Decomposing the Investment Channel of Monetary Policy*

Mathis Momm Lennart Niermann

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Abstract

Monetary policy announcements simultaneously change borrowing costs and firms' expectations about future economic conditions. Conventional estimates of the investment response to monetary policy therefore conflate price effects with belief revisions and fail to identify the structural sensitivity of investment to financing conditions. We exploit quasi-experimental variation from the ECB's 2016 Corporate Sector Purchase Programme (CSPP), which created persistent cross-sectional differences in firms' financing costs through bond market segmentation, to isolate the causal effect of borrowing costs on firm investment. We find that investment elasticities estimated from aggregate monetary policy shocks understate the effect of a pure cost-of-debt shock by 33 percent over four quarters. Our results imply that the structural sensitivity of investment to borrowing costs is larger than previously thought, with direct implications for the calibration of quantitative macroeconomic models. The gap reflects a sizable firm-level information effect. A model of lumpy firm investment with an information effect rationalizes this attenuation: information effects are quantitatively important when a sufficient mass of firms lies near their investment threshold.

Keywords: Investment, Information Effect, Monetary Policy, Monetary Non-Neutrality

JEL-Codes: E52, E44, G12, G32

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Momm: Faculty of Economics, University of Cambridge. mathisyaron@momm.ch
Niermann: Faculty of Economics, University of Cambridge. lcn29@cam.ac.uk

1 INTRODUCTION

A central question in macroeconomics is how monetary policy transmits to real economic activity. Firm investment is a natural focus because it is among the most interest-sensitive components of GDP (Ottonello and Winberry, 2020) and its muted response in downturns is one of the leading explanations for why monetary policy transmission is weakest in recessions, when stimulus is most valuable (Tenreyro and Thwaites, 2016). Empirically, firms' sensitivity is typically measured using aggregate investment responses to monetary policy shocks. Yet such shocks affect firms' borrowing costs while simultaneously altering beliefs about future economic fundamentals – an effect commonly referred to as the *information effect* of monetary policy announcements (Romer and Romer, 2000; Nakamura and Steinsson, 2018). As a result, observed aggregate investment responses conflate price effects with belief revisions and fail to isolate firms' underlying sensitivity to financing conditions.

Disentangling the borrowing-cost channel from information effects using conventional monetary policy shock series in practice requires additional – and potentially restrictive – identifying assumptions, such as sign restrictions (Jarociński and Karadi, 2020). In this paper, we exploit quasi-experimental cross-sectional variation generated by the ECB's Corporate Sector Purchase Programme (CSPP) to isolate the borrowing-cost channel. The CSPP announcement created a yield wedge between eligible and ineligible bonds, while firms differed in their exposure to this wedge due to persistent pre-CSPP financing structures. The resulting cross-sectional variation in firms' financing costs is plausibly exogenous, orthogonal to belief revisions at the time of the announcement, and unavailable in conventional monetary policy shocks.

This paper provides evidence that standard monetary policy shocks identify a composite object that substantially understates firms' true sensitivity to financing costs. We find that aggregate investment elasticities over four quarters are approximately one third smaller than the response implied by a pure cost-of-debt shock. This dampening reflects belief revisions connected to monetary policy announcements and contributes new evidence to the ongoing debate on the quantitative importance of information effects in monetary policy (Cochrane and Piazzesi, 2002; Jon et al., 2004; Bauer and Swanson, 2023).

The distinction matters because macroeconomic models rely on investment elasticities as behavioral primitives. By isolating the borrowing-cost channel, our results suggest that calibrations based on conventional high-frequency estimates (e.g. Cook and Hahn, 1989; Ottonello and Winberry, 2020) may systematically understate the true responsiveness of firm investment to changes in borrowing costs.

To guide intuition, we introduce a simple conceptual framework of lumpy firm investment with an information channel. Monetary policy surprises convey signals about other economic fundamentals, with implications for expected long-run prospects. Following a monetary policy surprise, firms close to the investment threshold find it valuable to extract the signal and adjust their investment decision. When a sufficiently large mass of firms lies near the threshold, the model can replicate attenuation of the borrowing-cost channel, consistent with our empirical estimates. The framework also clarifies how conventional monetary policy shocks and the CSPP-based design load on different sources of variation.

Related Literature Our paper relates to three strands of literature. First, we provide new evidence on information effects in monetary policy transmission. Following the seminal contribution by Romer and Romer (2000), a growing body of empirical work like Campbell et al. (2012) and Nakamura and Steinsson (2018) argue that policy announcements convey information about macro fundamentals beyond their mechanical effect on interest rates.[1] However, the existence and magnitude of information effects remain contested (Cochrane and Piazzesi, 2002; Jon et al., 2004; Bauer and Swanson, 2023). By exploiting cross-sectional variation in CSPP-exposure, we provide estimates that separate information effects from the borrowing-cost effect. We find a sizable wedge between the aggregate investment response and the

[1]There is a large theoretical literature discussing central bank signaling through its actions (e.g. changes in the federal funds rate), or communication (FOMC statements). Early contributions include Cukierman and Meltzer (1986) and Ellingsen and Söderström (2001). More recent contributions include Berkelmans (2011) Melosi (2016), and Andrade et al. (2019).

isolated cost-of-debt channel.

Second, this paper connects to the vast literature on monetary non-neutrality, particularly on monetary transmission through financing conditions. At least since the introduction of the *financial accelerator* mechanism by Bernanke et al. (1999), models of monetary transmission have emphasized how borrowing costs and financial constraints shape firms' investment decisions. Empirically, a large literature estimates firm-level responses to monetary policy surprises and documents substantial heterogeneity across firms depending on their characteristics. More recently, Ottonello and Winberry (2020) document that firms facing tighter financial constraints respond more strongly to monetary policy surprises. Other dimensions shown to matter include liquidity (Jeenas, 2023), age (Cloyne et al., 2023), and size (Gertler and Gilchrist, 1994). These estimates are typically treated as structural financing elasticities in quantitative models. Yet conventional monetary policy shocks simultaneously move borrowing costs and expectations, so they identify a composite object. By isolating a pure cost-of-debt shock, we show that standard estimates understate firms' true responsiveness to financing conditions, with direct implications for the calibration of macroeconomic models.

Third, our paper relates to previous work on the transmission of large-scale asset purchasing programs. A broad theoretical literature emphasizes that market segmentation and limits to arbitrage can generate localized price effects when the supply or demand of specific assets shifts, often rationalized through a *preferred habitat* view following Culbertson (1957) and Modigliani and Sutch (1966). Closest to our work, Greenwood et al. (2018) develop a market segmentation model where capital can move quickly within asset classes, but slowly between asset classes to study spillovers from sovereign bond purchases to the corporate bond sector.[2] Our identification exploits firms' heterogeneous exposure to such segmented markets.

Empirical work, like Krishnamurthy et al. (2011) for FED sovereign bond purchases, con-

[2]This approach firstly builds on the idea that there are limits to arbitrage due to high specialization of front-line arbitrageurs in financial markets (Merton, 1987; Grossman and Miller, 1988; Long et al., 1990; Shleifer and Vishny, 1997). Secondly, it relies on a notion of slow moving capital, which reflects the concept that capital does not flow to attractive investment opportunities as swiftly as textbook theories would suggest (Mitchell et al., 2007; Duffie, 2010). A recent application of the preferred habitat approach to the purchase of long-maturity sovereign bonds can be found in Vayanos and Vila (2021).

firms the distorting effect of large scale asset purchasing programs on prices. In the context of the CSPP, Todorov (2020) documents that yields on eligible bonds relative to ineligible ones fell by approximately 30 basis points following the announcement, and De Santis and Zaghini (2021a) show that the issuance of eligible bonds increased. Similar evidence for the Bank of England's corporate bond purchase announcement can be found in Boneva et al. (2018). We combine estimates of CSPP-induced yield changes with a stylized model of segmentation within corporate bond markets. We document segmentation along currency denomination and country of incorporation, providing empirical support for our identification strategy based on exposure to distinct market segments.

Outline The rest of the paper is organized as follows. Section 2 presents a simple framework that illustrates how monetary policy surprises generate an information effect in firms' investment decisions and clarifies why conventional monetary policy shocks identify a composite object, conflating borrowing-cost and belief channels. Section 3 lays out the institutional background for the CSPP that enables our empirical decomposition, as well as the different data sources and their descriptive statistics. Section 4 provides some motivating illustrative evidence that the CSPP has driven a wedge between the bond prices of eligible- and ineligible bonds to which firms are persistently exposed. Then section 5 lays out the empirical decomposition approach by using firms' CSPP pre-exposure as an instrument for their change in borrowing cost to estimate investment responses from a pure cost-of-debt shock as well as from an aggregate monetary policy shocks. Section 6 discusses results. Section 7 concludes.

2 A Simple Model of Firm Investment and Information Effects

We develop a tractable two-period model to illustrate the role of the information effect on firms' investment responses to monetary policy announcements. The model features a continuum of heterogeneous firms that make a lumpy, binary investment decision across two periods as well as a central bank. When the central bank announces a surprise change in the policy rate, this affects borrowing costs, but simultaneously transmits a signal about other economic fundamentals. In response, we show that the aggregate firm investment response can be broken down into a cost-of-debt channel - the response only due to the changed user cost of capital - as well as an offsetting information effect related to firms updating their beliefs about the future.

2.1 Environment

The model comprises two periods $t \in \{1, 2\}$. One can think of the two periods as the short- and long-run. Importantly, in period two the state of the economy θ can be good (G) or bad (B), but there is uncertainty about the realization of this state.

Aggregate investment in the model is determined by a continuum of firms who each make a binary lumpy-investment decision in the first period such that $I_f \in \{0, 1\}$. If they invest ($I_f = 1$), they receive a common state-dependent return $R(\theta)$ next period and have to pay a firm idiosyncratic user cost of capital $(i + r_f)$. The return on invest is a function of θ:

$$\pi_f(\theta) = R(\theta) - (i + r_f). \tag{1}$$

We assume that $R(B) < R(G)$ for some fixed, known values. Notably, firms are heterogeneous in the sense that they differ in the credit spread r_f at which they can borrow relative to the policy rate i.

The policy rate is governed by a central bank. At the start of period one, the central bank receives a noisy private signal $s \in \{H, L\}$ about the good state of the economy arising in the long run:[3]

$$p_s \ := \ P(\,\theta = G \mid s\,), \tag{2}$$

where $p_H > p_L$. After receipt of the signal, the central bank announces a monetary policy surprise: A change in the economy-wide interest rate from the expected i_{old} to the realized i. The direction of the interest surprise is a perfect signal about the central bank's previously private information: a surprise-cut signals $s = L$ associated with belief-updating to p_L, while a surprise-hike signals $s = H$ associated with p_H. Before the monetary policy surprise, firms hold ex-ante beliefs that $P(s = H) = \zeta$, so firm's unconditional expectation before observing the monetary policy shock equals:

$$p := \zeta p_H + (1 - \zeta) p_L$$

After observing the monetary policy surprise, firms update their beliefs about the future of the economy to p_H or p_L depending on the direction of the surprise before choosing whether or not to invest.

2.2 The Investment Decision

Firms in period one face the following problem. After they observe the central bank signal, they must choose whether or not to invest, based on their borrowing spread r_f. Conditional on some belief $\tilde{p} \in \{p, p_H, p_L\}$ about the long-run state of the economy, a firm will invest if and only if the expected return on invest is positive. The expected return on invest under beliefs

[3] It is well documented that the private sector updates its expectations around monetary policy surprises (Nakamura and Steinsson, 2018; Jarociński and Karadi, 2020; Andrade and Ferroni, 2021). There are two complementary ways to rationalize this with private information held by the central bank. First, the *delphic* view proclaims that the central bank has an information advantage over the private sector through superior forecasts about inflation and growth (Romer and Romer, 2000). Another *odyssian* explanation is that the central bank has private information about its own policy path, which determines long run growth. For example, the central bank plausibly has an information advantage on whether it places a large value on inflation rate-stabilization over closing the output gap in the short-run, which has been shown to have a big effect on long-run growth (see e.g. Rogoff (1985); Kydland and Prescott (1977); Barro and Gordon (1983); Walsh (1995); Fischer (1993)). In this paper we remain agnostic about the precise weight of the convex combination of the two channels. What we assume is that the central bank reveals some private information previously unbeknownst to firms via monetary policy surprises that affect expectations for long-run growth.

$\tilde{p}$ can be expressed as:

$$E\left[\pi_f(\theta) \mid \tilde{p}\right] \;=\; \tilde{p}R_G + (1-\tilde{p})R_B - (i + r_f). \tag{3}$$

This implies that a firm with beliefs $\tilde{p}$ will invest if and only if its credit spread r_f lies below its expected excess return:

$$r^*(\tilde{p}) \equiv \tilde{p}R_g + (1-\tilde{p})\, R_B - i\,. \tag{4}$$

In the following we denote r^*, r^*_H, r^*_L as the as the expected asset returns associated with beliefs p, p_H, and p_L, where it immediately follows that $p^*_L < p^* < p^*_H$. Now we can express the share of firms that invest following the monetary policy surprise entailing signal s by:

$$S(s) := \int_0^{r^*_L} dF(r) + \int_{r^*_L}^{r^*_H} \mathbb{1}\{s = H\} dF(r) \tag{5}$$

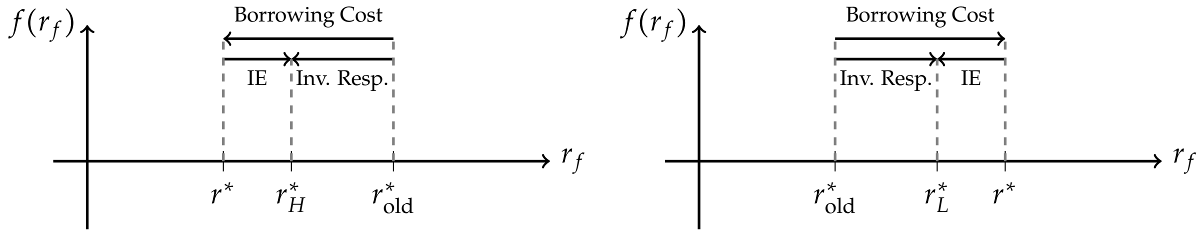
Here, we let r_f be described by some CDF denoted by F with the corresponding PDF denoted by f. Now we can further study the change in investment induced by the monetary policy surprise i, relative to the previous interest rate i_{old}. Namely, we can define $\Delta_{i_{\text{old}}\to i}S(s)$ as the change in firm investment associated with the monetary surprise, relative to the counterfactual case where the central bank would have kept i_{old} and did not send a signal, such that firms would have invested up until an $r^*_{\text{old}} \equiv pR_G + (1-p)\, R_B - i_{\text{old}}$:

$$\Delta_{i_{\text{old}}\to i}S(\gamma) := S\,(\gamma) - F(r^*_{\text{old}}) \tag{6}$$

Lemma 1. *If the central bank cuts rates from i_{old} to $i < i_{old}$, the aggregate change in investment can be decomposed into an information effect and a borrowing cost channel:*

$$\Delta_{i_{old}\to i}S = \int_{r^*_{old}}^{r^*_L} dF(r) = \underbrace{\left[F\,(r^*) - F\,(r^*_{old})\right]}_{\textit{Borrowing Cost Channel}} - \underbrace{\left[F\,(r^*) - F\,(r^*_L)\right]}_{\textit{Information Effect}} \tag{7}$$

FIGURE 1: DECOMPOSITION OF THE INVESTMENT RESPONSE



(A) A rate hike (B) A rate cut

Note: The left figure decomposes the aggregate investment response of a rate hike that signals $s = H$, while the right depicts a rate cut that signals $s = L$. The aggregate investment response (Inv. Resp.) of the monetary policy surprise can be decomposed into a borrowing cost channel, as well as an offsetting information effect (IE) as laid out in Lemma 1.

