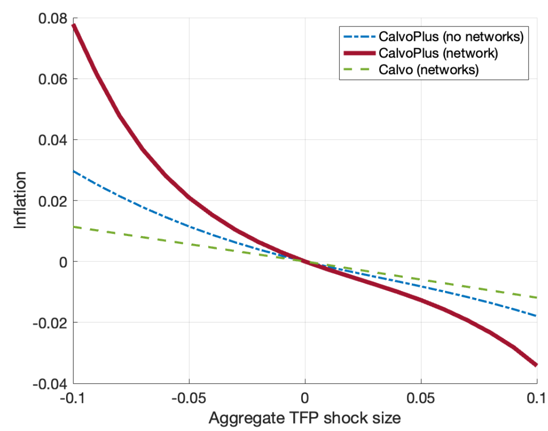
***Notes**: the figure shows the impact responses of aggregate frequency of adjustment to monetary shocks of different sizes, with and without networks.*

Figure D.6: **Frequency and inflation responses to agg. TFP shocks: CalvoPlus**

(a) Aggregate frequency response



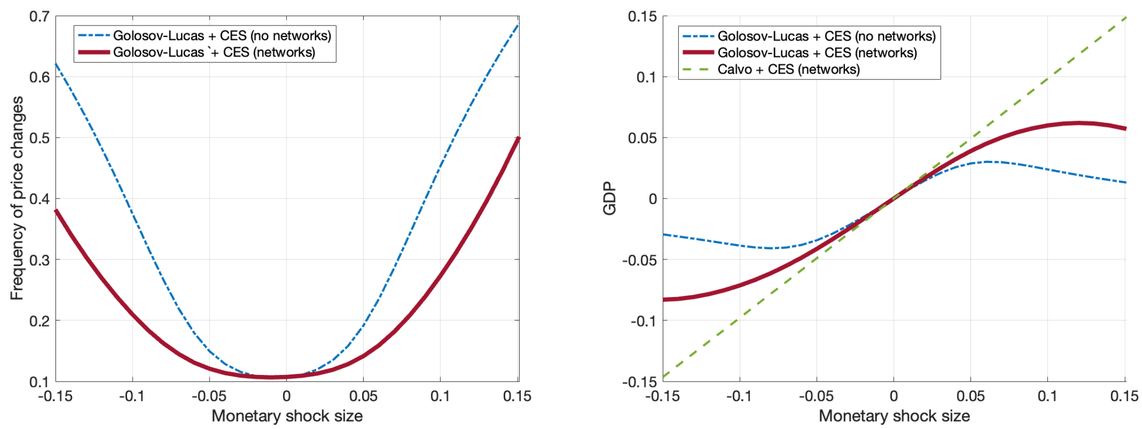
(b) Impact inflation response



***Notes**: the figure shows the impact responses of aggregate frequency of adjustment to monetary shocks of different sizes, with and without networks.*

Figure D.7: **Frequency and GDP responses to monetary shocks: CES aggregation**

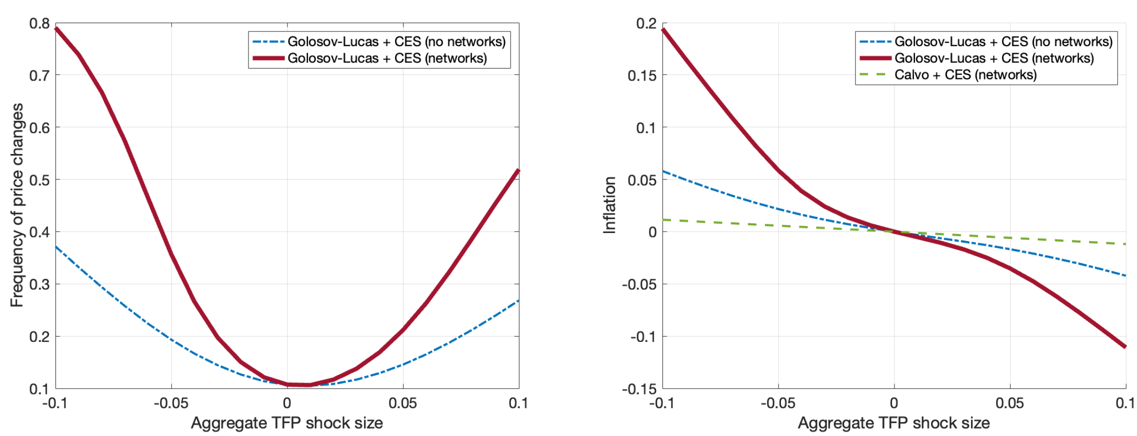
(a) Aggregate frequency response (b) Impact GDP response



***Notes**: the figure shows the impact responses of aggregate frequency of adjustment to monetary shocks of different sizes, with and without networks.*

Figure D.8: **Frequency and inflation responses to agg. TFP shocks: CES aggregation**

(a) Aggregate frequency response (b) Impact inflation response



***Notes**: the figure shows the impact responses of aggregate frequency of adjustment to monetary shocks of different sizes, with and without networks.*