

# Exit Strategies from Quantitative Easing: The role of the fiscal-monetary policy mix\*

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## Abstract

As a consequence of the policy responses to the COVID-19 crisis, central bank balance sheets, public debt, and liquidity increased in many developed economies. As the economies recover and inflation far exceeds the target, central banks face a challenge in how to manage their balance sheet. I study the macroeconomic effects of reducing the central bank balance sheet size, i.e., Quantitative Tightening (QT). I construct a Regime-Switching New Keynesian DSGE model calibrated to the US economy. The economy fluctuates between a monetary-led regime, a fiscally-led regime, and the zero lower bound on the monetary policy interest rate. The macroeconomic effects of QT crucially depend on the fiscal-monetary policy mix. In a monetary-led regime, QT effectively reduces inflation at the cost of increasing the government debt-to-GDP ratio. In contrast, unwinding the central bank balance sheet in a fiscally-led regime has little impact on inflation. The negative demand effect driven by QT is not enough to counteract the stimulative impact of negative real interest rates and fiscal stimulus.

**Keywords:** Monetary Policy; Fiscal and monetary policy mix; Quantitative Easing; Quantitative Tightening

**JEL Classification:** E31, E52, E58, E62, E63

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*“...I would just stress how uncertain the effect is of shrinking the balance sheet...”*

J. Powell, Federal Reserve Chairman, press conference May 2022.

## 1 Introduction

In the Great Recession and the COVID-19 crisis, short-term interest rates in the US approached the zero lower bound (ZLB), leaving the Federal Reserve without its conventional monetary policy instrument. Due to this, the Federal Reserve began applying unconventional monetary policies, such as Quantitative Easing (QE), to stimulate the economy by lowering long-term interest rates.<sup>1</sup> QE consists of expanding the central bank balance sheet by purchasing large amounts of assets and paying by issuing reserves.<sup>2</sup> At the same time, the government embarked on a fiscal expansion that increased debt-to-GDP ratios to levels not seen since World War II.

As the economy recovers and inflation reaches levels not seen in forty years, the Federal Reserve faces the challenge of how to combine its two main policy tools (short-term interest rate and balance sheet size) to stabilize the inflation rate. In this paper, I quantify the impact of reducing the central bank balance sheet size on macroeconomic and financial variables such as inflation, public debt, and yield spreads. To this end, I construct a Regime-Switching New Keynesian Dynamic Stochastic General Equilibrium (NK-DSGE) model where fiscal and monetary authorities are subject to regime changes in their policy rules.

The main contribution of this paper is to show that the macroeconomic impact of QT depends on how the public debt will be stabilized, i.e., through fiscal surpluses or inflation. When the fiscal authority adjusts taxes to stabilize the debt, QT reduces inflation, even under smaller increases in the short-term interest rate. However, shrinkage of the central bank balance sheet has little impact on inflation if there is no appropriate fiscal framework to stabilize government debt.

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<sup>1</sup>Other central banks in developed economies, like the Bank of England, the central bank of Sweden, and the ECB, among others, implemented similar policies.

<sup>2</sup>In practice, QE includes distinct policies. In this paper, QE consists of central bank purchases of long-term public bonds by selling short-term assets (bank reserves). The main objective of this policy is to stimulate output and inflation by lowering long-term yields. This policy was defined as QE-type 2 by Ricardo Reis in his talk “The original sin of QE.”

The distinctive features of my model rely on the design of the financial intermediaries and the fiscal and monetary authorities. The model exhibits market segmentation in the public debt market and a leverage constraint for financial intermediaries, as in Elenev et al. (2021). The fiscal authority issues long and short-term bonds, while the central bank issues reserves. The central bank does conventional monetary policy, which consists in setting the short-term interest rate, and unconventional monetary policy, which consists of central bank purchases of long-term government bonds, paid by issuing reserves.

QE operates through two channels in this model. First, bank reserves and deposits increase when the central bank expands its balance sheet. Second, the long-term yield falls due to the central bank's intervention in the long-term bond market. This yield decline gives households incentives to rebalance their portfolio, selling long-term bonds in exchange for deposits. QE transmits into the economy as a positive demand shock. The portfolio revaluation effect provides households with a wealth effect, increasing consumption demand. The fall in savings return generates a substitution effect from savings to consumption. Both channels operate by increasing aggregate demand. The extent to which QE increases output and inflation depends on the fiscal-monetary policy mix.

Conventional monetary and fiscal policies consist of a Taylor rule for the short-term interest rate (the return on short-term public bonds and reserves) and a fiscal rule for taxes. These rules switch between three policy regimes, as in [Bianchi and Melosi \(2017\)](#) and [Bianchi and Melosi \(2022\)](#): a monetary-led regime, a fiscal-led regime, and the ZLB regime. In the first regime, the central bank reacts strongly to inflation deviations from the target, and the fiscal authority adjusts the primary fiscal surplus to stabilize the public debt.<sup>3</sup> The opposite happens in the second regime, where the fiscal authority does not adjust the fiscal surplus enough to stabilize the debt. As a result, the monetary authority allows the inflation rate to deviate from the target to stabilize debt in real terms. Finally, the ZLB regime represents a crisis regime, where the monetary authority is constrained by the effective lower bound.

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<sup>3</sup>This behaviour was characterized as *active* monetary policy and *passive* fiscal policy in [Leeper \(1991\)](#) and the following literature. The opposite happens in the fiscally-led regime, where monetary policy is *passive* and fiscal policy *active*. The literature has also called the regimes *monetary dominance* and *fiscal dominance*, respectively. These regimes represent, in a parsimonious way, the outcome of a game between fiscal and monetary authorities.

The economy presents recurrent regime switches following a transition matrix. Differently from [Bianchi and Melosi \(2017\)](#) and [Bianchi and Melosi \(2022\)](#), the transition to and from the ZLB regime is endogenous and depends on macroeconomic conditions. This endogeneity allows agents to form rational expectations regarding the occurrence of this regime based on their knowledge of macroeconomic variables such as inflation, output, and nominal interest rates. The endogenization of this transition probability constitutes one of the contributions of this paper.

I calibrate the model for the US economy and show it can match volatility and cyclicity features in the data. Thus, I use it as a laboratory to study central bank balance sheet policies. I simulate a crisis that resembles the COVID-19 pandemic and the policy response. I show that a QE program that increases central bank long term bonds purchases by 10p.p. of GDP reduces the severity of the crisis. Output growth falls around 1p.p. less than without the program and recovers much faster, allowing a shorter duration of the ZLB regime. QE also has expansionary effects in terms of inflation. Prices fall by 2.5p.p. less than without the central bank's intervention, at the cost of more significant inflationary dynamics during the recovery, where annualized inflation reaches a peak of 8.1. Without QE, inflation would have still overshoot the target, but with a lower peak of 6.6%.

I show that how the central bank balance sheet's size is reduced matters for inflation, output, and debt dynamics. In the crisis recovery, I study different strategies regarding the size of the central bank balance sheet: 1) maintaining the enlarged size after the crisis (*tapering*), 2) decreasing the balance sheet size: a) by letting the bonds that mature, run off the balance sheet (*Quantitative Tightening (QT)*), and b) at a faster speed by selling bonds (*Aggressive QT*).

When the central bank reduces its purchases of long-term public bonds, it triggers a fall in their price and an increase in long-term yields. Increased returns result in a substitution effect from consumption to savings, reducing aggregate demand and output. At the same time, public debt issuance increases for three reasons. First, the increase in interest rates increases debt service. The fall in output triggers automatic stabilizers that increase the fiscal deficit. The fall in long-term bonds price reduces central bank remittances to the treasury.

In the monetary-led regime, the increase in debt causes a rise in taxes due to the fiscal rule. This exacerbates the fall in aggregate demand and disinflationary effects of Quantitative Tightening. In this regime, inflation falls even with lower increases in the policy rate. In the fiscally-led regime, public debt increases actual and expected inflation, exacerbating the rise in yield spreads and mitigating the disinflationary effects of QT.

In a fiscally-led regime, thus, there are no clear advantages in reducing the balance sheet size since it increases debt and spreads without helping to stabilize inflation. The negative demand shock generated by QT is not enough to counteract the stimulative effect of real negative interest rates and fiscal deficits.

Related Literature. This paper contributes to several strands of the literature. In particular, it contributes to the literature that studies the real and inflationary effects of QE through different channels.<sup>4</sup> Many authors studied the transmission mechanisms of QE in general equilibrium models, either with financial frictions, like in [Gertler and Karadi \(2011\)](#), [Sims and Wu \(2021\)](#), [Sims et al. \(2020\)](#), [Del Negro et al. \(2017\)](#), among others; with market segmentation or portfolio adjustment costs as the main mechanisms to break the non-arbitrage condition between short-term and long-term bonds, as in [Chen et al. \(2012\)](#), [Harrison \(2017\)](#), or information frictions as in [Gaballo and Galli \(2022\)](#). Furthermore, a recent paper by [Cui and Sterk \(2021\)](#) studies the liquidity effects of asset purchase programs in a model with heterogeneous agents. The main contribution to this strand of the literature is allowing the fiscal-monetary policy mix to play a role in shaping the macroeconomic effects of QE and showing alternative scenarios where fiscal and monetary policy interact.

In a related paper, [Elenev et al. \(2021\)](#) ask whether monetary policy can create fiscal capacity. In their setting, fiscal capacity depends on the probability of shifting fiscal policy from active to passive. In this sense, I share with them the objective of studying the effects of unconventional monetary policies while allowing fiscal policy to shift between regimes. The main difference with this paper is that [Elenev et al. \(2021\)](#) assume and estimate a fiscal limit beyond which the Treasury starts increasing taxes to ensure debt sustainability. At the same time, the conventional monetary policy is active at all periods. These assumptions prevent inflationary dynamics from arising in the model since all the agents in the economy

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<sup>4</sup>See [Bhattarai and Neely \(Forthcoming\)](#) and [Kuttner \(2018\)](#) for reviews of this literature.

are rational and know the government will never inflate away part of the debt. My main contribution here is to study QT when fiscal and monetary authorities can follow a different policy configuration.

By giving a central role to the interaction or coordination of fiscal and monetary policies, this paper also relates to the literature that studies fiscal-monetary policy mix, as in [Sims \(1994\)](#), [Leeper \(1991\)](#), [Schmitt-Grohe et al. \(2007\)](#), [Leeper and Leith \(2016\)](#), [Reis \(2017\)](#), [Bassetto and Sargent \(2020\)](#), [Barthélemy et al. \(2021\)](#), among others, and to the literature of the Fiscal Theory of the Price Level as in [Sargent et al. \(1981\)](#), [Bassetto \(2002\)](#), [Cochrane \(2001\)](#), [Sims \(2016\)](#), [Cochrane \(2021\)](#), [Brunnermeier et al. \(2020\)](#). Allowing the policy mix to alternate, this paper also relates to the literature on regime switches in policy rules, as in [Bianchi \(2013\)](#), [Bianchi and Melosi \(2017\)](#), [Bianchi and Melosi \(2022\)](#), among others. The contribution to this branch of the literature is the study of unconventional monetary policies together with the conventional Taylor rule on short-term interest rates. By considering an endogenous transition probability to get in and out of the ZLB, this paper relates to the literature that studies monetary policy rules that depend on endogenous variables, as in [Barthélemy and Marx \(2017\)](#), [Davig and Leeper \(2008\)](#).

This paper focuses on the macroeconomic effects of reducing the central bank balance sheet. In a broad sense, I share the question with, for example, [Wen et al. \(2014\)](#), [Harrison \(2017\)](#), [Sims et al. \(2020\)](#), [Bonciani and Oh \(2021\)](#), [Hall and Reis \(2016\)](#), [Foerster \(2015\)](#) and [Benigno and Benigno \(2022\)](#). However, this paper differs from these in many aspects. In [Wen et al. \(2014\)](#), the focus relies on the impact on firms, while there is no role for the fiscal authority. [Harrison \(2017\)](#) studies the optimal QE policy in a DSGE-NK model with portfolio adjustment costs, and [Sims et al. \(2020\)](#) studies optimal simple and implementable QE rules through minimizing a quadratic loss-function in a DSGE-NK model with financial frictions. [Bonciani and Oh \(2021\)](#) extends the work of [Sims et al. \(2020\)](#) by showing that the central bank's loss function depends on its asset purchase volatility. Crucial differences are that these articles focus on central bank purchases of corporate/ private sector bonds, assume a limited role in fiscal policy, and do not allow for changes in the conduct of the fiscal policy rule. [Hall and Reis \(2016\)](#) study how different strategies for the exit from quantitative easing in the face of interest-rate risk, as I do. However, the authors' main objective is to

study the implications on central bank financial stability, and they assume a passive fiscal policy. [Foerster \(2015\)](#) examines the macroeconomic effects of unwinding the central bank balance sheet during and after a financial crisis. He shows that private agents' expectations about the exit strategy from a QE program impact the initial effectiveness of the policy in a MS-DSGE model with a financial sector. However, there is no role for parameter switches in conventional policy rules, abstracting from the possibility of alternative fiscal-monetary policy interactions, which is the main contribution of this paper.

Finally, in a recent related paper, [Benigno and Benigno \(2022\)](#) study monetary policy normalization, defined as the combination of lifting the policy rate and reducing the size of central bank balance sheets. They study how these monetary policies interact and how they depend on the behavior of the fiscal authority. However, they focus on optimal policy analysis and does not account for policy uncertainty in the configuration of fiscal and monetary policy interactions, which constitutes a central contribution of this paper.

The remainder of the paper is as follows. In the following section, I provide motivating evidence of the importance of the fiscal and monetary policies interaction during Quantitative Easing programs, looking at data for the US during the COVID-19 crisis. Section 3 presents the model, Section 4 discusses the calibration, functional forms, and solution method. In Section 5, I present the quantitative results of the model, and in Section 6 explain the main transmission mechanism of QE. Section 7 presents the paper's main results, where I simulate the crisis and present the different exit strategies from QE. Finally, Section 8 concludes.

## 2 COVID-19 crisis and policy response in the US

This section provides data on the COVID-19 crisis and the policy response in the US. The objective is to show the behavior of the main players in the bonds market, which motivates some model decisions.

In March 2020, the COVID-19 pandemic hit economies worldwide, generating an unprecedented macroeconomic crisis. As a response, central banks in leading developed economies, particularly the US, started to stimulate the economy through cuts in short-term interest rates until they hit the zero lower bound. As a result, central banks began Quantitative

Easing programs. They consisted of purchasing assets, mainly government bonds of long maturity, expanding their balance sheets at a breakneck speed. As shown in figure 1, the assets in the Federal Reserve increased from \$4 trillion in the third quarter of 2019 to over \$8 trillion one year later. Treasuries are the main component of this assets' expansion at the Federal Reserve, which increased from about \$2 to \$6 trillion in less than two years.

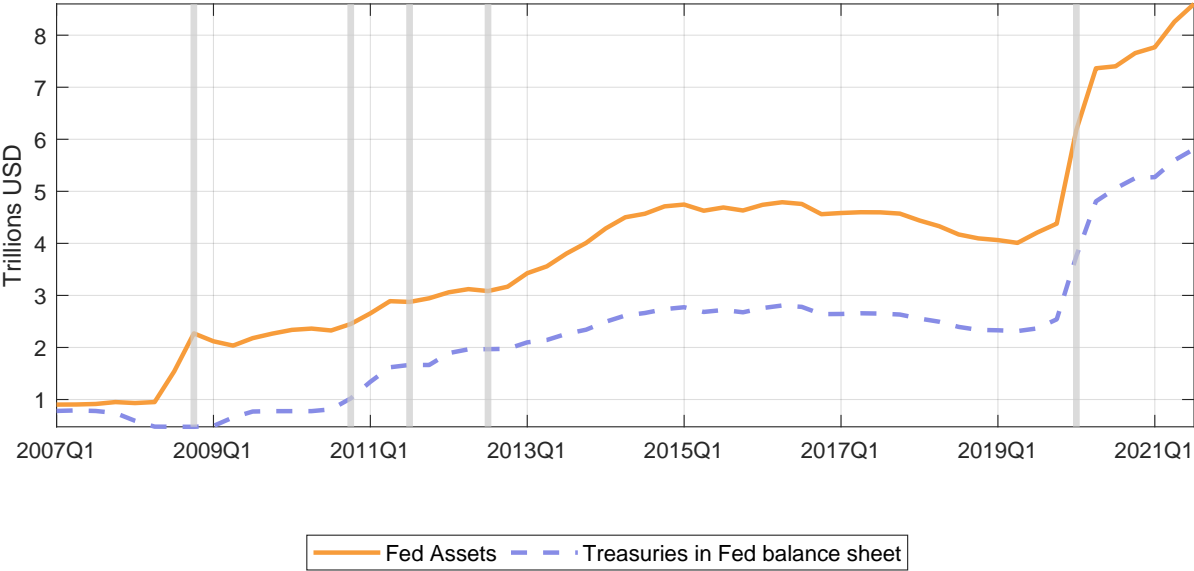


Figure 1: Federal Reserve Balance Sheet

Assets in the Federal Reserve balance sheet (orange line) and treasuries in the balance sheet (dotted light-blue line). Variables in trillions of dollars. Source: FRED and US Financial Accounts. Grey vertical lines show the start of a Quantitative Easing program.

The Quantitative Easing program took place with a substantial expansion in total debt issuance from the Treasury. Figure 2 shows net purchases of treasuries from the start of the COVID-19 crisis. They are net flows of treasuries, i.e., net of revaluation effects that take into account the change in treasuries' prices.<sup>5</sup> Different colors represent different agents in the economy. When the bar is above zero, the agent had net purchases of treasuries during the period, while if it is below zero, it represents net sales. The blue line represents the net

<sup>5</sup>The conclusions do not change when we consider changes in treasuries. See Appendix, section 9.1.



debt issuance from the US Treasury, which is the sum of all the bars in a corresponding period.

The figure shows the significant increment in debt issuance during the whole period. Furthermore, the monetary authority’s purchases of government bonds played a vital role during this period, sustaining the demand for treasuries when private sectors like households, mutual funds, and the rest of the world were selling their bond holdings.

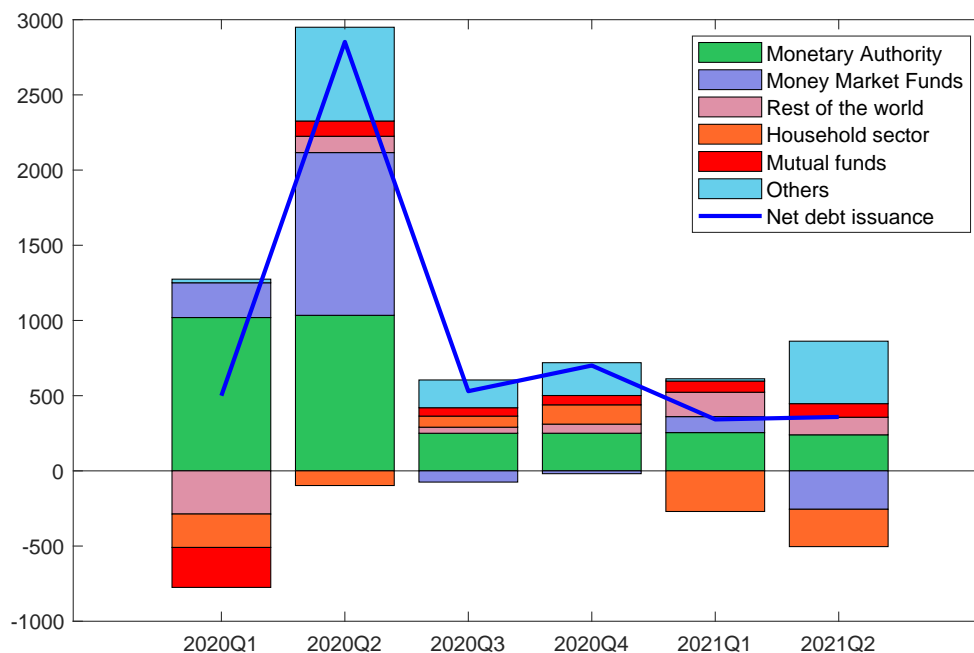


Figure 2: Net purchases of treasuries

Source: US Financial Accounts. Data in billions of dollars. Flows, net of revaluation effects.