If the central bank raises rates from i_{old} to $i > i_{old}$,

$$\Delta_{i_{old}\to i} S = \int_{r^*_{old}}^{r^*_H} dF(r) = \underbrace{\left[F\left(r^*\right) - F\left(r^*_{old}\right)\right]}_{\textit{Borrowing Cost Channel}} - \underbrace{\left[F\left(r^*\right) - F\left(r^*_{H}\right)\right]}_{\textit{Information Effect}} \tag{8}$$

Lemma 1 - visualized in figure 1 - outlines that in this setup one can decompose the aggregate investment response into a borrowing cost channel, as well as an offsetting information effect. Here, we define the borrowing cost channel as the measure of firms that would have started to invest only due to the change in borrowing cost from i_{old} to i if this did not incorporate any noisy signal about the future state of the economy. However, as the change in the safe rate is accompanied by such a signal, this partially offsets the borrowing cost channel, giving rise to an information effect.

Namely, under a rate cut that signals p_L, the information effect equals the measure of firms that would have invested under p, but have a credit spread r_f so high that they don't find this profitable any longer under p_L. Similarly for a rate hike that signals p_H, the information effect equals the measure of firms with an r_f low enough that they would not have invested under p, but do find it profitable to invest under p^H. Thus, in both directions, the information effect offsets the borrowing cost channel, leading to a dampened aggregate investment response. In Supplementary materials B, we show that these model properties are qualitatively sustained even under a rational inattention framework where it is costly for firms to extract the signal.

Such behavior is in line with the idea of an information effect for the overall economy as proposed by Romer and Romer (2000); Nakamura and Steinsson (2018). Yet whether this effect is quantitatively meaningful for firm investment decisions remains ambiguous from the model, as the size of the information effect is governed by how much information monetary policy surprises transmit and what measure of firms lies in the respective interval. This is uniquely determined by the distribution of firm's credit spreads r_f relative to the expected excess return after the change in the interest rate. One would only expect to see a meaningful information effect if enough firms are close to their extensive margin of investment, so that extracting the signal would sway their investment decision. In the remainder of this paper, we will provide empirical evidence for whether this offsetting information channel exists in practice and how quantitatively meaningful it is.

Identification The model clarifies why conventional monetary policy shocks do not identify the structural effect of borrowing costs on investment. Variation in the user cost of capital, $i + r_f$, driven by changes in the policy rate i, is correlated with revisions in expected fundamentals. Because these information effects are unobservable and enter the error term of a standard investment regression, the borrowing-cost variation induced by i is endogenous. As a result, the estimated elasticity reflects a composite object that conflates price effects with belief revisions. The magnitude of this bias increases the more information is transmitted through the monetary policy surprise.

In the remainder of this paper, instead we exploit cross-sectional variation in the firm-specific component r_f induced by quasi-random heterogeneous firm exposure to the ECB's Corporate Sector Purchasing Program (CSPP). This shifts borrowing costs in a manner orthogonal to firms' revisions in expected fundamentals, allowing us to decompose the aggregate response into a pure cost-of-debt channel as well as an offsetting information effect. We show that this information effect is quantitatively important for aggregate dynamics. Namely, we show that the aggregate firm investment response to a monetary policy shock typically estimated is 33 percent smaller than the elasticity of an isolated cost-of-debt shock.

3 Institutional Background and Data

The CSPP is one of a series of quantitative easing efforts launched by the ECB since 2012. These efforts comprise of the ECB's Public Sector Purchasing Program (PSPP), its Asset-Backed Securities Purchase Program (ABSPP), three rounds of Covered Bond Purchasing Programs (CBPP), and finally the CSPP. While the PSPP focusses on purchases of sovereign bonds issued by Euro-Area governments, and the ABSPP, CBPP1, CBPP2, and CBPP3 chiefly focussed on bonds backed or covered by pools of loans such as residential mortages, auto loans, or public sector loans, the CSPP was the first program allowing the ECB to purchase large quantities of corporate bonds on primary and secondary markets. On 10 March 2016, the CSPP was launched and was expected to last at least until March 2017:

> "Third, we decided to include investment-grade euro-denominated bonds issued by non-bank corporations established in the euro area in the list of assets that are eligible for regular purchases under a new corporate sector purchase programme. This will further strengthen the pass-through of our asset purchases to the financing conditions of the real economy. Purchases under the new programme will start towards the end of the second quarter of this year."
>
> — ECB Monetary policy decisions, 10 March 2016, 14:30 CET

Notably, the decision to include investment-grade non-bank corporate bonds as part of the ECB's asset purchases caught the market by complete surprise (Todorov, 2020). The CSPP was a very novel measure of QE, as the ECB had so far almost exclusively purchased government bonds and the only other central bank that had ever purchased corporate bonds at scale as a means of unconventional monetary policy had been the bank of Japan. The CSPP's novelty, however, is not its only feature that makes it attractive for our analysis. Indeed, the CSPP also constituted a very sizable program. Asset holdings under the CSPP peaked at €345 billion which equaled about a third of the market capitalization of all outstanding eligible European Corporate bonds at the time. Lastly, another attractive feature of studying bonds around the CSPP announcement is that 2016 and following years were a period of relative market calm, as the main goal was not to provide new monetary stimulus, but rather to strengthen pass-through to financing conditions. Unlike many other QE programs set up in response to the great financial crisis 2008/09, or COVID-19, the QE announcement did not coincide with a major global financial crisis. Thus the CSPP announcement constitutes a large shock to the European corporate bond market that was largely unanticipated and occured in a period with few other coinciding megatrends distorting investment decisions.

The core eligibility criteria for corporate bonds were disclosed on the initial announcement on March 10. For a corporate bond to be eligible it must satisfy the following conditions: (i) be denominated in Euro, (ii) have a minimum rating of BBB- or equivalent, (iii) have a remaining maturity of at least six months and at most 30 years, (iv) be issued by a firm incorporated in the Euro Area, (v) be issued by a non-financial firm. Purchases were outlined to start at the beginning of June 2016. On 21 April 2016 there was a further announcement on the implementation aspects of the CSPP, yet the core eligibility requirements were released in March.

3.1 Data

Our empirical analysis leverages three types of data. To study financial market dynamics, we use daily financial data on corporate bond yields at the ISIN level sourced from Bloomberg and Refinitiv Eikon. Second, on the firm level, we source annual balance sheet data from Moodle's Orbis database. Lastly, we compare estimates of our new cost-of-debt shock to conventional investment elasticity estimation approaches, so we also leverage an existing series on high-frequency monetary policy shocks.

Bond Data To document relevance of the CSPP for bond borrowing costs, we study the reaction of corporate bond yields in a 6 month window surrounding the CSPP announcement on 10 March 2016. Specifically, we collect data from January to June of that year. We only collect data on bonds that were eligible under the CSPP, as well as other European bonds ineligible due to their currency of denomination or their country of incorporation.[4] Namely, the universe of bonds that our analysis is based on consists of bonds that satisfy the following five criteria: (i) they must be investment grade with a rating of at least BBB-, (ii) they must be issued by a non-financial firm, (iii) they must have a maturity of between 6 months and 30 years, (iv) they must be denominated in the currency of a European country, and (v) they must be issued by a firm incorporated in a European country. According to Bloomberg, as of 10 March 2016, there were 1,956 active corporate bonds satisfying these criteria, of which 1,808 have daily price- and yield data available on Bloomberg or Refinitive Eikon. These are composed of 911 eligible bonds, as well as 897 ineligible bonds.

TABLE 1: SUMMARY STATISTICS OF CORPORATE BONDS

					Deciles				
		Mean	Std	Min	20%	40%	60%	80%	max
Maturity, Years	Eligible	5.40	4.28	0.52	1.89	3.56	5.51	8.07	29.60
	Incorp-ineligible	6.00	4.44	0.50	2.26	4.17	5.98	9.03	29.0
	Currency-ineligible	5.33	5.62	0.52	1.45	2.37	4.26	8.30	26.50
	Dual-ineligibility	10.70	7.92	0.52	3.10	6.53	11.3	19.10	30.00
Coupon, %	Eligible	2.79	1.78	0.00	1.12	2.12	3.30	4.50	8.62
	Incorp-ineligible	2.59	1.66	0.00	1.12	1.88	2.75	4.12	7.38
	Currency-ineligible	3.43	2.30	0.00	1.23	2.38	3.85	5.80	10.0
	Dual-ineligibility	3.97	2.29	0.00	1.65	3.12	5.00	6.00	10.0
Par, mil $	Eligible	769	572	5	284	634	812	1,091	4,180
	Incorp-ineligible	723	425	22	388	645	782	1,004	2,264
	Currency-ineligible	531	487	38	134	305	472	818	2,485
	Dual-ineligibility	407	325	2	134	281	414	607	2,009

Note: The Table shows summary statistics of outstanding corporate bonds active on the CSPP announcement date. Data is shown for bonds that can be purchased under the CSPP ('Eligible') as well as bonds ineligible due to their country of incorporation only ('Incorp-ineligible'), bonds ineligible due to currency denomination only ('Currency-ineligible'), or bonds ineligible due to both incorporation and denomination ('Dual-ineligibility').

[4]In theory, it would also be possible to consider other cutoffs, such eligibility around a credit-rating of BBB-. The reason why we do not explore this direction is that previous studies have documented large spillovers from corporate sector purchasing programs to ineligible lower-rated corporate bonds (De Santis and Zaghini, 2021b), which would result in no borrowing cost wedge between exposed and unexposed firms.

For every bond we collect two daily yield measures. First, the bonds raw yield to maturity rate from Bloomberg or Refinitive Eikon. Second, we further construct daily government-yield spreads between the corporate bond's yield and the currency and maturity matched government bond at the time. As our sample of bonds includes bonds denominated in Euro, Swiss Franc, Danish Krona, Swedish Krona, Norwegian Krona, and Great British Pound, we obtain daily estimations for the respective currency yield curves from the corresponding central banks. In the case of bonds denominated in Euro, we use the ECB estimations for the Euro Area yield curve.[5]

Table 1 presents simple summary statistics for the final sample of corporate bonds used in the analysis. We partition the sample into four groups: (i) Eligible bonds, (ii) Incorp-ineligible bonds that are ineligible only due to incorporation of the issuing firm in a European country that is not in the Euro Area, (iii) Currency-ineligible bonds that are ineligible only due to their denomination in a European currency that is not the Euro, and (iv) Duel-ineligible bonds that are ineligible due to both denomination and incorporation. Figure 2 shows the rating distribution of the bonds across groups, which appears to be evenly distributed.

Firm Data We draw firm-level variables from annual Orbis, a panel of public and privately listed firms with global coverage. The scope of our analysis is large European non-financial companies that issue investment-grade bonds. To identify the universe of such companies, we first collect all distinct issuer names from Bloomberg for firms which feature as issuers in our previously defined sample of 1,956 European investment-grade bonds. This yields a sample of 402 distinct issuer firms. We use Orbis data on firm ownership structures to map the issuers to unique global ultimate corporate owners. This is necessary as many issuing firms are financial subsidiaries of their bigger manufacturing parent companies.[6] We identify a total of 292 unique global ultimate owner companies. However, while all of these companies

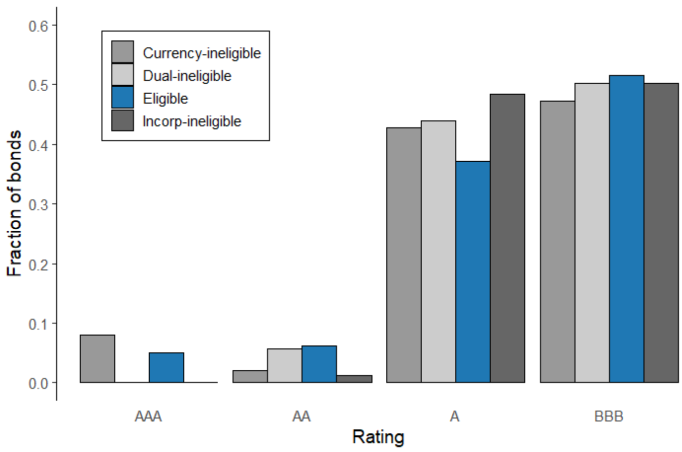
[5]The ECB calculates the Euro Area Yield Curve from the universe of outstanding government bonds denominated in Euro. We use the ECB estimates for the sample which contains only 'AAA' rated Euro Area government bonds.

[6]To give one example, the Volkswagen AG is issuing its bonds via six subsidiaries incorporated in Australia, the Netherlands, Germany, Japan, the US, and Canada with bonds denominated in 13 different currencies.

issue investment grade bonds in Europe, not all of them are incorporated in the European Single Market. For example if a non-European firm establishes a subsidiary in the euro area to issue bonds, or issues bonds in a European currency, the firm will feature in our sample. To keep comparability of firms in our sample, we exclude all global ultimate owners that are not incorporated in the European Single Market. This leaves us with 183 distinct firms. While this is only a small fraction of the over 40,000 public and private companies incorporated in the European Single Market and listed on Moody's Orbis, due to the heavy skewness of firm size, the 183 firms account for over 40 percent of aggregate revenue generation of all firms in the European Single market, making our sample relevant for aggregate dynamics.

For the set of 183 firms, we collect annual data on firm characteristics from Moody's Orbis database between 2000 and 2023. We collect data on firm investment, the cost of debt, and a number of other firm characteristics. For investment, we rely on three different measures: (i) growth in total tangible assets, (ii) growth in tangible fixed assets, and (iii) the change in the capital expenditure (CapEx) as a share of total assets, where we calculate CapEx from the change in tangible fixed assets plus depreciation. To proxy for firms' effective cost of debt, we construct the ratio of total interest paid each year divided by end-of-year total debt as cap-

FIGURE 2: SUMMARY STATISTICS ON BOND RATINGS



Note: The Figure shows the fraction of bonds by rating category just before the CSPP announcement on 10 March 2016 for bonds that could be purchased under the CSPP ('Eligible') as well as bonds ineligible due to their country of incorporation only ('Incorp-ineligible'), bonds ineligible due to currency denomination only ('Currency-ineligible'), or bonds ineligible due to both incorporation and denomination ('Dual-ineligibility').

