Optimal Trend Inflation

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- Productivity at firm level displays *systematic* trends:
  - life cycle: firms start small/unproductive, become productive, exit
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- Productivity at firm level displays *systematic* trends:
  - life cycle: firms start small/unproductive, become productive, exit
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- Productivity trends at the firm level
  $\Longrightarrow$ strongly affect optimal inflation dynamics
  & rationalizes positive steady state inflation

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  abstracts from firm level heterogeneity, except for price heterogeneity

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  Strong economic implications: zero inflation optimal

- Productivity of price adjusting firms equal to productivity of non-adjusting firms

- Adjusting firms' price = price of non-adjusting firms

  $\Longrightarrow$ strong force towards zero inflation

  Woodford(2003), Kahn, King & Wolman(2003), Schmitt-Grohé & Uribe(2010)

Introduction

- Golosov&Lucas (2007), Nakamura&Steinsson (2010)

  idiosyncratic firm level productivity $\Leftrightarrow$ without systematic trend

- Do not look at optimal inflation

- Results sugests zero inflation optimal:
  av. prod. of adjusting firm $\approx$ av. prod. of non-adjusting firm

Introduction

Enrich basic homogeneous firm setup by adding:

- Firm entry & exit
- Measure δ of randomly selected firms: very negative productivity shock & exit
- Exiting firms replaced by same measure of newly entering firms
- Alternative interpretations of setup possible (product substitution, quality improvements)

Introduction

Firm-level productivity trends driven by 3 underlying trends:

- **aggregate trend**: productivity gains experienced by all firms
- **experience trend:** firms become more productive over time
- **cohort trend:** productivity level for new cohort of firms

- Production function of firm $j \in [0, 1]$:

$$Y_{jt} = A_t Q_{t-s_{jt}} G_{jt} \left(K_{jt}^{1-\frac{1}{\phi}} L_{jt}^{\frac{1}{\phi}} - F_t \right),$$

where s_{jt} is time since last δ-shock

$$\begin{aligned} A_t &= a_t A_{t-1}, \\ Q_t &= q_t Q_{t-1}, \\ G_{jt} &= \begin{cases} 1 & \text{if } s_{jt} = 0, \\ g_t G_{jt-1} & \text{otherwise.} \end{cases} \end{aligned}$$

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  (a_t, q_t, g_t) arbitrary stationary process w mean $\mathbf{a}, \mathbf{q}, \mathbf{g}$

- Three productivity trends: $\mathbf{a}$, $\mathbf{q}$ and $\mathbf{g}$
- Measure δ of firms: productivity drops to zero & exit
- Special cases w/o firm level trends: $\delta = 0$ or if $\mathbf{q}_t \equiv \mathbf{g}_t$

Introduction

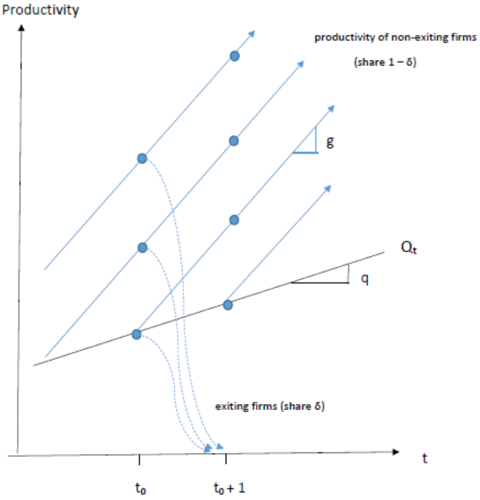

Figure: Productivity dynamics in a setting with firm entry and exit

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- **Strength of effect independent of turnover rate** $\delta > 0$

  Discontinous jump of optimal inflation: $\delta = 0 \ \rightarrow \delta > 0$

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- cannot be inferred from aggregate productivity trends

- has to know firm level trends & shocks to these trends

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- Optimal inflation $\Pi^* = 1$ if $\delta = 0$.