Who sells treasuries to the central bank during a QE operation has implications regarding the primary macroeconomic aggregates in the economy. When the Federal Reserve performs quantitative easing policies, it purchases assets. The counterpart of this operation is the issuance of bank reserves (or money in commercial banks’ hands) that increases the central bank’s liabilities, expanding the central bank’s balance sheet. If commercial banks were the original owners of the bonds (case 1), this operation would leave its balance sheet unchanged since they increase one asset (reserves) while decreasing another (treasuries). However, suppose the original owner of the bond was the non-bank private sector (case 2), like households

or mutual funds. In that case, the bank is a intermediary of this policy. The result would be an increase in deposits or currency (money in private hands), together with an increase in reserves and the central bank's balance sheet. As a result, different measures of money increase after quantitative easing policies under different cases; either bank money in the form of reserves (case 1) or non-bank private money (case 2).

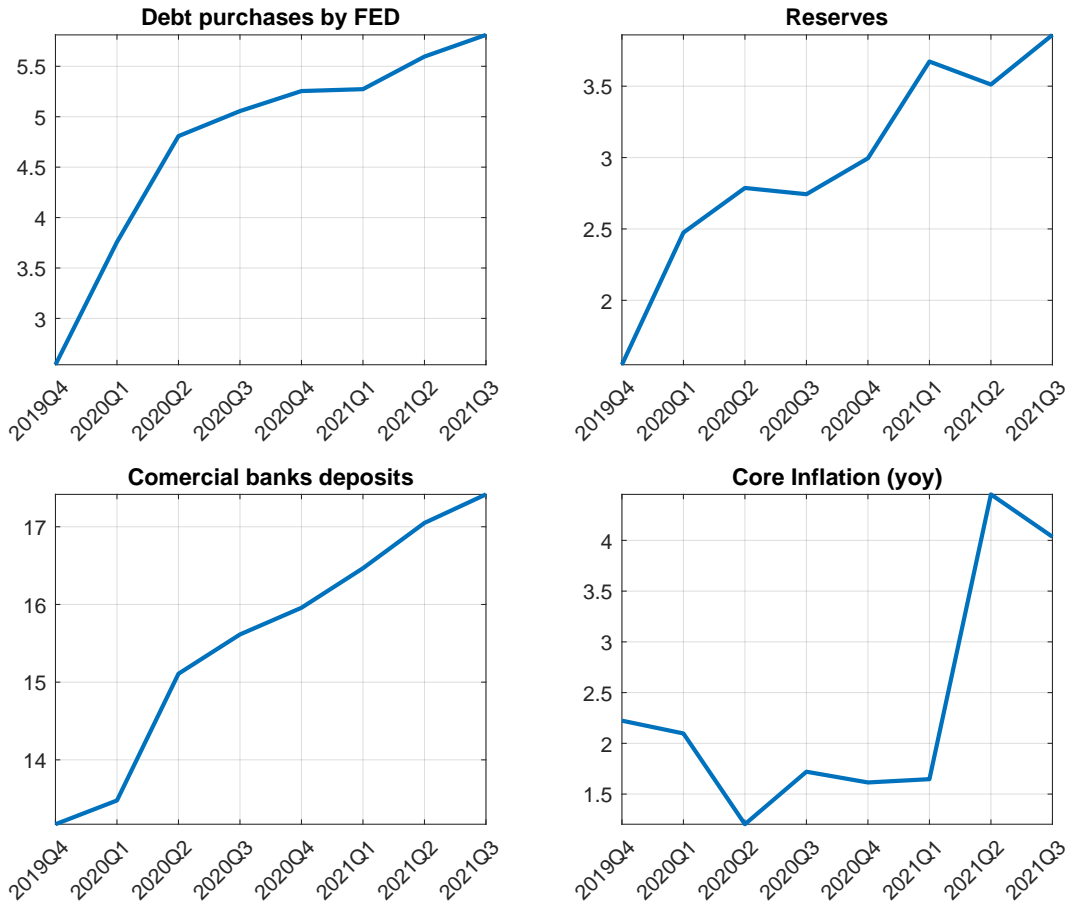


Figure 3: Macroeconomic variables: public debt, liquidity, inflation

US Federal Debt, Reserves and Deposits, in trillions of dollars. Core inflation in %. Source debt, deposits, and inflation data: FRED. Source reserves: US financial accounts, released December 2021.

In figure 3, we can see a great expansion in liquidity, both in the form of bank reserves and deposits.<sup>6</sup> Other measures of monetary aggregates present significant increases during

<sup>6</sup>In the appendix, section 9.1, I show this last feature differs from what happened after the Quantitative Easing programs after the Great Recession.

this period too. Some of these variables, like public debt, are at values over GDP not seen since the Second World War.<sup>7</sup>

As economies recover and face inflationary dynamics not seen in the last 40 years, it is unclear how to unwind the stimulus injected during the crisis. The combination of high inflation rates, large debt-to-GDP ratios, elevated liquidity, and expanded central bank balance sheet presents an extra challenge for policymakers.

In the following section, I present a model able to generate the comovements that we observed in aggregate macroeconomic variables during the COVID-19 crisis and the consequent policy response and use it as a laboratory to study balance sheet policies.

### 3 Model

The model is a NK-DSGE model with five agents: Firms, households, Monetary Authority, Fiscal Authority, and financial intermediaries (FI). There are three assets in the economy: Short-term public bonds  $B_t^S$  with one-period maturity and price  $Q_t^S$ ; deposits  $D_t$  which provide liquidity services to households and with price  $Q_t^D$ ; and long-term public bonds:  $B_t^L$ , with geometrically decaying maturity  $\delta$ , as in [Hatchondo and Martinez \(2009\)](#), that pay a coupon  $\kappa$  every period, and their price is  $Q_t^L$ .

As is standard in the literature that assesses the transmission mechanisms of quantitative easing policies, there is segmentation in bond markets.<sup>8</sup> I follow [Elenev et al. \(2021\)](#) in assuming that households cannot invest in short-term public bonds, which are held exclusively by financial intermediaries and the central bank. In contrast, financial intermediaries do not invest in long-term public bonds. These assumptions align with the data presented in [section 2](#).

Short-term bonds in the model stand for treasury bills and central bank reserves indistinctly since both assets share liquidity and return properties. The only difference between them is which governmental institution issues them (i.e., central bank or treasury). The

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<sup>7</sup>See [section 9.1](#) for a historical plot of this variable.

<sup>8</sup>See, for instance, [Chen et al. \(2012\)](#), and [Bhattarai and Neely \(Forthcoming\)](#) for a comprehensive analysis on different mechanisms to break the non-arbitrage condition between different assets in the economy, and thus, the neutrality result from [Wallace \(1981\)](#).

assumption that financial intermediaries exclusively hold them in the model makes this asset close to “money in banks’ hands.”

Market segmentation interacts in the model with the following frictions. First, prices are sticky, as in [Rotemberg \(1982\)](#) and the New-Keynesian literature. Second, households pay a portfolio adjustment cost when purchasing long-term public bonds, and financial intermediaries pay a convex cost when they raise new equity. Finally, financial intermediaries are subject to a leverage constraint that states that their debt (deposits) cannot be larger than their assets (reserves and treasury bills). On top of this, the real value of deposits provides liquidity services to households increasing their utility, as in the tradition of Money in the Utility (MIU) function models. Including variables that stand for reserves and monetary aggregates in the model, together with the frictions above, allows the model to generate the main macroeconomic effects of QE policies highlighted in the empirical literature and the comovements in macroeconomic aggregates that we observed in [section 2](#).

Two authorities constitute the government: the treasury and the central bank. Each authority has its budget constraint and policy instrument(s). The presence of the two governmental sectors allows the model to generate realistic policy interactions, which are at the center of macroeconomic dynamics. I follow [Bianchi and Melosi \(2017\)](#), and [Bianchi and Melosi \(2022\)](#) and assume that parameters in policy rules for conventional monetary policy, the Taylor rule, and fiscal rule alternate between regimes.

Next, I describe each agent and its optimization problem.

### 3.1 Households

There is a continuum of measure one of homogeneous households that live infinite periods in the economy. A representative agent chooses consumption  $c_t$ , labor  $n_t$ , deposits  $D_t^H$ , and long-term public bonds  $B_t^{L,H}$  to maximize its lifetime utility:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \nu_t \beta^t U \left( c_t, \frac{D_t^H}{P_t}, n_t \right)$$

$\nu_t$  is a preference shock that follows an AR(1) process. Deposits are the most liquid asset that a household can purchase. I assume agents derive utility from the liquidity services

this asset provides. The utility function is a monotone increasing function in consumption and deposits, a monotone decreasing function in labor, and satisfies Inada conditions on all variables.

Long-term bonds pay geometrically decaying coupons, as in [Hatchondo and Martinez \(2009\)](#). A bond  $B_t^L$  issued at time  $t$  pays the sequence of coupons:  $\kappa, \kappa(1 - \delta), \kappa(1 - \delta)^2, \dots$ , where  $\kappa > 0$  and  $\delta \in (0, 1)$ . This last parameter controls the debt maturity, where  $\delta = 1$  corresponds to a short-term bond (i.e., 1-period maturity), and  $\delta = 0$  represents a consol. This maturity specification reduces the number of state variables in the model. A bond issued at time  $j - k$  is equivalent to  $(1 - \delta)^k$  bonds issued at period  $t$ , and hence the state variable  $B_{t-1}^L$  represents total long-term debt in equivalent newly issued long-term bonds. Their price at period  $t$  is  $Q_t^L$ .<sup>9</sup>

Every period, the representative household pays lump-sum taxes  $\tau_t$ , receives nominal dividend payments from financial intermediaries  $Div_t$ , firm's profits since households are the owners  $\Pi_t^f$ , and rebates  $\tilde{\Pi}_t$ . When she invests in long-term bonds, she pays a portfolio adjustment cost of  $\Phi(\cdot)$  on top of the asset's price.<sup>10</sup>

The optimization problem of a representative household is the following.

$$\begin{aligned}
& \max_{c_t, n_t, D_t^H, B_t^{L,H}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \nu_t U \left( c_t, \frac{D_t^H}{P_t}, n_t \right) \\
& P_t c_t + Q_t^D D_t^H + B_t^{L,H} Q_t^L + \Phi_L \left( \frac{B_t^{L,H}}{P_t} \right) P_t = W_t n_t + D_{t-1}^H + \dots \\
& + B_{t-1}^{L,H} [\kappa + (1 - \delta) Q_t^L] + \Pi_t^f + Div_t + \tilde{\Pi}_t - \tau_t P_t \tag{1} \\
& D_t^H \geq 0 \\
& B_t^{L,H} \geq 0
\end{aligned}$$

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<sup>9</sup>Notice that the deflated number of nominal bonds by the price level,  $\frac{B_t^{L,H}}{P_t} = b_t^{L,H}$  is not the real value of public debt. Instead, the real value of public bonds held by households is  $Q_t^L \frac{B_t^{L,H}}{P_t}$ . As discussed in [Leeper et al. \(2021\)](#), introducing the variable  $b_t^{L,H}$  allows to dispose of one variable, by working with  $b_t^{L,H}$  instead of  $P_t$  and  $B_t^{L,H}$ .

<sup>10</sup>This assumption helps the model generate a positive term premium in long-term public bonds.

where  $P_t$  is the price level, and  $W_t$  is the nominal wage. Under this specification, the expected return on a long-term bond purchased at period  $t$  is  $R_{t,t+1}^L = \mathbb{E}_t \frac{\kappa + (1-\delta)Q_{t+1}^L}{Q_t^L}$ .

Notice that the non-negativity condition on deposits does not bind at the optimum even if they pay a lower return, given the assumption that deposits increase utility. Finally, the last constraint is the non-negativity conditions for public bonds since the household cannot go short on them.

Associate the multiplier  $\beta^t \lambda_t$  to the budget constraint. The system of equilibrium conditions that characterize the households' optimization problem solution is the following:

$$\frac{-U_n \left( c_t, \frac{D_t^H}{P_t}, n_t \right)}{U_c \left( c_t, \frac{D_t^H}{P_t}, n_t \right)} = \frac{W_t}{P_t}$$

$$Q_t^D = \mathbb{E}_t \mathcal{M}_{t,t+1} + \frac{U_d \left( c_t, \frac{D_t^H}{P_t}, n_t \right) P_t}{U_c \left( c_t, \frac{D_t^H}{P_t}, n_t \right)}$$

$$Q_t^L + \Phi'_L \left( \frac{B_t^{L,H}}{P_t} \right) = \mathbb{E}_t \mathcal{M}_{t,t+1} [\kappa + (1-\delta)Q_{t+1}^L]$$

Where  $\mathcal{M}_{t,t+1}$  is the stochastic discount factor between period  $t$  and  $t+1$ .

$$\mathcal{M}_{t,t+1} = \beta \frac{\lambda_{t+1}}{\lambda_t} = \beta \frac{\nu_{t+1}}{\nu_t} \frac{U_c \left( c_{t+1}, \frac{D_{t+1}^H}{P_{t+1}}, n_{t+1} \right)}{U_c \left( c_t, \frac{D_t^H}{P_t}, n_t \right)} \frac{1}{\pi_{t+1}}$$

Together with the budget constraint (1), they characterize the solution to the household's optimization problem.<sup>11</sup> Define  $\pi_t = \frac{P_t}{P_{t-1}}$  as the inflation rate of period  $t$ . Notice that a spread will exist between the prices of deposits and long-term bonds even without the presence of portfolio adjustment cost. Households are willing to invest in deposits, even though they provide a lower return, because they derive utility from them. The presence of portfolio adjustment costs in bonds generates a term spread and prevents the household from fully exploiting the arbitrage opportunities in the assets' markets. The literature has shown this feature is vital for the transmission mechanism of QE policies.

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<sup>11</sup>The non-negativity condition in public bond purchases ensures the transversality condition is satisfied.

For future reference, it is convenient to define the return on a risk-free private asset in this economy, which is in zero net supply, as given by:

$$R_t^N = \mathbb{E}_t \frac{1}{\mathcal{M}_{t,t+1}}$$

This expression is defined as the natural interest rate in [Benigno and Benigno \(2022\)](#), which differs from the policy rate, and is key in the consumption-savings trade-off.

## 3.2 Firms

The productive sector comprises two levels: final goods and intermediate goods producers.

### 3.2.1 Final good producer

A representative firm produces the domestic final good  $y_t$  from varieties  $y_i$ , for  $i \in [0, 1]$ .

$$y_t = \left[ \int_0^1 y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

Where  $\varepsilon$  is the elasticity of substitution between varieties. The optimization problem of the representative firm is the following:

$$\begin{aligned} \max_{y_t, \{y_{i,t}\}_{i \in [0,1]}} \quad & P_t y_t - \int_0^1 P_{i,t} y_{i,t} di \\ \text{s.t } y_t = \quad & \left[ \int_0^1 y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \end{aligned}$$

And the optimal demand function for variety  $i$  is given by the following expression:

$$y_{i,t} = y_t \left( \frac{P_{i,t}}{P_t} \right)^{-\varepsilon} \tag{2}$$

### 3.2.2 Intermediate goods firms

Intermediate goods firms are monopolistically competitive in the goods market. Each firm produces a variety  $i$  according to a linear production function:

$$y_{i,t} = z_t n_{i,t} \quad (3)$$

Where  $z_t$  is a mean reverting TFP shock, common to all varieties, with the law of motion:

$$\ln(z_t) = \rho_z \ln(z_{t-1}) + \sigma_z \varepsilon_t^z, \varepsilon_t^z \sim N(0, 1)$$

When changing prices, firm  $i$  is subject to a quadratic adjustment cost in prices, as in Rotemberg (1982):

$$\frac{\phi^P}{2} \left( \frac{P_{i,t}}{P_{i,t-1}} - \pi^* \right)^2 y_t$$

Where  $\phi^P$  measures the degree of nominal price rigidity, and  $y_t$  is aggregate output, given by:

$$y_t = \int_0^1 y_{i,t} di$$

The nominal profit of firm  $i$  at period  $t$ , transferred to households, is given by:

$$\Pi_{i,t}^f = P_{i,t} y_{i,t} - W_t n_{i,t} - \frac{\phi^P}{2} \left( \frac{P_{i,t}}{P_{i,t-1}} - \pi^* \right)^2 y_t P_t \quad (4)$$

Firms maximize the discounted sum of profits using the households' discount factor, and subject to technology 3, and demand 2.



Their optimization problem is the following:

$$\begin{aligned}
& \max_{P_{i,t}, n_{i,t}} \mathbb{E}_0 \sum_{k=0}^{\infty} \mathcal{M}_{t,t+k} \Pi_{i,t+k}^f \\
& \text{s.t. } y_{i,t} = z_t n_{i,t} \\
& y_{i,t} = y_t \left( \frac{P_{i,t}}{P_t} \right)^{-\varepsilon} \\
& \Pi_{i,t}^f = P_{i,t} y_{i,t} - W_t n_{i,t} - \frac{\phi^P}{2} \left( \frac{P_{i,t}}{P_{i,t-1}} - \pi^* \right)^2 P_t y_t
\end{aligned}$$

In equilibrium, all firms behave symmetrically, and we have  $P_{i,t} = P_t$ . Thus, aggregate profits are given by:

$$\Pi_t^f = P_t y_t - W_t n_t - \frac{\phi^P}{2} (\pi_t - \pi^*)^2 y_t P_t$$

Their optimization problem is characterized by the following conditions, where  $MC_t$  is the multiplier of equation 2, and it is the marginal cost.

$$1 - \varepsilon + \varepsilon \frac{MC_t}{P_t} = \phi^P (\pi_t - \pi^*) \pi_t - \phi^P \mathbb{E}_t \left[ \mathcal{M}_{t,t+1} \frac{y_{t+1}}{y_t} \pi_{t+1}^2 (\pi_{t+1} - \pi^*) \right] \quad (5)$$

$$W_t = MC_t z_t \quad (6)$$

The first condition is the New-Keynesian Phillips curve.

### 3.3 Financial intermediaries

In this section, I follow a simplified version of the financial intermediaries' description in [Elenev et al. \(2021\)](#).

A representative agent in this sector starts the period  $t$  with a net worth of  $W_t^I$ . Every period, it pays a fraction  $\tau^I$  of its net worth to households and raises new equity from them,  $A_t$ . The net payout to shareholders is then:

$$Div_t = \tau^I W_t^I - A_t \quad (7)$$

This agent can invest in short-term public bonds  $B_t^{S,I}$  for the price  $Q_t^S$ .  $B_t^{S,I}$  is the sum of treasury bills (treasuries with a maturity of up to a year,  $B_t^S$ ) and central bank reserves ( $B_t^{S,CB}$ ). The reason for adding these assets into one variable is that they have the same risk and return properties in the model, being perfect substitutes from the financial intermediaries' point of view. Its liabilities are given by deposits from households  $D_t^I$ . The following expression then gives the balance sheet:

$$(1 - \tau^I)W_t^I + A_t - \Phi_A(A_t) + Q_t^D D_t^I = Q_t^S B_t^{S,I} \quad (8)$$

Where  $\Phi_A(\cdot)$  is a convex cost of issuing new equity, rebated lump-sum to Households. This cost prevents financial intermediaries from raising funds only through equity, going to a corner solution with  $D_t^I = 0$ .