TABLE 2: SUMMARY STATISTICS OF 2015 FIRM CHARACTERISTICS

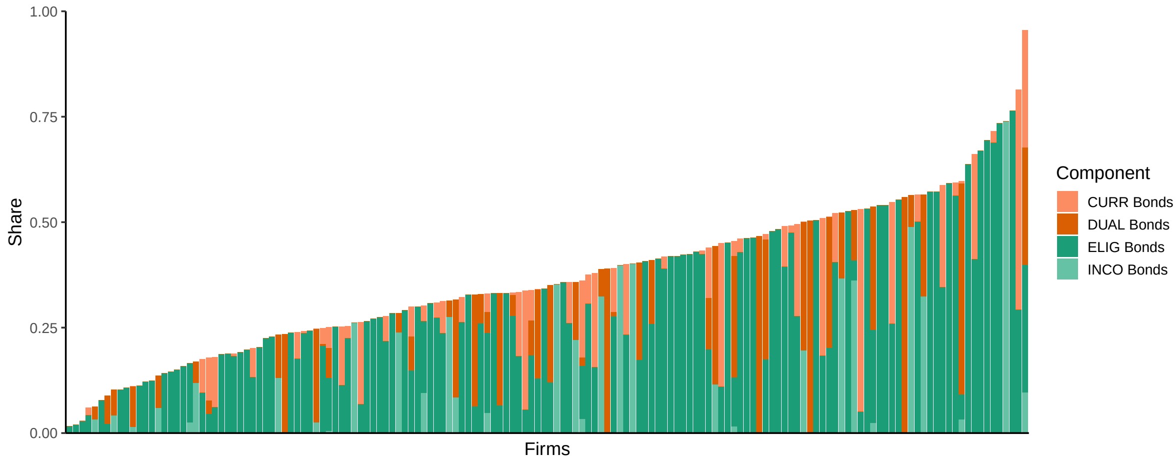
	Mean	Median	Std	Min	Max	# Obs
Total Assets (m EUR)	41,060	17,058	62,374	726	415,812	150
Total Debt (m EUR)	27,935	10,015	47,841	432	319,713	150
Interest-to-Debt-ratio (ppt)	2.17	2.02	1.35	0.05	12.40	141
Age (years)	53.9	40.0	43.79	6.0	180	145
Profitability (ppt)	10.91	10.51	5.32	-1.99	31.70	144
Liquidity (ppt)	9.76	7.75	9.87	0.00	92.06	150
Leverage (ppt)	64.77	64.69	14.61	23.61	134.75	150

Note: The table shows summary statistics for firms in our sample. Summary statistics are on the cross-section of firm data for 2015, which serves as a benchmark in our main specification. Data is based on a total of 183 firms.

tured by the sum of current and non-current liabilities. We further collect data on (i) firm age based on the year of incorporation, (ii) firm size proxied by the log of total assets, (iii) profitability, as the ratio between earnings-before-interest-taxes-deprechiation-and-amortization (EBITDA) and total assets, (iv) liquidity as the ratio between firms total cash-, or cash equivalent holdings over total assets, and (v) leverage captured by the fraction of total debt over total assets. Table 2 presents summary statistics for the distribution of key firm characteristics in the final sample of firms used in the analysis.

Construction of CSPP exposure Our later empirical identification leverages the fact that otherwise similar, large European firms differed in the extent to which their borrowing consisted of corporate bonds eligible under the CSPP whose yields were compressed by the CSPP relative to other forms of borrowing. To capture this, we construct pre-CSPP firm-exposure to borrowing under CSPP-eligible bonds by merging firms' balance sheet data with Bloomberg data on firms' outstanding bond series. In addition to the price data for our sample of 1,958 bonds, we further collect the nominal outstanding amount of all bonds issued by the 183 firms we study, as well as their subsidiaries' outstanding bonds. Here we collect data on the hole universe of bonds issued by these firms, not just the ones placed in a European country and in a European currency. We construct a firms' total pre-CSPP exposure as the ratio between a firm's Q4 2015 borrowing in corporate bonds eligible under the CSPP and the firm's Q4 2015 total debt balance.

FIGURE 3: FIRM CSPP-EXPOSURE



Note: This figure shows the debt portfolio composition before the CSPP in Q4 2015 for all 152 firms with Orbis data on their total debt amount and which issued investment-grade bonds in Europe and whose global parent company is incorporated in a European country. Total debt is measured using Orbis data (current plus non-current liabilities), while the share of different corporate bond segments is calculated using Bloomberg data on the outstanding amount of the global universe of active bonds for each such firm. ELIG bonds, CURR bonds, INCO bonds, and DUAL bonds are defined as in previous sections.

Figure 3 summarizes the distribution of firms' CSPP exposure in our final sample. Namely for each firm, we plot the share of debt raised via corporate bond borrowing broken down into the four segments of corporate bond borrowing previously defined: eligible, currency-ineligible, incorp-ineligible, and dual-ineligible. We can see that firm reliance on non-bank borrowing via bond issuance exhibits rich heterogeneity with most firms falling between 10 and 65 percent for our sample. Furthermore, there is also considerable heterogeneity in how reliant firms are on eligible bond issuance, conditional on their overall bond reliance. This allows us to compare the investment response of firms that are more exposed to the wedge in bond yields induced by the CSPP, but also allows us to compare firms that differ in this exposure, but are otherwise similar in their reliance on corporate bond financing.

Monetary Policy Shocks In addition to our novel identification strategy that isolates investment elasticities to a cost of debt shock by leveraging heterogeneous firm exposure to the CSPP, we also run a conventional estimation of firm elasticities on the same sample of firms over a comparable time period using a series of high-frequency monetary policy shocks. To

TABLE 3: SUMMARY STATISTICS OF MONETARY POLICY SHOCKS

	High Frequency	Sum
Mean	0.06	2.1
Median	0.0	0.5
S.D.	2.1	7.2
Min	-9.4	-19.4
Max	13.9	16.9
Observations	151	27

Note: Summary Statistics of monetary policy shocks from 01/01/2000 to 31/12/2023. 'High frequency' shocks are estimated based on equation 9 and taken from Jarociński and Karadi (2020). 'Sum' shocks are aggregated to a yearly frequency. Surprises in the EONIA rate are in basis points.

this end, we leverage the event study approach pioneered by Cook and Hahn (1989). We use the high-frequency shock series constructed for the Euro area by Jarociński and Karadi (2020). Shocks ε_t^m are constructed such that

$$\varepsilon_t^m = (\text{EONIA}_{t+\Delta+} - \text{EONIA}_{t-\Delta-}) \tag{9}$$

where t is the time of the monetary announcement, and EONIA is the Overnight Index Swap (OIS) based on the Euro OverNight Index Average (EONIA) rate. Jarociński and Karadi (2020) focus on swap rates for the Euro Area instead of future contracts, as European swap markets are more liquid and have a longer history than their future market counterparts. We also follow Jarociński and Karadi (2020) in focusing on the 3-month swap rate. Δ_+ and Δ_- determine the size of the event window and are chosen as 10 minutes before and 20 minutes after the monetary policy announcement yielding a 30 minute event window. Our shock series starts in 2000 and ends in 2023 similar to our firm data. We aggregate the high-frequency shocks to the annual frequency in order to merge them with our firm level data. Table 3 presents summary statistics for the high-frequency and aggregated shock series.

4 Evidence on CSPP Exposure

One cornerstone of the analysis is the relevance of firms' CSPP exposure for their cost of debt. In the following, we perform a number of illustrative exercises to show how the CSPP has persistently changed borrowing conditions for European firms. To this end, we first show that in line with the preferred habit view (see e.g. Culbertson (1957); Modigliani and Sutch (1966); Vayanos and Vila (2021)), the CSPP has reduced interest rates for borrowing in corporate bonds eligible for purchase under the CSPP. We show this both empirically, as well as in a stylized model of a segmented corporate bond market. Second, we show that pre-exposure to certain types of firm borrowing is sticky, so firms do not frictionlessly shift towards cheaper CSPP-eligible corporate bond borrowing. As a result, the CSPP induces a wedge between different types of borrowing to which firms were persistently exposed to.

Empirical Framework To document the effect of the CSPP announcement on bond yields, we estimate a before-after comparison between bonds that are eligible under the CSPP and ineligible corporate bonds around the announcement date:

$$y_{it} = \alpha_i + \lambda_t + \gamma_{Treat \times Post} \times Elig_i \times Post_t + \varepsilon_{it}. \tag{10}$$

Here, y_{it} is either the corporate bond's yield to maturity, or the spread of this yield to the maturity- and currency-matched government bond. α_i is an individual bond fixed effect and λ_t is a time-fixed effect. The two dummies $Elig_i$ and $Post_t$ indicate whether a bond is eligible for purchase under the CSPP and whether an observation date lies on or after the CSPP announcement, respectively. The sample period analyzed in this study is January 2016 to June 2016, which incorporates 127 trading days. The methodology is akin to Todorov (2020). To the best of our knowledge no other major central bank announcements or makroeconomically relevant events occurred in the Euro Area, or non-EA European economies throughout this time frame.

Table 4 reports the results from estimating the baseline regression (10) for two yield meas-

TABLE 4: IMPACT OF THE CSPP ON BOND YIELDS

Dep. Variable	Yields				Yield Spreads			
Control Group	ALL	INCO	CURR	DUAL	ALL	INCO	CURR	DUAL
Post × Treat	-8.6*	7.5	-2.8	-17.5***	-4.8	6.2	5.0	-13.1***
	(4.2)	(15.2)	(4.8)	(2.0)	(4.1)	(15.1)	(4.7)	(1.9)
# Observations (Bond-Time)	221k	137k	133k	173k	221k	137k	133k	173k
Bond FEs	YES	YES	YES	YES	YES	YES	YES	YES
Time FEs	YES	YES	YES	YES	YES	YES	YES	YES
F-Statistic	67.9	12.6	26.9	2974.2	21.3	8.7	86.0	1698.9

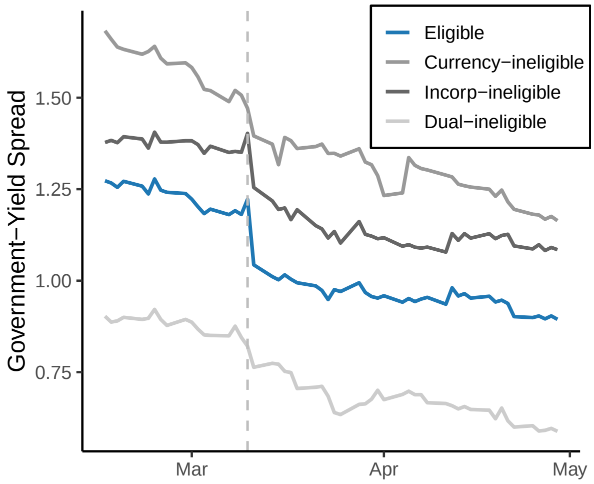
Note: Columns one to four estimate equation (10) for corporate bond yields as the dependent variable. Columns five through eight estimate equation (10) for corporate government-yield spreads as the dependent variable. Regressions are estimated based on eligible bonds as well as a varying group of ineligible bonds. Columns indicated by ALL are based on all eligible and all ineligible bonds, while columns indexed by INCO, CURR, and DUAL are based on eligible bonds and incorp-ineligible bonds, currency-ineligible bonds, and dual-ineligible bonds only, respectively. We report the coefficient of *Treat* × *Post*. Standard errors as shown in parentheses are heteroskedasticity robust and double-clustered by bond and time. Statistical significance is indicated by *, **, and *** at the 5%, 1% and 0.1% level. Units for bond yields and government-yield spreads is in basis points.

ures, as well as for four different sets of control groups: (i) all ineligible bonds, (ii) only currency-ineligible bonds, (iii) incorp-ineligible bonds, and (iv) dual-ineligible bonds. We report estimates for the coefficient $\hat{\gamma}$, which can be interpreted as the effect of the CSPP on eligible corporate bonds relative to the respective control group of ineligible bonds. In line with the preferred habit view as well as previous work on corporate sector purchasing programs (Todorov, 2020; De Santis and Zaghini, 2021a), the CSPP appears to have reduced bond yields of eligible bonds relative to ineligible ones. However, two interesting novel findings stand out. First, we see that a majority of the total effect on yields does not come from the CSPP affecting the underlying yield curve, but rather from compressing government yield spreads. For the comparison with dual ineligible bonds, about two-thirds of the total effect is driven by a reduction in yield spreads. Second, while the CSPP has reduced bond yields of eligible bonds relative to ineligible bonds, this effect on the yield of eligible bonds varies across control groups and is largest when comparing eligible bonds to dual-ineligible bonds.[7]

We explain the latter finding by varying spillovers induced by heterogeneous portfolio

[7]This finding that the CSPP has reduced yields of eligible bonds relative to ineligible ones is in line with Krishnamurthy et al. (2011) who similarly document a disproportionately large price effect for asset classes targeted in the FED's QE1 and QE2.

FIGURE 4: GOVERNMENT YIELD SPREADS AROUND THE CSPP ANNOUNCEMENT



Note: The figures show average trajectories of our corporate bond sample around the CSPP announcement on 10 March 2016 (dashed vertical line). We partition our sample of bonds into bonds that could be purchased under the CSPP ('Eligible') as well as bonds ineligible due to their country of incorporation only ('Incorp-ineligible'), bonds ineligible due to currency denomination only ('Currency-ineligible'), or bonds ineligible due to both incorporation and denomination ('Dual-ineligibility').

rebalancing effects. Figure 4 illustrates this point in a simple, purely descriptive way. It plots the average yield spreads around the CSPP announcement in the four bond market segments. While eligible bonds' government-yield spreads experienced a sharp drop on the CSPP announcement date of 17.9 basis points - equivalent to 6.0 standard deviations of their typical daily changes - so do incorp-ineligible bonds, which fell by 14.8 bps (4.9 standard deviations of their typical daily changes). Duel-ineligible bonds' government-yield-spread, meanwhile, only dropped by 5.7 (3.1 standard deviations), indicating a smaller amount of spillover.

A stylized Model Empirically, we see varying degrees of spillover of the CSPP effect to different ineligible bond market segments throughout our observation window. This is in line with the preferred habit view (see e.g. Culbertson (1957); Modigliani and Sutch (1966); Vayanos and Vila (2021)). One explanation for this heterogeneity in spillover to different market segments are different degrees of market segmentation akin to Greenwood et al. (2018). To strengthen this hypothesis, we show that a stylized model of a segmented corporate bond market can closely replicate observed empirical yield dynamics following the CSPP announcement for varying degrees of market segmentation. Here, market segmentation means how frictionless capital can flow from one segment to the next. Further, we can show that in such a stylized model, such effects are persistent.

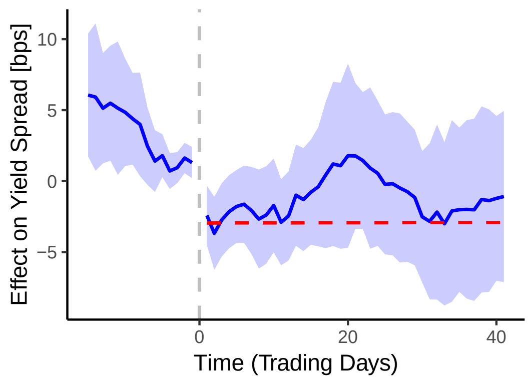
Supplementary materials C lay out the model structure of a partially segmented corporate bond market in detail. It describes a stylized model for pricing two long-term defaultable bonds A and B that reflect bonds eligible under the CSPP, as well as some segment of the ineligible corporate bond market. We extend the model from Greenwood et al. (2018) by allowing for default risk in both bond market segments to study market segmentation within the corporate bond market. The model features two key asset pricing frictions: partial market segmentation and slow-moving capital across markets. Markets are segmented in the sense that there are three types of traders: specialists A and B, who can only trade in bonds A and B respectively, as well as generalist traders, who can trade both bonds. Capital is slow-moving in the sense that generalists can only adjust their portfolio every k periods akin to a Calvo (1983) friction. Bond Supply in the model is stochastic and we model the announcement of the CSPP as a shock to the supply of eligible bonds A under different degrees of market segmentation, which we proxy by the measure of generalist traders q_G in the model.

For the three pairwise degrees of market segmentation between eligible bonds and the three ineligible bond market segments (currency-ineligible, incorp-ineligible, and dual-ineligible), we structurally estimate the measure of generalist traders q_G that minimizes the distance between the model-implied A and B yield differences following the CSPP shock relative to the following empirical local projection à la Jordà (2005):

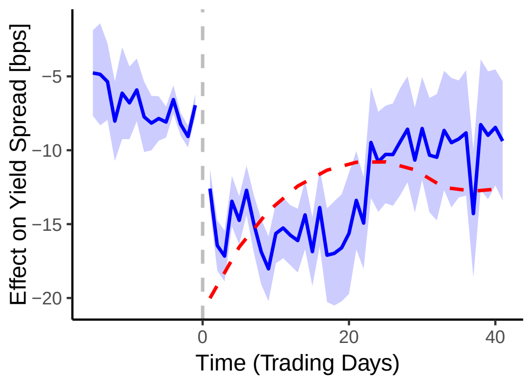
$$y_{i,t_{CSPP}+h} - y_{i,t_{CSPP}} = \alpha + \gamma_h \times D_i + \varepsilon_i. \tag{11}$$

Here, $D_{i,t}$ is a dummy variable that is one if bond i is eligible, and t equals the announcement date of the CSPP and is zero otherwise. What the simulated method of moments aims to minimize is the difference between $\hat{\gamma}_t$ and model-implied difference between the yields of

FIGURE 5: TRANSITIVE EFFECT OF THE CSPP ($\hat{\gamma}_k$)



(A) Eligible spread to Incorp-ineligible



(B) Eligible spread to Dual-ineligible

Note: The two figures compare empirical estimates (solid blue line) of the transitive effect of the CSPP to model outcomes from matched moments thereof (dashed red line). We do so for the relative effect of elegible bonds relative to incorp-ineligible bonds (left) and dual-ineligible bonds (right). Empirical estimates are based on parameter γ_h from equation (11), where the confidence bands reflect heteroskedasticity consistent standard errors. Model predictions reflect the difference in yield spread $y_{A,t} - y_{B,t}$, h periods after the supply contraction.

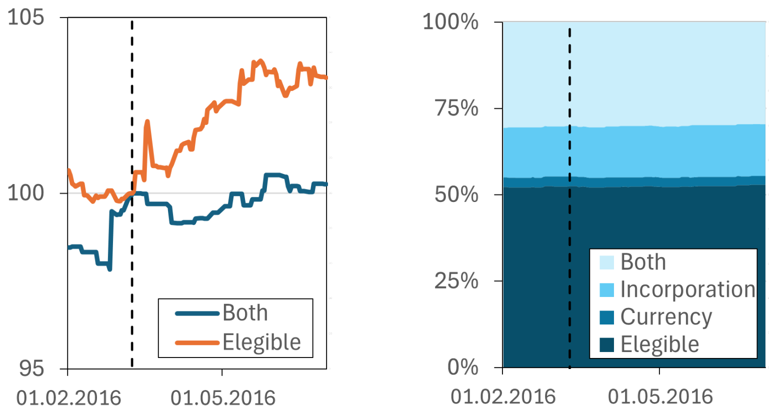
bond A and B following the model-CSPP announcement.

Figure 5 shows the local projection results ($\hat{\gamma}_t$) as well as the model-implied yield spread for the degree of market segmentation that most closely replicates the local projection on yield spreads. Results are presented for the pairwise relationship between eligible bonds and incorp-ineligible bonds, which exhibit the largest spillovers, and eligible and dual-ineligible bonds, which exhibit the lowest spillovers. We can closely replicate the former under no market segmentation, where all traders are generalist traders. We can replicate the latter under partial market segmentation, where only 40 percent of all traders are generalist traders. The effect on yield spreads in the latter case also is persistent and yields of ineligible bonds stay depressed ten basis points below eligible yields.

Stickt Firm Exposure Lastly we document a degree of stickyness in firms' CSPP exposure. Pre-CSPP exposure to borrowing under CSPP eligible bonds would not be indicative of actual borrowing costs if firms could switch between different credit lines frictionlessly. However, we find that similar to the notion of relationship lending in bank-based borrowing (see e.g. Boot (2000); Ongena and Smith (2000); Kysucky and Norden (2016)), there appear to be substantial frictions and fixed costs associated with switching from bank borrowing to corporate bond issuance, as well as with switching market segments within the corporate bond market. In particular, we find that the CSPP led to some increased issuance of eligible bonds on the intensive margin, but no noticeable change on the extensive margin.

To show this, we study the universe of European bonds as defined earlier that were active between 01 February 2016 and 31 June 2016. Figure 6b shows that the average within firm reliance on bonds eligible under the CSPP remains stable both before the CSPP, as well as up to three months after the CSPP announcement. This indicates that firms do not shift to borrowing under comparatively cheaper CSPP-eligible bonds. We further confirm this by analyzing the bond issuance behaviour of firms that have a 100 percent reliance on dual ineligible bonds. Over the three months following the CSPP announcement, none of these firms that had only issued dual-ineligible bonds before the announcement issued any bonds from other bond market segments. This indicates that while firms appear to increase issuance of eligible bonds on the intensive margin, firms do not appear to shift towards eligible bonds on

FIGURE 6: DAILY EVOLUTION OF OUTSTANDING CORPORATE BONDS



(A) TOTAL VOLUME GROWTH (B) WITHIN-FIRM SHARES

Note: Figures are based on the daily universe of bonds that are investment grade, denominated in a European currency, whose issuer is incorporated in a European country by a non-financial firm, and whose outstanding maturity is between 6 months and 30 years. Figure (A) plots the trajectory of total outstanding eligible bond volume over time relative to total dual-ineligible bond volume. Figure (B) plots the trajectory of within-firm reliance on the four partitions of the European corporate bond market. Here, firms are clustered based on their Bloomberg ticker, which exhibits a near one-to-one mapping with the global ultimate owners of issuing companies identified via Orbis ownership structures. The dashed line reflects the CSPP announcement on 10 March 2016.

the extensive margin (Figure 6a).

Such stickiness of firms' debt portfolios is also in line with the reality of corporate bond placements. Such placements often include substantial fixed costs, chiefly driven by large underwriter fees. According to Dalal (2018), the average underwriting fee received by intermediary banks for the placement of new corporate bonds averaged around 0.7 percent (70 bps) of the par-value for investment grade bonds in the United States around the announcement of the CSPP. In addition to underwriting fees, firms have to pay legal fees, marketing and road-show costs, as well as regulatory filing fees, especially if they are planning on setting up a new subsidiary in a European Economic Area country. Relative to our estimated 17 basis points reduction in bond yields, such fixed costs are a severe deterrent for switching bond markets.

5 Econometric Approach

Thus far, we have shown two key features in the data that enable our analysis. First the CSPP has induced a global shift in borrowing costs: borrowing in bonds eligible for purchase under the CSPP has become cheaper relative to other forms of borrowing. We further show evidence for the persistance of this wedge in borrowing costs. Second, we find that the share to which firms are exposed to this shift varies as shown in Figure (3). Even conditional on a firm's reliance on borrowing in corporate bonds, the CSPP's eligibility cutoffs along lines of incorporation and denomination induce quasi-experimental variation in how exposed otherwise comparable multinational companies from the European Single market are to the wedge. Thus, we will use the pre-exposure as a source of variation in the cost of debt that is orthogonal to other co-moving forces of the monetary policy shock that usually cannot be partialled out. We will discuss this assumption in more detail below in the 'threats to identification' section.

However, even-though pre-CSPP exposure has many desirable properties, it is only an imperfect shock measure for a cost-of-debt shock. Thus, we will not use it as a direct shock measure, but rather as an instrument for the observed change in the interest-to-debt ratio at the firm level. This is in spirit the Bartik (1991) style shift-share-IV literature. Namely we run the following two regressions:

$$\Delta_h r_i = \alpha Z_i + \gamma X_{i0} + \varepsilon_i, \tag{12}$$

$$\Delta_h Y_i = \delta \Delta_h \hat{r}_i + \tilde{\gamma} X_{i0} + \tilde{\varepsilon}_i. \tag{13}$$

Here, Z_i is a firms' CSPP exposure share defined as a firm's 2015 debt share raised in CSPP eligible corporate bonds, $\Delta_h r_i$ is the percentage change in company i's effective interest rate (interest-to-debt ratio) over the h year period following 2015, and X_{i0} is a vector of controls which includes 2015 values of firm size (log of total assets), profitability (ebitda-to-asset-ratio), leverage (debt-to-asset-ratio), liquidity (cash-to-asset-ratio), firm age in years, as well firm bond reliance proxied by the 2015 share of debt a firm holds via corporate bonds. Lastly, $\Delta_h Y_i$ denotes the percentage change in our investment measure over the h year period following our benchmark year 2015.

In addition to the estimation of investment elasticities, our setup also allows us to conduct a heterogeneity analysis to study which firm characteristics are associated with amplifying or dampening investment elasticities to a cost-of-debt shock. To this end, we interact the instrumented effective interest rate of the firm in the second-stage regression with the firm characteristic I_t:

$$\Delta_h Y_i = \alpha \Delta_h \hat{r}_i \times I_i + \beta \Delta_h \hat{r}_i + \zeta I_t + \tilde{\gamma} X_{i0} + \tilde{\varepsilon}_i \tag{14}$$

Here, I_t is the 2015 value of the firm characteristic whose interaction with investment elasticity we are studying. If the coefficient α of the interaction term is of opposite (same) sign to the estimated average elasticity β, the feature is associated with a dampened (amplified) investment-elasticity.

Threats to Identification Our identification strategy relies on the assumption that firms that differ in how much of their bond debt is raised via CSPP-eligible bonds do not meaningfully differ across other dimensions that affect their investment-elasticity, or how responsive they are to the information effect, or other forces co-moving with cost-of-debt shocks induced by monetary policy shocks. The core rationale for why this should hold is that we only consider large firms issuing investment-grade corporate bonds which are all incorporated in the European Single Market. Due to their bond issuance choices in non-Euro-Area currencies or countries, some firms will be more or less exposed to the CSPP, yet due to the setup of the European Single market, their common incorporation should allow all firms to sell goods and services frictionlessly within the single market, access capital markets across borders, and hire workers from any member country. In other words, since the single market functions like "one country" for firms in many important ways, they should be similarly exposed to macro shocks and monetary policy in the Euro area.

While we cannot test for the assumption directly, we will now provide some motivating evidence that firm exposure to the CSPP is not meaningfully correlated with other firm characteristics that matter. To this end, we first run the following regression of firm CSPP exposure on overall firm bond-exposure in 2015:

$$Z_i = \alpha + \beta Bond_i + e_i. \tag{15}$$

Here, Z_i is the firm's share of debt raised under CSPP-eligible corporate bonds, while $Bond_i$ is the firm's share of debt raised under any corporate bond. We run this regression on our sample of firms to derive the residual $\hat{e}_i$ and then test whether this residual in the CSPP exposure that can't be explained by the bond exposure - which we control for in the first stage regression - is correlated with other firm characteristics in a meaningful way.

Table 5 summarizes the correlation between $\hat{e}_i$ and some firm characteristics. As shown, no firm characteristic exhibits statistically significant correlation with the share of CSPP-exposure that can't be explained by bond reliance ($\hat{e}_i$). In addition to standard firm characteristics, which we summarize in the data section of the paper, we also construct a firm-level procyclicality measure with Euro-Area output. Namely, we calculate firm-level procyclicality as the

TABLE 5: CORRELATION BETWEEN CSPP EXPOSURE AND FIRM CHARACTERISTICS

	Procyclicality	Size	Leverage	Liquidity	Profitability	Age
Correlation with $\hat{e}_i$	0.018	-0.155	-0.148	0.100	-0.008	-0.038
	(0.082)	(0.078)	(0.079)	(0.080)	(0.084)	(0.083)
P-Value	0.832	0.056	0.071	0.227	0.922	0.6476
Observations	148	149	148	148	141	144

Note: This table summarizes how different firm characteristics correlate with the share of firms' CSPP exposure that can't be explained via firms' bond-exposure. We report Pearson correlation coefficients with standard deviations in brackets below. Statistical significance is based on a two-tailed test, and p-values are reported. Procyclicality refers to a firm's revenue procyclicality with EEA GDP since the incorporation of the firm. Size, leverage, liquidity, profitability, and age are defined in the data section. Statistical significance is indicated by *,**, and *** at 5%, 1% and 0.1%.

correlation between the HP-filtered cyclical component of each firm's revenue growth and the cyclical component of the European Economic Area's aggregate GDP growth. Importantly, we find that this measure also exhibits no correlation with $\hat{e}_i$, which indicates that firms that are more exposed to the CSPP do not have higher incentives to care ECB monetary policy shocks due to a higher reliance on the EEA market.

High-Frequency Monetary Policy Shocks We argue that the previously described setup allows us to estimate the investment elasticity in response to an isolated cost-of-debt shock. Yet, for a decomposition of the aggregate investment elasticity into a cot-of-debt component and a residual, we still require information about the aggregate response. To this end, we estimate a different first stage regression using a series of high-frequency monetary policy shocks as the instrument for the change in the interest-to-debt ratio:

$$\Delta r_{f,t} = \alpha MP_t + \gamma X_{f,t-1} + \varepsilon_{f,t}, \tag{16}$$

$$\Delta Y_{f,t} = \delta \Delta \hat{r}_{f,t} + \tilde{\gamma} X_{f,t-1} + \tilde{\varepsilon}_{f,t}. \tag{17}$$

Here, MP_t reflects aggregated high-frequency changes in the 3-month EONIA swap rate around ECB monetary policy announcements following Jarociński and Karadi (2020), t indexes the year at an annual frequency, and f indexes the firm. We consider the same sample of

183 firms for which we previously calculated CSPP-pre-exposure. We consider a time period between 2005 and 2019, which most closely resembles the time following the CSPP announcement studied earlier. We only study one-year changes over the year in which the monetary policy shocks to the EONIA swap rate materialized. Other variables are defined as for equation (12) and (13).

Importantly, using conventional monetary policy shocks as an instrument allows us to estimate the aggregate investment elasticity to a monetary policy shock, not just the cost of capital component. Consider for example the case, where the rate change in the EONIA around a monetary policy announcement correlates with revisions of the economic outlook. Then in the second stage $\tilde{\varepsilon}_{f,t}$ would include a changed economic outlook which will matter for the investment decision, but will also be correlated to $\Delta\hat{r}_{f,t}$ as years in which many rat cuts led to a reduced cost of debt, will also be years in which the economic outlook improved. Thus, here δ is not the pure investment elasticity due to a pure cost-of-debt-shock, but rather also incorporates all other correlated channels of the monetary policy announcement and should be interpreted as the aggregate investment elasticity induced by a monetary policy shock. Comparing this investment elasticities to our previous estimate leveraging variation in the cost-of-debt orthogonal to such co-moving channels allows us to implicitly estimate the drag of these channels - such as the information effect (Nakamura and Steinsson, 2018; Romer and Romer, 2000) - on the investment elasticity of a pure cost-of-debt shock.

Lastly, we can also introduce an interaction term for this setting to study whether certain firm characteristics are associated with an amplified or dampened investment elasticity in response to a monetary policy shock. This similarly allows us to decompose firm heterogeneity in their investment elasticity into a cost-of-capital channel and a residual. To this end, we can run the following second stage:

$$\Delta Y_{f,t} = \alpha \Delta\hat{r}_{f,t} \times I_{f,t-1} + \beta \Delta\hat{r}_{f,t} + \zeta I_{f,t-1} + \tilde{\gamma} X_{f,t-1} + \tilde{\varepsilon}_{f,t} \tag{18}$$

TABLE 6: DECOMPOSING THE INVESTMENT ELASTICITY

	Borrowing Cost Shock		MP Shock	
	2SLS	Fuller	2SLS	Fuller
Total Asset Growth	0.98**	0.83**	0.30**	0.27**
	(0.47)	(0.38)	(0.14)	(0.12)
First stage F	5.05		6.94	
Obs	138		998	

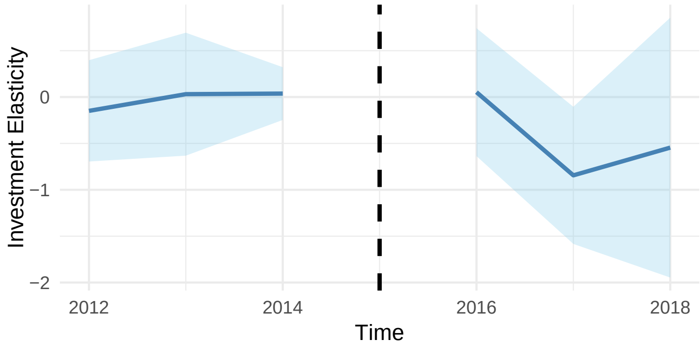
Note: The table reports the Two-Stage Least Squares (2SLS) estimates for δ. The borrowing cost shock specification estimates equations (12) and (13) over the two year 2015-2017 window. The MP Shock specification on the other hand estimates equations (16) and (17). In addition to the 2SLS estimate, we also report the Fuller estimates which are more robust to weak instruments. To further account for the possibility of a weak instrument, we compute Conditional Likelihood Ratio (CLR)–based confidence intervals for both Two-Stage Least Squares (2SLS) and Fuller estimators. Statistical significance is indicated by *, **, and *** at the 5%, 1% and 0.1% level.