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- Model-consistent approach for estimating SS inflation rate from firm level trends: 147million firm observations from the LBD database (US Census)

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- Model-consistent approach for estimating SS inflation rate from firm level trends: 147million firm observations from the LBD database (US Census)
- **Estimated optimal infl. rate steadily declined:**

1986: $\approx$2% $\Longrightarrow$ **2013: $\approx$1%**

Related Literature

- Few papers: inflation $\Leftrightarrow$ productivity dynamics
- All of them find negative inflation rates optimal:
  - Wolman (JMCB, 2011): two sector economy with different sectorial productivity trends, homogeneous firms in each sector, neg. inflation optimal despite monetary frictions being absent
  - Amano, Murchison & Rennison (JME, 2009): homogeneous firm model with sticky prices and wages & aggregate growth; wages more sticky than prices; to depress wage-markups deflation turns out optimal.

Related Literature

- Zero inflation approx. optimal in models w homogeneous firms Woodford (2003), Kahn, King & Wolman (2003), Schmitt-Grohé and Uribe (2010)
- Zero lower bound cannot justify positive average rates of inflation: Adam & Billi (2006), Coibion, Gorodnichenko & Wieland (2012)
- Brunnermeier and Sannikov (2016): idiosyncratic risk -> positive inflation increasingly optimal
- Downward nominal wage rigidity may justify positive inflation rates Kim & Ruge-Murcia (2009), Benigno & Ricci (2011), Schmitt-Grohe & Uribe (2013), Carlsson & Westermark (2016)
- Positive inflation possibly optimal in models with endogenous entry: Corsetti & Bergin (2008), Bilbiie, Ghironi & Melitz (2008), Bilbiie, Fujiwara & Ghironi (2014)

Outline of Remaining Talk

1. **Sticky price model with δ-shocks**
2. Aggregation & optimality of flex price equilibrium
3. Optimal inflation: main result
4. Multi-sector extension & empirical strategy

Sticky Price Model

- Consider a Calvo sticky price setup: price stickiness parameter α (main results extend to menu cost setting)
- Continuum of sticky price firms, Dixit-Stiglitz aggregate Y_t
- Random sample δ receives δ-shocks
- Firm productivity dynamics as described before
- Competitive labor and capital markets

Sticky Price Model

- Household problem

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \xi_t \left(\frac{\left[C_t V(L_t)\right]^{1-\sigma} - 1}{1-\sigma} \right)$$

$$s.t.$$

$$C_t + K_{t+1} + \frac{B_t}{P_t} =$$

$$(r_t + 1 - d) K_t + \frac{W_t}{P_t} L_t + \int_0^1 \frac{\Theta_{jt}}{P_t} \, \mathrm{dj} + \frac{B_{t-1}}{P_t}(1 + i_{t-1}) - T_t$$

- Existence of balanced growth path:

$$\beta < (aq)^{\phi\sigma} \quad and \quad (1-\delta)\,(g/q)^{\theta-1} < 1$$

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Aggregation

- Highlight the differences relative to a model with homogeneous firms
- Will spare you the derivation behind the results...

Aggregate Output, Capital & Labor

- Aggregate output Y_t :

$$Y_t = \frac{A_t Q_t}{\Delta_t} \left(K_t^{1-\frac{1}{\phi}} L_t^{\frac{1}{\phi}} - F_t \right),$$

  with K_t, L_t aggregate capital, labor and $F_t \geq 0$ fixed costs

- Δ_t: captures joint distribution of prices **&** productivities:

$$\Delta_t = \int_0^1 \left(\frac{Q_t}{G_{jt} Q_{t-s_{jt}}} \right) \left(\frac{P_{jt}}{P_t} \right)^{-\theta} dj \qquad (1)$$

Price Level

- Price level: exp.-weighted average of product prices

$$P_t = \left(\int_0^1 (P_{jt})^{1-\theta} \, \mathrm{dj} \right)^{\frac{1}{1-\theta}}$$
$$= \int_0^1 \left(\frac{Y_{jt}}{Y_t} \right) P_{jt} \, \mathrm{dj}$$

Price level accounts for product substitution (as statistical agencies do)

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  Price level accounts for product substitution (as statistical agencies do)