The net wealth at period t is:

$$W_t^I = B_{t-1}^{S,I} - D_{t-1}^I \quad (9)$$

In line with Basel regulation, financial intermediaries are subject to a leverage constraint. It states that their debt (in this case, deposits) can be, at most, a fraction  $\zeta$  of its assets.<sup>12</sup>

$$D_t^I \leq \zeta B_t^{S,I} \quad (10)$$

The optimization problem of a financial intermediary consists of maximizing the discounted sum of dividends subject to restrictions 8, 9, and 10. I assume the financial intermediary discounts its future flows using the household's stochastic discount factor.

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<sup>12</sup>I assume the financial intermediary sector's assets are composed only of treasury bills and central bank reserves for simplicity. In a more complicated model, they could also purchase firms' bonds or provide loans to firms to finance capital purchases. This is the case in [Elenev et al. \(2021\)](#), and [Benigno and Benigno \(2021\)](#). The qualitative conclusions of this paper do not change when considering a more complicated version of the model, reducing to quantitative differences.

$$\begin{aligned}
& \max_{A_t, D_t^I, B_t^{S,I}} \mathbb{E}_0 \sum_{k=0}^{\infty} \mathcal{M}_{t,t+k} (\tau^I W_{t+k}^I - A_{t+k}) \\
& \text{s.t. } (1 - \tau^I) W_t^I + A_t - \Phi_A(A_t) + Q_t^D D_t^I = Q_t^S B_t^{S,I} \\
& \quad W_t^I = B_{t-1}^{S,I} - D_{t-1}^I \\
& \quad D_t^I \leq \zeta B_t^{S,I}
\end{aligned}$$

Define  $\eta_t$  as the Balance sheet multiplier and  $\mu_t$  as the Leverage constraint multiplier. Using the first order condition to equity  $A_t$  to substitute out the multiplier  $\eta_t$ , we obtain the system of equations that characterize the financial intermediary's optimization problem, together with 7, 8, 9, 10:<sup>13</sup>

$$Q_t^D = \mathbb{E}_t \tilde{\mathcal{M}}_{t,t+1} + \mu_t (1 - \Phi'_A(A_t)) \quad (11)$$

$$Q_t^S = \mathbb{E}_t \tilde{\mathcal{M}}_{t,t+1} + \zeta \mu_t (1 - \Phi'_A(A_t)) \quad (12)$$

Where  $\tilde{\mathcal{M}}_{t,t+1}$  is the stochastic discount factor for financial intermediaries, defined as:

$$\tilde{\mathcal{M}}_{t,t+1} \equiv \mathcal{M}_{t,t+1} (1 - \Phi'_A(A_t)) \left( \tau^I + \frac{1 - \tau^I}{1 - \Phi'_A(A_{t+1})} \right)$$

Notice that the spread between deposits and short-term bonds is a function of the leverage constraint multiplier  $\mu_t$ . Since  $\zeta < 1$ , the return on short-term bonds is higher than the short-term bonds when the leverage constraint binds. If the leverage constraint does not bind, then we get  $Q_t^S = Q_t^D$ , and from the Households' problem, this could only be possible in a situation where liquidity services are zero.

### 3.4 Monetary Authority

The Central Bank performs conventional and unconventional monetary policies. The conventional monetary policy sets the short-term nominal interest rate  $R_t$ , subject to a ZLB

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<sup>13</sup>See appendix, section 9.2 for further details.

restriction. This rate is the inverse of the short-term public debt price:

$$R_t \equiv \frac{1}{Q_t^S} \quad (13)$$

The unconventional monetary policy consists of central bank balance sheet policies. In particular, the central bank purchases long-term government debt  $B_t^{L,CB}$  in exchange for reserves ( $B_t^{S,CB}$ ), as in the data since the Great Recession. The rules for setting these two instruments are provided in a following section.

The following expression gives the Central Bank's budget constraint:

$$\frac{B_{t-1}^{S,CB}}{P_t} + \frac{B_{t-1}^{L,CB}}{P_t} [\kappa + (1 - \delta)Q_t^L] = Q_t^S \frac{B_t^{S,CB}}{P_t} + Q_t^L \frac{B_t^{L,CB}}{P_t} + \Lambda_t^{CB}$$

where  $\Lambda_t^{CB}$  are central bank remittances to the fiscal authority. I assume that profits  $\Lambda_t^{CB}$  are transferred to the fiscal authority.<sup>14</sup>

Finally, the central bank is subject to a revenue neutrality constraint, which is in line with the data. It states that when the central bank increases its assets by purchasing long-term bonds, it has to offset the operation by decreasing its net position of short-term assets.

$$Q_t^L B_t^{L,CB} + Q_t^S B_t^{S,CB} = 0 \quad (14)$$

When it performs QE, we have  $B_t^{S,CB} < 0$ , representing the increase in reserves issuance.

### 3.5 Fiscal Authority

The treasury consumes  $g_t$  and obtains resources from three different sources. It collects tax revenues from households in a lump-sum fashion,  $\tau_t$ , receives dividends from the central bank  $\Lambda_t^{CB}$ , and issues debt, whose total real value is  $\frac{B_t}{P_t}$ . This debt comprises short-term bonds

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<sup>14</sup>For simplicity, in this paper, I abstract from asymmetries in the transfer of the central bank's profits to the treasury, like the ones discussed in [Hall and Reis \(2016\)](#).

$B_t^S$  and long-term bonds  $B_t^L$ . The total debt issuance is:<sup>15</sup>

$$B_t = Q_t^S B_t^S + Q_t^L B_t^L \quad (15)$$

The period budget constraint of the fiscal authority, in real terms, is:

$$\underbrace{\tau_t - g_t}_{\equiv s_t} + \frac{B_t}{P_t} + \Lambda_t^{CB} = \frac{B_{t-1}^S}{P_t} + \frac{B_{t-1}^L}{P_t} [\kappa + (1 - \delta)Q_t^L]$$

Where  $s_t$  is the real primary fiscal surplus of period t. Replacing  $\Lambda_t^{CB}$  from the central bank balance sheet and using 14, I obtain a consolidated budget constraint:

$$s_t + \frac{B_t}{P_t} = \underbrace{\frac{B_{t-1}^S - B_{t-1}^{S,CB}}{P_t}}_A + \underbrace{\frac{B_{t-1}^L - B_{t-1}^{L,CB}}{P_t}}_B [\kappa + (1 - \delta)Q_t^L] \quad (16)$$

The terms 'A' and 'B' are the outstanding short and long-term public debt in the private's hands. Notice that through the purchases of long-term public bonds, the central bank relaxes the budget constraint for the consolidated government. However, this is not necessarily the case when considering the net position of short-term assets. For instance, if  $B_t^{S,CB} < 0$ , implying reserves issuance, then the consolidated government debt of short maturity is increasing with this policy. In this sense, quantitative easing policies can be interpreted as a maturity swap, exposing the government to interest rate risk.

The following expression gives government consumption:

$$g_t = \theta(y^* - y_t) + (1 - \rho_g)\bar{g} + \rho_g g_{t-1} + \sigma_g \varepsilon_t^g, \varepsilon_t^g \sim N(0, 1)$$

$0 < \theta < 1$  represents the government spending reaction to output deviations from its steady state. This term generates a counter-cyclical behavior of government consumption, introducing fiscal stimulus when the economy is in a recession. Government consumption goods are thrown into the ocean. Finally, the maturity composition of newly issued government debt is constant in book value terms, with a fraction  $\bar{\mu}$  of debt being long-term.

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<sup>15</sup>Notice that the real amount of short-term bonds is  $Q_t^S \frac{B_t^S}{P_t}$  and the one of long-term bonds is  $Q_t^L \frac{B_t^L}{P_t}$ .

$$\frac{B_t^S}{B_t^L} = \frac{1 - \bar{\mu}}{\bar{\mu}} \quad (17)$$

### 3.6 Market clearing conditions

Market clearing conditions are the following:

$$\begin{aligned} c_t + g_t + \frac{\phi^P}{2} (\pi_t - \pi^*)^2 y_t &= y_t \\ D_t^H &= D_t^I \\ B_t^S - B_t^{S,CB} &= B_t^{S,I} \\ B_t^L &= B_t^{L,H} + B_t^{L,CB} \end{aligned}$$

And households' rebates by other agents in the model are equal to:

$$\tilde{\Pi}_t = \Phi_L \left( b_t^{L,H} \right) + \Phi_A(A_t)$$

### 3.7 Policy rules

To close the model, I assume that fiscal and monetary authorities follow policy rules to set their instruments:  $\tau_t$ ,  $R_t$ , and  $b_t^{L,CB}$ . First, I assume that central bank purchases of long-term bonds follow an AR(1) process:

$$b_t^{L,CB} = (1 - \rho^{QE}) b_*^{L,CB} + \rho^{QE} b_{t-1}^{L,CB} + \sigma^{QE} \epsilon_t^{QE} \quad (18)$$

Where  $b_*^{L,CB}$  is the average amount of long-term bonds at the steady state. Increases in the central bank balance sheet are random and unrelated to the economic conditions. I follow this assumption for two reasons. First, this paper aims to study the effects of QE policies under different interactions of conventional fiscal and monetary policies. Second, there is no evidence or consensus of a clear rule for central bank purchases, a policy with a great discretionary component. This assumption implies that QE constitutes a complementary policy instrument of the central bank and not necessarily a substitute, in line with what we

observe in the data. Furthermore, it prevents the instrument from altering the determinacy properties of the model.

I follow the literature on fiscal-monetary policy interactions and assume that fiscal and monetary authorities follow rules to determine the conventional policy instruments. In particular, the nominal short-term interest rate follows the Taylor rule 20, reacting to output and inflation deviations from its steady state values, together with an autoregressive parameter  $\rho_R$  and a monetary shock  $\epsilon_t^m$  with standard deviation  $\sigma_t^m$ . The monetary shock stands for interest rate deviations from the reaction function. The intensity of interest rate reaction to output and inflation deviations are characterized by policy parameters  $\alpha_y$  and  $\alpha_\pi$ , respectively. Finally,  $\bar{R}$  is the mean of the nominal interest rate.

For fiscal policy, I assume that taxes  $\tau_t$  react to total real debt deviations from its steady state value  $b^*$  and have an autoregressive coefficient  $\rho_\tau$ , as in 19. The elasticity of tax deviations to debt deviations is characterized by the parameter  $\gamma$ .<sup>16</sup>

$$\tau_t - \tau^* = \rho_\tau (\xi_t) (\tau_{t-1} - \tau^*) + (1 - \rho_\tau (\xi_t)) \gamma (\xi_t) (b_{t-1} - b^*) \quad (19)$$

$$\frac{R_t}{\bar{R}(\xi_t)} = \left( \frac{R_{t-1}}{\bar{R}(\xi_t)} \right)^{\alpha_R(\xi_t)} \left[ \left( \frac{\pi_t}{\pi^*} \right)^{\alpha_\pi(\xi_t)} \left( \frac{y_t}{y^*} \right)^{\alpha_y(\xi_t)} \right]^{1-\alpha_R(\xi_t)} e^{\sigma_M(\xi_t)\epsilon_t^M} \quad (20)$$

The parameters above depend on a discrete shock  $\xi_t$  that follows a Markov process. I follow Bianchi and Melosi (2017) and assume that this shock can take three values, representing the three regimes through which the economy fluctuates.

The first regime, the *monetary-led (M)* regime, is characterized by a strong interest rate reaction to inflation deviations (high  $\alpha_\pi$ ) and a strong tax reaction to debt (high enough  $\gamma$ ).<sup>17</sup> This regime is associated to an *active monetary policy* and *passive fiscal policy*, in Leeper (1991) terminology. Under this regime, the monetary authority adjusts the nominal

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<sup>16</sup>Assuming the fiscal rule reacts to real public debt in private hands, i.e.,  $Q_t^L b_t^{L,H} + Q_t^S b_t^{S,I}$  does not change the conclusions in this paper. These results are available under request.

<sup>17</sup>In Appendix, I present, for a given calibration, the combinations of values  $\alpha_\pi$  and  $\gamma$  that give rise to determinacy at each regime. Notice, however, that the global stability of the system does not require determinacy at each regime.

interest rate more than proportionally to changes in the inflation rate to stabilize inflation. At the same time, the treasury passively adjusts taxes to stabilize the real debt.

The second regime is the *fiscally-led regime (F)* where the fiscal authority does not adjust taxes enough to stabilize the real debt (low  $\gamma$ ). Then, the central bank allows the inflation rate to deviate from the target to stabilize debt in real terms. This regime can be defined as *passive monetary policy* and *active fiscal policy* in the sense of [Leeper \(1991\)](#).

The third regime is the *Zero Lower Bound regime (ZLB)*. It is characterized by an unreactive nominal short-term interest rate that remains fixed at its effective lower bound and by a fiscal policy that is unreactive to the real debt level. This regime is an extreme form of the fiscally led regime since we have  $\alpha_\pi = \gamma = 0$ . As in [Bianchi and Melosi \(2017\)](#), this regime represents a *crisis regime*, where the economy enters due to the realization of bad shocks that drive the economy to a recession. However, differently from [Bianchi and Melosi \(2017\)](#), the economy enters the ZLB regime endogenously, as I explain in the next section.

### 3.8 Transition probabilities

Assume the Markov-switching shock  $\xi_t$  depends on the realization of two random variables  $\xi_t^P$  and  $\xi_t^C$ . When  $\xi_t^C = 1$ , the economy suffers a crisis and moves to the ZLB regime. When  $\xi_t^C = 0$ , the government can set fiscal and monetary policies without restriction. In this case, the variable  $\xi_t^P$  determines the regime in place stochastically.  $\xi_t^P = M$  stands for the monetary-led regime,  $\xi_t^P = F$  for the fiscally-led regime, and it evolves according to the transition matrix:

$$P = \begin{bmatrix} p_{mm} & 1 - p_{mm} \\ 1 - p_{ff} & p_{ff} \end{bmatrix}$$

where  $p_{ij} = P(\xi_{t+1}^P = j | \xi_t^P = i)$ .

As seen from the transition matrix  $P$ , the probability of being in one regime or another is constant and entirely exogenous. The realization of this shock represents, in a simplified way, the outcome of a policy game between the fiscal and the monetary authority, where the winner is the *active* authority.



Define  $q$  as the probability of entering to the ZLB regime,  $q = P(\xi_{t+1}^C = 1 | \xi_t^C = 0)$  and  $r$  as the probability of moving out of the crisis regime  $q = P(\xi_{t+1}^C = 0 | \xi_t^C = 1)$ .

The transition matrix for  $\xi_t$  is then:

$$T = \begin{bmatrix} (1 - q)P & q[1; 1] \\ r[p_{zm}; (1 - p_{zm})] & (1 - r) \end{bmatrix}$$

Where  $p_{zm}$  is the probability of exiting the ZLB regime towards a monetary-led regime.  $q$  and  $r$  are endogenous processes. The probability of entering the ZLB regime is a decreasing function of the nominal interest rate:

$$q = P(\xi_{t+1}^C = 1 | \xi_t^C = 0) = f(R_t)$$

Intuitively, it is a function that generates zero probability of switching when the gross nominal interest rate is higher than one, and it increases as  $R_t$  approaches the value of 1.

The probability of leaving the crisis regime,  $r$ , is a function of a shadow interest rate  $R_t^S$ :

$$r = P(\xi_{t+1}^C = 0 | \xi_t^C = 1) = g(R_t^S)$$

The shadow interest rate represents the interest rate that would hold in the economy if this were always in the monetary-led regime, without a lower bound restriction:

$$\frac{R_t}{\bar{R}(\xi_t^P = M)} = \left( \frac{R_{t-1}}{\bar{R}(\xi_{t-1}^P = M)} \right)^{\alpha_R(\xi_t^P = M)} \left[ \left( \frac{\pi_t}{\pi^*} \right)^{\alpha_\pi(\xi_t^P = M)} \left( \frac{y_t}{y^*} \right)^{\alpha_y(\xi_t^P = M)} \right]^{1 - \alpha_R(\xi_t^P = M)} e^{\sigma_M(\xi_t^P = M)\epsilon_t^M} \quad (21)$$

This assumption reflects that the probability of leaving the ZLB regime is not independent of the economic conditions. For instance, when output and(or) inflation recovers, the Shadow interest rate increases, increasing the likelihood that the central bank would start raising interest rates.

Endogenous transition probabilities to and out of the ZLB matter for agents' expectations. Contrary to an entirely exogenous transition matrix model, agents know the likelihood of tightening monetary policy increases with output and inflation, even at the ZLB.

## 4 Functional forms, calibration and solution

### 4.1 Functional forms

In this section, I present the functional forms assumed in the numerical exercise. I assume the following CRRA utility function for households, which depends positively on consumption and deposits and negatively on labor.

$$U(c_t, d_t^H, n_t) = \frac{\left[ c_t^{1-\varphi} (d_t^H)^\varphi \right]^{1-\sigma}}{1-\sigma} - \psi \frac{n_t^\eta}{\eta}$$

Where  $d_t^H = \frac{D_t^H}{P_t}$  are deposits deflated by the price level.

The portfolio adjustment cost for long-term bonds is quadratic:

$$\Phi_L \left( b_t^{L,H} \right) = \frac{\phi_L}{2} \left( \frac{b_t^{L,H}}{b^{L,H}} \right)^2$$

where  $b_t^{L,H} = \frac{B_t^{L,H}}{P_t}$ , and  $b^{L,H}$  is the steady state value of public long-term bonds held by households.

The convex cost of issuing equity is the following:

$$\Phi_A(A_t) = \frac{\chi}{2} \frac{A_t^2}{P_t}$$

Finally, the endogenous transition probabilities to and out of the ZLB regime are assumed to follow a logistic distribution as in [Benigno et al. \(2020\)](#), and [Bocola \(2016\)](#). As in their models, the economy's transition between regimes is a logistic function of a subset of the model's endogenous variables. In this case, they are given by the following expressions:

$$q = P(\xi_{t+1}^C = 1 | \xi_t^C = 0) = \frac{\exp \{ -\gamma^q (R_t - 1) \}}{1 + \exp \{ -\gamma^q (R_t - 1) \}}$$

where  $\gamma^q > 0$  is a constant. And,

$$r = P(\xi_{t+1}^C = 0 | \xi_t^C = 1) = \frac{\exp \{ -\gamma^r (R_t^S - 1) \}}{1 + \exp \{ -\gamma^r (R_t^S - 1) \}}$$

with  $\gamma^r < 0$ .

## 4.2 Calibration

I work with quarterly data for the US. In this section, I present the calibration. In table 1, I present the calibration of structural parameters with their source or target. Some parameters have direct counterparts in the data. For instance,  $\bar{R}$ , the average gross short-term nominal interest rate is set equal to 1.011, as in quarterly data for the period 1980-2020.<sup>18</sup> The average of long-term public bonds purchased by the central bank,  $b_*^{L,CB}$  is set to 0.014 to match the average 7% annual ratio of Federal Reserve total treasuries to output ratio, in market value terms, for the period 1980-2020. Finally, the parameter that characterizes the collateral ratio in the financial intermediaries' leverage constraint,  $\zeta$ , is set to 0.97 as in the Basel regulation.<sup>19</sup>

Some parameters are taken from the literature. For instance, the risk aversion parameter  $\sigma$  is set to 2, as is standard in the literature. The inverse of Frisch elasticity  $\eta$ , is set to 3 as in [Leeper et al. \(2021\)](#),  $\bar{\mu}$ , that is the proportion of long-term bonds in book values issued by the treasury, is 0.67 as in [Elenev et al. \(2021\)](#) and  $\theta$ , that is 0.27 as in [Bianchi and Melosi \(2017\)](#). The convex cost of issuing equity for financial intermediaries,  $\chi$  is 22 as in [Elenev et al. \(2021\)](#). The coupon payment of long-term bonds,  $\kappa$ , that includes the interest and the matured part fraction of bonds is normalized to 1.