6 RESULTS

Our empirical headline finding is summarized in table 6. Here we compare the investment elasticity induced by a pure cost-of-debt shock, as well as the aggregate elasticity to the series of monetary policy announcements. Namely, the borrowing cost specification estimates equations (12) and (13) using CSPP exposure as an instrument, while the MP Shock specification estimates equations (16) and (17) using aggregated high-frequency monetary policy shocks as an instrument. We present estimates for the investment elasticities $\hat{\delta}$. As we can see from the first stages, in both instances, we do not have a strong IV, indicated by an F statistic below ten. To account for this, we report estimates based on Fuller (1977) in our main specification, which reduce the small sample bias of limited information maximum likelihood estimations, while maintaining low variance, giving a more robust estimator under a weak instrument. Additionally, we also report confidence intervals based on the conditional likelihood ratio (CLR) approach (Moreira, 2003) which is robust to weak instruments.

In the 'borrowing cost shock' specification of table 6, results are based on differences over the two years following the benchmark year 2015. We focus on the two-year specification, as the first stage is strongest. In other words, CSPP-pre-exposure is most relevant for the 2015-2017 change in firms' interest-to-debt ratios. Table 8 in supplemantary materials A provides full regression results over one-two-and three-year windows following 2015 for our main

FIGURE 7: INVESTMENT RESPONSE



Note: The figure plots Fuller (1977) estimates for the investment elasticity $\hat{\delta}$ from equation (13). The time axis corresponds to the window over which investment elasticities are estimated relative to the benchmark year 2015. Years below 2015 correspond to a negative h. Confidence bands Conditional Likelihood-Ratio based and reflect the 95% confidence interval.

measure of investment, as well as the two other proxies: tangible fixed asset growth, and total capital expenditure growth. Figure 7 further plots investment elasticities for our main investment measure over different time windows. Similarly, we can see that due to the weak first stage over one-, and three years, results are not statistically significant. There are two possible explanations for this. First, given that much of firm debt is locked in fixed-rate debt contracts, it takes some time until lower borrowing costs in secondary markets affect the average interest rate of a firm-loan portfolio. In our sample, the average maturity of outstanding corporate bonds is lies between 7.05 years, which could explain why the effect of the CSPP announcement on actual interest payments was muted between 2015 and 2016. Second, while we do document stickiness in firms' CSPP exposure via their bond portfolio, the longer the time window, the more likely it is that some firms do end up issuing bonds in different market segments, which weakens the link between pre-CSPP exposure and firms' effective cost of borrowing. The two-year window appears to be the sweet spot between allowing for enough lag for effects to pass through to debt portfolios, without being too long for debt portfolios to rebalance. Figure 7 summarizes these considerations by showing estimates for the investment elasticity in the borrowing cost shock specification based on a h year window following the baseline year 2015 for varying h. Here, we further consider negaive h, which represents a retrospective application of our methodology to years before 2015. Since we do not find an effect here, this suggests that there were no pre-trends that varied and drove our 2SLS results.

When annualizing investment elasticities from table 6, we find that the elasticity from a cost-of-debt shock is much larger than the investment elasticity induced by a monetary policy shock. The former estimate of an 0.83 investment elasticity over two years corresponds to a conservative, annualized investment elasticity of 0.41, which exceeds the investment elasticity to an MP shock of 0.27 by 51.8 percent.[8] This interpretation is in line with the existance of an information effect. If a monetary policy announcement also changes firm's economic outlook in a way that affects the investment response contrary to how the rate cut would affect it, this should mute the investment elasticity relative to a raw cost-of-debt shock that does not coincide with a correlated revision of the economic outlook.

Our setup also allows us to study which firm characteristics are associated with an amplified or dampened investment elasticity in response to both a cost-od-debt shock, as well as an aggregate monetary policy shock. Table 7 summarizes regression results for equations (14) and (18), which interact investment elasticities in the second stage regression with firm

TABLE 7: DETERMINANTS OF THE INVESTMENT ELASTICITY

	Borrowing Cost Shock				Joint MP Shock			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\Delta r_i \times$ Leverage	0.19				-0.02			
	(0.21)				(0.05)			
$\Delta r_i \times$ Size		0.13				0.07		
		(0.22)				(0.06)		
$\Delta r_i \times$ Liquidity			0.35**				-0.07	
			(0.17)				(0.05)	
$\Delta r_i \times$ Profitability				0.39**				0.05
				(0.18)				(0.04)
First Stage F	5.05	5.05	5.05	5.05	6.94	6.94	6.94	6.94
N Firms	138	138	138	138	998	998	998	998

Note: The table reports the estimates for the coefficient of the interaction term $\Delta\hat{r} \times I$ for equations (14) and (18). Institutional features are lagged by one priod in the MP Shock specification, while values from 2015 are used for the Borrowing Cost Shock specification. Statistical significance is indicated by .,*, **, and *** at the 10%, 5%, 1%, and 0.1% level.

[8] We annualize the elasticity over the two year window by as follows: $(1+\hat{\delta})^{0.5}$. This is conservative in the sense that it assumes that the reduction in borrowing cost has persisted over the whole 2-year period equally, although we see from the weak first stage over the one-year period, that this likely is not the case.

characteristics which the literature has found to matter for investment responses. Namely, we interact the investment elasticity with a proxy for leverage, firm size, liquidity, and profitability. We do so both for the first stage, which uses CSPP exposure as an instrument, as well as for the other case, which uses aggregated monetary policy surprises as an instrument.

We find that firms that are more liquid and more profitable exhibit a larger investment response to a cost-of-debt shock. A one standard deviation increase in liquidity and profitability is associated with an increase in the investment elasticity of 0.35 and 0.39, respectively. Interestingly we do not find the same mechanism for our joint monetary policy shock specification. This indicates, that while firms that are more profitable and more liquid do react more strongly in their investment to a cost-of-debt shock, they are also disproportionately more disincentivized by other features of a monetary policy shock, such as the information effect.

7 Conclusion

This paper provides firm-level evidence that the investment channel of monetary policy is materially attenuated by an offsetting information effect. Using quasi-experimental variation from the ECB's Corporate Sector Purchase Programme (CSPP), we isolate changes in firms' financing costs that are plausibly orthogonal to contemporaneous belief revisions. The central empirical finding is that the investment elasticity typically inferred from aggregate monetary policy shocks is substantially smaller than the elasticity to a "pure" cost-of-debt shock: over four quarters, the observable aggregate investment response is dampened by about one third relative to the borrowing-cost channel in isolation.

To interpret these results, we develop a tractable model of lumpy firm investment with costly attention to the central bank's signal. Monetary policy surprises both move the user cost of capital and reveal information about the policy regime with implications for long-run growth prospects. The model clarifies why information effects can be quantitatively important precisely when many firms are near their extensive margin of investment: in that region, even modest belief updating can overturn discrete investment decisions, thereby offsetting the borrowing-cost channel. This mechanism delivers a direct conceptual link between cross-sectional financial conditions and aggregate monetary transmission. In particular, the model implies that the strength of the information effect—and therefore the net potency of monetary policy—depends on the distribution of firm-level credit spreads relative to investment thresholds.

The results have several implications. First, calibrations of investment elasticities based on standard high-frequency identification of monetary policy shocks may understate the true sensitivity of firm investment to financing conditions, because they embed the net effect of borrowing-cost changes and belief revisions. Second, the size relative strength of offsetting via an information channel can plausibly become state dependent and can help explain monetary policy's state dependent transmittion to the real economy. Namely, the distribution of credit spreads emerges as a sufficient statistic for how strongly information effects can offset the borrowing-cost channel, suggesting a parsimonious way to incorporate state-dependent transmission into macro-finance models.

Several directions for future work follow naturally. Empirically, richer measurement of expectation revisions at the firm level—through analyst forecasts, earnings guidance, or text-based measures of managerial sentiment—could sharpen the mapping from the residual component of monetary policy shocks to belief updating. Relatedly, extending the decomposition to other real outcomes (employment, R&D, or intangibles) could clarify whether information effects are particularly salient for irreversible, lumpy expenditures. On the policy side, the results motivate closer attention to how central bank communication interacts with unconventional policies that directly affect private financing costs. If real transmission depends on where firms sit in the cross-section of spreads and investment thresholds, then the same policy action may have different real effects across episodes even when aggregate conditions look similar.

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A Supplementary Materials

Table 8: Investment Elasticities

	$\Delta_1 Y_i$		$\Delta_2 Y_i$		$\Delta_3 Y_i$	
	2SLS	Fuller	2SLS	Fuller	2SLS	Fuller
Total Asset Growth	-0.26	0.05	0.98**	0.83**	3.67	0.57
	(0.75)	(0.33)	(0.47)	(0.38)	(8.83)	(0.77)
Tangible Fixed Asset Growth	0.26	0.12	1.27	1.09	4.29	0.84
	(0.43)	(0.22)	(0.80)	(0.69)	(9.29)	(1.09)
Total Capital Expenditure	-0.25	0.07	0.36*	0.31**	0.46	0.09
	(0.45)	(0.14)	(0.19)	(0.15)	(1.21)	(0.13)
First Stage F	0.43		4.68		0.17	
N (Firms)	138		138		140	

Note: The table reports the Two-Stage Least Squares (2SLS) estimates for δ from equation 13 for three measures of investment (rows) and three different time-windows (columns). We estimate the percentage change of investment for a one percentage point drop in borrowing costs. In addition to the 2SLS estimate, we also report the Fuller estimates which are more robust to weak instruments. To further account for the possibility of a weak instrument, we compute Conditional Likelihood Ratio (CLR)–based confidence intervals for both Two-Stage Least Squares (2SLS) and Fuller estimators. This approach allows us to mitigate small-sample bias and obtain valid inference even when instrument strength is limited. Statistical significance is indicated by *, **, and *** at the 5%, 1% and 0.1% level.

B A Model of Signal Extraction Costs

In the following section, we will incorporate an arbitrarily large attention cost for the firm into the model laid out in section 2. Now firms have to pay an exogenous attention cost k to extract the central bank signal s from the monetary policy surprise.

Firms in period zero face the following problem. Based on their borrowing spread r_f they must first choose whether or not to pay attention to the central bank signal. Following this, they then must make the decision of whether or not to invest. As before, under certain beliefs about the good state of the economy arising $\tilde{p}$, the firm will choose to invest if and only if $r_f < r^*(\tilde{p})$.

Given this investment decision of the firm, we can now define the expected payoff for (i) not paying attention to the central bank signal (V^N), and (ii) paying attention to the signal (V^S). Ex-ante - when choosing whether or not to pay attention - the firm beliefs with probability ζ that it would discover with certainty that the central bank received private signal $s = H$, and with probability $1-\zeta$ that it received $s = L$.

We will start with the former case, where the firm does not pay attention to the signal and relies on beliefs p as defined in equation (2.1):

$$V^N(r_f) = \begin{cases} pR_G + (1-p)R_B - (i + r_f) & \text{if} \quad r_f < r^* \\ 0 & \text{if} \quad r_f \geq r^* \end{cases} \tag{19}$$

Here, for r_f smaller than r^* the firm invests independent of the realization of the central bank's type and expects the corresponding unconditional payoff. For higher r_f the firm does not invest and receives zero payoff.

Now consider the second case in which the firm invests k to infer the extract the private signal from the central bank monetary policy surprise. With probability ζ it expects to update its beliefs to p_H and invest if its r_f is below r_H^*. With probability $1-\zeta$ the firm expects to update its beliefs to p_L and only invest if its credit spread r_f is below r_L^*. The associated conditional expected return on invest under the updated belief is specified in equation (3). Notably, since $p_H > p > p_L$ we know that $r_H^* > r^* > r_L^*$. Thus, the ex-ante value of paying attention to the central bank signal is:

$$V^S(r_f) = \begin{cases} pR_G + (1-p)R_B - (i + r_f) - k & \text{If } r_f \leq r_L^* \\ \zeta \times \left[p_H R_G + (1-p_H)R_B - (i + r_f)\right] - k & \text{If } r_L^* < r_f \leq r_H^* \\ -k & \text{If } r_H^* < r_f \end{cases} \tag{20}$$

Intuitively, ex ante, the firm knows that for an r_f below r_L^*, it will invest no matter what it finds out about the central bank's private signal, so the payoff equals the unconditional payoff net the attention cost k. If r_f is such that the firm will invest if and only if the central bank has positive private information ($s = H$), the ex-ante value equals the probability that $s = H$, which is ζ, times the expected return of investment conditional on the central bank being a hawk net the cost k. Lastly, if r_f is so high that the firm will not invest no matter what type the central bank is, the value is just the negative cost of paying attention.

Now, the gross benefit of interpreting the central bank signal and understanding its type can be defined as:

$$\Delta(r_f) := V^S(r_f) - V^N(r_f)$$
$$= \begin{cases} -k & \text{If } r_f < r_L^* \\ -(1-\zeta) \times \left[p_L R_G + (1-p_L)R_B - (i+r_f)\right] - k & \text{If } r_L^* \leq r_f \leq r^* \\ \zeta \times \left[p_H R_G + (1-p_H)R_B - (i+r_f)\right] - k & \text{If } r^* < r_f \leq r_H^* \\ -k & \text{If } r_H^* < r_f \end{cases} \tag{21}$$

Equation (21) is visualized in figure 8. The intuition for this is as follows. If r_f is below r_L^*, or above r_H^*, there is no value to paying attention. The firm would invest under either private signal, or under no private signal respectively, so the gross benefit of paying attention equals the negative attention cost k. Furthermore, there are two more cases. If r_f is above r_L^*, but below r^* then by learning the private signal, the firm can avoid making an unprofitable investment if the central bank turns out to hold a downturn indicator ($s = L$). Likewise, if r_f is bigger than r^*, but smaller than r_H^*, by learning the central bank type, the firm gains by being able to make a profitable investment if the central bank turns out to hold an upswing indicator ($s = H$). Gains are largest when r_f is closest to the public knowledge investment cutoff r^*, as for r_f just below r^* learning the central bank information maximizes the value of avoided losses from an unprofitable investment. Equivalently, for an r_f just above r^*, knowing the central bank information maximizes the profit from allowing the investment under an upswing indicator.

Lemma 2. $\Delta(r_f)$ *is continuous and single-peaked with a unique maximum at* $r_f = r^*$

Proof. It is immediate from the piece-wise expression above that $\Delta(r_L^*) = \Delta(r_H^*) = -k$. Further, $\Delta(r_f)$ linearly increases for $r_f \in [r_L^*, r^*]$ and linearly decreases for $r_f \in [r^*, r_H^*]$. At $r = r^*$, the piece-wise expressions coincide at $\zeta(1-\zeta)\left[(p_H - p_L)R_G + (1-(p_H - p_L))R_B\right] - k$. □

Figure 8 shows a stylized example for the gross benefit of interpreting the central bank information $\Delta(r_f)$. A firm will choose to extract the signal if $\Delta(r_f) > 0$. Thus, the policy function for paying attention to the central bank signal is governed by two cutoff points: $\underline{r}, \bar{r}$. If and only if $r_f \in [\underline{r}, \bar{r}]$ then the expected added value from knowing the central bank type exceeds the attention cost. Thus, only such firms will choose to extract the signal. We can find the following two closed form expressions for $\underline{r}, \bar{r}$:

$$\underline{r} = \frac{k}{1-\zeta} + p_L R_G + (1-p_L) R_B - i \tag{22}$$

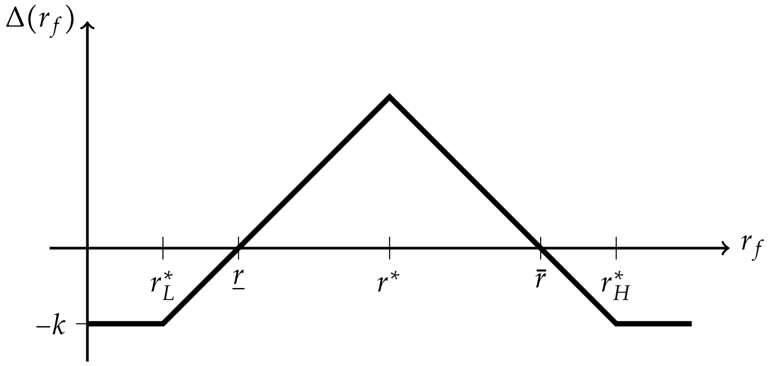
$$\bar{r} = -\frac{k}{\zeta} + p_H R_G + (1-p_H) R_B - i \tag{23}$$

Similarly to the main framework, we can define the share of firms that invest following the monetary policy surprise denoted by $S(s)$, as well as the change in investment induced by the monetary policy surprise i and the associated signal for s denoted by $\Delta_{i_{\text{old}} \to i} S(s)$:

$$S(s) := \int_0^{\underline{r}} dF(r) + \int_{\underline{r}}^{\bar{r}} \mathbb{1}\{s = H\} dF(r) \tag{24}$$

$$\Delta_{i_{\text{old}} \to i} S(s) := S\,(s) - F(r^*_{\text{old}}) \tag{25}$$

FIGURE 8: VALUE OF INFORMATION



Note: The figure displays the value of extracting the central bank's signal about its private type *dovish*, or *hawkish*. Firms with an r_f below $\underline{r}$, or above $\bar{r}$ will not extract the signal. Firms with an r_f between $\underline{r}$ and $\bar{r}$ will extract the signal.

Lemma 3. *If the central bank cuts rates from i_{old} to $i < i_{old}$, the aggregate change in investment can be decomposed into an information effect and a borrowing cost channel:*

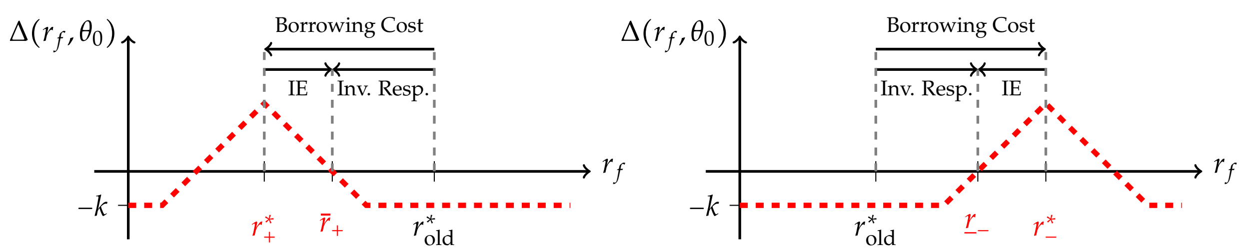
$$\Delta_{i_{old}\to i} S = \int_{r^*_{old}}^{\underline{r}} dF(r) = \underbrace{[F(r^*) - F(r^*_{old})]}_{\text{Borrowing Cost Channel}} - \underbrace{[F(r^*) - F(\underline{r})]}_{\text{Information Effect}} \tag{26}$$

If the central bank raises rates from i_{old} to $i > i_{old}$,

$$\Delta_{i_{old}\to i} S = \int_{r^*_{old}}^{\bar{r}} dF(r) = \underbrace{[F(r^*) - F(r^*_{old})]}_{\text{Borrowing Cost Channel}} - \underbrace{[F(r^*) - F(\bar{r})]}_{\text{Information Effect}} \tag{27}$$

Similarly to the main framework, Lemma 3 - visualized in figure 9 - outlines that the aggregate investment response can be decomposed into a borrowing cost channel, as well as an offsetting information effect. Namely as shown in figure 9b, if a surprise rate cut (variables sub-scripted by '-') signals an upturn, firms with an $r_f \in [\underline{r}_-, r^*_-]$ will extract the signal and choose not to invest upon learning that the central bank's private information. This information effect offsets the borrowing cost channel and leads to a dampened investment response. Vice-versa as shown in figure 9a, if a surprise rate hike signals a downturn (variables sub-scripted by '+'), firms with an $r_f \in [r^*_+, \bar{r}_+]$ will extract this signal and choose to invest upon learning the central bank's information. Again, this offsets the borrowing cost channel un-

FIGURE 9: DECOMPOSITION OF THE INVESTMENT RESPONSE



(A) A rate hike that signals a hawk

(B) A rate cut that signals a dove

Note: Variables with a subscript '-' correspond to a surprise rate cut, while variables with a '+' subscript denote a surprise rate hike. The aggregate investment response (Inv. Resp.) of the monetary policy surprise can be decomposed into a borrowing cost channel, as well as an offsetting information effect (IE). as laid out in lemma 3.

der which the same firms would not have invested, had they not attained certainty about the central bank's type. Thus, in both directions, the information effect offsets the borrowing cost channel, leading to a dampened aggregate investment response.

C A Model of Market Segmentation

In section 4, we established that the CSPP reduced the yields of eligible bonds primarily by lowering government-yield spreads. The degree to which this effect spilled over to different market segments of ineligible bonds, however, differed. We argue that this can be explained by different degrees of market segmentation between eligible bonds and the different segments of ineligible corporate bonds. To test this hypothesis, in this section, we build on Greenwood et al. (2018), to develop a model of a segmented corporate bond market. For different degrees of market segmentation and under a reasonable model calibration, we can indeed replicate the empirical findings presented in section 4 and account for the heterogeneity observed in the yield response across different market segments.

The model presented in this section is a stylized model for pricing two long-term defaultable bonds A and B that reflect bonds eligible under the CSPP, as well as some control groups of ineligible bonds. We extend the model from Greenwood et al. (2018) by allowing for default risk in both bond market segments to study market segmentation within the corporate bond market, instead of studying market segmentation between sovereign and corporate bond markets, which is the focus of Greenwood et al. (2018). The model features two key asset pricing frictions: partial market segmentation and slow-moving capital across markets. Bond Supply in our model is stochastic and we model the announcement of the CSPP as a shock to the supply of eligible bonds A. This allows us to analyze the counterfactual effect of a drop in the active bond supply on the yield spread between bonds under different degrees of market segmentation.

C.1 Bonds

We consider two types of bonds in our model: A and B. Both bonds are exposed to interest risk and bond-idiosyncratic default risk. We will think of bond A as bonds eligible under the CSPP, while we will think of bond B as bonds ineligible for the CSPP.

For $i \in \{A, B\}$, the i perpetuities pay a coupon C_i each period, but a random fraction $h_{i,t+1}$ of the bonds default at $t+1$ and are worth $(1 - L_{i,t-1})(P_{L,t+1} + C_i)$, where $0 \leq 1 - L_{i,t+1} \leq 1$ is the recovery rate of the bond. As a consequence, in the next period, $Z_{i,t+1} \equiv h_{i,t+1} L_{i,t+1}$ of the bond value is lost due to default. The return of bond i is

$$1 + R_{i,t+1} = \frac{(1 - Z_{t+1})\,(P_{i,t+1} + C_i)}{P_{i,t}} \tag{28}$$

Using the Campbell and Shiller (1988) log-linear approximation, we find that the excess return on the i perpetuity can be written as

$$rx_{i,t+1} \equiv ln\,(1 + R_{i,t+1}) - r_t \approx \frac{1}{1-\theta_i} y_{i,t} - \frac{\theta_i}{1-\theta_i} y_{i,t+1} - z_{i,t+1} - r_t \tag{29}$$

where $\theta_i \equiv 1/(1 + C_A) < 1$, and $y_{i,t}$ is the log yield-to-maturity on bond i at time t, and $z_{i,t} \equiv -ln(1 - Z_t)$.

C.2 Risk Factors

The model incorporates three different sources of risk: (i) short-term interest risk r_t, (ii) default risks $z_{i,t}$, and (iii) supply risks $s_{i,t}$, which all evolve randomly according to an AR(1) process as

$$r_{t+1} = \bar{r} + \rho_r\,(r_t - \bar{r}) + \varepsilon_{r,t+1} \tag{30}$$

$$z_{i,t+1} = \bar{z}_i + \rho_{z_i}\,(z_{i,t} - \bar{z}_i) + \varepsilon_{z_i,t+1} \tag{31}$$

$$s_{i,t+1} = \bar{s}_i + \rho_{s_i}\,(s_{i,t} - \bar{s}_i) + \varepsilon_{s_i,t+1}, \tag{32}$$

where $Var(\varepsilon_{r,t+1}) = \sigma_r^2$, $Var(\varepsilon_{z_i,t+1}) = \sigma_{z_i}^2$, $Var(\varepsilon_{s_i,t+1}) = \sigma_{s_i}^2$. This means that both bonds are in an exogenous time-varying supply, governed by independent shocks. At the same time, the log short-term interest rate and the default processes similarly follow an exogenous AR(1) process. At this point we do not assume the correlation structure of the underlying shocks $\varepsilon_{r,t}, \varepsilon_{z_i,t+1}, \varepsilon_{s_i,t+1}$, yet for simplicity, in the numerical illustration, we will solve for underlying orthogonal shocks.

C.3 Market Structure

The model features three types of investors: (i) A specialists, (ii) B specialists, and (iii) Generalists. A specialists can only trade bond A, while B specialists can only trade bone B. Generalists can trade both bonds, but can only adjust their portfolio every k periods due to the financial friction in the model.

For $i \in \{A, B\}$, there exists a measure q_i of i specialists, whose demand for bond i in period t is denoted by $b_{i,t}$. Fast moving i specialists have mean-variance preferences over one-period portfolio log returns

$$b_{i,t} = \tau \frac{E_t\left[rx_{i,t+1}\right]}{\text{Var}_t\left[rx_{i,t+1}\right]}. \tag{33}$$

There also is a measure of $1 - q_A - q_B$ slow-moving generalist investors who can adjust their holdings of both A and B bonds, but only a fraction $1/k$ of them is active each period. When they are active, they need to maintain their portfolio for the next k periods. As a result, they have mean-variance preferences over their k period cumulative excess return. Defining $rx_{i,t\to t+k} \equiv \sum_{j=1}^{k} rx_{i,t+j}$ the excess return on a generalist's portfolio is

$$rx_{d_t,t\to t+k} \equiv d_{A,t} \times rx_{A,t\to t+k} + d_{B,t} \times rx_{B,t\to t+k}, \tag{34}$$

where $d_{i,t}$ denote the generalist's demand for bond i at time t. Generalists now choose their bond demand such that

$$\max_{d_{A,t},d_{B,t}} \left\{E_t\left[rx_{d_t,t\to t+k}\right] - (2\tau)^{-1}\left(\text{Var}_t\left[rx_{d_t,t\to t+k}\right]\right)\right\}, \tag{35}$$

which implies that

$$\begin{bmatrix} d_{A,t} \\ d_{B,t} \end{bmatrix} = \tau \begin{bmatrix} \text{Var}_t\left[rx_{A,t\to t+k}\right] & \text{Cov}_t\left[rx_{A,t\to t+k}, rx_{B,t\to t+k}\right] \\ \text{Cov}_t\left[rx_{A,t\to t+k}, rx_{B,t\to t+k}\right] & \text{Var}_t\left[rx_{B,t\to t+k}\right] \end{bmatrix}^{-1}$$

$$\times \begin{bmatrix} E_t\left[rx_{A,t\to t+k}\right] \\ E_t\left[rx_{B,t\to t+k}\right] \end{bmatrix}. \tag{36}$$

C.4 Market Clearing

In equilibrium of the model, markets for both bonds must clear in a way that is consistent with the optimization of specialists A and B, generalists, and the law of motion for all exogenous and endogenous variables. Since the measure $q_G \equiv 1 - q_A - q_B$ of generalists can only adjust their portfolio every k periods, the active supply of bonds equals the total supply of bonds net what generalists have purchased over the past $k-1$ periods, which is now being kept fixed. The market clearing condition for bond $i \in \{A, B\}$ is

$$\underbrace{q_i b_{i,t}}_{\text{Specialist demand}} + \underbrace{q_G k^{-1} d_{i,t}}_{\text{Active generalist demand}} = \underbrace{s_{i,t}}_{\text{Total bond supply}} - \underbrace{(1 - q_A - q_B)\left(k^{-1}\sum_{i=1}^{k-1} d_{i,t-i}\right)}_{\text{Inactive generalist holdings}}. \tag{37}$$

Notably, in the model, the parameters k and q_G are the exogenous parameters that determine the degree of market segmentation in the model. They in down how many traders there are that operate in both market segments and at what speed they can adjust their portfolio.

C.5 Equilibrium Conjecture

Similar to Greenwood et al. (2018), we solve for a rational expectations equilibrium in which equilibrium yields and generalist demands are linear functions of a state vector x_t which includes steady state deviations of the short term interest rate, default realizations, the supply of A bonds, the supply of B bonds, and inactive generalist holdings of both bonds. Concretely, we assume that

$$y_{A,t} = \alpha_{A0} + \boldsymbol{\alpha}'_{A1}\boldsymbol{x}_t \tag{38}$$

$$y_{B,t} = \alpha_{B0} + \boldsymbol{\alpha}'_{B1}\boldsymbol{x}_t \tag{39}$$

$$d_{A,t} = \delta_{A0} + \boldsymbol{\delta}'_{A1}\boldsymbol{x}_t \tag{40}$$

$$d_{A,t} = \delta_{B0} + \boldsymbol{\delta}'_{B1}\boldsymbol{x}_t \tag{41}$$

where the $(5 + 2(k-1)) \times 1$ dimensional vector $\boldsymbol{x}_t$ is given by

$$\boldsymbol{x}_t = [r_t - \bar{r}, z_{A,t} - \bar{z}_A, z_{B,t} - \bar{z}_B, s_{A,t} - \bar{s}_A, s_{B,t} - \bar{s}_B,$$

$$d_{A,t-1} - \delta_{A0}, ..., d_{A,t-k} - \delta_{A0}, d_{B,t-1} - \delta_{B0}, ..., d_{B,t-k} - \delta_{B0}] \tag{42}$$

which implies that the state vector $\boldsymbol{x}_t$ follows an AR(1) process as well:

$$\boldsymbol{x}_{t+1} = \boldsymbol{\Gamma}\boldsymbol{x}_t + \boldsymbol{\varepsilon}_{t+1} \tag{43}$$

where $\varepsilon_{t+1} \equiv [\varepsilon_{r,t+1}, \varepsilon_{z_A,t+1}, \varepsilon_{z_B,t+1}, \varepsilon_{s_A,t+1}, \varepsilon_{s_B,t+1}, 0, ..., 0]$, $Var(\boldsymbol{\varepsilon}_{t+1}) \equiv \boldsymbol{\Sigma}$, and the transition matrix $\boldsymbol{\Gamma}$ depends on generalist demand functions (δ_{A1}, and δ_{B1}) as described in detail in the appendix (equations 49 and 50).