- Inflation:

$$\Pi_t = P_t / P_{t-1}.$$

Aggregate Price Level Dynamics

Evolution of the aggregate price under opt. price setting:

$$P_t^{1-\theta} = (\underbrace{\delta}_{\substack{\text{new}\\ \text{firms}}} + \underbrace{(1-\alpha)(1-\delta)}_{\text{old adj.firms}} \underbrace{\frac{(p_t^n)^{\theta-1} - \delta}{1-\delta}}_{\substack{\text{rel. price}\\ \text{factor}}}) \underbrace{P_{t,t}^{\star}}_{\substack{\text{opt}\\ \text{price}\\ \text{new}\\ \text{firm}}}{}^{1-\theta} + \underbrace{\alpha(1-\delta)}_{\substack{\text{old firms,}\\ \text{w/o adj.}}} P_{t-1}{}^{1-\theta}$$

$$(p_t^n)^{\theta-1} = \delta + (1-\delta)\left(p_{t-1}^n \frac{g_t}{q_t}\right)^{\theta-1}.$$

Aggregate Price Level Dynamics

$g_t \equiv q_t \implies$ no firm level trends and $(p_t^n)^{\theta-1} \to 1$ and

$$P_t^{1-\theta} = (\delta + (1-\alpha)(1-\delta))(P_{t,t}^{\star})^{1-\theta} + \alpha(1-\delta)(P_{t-1})^{1-\theta}$$

If - in addition - $\delta = 0$:

$$P_t^{1-\theta} = (1-\alpha)(P_{t,t}^{\star})^{1-\theta} + \alpha(P_{t-1})^{1-\theta}$$

Standard price evolution equation in homogeneous firm models.

Conditions Insuring Efficiency

- Attaining efficiency requires

  - eliminating firm's monopoly power by an output subsidy

  - choosing Δ_t in the production function

$$Y_t = \frac{A_t Q_t}{\Delta_t} \left(K_t^{1-\frac{1}{\phi}} L_t^{\frac{1}{\phi}} - F_t \right),$$

  equal to

$$\Delta_t = \Delta_t^e = \left(\int_0^1 \left(\frac{Q_t}{G_{jt} Q_{t-s_{jt}}} \right)^{1-\theta} \mathrm{dj} \right)^{\frac{1}{1-\theta}}$$

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- $\Delta_t = \Delta_t^e$ decentralized by prices satisfying

$$\frac{P_{jt}}{P_t} = \frac{1}{\Delta_t^e} \frac{Q_t}{G_{jt} Q_{t-s_{jt}}}$$

Efficiency of Flex Price Equilibrium

Proposition: With flexible prices ($\alpha = 0$) & appropriate output subsidy, the equilibrium allocation is efficient.

The optimal inflation rate is indeterminate....

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- Price level equation
$$P_t^{1-\theta} = (\delta + (1-\alpha)(1-\delta)\frac{(p_t^n)^{\theta-1} - \delta}{1-\delta})(P_{t,t}^{\star})^{1-\theta} + \alpha(1-\delta)(P_{t-1})^{1-\theta},$$

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$\Longrightarrow$ since $1-\theta < 0$: old firms to set lower prices than new firms

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  $\Longrightarrow$ since $1-\theta < 0$: old firms to set lower prices than new firms
- Efficiency: old firms that adjust must choose same price as old firms that do not adjust
- Need to allow for inflation to achieve efficiency!

Efficiency under Sticky Prices

- **Proposition:** Suppose (1) there is an appropriate output subsidy and (2) initial prices in $t = -1$ reflect firms' relative productivities, i.e., $P_{j,-1} \propto 1/\left(Q_{-1-s_{j,-1}} G_{j,-1}\right)$ for all $j \in [0, 1]$. The eq. allocation is efficient under sticky price if

$$\Pi_t^\star = \left(\frac{1 - \delta / \left(\Delta_t^e\right)^{1-\theta}}{1 - \delta} \right)^{\frac{1}{\theta - 1}} \tag{2}$$

for all $t \geq 0$, where $\left(\Delta_t^e\right)^{1-\theta} = \delta + (1 - \delta) \left(\Delta_{t-1}^e q_t / g_t\right)^{1-\theta}$.

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- Prop holds for arbitrary initial prod. distributions & arbitrary shock processes (consistent with balanced growth)
- Proof works as follows: under the inflation rate (2)
  1. new firms choose relative price as in the flex price economy
  2. existing firms do not want to adjust their price.
  3. with initial prices 'right' & output subsidy $\Longrightarrow$ flex price alloc.

Efficiency under Sticky Prices

$$\Pi_t^\star = \left(\frac{1 - \delta / \left(\Delta_t^e\right)^{1-\theta}}{1 - \delta} \right)^{\frac{1}{\theta - 1}}$$

- **In the absence** δ**-shocks/firm level trends** ($\delta = 0$ and/or $g_t \equiv q_t$) get familiar result:

$$\Pi_t^* \equiv 1$$

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$$\Pi_t^* \equiv 1$$

- Price stability optimal, independently of realized productivity shocks.

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- **With firm level trends** ($\delta > 0$), steady state inflation is

$$\lim \Pi_t^{*} = \frac{g}{q}$$

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- **With firm level trends** ($\delta > 0$), steady state inflation is

$$\lim \Pi_t^* = \frac{g}{q}$$

- SS inflation positive when $g > q$
- SS independent of δ:
  - fewer unproductive firms enter $\longrightarrow$ lower inflation
  - existing firms accumulated more experience $\longrightarrow$ higher inflation

Efficiency under Sticky Prices

$$\Pi_t^\star = \left(\frac{1 - \delta / \left(\Delta_t^e\right)^{1-\theta}}{1-\delta} \right)^{\frac{1}{\theta-1}} \tag{4}$$

where $\left(\Delta_t^e\right)^{1-\theta} = \delta + (1-\delta)\left(\Delta_{t-1}^e q_t / g_t\right)^{1-\theta}$.

Linearization:

$$\pi_t^\star = (1-\delta)\pi_{t-1}^\star + \delta\left(\frac{g_t}{q_t} - 1\right) + O(2)$$

- Positive experience shock (g_t): persistent rise in opt. inflation

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Linearization:

$$\pi_t^\star = (1-\delta)\pi_{t-1}^\star + \delta\left(\frac{g_t}{q_t} - 1\right) + O(2)$$

- Positive experience shock (g_t): persistent rise in opt. inflation
- Positive chohort shock (q_t): persistent drop in opt inflation
- $\lim_{\delta \to 0}$: $\pi_t^\star$ random walk, but $Var(\pi_t^\star) \to 0$.

The Welfare Costs of Strict Price Stability

- Suppose MP implements $\Pi = 1$ in an economy where $\Pi^{\star} \neq 1$
- Analytical result: strictly positive welfare costs even in the limit $\delta \longrightarrow 0$
- Numerical illustration highlighting the source of welfare distortions

The Welfare Costs of Strict Price Stability

Assumptions for the analytical result:

- there is an optimal output subsidy and initial prices reflect initial productivities
- there are no aggregate productivity disturbances and $\delta > 0$
- fixed costs of production are zero ($f = 0$)
- disutility of work is given by

$$V(L) = 1 - \psi L^{\nu}, \text{ with } \nu > 1, \psi > 0.$$

- $g/q > \alpha(1-\delta)$, so that a well-defined steady state with strict price stability exists
- consider the limit $\beta(\gamma^{e})^{1-\sigma} \to 1$

The Welfare Costs of Strict Price Stability

Proposition: Consider a policy implementing the optimal inflation rate $\Pi_t^\star$, which satisfies $\lim_{t\to\infty}\Pi_t^\star = \Pi^\star = g/q$. Let $c(\Pi^\star)$ and $L(\Pi^\star)$ denote the limit outcomes for $t \to \infty$ for consumption and hours under this policy. Similarly, let $c(1)$ and $L(1)$ denote the limit outcomes under the alternative policy of implementing strict price stability. Then,

$$L(1) = L(\Pi^\star)$$

and

$$\frac{c(1)}{c(\Pi^\star)} = \left(\frac{1-\alpha(1-\delta)(g/q)^{\theta-1}}{1-\alpha(1-\delta)}\right)^{\frac{\phi\theta}{\theta-1}} \left(\frac{1-\alpha(1-\delta)\left(g/q\right)^{-1}}{1-\alpha(1-\delta)(g/q)^{\theta-1}}\right)^{\phi} \leq 1. \tag{5}$$

For $g \neq q$ the previous inequality is strict and

$$\lim_{\delta\to 0} c(1)/c(\Pi^\star) < 1$$

.