Other parameters are calibrated to match first-order moments in the data.  $\beta$  takes the value 0.996 to match the average return of a real risk-free asset from [Jordà et al. \(2017\)](#).  $\delta$  is set to 0.0357 to roughly match the average maturity of public bonds with a maturity longer than one year, seven years. The parameter that characterizes the quadratic portfolio adjustment cost of long-term bonds,  $\phi_L$  is calibrated to 0.0039 to match the average spread between long and short-term bonds of 0.32% in the period 1980Q1-2020Q1 at the steady state.<sup>20</sup>

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<sup>18</sup>Notice that, in the data, this value corresponds to the average quarterly nominal interest rate, including periods where the interest rate was at the effective lower bound.

<sup>19</sup>In the data, the balance sheet of financial intermediaries includes a broader set of assets that can be used as collateral, than the ones included in the model (T-bills and central bank reserves). This assumption, although restrictive since it introduces a tight relationship between deposits and reserves, is maintained for simplicity.

<sup>20</sup>This spread in the model is the difference between the return on the long-term bond  $R^L = \frac{\kappa + (1-\delta)Q^L}{Q^L}$  and the short-term interest rate .

	Description	Value	Source or target
$\sigma$	Risk aversion	2	Standard
$\eta$	Inverse Frisch elasticity	3	Leeper et al. (2021)
$\kappa$	Coupon Payment	1	Normalization
$\chi$	Equity cost	22	Elenev et al (2021)
$\theta$	Government spending	0.27	Bianchi and Melosi (2017)
$\zeta$	Leverage constraint FI	0.97	Basel regulation
$\psi$	Preference parameter	1.339	Normalization labor
$\beta$	Discount factor	0.996	Av. Inflation rate
$R^*$	Average interest rate	1.011	Av. E. Fed Funds
$\bar{\mu}$	Proportion of long-debt	0.67	Elenev et al. (2021)
$\delta$	Maturity parameter	0.0357	Maturity long bonds (7 years)
$\phi_L$	Portfolio adjustment cost	0.004	10-year yield
$\varphi$	Preference parameter	0.0023	Debt/GDP $\frac{b}{4y} = 68\%$
$\tau^I$	Dividends distribution	0.84	Spread T-bill to deposits
$\phi^P$	Prices adjustment cost	150	Inflation volatility
$\epsilon$	Elasticity of subst. varieties	7	Markup 17%
$b_*^{L,CB}$	Average CB Balance sheet	0.0140	$\frac{Q^L b_*^{L,C,B}}{4y} = 7\%$

Table 1: Calibration: model parameters

The preference parameter  $\varphi$  is set to 0.0023 to generate a simulated mean of annualized debt to GDP ratio as in the average data before the COVID-19 crisis, around 70% until 2020Q1. The fraction of financial intermediaries' wealth paid to households as dividends is calibrated to 0.84 to match the average spread between short-term interest rate and deposits of 0.31% from Drechsler et al. (2017)<sup>21</sup>. The elasticity of substitution between varieties  $\epsilon$  is set to 7, generating an average markup of 17%, and the parameter  $\phi^P$  that characterizes Rotemberg adjustment costs is set to 150 to roughly match the average inflation standard deviation during the period 1980-2020. Finally, the preference parameter  $\psi$  is calibrated to normalize labor to one at the steady state.

Table 2 presents the calibration for persistence and standard deviation of the exogenous processes in the model. They were jointly calibrated to match some second-order moments in the data for the period 1980-2020. The comparison between moments in the data with the simulated moments is presented in the following section.

<sup>21</sup>The model does not prevent the net return on deposits to be negative at the zero lower bound regime. This could be motivated by the data, with the fact that during this period, although banks did not charge fees on deposits, some of them increased their account maintenance charges to customers.

Parameter	Description	Value
$\rho_{QE}$	Persistence QE	0.9
$\rho_\nu$	Persistence preference	0.89
$\rho_z$	Persistence TFP	0.92
$\rho_G$	Persistence gov. spending	0.96
$\sigma_{QE}$	Dispersion QE	0.0025
$\sigma_\nu$	Dispersion preference	0.008
$\sigma_z$	Dispersion TFP	0.0021
$\sigma_G$	Dispersion gov. spending	0.0026

Table 2: Calibration: exogenous processes

Table 3 presents the regime-switching parameters correspondent to the fiscal and monetary policy rules from section 3.7. The first set of parameters corresponds to the Taylor rule 20, and the correspondent parameters at each regime. The second set corresponds to the parameter values for the Shadow interest rate 21. Parameters in the Shadow interest rate are equal to the Taylor rule’s parameters at the monetary regime, and they are not regime-dependent. They were included in the table for completeness. The third set of parameters in this table corresponds to the fiscal rule 19. The parameters in this table, with the exception of mean interest rates in all regimes, come from Bianchi and Melosi (2017). Since a parameter estimation goes beyond the scope of this paper, I take the parameters correspondent to conventional fiscal and monetary policy rules from this article that presents the same regimes and performs a Bayesian Estimation of the correspondent parameters using data for the US until the Great Recession. The average interest rate out of the ZLB is  $\bar{R}$ , explained in table 1. At the zero lower bound, I set the average interest rate to 1.0005, as the average quarterly Effective federal funds rate observed in the period: 2008Q4-2017Q1 and 2020Q1-2022Q1.

	Description	MD	FD	ZLB
$\alpha_R$	Taylor rule	0.86	0.67	0.2
$\alpha_\pi$	Taylor rule	1.6	0.64	0
$\alpha_y$	Taylor rule	0.51	0.27	0
$\sigma^M$	Taylor rule	0.25/100	0.25/100	0.25/1000
$R$	Taylor rule	$R^*$	$R^*$	1.0005
$\alpha_{R,s}$	Shadow R	-	-	0.86
$\alpha_{\pi,s}$	Shadow R	-	-	1.6
$\alpha_y$	Shadow R	-	-	0.9
$\sigma^{M,s}$	Shadow R	-	-	0.0025
$R^S$	Shadow R	$R^*$	$R^*$	$R^*$
$\gamma$	Fiscal rule	0.0712	0	0
$\alpha_\tau$	Fiscal rule	0.96	0.69	0.69

Table 3: Calibration: regime-dependent policy parameters .

Table 4 presents parameters relative to transition probabilities between regimes. Exogenous parameters from matrix P,  $p_{mm}$  and  $p_{ff}$  come from [Bianchi and Melosi \(2022\)](#). The probability of exiting the ZLB towards a monetary-led regime,  $p_{zm}$ , is set to 0.7031, and it comes from [Bianchi and Melosi \(2022\)](#). In this paper, the authors show that this probability significantly decreased after the COVID-19 crisis. Since this paper aims to study the exit strategies from the crisis and policies applied during that period, I considered the latest value of this estimated probability. However, the results do not significantly differ in a model with lower  $p_{zm}$ .

Parameters  $\gamma^r$  and  $\gamma^q$  give the steepness of the logistic function. They were calibrated to obtain a similar ergodic probability of the ZLB regime as in [Bianchi and Melosi \(2022\)](#) and to minimize the cases in which the economy is at this regime with a gross interest rate below one.

Parameter	Value	Source or target
$p_{mm}$	0.9923	<a href="#">Bianchi and Melosi (2022)</a>
$p_{ff}$	0.9923	<a href="#">Bianchi and Melosi (2022)</a>
$p_{zm}$	0.7031	<a href="#">Bianchi and Melosi (2022)</a>
$\gamma^q$	500	Average prob. of ZLB regime
$\gamma^r$	-200	Average prob. of ZLB regime

Table 4: Calibration: transition probabilities

Figure 4 shows the corresponding probabilities for values of the nominal interest rate (left) or shadow interest rate (right) between 0.95 and 1.02. A positive value for  $\gamma^q$  generates a higher probability of entering the ZLB when the interest rate is below one and lower when it is above one.

Notice that for this calibration, the probability of entering this regime when  $R$  is 0.987 or lower is one.

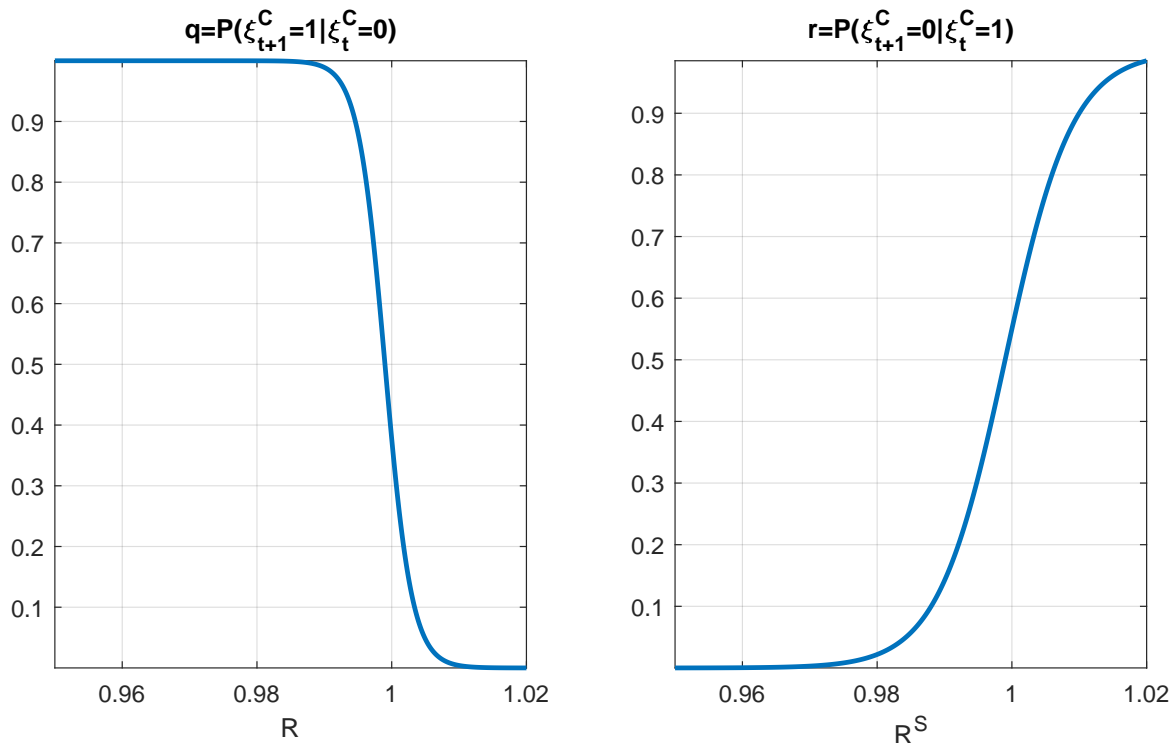


Figure 4: Endogenous transition probabilities to and out the zero lower bound.

For the probability of exiting the zero lower bound, the parameter  $\gamma^r$  is negative, generating a monotone increasing probability of leaving the crisis regime when the Shadow interest rate is higher.

### 4.3 Solution method

I solve the model in real terms through second-order perturbation methods for endogenous Markov Switching DSGE models, following [Benigno et al. \(2020\)](#). In the solution, I assume the leverage constraint for financial intermediaries is binding in all regimes. The approximation point for the solution method consists of a weighted average of steady states in different regimes, as authors in [Benigno et al. \(2020\)](#) explain. The weights are the ergodic means of corresponding regimes. In [appendix 9.4](#), I present the steady state equations and describe the approximation point.

## 5 Quantitative Results

### 5.1 Second order moments

In this section, I present some second-order moments for selected variables to evaluate how the model performs in terms of generated volatility and cyclical properties. Data variables were demeaned to make them comparable with their model counterpart, where there is no growth. Empirical moments were calculated using quarterly data for the period 1980Q2-2020Q1. NIPA variables are real and per capita. Debt, inflation, and interest rate are annualized, both in the model simulations and in the data. Data moments were calculated from a simulation of one million periods, and they correspond to the averages of the three regimes.

The following table compares the standard deviation (in %) and correlation with output growth. Output and public debt are logarithmic differences.

	$\Delta$ GDP	$\Delta$ public debt	Bal. Sheet	Inflation	Spread
Standard deviation (in %)					
Data	0.8	1.7	3.7	2.8	1.6
Model	0.7	1.5	3.1	2.4	1.5
Correlation with $\Delta$ GDP					
Data	-	-0.33	0.71	0.44	-0.11
Model	-	-0.27	0.22	0.25	-0.04

Selected second-order moments in data and model

Note: Growth rates for output and debt. Inflation is gross and annualized. The term spread is the annualized difference between the 10-year treasury yield and the federal funds rate. Model moments were obtained from a simulation with one million periods.

As can be seen from the table, the model generates the correct ranking in volatilities and standard deviations similar to the ones in the data for the main endogenous variables. In terms of cyclical properties, the model generates the correct sign in correlation with output for all the variables in the table and close magnitudes to the empirical ones.

### 5.2 Conditional second order moments at different regimes

Table 5 show means and volatility measures for debt to GDP ratio, inflation, nominal short-term interest rate ( $R_t$ ) and nominal return on long-term bonds ( $R_t$ ), conditional on regimes. They reflect



features typically highlighted by the literature that studies fiscal-monetary policy mix.<sup>22</sup> The fiscal regime is characterized by higher debt to GDP ratio, interest rates, and more volatile inflation and interest rate. Real debt to GDP, on the contrary, is more volatile in the monetary-led regime. The ZLB is characterized by a very stable short-term interest rate close to one and almost nil inflation but quite volatile (standard deviation 2.7%).

	M		F		ZLB	
	Mean	Std(%)	Mean	Std(%)	Mean	Std(%)
Debt to GDP	72%	7.0	80%	3.8	74%	5.9
Inflation	1.02	1.7	1.02	3.8	1.01	2.7
Interest rate (R)	1.03	1.5	1.04	2.5	1.00	0.2
Long-run return ( $R^L$ )	1.04	1.5	1.05	3.1	1.02	1.8

Table 5: Data moments conditional on regimes

Note: Data generated moments, from a sample of one million periods. The model is simulated for a long sample where the regime is stochastic. Moments at each regime are obtained conditioning the economy being on the corresponding regime at a given period. Debt to GDP is  $\frac{b}{4y}$ , inflation, and returns are annualized since the model is solved quarterly.

## 6 The transmission mechanism of Quantitative Easing

This section sheds light on the model’s transmission mechanism of an increase in central bank bond purchases,  $b_t^{L,CB}$ , given by an exogenous shock  $\epsilon_t^{QE}$ , following 18. I show the log deviations (in %) of a simulated path for endogenous variables when there is a one standard deviation shock in the central bank purchases of long-term bonds to a counterfactual without the shock. This shock implies increasing the real balance sheet to GDP ratio by 1.3p.p., i.e., rising from its steady state of 7% to 8.3%, a conservative shock. I consider three different scenarios, where the economy is at a given regime and remains at the same regime for 16 quarters in both paths (with and without the shock). Even though there is no regime change in the simulation exercise, agents in the economy expect the economy to evolve according to the transition matrix 3.8.

<sup>22</sup>See, for instance Bianchi et al. (2020), where the authors estimate a Markov-Switching VAR with three regimes for the period 1960-2014.

## 6.1 Regime dependent Quantitative Easing shock

Figures 5 and 6 show the evolution of endogenous variables at monetary-led regime (continuous blue line, named ‘M’), fiscally-led regime (dotted red line, named ‘F’) and ZLB regime (dashed yellow line, named ‘ZLB’). They are presented as log deviations from a path without an increase in central bank purchases, in percentage<sup>23</sup>.

When the central bank increases its purchases of long-term bonds ( $b_t^{L,CB}$ ), it increases the size of its balance sheet, defined as  $b_t^{L,CB} Q_t^L$ . Due to the revenue neutrality constraint, it does it through the issuance of reserves to financial intermediaries. QE transmission mechanism operates through two channels in this model. First, when the central bank introduces reserves in the intermediary sector, it relaxes its leverage constraint 10, allowing them to increase the deposit supply to households.<sup>24</sup> Second, the central bank intervention in the long-term bond market drives its price up, and then the long-term yield ( $\mathbb{E}_t R_{t,t+1}^L$ ) falls. This price increase gives households incentives to rebalance their portfolio, selling their long-term bonds and exchanging them for deposits. These mechanisms operate as a demand shock in this economy. The portfolio revaluation effect provides households with a wealth effect, increasing consumption. The fall in savings returns generates a substitution effect from savings to consumption. Both channels operate, increasing aggregate demand in this economy.<sup>25</sup>

The increase in aggregate demand is inflationary. Since firms operate under monopolistic competition, they can react in two ways to the rise in demand: either increase prices or produce more. In the first case, they have to pay a price adjustment cost due to the presence of nominal rigidities in this model. In the second case, given a constant TFP, they need to hire more labor. This pressure

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<sup>23</sup>Notice that, as shown in the previous section, the ergodic mean of endogenous variables differs among regimes. In the absence of shocks, the resting point of these variables would be the ergodic mean. For this reason, I show the log deviations from the correspondent mean, not from the steady state, broadly defined as an equilibrium without shocks.

<sup>24</sup>Given their leverage constraint, financial intermediaries increase deposits for a fraction  $\zeta$  of the new amount of reserves and raise equity from their shareholders, households, for the remainder fraction, net of cost of issuing equity.

<sup>25</sup>A complementary interpretation for the substitution between consumption and saving decision in the model comes from looking at the "natural interest rate" as defined in Benigno and Benigno (2022). As the authors highlight in their paper, a key reason why QE policies have real effects on the economy is that the policy rate differs from the natural interest rate. The later is the one that drives consumption/savings decision, defined as  $R_t^N = \frac{1}{\mathbb{E}_t \mathcal{M}_{t,t+1}}$ , i.e., the return on a risk-free private asset assumed to be in zero net supply in this paper. When the central bank increases its purchases of long-term bonds, the natural interest rate falls, even though it is not under the direct control of the central bank. This shifts households' savings towards consumption and explains the increase in aggregate demand. The most significant fall in this variable occurs at the ZLB, where we can see that QE is more expansionary.

on labor markets generates an increase in real wages and, thus, in firms' marginal costs. We see a combination of these effects in equilibrium, so salaries, labor, and output also increase.

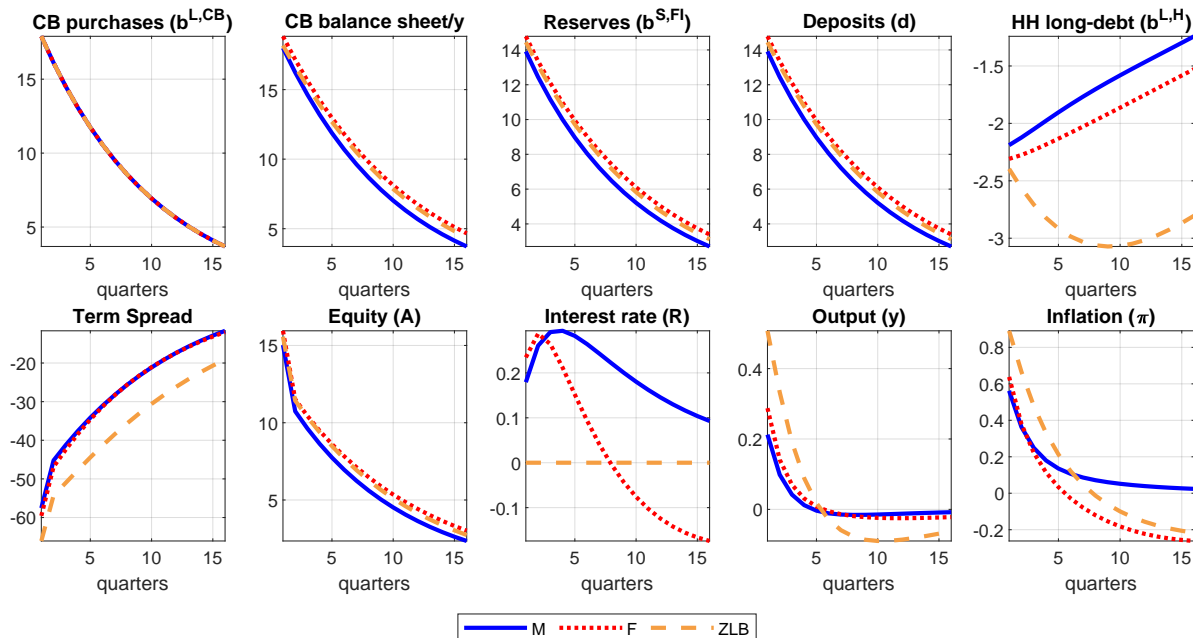


Figure 5: Impact of a Quantitative Easing shock conditional on a regime

Note: log deviations (in %) in a simulated path with a one standard deviation shock in the central bank purchases of long-term bonds ( $\epsilon_t^{QE} = 1$ ) to the counterfactual path without shock ( $\epsilon_t^{QE} = 0$ ). Path conditional on the monetary-led regime (continue blue line), fiscally-led regime (dotted red line), and zero lower bound regime (dashed yellow line) for 16 periods. The balance sheet to output ratio, interest rates, and inflation are annualized.

The intensity of these expansionary effects critically depends on the regime, being more expansive at the ZLB and, in second place, in the fiscally-led regime. At the ZLB, the fixed short-term interest rate exacerbates the substitution effect between savings and consumption. The short-term nominal interest rate increases in the fiscal and monetary-led regimes, following the corresponding Taylor rule. This return increase ameliorates the expansionary and inflationary effects of an increase in central bank purchases, especially in the monetary-led regime where the real short-term interest rate becomes positive.

Figure 6 shows the fiscal variables. Government debt behavior depends on the evolution of the three components of the government budget constraint: fiscal surplus, central bank remittances to the treasury, and debt service. First, the expansionary effect of central bank purchases triggers a

fall in government spending due to automatic stabilizers. Hence, primary fiscal surplus increases on impact in all regimes. This effect is more significant in the ZLB, where output increases more. Central bank profits ( $\Lambda_t^{CB}$ ) increase from period two in all scenarios, alleviating the pressure on fiscal accounts. Finally, total debt service behavior differs among regimes. Initially, it falls due to larger fiscal surpluses and remittances from the central bank, allowing the treasury to issue less debt ( $b_t^L, b_t^S$ ). This effect is larger in the ZLB and fiscally led regime, where higher inflation rates give the government seigniorage revenues. However, the revaluation of long-term bonds is so strong in the fiscally-led regime that it dominates the effect on real debt ( $b_t$ ), which increases.

The final result is an increase in fiscal space, interpreted as a fall in real debt ( $b_t$ ) at the ZLB and the monetary regime. This effect is milder in the second, where taxes fall following debt issuance. In the fiscally-led regime, however, the revaluation effect on long-term bonds is so strong that the debt ratio to GDP increases.

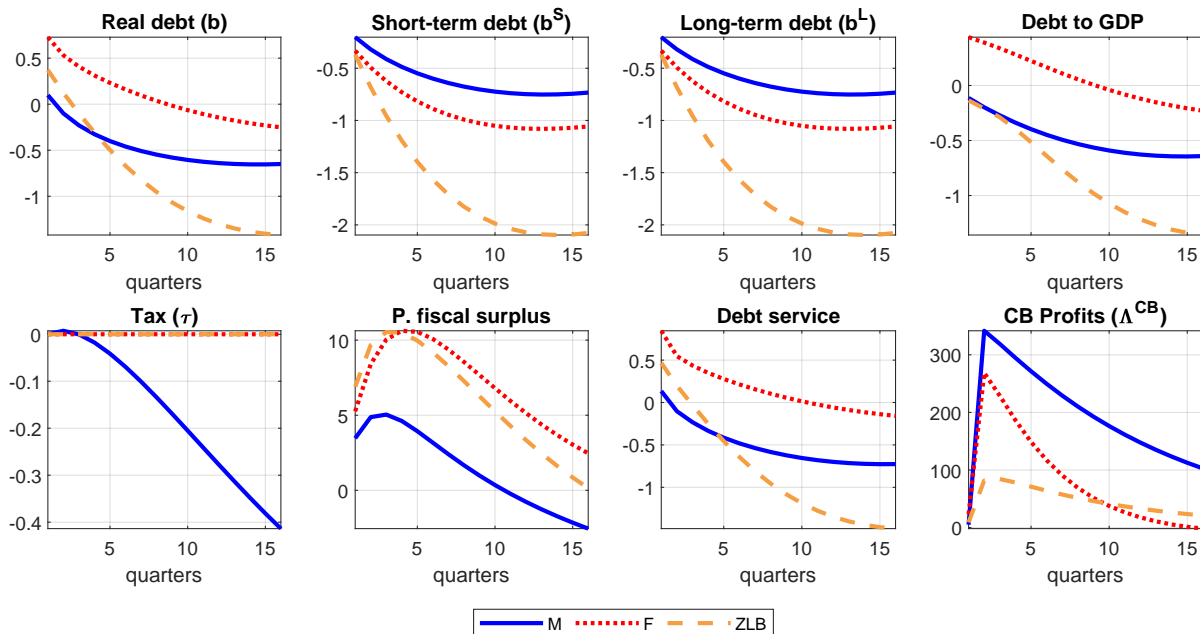


Figure 6: Impact of a Quantitative Easing shock conditional on a regime: fiscal variables

Note: log deviations (in %) in a simulated path with a one standard deviation shock in the central bank purchases of long-term bonds ( $\epsilon_t^{QE} = 1$ ) to the counterfactual path without shock ( $\epsilon_t^{QE} = 0$ ). Path conditional on the monetary-led regime (continue blue line), fiscally-led regime (dotted red line), and zero lower bound regime (dashed yellow line) for 16 periods. Debt to GDP is annualized.

This exercise shows that the expansionary effects of QE policies are of considerable size under a crisis regime. At the ZLB, 1.3p.p. temporal increase in the central bank balance sheet increases output by almost 0.6 p.p. and inflation 0.9p.p. to the counterfactual scenario without intervention. However, its effects on output and inflation are milder in fiscal and monetary dominance regimes, where output increases by 0.3 and 0.2 p.p. and inflation by 0.6 and 0.5p.p., being considerably less persistent in the first case. This analysis provides some insights into the power of QE to expand the economy and how it differs among regimes. Furthermore, it highlights the fact that the fiscal implication of this policy entails substantial differences across policy scenarios.

## 7 Exit strategies from Quantitative easing programs

As stated by policymakers, financial analysts, and academics, there is high uncertainty regarding the impact of implementing Quantitative Easing programs and, even more, the unwinding of the economic stimulus introduced through these measures. The main objective of this paper is to contribute to understanding the macroeconomic effects of different strategies regarding the size of the central bank balance sheet. In this section, I simulate the economy to generate a crisis that resembles the one in the US from the first quarter of 2020 when the COVID-19 pandemic became widespread worldwide. First, I show the model can generate a crisis with similar characteristics to the data: a substantial fall in output and short-term interest rate until it reaches the zero lower bound, together with a rise in the debt-to-GDP ratio and a fall in the inflation rate. In this context, I simulate a Quantitative Easing program that increases the ratio of the central bank balance sheet over output by 10p.p. and compare the macroeconomic dynamics with and without this program. Then, I study three different strategies for managing the central bank balance sheet size: 1) maintaining the enlarged size after the crisis (*Tapering*), 2) decreasing the balance sheet size: a) by letting the bonds that mature, run off the balance sheet (*Quantitative Tightening (QT)*), and b) at a faster speed by selling bonds (*Aggressive QT*).

*Tapering* is defined as a situation where the stock of long-term bonds at the central bank ( $b_t^{L,CB}$ ) remains constant, implying  $b_t^{L,CB} = b_{t-1}^{L,CB}$ . With that objective, the central bank engages in new purchases of bonds to make up for maturing bonds. The size of the central bank balance sheet over output ratio is, however, endogenous since it is affected by the price of long-term bonds ( $Q_t^L$ ) and output  $y_t$ , which are not under the control of the central bank.

*Quantitative tightening* is a strategy where the central bank does not repurchase the bonds that mature every period, letting them run off the balance sheet. The stock of bonds at the central bank will then decrease at rate  $\delta$ , the average maturity of long-term bonds, the unique kind of bonds the monetary authority purchases in the model. This implies the stock of bonds follows:  $b_t^{L,CB} = (1 - \delta) b_{t-1}^{L,CB}$ .

Finally, the strategy called *Aggressive QT* consists of the central bank actively selling public bonds. It is any path of shocks that generates  $b_t^{L,CB} < (1 - \delta) b_{t-1}^{L,CB}$ , implying that the stock of bonds at the central bank decreases at a faster rate than the maturity rate.

## 7.1 The crisis development and the QE program

I simulate the model in 50,000 samples of 40 periods under two scenarios, “Baseline” and “Quantitative Easing,” with the following characteristics. In the baseline, the economy is at the approximation point at  $t=1$ , and in the monetary-led regime. During periods 2 to 4, the economy is hit by strong negative preference and TFP shocks. Since both shocks are persistent, they remain below their steady state value for the whole sample. From  $t=2$  onward, the regime at place is stochastic, following the transition matrix presented in section 3.8. This scenario makes QE, monetary policy, and fiscal policy shock random. After the initial hit during periods 2 to 4, preference and TFP shocks follow random paths.

The quantitative easing scenario shares the same characteristics as the baseline, except that the path for QE shocks is not random. It is imposed to generate an increase in the annualized central bank balance sheet to 20% of GDP. in the first six periods. After period 6, the balance sheet remains constant (tapering strategy).

Figures 7 and 8 show the mean of simulated variables for the first 20 periods for the economy without QE (continuous blue line) and with QE (dotted orange line). The negative preference and technology shocks generate a substantial fall in output growth that reaches its bottom of -8.8p.p. in period four. Besides, it does not fully recover for eight periods. The short-term interest rate starts falling immediately until it reaches the ZLB. The primary fiscal surplus falls by almost 6p.p. in the baseline scenario.<sup>26</sup>

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<sup>26</sup>This exercise aims to analyze the effects of the quantitative easing program. This baseline exercise excludes the increase in fiscal transfers that took place during that period of turmoil. In the model, it can be easily generated through a fiscal shock in  $g_t$ , generating more significant primary fiscal deficits.

The main features of the crisis with quantitative easing are the following. First, there is a slightly milder fall in output than without the program. Second, there is a faster recovery within less than a year. QE also has expansionary effects in terms of inflation. Prices fall less during the crisis than in a scenario without the intervention of the central bank, however, this is at the cost of more important inflationary dynamics in the recovery. Inflation reaches a peak of 8.4% in period nine, two periods after the balance sheet reaches its maximum value of 20.8% of GDP. Without this program, inflation increases less during the recovery and reaches a peak of 6.9% three periods later.

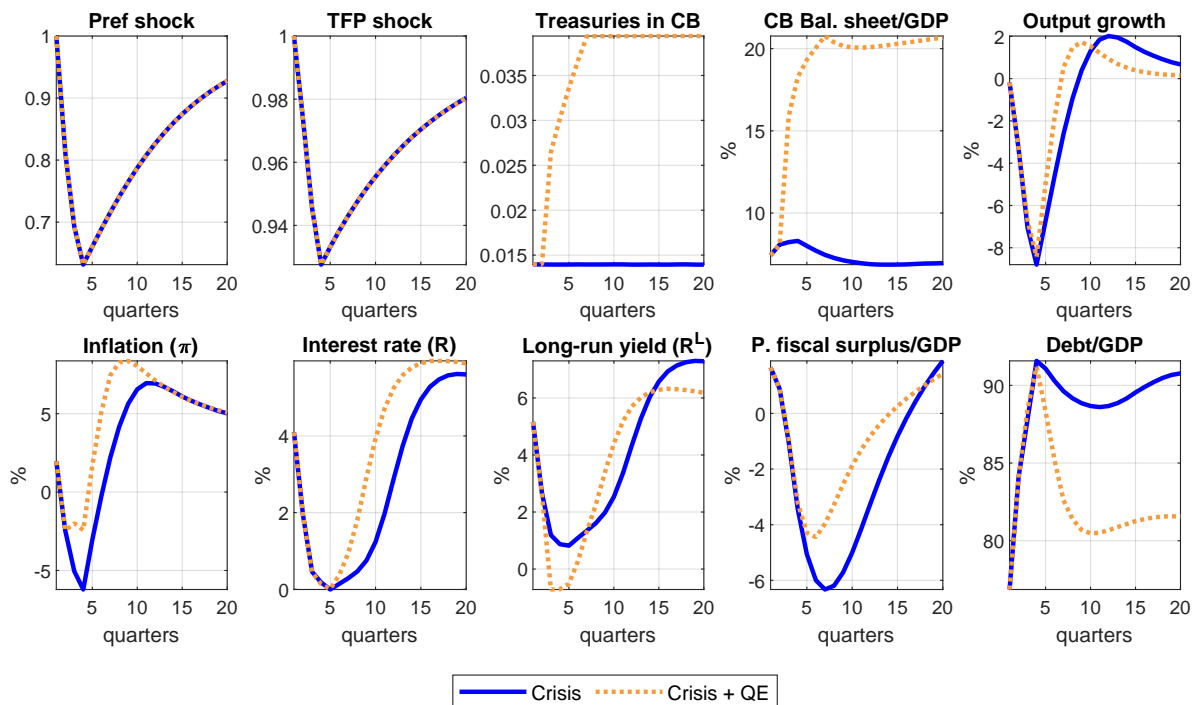


Figure 7: Simulated crisis

Note: Average from 50,000 samples. The continuous blue line is the scenario without the QE program. The Orange dotted line is the scenario with a QE program that increases the annualized central bank balance sheet to output by around 13p.p. Interest rate ( $R$ ), long-run return ( $R^L$ ), and inflation are annualized gross rates. Debt-to-GDP is annualized.

In both cases, real debt increases quickly when the crisis starts, increasing almost 20p.p. of GDP. However, its real value decreases fast following the output and price recovery. The fall in real debt is more significant under the QE program for three reasons: first, central bank purchases increase central bank profits transferred to the treasury; second, the central bank intervention decreases the long-run yield, and thus interest payments on debt are lower; and third, the faster recovery

triggers a fall in government spending driven by automatic stabilizers so that the primary fiscal deficit is lower. These three factors allow the government to issue less debt during the crisis than in a counterfactual without the QE program.

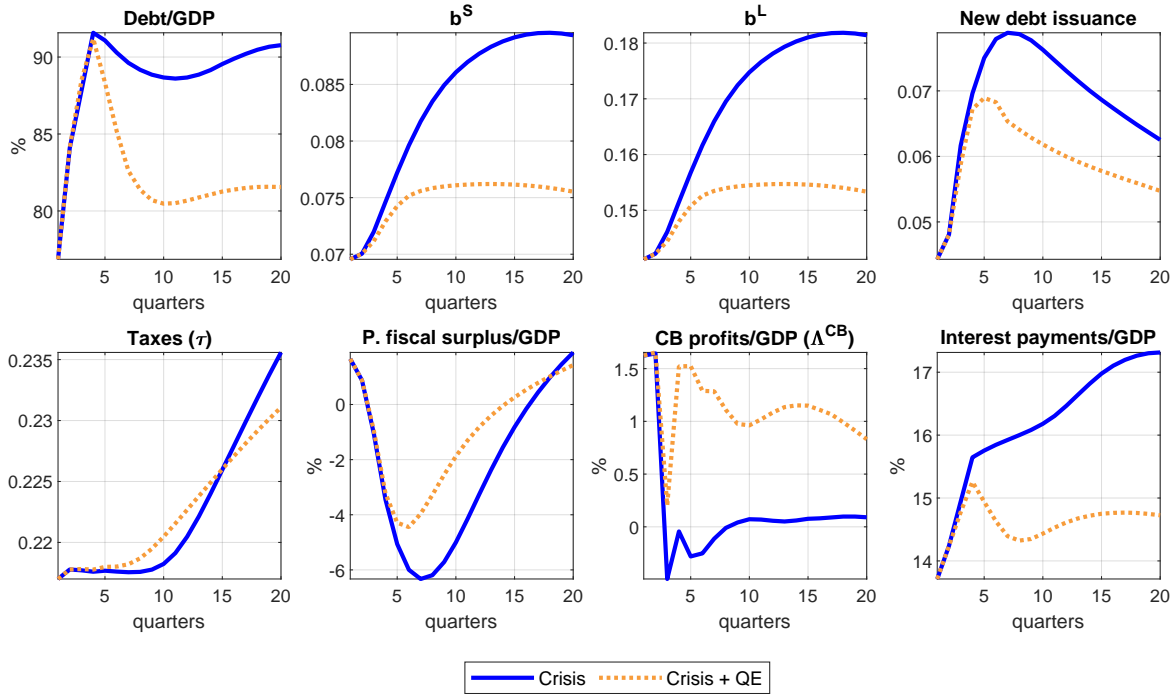


Figure 8: Simulated crisis

Note: Average from 50,000 samples. The continuous blue line is the scenario without the QE program. The Orange dotted line is the scenario with a QE program that increases the annualized central bank balance sheet to output by around 10p.p.

Finally, figure 9 shows, per period, the percentage of samples in which the economy is at the ZLB regime. Notice it is zero in periods one (initial condition) and two. The reason is that the endogenous probability of moving to the zero lower bound regime at  $t + 1$  depends on the interest rate at  $t$ . From period three onward, the economy is at the ZLB with a high probability. The frequency of this regime is considerably lower under the scenario with the QE program, and it is almost nil from period twelve.



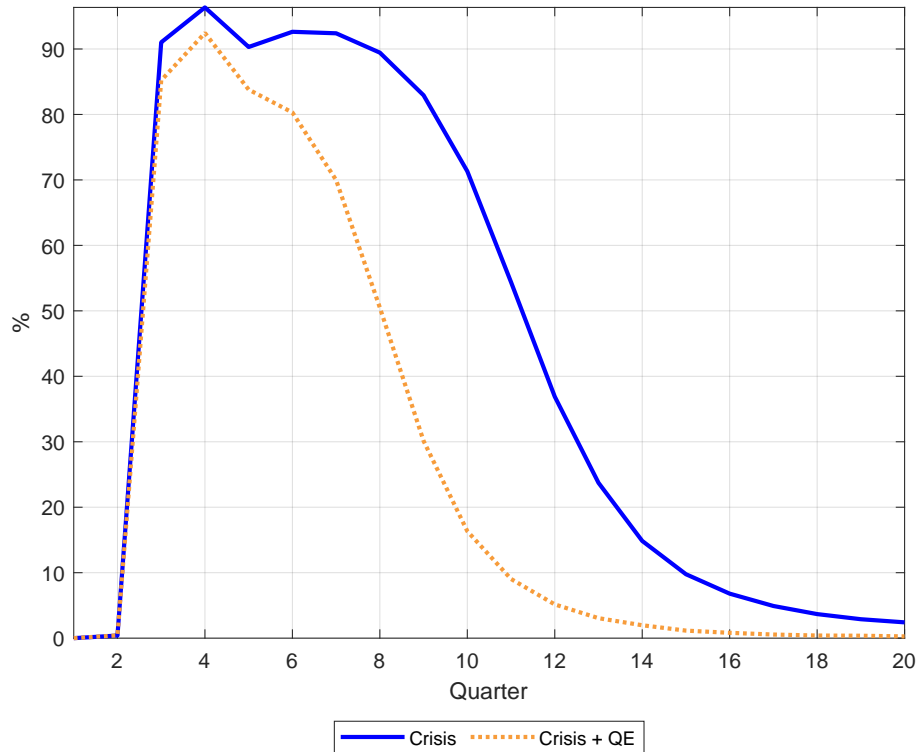


Figure 9: Frequency ZLB regime

Note: Percentage of simulated samples at the ZLB regime, per period, from 50,000 samples, in the scenario without the QE (continuous blue line), and with QE program (orange dotted line).

To summarize, unconventional monetary policy intervention reduces the severity of the crisis and implies a faster recovery and a shorter duration of the ZLB regime. Additionally, the debt stock is lower while inflation is higher.

## 7.2 Central bank balance sheet exit strategies

This section compares the path for endogenous variables under three scenarios after the initial crisis. The continuous green line is the scenario with Quantitative Tightening (QT) from period 9 onward, where the central bank allows mature treasuries to run off its balance sheet. Light-blue dashed line

is the scenario with sales of bonds from period 9 onward.<sup>27,28</sup> The orange dotted line corresponds to Tapering, identical to the one in figure 7. Figure 10 shows the evolution of the treasuries in the central bank ( $b_t^{L,CB}$ ) and the central bank balance sheet ( $Q_t^L b_t^{L,CB}$ ) as a GDP ratio.

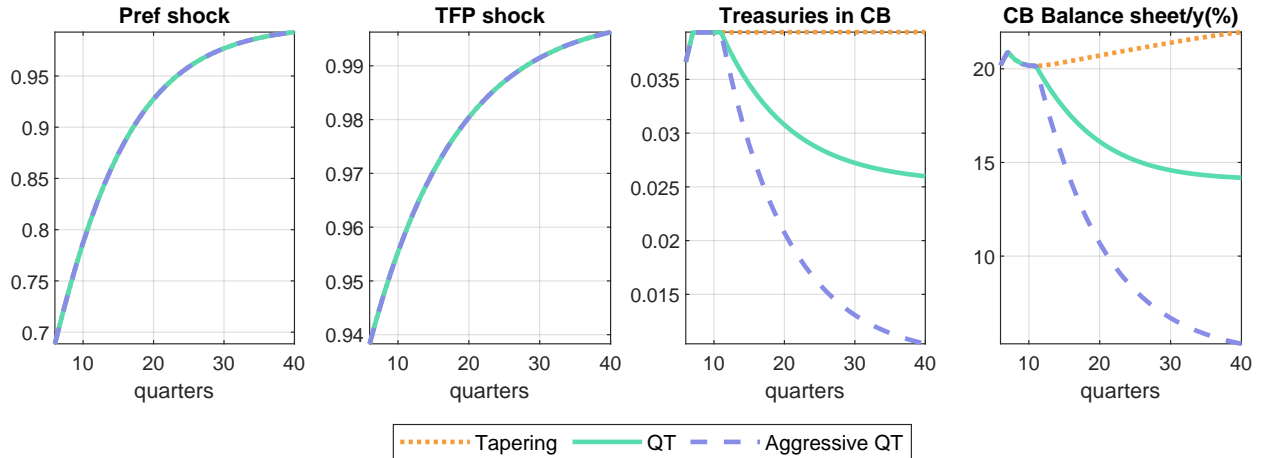


Figure 10: Crisis and exit strategies from QE.

Note: Average from 50,000 samples, plots since period  $t=8$ . The Orange dotted line is the tapering scenario. The continuous green line is the scenario with Quantitative Tightening (QT) from period 9 onward. Light-blue dashed line is the scenario with Aggressive QT or sales of bonds from period 9 onward.

The figure shows mean values for each variable across samples. This implies an average behavior where the regime is stochastic, i.e., which could leave the ZLB regime towards a fiscally-led regime or a monetary-led regime following the ergodic probabilities of matrix presented in section 3.8. For clarity of exposition, I show the variables from period  $t=8$ . The unwinding of the balance sheet is assumed to start almost two years after the crisis when most of the simulated samples are endogenously out of the ZLB (as shown in figure 4).<sup>29</sup>

When the central bank unwinds its balance sheet, it decreases the total demand for long-term bonds, pushing their price down, and generating a rise in the long-term yield. This price effect causes incentives in households to rebalance its portfolio away from deposits and toward long-term public bonds. At the same time, since households were already investors in these assets, they perceive a

<sup>27</sup>For this scenario, I assume a QE shock equal to -0.2 for every  $t$  from  $t=9$ .

<sup>28</sup>The policies denominated “QT” and “Aggressive QT” in this paper have been classified as “Passive QT” and “Active QT” by the Bank of England. See, for instance, the speech of Andrew Hauser at the ECB’s 2022 Conference on Money Markets, available at: [www.bis.org/review/r221104k.pdf](http://www.bis.org/review/r221104k.pdf).

<sup>29</sup>The main results are robust to start the unwinding of the central bank balance sheet some periods after (period  $t = 12$  or  $t = 15$ ). These exercises are available under request.

negative wealth effect from the revaluation of their portfolio, decreasing consumption and aggregate demand in the economy, to a situation without the QT. This effect complements the substitution effect that generates incentives to save more in the context of rising interest rates.

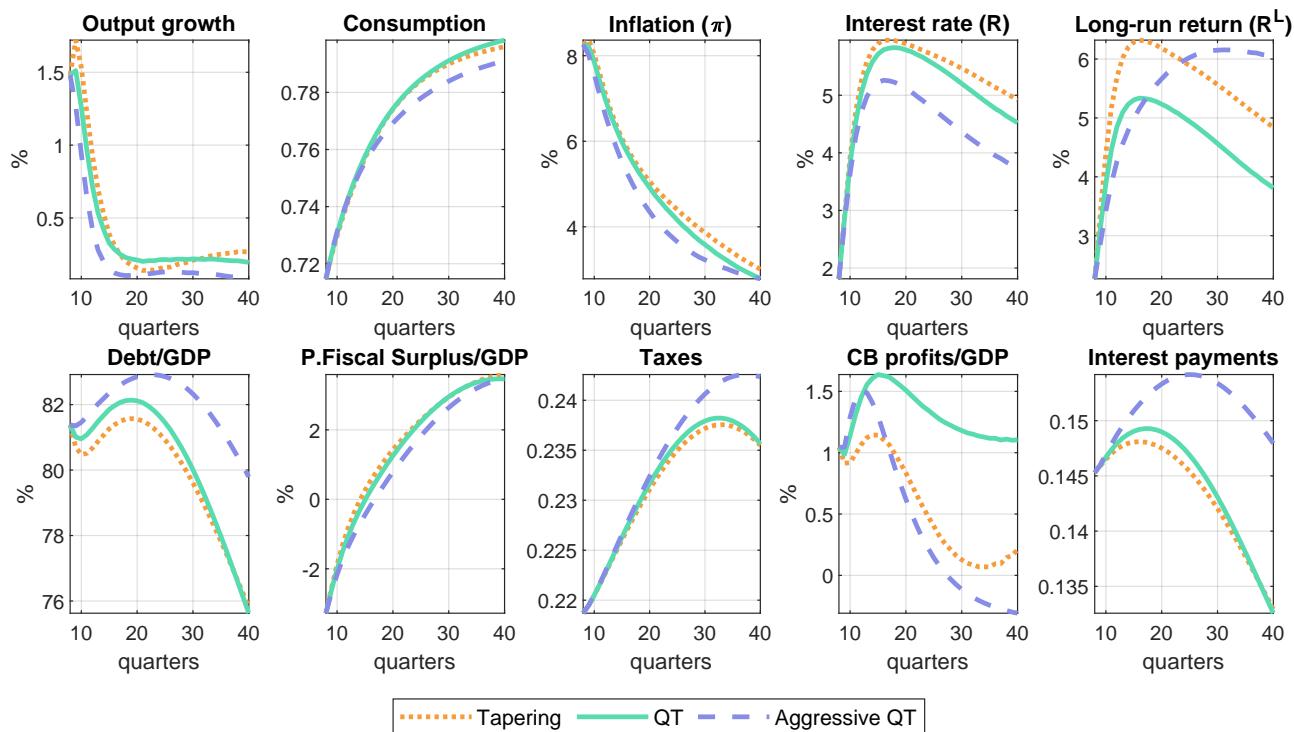


Figure 11: Crisis and exit strategies from QE.

Note: Average from 50,000 samples, since period  $t=8$ . The Orange dotted line is the tapering scenario. The continuous green line is the scenario with Quantitative Tightening (QT) from period 9 onward. Light-blue dashed line is the scenario with Aggressive QT or sales of bonds from period 9 onward. Output growth, balance sheet, and Debt to GDP are in percentages. Interest rate (R), long-run return ( $R^L$ ), inflation, and debt-to-GDP are annualized.

Firms respond to this fall in aggregate demand through a combination of a fall in production and prices. In this context, unwinding the central bank balance sheet generates a fall in output growth and faster inflation stabilization, allowing the central bank to stop increasing the short-term nominal interest rate. The disinflationary forces of reducing the central bank balance sheet are more substantial as the unwinding speed gets faster. Under an aggressive QT, after four years of applying this strategy (period  $t=27$ ) and in the absence of further shocks, the inflation rate is 3.3%, while it remains at 4.1% under Tapering. Moreover, this is accomplished with a short-term interest rate

of 1p.p. lower than in the tapering scenario. In a longer horizon (40 periods from the beginning of the simulation), we can see that the inflation rate remains at a higher level when the size of the balance sheet remains constant after the expansion during the crisis. In other words, more liquidity permanently circulating in the economy increases the long-term inflation rate.

On the fiscal side, we observe that the debt stock increases with the balance sheet unwinding. There are three reasons. First, the negative effects on output trigger government spending through automatic stabilizers, decreasing the fiscal surplus even as taxes increase. Second, interest payments increase due to higher interest rates. Third, central bank remittances to the treasury decrease in the scenario of sales of bonds. Notice that when the short-term interest rate increases, the central bank starts paying interest rates on reserves, decreasing its profits. With the unwinding, bank reserves which constitute the liabilities in its balance sheet, decline, reducing this cost. However, at the same time, it generates a fall in the return it receives for its holdings of long-term bonds. Under QT, the first effect dominates, and central bank profits increase compared to the tapering scenario. The opposite happens in an aggressive QT, where the unwinding is faster.

In conclusion, selling bonds from the central bank effectively reduce the inflation rate to the target, requiring smaller increases in the short-term interest rate. However, it has the downside of a slower deleveraging of the economy. As I show in the next section, these conclusions are not independent of the policy regime at the time of the unwinding.

### 7.3 Unwinding the balance sheet in a fiscally-led regime

In this section, I restrict the analysis to the samples that exit the ZLB toward a fiscally-led regime and stay there for at least one year (4 quarters). I compare it with the selected samples where the exit from the ZLB is toward the monetary regime. Since both fiscally and monetary-led regimes are substantially persistent, the probability of staying there for a good proportion of the sample is high. The exit of the crisis regime is stochastic and could happen at any period. I present the dynamics corresponding to these subsamples in figures 12 and 13.

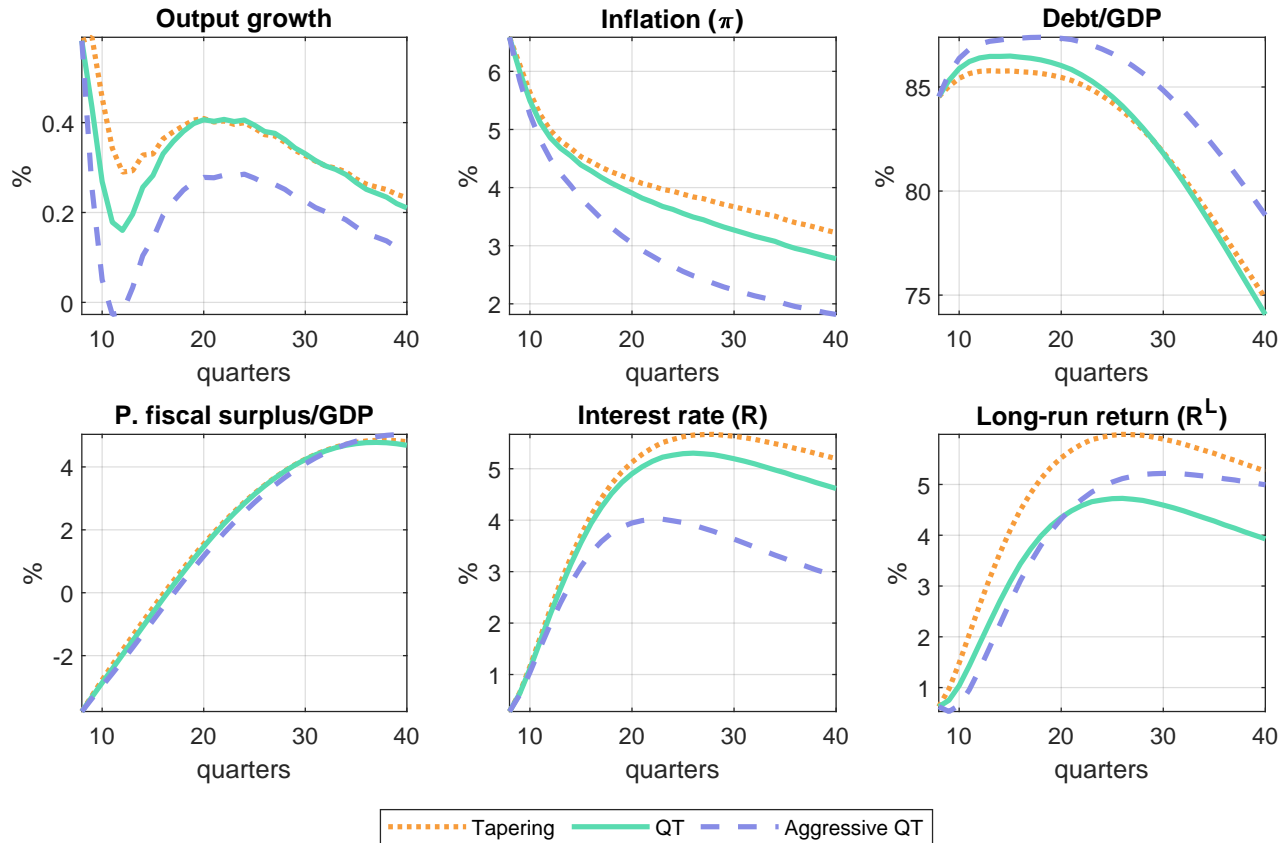


Figure 12: Quantitative Tightening in the monetary-led regime.

Note: Simulation of 50,000 samples, since period  $t=8$ . The figure restricts the analysis to samples that exit the zero lower bound toward a monetary-led regime and stay there for at least one year (4 quarters). The Orange dotted line is the tapering scenario, the same as the one in figure 7. The continuous green line is the scenario with Quantitative Tightening (QT) from period 9 onward. Light-blue dashed line is the scenario with Aggressive QT or sales of bonds from period 9 onward. Debt to GDP is in percentages. Interest rate ( $R$ ), long-run return ( $R^L$ ), inflation, and debt-to-GDP are annualized.

Exiting the crisis towards a fiscally-led regime brings a higher inflation rate that peaks at 11% in period nine and an expansionary period where output growth is above trend. As a result, interest rates are higher, even under lower reactions to inflation and output deviations. Higher inflation rates help to quickly reduce the stock of real debt, which falls 20p.p. of GDP in 7 quarters.

The most remarkable difference between regimes is that QT in a fiscally-led regime has little impact on inflation, while it has a sizable disinflationary impact in a monetary-led regime. In a monetary-led regime, the inflation rate in scenarios with QT is persistently below the one in the case with tapering. This difference tops at 1.5p.p. Notice that this is attained with lower increases

in the short-term interest rate. In the case of aggressive QT, the conventional monetary policy tool changes the stance from contractionary to expansionary 10 quarters before the case of constant size in the central bank balance sheet. Figure 14 shows the accumulated inflation since period  $t = 8$

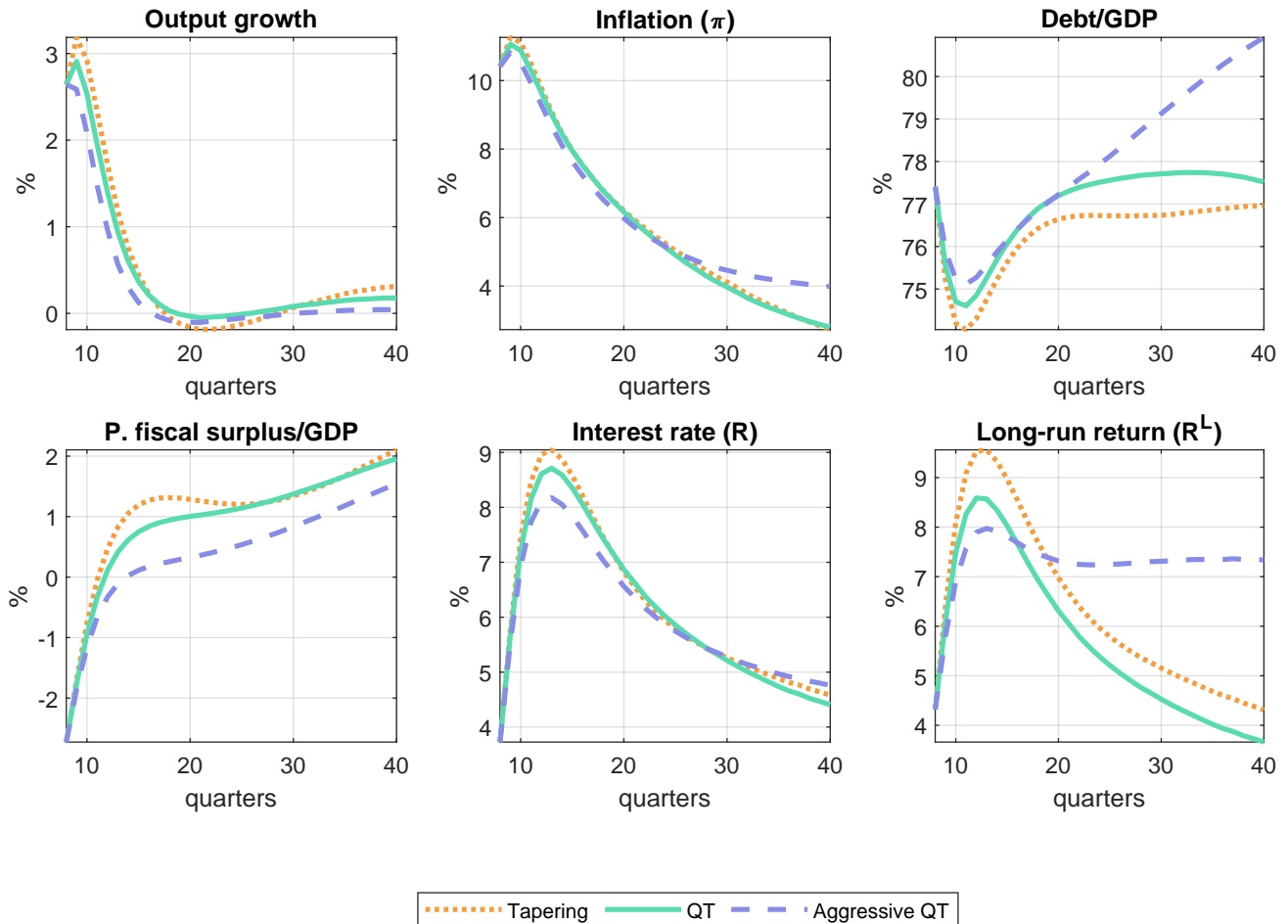


Figure 13: Quantitative Tightening in the fiscally-led regime.

Note: Simulation of 50,000 samples, since period  $t=8$ . The figure restricts the analysis to samples that exit the zero lower bound toward a fiscally-led regime and stay there for at least one year (4 quarters). The Orange dotted line is the tapering scenario, the same as the one in figure 7. The continuous green line is the scenario with Quantitative Tightening (QT) from period 9 onward. Light-blue dashed line is the scenario with Aggressive QT or sales of bonds from period 9 onward. Debt to GDP is in percentages. Interest rate (R), long-run return ( $R^L$ ), inflation, and debt-to-GDP are annualized.

The difference in inflationary dynamics after QT programs is the following. As shown in the average scenario, real debt increases with QT. In the monetary-led regime, the increase in debt causes a rise in taxes, given by the fiscal rule. This exacerbates the fall in aggregate demand and

the disinflationary effects of QT. In the fiscally-led regime, the rise in public debt increases actual and expected inflation, aggravating the rise in yield spreads and mitigating the disinflationary effects of QT. In this case, the negative demand effect driven by the reduction in central bank purchases is not enough to counteract the stimulative impact of negative interest rates and fiscal stimulus.

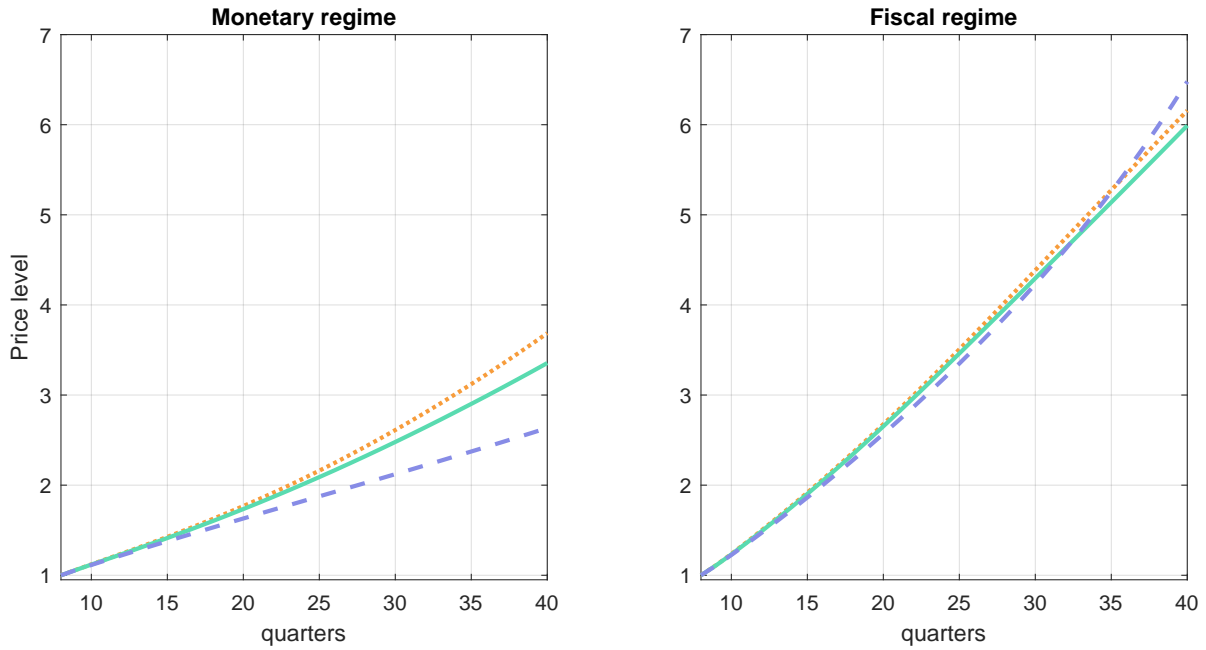


Figure 14: Accumulated inflation under different regimes and exit strategies from QE.

Note: Simulation of 50,000 samples, since period  $t=6$ . The figure restricts the analysis to samples that exit the zero lower bound toward a monetary (left) or fiscally-led regime (right) and stay there for at least one year (4 quarters). The continuous green line is the scenario with Quantitative Tightening (QT) from period 9 onward. Light-blue dashed line is the scenario with Aggressive QT or sales of bonds from period 9 onward. This figure shows the CPI in the monetary vs. fiscal regime, with base  $P_8 = 1$ .

In conclusion, in a regime where the fiscal authority is not committed to stabilizing the government debt, unwinding the central bank balance sheet does not generate clear economic advantages.

## 8 Concluding Remarks

I study different exit strategies for reducing the central bank balance sheet in a model that generates fiscal-monetary policy trade-offs.

The impact of central bank balance sheet policies (QE or QT) depends on the fiscal-monetary policy mix. First, QE reduces the need for new debt issuance. Second, as a counterpart of the central bank's balance sheet expansions, liquidity increases substantially, both in the hand of commercial banks (reserves) and non-bank private agents (deposits). The extent to which the increase in liquidity affect output and inflation rate depends on the fiscal-monetary policy mix. QE is more expansive at the ZLB and under the fiscally-led regime when the inflationary impact is also more significant.

How the central bank balance sheet's size is reduced matters for inflation, output, and debt dynamics. In an average simulation, when the monetary authority starts shrinking its holdings of long-term government bonds, it decreases inflation at the cost of an increase in the ratio of debt to GDP. These effects increase when the exit strategy is more aggressive.

When the unwinding occurs in a fiscally-led regime, reducing the balance sheet's size does not help bring the inflation back to target. In this regime, the rise in public debt increases actual and expected inflation, exacerbating the rise in yield spreads and mitigating the disinflationary effects of QT. This implies that, with unconventional policies, coordination between fiscal and monetary authorities is necessary to stabilize the inflation rate.



## 9 Appendix

### 9.1 Extra plots for US case

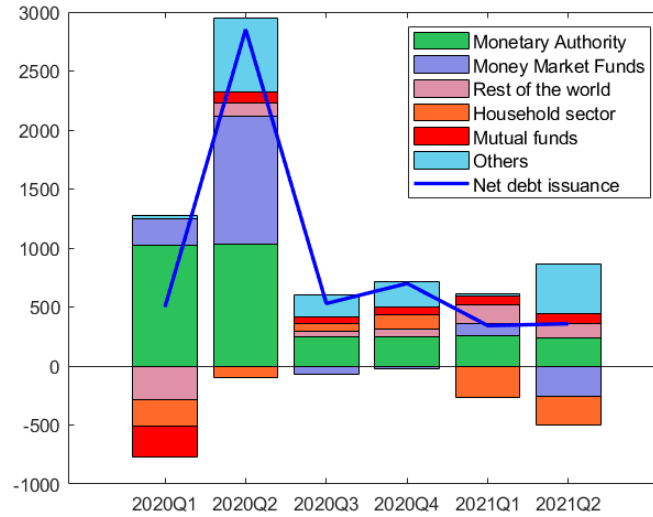


Figure 15: Change in Treasuries' holdings

Source: US Financial Accounts. Data in billions of dollars. Contain revaluation effects.

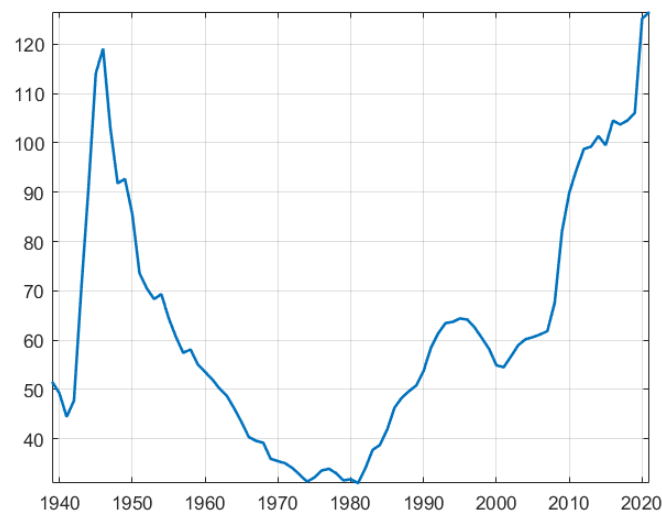


Figure 16: Public debt to GDP in US: historical perspective

Source: FRED. Annual Gross Federal Debt as a Percent of GDP.

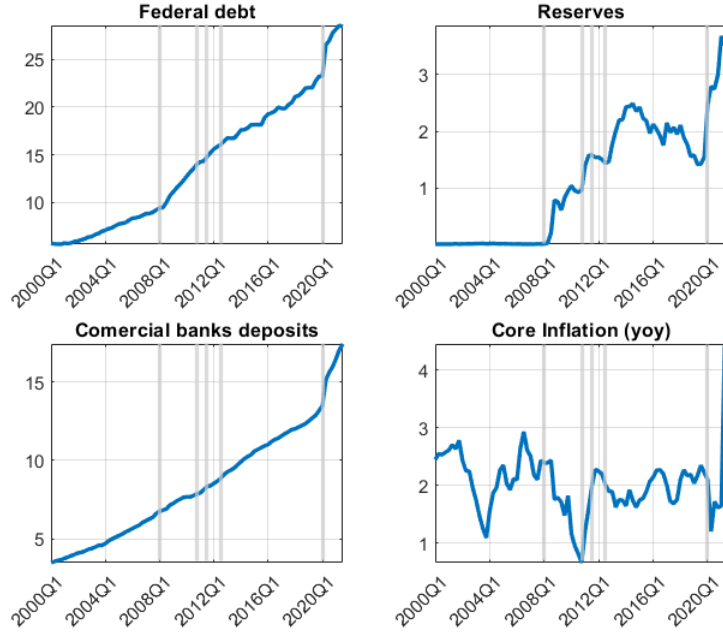


Figure 17: Macroeconomic variables: public debt, liquidity, inflation. Great recession and COVID-19 crisis.

US Federal Debt, Reserves and Deposits, in trillions of dollars. Core inflation in %. Source debt, deposits and inflation data: FRED. Source reserves: US financial accounts, release December 2021.

## 9.2 Model: Financial intermediary optimization problem

The optimization problem of a representative financial intermediary can be written in sequential form as follows:

$$\begin{aligned}
 V^I(W_t^I) &= \max_{A_t, D_t^I, F_t^I, B_t^{S,I}} \tau^I W_t^I - A_t + \mathbb{E}_t [\mathcal{M}_{t,t+1} V^I(W_{t+1}^I)] \\
 \text{s.t. } &(1 - \tau^I)W_t^I + A_t - \Phi_A(A_t) + Q_t^D D_t^I = Q_t^S B_t^{S,I} \\
 &W_t^I = B_{t-1}^{S,I} - D_{t-1}^I \\
 &D_t^I \leq \zeta B_t^{S,I}
 \end{aligned}$$

Define  $\eta_t$  as the Balance sheet multiplier and  $\mu_t$  as the Leverage constraint multiplier. The first order conditions are given by the following system:

$$\begin{aligned} A_t : -1 + \eta_t - \eta_t \chi \frac{A_t}{P_t} &= 0 \\ D_t^I : \eta_t Q_t^D - \mu_t - \mathbb{E}_t \mathcal{M}_{t,t+1} V_w^I(W_{t+1}^I) &= 0 \\ B_t^{S,I} : -\eta_t Q_t^D + \mu_t \zeta + \mathbb{E}_t \mathcal{M}_{t,t+1} V_w^I(W_{t+1}^I) &= 0 \end{aligned}$$

And the envelope condition:

$$V_w^I(W_{t+1}^I) = \tau^I + \eta_t(1 - \tau^I)$$

Using the first order condition with respect to  $A_t$  to substitute out the multiplier  $\eta_t$ , we obtain the system of equations that characterize the financial intermediary's optimization problem, together with 7, 8, 9, ??:

$$Q_t^D = \mathbb{E}_t \tilde{\mathcal{M}}_{t,t+1} + \mu_t(1 - \chi a_t) \quad (22)$$

$$Q_t^S = \mathbb{E}_t \tilde{\mathcal{M}}_{t,t+1} + \zeta \mu_t(1 - \chi a_t) \quad (23)$$

Where  $\tilde{\mathcal{M}}_{t,t+1}$  is the stochastic discount factor for financial intermediaries, defined as:

$$\tilde{\mathcal{M}}_{t,t+1} \equiv \mathcal{M}_{t,t+1} \left(1 - \chi \frac{A_t}{P_t}\right) \left(\tau^I + \frac{1 - \tau^I}{1 - \chi \frac{A_{t+1}}{P_{t+1}}}\right)$$

### 9.3 Equilibrium conditions

In this section I present the system of equilibrium conditions with functional forms and in real terms.

Definitions:  $d_t = \frac{D_t}{P_t}$ ,  $b_t^{j,i} = \frac{b_t^{j,i}}{P_t}$ ,  $w_t = \frac{W_t}{P_t}$ ,  $div_t = \frac{Div_t}{P_t}$ ,  $\pi_t^f = \frac{\Pi_t^f}{P_t}$ ,  $a_t = \frac{A_t}{P_t}$ ,  $mc_t = \frac{MC_t}{P_t}$ ,  $w_t^I = \frac{W_t^I}{P_t}$ ,  $\pi_t = \frac{P_t}{P_{t-1}}$ , for  $j \in \{L, S\}$ ,  $i \in \{H, CB, FI\}$ .

#### 9.3.1 Households

$$c_t + Q_t^D d_t^H + b_t^{L,H} Q_t^L = w_t n_t - \tau_t + \frac{d_{t-1}^H}{\pi_t} + \frac{b_{t-1}^{L,H}}{\pi_t} [\kappa + (1 - \delta) Q_t^L] + div_t + \pi_t^f + \frac{\chi}{2} a_t^2 \quad (1)$$

$$\frac{\gamma n_t^\eta}{\left[ c_t^{1-\varphi} (d_t^H)^\varphi \right]^{-\sigma} (1-\varphi) c_t^{-\varphi} (d_t^H)^\varphi} = w_t \quad (2)$$

$$Q_t^D = \mathbb{E}_t \mathcal{M}_{t,t+1} + \frac{\varphi c_t}{(1-\varphi) d_t^H} \quad (3)$$

$$Q_t^L + \phi^L \frac{b_t^{L,H}}{(b^{L,H})^2} = \mathbb{E}_t \mathcal{M}_{t,t+1} [\kappa + (1-\delta) Q_{t+1}^L] \quad (4)$$

$$\mathcal{M}_{t,t+1} = \frac{\beta \lambda_{t+1}}{\lambda_t} = \beta \mathbb{E}_t \frac{\nu_{t+1}}{\nu_t} \frac{\left[ c_{t+1}^{1-\varphi} (d_{t+1}^H)^\varphi \right]^{-\sigma} c_{t+1}^{-\varphi} (d_{t+1}^H)^\varphi}{\left[ c_t^{1-\varphi} (d_t^H)^\varphi \right]^{-\sigma} c_t^{-\varphi} (d_t^H)^\varphi} \frac{1}{\pi_{t+1}} \quad (5)$$

### 9.3.2 Firms

$$y_t = z_t n_t \quad (6)$$

$$\pi_t^f = y_t - w_t n_t - \frac{\phi^P}{2} (\pi_t - 1)^2 y_t \quad (7)$$

$$1 - \varepsilon + \varepsilon m c_t = \phi^P (\pi_t - \pi^*) \pi_t - \phi^P \mathbb{E}_t \left[ \mathcal{M}_{t,t+1} \frac{y_{t+1}}{y_t} \pi_{t+1}^2 (\pi_{t+1} - \pi^*) \right] \quad (8)$$

$$w_t = m c_t z_t \quad (9)$$

### 9.3.3 Financial intermediaries

$$div_t = \tau^I w_t^I - a_t \quad (10)$$

$$(1 - \tau^I) w_t^I + a_t - \frac{\chi}{2} a_t^2 + Q_t^D d_t^I = Q_t^S b_t^{S,I} \quad (11)$$

$$w_t^I = \frac{b_{t-1}^{S,I}}{\pi_t} - \frac{d_{t-1}^I}{\pi_t} \quad (12)$$

$$d_t^I \leq \zeta b_t^{S,I} \quad (13)$$

$$Q_t^D = \mathbb{E}_t \tilde{\mathcal{M}}_{t,t+1} + \mu_t (1 - \chi a_t) \quad (14)$$

$$Q_t^S = \mathbb{E}_t \tilde{\mathcal{M}}_{t,t+1} + \zeta \mu_t (1 - \chi a_t) \quad (15)$$

$$\tilde{\mathcal{M}}_{t,t+1} = \mathcal{M}_{t,t+1} (1 - \chi a_t) \left( \tau^I + \frac{1 - \tau^I}{1 - \chi a_{t+1}} \right) \quad (16)$$

### 9.3.4 Government

$$\frac{1}{R_t} = Q_t^S \quad (17)$$

$$Q_t^S b_t^{S,CB} + Q_t^L b_t^{L,CB} = 0 \quad (18)$$

$$b_t = Q_t^S b_t^S + Q_t^L b_t^L \quad (19)$$

$$b_t^S = (1 - \bar{\mu}) / \bar{\mu} b_t^L \quad (20)$$

$$g_t = \theta(y^* - y_t) + (1 - \rho_g)\bar{g} + \rho_g g_{t-1} + \sigma_g \varepsilon_t^g, \varepsilon_t^g \sim N(0, 1) \quad (21)$$

### 9.3.5 Market clearing conditions

$$c_t + g_t + \frac{\phi_P}{2} (\pi_t - \pi^*)^2 y_t = y_t \quad (22)$$

$$d_t^H = d_t^I \quad (23)$$

$$b_t^S = b_t^{S,I} + b_t^{S,CB} \quad (24)$$

$$b_t^L = b_t^{L,H} + b_t^{L,CB} \quad (25)$$

### 9.3.6 Policy rules

$$\tau_t - \tau^* = \rho_\tau (\xi_t) (\tau_{t-1} - \tau^*) + (1 - \rho_\tau (\xi_t)) \gamma (\xi_t) (b_{t-1} - b^*) \quad (16)$$

$$\frac{R_t}{R(\xi_t)} = \left( \frac{R_{t-1}}{R(\xi_t)} \right)^{\alpha_R(\xi_t)} \left[ \left( \frac{\pi_t}{\pi^*} \right)^{\alpha_\pi(\xi_t)} \left( \frac{y_t}{y^*} \right)^{\alpha_y(\xi_t)} \right]^{1 - \alpha_R(\xi_t)} e^{\sigma_M(\xi_t)\varepsilon_t^M} \quad (27)$$

$$b_t^{L,CB} = (1 - \rho^{QE}) b_*^{L,CB} + \rho^{QE} b_{t-1}^{L,CB} + \sigma^{QE} \varepsilon_t^{QE} \quad (28)$$

Law of motion for exogenous processes:  $z_t, \nu_t$ .

Transition matrix for Markov-switching shock  $\xi_t$ .

### 9.3.7 Definition of auxiliary variables used in figures and tables

Return on long-term bond:

$$R_{t,t+1}^L = \mathbb{E}_t \frac{\kappa + (1 - \delta) Q_t^L}{Q_{t+1}^L}$$

Convenience yield:

$$cy_t = R_t - \mathbb{E}_t \frac{1}{\mathcal{M}_{t,t+1}}$$

Annual debt to GDP ratio:

$$\frac{b_t}{4y_t}$$

Primary fiscal surplus:

$$s_t = \tau_t - g_t$$

Total interest rate payments:

$$(R_{t-1} - 1) \frac{b_{t-1}^S}{\pi_t} + (\kappa - \delta) \frac{b_{t-1}^L}{\pi_t}$$

Private interest rate payments:

$$(R_{t-1} - 1) \frac{b_{t-1}^{S,FI}}{\pi_t} + (\kappa - \delta) \frac{b_{t-1}^{L,H}}{\pi_t}$$

Total debt service:

$$\frac{b_{t-1}^S}{\pi_t} + \kappa \frac{b_{t-1}^L}{\pi_t}$$

## 9.4 Steady state and approximation point

At the steady state, shocks are equal to zero:  $\epsilon_t^v = 0$ ,  $\epsilon_t^z = 0$ ,  $\epsilon_t^g = 0$ ,  $\epsilon_t^m = 0$ ,  $\epsilon_t^{QE} = 0$ .

Then:  $z^{ss} = 1$ ,  $\nu^{ss} = 1$ ,  $g^{ss} = \bar{g}$ ,  $b^{L,CB,ss} = b_*^{L,CB}$ .

There is one endogenous variable which steady state value differs among regimes. For monetary ( $\xi_t^C = 0$  and  $\xi_t^P = M$ ) or fiscal ( $\xi_t^C = 0$  and  $\xi_t^P = F$ ) dominance regimes, we have:  $R_t = R^*$  in absence of shocks. For the Zero lower bound regime ( $\xi_t^C = 1$ ),  $R_t = 1.0005$ . Define  $p^M$ ,  $p^F$ ,  $p^C$  as the ergodic probability of monetary dominance, fiscal dominance or crisis regime, respectively. Where:  $p^M + p^F + p^C = 1$ . The short-term nominal interest rate at the approximation point is obtained as follows:

$$R^{ss} = (p^M + p^F) R^* + (1 - p^M - p^F) 1.0005$$

## 9.5 Additional results

### 9.5.1 Determinacy regions for fiscal and monetary policy rules

The following figure shows combinations for policy parameters  $\gamma$  and  $\alpha_\pi$  that give rise to a unique equilibrium (determinacy, yellow areas), given the calibration for the rest of the parameters, in the monetary-led regime. The blue areas represent parameter combinations that generate either no equilibrium, or multiple equilibria.

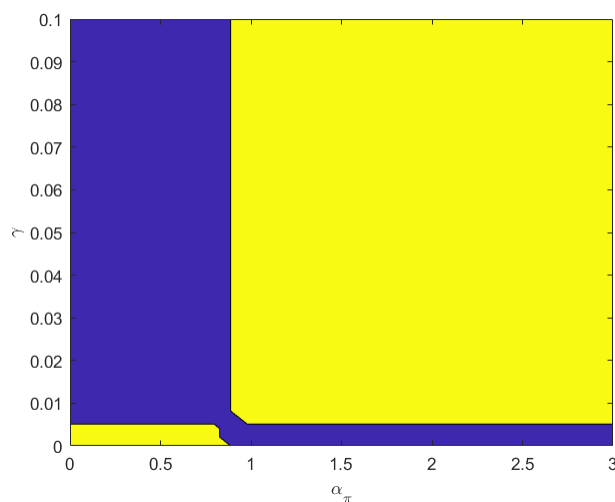


Figure 18: Determinacy regions

Note: Yellow areas represent combinations of parameters  $\alpha_\pi$  and  $\gamma$  that give rise to determinacy. Blue areas represent either multiple equilibria or no equilibrium.

### 9.5.2 Impulse response functions for additional shocks

This section presents figures showing the impact of the different shocks in the model, under different regimes. They show log deviations (in %) in a simulated path with a one standard deviation shock, to the counterfactual path without the shock.

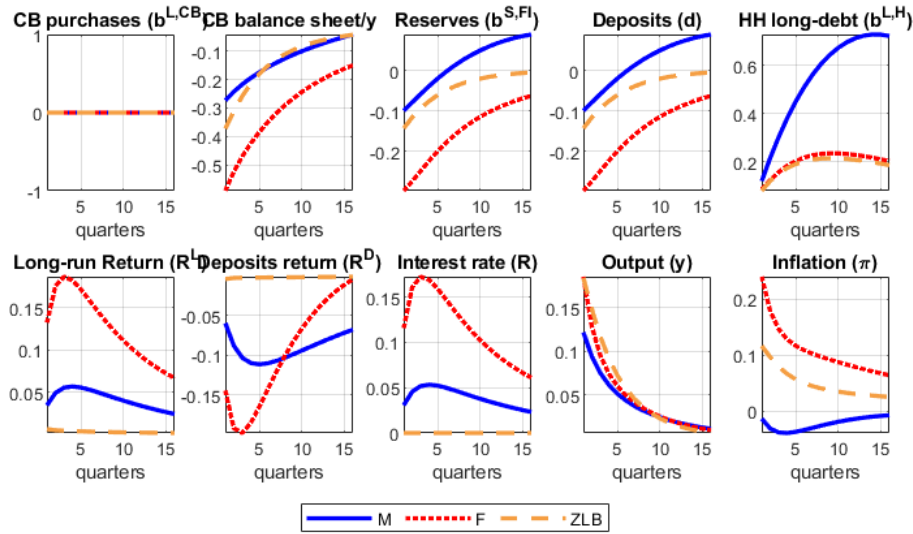


Figure 19: Impact of a Quantitative Easing shock conditional on a regime

Note: log deviations (in %) in a simulated path with a one standard deviation shock in the government spending ( $\epsilon_t^g = 1$ ) to the counterfactual path without shock ( $\epsilon_t^g = 0$ ). Path conditional on the monetary-led regime (continue blue line), fiscally-led regime (dotted red line), and zero lower bound regime (dashed yellow line) for 16 periods.

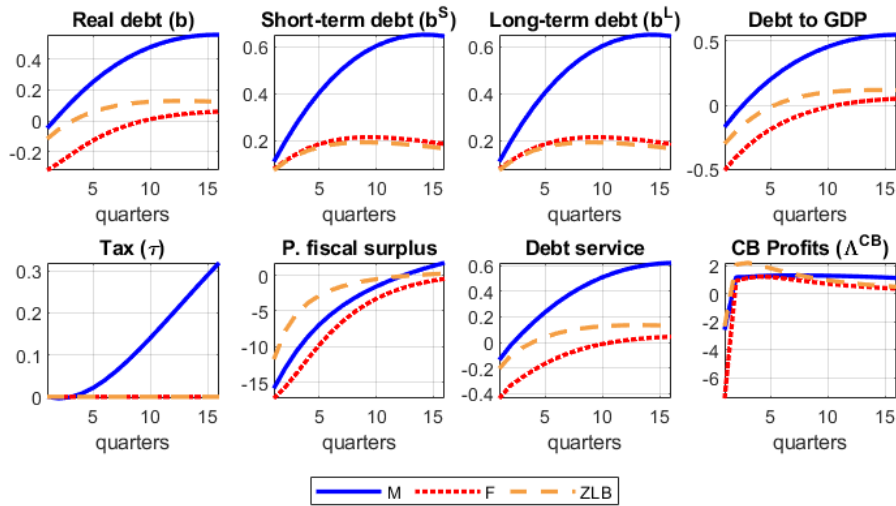


Figure 20: Impact of a Quantitative Easing shock conditional on a regime

Note: log deviations (in %) in a simulated path with a one standard deviation shock in the government spending ( $\epsilon_t^g = 1$ ) to the counterfactual path without shock ( $\epsilon_t^g = 0$ ). Path conditional on the monetary-led regime (continue blue line), fiscally-led regime (dotted red line), and zero lower bound regime (dashed yellow line) for 16 periods.



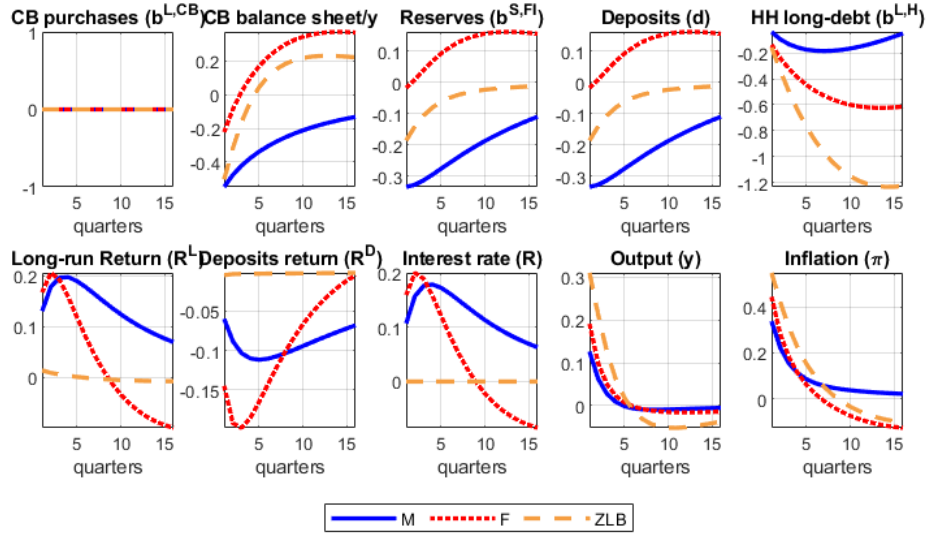


Figure 21: Impact of a Quantitative Easing shock conditional on a regime

Note: log deviations (in %) in a simulated path with a one standard deviation shock in the preferences ( $\epsilon_t^y = 1$ ) to the counterfactual path without shock ( $\epsilon_t^y = 0$ ). Path conditional on the monetary-led regime (continue blue line), fiscally-led regime (dotted red line), and zero lower bound regime (dashed yellow line) for 16 periods.

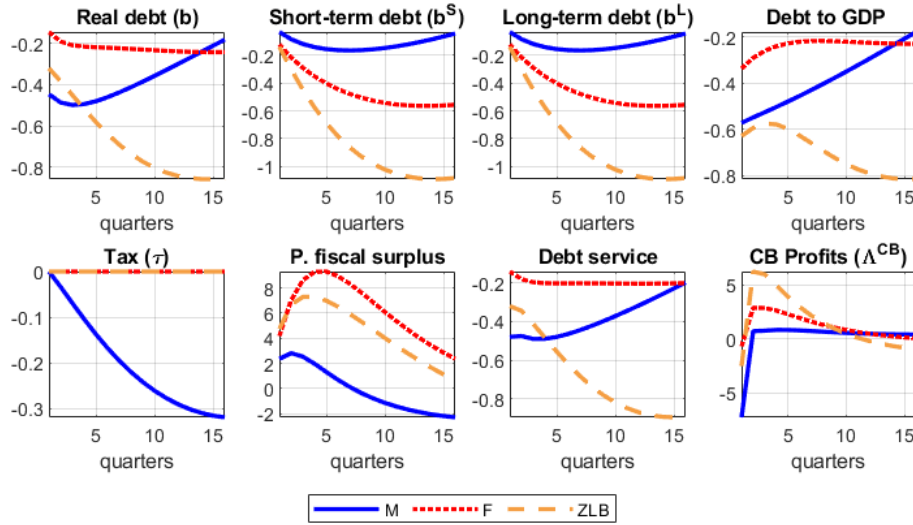


Figure 22: Impact of a Quantitative Easing shock conditional on a regime

Note: log deviations (in %) in a simulated path with a one standard deviation shock in the preferences ( $\epsilon_t^y = 1$ ) to the counterfactual path without shock ( $\epsilon_t^y = 0$ ). Path conditional on the monetary-led regime (continue blue line), fiscally-led regime (dotted red line), and zero lower bound regime (dashed yellow line) for 16 periods.

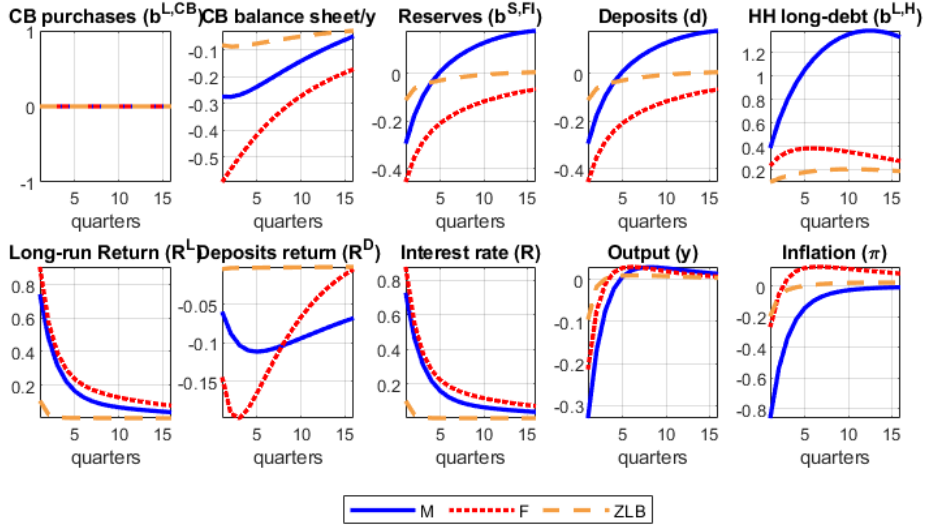


Figure 23: Impact of a Quantitative Easing shock conditional on a regime

Note: log deviations (in %) in a simulated path with a one standard deviation monetary shock ( $\epsilon_t^m = 1$ ) to the counterfactual path without shock ( $\epsilon_t^m = 0$ ). Path conditional on the monetary-led regime (continue blue line), fiscally-led regime (dotted red line), and zero lower bound regime (dashed yellow line) for 16 periods.

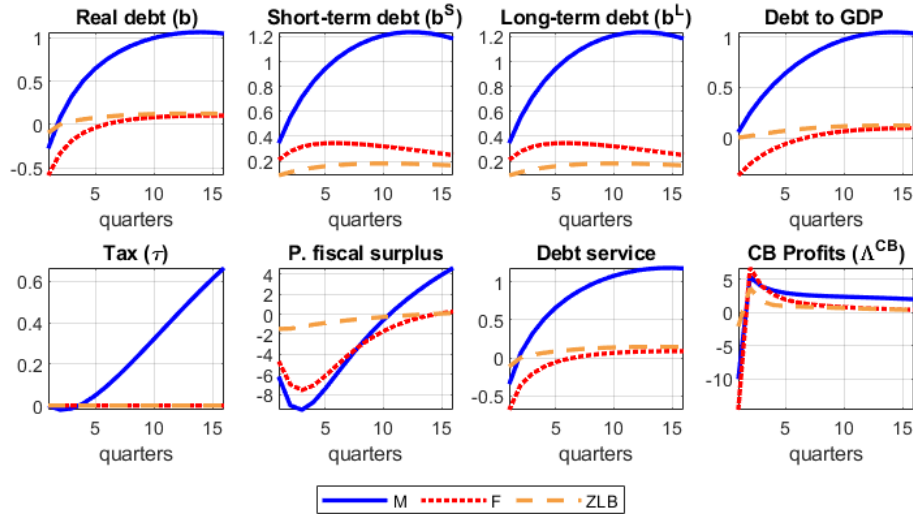


Figure 24: Impact of a Quantitative Easing shock conditional on a regime

Note: log deviations (in %) in a simulated path with a one standard deviation monetary shock ( $\epsilon_t^m = 1$ ) to the counterfactual path without shock ( $\epsilon_t^m = 0$ ). Path conditional on the monetary-led regime (continue blue line), fiscally-led regime (dotted red line), and zero lower bound regime (dashed yellow line) for 16 periods.

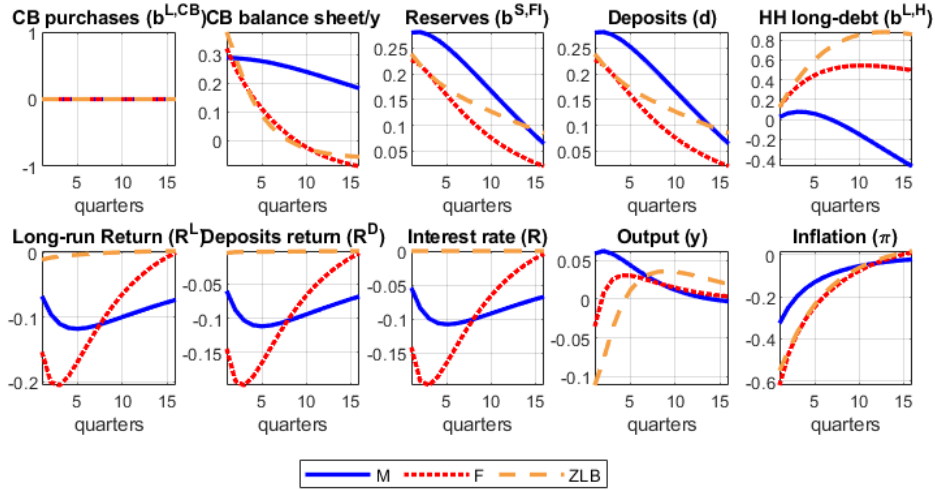


Figure 25: Impact of a Quantitative Easing shock conditional on a regime

Note: log deviations (in %) in a simulated path with a one standard deviation TFP shock ( $\epsilon_t^z = 1$ ) to the counterfactual path without shock ( $\epsilon_t^z = 0$ ). Path conditional on the monetary-led regime (continue blue line), fiscally-led regime (dotted red line), and zero lower bound regime (dashed yellow line) for 16 periods.

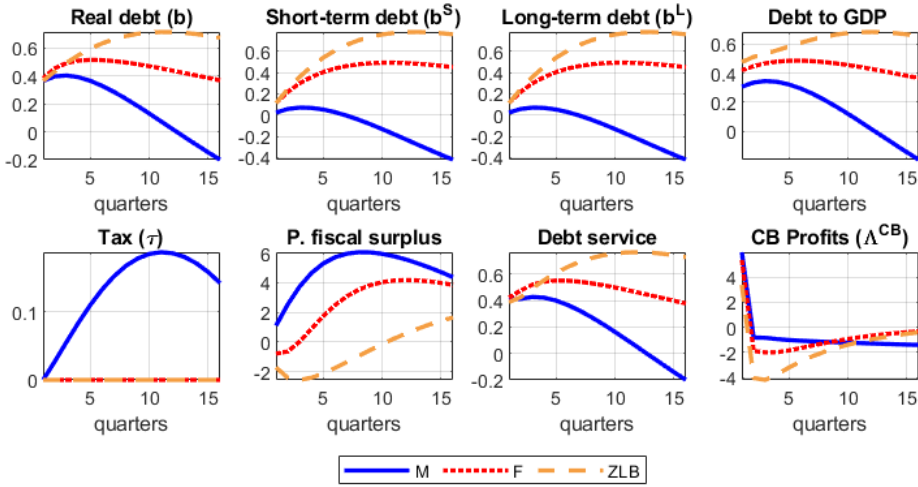


Figure 26: Impact of a Quantitative Easing shock conditional on a regime

Note: log deviations (in %) in a simulated path with a one standard deviation TFP shock ( $\epsilon_t^z = 1$ ) to the counterfactual path without shock ( $\epsilon_t^z = 0$ ). Path conditional on the monetary-led regime (continue blue line), fiscally-led regime (dotted red line), and zero lower bound regime (dashed yellow line) for 16 periods.

## 9.6 Data sources

The following table presents data sources for plots in section 2 and calibration.

Variable	Source	Table
Deposits in depository institutions	FRED	
Monetary aggregate M1	FRED	
CPI	FRED	
US GDP Implicit Price Deflator, Index 2015=100	FRED	
Federal reserve Assets	US Financial Accounts	L109
Federal Reserve Total treasuries	US Financial Accounts	L110
Federal Reserve Treasury bills	US Financial Accounts	L111
Federal Reserve Other treasuries	US Financial Accounts	L112
Federal Reserve Total liabilities	US Financial Accounts	L113
Federal Reserve Reserves	US Financial Accounts	L114
Checkable Accounts in Federal Reserve	US Financial Accounts	L115
Total public debt	FRED	
Effective federal funds rate	FRED	
10-year Yield	FRED	
1-year Yield	FRED	
US GDP	BEA	NIPA
Private consumption	BEA	NIPA
Government spending	BEA	NIPA
US Population	FRED	

Table 6: Data sources

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