# Cocos, Contagion and Systemic Risk

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**Abstract**

Cocos (contingent convertible capital) are designed to convert from debt to equity when banks need it most. Using a Diamond-Dybvig model cast in a global games framework, we show that while the coco conversion of the issuing bank may bring the bank back into compliance with capital requirements, it will nevertheless raise the probability of the bank being run, because conversion is a negative signal to depositors about asset quality. Moreover, conversion imposes a negative externality on other banks in the system in the likely case of correlated asset returns, so bank runs elsewhere in the banking system become more probable too and systemic risk will actually go up after conversion. Cocos thus lead to a direct conflict between micro- and macroprudential objectives. We also highlight that *ex ante* incentives to raise capital to stave off conversion depend critically on coco design. In many currently popular coco designs, wealth transfers after conversion actually flow from debt holders to equity holders, destroying the latter's incentives to provide additional capital in times of stress. Finally the link between coco conversion and systemic risk highlights the tradeoffs that a regulator faces in deciding to convert cocos, providing a possible explanation of regulatory forbearance.

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**Keywords:** Contingent Convertible Capital; Contagion; Systemic Risk; Bank Runs; Global games

## 1 Introduction and literature review

As early as 2002, Flannery proposed an early form of contingent convertible (coco) capital that he called reverse convertible debentures [1]. The idea was simple: whenever the bank issuing such debentures reaches a market-based capital ratio which is below a pre-specified level (say, 8% of assets), a sufficient number of said debentures would automatically convert to equity at the prevailing market price of the bank's shares. The automatic conversion feature frees the issuing bank from having to raise additional capital immediately when its capital ratio is lower than the minimum requirement. For larger shocks, conversion may not be enough to restore compliance with capital requirements, but it would make banks merely undercapitalized instead of bankrupt.

Flannery's initial coco design proposal was attractive, as its automatic conversion feature had the potential to avoid socially costly bailouts. After the 2007 financial crisis, regulators realized that even though systemically important financial institutions (SIFIs) held Tier 2 Capital, that type of capital failed to be loss-absorbing during the time of distress. Instead, some of the SIFIs were bailed out while others were allowed to fail. Yet despite having Tier 2 status, many of the subordinated loans continued to be serviced. In response, the Basel Committee on Banking Supervision (BCBS) made a number of changes to what is now known as the Basel III framework. Among the changes were the redefinition of "gone concern" to include potential bailout situations[2], and the inclusion of coco-like instruments as part of Additional Tier 1 Capital.[3] Also, while not yet finalized, the Basel III document suggested that cocos might play a role in ensuring that SIFIs would have higher loss absorption capacities than regular financial institutions.

[1] Unlike ordinary convertible bonds, reverse convertible debentures expose the holder to the potential downside of holding equity

[2] "Proposal to ensure the loss absorbency of regulatory capital at the point of non-viability", Basel Committee on Banking Supervision (2010)

[3] which must meet several requirements as set forth in the Basel III framework

The inclusion of cocos as part of Additional Tier 1 Capital is a likely factor in the increase of coco issuance. Coco issuances totaled 42.5bn EUR in 2013, up from only 2.5bn EUR in 2010.[4] Also, this year (2014) saw a number of banks issuing cocos, including Deutsche Bank and Mizuho Financial Group. Within the same period, the academic literature branched off in three different directions. Flannery (2005) and McDonald (2013) were among those that dealt with design features such as triggers and bases. Pennacchi (2010) dealt with the pricing and valuation of cocos. Finally, Martynova and Perotti (2012), Hilscher and Raviv (2014) and Berg and Kaserer (2014) consider the effect of cocos on risk-taking incentives of banks. Moreover, several survey articles have been written about cocos. Maes and Schoutens (2012) provide an overview of cocos and enumerate the potential downside of coco issuance such as contagion from the banking to the insurance sector, and the creation of a "death spiral" where coco holders short-sell the stock of the issuing bank in order to profit from potential conversion. Avdjiev et al. (2013) discuss the features of the coco trend - from the reason why banks issue them to the main groups of investors that are interested in buying cocos, as well as the pricing of cocos. Wilkens and Bethke (2014) summarize and empirically assess some of the pricing models' performance. There is disagreement in the literature on whether coco conversion should be triggered based on market prices or book values (e.g. capital ratios used under Basel III). On one side are authors like Sundaresan and Wang (2014), who argue that using market prices in calculating trigger values might lead to multiple equilibria problems and potentially destabilizing bear runs on bank stock. On the other side, Calomiris and Herring (2013) argue that this problem can be mitigated by using 90-day moving averages of what they call "quasi-market data"[5], arguing that using book values creates room for creative accounting - for example pressure to delay recognition of losses. We do not take a position in this debate, our analysis applies to both types of triggers. Anyhow there are beyond doubt banks that have no choice because they are not listed (for example in the Netherlands two of the largest four banks are completely state owned (ABNAMRO and SNS Bank) while one of the remaining two has no listing either, being a cooperative (RABO).

[4]ECB Financial Stability Report, