

NBER WORKING PAPER SERIES

BANKING ON UNINSURED DEPOSITS

Itamar Drechsler  
Alexi Savov  
Philipp Schnabl  
Olivier Wang

Working Paper 31138  
<http://www.nber.org/papers/w31138>

NATIONAL BUREAU OF ECONOMIC RESEARCH  
1050 Massachusetts Avenue  
Cambridge, MA 02138  
April 2023, Revised June 2023

We thank Viral Acharya, Xavier Gabaix, Sam Hanson, Thomas Philippon, Matt Richardson, Andrei Shleifer, Jeremy Stein, Bruce Tuckman and seminar participants at NYU, Stanford, the Federal Reserve Board, UCLA, and IMF for helpful comments. The authors have nothing to disclose. The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

NBER working papers are circulated for discussion and comment purposes. They have not been peer-reviewed or been subject to the review by the NBER Board of Directors that accompanies official NBER publications.

© 2023 by Itamar Drechsler, Alexi Savov, Philipp Schnabl, and Olivier Wang. All rights reserved. Short sections of text, not to exceed two paragraphs, may be quoted without explicit permission provided that full credit, including © notice, is given to the source.

Banking on Uninsured Deposits

Itamar Drechsler, Alexi Savov, Philipp Schnabl, and Olivier Wang

NBER Working Paper No. 31138

April 2023, Revised June 2023

JEL No. E52,G12,G21

### **ABSTRACT**

We model the impact of interest rates on the liquidity risk of banks. Banks hedge the interest rate risk of their assets with their deposit franchise: when rates rise the value of their assets falls, but the value of their deposit franchise rises. Yet the deposit franchise is only valuable if deposits remain in the bank. This makes the deposit franchise runnable if deposits are uninsured. We show there is no run equilibrium at low interest rates, but a run equilibrium emerges as rates rise. This is because the value of the deposit franchise rises with rates, making a run more destructive, and hence more likely. To prevent a run, the bank needs to keep the value of its uninsured deposit franchise from exceeding its equity. It can do so by shortening the duration of its assets, so that their value falls less if rates rise. However, this undoes the bank's interest rate hedge, which can make it insolvent if rates fall. The uninsured deposit franchise therefore poses a risk management dilemma: the bank cannot simultaneously hedge its interest rate and liquidity risk exposures. We show banks can address the dilemma by buying claims with option-like payoffs to interest rates, or by raising additional capital as interest rates rise. These strategies minimize the additional capital needed to prevent a run if rates rise and avoid insolvency if rates fall.

Itamar Drechsler  
Wharton School  
University of Pennsylvania  
3620 Locust Walk  
Philadelphia, PA 19104  
and NBER  
idrechsl@wharton.upenn.edu

Philipp Schnabl  
Stern School of Business  
New York University  
44 West Fourth Street  
New York, NY 10012  
and NBER  
schnabl@stern.nyu.edu

Alexi Savov  
Stern School of Business  
New York University  
44 West Fourth Street, Suite 9-80  
New York, NY 10012  
and NBER  
asavov@stern.nyu.edu

Olivier Wang  
New York University  
Kaufman Management Center  
44 West Fourth Street  
New York, NY 10012  
olivier.wang@nyu.edu

# 1 Introduction

Why did the recent rise in interest rates destabilize a number of banks like Silicon Valley Bank (SVB)? One concern is that these banks were insolvent due to losses on long-term loans and securities. Yet, alongside these losses banks were making record profits on deposits by paying far below market rates. This is not by chance: prior work shows that banks invest in long-term assets to hedge the interest rate exposure of their deposit franchise (Drechsler et al., 2021). This explains why bank stocks held up well and tracked the market index as interest rates increased prior to SVB's failure, as Figure 1 shows. But if banks are hedged to interest rates, why *did* they become unstable?

We provide a model of the impact of interest rates on the liquidity risk of banks. We model a bank that issues deposits and invests in loans and securities. The bank has market power over deposits, which allows it to pay a deposit rate that rises less than one-for-one with the market interest rate. The amount by which it does so, the bank's "deposit beta," is thus less than one. The bank thus earns a deposit spread that rises in proportion with the interest rate. This is the source of its deposit franchise. The deposit franchise does not come for free: the bank pays an operating cost to maintain it. The deposit franchise is effectively an interest rate swap in which the bank pays fixed (the operating cost) and receives floating (the deposit spread). This swap has negative duration. The bank hedges it by investing in assets with positive duration; by holding long-term loans and securities.

For the deposit hedge to work, deposits must stay in the bank. If they leave, the deposit franchise is destroyed. This creates run incentives if deposits are uninsured. The run incentives intensify when the value of the deposit franchise rises relative to the value of the bank's assets. This happens when interest rates are high, because that is when deposit profits are large relative to the income from the bank's assets. Thus, the risk of a run increases with interest rates even though the bank is hedged to interest rates absent a run. The 2023 regional bank crisis can be understood as stemming from the impact of interest rates on the risk of a run on the deposit franchise by uninsured depositors.

We use the model to study bank risk management and capital regulation. We build on Drechsler et al. (2021) by introducing liquidity risk in the form of deposit outflows. They can be due to two reasons. The first is the direct impact of interest

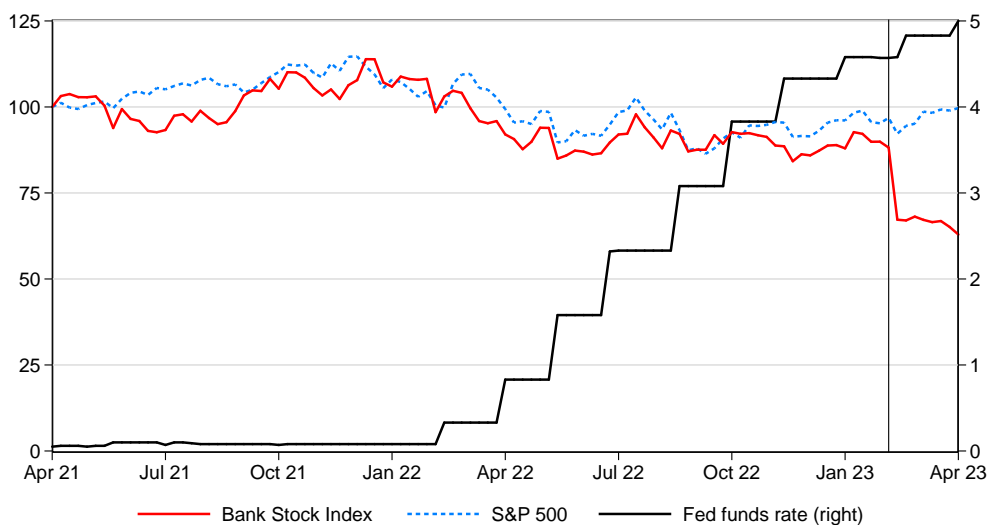


Figure 1: KBE Bank Stock Index (red, left scale), S&P500 (blue dashed, left scale) and Fed funds rate (black, right scale). Vertical line shows the failure of Silicon Valley Bank.

rates: as interest rates rise, banks widen their deposit spreads, which leads some depositors to shift toward higher-paying alternatives like money market funds. This is the “deposits channel” of monetary policy of [Drechsler et al. \(2017\)](#). Deposits channel outflows are relatively mild because the bank effectively chooses to have them by setting a profit-maximizing deposit spread. Nevertheless, we show that they shorten the negative duration of the deposit franchise relative to the no-outflows case. This leads the bank to choose a shorter asset duration. We provide a formula for how much; it shows that the bank should act as if its deposit beta is slightly higher than it actually is. Using estimates of the strength of deposits channel outflows from [Drechsler et al. \(2017\)](#), we find that an average bank with a beta of 0.3 should use an effective beta of 0.35. This is a small, but non-negligible, adjustment. The adjustment is larger for banks with more market power because their deposits channel outflows are bigger.

The second reason for outflows is a run by uninsured depositors. Uninsured depositors have an incentive to run if their deposits exceed the value of the bank’s assets in case they run. In standard models ([Diamond and Dybvig, 1983](#)), deposits can exceed asset values due to fire sales of the bank’s loans, which are illiquid. There are no such fire sales in our model; the bank’s loans and securities are fully

liquid. Runs are instead due to the nature of the deposit franchise, which is intangible. When a deposit is withdrawn, the bank loses the stream of deposit spreads it would have earned on that deposit. Hence, the deposit franchise is maximally exposed to Diamond-Dybvig runs because its recovery rate is zero. Moreover, since its value is increasing in interest rates, a run is more destructive – and hence more likely – at high interest rates.

Our model thus implies that a bank’s liquidity risk is increasing in interest rates. When interest rates are low, the value of the deposit franchise is small relative to the value of the bank’s assets. There is no incentive to run because it would have little effect on the bank. But when interest rates rise and the deposit franchise comes to dominate the value of the bank, a run equilibrium emerges. This is true even if the bank is fully hedged to interest rates outside of a run.

We stress that only uninsured depositors have an incentive to run, and hence the run risk only applies to the portion of the deposit franchise that comes from uninsured deposits. The larger is this portion, the more likely is a run.

Uninsured depositors do not need to understand the notion of franchise value to run. Once a run equilibrium exists, a run can be triggered by low earnings, a fall in the stock price, or even a rumor on social media. The key insight is that a run equilibrium only emerges at high interest rates. Once it does, even a small change in fundamentals or sentiment can trigger a run.<sup>1</sup>

To offset the impact of interest rates on liquidity risk, the bank can shorten its asset duration further. This limits the fall in the value of its assets when interest rates rise, and hence prevents the deposit franchise from becoming too large relative to the bank’s assets. To prevent a run using this approach, the bank again needs to act as if its deposit beta is higher than measured. However, this time the adjustment is much larger: the bank’s adjustment rises one-for-one with its share of uninsured deposits. Intuitively, the bank treats its uninsured deposits as if their beta is one, regardless of their actual beta.

Shortening duration to hedge liquidity risk has a downside: it undoes the bank’s interest-rate hedge. If interest rates fall instead of rising, the shorter-duration assets fail to appreciate enough to offset the decline in the value of the deposit fran-

---

<sup>1</sup>In the case of SVB the run appears to have been triggered by a large quarterly loss, which was the result of a reduction in deposits from the high-tech industry, SVB’s main customer base. The initial reduction in high-tech deposits was due to a down cycle in the high-tech industry that was independent of SVB.

chise. In cash flow terms, deposit spreads and interest income shrink such that the bank may not be able to cover its operating costs. If the drop in rates is large enough, the bank becomes insolvent. The bank turns into a “zombie bank”; it may continue to operate but its equity-holders are underwater and hence face distorted incentives. While the literature stresses regulatory forbearance, in our framework zombie banks arise due to unhedged exposure to a decline in interest rates.

At the heart of our model is a risk management dilemma: the bank can hedge itself to interest rates or liquidity risk, but not both. If it hedges to interest rates, it becomes exposed to a run if interest rates rise. If it hedges to liquidity risk, it becomes exposed to insolvency if rates fall. The dilemma disappears only if uninsured deposits do not contribute to the bank’s deposit franchise. This happens if they do not earn a deposit spread (their deposit beta is one), i.e., they are effectively wholesale funding. In this case the bank’s interest rate risk and liquidity risk management objectives align. The bank invests such deposits in short-term assets.

The risk management dilemma thus arises from low-beta uninsured deposits. This is interesting because it explains why previous interest rate cycles did not trigger similar instability. In the past, uninsured deposits were primarily large time deposits and other forms of wholesale funding. These deposits have a beta close to one and do not require expensive branch networks. As [Drechsler et al. \(2023c\)](#) show, this changed during the zero lower bound period when uninsured deposits flowed into low-beta checking and savings accounts.<sup>2</sup> It is this decoupling of interest rate and liquidity risk that creates the bank’s risk management dilemma. Our framework therefore implies that uninsured checking and savings accounts pose an ongoing risk to the banking system.

How can banks or regulators address this risk? First, we show that banks benefit from investing in assets with a convex payoff with respect to interest rates. For example, they can embed option features such as interest rate floors in their loans, or buy interest rate options such as swaptions or puttable bonds.<sup>3</sup> These convex

---

<sup>2</sup>In the case of Silicon Valley Bank (SVB), almost all of its deposits were uninsured corporate checking and savings accounts. These had a low beta and a high operating cost, which calls for a long asset duration. However, because they were uninsured, their liquidity risk calls for a short duration. One interpretation of SVB’s failure is that it ignored the second consideration. It also likely overdid the first, see [U.S. Federal Reserve \(2023\)](#).

<sup>3</sup>The buyer of a swaption has the right to enter into an interest rate swap at a fixed rate. The buyer of a receiver swaption receives the fixed rate and pays floating. The buyer of a payer swap-

payoffs ensure that the bank's interest income rises enough to deter a run if interest rates rise, while remaining sufficiently high to keep the bank solvent if interest rates fall. Of course, options require capital up-front to pay for the premium. Banks may resist raising this capital if they face equity issuance costs. In this case, regulators may need to require them to do so as part of liquidity regulation.

An alternative approach to addressing the bank's risk management dilemma is to directly adjust equity capital as interest rates change. A large enough capital buffer deters a run and allows the bank to set its duration to hedge its interest rate risk. We solve for the optimal capital buffer and find it is "rate-cyclical", i.e. it increases with interest rates. This is because the bank's liquidity risk increases with interest rates. This result provides a new rationale for time-varying capital buffers. Different from the literature, it is one based purely on interest rates, not the state of the economy.

We show that the bank needs more options or rate-cyclical capital if it faces other unhedged risk exposures, for example to credit shocks. The reason is that these shocks are more likely to trigger a run at high interest rates, when the bank's liquidity risk is already high. Our model implies that credit losses are themselves more damaging at high interest rates.

The banks' risk management dilemma can also be addressed by a lender of last resort. The central bank must be willing to lend against the banks' uninsured deposit franchise. Doing so deters a run but raises concerns about bank moral hazard. An example is the Bank Term Funding Program introduced in March 2023. Under this program, the Federal Reserve lent against banks' mortgage backed securities up to their par value. Since the market value of these securities was below their par value, this can be interpreted as effectively lending against the banks' deposit franchise.

What are the implications for the central bank's interest rate policy? If banks are hedging interest rates, an increase in interest rates can create financial instability. On the other hand, if they are hedging liquidity risk, an interest rate cut can make them insolvent. This reinforces the case for options-based or capital regulation because it allows banks to hedge interest rates without opening the door to

---

tion pays fixed and receives floating. To implement risk management using swaptions, the bank should buy long-term assets and payer swaptions, or equivalently, short-term assets and receiver swaptions.

runs. Otherwise, the banks' risk management dilemma creates a financial stability dilemma for the central bank in setting interest rates.

## 2 Related Literature

Our paper belongs to the literature on banks' liquidity risk and interest rate risk management (e.g., Freixas and Rochet 2008, English et al. 2018, Nagel and Puranandam 2020, Gomez et al. (2021), Di Tella and Kurlat 2021). Closest is Drechsler et al. (2021), which shows that when banks have deposit market power, deposits are effectively a long-term liability that should be hedged with long-term assets. Drechsler et al. (2021) focuses on a simplified setting without deposit outflows. Our paper starts from the observation that there are two kinds of outflows that affect the value of the deposit franchise. First, interest rate increases induce outflows when banks do not adjust deposit rates one-for-one; this is the "deposits channel" of monetary policy (Drechsler et al., 2017). The outflows flow into higher-rate substitutes such as money market funds (Xiao, 2020).<sup>4</sup> While these outflows are important to consider and call for a shorter asset duration, they are "smooth" and hence manageable compared to the second kind of outflows: runs on uninsured deposits. As Hanson et al. (2015) argue, these types of outflows have a larger impact on the ability of financial institutions to invest in long-term assets.

Our analysis of runs on the deposit franchise builds on Diamond and Dybvig (1983) and the subsequent theoretical literature, reviewed extensively in Gorton and Winton (2003) and Allen and Gale (2009). In line with recent theoretical work on optimal liquidity regulation (Diamond and Kashyap 2016, Dewatripont and Tirole 2018), we add a hedging perspective by studying what assets the bank should hold to prevent runs, and how liquidity risk management is in tension with interest rate risk management.<sup>5</sup>

The key new point in our paper is that an uninsured deposit franchise is a large but runnable part of a bank's value. This creates important differences with

---

<sup>4</sup>Other recent models of deposit outflows include Bolton et al. (2020) and Jermann and Xiang (2023).

<sup>5</sup>Other recent work includes empirical analyses of bank runs (e.g. Iyer and Puri 2012, Iyer et al. 2016) and liquidity risk more broadly (Bai et al., 2018), structural estimation of run incentives (Egan et al., 2017), and embedding runs in macroeconomic models (Gertler and Kiyotaki 2015, Amador and Bianchi 2021).



the standard source of runs in the literature: asset illiquidity as in [Diamond and Dybvig \(1983\)](#). Runs in our model do not rely on such illiquidity, hence they can happen even if a single bank is affected and if it holds only high quality liquid assets (as SVB did). Moreover, in our framework run risk endogenously rises with interest rates. This is important because it affects how banks should set their asset duration and how their capital ratios should adjust with interest rates. Another implication is that bank runs are less likely in a low-rate environment.

Our emphasis on the deposit franchise as an important part of banks' business model builds on the large literature on the role of banks as providers of liquid assets, e.g., [Gorton and Pennacchi \(1990\)](#), [Kashyap et al. \(2002\)](#), [Stein \(2012\)](#), [DeAngelo and Stulz \(2015\)](#), [Hanson et al. \(2015\)](#), [Dang et al. \(2017\)](#), [Moreira and Savov \(2017\)](#), [Egan et al. \(2021\)](#), [Donaldson and Piacentino \(2022\)](#). Motivated by the zero lower bound period after the 2008 financial crisis, recent work has focused on the harmful effect of persistently low interest rates on the deposit franchise (e.g., [Abadi et al. 2022](#), [Ulate 2021](#), [Wang 2022](#), [Wang et al. 2022](#)). This channel is why short asset durations are problematic in our model.

Our paper is motivated by the 2023 regional bank crisis, which was sparked by asset losses due to an increase in interest rates as opposed to credit shocks. This makes it reminiscent of the Savings & Loans (S&L) crisis of the 1980s. S&Ls held long-duration mortgages, which they hedged with near-zero beta deposits due to the deposit rate ceilings of Regulation Q. When Reg Q was repealed at the end of the 1970s, S&Ls' deposit betas shot up, making them insolvent (FDIC, [1997](#), [Drechsler et al., 2022](#)).

For the current crisis, [Jiang et al. \(2023\)](#) use detailed bank data and estimate a total decline in asset values of \$2.1 trillion. They study several failure scenarios depending on the extent of mark-to-market losses, banks' capitalization, and depositors' withdrawals. [Drechsler et al. \(2023b\)](#) find a similar decline in asset values using aggregate data. [Drechsler et al. \(2023a\)](#) estimate that the increase in the value of the deposit franchise roughly offsets the decline in asset values though they emphasize that this valuation is uncertain given the behavioral assumptions on depositors.

Finally, our paper also brings interesting connections to the international finance literature on reserve assets and the [Triffin \(1961\)](#) dilemma (more recently, [Farhi and Maggiori, 2017](#), and [He et al., 2019](#)). [Triffin \(1961\)](#) warned that persis-

tent U.S. current account deficits would ultimately lead to a collapse of the dollar, while [Despres et al. \(1966\)](#) argued that the U.S. deficits could remain stable as they were implicitly backed by the present value of risk premia on its external assets and, closer to us, liquidity premia on its liabilities. In this sense, one can think of U.S. debt as the world's uninsured deposit franchise.

### 3 A Model of the Deposit Franchise with Outflows

We model a bank that seeks to hedge the interest rate risk and liquidity risk of its deposit franchise. We begin by focusing on interest rate risk by assuming that all deposits are insured. Since insured depositors have no incentive to run, the bank does not face liquidity risk.

The deposit franchise is the present value of the profits the bank earns on its deposits. These profits come from paying below-market rates on deposits, i.e. from charging a deposit spread. When interest rates increase, the deposit spread widens and the value of the deposit franchise rises. It does so by less if the rise in rates triggers deposit outflows. Outflows can happen either because depositors substitute to other assets like money market funds, or because they are uninsured and become concerned about the bank's solvency. We start with the first type of outflows and introduce the second in [Section 4](#).

**Timing.** Time is discrete,  $t = 0, 1, \dots$ . The initial interest rate is  $r$ . Then at the end of period  $t = 0$ , an interest rate shock is realized: the interest rate becomes  $r'$ , which can be above or below  $r$ , and remains constant forever after.<sup>6</sup> In [Section 4.3](#) we introduce other shocks such as credit losses.

At the beginning of  $t = 0$ , before the interest rate shock is realized, the bank chooses a portfolio of assets to hedge against the shock, taking into account expected deposit withdrawals.

**Deposit outflows.** The bank starts with a deposit base  $D_{-1} = D$ . It faces interest-rate driven deposit outflows at the time of the interest rate shock. Thus, at the end

---

<sup>6</sup>In the context of the 2023 regional bank crisis, the shock is an increase in interest rates that hurts the value of banks' assets. As we explain below, banks must also hedge against decreases in rates that hurt the profitability of their deposit franchise.

of  $t = 0$ , after the interest rate  $r'$  and deposit rate  $r'_d$  (specified below) are realized, depositors withdraw a fraction  $\omega(s'_d, r')$ , i.e.,

$$D_0 = [1 - \omega(s'_d, r')] D,$$

where  $s'_d = r' - r'_d$  is the deposit spread set by the bank. The outflow rate  $\omega$  is increasing in the deposit spread  $s'_d$  and, holding  $s'_d$  fixed, is decreasing in  $r'$ . The two arguments capture the “deposits channel” of monetary policy (Drechsler et al., 2017): depositors withdraw if the deposit rate  $r'_d$  is too low relative to  $r'$ , in order to substitute toward higher-yield assets such as money market funds. The second argument captures the fact that deposit demand becomes less elastic at higher rates as the opportunity cost of cash rises. The lower elasticity of deposit demand allows banks to charge a larger spread  $s'_d$  when rates  $r'$  are high, as we see in practice.

In addition to the interest rate-driven outflows  $\omega$ , in periods  $t \geq 1$  the bank experiences exogenous outflows at a rate  $\delta$ :

$$D_t = (1 - \delta)^t D_0,$$

hence deposits have an average maturity  $1/\delta$ . These outflows capture the natural decay of a bank’s deposit franchise in the absence of further fixed investment.

The bank meets its deposit outflows by selling assets of equal value.

**Deposit pricing and operating costs.** The bank sets its deposit rate  $r'_d$  after the realization of the  $r'$  shock, according to a deposit pricing function  $r_d(r')$ . We take as given a linear pricing function

$$r_d(r') = \beta r',$$

where  $\beta \in [0, 1]$  is the “deposit beta.” Banks in practice issue different types of deposits (e.g., checking, savings, time), hence  $\beta$  is the average beta across all products. The deposit pricing policy and thus  $\beta$  are chosen endogenously by the bank, for instance to maximize profits on deposits, but when analyzing hedging we can take it as given.

Given  $\beta$ , the date-0 withdrawal rate can be rewritten as a function of  $r'$  only:

$$w(r') \equiv \omega((1 - \beta)r', r').$$

We normalize date-0 outflows without the interest rate shock to  $w(r) = 0$ . We discuss below the relation between  $w$  and  $\beta$  arising from banks' optimal deposit pricing policy, and how our results extend to a nonlinear pricing policy  $\beta(r')$ .

The ability to charge a deposit spread does not come for free. The bank must pay an operating cost of  $c$  per dollar of deposits starting from period  $t = 1$ . This operating cost is again averaged across all deposit products. In Appendix B we extend the framework to allow for fixed costs  $\kappa$  that must be paid even if deposits are withdrawn, in addition to the variable cost  $c$ .

We adopt the timing convention that date- $t$  interest expenses and operating costs are paid on the outstanding deposits  $D_{t-1}$  at the start of the period, while withdrawals take place at the end of the period.

### 3.1 Valuing the Deposit Franchise

The bank uses the deposits it has and any additional equity it has raised to buy assets with value  $A$ .<sup>7</sup> Then at  $t = 0$ , after the realization of the shock  $r'$ , the market value of the bank is

$$V = A - L,$$

where  $A = A(r')$  is the total market value of assets such as loans and securities as a function of the market interest rate  $r'$ , and  $L = L(r')$  is the market value of liabilities. This value is the present value of all future deposit withdrawals, interest expenses, and operating costs:

$$L(r') = X_0 + \sum_{t=1}^{\infty} \frac{X_t}{(1+r')^t} + \sum_{t=1}^{\infty} \frac{(\beta r' + c) D_{t-1}}{(1+r')^t}, \quad (1)$$

where  $X_t = D_{t-1} - D_t$  are outflows in period  $t$  and are given by

$$\begin{aligned} X_0 &= w(r')D, \\ X_t &= \delta(1-\delta)^{t-1} [1 - w(r')] D \quad \text{for } t \geq 1. \end{aligned}$$

The first term in (1) corresponds to date-0 withdrawals that depend on the interest rate shock, and the second term to exogenous withdrawals in periods  $t \geq 1$ .

---

<sup>7</sup> $A$  denotes assets after the cost to acquire and maintain the initial deposit base  $cD$  as well as the interest expense  $\beta rD$  have been paid, as these do not depend on  $r'$ .

The third term captures interest expenses and operating costs on the remaining deposits in each period.

It is convenient to rewrite the value of the bank as

$$V = A - D + DF,$$

where  $DF$  is the value of the bank's deposit franchise:

$$DF = D - L.$$

The deposit franchise is an intangible asset that arises from the bank's ability to pay below-market rates on deposits. To see why, consider a bank that pays the market rate ( $\beta = 1$ ) and incurs no operating cost ( $c = 0$ ), e.g. a money market fund. In this case (1) gives  $L = D$ , hence the value of the deposit franchise is  $DF = 0$ . The value of the bank is then just  $V = A - D$ .<sup>8</sup>

Our first main result shows why the deposit franchise provides a natural hedge against interest rate increases and how rate-driven outflows reduce this hedge:

**Proposition 1.** *The value of the deposit franchise at  $r'$  is*

$$DF(r') = D [1 - w(r')] \left[ \frac{(1 - \beta)r' - c}{r' + \delta} \right]. \quad (2)$$

*The modified duration of the deposit franchise at  $r$ , defined when  $DF(r) \neq 0$ , is*

$$T_{DF}(r) \equiv -\frac{DF'(r)}{DF(r)} = -\frac{1}{r + \delta} \left[ \frac{c + (1 - \beta)\delta}{(1 - \beta)r - c} \right] + w'(r). \quad (3)$$

Equation (2) is a simple valuation equation similar to a Gordon Growth formula. The first term,  $D [1 - w(r')]$ , is the size of the deposit base after the shock. The second term is the net cash flow,  $(1 - \beta)r' - c$  (the deposit spread net of operating costs) divided by the discount rate  $r'$  minus the growth rate  $-\delta$ . The deposit franchise value can be positive or negative, depending on the interest rate  $r'$ . It is positive when  $r'$  is high enough so the deposit spread exceeds the operating cost,  $(1 - \beta)r' > c$ . But importantly,  $DF$  can become negative if rates fall enough and  $(1 - \beta)r' < c$ .<sup>9</sup>

---

<sup>8</sup>We focus on the role of banks as deposit providers and abstract from the loan franchise value coming from, e.g., monitoring (Diamond, 1984) or relationship lending (Petersen and Rajan, 1994).

<sup>9</sup>In a richer model with a yield curve that is not perfectly flat the deposit franchise can remain

Equation (3) shows the duration of the deposit franchise. Suppose first that there are no rate-driven outflows,  $w(r') = 0$ . Then, assuming  $DF(r) > 0$ , the deposit franchise has *negative duration*, i.e. it appreciates when interest rates rise.<sup>10</sup> There are two reasons. First, a higher  $r'$  lowers the present value of operating costs  $c$ . Second, a higher  $r'$  raises the present value of deposit spreads  $(1 - \beta)r'$  as long as  $\delta > 0$ . In the limit  $\delta \rightarrow 0$  as in Drechsler et al. (2021), the present value of spreads is  $1 - \beta$  and independent of  $r'$ , and only the cost effect remains. But for plausible parameter values, the increase in the present value of deposit spreads is the dominant force: our baseline calibration described below implies  $(1 - \beta)\delta = 7\%$  which is several times larger than  $c = 1.5\%$ .

Rate-driven outflows,  $w'(r) > 0$ , act in the opposite direction to raise the duration of the deposit franchise. If the bank loses deposits as rates rise, this hurts its value if those deposits are profitable, i.e.  $(1 - \beta)r > c$ . One extreme case is when  $w(r')$  jumps to 1 ( $w'(r) \rightarrow \infty$ ), which corresponds to a run at  $t = 0$ . In this case the deposit franchise value falls to 0.

In the rest of the paper we focus on the case when the deposit franchise has negative duration,  $T_{DF}(r) < 0$ . From (3), this requires that rate-driven outflows  $w'(r)$  are not too large:

$$\frac{c + (1 - \beta)\delta}{r + \delta} \geq w'(r) [(1 - \beta)r - c]. \quad (4)$$

This condition says that when rates rise the bank gains more on the deposits that remain than it loses on the deposits that leave. As we discuss below, this condition is easily satisfied in the data.

### 3.2 Hedging Interest Rate Risk with Rate-Driven Outflows

Suppose that the bank seeks to hedge interest rate risk, by making  $V$  insensitive to the shock  $r'$ , for instance to avoid costs of financial distress in case  $V$  becomes too low or negative. Interest rate risk can be hedged by holding assets  $A$  whose

---

valuable even if deposit spreads are temporarily below operating costs as long as they are expected to eventually rise above them.

<sup>10</sup>More generally, even if  $DF(r) < 0$  then  $DF'(r) > 0$  and an increase in rates raises the deposit franchise value.

duration offsets the negative duration of the deposit franchise, i.e., such that

$$dV = [A'(r) + DF'(r)] dr = 0.$$

Letting  $T_A$  be the modified duration of the bank's assets, this requires

$$T_A = -\frac{DF}{A} T_{DF}.$$

The bank must set its asset duration  $T_A$  to the negative of its deposit franchise duration  $T_{DF}$ , taking into account the relative size of the deposit franchise to its assets,  $DF/A$ . Doing so immunizes the value of its equity  $V$  to interest rate shocks. The following proposition focuses on the natural case when  $A(r) = D$ , i.e. the bank has raised  $D$  deposits and purchased assets of equal value. In this case its equity is  $V = DF(r)$ .

**Proposition 2** (Hedging with Deposit Outflows). *The modified asset duration that hedges interest rate risk for a bank with initial assets equal to deposits,  $A(r) = D$ , is*

$$T_A = \frac{1}{r + \delta} \left( \frac{c + (1 - \beta) \delta}{r + \delta} - w'(r) [(1 - \beta) r - c] \right). \quad (5)$$

For hedging purposes, rate-driven outflows  $w'(r) > 0$  are equivalent to assuming  $w'(r) = 0$  with an effective deposit beta

$$\tilde{\beta} = \beta + w'(r) [(1 - \beta) r - c] (1 + r/\delta). \quad (6)$$

The effective beta is higher than the actual beta,  $\tilde{\beta} > \beta$ , if and only if the deposit franchise is valuable,  $(1 - \beta)r > c$ .

Proposition 2 shows that the optimal asset duration for hedging is positive given condition (4) that rate-driven outflows are not too large. The reason is that the bank must protect itself against a decline in interest rates which hurts the deposit franchise, as in Drechsler et al. (2021). A positive asset duration hedges the bank to such a decline.

Proposition 2 further shows that rate-driven outflows  $w'(r) > 0$  reduce asset duration whenever deposits are profitable,  $(1 - \beta)r > c$ . These outflows limit the appreciation of the deposit franchise when interest rates rise. To offset this, the bank makes sure its assets decline by less, i.e. it chooses a shorter asset duration.

Equation (6) gives another way of looking at the impact of outflows. It shows that when deposits are profitable, a bank facing outflows should act as if its deposit beta is higher than it actually is. The effective beta,  $\tilde{\beta}$ , captures the additional expense the bank faces from rate-driven outflows. The bank has to make sure it generates enough income to cover this expense if interest rates rise, i.e. it needs a shorter asset duration.

A surprising implication of (5) is that the impact of outflows on asset duration reverses sign at low rates. When rates are so low that deposits are unprofitable,  $(1 - \beta)r < c$ , rate-driven outflows call for *longer* asset duration. The reason is that they benefit the bank by shrinking its unprofitable deposit business. The bank hedges this increase in value by choosing a longer asset duration.

The positive impact of outflows on duration at low rates is particularly stark because we assume that all operating costs are variable. When depositors leave, the bank recoups the costs it would have paid to service them. In Appendix B we introduce fixed costs that still need to be paid even if depositors leave. We show that if the fixed costs are large enough, outflows always shrink asset duration.<sup>11</sup>

**Endogenous deposit pricing and the relation between  $\beta$  and  $w'$ .** Our results take advantage of the fact that the endogenous variables  $\beta$  and  $w'$  can be used as “sufficient statistics” for the bank’s hedging problem. In Section 3.3 below we use estimates from Drechsler et al. (2017) to calibrate them. Here we discuss how they are related.

While  $\beta$  depends on  $w'$  through the bank’s deposit pricing problem, and  $w'$  depends on  $\beta$  through the deposit demand function, in general neither is fully pinned down by the other, which is why we treat them as separate. Indeed, without additional restrictions on the deposit demand function  $\omega(s_d, r)$ , the correlation between  $\beta$  and  $w'(r)$  can take any sign, as  $w'$  is given by

$$w' = (1 - \beta)\omega_s + \omega_r,$$

where  $\omega_s = \partial\omega/\partial s_d$  and  $\omega_r = \partial\omega/\partial r$ . Banks facing relatively inelastic depositors

---

<sup>11</sup>It is difficult to know the mix of fixed and variable costs in practice. Some costs, like branches, are relatively fixed while others, like transaction services, are variable. One component of variable costs are regulations like the supplementary leverage ratio (SLR), which led large banks to turn away deposits in 2021 when rates were low and deposits were not profitable (see Bolton et al., 2020).



(low  $\omega_s$ ) have more market power, which allows them to set a low  $\beta$ . But given their low  $\beta$ , these banks could see more or less outflows when rates go up, as  $w'$  depends on the product  $(1 - \beta)\omega_s$ . Empirically, [Drechsler et al. \(2017\)](#) find a negative correlation between  $\beta$  and  $w'$ : low  $\beta$  banks face stronger rate-driven outflows  $w'$ .

**Variable deposit beta.** We can easily extend the model to allow for non-linear deposit pricing with a deposit beta  $\beta(r)$  that depends on  $r$ . In that case, the optimal modified duration is

$$T_A = T_A|_{\beta'(r)=0} - \beta'(r) \frac{r}{r + \delta},$$

where  $T_A|_{\beta'(r)=0}$  is given by (5). Betas tend to increase with rates: as rates rise the composition of deposits shifts from low-beta checking and savings accounts to higher-beta time deposits ([Drechsler et al., 2017](#)); betas also increase for given products ([Wang, 2022](#)). An increasing beta,  $\beta'(r) > 0$ , makes the duration of the deposit franchise less negative and hence calls for a shorter asset duration. Unlike for outflows, this duration-shortening effect is present at both low and high rates because an increase in  $\beta$  always hurts the deposit franchise value, even if it is negative.

### 3.3 Quantification: U.S. Banks on the Eve of the Regional Bank Crisis

Total U.S. deposits  $D$  stood at \$17.5 trillion at the end of 2022. We use an average deposit maturity  $1/\delta = 10$  years, deposit beta  $\beta = 0.3$ , and operating cost  $c = 1.5\%$ .<sup>12</sup> Given these assumptions, the aggregate value of the deposit franchise of all U.S. banks with a long-term interest rate of  $r = 4\%$  is

$$DF = \$1.6 \text{ trillion,}$$

---

<sup>12</sup>For maturity  $1/\delta$ , we use a recent FDIC study, see <http://fdic.gov/regulations/reform/coredeposit-study.pdf>. The number we use, 10 years, is based on the last column of Table 5. Note that it is in the lower range of estimates. For the recent betas, see [https://www.gobaker.com/wp-content/uploads/articles/TBG-B2111-Article\\_Series.pdf](https://www.gobaker.com/wp-content/uploads/articles/TBG-B2111-Article_Series.pdf). For the operating cost, see [Drechsler et al. \(2021\)](#).

up from  $DF = -\$680$  billion in early 2021 when  $r$  was 1.5%. The increase in banks' deposit franchise value is similar to their unrealized losses on loans and securities. Banks had \$17 trillion of loans and securities with an average duration of 3.9 years, hence a 2.5% increase in interest rates leads to a loss of \$1.7 trillion ( $= .025 \times 3.9 \times 17$ ).<sup>13</sup>

The strength of the rate-driven outflows  $w'$  can be calibrated using estimates from Drechsler et al. (2017). They show that the typical 400 bps Fed hiking cycle corresponds to a 12% outflow, hence  $w' \approx 3$ .<sup>14</sup> With these numbers, the upper bound on  $w'$  in (4) is easily satisfied: the left-hand side of (4) is 0.6, an order of magnitude larger than the right-hand side of 0.04.

Figures 2 and 3 show the optimal asset duration and effective beta  $\tilde{\beta}$  as functions of  $w'$  for three values of the deposit beta. For an average bank with  $\beta = 0.3$ , going from  $w' = 0$  to  $w' = 3$  implies reducing asset duration from 4.5 years to 4.2 years, or equivalently using an effective beta of

$$\tilde{\beta} = 0.35.$$

The typical correction  $\tilde{\beta} - \beta$  is not negligible. There is also substantial heterogeneity across banks. Most importantly, equation (6) shows that the correction  $\tilde{\beta} - \beta$  is higher for low- $\beta$  banks (as they earn higher deposit spreads hence stand to lose more from outflows) and high  $w'$  banks, which also tend to be the low- $\beta$  banks (Drechsler et al., 2017). For instance, their estimates imply that going from  $\beta = 0.2$  (high market power) to  $\beta = 0.4$  (low market power), outflow betas decline from  $w' = 3.5$  to  $w' = 2.5$ . Therefore, the effective betas for setting asset duration are

$$\beta = 0.2 \rightarrow \tilde{\beta} = 0.28 \quad (\text{high market power})$$

$$\beta = 0.4 \rightarrow \tilde{\beta} = 0.43 \quad (\text{low market power})$$

Thus correcting for outflows has a modest effect for competitive banks, but a strong effect for banks with high deposit market power.<sup>15</sup>

<sup>13</sup>These numbers are examples meant to give a sense of the magnitudes. Exact values of  $DF$  depend on estimates of  $\beta$  and  $c$ ; for instance using a lower deposit beta  $\beta = 0.2$  (as was the case in 2022) implies  $DF = \$2.1$  trillion, while using a higher cost  $c = 2\%$  implies  $DF = \$1$  trillion.

<sup>14</sup>The numbers for the current hiking cycle are similar. Between March 2022 and May 2023, the Fed funds rate rose by 5% and deposits declined by 9%. This gives  $w' \approx 1.8$ . Deposit outflows tend to lag, hence  $w'$  is likely to increase toward the historical norm.

<sup>15</sup>Interestingly, Drechsler et al. (2021) show that banks' income is slightly more interest-sensitive

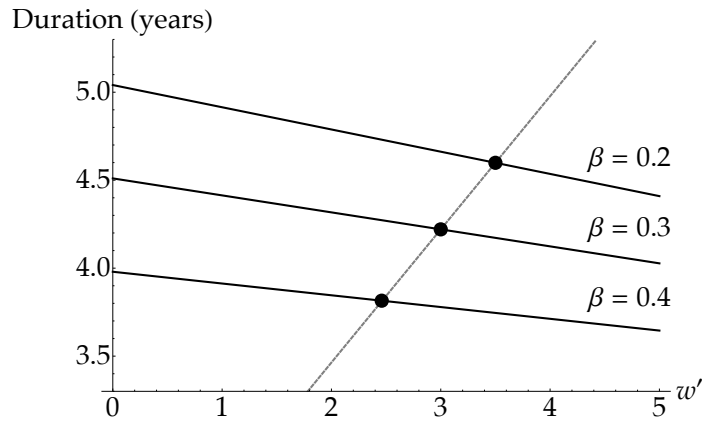


Figure 2: Optimal hedging duration  $T_A \times (1 + r)$  as a function of outflow elasticity  $w'$  for three values of rate beta  $\beta = 0.2, 0.3, 0.4$ . The dotted gray line captures the negative correlation between  $\beta$  and  $w'$  estimated by Drechsler et al. (2017).

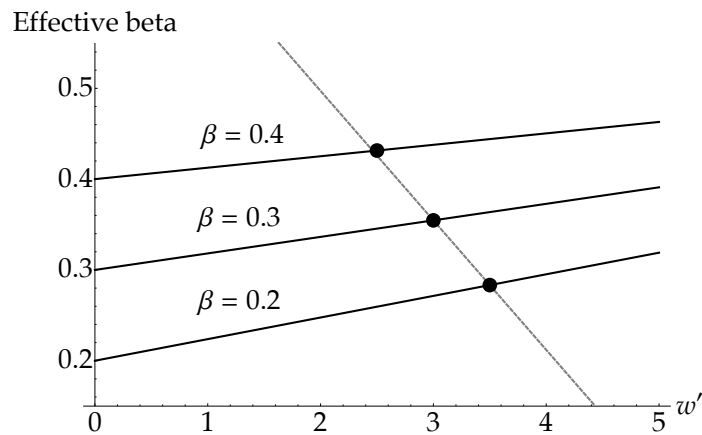


Figure 3: Effective beta as a function of outflow elasticity  $w'$  for three values of rate beta  $\beta = 0.2, 0.3, 0.4$ . The dotted gray line captures the negative correlation between  $\beta$  and  $w'$  estimated by Drechsler et al. (2017).

## 4 Uninsured Deposits and Liquidity Risk

A key assumption so far is that deposits are insured, hence outflows are only driven by substitution toward higher-paying assets such as money market funds. Concerns about the strength of the bank play no role. We introduce such concerns via uninsured deposits. Uninsured deposits create the possibility of a run because by running they destroy the deposit franchise value.

There are now two kinds of deposits, insured  $D_I$  and uninsured  $D_U$ :

$$D = D_I + D_U,$$

The two have respective deposit betas  $\beta^I$  and  $\beta^U$  and operating costs  $c^I$  and  $c^U$ . The value of the bank is then

$$V = A - D + DF_I + DF_U,$$

where  $DF_I$  and  $DF_U$  are the deposit franchise values corresponding to insured and uninsured deposits, respectively. The previous analysis applies to insured deposits, whose rate-driven outflows are given by  $w_I(r)$  as before.

Note that if the uninsured deposits are effectively wholesale funding with  $\beta^U = 1$  and  $c^U = 0$ , then  $DF_U = 0$  and the model is equivalent to before. For uninsured deposits to make a difference they must be low-beta, i.e. they must contribute to the bank's deposit franchise.

**Run incentives.** The key difference between insured and uninsured deposits is that uninsured deposit outflows  $w_U(r, v)$  depend not only rates  $r$  but also on the bank's solvency ratio  $v = V/D$ , equal to bank value per dollar of deposits.<sup>16</sup> The sensitivity of uninsured outflows  $w_U$  to bank solvency captures the possibility of runs. We assume that  $w_U$  is given by

$$w_U(r, v) = [1 - \lambda(v)] \times 1 + \lambda(v) \times \underline{w}_U(r),$$

---

than their deposits. This is consistent with banks setting asset duration using an effective beta that is slightly higher than their deposit beta, as in Proposition 2.

<sup>16</sup>We assume that all deposits, both insured and uninsured, enter the denominator equally; our results would be reinforced if only the runnable uninsured deposits entered the denominator.

where  $\underline{w}_U$  is a baseline runoff rate absent any runs as for insured depositors, but with a potentially different sensitivity  $\underline{w}'_U \neq w'_I$  that also satisfies (4). The increasing function  $\lambda(v)$  denotes the fraction of uninsured depositors who do *not* run, and  $w_U$  is a weighted average of a full withdrawal rate (i.e., 1) and the normal withdrawal rate  $\underline{w}_U$ . When  $\lambda = 1$ , there is no run and  $w_U = \underline{w}_U$ ; when  $\lambda = 0$  there is a full run and  $w_U = 1$ . Therefore, uninsured outflows can be higher than insured ones both because uninsured depositors are more rate-sensitive (high  $\underline{w}'_U$ ) and because they are concerned about bank health.

We work directly with  $\lambda$  and assume a step function around an insolvency threshold  $\underline{v}$ :

$$\lambda(v) = \begin{cases} 0 & v < \underline{v} \\ 1 & v \geq \underline{v} \end{cases}$$

A natural threshold is  $\underline{v} = 0$ , but  $\underline{v}$  may be positive if, e.g., uninsured depositors all run when the bank is distressed enough even if it still solvent, or negative if, e.g., the bank has a positive loan franchise value. Figure 4 displays the function  $\lambda$ . For an uninsured solvency ratio  $v$  below  $\underline{v}$  all uninsured depositors run, hence  $\lambda = 0$  and  $w_U = 1$ ; if  $v$  is above  $\underline{v}$  then no uninsured depositor runs, hence  $\lambda = 1$  and  $w_U = \underline{w}_U$ .<sup>17</sup>

Our theory does not require all depositors to be extremely attentive to the bank's value, nor do they need to understand the notion of deposit franchise value. In general,  $\lambda$  could be an increasing function with a finite slope capturing heterogeneity in depositors' attention to bank fundamentals such as earnings and the stock price. As  $v$  starts falling, the most attentive uninsured depositors notice a decline in fundamentals, e.g., due to rate-driven outflows, and start withdrawing. A larger fall in  $v$  then triggers withdrawals by depositors with intermediate attention, and so on. The slope of  $\lambda$  should also increase with depositor concentration, social media usage, and access to mobile banking (Cookson et al., 2023; Koont et al., 2023). When the depositor base is diversified across sectors and demographic categories and includes relatively slow-moving depositors, withdrawals are less responsive to news about bank health. Our step function corresponds to the conservative case of an extremely concentrated and coordinated uninsured depositor

---

<sup>17</sup>Following the literature on fundamental-based runs (e.g., Rochet and Vives 2004; Goldstein and Pauzner 2005) the model could be extended a step further by deriving the probability of a run as a decreasing function of  $v$ , using a richer game with imperfect information among depositors.

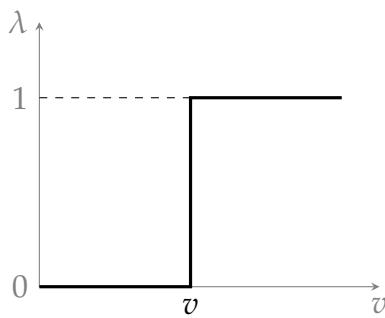


Figure 4: Fraction of remaining depositors  $\lambda$  as a function of solvency ratio  $v$ .

base with access to an almost instantaneous withdrawal technology, as in the case of corporate checking accounts at SVB.

*Remark 1* (Insured vs. uninsured depositors). Different types of depositors are better defined in terms of their actual withdrawal behavior, but for clarity we label the sleepy depositors “insured” and the flighty ones “uninsured”. In practice the distinction between insured and uninsured depositors is not as clear: some uninsured depositors are “sleepy” (i.e., they do not monitor the bank’s health) while some insured depositors may be flighty because they want to avoid the inconvenience of going through the FDIC resolution process. Insured depositors’ behavior also depends on the perceived soundness of deposit insurance and the country’s fiscal capacity.

## 4.1 Runs on the Deposit Franchise

Let  $u = D_U/D$  be the share of uninsured depositors. The solvency ratio of the bank after the interest rate shock as a function of  $\lambda$  around the usual normalization  $\underline{w}_U(r) = 0$  is:

$$v(\lambda, r') = v(0, r') + \underbrace{u\lambda (1 - \underline{w}_U(r'))}_{=DF_U(\lambda, r')/D} \frac{(1 - \beta^U)r' - c^U}{r' + \delta}, \quad (7)$$

where

$$v(0, r') = \frac{A(r') - D + DF_I(r')}{D} \quad (8)$$

is the solvency ratio when all uninsured depositors run ( $\lambda = 0$ ).  $v$  is monotone in the fraction of remaining depositors  $\lambda$ : increasing if  $DF_U > 0$  and decreasing if  $DF_U < 0$ . Inverting  $v(\cdot, r')$  yields a function  $\Lambda(v, r')$ , defined as satisfying  $v(\Lambda(v, r'), r') = v$ , and capturing the fraction of remaining uninsured depositors that is consistent with a solvency ratio  $v$ .

**Definition 1** (Equilibrium). Given  $r'$  and bank assets  $A(r')$ , a stable equilibrium (henceforth, equilibrium) is given by a pair of bank value  $v$  and fraction of remaining depositors  $\lambda$  such that  $\lambda(v) = \Lambda(v, r')$  and  $\Lambda$  crosses  $\lambda$  from below.

Our next main result shows that relying on a large uninsured deposit franchise, for instance at high interest rates, creates the potential for runs:

**Proposition 3** (Interest Rates and Liquidity Risk). *Suppose that the uninsured deposit franchise is profitable:  $(1 - \beta^U)r' > c^U$ .*

- *If  $v(0, r') \geq \underline{v}$  then the unique equilibrium is run-free, i.e.,  $\lambda = 1$ .*
- *If  $v(1, r') < \underline{v}$  then the unique equilibrium features  $\lambda = 0$ , i.e., all uninsured depositors run.*
- *If  $v(0, r') < \underline{v} \leq v(1, r')$ , then both equilibria  $\lambda = 0$  and  $\lambda = 1$  exist.*

*The run equilibrium is more likely to exist when the share of uninsured deposits  $u$  is higher, the interest rate  $r'$  is higher, and the uninsured deposit beta  $\beta^U$  is lower.*

At high rates, the uninsured deposit franchise is a large but runnable part of the bank's value: it relies on depositors remaining with the bank and tolerating low deposit rates. This creates important differences with the standard source of runs in the literature: asset illiquidity as in [Diamond and Dybvig \(1983\)](#) and the subsequent literature. Notably, runs in our model do not rely on fire sales, hence they can happen even when a single bank is affected and when it holds only high quality liquid assets (as SVB did). Moreover, in our framework liquidity risk naturally increases with the level of interest rates, and banks with a more valuable uninsured deposit franchise (low  $\beta^U$ ) face higher liquidity risk.

Figures 5 and 6 illustrate the shift from a unique run-free equilibrium to equilibrium multiplicity when rates increase. The natural interpretation of the condition  $v(0, r') > \underline{v}$  is that the bank has enough capital; we return to this issue in [Section 5.2](#) on bank capital.

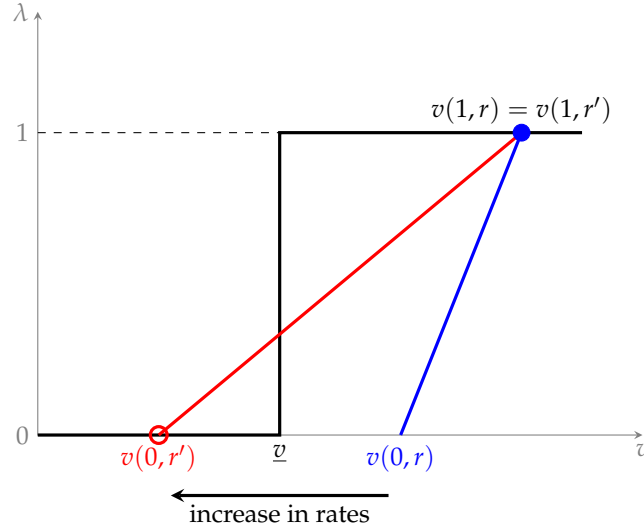


Figure 5: Unique equilibrium (blue dot) at rate  $r$ , two equilibria (blue dot and red circle) at higher rate  $r' > r$ .

## 4.2 Hedging Interest Rate Risk versus Liquidity Risk: the Dilemma

A key insight of the model is that runs can happen as rates rise *even if the bank is fully hedged against interest rate risk in the good equilibrium*. In other words, the bank faces a risk management dilemma, illustrated in Figure 5. Suppose the bank initially hedges interest rate risk, assuming the good equilibrium ( $\lambda = 1$ ) prevails. Then it chooses assets such that

$$A'(r) + DF'_I(r) + DF'_U(1, r) = 0$$

and the optimal duration follows Proposition 2. The bank is hedged as long as uninsured depositors do not run, which is shown in Figure 5 by the fact that  $v(1, r) = v(1, r')$  in spite of an increase in rates from  $r$  to  $r'$ . However, a byproduct of this hedging strategy is that the value under a run  $v(0, r')$  is unhedged and can potentially fall below  $\bar{v}$ . The reason is that hedging against rates requires positive asset duration. When rates rise, the value of assets falls while the value of the deposit franchise rises. But since the deposit franchise is runnable, the liquidity risk of the bank increases.

Conversely, hedging liquidity risk requires the bank to invest in short-term assets. Starting from the run-free equilibrium, hedging liquidity risk requires en-



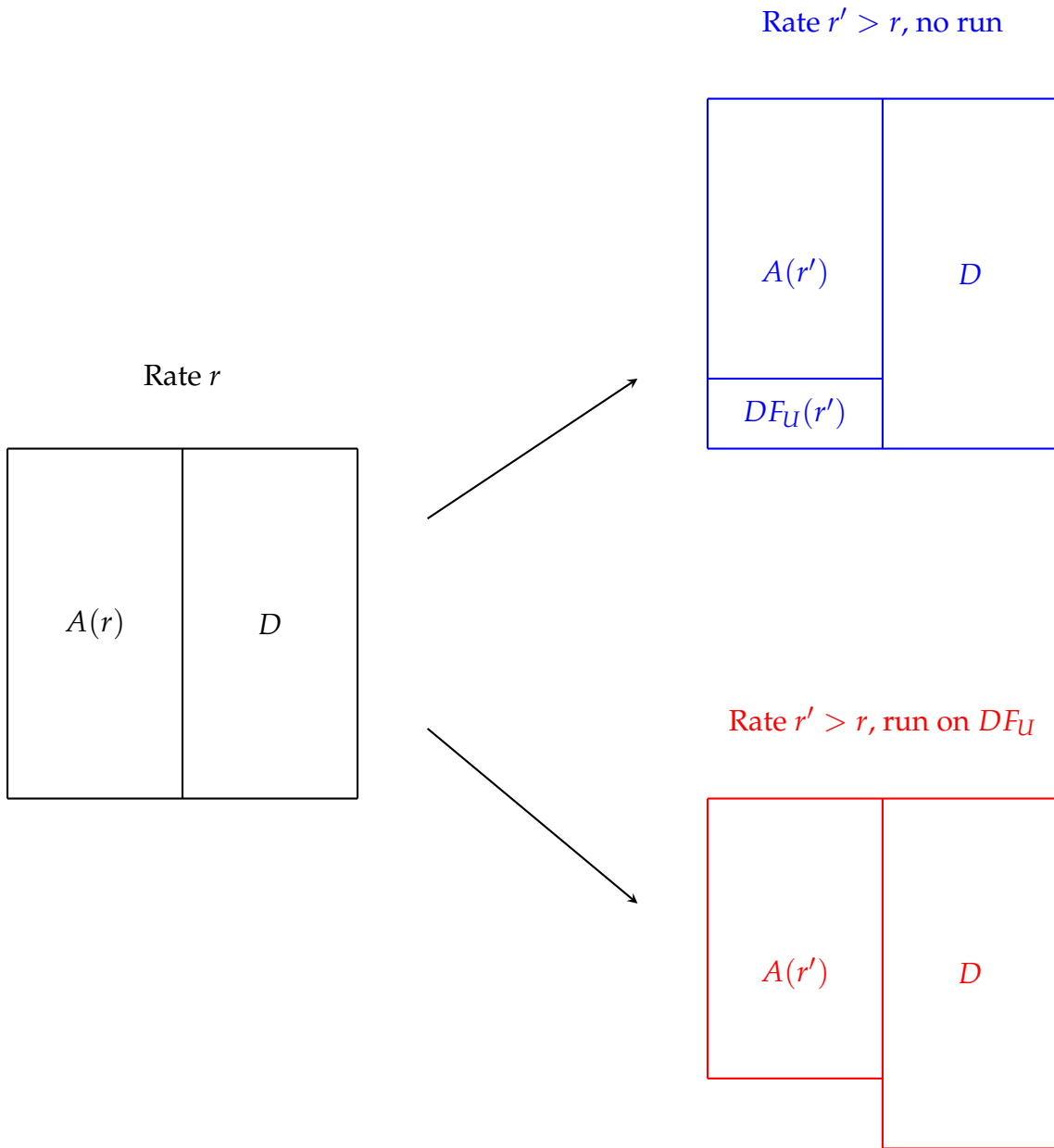


Figure 6: Bank balance sheet: one equilibrium at old rate  $r$ , two equilibria at new rate  $r' > r$ .

sureing that  $v(0, r')$  never falls below  $\underline{v}$ . This can be done by choosing assets such that

$$A'(r) + DF_I'(r) = 0,$$

effectively ignoring the negative duration of the uninsured deposit franchise. The bank must invest in assets with modified duration

$$T_A = \frac{DF_I}{A} T_{DF_I},$$

which is lower than the duration that hedges the bank in the good equilibrium,  $T_A = \frac{DF_I}{A} T_{DF_I} + \frac{DF_U}{A} T_{DF_U}(1, r)$ , assuming the uninsured deposit franchise is profitable and hence has negative duration.

Hedging liquidity risk can again be reinterpreted as hedging without deposit outflows, but against an effective beta that exceeds the actual deposit beta. Ignoring the “smooth” (in contrast to runs) rate-driven outflows studied extensively in the previous section, we obtain a very simple formula for the effective beta:

**Proposition 4** (Hedging Liquidity Risk). *Suppose there are no smooth rate-driven outflows,  $w_I' = w_U' = 0$ , and initially there is no run equilibrium:  $v(0, r) > \underline{v}$ .*

*Then the bank can hedge against liquidity risk by using an effective duration*

$$T_A = (1 - u) \frac{(1 - \beta^I)\delta + c^I}{(r + \delta)^2},$$

*or equivalently an effective beta*

$$\hat{\beta} = (1 - u) \times \beta^I + u \times 1 + uc^U/\delta, \quad (9)$$

*The effective beta  $\hat{\beta}$  increases with the share of uninsured deposits and the operating cost on uninsured deposits  $c^U$ .*

A bank with a larger share of uninsured deposits can hedge against the liquidity risk arising from higher rates by shortening the duration of its assets. Equation (9) shows that when  $c^U = 0$ , the correction is extremely simple: instead of using an average beta  $(1 - u)\beta^I + u\beta^U$  the bank should act as if its uninsured deposits have a beta  $\beta^U = 1$ . In addition, there is a correction increasing with  $c^U$  because operating costs are one reason the bank normally holds long duration assets, but this term is small given  $\delta \gg c^U$ .

The correction  $\hat{\beta} - \beta$  can be large. When most deposits are uninsured ( $u \rightarrow 1$ ) as was the case for SVB, hedging liquidity risk requires holding only zero-duration assets (cash and floating-rate loans). By contrast, SVB had an unusually high share of long-duration assets.

Banks face a dilemma as there is no duration that allows them to simultaneously hedge the interest rate risk in the good equilibrium and the liquidity risk arising from the bad equilibrium:

**Proposition 5 (Dilemma).** *Suppose that initially the uninsured deposit franchise is profitable ( $(1 - \beta^U)r > c^U$ ), only the good equilibrium exists*

$$v(1, r) = v^* > v(0, r) = v_* \geq \underline{v},$$

and for simplicity  $\underline{w}_U = 0$ .

- If the bank perfectly hedges interest rate risk in the good equilibrium by holding long-term assets  $A$  such that  $v(1, r')$  is constant in  $r'$ , then there exists a threshold

$$\bar{r} = \frac{c^U + \delta \frac{v^* - \underline{v}}{u}}{1 - \beta^U - \frac{v^* - \underline{v}}{u}} \quad (10)$$

such that for  $r' > \bar{r}$  the run equilibrium exists. The threshold  $\bar{r}$  is decreasing in the uninsured deposit ratio  $u$  and increasing in the uninsured beta  $\beta^U$ .

- If the bank perfectly hedges liquidity risk by choosing short-term assets  $A$  such that  $v(0, r')$  is constant in  $r'$ , then there is no run equilibrium but there exists a threshold

$$\underline{r} = \frac{c^U - \delta \frac{v_* - \underline{v}}{u}}{1 - \beta^U + \frac{v_* - \underline{v}}{u}} < \bar{r}$$

such that for  $r' < \underline{r}$  the equilibrium bank value is  $\underline{v}$ . The threshold  $\underline{r}$  is increasing in the uninsured deposit ratio  $u$  and the uninsured beta  $\beta^U$ .

Proposition 5 formalizes the bank's risk management dilemma. Hedging interest rate risk requires it to hold long-term assets to offset the negative duration of the deposit franchise. But since the deposit franchise is partly uninsured, it is runnable. As rates rise and the deposit franchise grows relative to the bank's assets, the liquidity risk of the bank increases. The first part of the proposition shows that if rates rise enough, a run equilibrium always emerges.

This result reveals that the dilemma arises from low-beta uninsured deposits: holding costs  $c^U$  fixed, the threshold  $\bar{r}$  becomes infinite as  $\beta^U$  grows towards 1 (at  $\beta^U = 1 - \frac{v^* - \underline{v}}{u}$ ); in Section 4.4 we extend this result to the case of endogenous costs  $c^U$ . Intuitively, a run on uninsured deposits is only a problem if they contribute to the bank's deposit franchise value, i.e., if  $\beta^U$  is low. If  $\beta^U$  is high then uninsured deposits bring neither value nor negative duration to the bank, hence the bank can simply hold cash against them, and there is no conflict between hedging interest rate risk and liquidity risk.

### 4.3 Risks from Holding Short-Duration Assets

The second part of Proposition 5 shows that holding short-duration assets does not solve the bank's risk management dilemma. It ensures there are no runs in equilibrium, but the bank becomes unhedged to interest rates. If rates fall, its assets do not appreciate enough to offset the decline in the deposit franchise, as shown in Figure 7.

Shortening duration to hedge liquidity risk is not a perfect solution because it hurts bank solvency if interest rates fall. At low rates deposit spreads are insufficient to cover operating costs. If the bank holds short-term assets then its interest income will also be low. This can make the bank insolvent even though the value of its assets is relatively high. The bank can become a "zombie bank"; the resulting incentives from debt overhang and risk-shifting can have adverse macroeconomic consequences if widespread (Caballero et al., 2008; Acharya et al., 2022).

**Uninsured deposit withdrawals at low rates.** When the interest rate declines by so much that  $v(\lambda = 1, r')$  falls below  $\underline{v}$ , uninsured depositors start withdrawing again, i.e.,  $\lambda < 1$ . But this kind of outflow is substantially different from the run that can happen at high rates. Low-rate withdrawals happen only when the deposit franchise is unprofitable, i.e., when rates are so low that  $v(\lambda, r)$  is actually *decreasing* in  $\lambda$  because the per-dollar profit on deposits,  $(1 - \beta^U)r - c^U$ , is negative. This is illustrated in Figure 7, where the threshold rate  $r' = c^U / (1 - \beta^U)$  is the rate such that the uninsured deposit franchise is worth exactly zero. When the rate falls further, for instance to  $r'' < r'$ , the franchise value becomes negative. Since the bank is losing money on each uninsured deposit, it actually welcomes

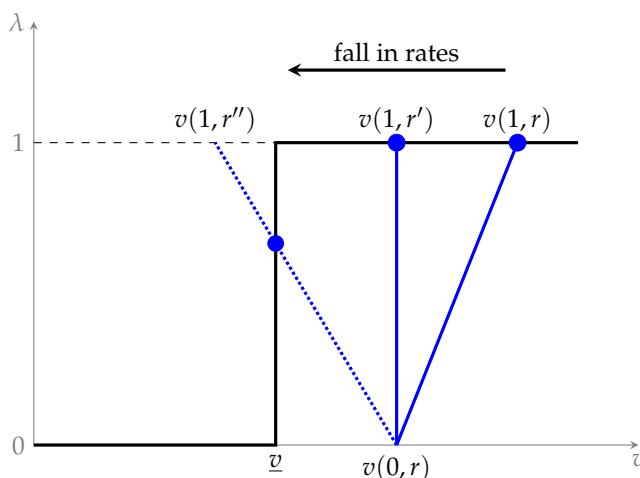


Figure 7: Unique equilibrium as interest rate falls from  $r$  to  $r'$  to  $r''$ .

outflows. In that case the bank does not really face liquidity risk: starting from  $\lambda = 1$ , uninsured deposit outflows increase the bank value  $v$  until it reaches  $\underline{v}$ . This highlights that in our framework liquidity risk is a feature of high interest rates.<sup>18</sup>

**Credit risk.** We focused on interest rate shocks thus far for clarity. When banks hold assets that are exposed to other unhedged sources of risk, for instance credit risk, then these other shocks can also trigger a run on the uninsured deposit franchise. We show that following the same logic as for interest rate shocks, this amplification is more likely when the uninsured deposit franchise is large, for instance when interest rates are high and the bank has a large share of uninsured deposits.

Let  $s$  denote the unhedged credit risk shock so that the value of assets is decreasing in  $s$  ( $\partial A/\partial s < 0$ ). Suppose that initially only the run-free equilibrium exists and

$$v(1, r, s) = \frac{A(r, s) - D + DF_I(r) + DF_U(1, r)}{D}.$$

In our framework the deposit franchise does not hedge credit shocks because they do not enter into the deposit demand function.<sup>19</sup> This leaves the bank overall

<sup>18</sup>As discussed above, in the presence of fixed costs as in Appendix B, the value of the bank increases less when unprofitable deposits leave.

<sup>19</sup>For large banks that are considered too-big-to-fail, adverse macroeconomic shocks may lead to an inflow of “flight-to-safety” deposits which can be captured through a net outflow function  $w(r, s)$ .

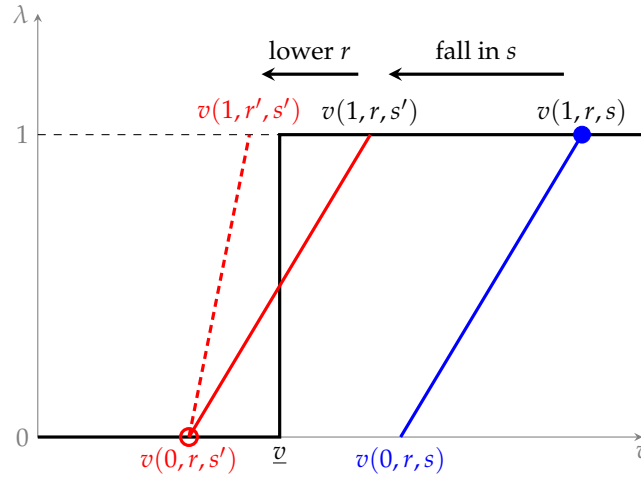


Figure 8: Effect of a shock to asset values  $s' < s$  and policy response  $r' < r$ .

unhedged. An adverse shock  $s$  triggers a fall in  $A$ , and hence lowers bank solvency  $v$ . If the fall in  $v$  is large enough, some uninsured depositors withdraw ( $\lambda < 1$ ), reducing the value of the deposit franchise  $DF_U$ . If the deposit franchise is large, as it is at high interest rates, the fall in its value hurts  $v$  further and triggers more withdrawals, and so on. The mechanism is shown in Figure 8. It highlights that credit risk in our framework is amplified by high interest rates.

**Constraints on monetary policy.** How should the central bank set interest rates, taking into account the impact on financial stability? Our framework implies that it depends on banks' risk management policy.

If banks hedge interest rates, then an interest rate increase can expose them to runs. Conversely, an interest rate cut makes runs less likely. However, the rate cut does not increase the value of banks in the no-run equilibrium (asset values rise but deposit franchise value falls). Thus, accommodative monetary policy, for example following credit losses, does not improve banks' solvency.

If banks hedge liquidity risk by holding short-term assets, then interest rate cuts actually hurt bank solvency. The reason is that asset values rise less than the amount by which the deposit franchise value shrinks. A rate cut then worsens the impact of a negative credit shock. If the deposit franchise is very valuable, accommodative monetary policy can even make the good equilibrium disappear, as depicted by the dotted red line in Figure 8.

This reinforces the case for strategies such as swaptions and capital requirements that allow banks to keep their deposit franchise valuable and hedge interest rate risk based on the good equilibrium. This allows the central bank to set interest rates without adversely affecting bank health.

An interesting extension outside the scope of our model would be to include imperfect and dispersed information about  $s$  and other characteristics such as  $\beta$  and  $w'$ . In that case, uninsured depositors could even interpret ordinary rate-driven withdrawals as a signal of poor asset quality  $s$ .

#### 4.4 High $\beta$ Uninsured Deposits: a “Divine Coincidence”

Drechsler et al. (2023c) find that prior to 2012 uninsured deposits consisted primarily of large CDs, effectively wholesale funding, which have a beta  $\beta^U$  close to one and operating cost  $c^U$  close to zero. Thus, uninsured deposits contributed little to banks’ deposit franchise value. Holding short-duration assets against this type of deposits both hedges the bank to interest rate risk and prevents runs. We refer to this as a “divine coincidence” in the spirit of the macro literature. It helps explain why SVB-type runs were relatively uncommon in the past.

Formally, Proposition 5 shows that for a given cost  $c^U$ , the threshold interest rate  $\bar{r}$  at which a run equilibrium emerges grows without bound when  $\beta^U \rightarrow 1$ . In other words, the run equilibrium fails to emerge. The reason is that in this case uninsured withdrawals do little to damage the value of the bank, hence they do not trigger further withdrawals. This is very different from models with illiquid assets where uninsured deposits always create the potential for runs. Our framework thus implies that the liquidity risk implications of uninsured deposits depend on their deposit betas.

While this argument holds  $c^U$  fixed as  $\beta^U$  rises, in principle we expect  $c^U$  to fall. This is because operating costs are increasing in the level of services that a bank provides (branches, apps, etc.), which in turn helps it attract low-beta deposits. A bank that provides a high level of services (high  $c$ ) is thus able to pay a lower deposit rate (low  $\beta$ ). We therefore expect operating costs and deposit betas to be negatively related, both across types of deposits and across banks. We now enrich the model to allow for endogenous operating costs and show that the divine coincidence goes through.

Suppose the bank can invest in the quality of its services. The initial investment determines its future operating costs. Following standard  $q$ -theory logic, the bank chooses a higher investment scale and thus higher operating costs when the deposit franchise is more valuable, for instance if the bank has more market power. Focusing on the uninsured deposit franchise, the optimal operating cost is given by a function  $c^U = \phi(DF_U)$  with  $\phi(0) = 0$ . Since  $DF_U$  itself depends on the operating cost, we get a solution  $c^U = \mathcal{C}((1 - \beta^U)r^*)$ , where  $r^*$  is the long-run interest rate and  $\mathcal{C}(\cdot)$  is an increasing function with  $\mathcal{C}(0) = 0$ ; the free-entry condition  $c^U = (1 - \beta^U)r^*$  in Drechsler et al. (2021) is a special case with  $\mathcal{C}(x) = x$ .<sup>20</sup>

Focusing on uninsured deposits (similar expressions hold for insured deposits), the resulting deposit franchise value is

$$DF_U(r') = D_U(1 - w_U(r')) \left( \frac{(1 - \beta^U)r' - \mathcal{C}((1 - \beta^U)r^*)}{r' + \delta} \right).$$

Suppose that  $r' \in (0, r_{\max})$  for some finite  $r_{\max}$ . We have the following result:

**Proposition 6.** *There exists a threshold  $\bar{\beta}^U < 1$  such that for a high-enough uninsured deposit beta,  $\beta^U \geq \bar{\beta}^U$ , if the bank hedges against interest rate risk then the good equilibrium is the unique equilibrium for any  $r' \in (0, r_{\max})$ . The threshold  $\bar{\beta}^U$  is increasing in the uninsured deposit ratio  $u$ .*

As  $\beta^U \rightarrow 1$ , the uninsured deposit franchise  $DF_U$  converges to zero and so does the interest rate risk due to the possibility of runs in the bad equilibrium. The “divine coincidence” is that by optimally hedging against the high  $\beta^U$  deposit franchise using short-duration assets, the bank also deters runs. This is possible because the uninsured franchise  $DF_U$  is small and does not bring much negative duration to total bank value. This result can hold in spite of a large uninsured deposit ratio  $u$ , but a higher  $u$  requires a higher  $\beta^U$ .

Drechsler et al. (2023c) further show that after 2012 uninsured deposits shifted toward checking and savings accounts. Unlike wholesale funding, these accounts have a low  $\beta$  and a high operating cost  $c$ , hence they contribute a lot to the deposit franchise. In the case of SVB, almost all of its deposits were uninsured corporate checking and savings accounts. This means that it had a large uninsured deposit

---

<sup>20</sup>For simplicity we take as given the demand curve for deposits and therefore the resulting  $r_d$  and  $\beta$ ; of course an even more complete version of this problem would acknowledge that the  $\beta$  is itself endogenous to investments in quality and thus  $c$ , but this would only change the function  $\phi$ .



franchise. As our model shows, this is the extreme case for a bank’s risk management dilemma. The low  $\beta$  and high cost call for a long asset duration, while the fact that the deposits are uninsured calls for a short duration. SVB’s decision to invest heavily in long-term MBS reveals that it ignored the second consideration, leaving it maximally exposed to a run.<sup>21</sup>

A broader implication of our analysis is that uninsured checking and savings accounts, more so than wholesale funding, expose banks to instability.

## 5 Resolving the Dilemma: Options and Capital

We propose three resolutions to the risk management dilemma that can be implemented by banks or regulators. The first is based on ex-ante hedging through options on interest rates. The second is based on ex-post capital raising that adjusts with the level of interest rates. The third is an ex post government intervention in the spirit of the Bank Term Funding Program (BTFP) introduced by the Federal Reserve in March 2023.

### 5.1 Options

We show that a hedging strategy that takes into account both interest rate and liquidity risks requires the bank to hold assets with strong convexity, for instance puttable bonds or loans featuring an interest rate floor. An equivalent implementation is a combination of long-duration assets with call options on interest rates (e.g., “payer swaptions”). Since options are costly, this requires the bank to raise more capital up front.

Suppose the bank starts from the good equilibrium with value  $v(1, r) = v^* > \underline{v}$ . As rates vary, the bank can ensure that its value does not fall below  $v^*$  in the good equilibrium, while preventing the bad equilibrium if

$$v(1, r') \geq v^* \tag{11}$$

$$v(0, r') \geq \underline{v} \tag{12}$$

---

<sup>21</sup>In addition SVB may have underestimated factors that made their outflow elasticity  $w'$  particularly high, due to withdrawals unrelated to the standard substitution towards MMFs. Most of its depositors were startups whose funding dried up with rising interest rates, which led to more withdrawals to finance their operating expenses even before the run.

for all  $r'$ . Under condition (12) runs never happen since even if all uninsured depositors withdraw, the value  $v(0, r')$  is still above  $\underline{v}$ . (11)-(12) then translate into a lower bound on asset values, which must remain above a minimal level  $A^*(r')$ :

**Proposition 7.** *Suppose that the bank starts from a solvency ratio  $v^* > \underline{v}$  and holds assets  $A^*$  such that*

$$A^*(r') = \underbrace{(1 + v^*)D - DF_I(r') - DF_U(\lambda = 1, r')}_{\text{long-term assets}} + \underbrace{\max\{0, DF_U(\lambda = 1, r') - (v^* - \underline{v})D\}}_{\text{swaptions}}. \quad (13)$$

Then for any  $r'$  only the run-free equilibrium  $\lambda = 1$  exists and the bank value never falls below its initial value  $v^*D$ .<sup>22</sup>

To first order the second term in (13) equals

$$N \cdot \max\{0, r' - \bar{r}\}$$

where (assuming  $w'_U = 0$  to simplify)  $\bar{r}$  is given by (10) and

$$N = uD \frac{c^U + (1 - \beta^U)\delta}{(\bar{r} + \delta)^2}. \quad (14)$$

Therefore the bank needs to buy an amount  $N$  of standard payer swaptions with strike  $\bar{r}$ .

Hedging against both risks requires a convex asset payoff  $A^*$  which can be interpreted as a puttable bond, i.e., the opposite of callable bonds which have negative convexity. It can also be implemented by holding long-term assets that hedge against interest rate risk in the good equilibrium—as if all deposits were insured—together with interest rate options that pay off when interest rates are high, i.e., payer swaptions. The swaptions are in the money if and only if the interest rate exceeds a strike  $\bar{r}$  equal to the threshold rate above which runs appear in Proposition 5. Ex ante the bank must spend  $N \times P$  on options, where the premium

$$P = \mathbf{E}^Q [\max\{r' - \bar{r}, 0\}]$$

<sup>22</sup>From the hedging dilemma in Proposition 5 we know that the bank cannot make (11) hold with equality while satisfying (12). Hence at high rates the bank's value must exceed  $v^*D$ .

can be computed using standard option pricing formulas, given a stochastic process for interest rates.  $P$  is increasing in the volatility of  $r'$  and decreasing in the strike  $\bar{r}$ . Importantly a bank with a more valuable uninsured deposit franchise (higher  $u$  and lower  $\beta^U$ ) must buy a larger amount  $N$  of options *and* use a lower strike  $\bar{r}$ , which increases the cost per option  $P$ .

Figure 9 plots the asset payoff  $A^*$  as a function of  $r$ . The key observation is that  $A^*$  must be extremely convex due to the kink at  $r' = \bar{r}$ .<sup>23</sup> In this numerical example this corresponds to a swaption strike  $\bar{r} \approx 3\%$ . For  $r' < \bar{r}$ , assets have a long duration and so  $A^*$  decreases strongly with  $r'$ . For  $r' > \bar{r}$ , assets have a short duration and  $A^*$  is insensitive to  $r'$ . Figure 10 shows the decomposition of  $A^*$  in (13) as a function of  $r'$ . Equivalently, the bank could hold short-term assets combined with receiver swaptions, i.e., put options on interest rates that insure against the loss in deposit franchise value as rates fall, as shown in Appendix Figure A.1.

Banks are already active in the swaption market in order to hedge the negative convexity of their MBS portfolio on the asset side: as rates fall, more mortgages are refinanced and the duration of MBS falls. Our model highlights another reason why banks with a large uninsured deposit base should buy—or be required to buy—swaptions: the deposit franchise asset has negative convexity, just like the MBS.

Hedging requires capital ex ante to pay for the options. Bank's equity holders may find it too costly to issue the capital needed to buy enough swaptions if there are equity issuance costs due to, e.g., adverse selection or debt overhang. In this case regulators would need to mandate a minimum level of swaptions (that depends on each bank's uninsured deposit franchise) as part of liquidity regulation.

## 5.2 Rate-Cyclical Capital

Instead of raising capital ex ante to buy options, the bank can address its risk management dilemma by raising capital (or changing its dividend) ex post as interest rates change. Our model has implications for how such an optimal capital buffer, which may again be required by regulators, should depend on interest rates:

---

<sup>23</sup>Smooth rate-driven outflows also induce negative convexity of the deposit franchise, but we focus on the more extreme negative convexity induced by the possibility of runs at high rates.

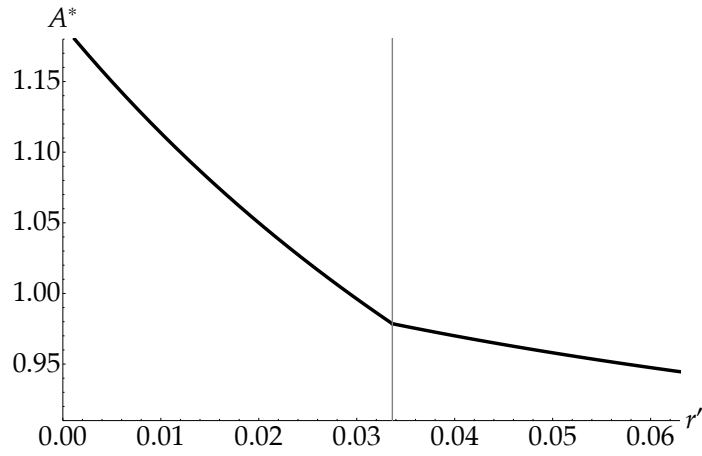


Figure 9:  $A^*$  as a function of  $r'$ . Parameters:  $c = 1.4\%$ ,  $\delta = 0.1$ ,  $r = 4\%$ ,  $\beta^l = \beta^u = 0.3$ ,  $u = 0.7$ ,  $v^* = 5\%$ ,  $\underline{v} = 0$ .

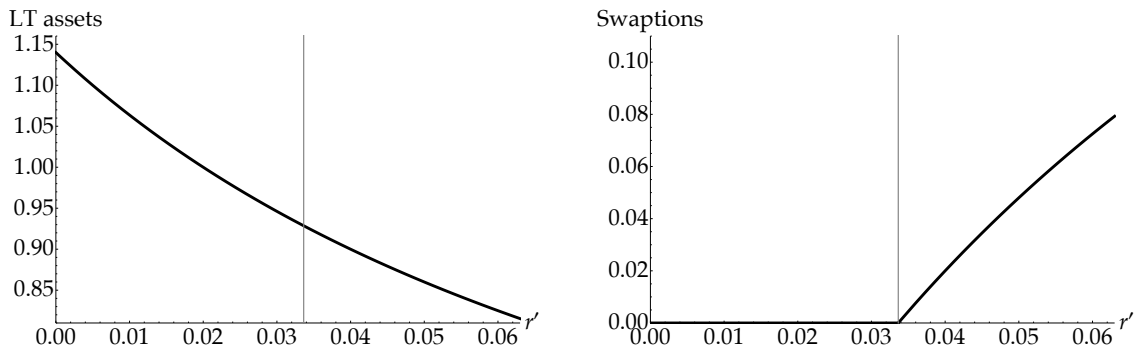


Figure 10: Decomposing  $A^*(r')$  according to (13) into long-term assets (left panel) and payer swaptions with strike  $r' = \bar{r}$  (right panel).

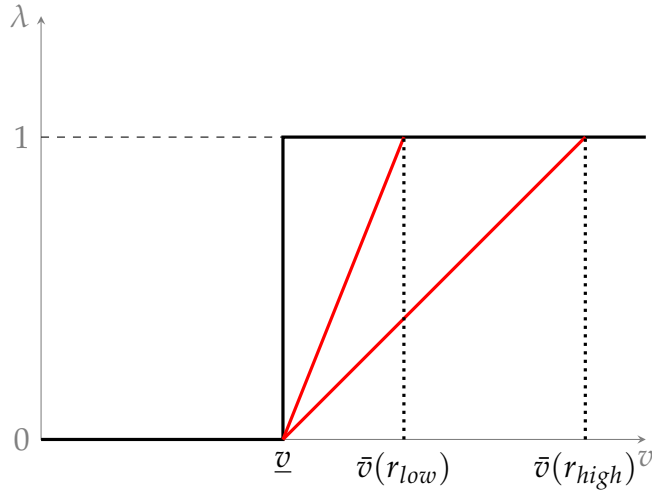


Figure 11: Capital requirement  $\bar{v}$  at low rate  $r_{low}$  and high rate  $r_{high}$ .

**Proposition 8.** *The minimum solvency ratio  $\bar{v}$  that prevents runs at  $r'$  is*

$$\bar{v}(r', u) = \underline{v} + u(1 - \underline{w}_U(r')) \left[ \frac{(1 - \beta^U)r' - c}{r' + \delta} \right].$$

*It increases with the interest rate  $r'$  and the uninsured deposit ratio  $u$ .*

Proposition 8 characterizes the level of capital the bank needs to deter runs at any given interest rate. This level is increasing in interest rates as illustrated in Figure 11. The reason is that the uninsured deposit franchise is more valuable at high rates. This makes the value of the bank more sensitive to the fraction of uninsured depositors who remain,  $\lambda$ . Given the increased sensitivity, bank value has to be higher ex ante in order to deter a run. The bank therefore needs to raise more capital (or reduce its dividend) as the interest rate rises. Once it has done so, the bank no longer faces a risk management dilemma and can focus on interest rate risk management.

Implementing  $v \geq \bar{v}$  with a capital requirement corresponds to attributing a risk-weight to the uninsured deposit franchise. The risk-weight accounts for the increased liquidity risk of the bank as interest rates rise. The behavior of liquidity risk thus yields a new rationale for procyclical capital requirements. Unlike standard arguments, the procyclicality here comes from a pure interest rate effect, not the state of the economy. While we frame the result in terms of a capital requirement, depending on the cost of issuing equity banks may wish to follow this

strategy independent of regulation in order to prevent runs.

One advantage of ex ante insurance through options before negative shocks are realized is that raising capital is difficult in times of stress. Incomplete information about bank strength means that going to the equity market is viewed as a signal of weakness. This stigma effect may be dampened, however, if equity issuance is mandated according to a well-understood rule that depends on the aggregate interest rate as in Proposition 8.

### 5.3 Lender of Last Resort

We end by showing how a lender of last resort can eliminate the run equilibrium ex post and thus avoid the risk management dilemma ex ante. We caution, however, that in a richer model this could lead to moral hazard or adverse selection.