Similar to insights from Greenwood et al. (2018) and as shown in appendix section D, a rational expectations equilibrium of the model is a fixed point of a specific operator which involves the price-impact coefficients $(\alpha_{A1}, \alpha_{B1})$ as well as generalists' demand impact coefficients $\delta_{A1}, \delta_{B1})$. Specifically, we let $\omega = (\alpha'_{A1}, \alpha'_{B1}, \delta'_{A1}, \delta'_{B1})$ and derive the operator $f(\omega_0)$ that gives the new price impact coefficients and demand impact coefficients that clear both bond markets when agents expect ω_0 to hold at all future dates. Thus we find our equilibrium as a fixed point of f where $f(\omega^*) = \omega^*$.

We follow Greenwood et al. (2018) in applying the Powell hynrid algorithm to solve for the fix point, which performs a quasi-Newton search for the fix-point from an initial guess. We account for possible equilibria-multiplicity by sampling over 10,000 initial guesses. While multiple equilibria can arise, we focus on the study of the unique stable equilibrium, defined by having all eigenvalues of the Jacobian $D_\omega f(\omega^*)$ less than one in magnitude.

C.6 An illustrative Comparison

After having set up the model and outlined how to solve for general equilibrium, in this section, we will model a change in the supply of bond A and how this affects bond yields in both market segments under given degree of market segmentation, as captured by a varying q_G. We do so to compare the heterogeneous effect of the CSPP on yield-spreads between eligible bonds and ineligible bonds from different market segments to the yield-spread dynamics arising from our model under different degrees of market segmentation. We summarize illustrative parameters for a numerical calibration of the model in table 9.

We model the CSPP announcement as a supply contraction in A bonds. We match the size of the supply contraction to the observed total purchases under the CSPP as a share of outstanding eligible bonds - a contraction by $1/3$ of the market capitalization. We then derive

TABLE 9: ILLUSTRATIVE MODEL PARAMETERS FOR NUMERICAL CALIBRATION

Parameters	Description	Value
$\bar{r}$	Average annualized short-term interest rate	4%
σ_r	Volatility of quarterly shocks to annualized short-term riskless rate	0.64%
ρ_r	Quarterly persistence of short-term riskless rate	0.9
$\bar{z}_A, \bar{z}_B$	Expected annualized default losses	0.4%
$\sigma_{z_A}, \sigma_{z_B}$	Volatility of quarterly shocks to annualized default losses	0.4%
ρ_{z_A}, ρ_{z_B}	Quarterly persistence of default losses	0.96
$\bar{s}_A, \bar{s}_B$	Average asset supplies	5
$\sigma_{s_A}, \sigma_{s_B}$	Volatility of quarterly supply shocks	1
ρ_{s_A}, ρ_{s_B}	Quarterly persistence of supply shock	0.999
D_A, D_B	Macaulay duration in years (implies $\theta_A = \theta_B = 0.95$)	5 years
τ	Aggregate investor risk tolerance	1.75

NOTE: This table presents the illustrative model parameters that we use throughout our numerical exercises. One period corresponds to one week. We report annualized values for the mean and standard deviation of shocks to both the short rate and default losses.

model estimates for bond yields over time for both A and B bonds. To match our empirical findings on the transitive effect on eligible yields relative to different market segments of ineligible yields, we then take the difference $y_A - y_B$ and choose the degree of market segmentation in the model, such that we minimize the distance between our empirical estimates from figure 5 and our model prediction $y_A - y_B$. To this end, we keep k fixed at 8 in line with Greenwood et al. (2018), but vary the measure of generalist traders q_G.

Figure 5 plots the empirical estimates of the transitive impact of the CSPP (solid lines) against our model estimates under varying degrees of market segmentation (dashed line). We find that we can match our empirical findings for the wedge between eligible and inco-ineligible bonds most closely under no market segmentation when $q_G = 1$. Likewise, we can match our empirical findings for the wedge between eligible and dual-ineligible bonds most closely by complete market segmentation ($q_G = 0$). For the wedge between eligible and currency-ineligible bonds, we find that a moderate degree of market segmentation ($q_G = 0.4$) replicates empirical findings most closely. This indicates, that dual-ineligible bonds are plausibly the most segmented ineligible bonds that experience the least amount of spillover and provide the best counterfactual in our empirical analysis from section 4. It further indicates some persistence of the wedge in borrowing costs on secondary markets following the CSPP, at least between eligible and dual-ineligible bonds that are not traded away by arbitrageurs instantaneously.

D Solution to the Baseline Segmented Market Model

In line with the solution method, we conjecture that the equilibrium yields in both markets at time t take the form

$$y_{A,t} = \alpha_{A0} + \boldsymbol{\alpha}'_{A1}\mathbf{x}_t, \tag{44}$$

$$y_{B,t} = \alpha_{B0} + \boldsymbol{\alpha}'_{B1}\mathbf{x}_t, \tag{45}$$

while generalists' demands for bonds A and B take the form

$$d_{A,t} = \delta_{A0} + \delta'_{A1}\mathrm{x}_t, \tag{46}$$

$$d_{B,t} = \delta_{B0} + \delta'_{B1}\mathrm{x}_t. \tag{47}$$

For concreteness and illustrative purposes, let us fix $k = 3$. To keep track of the inactive bond holdings of generalist traders, the state vector must include the past two lags of $d_{A,t}$ and $d_{B,t}$, as well as the fundamentals. In general x_t is the $2(k-1)+5$ dimensional state vector.

$$x_t = \begin{bmatrix} r_t - \bar{r} \\ z_{A,t} - \bar{z}_A \\ z_{B,t} - \bar{z}_B \\ s_{A,t} - \bar{s}_A \\ s_{B,t} - \bar{s}_B \\ d_{A,t-1} - \delta_{A0} \\ d_{A,t-2} - \delta_{A0} \\ d_{B,t-1} - \delta_{B0} \\ d_{B,t-2} - \delta_{B0} \end{bmatrix} \tag{48}$$

Based on the model assumptions as well as the conjecture for the generalists' demand, we can define the transition matrix Γ for the state vector as follows:

$$\mathbf{x}_{t+1} = \Gamma(\delta)\mathbf{x}_t + \boldsymbol{\epsilon}_{t+1} \tag{49}$$

$$= \begin{bmatrix} \rho_r & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho_{z_A} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho_{z_B} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \rho_{s_A} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \rho_{s_B} & 0 & 0 & 0 & 0 \\ \delta_{A1} & \delta_{A2} & \delta_{A3} & \delta_{A4} & \delta_{A5} & \delta_{A6} & \delta_{A7} & \delta_{A8} & \delta_{A9} \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \delta_{B1} & \delta_{B2} & \delta_{B3} & \delta_{B4} & \delta_{B5} & \delta_{B6} & \delta_{B7} & \delta_{B8} & \delta_{B9} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} r_t - \bar{r} \\ z_{A,t} - \bar{z}_A \\ z_{B,t} - \bar{z}_B \\ s_{A,t} - \bar{s}_A \\ s_{B,t} - \bar{s}_B \\ d_{A,t-1} - \delta_{A0} \\ d_{A,t-2} - \delta_{A0} \\ d_{B,t-1} - \delta_{B0} \\ d_{B,t-2} - \delta_{B0} \end{bmatrix} + \begin{bmatrix} \varepsilon_{r,t+1} \\ \varepsilon_{z_A,t+1} \\ \varepsilon_{z_B,t+1} \\ \varepsilon_{s_A,t+1} \\ \varepsilon_{s_B,t+1} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Under the assumption that the error terms are mutually orthogonal, we have

$$\mathbf{\Sigma} \equiv \mathrm{Var}_t\left[\epsilon_{t+1}\right] = \begin{bmatrix} \sigma_r^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_{z_A}^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_{z_B}^2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_{s_A}^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_{s_B}^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{50}$$

We further adopt the following notation: Let $\mathbf{e}_r$ be the basis vector with a 1 corresponding to the first $r_t - \bar{r}$ column and zeros otherwise, let $\mathbf{e}_{z_A}$ be the basis vector with a 1 at the second row and zeros otherwise, and let $\mathbf{e}_{z_B}$ be the basis vector with a one in the third row and zeros otherwise. Finally, denote $C_{[t+i,t+j]} \equiv Cov(x_{t+i}, x_{t+j}|x_t)$ and note that

$$\mathbf{C}_{[t+i,t+j]} = \sum_{s=1}^{\min\{i,j\}} \left[\mathbf{\Gamma}^{i-s}\right] \mathbf{\Sigma} \left[\mathbf{\Gamma}^{j-s}\right]', \tag{51}$$

so $C_{[t+i,t+j]} = C'_{[t+i,t+j]}$.

D.1 Specialist Demand

Given the conjectured forms of equilibrium yields from (44) and (45), we have

$$\begin{aligned} rx_{i,t+1} &= \frac{1}{1-\theta_i} y_{i,t} - \frac{\theta_i}{1-\theta_i} y_{i,t+1} - z_{i,t+1} - r_t \\ &= (\alpha_{i0} - \bar{r} - \bar{z}_i) + \left(\frac{1}{1-\theta_i}\boldsymbol{\alpha}_{i1} - \mathbf{e}_r\right)' \mathbf{x}_t - \left(\frac{\theta_i}{1-\theta_i}\boldsymbol{\alpha}_{i1} + \mathbf{e}_{z_i}\right)' \mathbf{x}_{t+1} \end{aligned} \tag{52}$$

From this, we can find the expectation of excess return as well as its variance

$$E_t\left[rx_{i,t+1}\right] = (\alpha_{i0} - \bar{r} - \bar{z}_i) + \left(\frac{1}{1-\theta_i}\boldsymbol{\alpha}_{i1} - \mathbf{e}_r\right)' \mathbf{x}_t - \left(\frac{\theta_i}{1-\theta_i}\boldsymbol{\alpha}_{i1} + \mathbf{e}_{z_i}\right)' \mathbf{\Gamma}\mathbf{x}_t \tag{53}$$

$$\mathrm{Var}_t\left[rx_{i,t+1}\right] = \left(\frac{\theta_i}{1-\theta_i}\boldsymbol{\alpha}_{i1} + \mathbf{e}_{z_i}\right)' \mathbf{\Sigma} \left(\frac{\theta_i}{1-\theta_i}\boldsymbol{\alpha}_{i1} + \mathbf{e}_{z_i}\right) \tag{54}$$

Due to the mean-variance preferences of the specialist agent, demand for asset A under the conjecture for equilibrium yields is

$$\begin{aligned} b_{i,t} &= \tau \frac{E_t\left[rx_{i,t+1}\right]}{\mathrm{Var}_t\left[rx_{i,t+1}\right]} \\ &= \left[\tau \frac{\alpha_{i0} - \bar{r} - \bar{z}_i}{\left(\frac{\theta_i}{1-\theta_i}\boldsymbol{\alpha}_{i1} + \mathbf{e}_{z_i}\right)' \mathbf{\Sigma} \left(\frac{\theta_i}{1-\theta_i}\boldsymbol{\alpha}_{i1} + \mathbf{e}_{z_i}\right)}\right] + \left[\tau \frac{\left(\frac{1}{1-\theta_i}\boldsymbol{\alpha}_{i1} - \mathbf{e}_r\right)' - \left(\frac{\theta_i}{1-\theta_i}\boldsymbol{\alpha}_{i1} + \mathbf{e}_{z_i}\right)' \mathbf{\Gamma}}{\left(\frac{\theta_i}{1-\theta_i}\boldsymbol{\alpha}_{i1} + \mathbf{e}_{z_i}\right)' \mathbf{\Sigma} \left(\frac{\theta_i}{1-\theta_i}\boldsymbol{\alpha}_{i1} + \mathbf{e}_{z_i}\right)}\right] \mathbf{x}_t \end{aligned} \tag{55}$$

D.2 Generalists' Demand

Generalists who can trade in both bonds, but are only allowed to adjust their portfolio every k periods solve

$$\max_{d_{A,t},d_{B,t}} \left\{ \begin{array}{c} d_{A,t}E_t\left[\sum_{i=1}^{k} rx_{A,t+i}\right] + d_{B,t}E_t\left[\sum_{i=1}^{k} rx_{B,t+i}\right] \\ -\frac{1}{2\tau}\left(\begin{array}{c} (d_{A,t})^2\,\mathrm{Var}_t\left[\sum_{i=1}^{k} rx_{A,t+i}\right] + (d_{B,t})^2\,\mathrm{Var}_t\left[\sum_{i=1}^{k} rx_{B,t+i}\right] \\ +2d_{A,t}d_{B,t}\,\mathrm{Cov}_t\left[\sum_{i=1}^{k} rx_{A,t+i}, \sum_{i=1}^{k} rx_{B,t+i}\right] \end{array} \right) \end{array} \right\}, \tag{56}$$

which implies

$$\begin{aligned} \begin{bmatrix} d_{A,t} \\ d_{B,t} \end{bmatrix} &= \tau \begin{bmatrix} V_A^{(k)} & C_{AB}^{(k)} \\ C_{AB,k} & V_B^{(k)} \end{bmatrix}^{-1} \begin{bmatrix} E_t\left[\sum_{i=1}^{k} rx_{A,t+i}\right. \\ E_t\left[\sum_{i=1}^{k} rx_{B,t+i}\right] \end{bmatrix} \\ &= \frac{\tau}{V_A^{(k)}V_B^{(k)} - \left(C_{AB}^{(k)}\right)^2} \begin{bmatrix} V_B^{(k)}E_t\left[\sum_{i=1}^{k} rx_{A,t+i}\right] - C_{AB}^{(k)}E_t\left[\sum_{i=1}^{k} rx_{B,t+i}\right] \\ V_A^{(k)}E_t\left[\sum_{i=1}^{k} rx_{B,t+i}\right] - C_{AB}^{(k)}E_t\left[\sum_{i=1}^{k} rx_{A,t+i}\right] \end{bmatrix}, \end{aligned} \tag{57}$$

where $V_i^{(k)} \equiv Var_t\left[\sum_{j=1}^{k} rx_{i,t+j}\right]$ and $C_{AB}^{(k)} \equiv Cov_t\left[\sum_{j=1}^{k} rx_{A,t+j}, \sum_{j=1}^{k} rx_{B,t+j}\right]$. Under the conjectured forms for equilibrium yields, the realized k-period cumulative excess returns for both bonds are

$$\begin{aligned} \sum_{i=1}^{k} rx_{i,t+i} &= \sum_{i=0}^{k-1}\left(y_{i,t+i} - r_{t+i} - z_{i,t+i+1}\right) - \frac{\theta_i}{1-\theta_i}\left(y_{i,t+k} - y_{i,t}\right) \\ &= k\left(\alpha_{i0} - \bar{r} - \bar{z}_i\right) + \left(\boldsymbol{\alpha}_{i1} - \mathbf{e}_r\right)'\left(\sum_{j=0}^{k-1}\mathbf{x}_{t+j}\right) - \mathbf{e}_{z_i}'\left(\sum_{j=1}^{k}\mathbf{x}_{t+j}\right) - \frac{\theta_i}{1-\theta_i}\left(\boldsymbol{\alpha}_{i1}'\mathbf{x}_{t+k} - \boldsymbol{\alpha}_{i1}'\mathbf{x}_t\right). \end{aligned} \tag{58}$$

Given this formulation for excess returns, we find expressions for expected excess returns, the variance of excess returns, as well as the covariance between excess returns on A and B bonds:

$$E_t\left[\sum_{i=1}^{k} rx_{i,t+i}\right] = k\left(\alpha_{i0} - \bar{r} - \bar{z}_i\right) + \left(\left(\boldsymbol{\alpha}_{i1} - \mathbf{e}_r\right)'\left(\mathbf{I} - \boldsymbol{\Gamma}\right)^{-1} + \frac{\theta_i}{1-\theta_i}\boldsymbol{\alpha}_{i1}' - \mathbf{e}_{z_i}'\left(\mathbf{I} - \boldsymbol{\Gamma}\right)^{-1}\boldsymbol{\Gamma}\right)\left(\mathbf{I} - \boldsymbol{\Gamma}^k\right)\mathbf{x}_t. \tag{59}$$

$$V_i^{(k)} = \text{Var}_t\left[(\boldsymbol{\alpha}_{i1} - \mathbf{e}_r - \mathbf{e}_{z_i})'\left(\sum_{j=1}^{k-1}\mathbf{x}_{t+j}\right) - \left(\frac{\theta_i}{1-\theta_i}\boldsymbol{\alpha}_{i1} + \mathbf{e}_{z_i}\right)'\mathbf{x}_{t+k}\right] \tag{60}$$

$$= (\boldsymbol{\alpha}_{i1} - \mathbf{e}_r - \mathbf{e}_{z_i})'\left(\sum_{i=1}^{k-1}\sum_{j=1}^{k-1}\mathbf{C}_{[t+i,t+j]}\right)(\boldsymbol{\alpha}_{i1} - \mathbf{e}_r - \mathbf{e}_{z_i})$$

$$+ \left(\frac{\theta_i}{1-\theta_i}\boldsymbol{\alpha}_{i1} + \mathbf{e}_{z_i}\right)'\mathbf{C}_{[t+k,t+k]}\left(\frac{\theta_i}{1-\theta_i}\boldsymbol{\alpha}_{i1} + \mathbf{e}_{z_i}\right)$$

$$- 2\,(\boldsymbol{\alpha}_{i1} - \mathbf{e}_r - \mathbf{e}_{z_i})'\sum_{i=1}^{k-1}\mathbf{C}_{[t+i,t+k]}\left(\frac{\theta_i}{1-\theta_i}\boldsymbol{\alpha}_{i1} + \mathbf{e}_{z_i}\right)$$

$$C_{AB}^{(k)} = \text{Cov}_t\left[\begin{array}{c}(\boldsymbol{\alpha}_{A1} - \mathbf{e}_r - \mathbf{e}_{z_A})'\left(\sum_{j=1}^{k-1}\mathbf{x}_{t+j}\right) - \left(\frac{\theta_A}{1-\theta_A}\boldsymbol{\alpha}_{A1} + \mathbf{e}_{z_A}\right)'\mathbf{x}_{t+k},\\ (\boldsymbol{\alpha}_{B1} - \mathbf{e}_r - \mathbf{e}_{z_B})'\left(\sum_{j=1}^{k-1}\mathbf{x}_{t+j}\right) - \left(\frac{\theta_B}{1-\theta_B}\boldsymbol{\alpha}_{B1} + \mathbf{e}_{z_B}\right)'\mathbf{x}_{t+k}\end{array}\right] \tag{61}$$

$$= (\boldsymbol{\alpha}_{A1} - \mathbf{e}_r - \mathbf{e}_{z_A})'\left(\sum_{i=1}^{k-1}\sum_{j=1}^{k-1}\mathbf{C}_{[t+i,t+j]}\right)(\boldsymbol{\alpha}_{B1} - \mathbf{e}_r - \mathbf{e}_{z_B})$$

$$- (\boldsymbol{\alpha}_{A1} - \mathbf{e}_r - \mathbf{e}_{z_A})'\sum_{i=1}^{k-1}\mathbf{C}_{[t+i,t+k]}\left(\frac{\theta_B}{1-\theta_B}\boldsymbol{\alpha}_{B1} + \mathbf{e}_{z_B}\right)$$

$$- (\boldsymbol{\alpha}_{B1} - \mathbf{e}_r - \mathbf{e}_{z_B})'\sum_{i=1}^{k-1}\mathbf{C}_{[t+i,t+k]}\left(\frac{\theta_A}{1-\theta_A}\boldsymbol{\alpha}_{A1} + \mathbf{e}_{z_A}\right)$$

$$+ \left(\frac{\theta_A}{1-\theta_A}\boldsymbol{\alpha}_{A1} + \mathbf{e}_{z_A}\right)'\mathbf{C}_{[t+k,t+k]}\left(\frac{\theta_B}{1-\theta_B}\boldsymbol{\alpha}_{B1} + \mathbf{e}_{z_B}\right)$$

Using our formulation for generalists bond demand from (57) and our closed form expressions for expectation, variance and covariance of excess returns, we can find a closed form for generalists bond demands and match these to the coefficients $\delta_{A1}, \delta_{A2}, \delta_{B1}, \delta_{B2}$ based on the conjectured form for bond demand:

$$\delta_{A0} = \tau\frac{V_B^{(k)}k\,(\alpha_{A0} - \bar{r} - \bar{z}) - C_{AB}^{(k)}k\,(\alpha_{B0} - \bar{r} - \bar{z})}{V_A^{(k)}V_B^{(k)} - \left(C_{AB}^{(k)}\right)^2} \tag{62}$$

$$\delta_{B0} = \tau \frac{V_A^{(k)} k \left(\alpha_{B0} - \bar{r} - \bar{z}\right) - C_{AB}^{(k)} k \left(\alpha_{A0} - \bar{r} - \bar{z}\right)}{V_A^{(k)} V_B^{(k)} - \left(C_{AB}^{(k)}\right)^2} \tag{63}$$

$$\delta'_{A1} = \tau \frac{\begin{pmatrix} V_B^{(k)} \left(\left(\alpha_{A1} - \mathbf{e}_r\right)' \left(\mathbf{I} - \mathbf{\Gamma}\right)^{-1} + \frac{\theta_A}{1-\theta_A} \alpha'_{A1} - \mathbf{e}'_{z_A} \left(\mathbf{I} - \mathbf{\Gamma}\right)^{-1} \mathbf{\Gamma} \right) \\ -C_{AB}^{(k)} \left(\left(\alpha_{B1} - \mathbf{e}_r\right)' \left(\mathbf{I} - \mathbf{\Gamma}\right)^{-1} + \frac{\theta_B}{1-\theta_B} \alpha'_{B1} - \mathbf{e}'_{z_B} \left(\mathbf{I} - \mathbf{\Gamma}\right)^{-1} \mathbf{\Gamma} \right) \end{pmatrix}}{V_A^{(k)} V_B^{(k)} - \left(C_{AB}^{(k)}\right)^2} \left(\mathbf{I} - \mathbf{\Gamma}^k\right) \tag{64}$$

$$\delta'_{B1} = \tau \frac{\begin{pmatrix} V_A^{(k)} \left(\left(\alpha_{B1} - \mathbf{e}_r\right)' \left(\mathbf{I} - \mathbf{\Gamma}\right)^{-1} + \frac{\theta_B}{1-\theta_B} \alpha'_{B1} - \mathbf{e}'_{z_B} \left(\mathbf{I} - \mathbf{\Gamma}\right)^{-1} \mathbf{\Gamma} \right) \\ -C_{AB}^{(k)} \left(\left(\alpha_{A1} - \mathbf{e}_r\right)' \left(\mathbf{I} - \mathbf{\Gamma}\right)^{-1} + \frac{\theta_A}{1-\theta_A} \alpha'_{A1} - \mathbf{e}'_{z_A} \left(\mathbf{I} - \mathbf{\Gamma}\right)^{-1} \mathbf{\Gamma} \right) \end{pmatrix}}{V_A^{(k)} V_B^{(k)} - \left(C_{AB}^{(k)}\right)^2} \left(\mathbf{I} - \mathbf{\Gamma}^k\right) \tag{65}$$

D.3 Rational Expectations Equilibrium

In equilibrium, we need to clear markets for both bonds in a way that is consistent with the optimization of specialists A, and B as well as generalists. For both $i \in \{A, B\}$, the market clearing condition is

$$\left(1 - q_A - q_B\right) k^{-1} d_{i,t} + q_i b_{i,t} = s_{i,t} - \left(1 - q_A - q_B\right) \left(k^{-1} \sum_{i=1}^{k-1} d_{i,t-i} \right). \tag{66}$$

Plugging in $d_{i,t} = \delta_{i0} + \delta'_{i1} \mathrm{x}_t$ as well as our closed form expression for $b_{i,t}$ this becomes

$$\frac{\tau q_i}{V_i^{(1)}} \left(\alpha_{i0} - \bar{r} - \bar{z}\right) + q_i \left[\tau \frac{\left(\frac{1}{1-\theta_i} \alpha_{i1} - \mathbf{e}_r\right)' - \left(\frac{\theta_i}{1-\theta_i} \alpha_{i1} + \mathbf{e}_z\right)' \mathbf{\Gamma}}{V_i^{(1)}} \right] \mathbf{x}_t$$

$$= \left(\bar{s}_i - \left(1 - q_A - q_B\right) \delta_{i0}\right) + \left(\mathbf{e}_{s_i} - \left(1 - q_A - q_B\right) k^{-1} \left(\mathbf{1}_{(i)} + \delta_{i1}\right)\right)' \mathbf{x}_t, \tag{67}$$

where $\mathbf{1}_{(A)} \equiv \mathbf{e}_6 + \mathbf{e}_7$ and $\mathbf{1}_{(B)} \equiv \mathbf{e}_8 + \mathbf{e}_9$ in the case where $k = 3$. Matching constants for α_{i0} we have

$$\alpha_{i0} = \bar{r} + \bar{z} + \frac{V_i^{(1)}}{q_i \tau} \left(\bar{s}_i - \left(1 - q_A - q_B\right) \delta_{i0}\right) \tag{68}$$

Matching slopes for α_{i1} yields

$$\alpha_{i1} = \frac{1-\theta_i}{1-\rho_r\theta_i}\mathbf{e}_r + \frac{1-\theta_i}{1-\rho_{z_i}\theta_i}\rho_{z_i}\mathbf{e}_z \tag{69}$$

$$+ \frac{V_i^{(1)}}{\tau q_i}\left(\frac{1-\theta_i}{1-\theta_i\rho_{s_i}}\mathbf{e}_{s_i} - (1-\theta_i)(1-q_A-q_B)\,k^{-1}\left[\mathbf{I}-\theta_i\mathbf{\Gamma}'\right]^{-1}\left(\mathbf{1}_{(i)}+\delta_{i1}\right)\right) \tag{70}$$

Now note that α_{i0} and δ_{i0} are closed form expressions, and we express α_{i1} and δ_{i1} as closed form expressions that only depend on other exogenous closed form components and one-another. Namely, an equilibrium solves the following system of equations

$$\alpha_{A1} = \frac{1-\theta_A}{1-\rho_r\theta_A}\mathbf{e}_r + \frac{1-\theta_A}{1-\rho_{z_A}\theta_A}\rho_{z_A}\mathbf{e}_{z_1}$$

$$+ \frac{V_A^{(1)}(\alpha)}{\tau q_A}\left(\frac{1-\theta_A}{1-\theta_A\rho_{s_A}}\mathbf{e}_{s_A} - (1-\theta_A)(1-q_A-q_B)\,k^{-1}\left[\mathbf{I}-\theta_A\mathbf{\Gamma}(\delta)'\right]^{-1}\left(\mathbf{1}_{(A)}+\delta_{A1}\right)\right) \tag{71}$$

$$\alpha_{B1} = \frac{1-\theta_B}{1-\rho_r\theta_B}\mathbf{e}_r + \frac{1-\theta_B}{1-\rho_{z_B}\theta_B}\rho_{z_B}\mathbf{e}_{z_B}$$

$$+ \frac{V_B^{(1)}(\alpha)}{\tau q_B}\left(\frac{1-\theta_B}{1-\theta_B\rho_{s_B}}\mathbf{e}_{s_B} - (1-\theta_B)(1-q_A-q_B)\,k^{-1}\left[\mathbf{I}-\theta_B\mathbf{\Gamma}(\delta)'\right]^{-1}\left(\mathbf{1}_{(B)}+\delta_{B1}\right)\right) \tag{72}$$

$$\delta'_{A1} = \tau\frac{\begin{pmatrix} V_B^{(k)}(\alpha,\delta)\left((\alpha_{A1}-\mathbf{e}_r)'(\mathbf{I}-\mathbf{\Gamma}(\delta))^{-1} + \frac{\theta_A}{1-\theta_A}\alpha'_{A1} - \mathbf{e}'_{z_A}(\mathbf{I}-\mathbf{\Gamma}(\delta))^{-1}\mathbf{\Gamma}(\delta)\right) \\ -C_{AB}^{(k)}(\alpha,\delta)\left((\alpha_{B1}-\mathbf{e}_r)'(\mathbf{I}-\mathbf{\Gamma}(\delta))^{-1} + \frac{\theta_B}{1-\theta_B}\alpha'_{B1} - \mathbf{e}'_{z_B}(\mathbf{I}-\mathbf{\Gamma}(\delta))^{-1}\mathbf{\Gamma}(\delta)\right)\end{pmatrix}}{V_A^{(k)}(\alpha,\delta)V_B^{(k)}(\alpha,\delta) - \left(C_{AB}^{(k)}(\alpha,\delta)\right)^2}\left(\mathbf{I}-\mathbf{\Gamma}(\delta)^k\right) \tag{73}$$

$$\delta'_{B1} = \tau\frac{\begin{pmatrix} V_A^{(k)}\left((\alpha_{B1}-\mathbf{e}_r)'(\mathbf{I}-\mathbf{\Gamma}(\delta))^{-1} + \frac{\theta_B}{1-\theta_B}\alpha'_{B1} - \mathbf{e}'_{z_B}(\mathbf{I}-\mathbf{\Gamma}(\delta))^{-1}\mathbf{\Gamma}(\delta)\right) \\ -C_{AB}^{(k)}\left((\alpha_{A1}-\mathbf{e}_r)'(\mathbf{I}-\mathbf{\Gamma}(\delta))^{-1} + \frac{\theta_A}{1-\theta_A}\alpha'_{A1} - \mathbf{e}'_{z_A}(\mathbf{I}-\mathbf{\Gamma}(\delta))^{-1}\mathbf{\Gamma}(\delta)\right)\end{pmatrix}}{V_A^{(k)}V_B^{(k)} - \left(C_{AB}^{(k)}\right)^2}\left(\mathbf{I}-\mathbf{\Gamma}(\delta)^k\right). \tag{74}$$

Here, we write $V_A^{(1)}(\alpha)$, $V_B^{(1)}(\alpha)$ to emphizize their dependence on α_{A0}, α_{A1}, we write $\mathbf{\Gamma}(\delta)$ to emphizize $\mathbf{\Gamma}$'s dependence on δ_{A1} and δ_{B1}, we write $V_A^{(k)}(\alpha, \delta)$, $V_B^{(k)}(\alpha, \delta)$, and $C_{AB}^{(k)}(\alpha, \delta)$ to emphizize $V_i^{(k)}$'s and C_{AB}^{k}'s dependence on α_{A1}, α_{B1}, δ_{A1}, and δ_{B1}. We can write the above equations compactly as

$$\alpha_{A1} = \mathrm{f}_{\alpha_{A1}}\left(\alpha_{A1}, \alpha_{B1}, \delta_{A1}, \delta_{B1}\right)$$

$$\alpha_{B1} = \mathrm{f}_{\alpha_{B1}}\left(\alpha_{A1}, \alpha_{B1}, \delta_{A1}, \delta_{B1}\right)$$

$$\delta_{A1} = \mathrm{f}_{\delta_{A1}}\left(\alpha_{A1}, \alpha_{B1}, \delta_{A1}, \delta_{B1}\right)$$

$$\delta_{A2} = \mathrm{f}_{\delta_{A2}}\left(\alpha_{A1}, \alpha_{B1}, \delta_{A1}, \delta_{B1}\right),$$

or simply as

$$\omega = f(\omega)$$

where $\omega = (\alpha'_{A1}, \alpha'_{B1}, \delta'_{A1}, \delta'_{B1})$. We solve this system of non-linear equations numerically in python by employing the Powell hybrid algorithm.