The Welfare Costs of Strict Price Stability

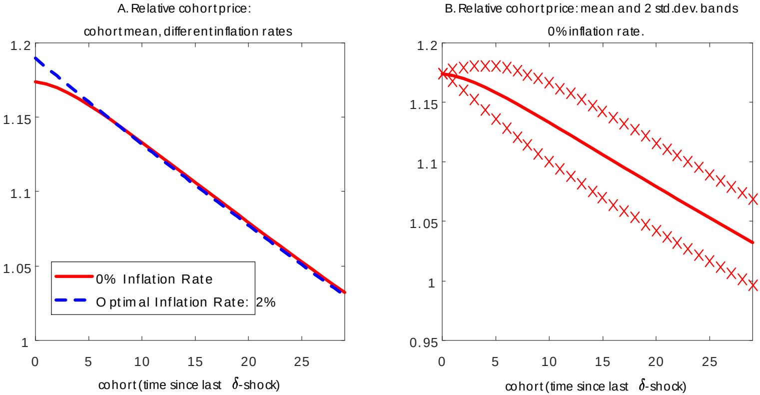

Figure: Relative prices and inflation

The Welfare Costs of Strict Price Stability

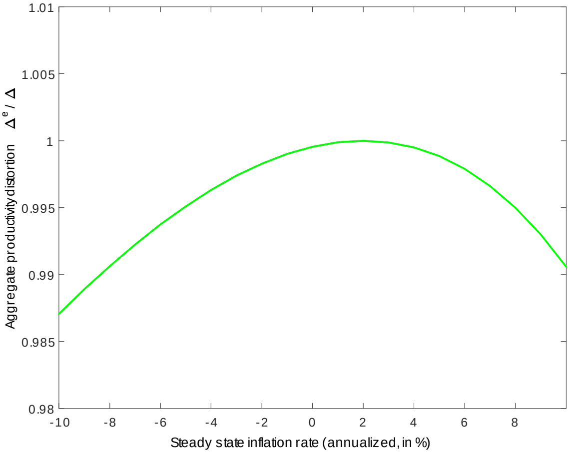

Figure: Aggregate productivity as a function of gross steady state inflation (optimal inflation rate is 1.02)

Outline of Remaining Talk

1. Sticky price model with δ-shocks
2. Aggregation & optimality of flex price equilibrium
3. Optimal inflation: main result
4. **Multi-sector extension & empirical strategy**

Multi-Sector Extension / Empirical Strategy

- Goal: quantify inflation rates arising from firm trends
- Take into account of sector-specific productivity trends: manufacturing vs services
- Present a multi-sector extension of our analytical results & model-consistent empirical strategy

Multi-Sector Extension / Empirical Strategy

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Multi-Sector Extension / Empirical Strategy

Proposition: Suppose initial prices reflect initial productivity, no economic disturbances, and an optimal output subsidy. Consider the limit $\beta(\gamma^e)^{1-\sigma} \to 1$ and suppose monetary policy implements $\Pi_t = \Pi$ for all t. The inflation rate Π that maximizes the resulting steady state utility is

$$\Pi^\star = \sum_{z=1}^{Z} \omega_z \left(\frac{g_z}{q_z} \frac{\gamma_z^e}{\gamma^e} \right),$$

where

$$\frac{\gamma_z^e}{\gamma^e} = \frac{a_z q_z}{\prod_{z=1}^{Z} (a_z q_z)^{\psi_z}}$$

is the growth trend of sector z relative to the growth trend of the aggregate economy in the efficient allocation.

The sector weights $\omega_z \geq 0$ sum to one and are given by

$$\tilde{\omega}_z = \frac{\psi_z \theta \alpha_z (1-\delta_z)(\Pi\gamma^e/\gamma_z^e)^\theta (q_z/g_z)}{\left[1-\alpha_z(1-\delta_z)(\Pi\gamma^e/\gamma_z^e)^\theta (q_z/g_z)\right] \left[1-\alpha_z(1-\delta_z)(\Pi\gamma^e/\gamma_z^e)^{\theta-1}\right]}.$$

Multi-Sector Extension / Empirical Strategy

- The optimal steady state inflation rate

$$\Pi^* = \sum_{z=1}^{Z} \psi_z \left(\frac{g_z \gamma_z^e}{q_z \gamma^e} \right) + O(2),$$

$O(2)$: second order approximation error.

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- How to estimate sector specific productivity trends g_z / q_z?

  - firm level productivity: not observed.....

  - firm level prices: not observed....