Suppose that the central bank (henceforth, the Fed) gives a long-term loan to the bank

$$B(\lambda, r')$$

contingent on the extent of the run  $1 - \lambda$ , with  $B(1, r') = 0$ . After borrowing from the Fed the solvency ratio entering uninsured depositors' run function  $\lambda(v)$  is  $v = \frac{A(r') + \lambda DF_U(1, r') + B(\lambda, r')}{D} - 1$ . Importantly the denominator is given by deposits  $D$  hence does not take into account the long-term loan from the Fed, which is not runnable and effectively junior to current uninsured deposits. This is why the long-term loan improves  $v$ . More generally, it is enough if the denominator includes the government loan  $B$  discounted by a "runnability" factor  $\alpha < 1$ .

By setting

$$B(\lambda, r') = (1 - \lambda)DF_U(1, r')$$

the Fed can ensure that

$$v(\lambda, r') = v(1, r')$$

for any  $\lambda$ . Therefore, the Fed policy eliminates the run equilibrium, in the sense that if the bank is hedged against interest rate risk in the good, run-free, equilibrium, then there is no run equilibrium. Moreover, since there is no run, the intervention ends up costless in equilibrium.

**Proposition 9.** *Suppose that for any  $\lambda$ , a bank facing an uninsured deposit run  $1 - \lambda$  can*

*borrow*

$$B(\lambda, r') = (1 - \lambda)DF_U(1, r') \quad (15)$$

*long-term from the Fed. Then  $\lambda = 1$  is the unique equilibrium and the equilibrium cost of the Fed intervention is zero.*

One particular implementation resembles the Bank Term Funding Program (BTFP) introduced by the Federal Reserve in March 2023 allowing banks to borrow from the Fed at par, that is against a collateral value  $A(r)$  instead of  $A(r')$ . Starting from an interest rate  $r$  such that  $DF_U(1, r) = 0$ , this corresponds to a loan size

$$B(\lambda, r') = (1 - \lambda) [A(r) - A(r')],$$

which is exactly (15).

In principle complete private insurance markets could also implement this allocation: banks would buy insurance contracts contingent on their idiosyncratic realization of  $\lambda$  and not just on the aggregate interest rate  $r'$  as in Section 5.1. In the model featuring only interest rate risk and purely idiosyncratic runs, these contracts would prevent runs from happening in the first place hence the equilibrium price of such insurance would be zero. However, in a richer setting featuring contagion and widespread runs, or in the presence of other aggregate shocks  $s$  as in Section 4.3, such private insurance markets would be insufficient, which is why we focus on government-provided insurance.

There are two main drawbacks of the Fed policy we describe: moral hazard and adverse selection, both of which are outside the model. The equilibrium cost for the Fed is zero because banks hold high-quality liquid assets such as agency MBS and Treasuries whose value depends only on  $r$ . Banks therefore have no “bad” assets to offload on the Fed. In a model with more complex bank incentives and assets, the intervention may lead to excessive risk-taking and adverse selection in asset purchases, in particular because it is targeted at the ex post weakest institutions instead of favoring the healthiest ones.<sup>24</sup> The expectation of ex-post Fed intervention may also reduce private incentives to buy swaptions ex ante. All else equal, these considerations favor the options-based and capital-based approaches discussed in Sections 5.1 and 5.2.

---

<sup>24</sup>See, e.g., Philippon and Skreta (2012) and Tirole (2012) for an analysis of the cost of intervention with adverse selection, and Philippon and Schnabl (2013) and Philippon and Wang (2022) for the design of ex post interventions that mitigate moral hazard.

## 6 Conclusion

Banks hedge the interest rate risk of their assets with their deposit franchise. When interest rates go up, the value of the assets falls but the value of the deposit franchise rises. For this to work deposits must stay in the bank. If they leave, whether in search of higher rates or out of concern about the bank's health, the hedge fails and the bank can become insolvent. This creates run incentives among uninsured depositors.

We present a model of runs on the deposit franchise and study its implications for bank risk management. The key ingredient is "low-beta" uninsured deposits that contribute to the bank's deposit franchise. Relying on such deposits makes the bank subject to runs even if its loans and securities are fully liquid. However, runs only appear at high interest rates. At low rates deposits are unprofitable, hence the bank is unharmed by a run and a run never occurs. At high rates, when bank assets are down and the deposit franchise is up, a run is a lot more harmful and hence more likely. Our key result therefore is that for banks with an uninsured deposit franchise, liquidity risk is increasing in interest rates.

We provide formulas for the bank's optimal risk management policy taking deposit outflows into account, but stress that the bank faces a new risk management dilemma: it cannot simultaneously hedge its interest rate risk and liquidity risk exposures. Hedging interest rates with long-duration assets opens the door to runs at high rates, while hedging liquidity risk with short-duration assets can lead to insolvency at low rates. This implies that low-beta uninsured deposits (e.g., uninsured checking and savings accounts) are a source of instability for banks. We show that a solution for banks, though not costless, is to buy options such as puttable bonds or swaptions, or to raise additional capital when interest rates rise.

Our results highlight that banks are in the business of providing optionality to their customers. They do so through their assets, for example by extending mortgages with the option to refinance, and they do so through their liabilities, by allowing depositors to withdraw at will. Both are options on interest rates: mortgage borrowers prepay when rates are low, depositors withdraw when rates are high. Both give banks negative convexity. It is well known that mortgage prepayment risk can be hedged using swaptions. Deposit prepayment risk can as well.



But who will take the other side? An intriguing answer, one we leave for future work, is that since in times of stress deposits mainly stay within the banking system, the natural providers of convexity are financial institutions that enjoy “flight to safety.” Like the hiker who need not outrun the bear provided he can outrun his companion, the ultimate deposit franchise is the one that can outlast the others.

## References

- Abadi, Joseph, Markus Brunnermeier, and Yann Koby** (2022), “The Reversal Interest Rate”, working paper.
- Acharya, Viral V., Simone Lenzu, and Olivier Wang** (2022), “Zombie Lending and Policy Traps”, working paper.
- Allen, Franklin and Douglas Gale** (2009), *Understanding Financial Crises*: OUP Oxford.
- Amador, Manuel and Javier Bianchi** (2021), “Bank Runs, Fragility, and Credit Easing”, Working Paper 29397, National Bureau of Economic Research.
- Bai, Jennie, Arvind Krishnamurthy, and Charles-Henri Weymuller** (2018), “Measuring Liquidity Mismatch in the Banking Sector”, *The Journal of Finance*, Vol. 73, pp. 51–93.
- Bolton, Patrick, Ye Li, Neng Wang, and Jinqiang Yang** (2020), “Dynamic Banking and the Value of Deposits”, Working Paper 28298, National Bureau of Economic Research.
- Caballero, Ricardo J., Takeo Hoshi, and Anil K. Kashyap** (2008), “Zombie Lending and Depressed Restructuring in Japan”, *American Economic Review*, Vol. 98, pp. 1943–77.
- Cookson, J Anthony, Corbin Fox, Javier Gil-Bazo, Juan Felipe Imbet, and Christoph Schiller** (2023), “Social Media as a Bank Run Catalyst”, Available at SSRN 4422754.
- Dang, Tri Vi, Gary Gorton, Bengt Holmström, and Guillermo Ordoñez** (2017), “Banks as Secret Keepers”, *American Economic Review*, Vol. 107, pp. 1005–29.
- DeAngelo, Harry and René M. Stulz** (2015), “Liquid-claim production, risk management, and bank capital structure: Why high leverage is optimal for banks”, *Journal of Financial Economics*, Vol. 116, pp. 219–236.
- Despres, E., E. Kindleberger, and W.S. Salant** (1966), *The Dollar and World Liquidity: A Minority View*: Brookings Institution.

- Dewatripont, Mathias and Jean Tirole** (2018), “Liquidity regulation, bail-ins and bailouts”, Technical report, TSE working paper.
- Di Tella, Sebastian and Pablo Kurlat** (2021), “Why Are Banks Exposed to Monetary Policy?”, *American Economic Journal: Macroeconomics*, Vol. 13, pp. 295–340.
- Diamond, Douglas W.** (1984), “Financial Intermediation and Delegated Monitoring”, *The Review of Economic Studies*, Vol. 51, pp. 393–414.
- Diamond, Douglas W. and Philip H. Dybvig** (1983), “Bank Runs, Deposit Insurance, and Liquidity”, *Journal of Political Economy*, Vol. 91, pp. 401–419.
- Diamond, D.W. and A.K. Kashyap** (2016), “Chapter 29 - Liquidity Requirements, Liquidity Choice, and Financial Stability”, Vol. 2 of *Handbook of Macroeconomics*: Elsevier, pp. 2263–2303.
- Donaldson, Jason Roderick and Giorgia Piacentino** (2022), “Money runs”, *Journal of Monetary Economics*, Vol. 126, pp. 35–57.
- Drechsler, Itamar, Alexi Savov, and Philipp Schnabl** (2017), “The Deposits Channel of Monetary Policy”, *The Quarterly Journal of Economics*, Vol. 132, pp. 1819–1876.
- (2021), “Banking on Deposits: Maturity Transformation without Interest Rate Risk”, *The Journal of Finance*, Vol. 76, pp. 1091–1143.
- (2022), “Credit Crunches and the Great Stagflation”, *Working Paper*.
- (2023a), “Valuing the deposit franchise”, working paper.
- (2023b), “Why do banks invest in MBS?”, working paper.
- Drechsler, Itamar, Alexi Savov, Philipp Schnabl, and Dominik Supera** (2023c), “Deposit beta risk”, working paper.
- Egan, Mark, Ali Hortaçsu, and Gregor Matvos** (2017), “Deposit Competition and Financial Fragility: Evidence from the US Banking Sector”, *American Economic Review*, Vol. 107, pp. 169–216.
- Egan, Mark, Stefan Lewellen, and Adi Sunderam** (2021), “The Cross-Section of Bank Value”, *The Review of Financial Studies*, Vol. 35, pp. 2101–2143.

- English, William B., Skander J. Van den Heuvel, and Egon Zakrajsek** (2018), "Interest rate risk and bank equity valuations", *Journal of Monetary Economics*.
- Farhi, Emmanuel and Matteo Maggiori** (2017), "A Model of the International Monetary System\*", *The Quarterly Journal of Economics*, Vol. 133, pp. 295–355.
- Federal Deposit Insurance Corporation** (1997), *History of the Eighties: Lessons for the Future. Vol. 1, An Examination of the Banking Crises of the 1980s and Early 1990s.*: Washington, DC: FDIC.
- Freixas, Xavier and Jean-Charles Rochet** (2008), *Microeconomics of banking*: MIT press.
- Gertler, Mark and Nobuhiro Kiyotaki** (2015), "Banking, Liquidity, and Bank Runs in an Infinite Horizon Economy", *American Economic Review*, Vol. 105, pp. 2011–43.
- Goldstein, Itay and Ady Pauzner** (2005), "Demand-Deposit Contracts and the Probability of Bank Runs", *The Journal of Finance*, Vol. 60, pp. 1293–1327.
- Gomez, Matthieu, Augustin Landier, David Sraer, and David Thesmar** (2021), "Banks' exposure to interest rate risk and the transmission of monetary policy", *Journal of Monetary Economics*, Vol. 117, pp. 543–570.
- Gorton, Gary and George Pennacchi** (1990), "Financial Intermediaries and Liquidity Creation", *The Journal of Finance*, Vol. 45, pp. 49–71.
- Gorton, Gary and Andrew Winton** (2003), "Chapter 8 - Financial Intermediation", in George M. Constantinides, Milton Harris, and René M. Stulz eds. *Corporate Finance*, Vol. 1 of Handbook of the Economics of Finance: Elsevier, pp. 431–552.
- Hanson, Samuel G., Andrei Shleifer, Jeremy C. Stein, and Robert W. Vishny** (2015), "Banks as patient fixed-income investors", *Journal of Financial Economics*, Vol. 117, pp. 449–469.
- He, Zhiguo, Arvind Krishnamurthy, and Konstantin Milbradt** (2019), "A Model of Safe Asset Determination", *American Economic Review*, Vol. 109, pp. 1230–62.

- Iyer, Rajkamal and Manju Puri** (2012), “Understanding Bank Runs: The Importance of Depositor-Bank Relationships and Networks”, *American Economic Review*, Vol. 102, pp. 1414–45.
- Iyer, Rajkamal, Manju Puri, and Nicholas Ryan** (2016), “A Tale of Two Runs: Depositor Responses to Bank Solvency Risk”, *The Journal of Finance*, Vol. 71, pp. 2687–2726.
- Jermann, Urban and Haotian Xiang** (2023), “Dynamic banking with non-maturing deposits”, *Journal of Economic Theory*, Vol. 209, p. 105644.
- Jiang, Erica Xuewei, Gregor Matvos, Tomasz Piskorski, and Amit Seru** (2023), “Monetary Tightening and US Bank Fragility in 2023: Mark-to-Market Losses and Uninsured Depositor Runs?”, Technical report, National Bureau of Economic Research.
- Kashyap, Anil K., Raghuram Rajan, and Jeremy C. Stein** (2002), “Banks as Liquidity Providers: An Explanation for the Coexistence of Lending and Deposit-taking”, *The Journal of Finance*, Vol. 57, pp. 33–73.
- Koont, Naz, Tano Santos, and Luigi Zingales** (2023), “Destabilizing Digital Bank Walks”, Working Paper 328, Chicago Booth.
- Moreira, Alan and Alexi Savov** (2017), “The macroeconomics of shadow banking”, *The Journal of Finance*, Vol. 72, pp. 2381–2432.
- Nagel, Stefan and Amiyatosh Purnanandam** (2020), “Banks’ risk dynamics and distance to default”, *The Review of Financial Studies*, Vol. 33, pp. 2421–2467.
- Petersen, Mitchell A. and Raghuram G. Rajan** (1994), “The Benefits of Lending Relationships: Evidence from Small Business Data”, *The Journal of Finance*, Vol. 49, pp. 3–37.
- Philippon, Thomas and Philipp Schnabl** (2013), “Efficient Recapitalization”, *The Journal of Finance*, Vol. 68, pp. 1–42.
- Philippon, Thomas and Vasiliki Skreta** (2012), “Optimal Interventions in Markets with Adverse Selection”, *American Economic Review*, Vol. 102, pp. 1–28.

- Philippon, Thomas and Olivier Wang** (2022), "Let the Worst One Fail: A Credible Solution to the Too-Big-To-Fail Conundrum\*", *The Quarterly Journal of Economics*, Vol. 138, pp. 1233–1271.
- Rochet, Jean-Charles and Xavier Vives** (2004), "Coordination Failures and the Lender Of Last Resort: Was Bagehot Right After All?", *Journal of the European Economic Association*, Vol. 2, pp. 1116–1147.
- Stein, Jeremy C.** (2012), "Monetary Policy as Financial Stability Regulation", *The Quarterly Journal of Economics*, Vol. 127, pp. 57–95.
- Tirole, Jean** (2012), "Overcoming Adverse Selection: How Public Intervention Can Restore Market Functioning", *American Economic Review*, Vol. 102, pp. 29–59.
- Triffin, Robert** (1961), *Gold and the Dollar Crisis: The future of convertibility*, New Haven: Yale University Press.
- Ulate, Mauricio** (2021), "Going Negative at the Zero Lower Bound: The Effects of Negative Nominal Interest Rates", *American Economic Review*, Vol. 111, pp. 1–40.
- U.S. Federal Reserve** (2023), "Review of the Federal Reserve's supervision and regulation of Silicon Valley Bank", *Federal Reserve Board of Governors*, Vol. 28.
- Wang, Olivier** (2022), "Banks, Low Interest Rates, and Monetary Policy Transmission", working paper.
- Wang, Yifei, Toni M. Whited, Yufeng Wu, and Kairong Xiao** (2022), "Bank Market Power and Monetary Policy Transmission: Evidence from a Structural Estimation", *The Journal of Finance*, Vol. 77, pp. 2093–2141.
- Xiao, Kairong** (2020), "Monetary transmission through shadow banks", *The Review of Financial Studies*, Vol. 33, pp. 2379–2420.

# Appendix

## A Proofs

### A.1 Proof of Proposition 1

From (1) and using  $D_{t-1} = (1 - \delta)^{t-1}(1 - w)D$  we have

$$\begin{aligned}
 L &= wD + \sum_{t=1}^{\infty} \frac{\delta D_{t-1}}{(1 + r')^t} + \sum_{t=1}^{\infty} \frac{(\beta r' + c) D_{t-1}}{(1 + r')^t} \\
 &= wD + (1 - w)D \left[ \sum_{t=1}^{\infty} \frac{(1 - \delta)^{t-1}}{(1 + r')^t} \right] (\delta + \beta r' + c) \\
 &= D \left[ w + (1 - w) \frac{(\delta + \beta r' + c)}{r' + \delta} \right].
 \end{aligned}$$

Therefore

$$\begin{aligned}
 DF &= D - L \\
 &= D(1 - w) \left[ 1 - \frac{(\delta + \beta r' + c)}{r' + \delta} \right] \\
 &= D(1 - w) \left[ \frac{(1 - \beta)r' - c}{r' + \delta} \right]
 \end{aligned}$$

and

$$\begin{aligned}
 DF'(r) &= D \left\{ (1 - w(r)) \left[ \frac{(1 - \beta)\delta + c}{(r + \delta)^2} \right] - w'(r) \left[ \frac{(1 - \beta)r - c}{r + \delta} \right] \right\} \\
 &= D \left\{ \frac{(1 - \beta)\delta + c}{(r + \delta)^2} - w'(r) \left[ \frac{(1 - \beta)r - c}{r + \delta} \right] \right\}
 \end{aligned}$$

using  $w(r) = 0$ . Thus the duration evaluated at  $r$  is

$$T_{DF}(r) = -\frac{DF'(r)}{DF(r)} = -\frac{1}{r + \delta} \frac{(1 - \beta)\delta + c}{(1 - \beta)r - c} + w'(r).$$

## A.2 Proof of Proposition 2

The optimal modified duration is

$$\begin{aligned}
 T_A &= \frac{DF'(r)}{A(r)} \\
 &= \frac{DF'(r)}{D} \\
 &= \frac{c + (1 - \beta)\delta}{(r + \delta)^2} - w'(r) \frac{(1 - \beta)r - c}{r + \delta}.
 \end{aligned}$$

The effective beta  $\tilde{\beta}$  solves

$$\begin{aligned}
 \frac{c + (1 - \beta)\delta}{(r + \delta)^2} - w'(r) \frac{(1 - \beta)r - c}{r + \delta} &= \frac{c + (1 - \tilde{\beta})\delta}{(r + \delta)^2} \\
 (\tilde{\beta} - \beta)\delta &= (r + \delta)w'(r) [(1 - \beta)r - c] \\
 \tilde{\beta} &= \beta + w'(r) [(1 - \beta)r - c] (1 + r/\delta)
 \end{aligned}$$

## A.3 Proof of Proposition 4

To hedge  $v(0, r')$  we need

$$\begin{aligned}
 A' + DF'_I &= 0 \\
 T_A &= \frac{1}{A} DF'_I \\
 &= (1 - u) \frac{c^I + (1 - \beta^I)\delta}{(r + \delta)^2}
 \end{aligned}$$

This corresponds to the optimal duration for a bank with an effective beta  $\hat{\beta}$  solving

$$\begin{aligned}
 (1 - u) \frac{c^I + (1 - \beta^I)\delta}{(r + \delta)^2} &= \frac{c + (1 - \hat{\beta})\delta}{(r + \delta)^2} \\
 (1 - u) [c^I + (1 - \beta^I)\delta] &= c + (1 - \hat{\beta})\delta \\
 (1 - u)(\hat{\beta} - \beta^I)\delta &= uc^U + u(1 - \hat{\beta})\delta \\
 \hat{\beta} &= (1 - u)\beta^I + uc^U/\delta + u
 \end{aligned}$$



## A.4 Proof of Proposition 5

If the bank perfectly hedges interest rate risk in the good equilibrium by choosing assets  $A$  such that  $v(1, r')$  is constant, i.e.,  $A(r') = v^*D - DF_I(r') - DF_U(\lambda = 1, r')$  for some constant  $v^* > \underline{v}$ , then

$$\begin{aligned} v(0, r') &= v(1, r') - u \frac{(1 - \beta^U)r' - c^U}{r' + \delta} \\ &= v^* - u \frac{(1 - \beta^U)r' - c^U}{r' + \delta} \end{aligned}$$

hence for

$$\begin{aligned} v(0, r') < \underline{v} &\Leftrightarrow v^* - u \frac{(1 - \beta^U)r' - c^U}{r' + \delta} < \underline{v} \\ &\Leftrightarrow \frac{v^* - \underline{v}}{u} < \frac{(1 - \beta^U)r' - c^U}{r' + \delta} \\ &\Leftrightarrow \frac{v^* - \underline{v}}{u} \delta + c^U < (1 - \beta^U - \frac{v^* - \underline{v}}{u})r' \\ &\Leftrightarrow r' > \bar{r} = \frac{c^U + \frac{v^* - \underline{v}}{u} \delta}{1 - \beta^U - \frac{v^* - \underline{v}}{u}} \end{aligned}$$

where  $\bar{r}$  decreases with the uninsured deposit ratio  $u$ .

If instead the bank perfectly hedges liquidity risk by making  $v(0, r')$  constant equal to  $v_*$  then

$$\begin{aligned} v(1, r') \leq \underline{v} &\Leftrightarrow v_* + u \frac{(1 - \beta^U)r' - c^U}{r' + \delta} \leq \underline{v} \\ &\Leftrightarrow \frac{v_* - \underline{v}}{u} \leq \frac{c^U - (1 - \beta^U)r'}{r' + \delta} \\ &\Leftrightarrow \frac{v_* - \underline{v}}{u} \delta + (1 - \beta^U + \frac{v_* - \underline{v}}{u})r' \leq c^U \\ &\Leftrightarrow r' \leq \underline{r} = \frac{c^U - \frac{v_* - \underline{v}}{u} \delta}{1 - \beta^U + \frac{v_* - \underline{v}}{u}} \end{aligned}$$

## A.5 Proof of Proposition 7

Satisfying (11)-(12) requires

$$\begin{aligned} A(r') + DF_I(r') + DF_U(\lambda = 1, r') &\geq (1 + v^*)D \\ A(r') + DF_I(r') &\geq (1 + \underline{v})D \end{aligned}$$

or

$$A(r') \geq A^*(r') \equiv \max \{ (1 + \underline{v})D - DF_I(r'), (1 + v^*)D - DF_I(r') - DF_U(\lambda = 1, r') \}.$$

Subtracting  $(1 + v^*)D - DF_I(r') - DF_U(\lambda = 1, r')$  from both sides and adding it back outside the bracket we get

$$A^*(r') = (1 + v^*)D - DF_I(r') - DF_U(\lambda = 1, r') + \max \{ -(v^* - \underline{v})D + DF_U(\lambda = 1, r'), 0 \}.$$

The term  $(1 + v^*)D - DF_I(r') - DF_U(\lambda = 1, r')$  corresponds to the long-term assets that hedge interest rate risk in the good equilibrium.

The second term in  $A^*$  can be approximated to first order as

$$\begin{aligned} &\max \{ 0, DF_U(\lambda = 1, \bar{r}) + DF'_U(\lambda = 1, \bar{r}) \times (r' - \bar{r}) - (v^* - \underline{v})D \} \\ &= DF'_U(\lambda = 1, \bar{r}) \times \max \{ 0, r' - \bar{r} \} \end{aligned}$$

since  $DF_U(\lambda = 1, \bar{r}) = (v^* - \underline{v})D$  by definition of  $\bar{r}$ . The amount of standard swaptions the bank needs to buy is thus

$$\begin{aligned} N &= DF'_U(\lambda = 1, \bar{r}) \\ &= uD \frac{c^U + (1 - \beta^U)\delta}{(\bar{r} + \delta)^2}. \end{aligned}$$

## B Fixed and Variable Operating Costs

Recall that in the baseline model we assume that  $c$  is a cost per dollar of remaining deposits hence the franchise value is

$$DF(r') = D (1 - w(r')) \frac{(1 - \beta)r' - c}{r' + \delta}.$$

In practice operating costs are a combination of pre-determined costs that do not fully respond to withdrawals, and costs that scale with the amount of deposits in each period. Here we extend the model by allowing the bank to decide the scale of the branch network and services offered before the interest rate shock, which corresponds to costs  $\kappa$  that must be paid even if deposits are withdrawn at  $t = 0$ . In that case the franchise value writes instead

$$DF(r') = D (1 - w(r')) \frac{(1 - \beta)r' - c}{r' + \delta} - D \frac{\kappa}{r' + \delta}.$$

For simplicity we still assume that costs  $\kappa$  stop being paid upon exogenous withdrawals (at rate  $\delta$ ); otherwise nothing substantial changes except that expressions are slightly more complex because the last term is  $-D\kappa/r'$  instead. Define the total cost

$$C = c + \kappa.$$

Results are mostly unchanged under this formulation except for two points.

First, since outflows do not help economize the fixed operating costs  $\kappa$ , if all costs are fixed ( $C = \kappa, c = 0$ ) then outflows always hurt the franchise value even when  $DF < 0$ . We can generalize Proposition 2 to:

**Proposition.** *Suppose that  $w'(r) ((1 - \beta)r - c) \leq \frac{C + \delta(1 - \beta)}{r + \delta}$  hence  $DF' \geq 0$ . The modified duration of assets for hedging interest rate-driven outflows for a bank with  $A(r) = D$  is*

$$T_A = \frac{(1 - \beta)\delta + C}{(r + \delta)^2} - w'(r) \left[ \frac{(1 - \beta)r - c}{r + \delta} \right].$$

As in Proposition 2 the effect of endogenous outflows still depends on the level of interest rates. If  $1 - \beta > c/r$  then outflows hurt the franchise value hence asset duration should be shorter. But now it is only the variable part of the cost  $c$ , not the fixed cost  $\kappa$ , that matters for this effect, whereas the franchise value is positive if and only if the deposit spread  $(1 - \beta)r$  exceeds the *total* cost  $C = c + \kappa$ .

Second, the analysis of uninsured depositor runs is unchanged, except that in a run  $\lambda = 0$  the uninsured deposit franchise may be negative instead of zero, since the fixed operating costs  $\kappa$  must still be paid. This only affects the expressions for  $v$  and  $\Lambda$  as follows. The solvency ratio of the bank after the interest rate shock as a

function of  $\lambda$  is still

$$v(\lambda, r') = v(0, r') + u\lambda (1 - \underline{w}_U(r')) \frac{(1 - \beta^U)r' - c}{r' + \delta}$$

as in (7), but the solvency ratio when all uninsured depositors run ( $\lambda = 0$ ) is

$$v(0, r') = \frac{A(r') - D + DF_I(r')}{D} - u \frac{\kappa}{r' + \delta}$$

instead of (8). The cost  $\kappa$  implies that the bank should hold some long-term assets to cover  $\kappa$  per period even to hedge liquidity risk, i.e., to stabilize  $v(0, r')$ . But the dilemma between hedging interest rate and liquidity risk persists for two reasons: first, a bank hedging interest rate risk in the no-run equilibrium must account for total operating costs  $C = c + \kappa$ , and second, the term  $\frac{(1 - \beta^U)r' - c}{r' + \delta}$  increases with  $r'$  even if  $c = 0$ .

## C Additional Figures

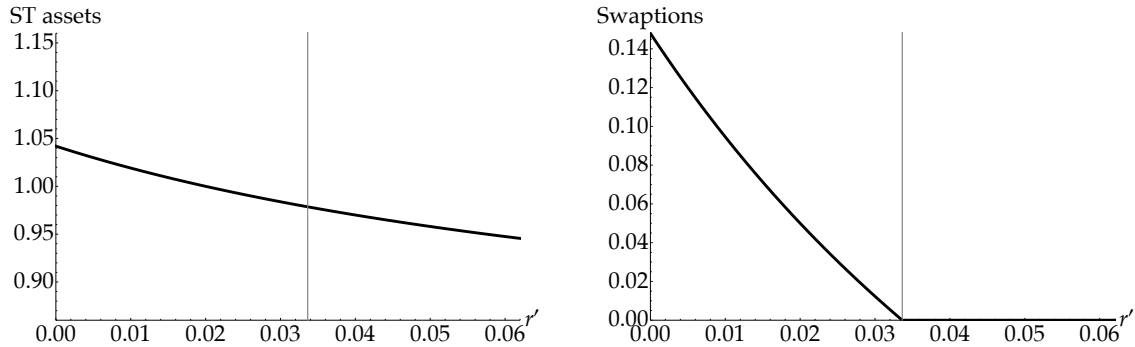


Figure A.1: Decomposing  $A^*(r')$  into short-term assets (left panel) and receiver swaptions with strike  $r = \bar{r}$  (right panel).