  - firm level employment: productivity->prices->demand/employment

- Model implies that g_z/q_z can be estimated **from firm level employment trends:**

$$\ln(L_{jzt}) = d_{zt} + \eta_z \cdot s_{jzt} + \epsilon_{jzt}, \tag{6}$$

d_{zt} :sector dummy, s_{jzt} the age of the firm j, and ϵ_{jzt} a stationary residual term, and

$$\eta_z = (\theta - 1)\ln(g_z/q_z).$$

Multi-Sector Extension / Empirical Strategy

- Estimate the firm level trends:
  - 65 BEA private industries
  - use LBD database for US Census Data: 147 million firm employment observations
  - use repeated cross-sections from 1986-2013 to estimate g_z / q_z

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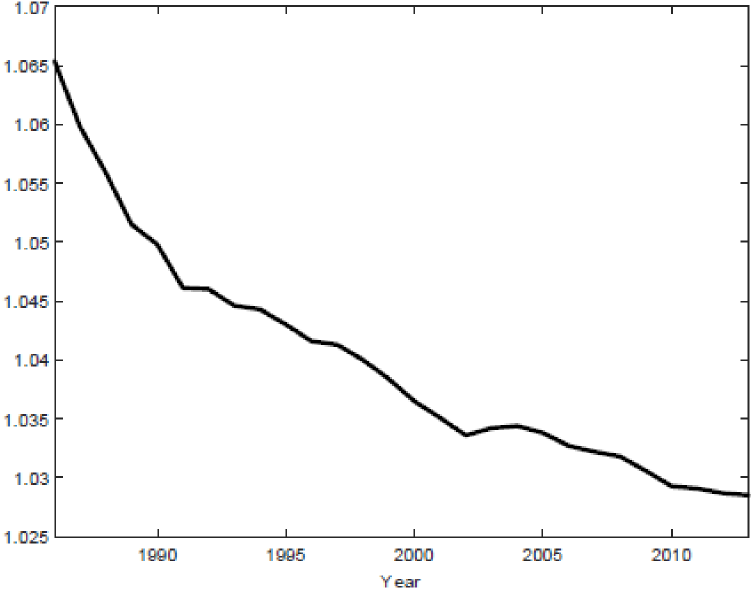
Optimal US Inflation times $(\theta - 1)$



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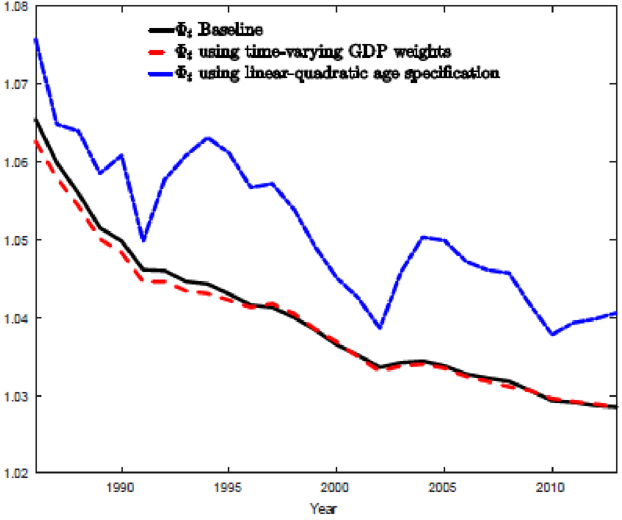

Multi-Sector Extension / Empirical Strategy

Table 1: Optimal Inflation Rate (Net)

	Baseline	TV Weights	LQ Specification	Baseline	TV Weights	LQ Specifica
	$\theta = 3.8$			$\theta = 5$		
Π^*_{1986}	2.34%	2.24%	2.70%	1.64%	1.57%	1.89%
Π^*_{2013}	1.02%	1.02%	1.45%	0.71%	0.71%	1.01%

Notes: "Baseline" refers to the baseline estimate of Φ_t with fixed GDP weights and age as single regressor. "TV Weights" refers to the estimate of Φ_t that is based on time-varying GDP weights. "LQ Specification" refers to the estimate of Φ_t that is based on a specification with both age and age squared as regressors. The parameter θ denotes the product demand elasticity.

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- Steady state inflation $\Pi^* = \frac{g}{q} > 1$
- Productivity disturbances have persistent effects on optimal inflation
- Optimal US inflation: dropped from approx 2% in 1986 to 1% in 2013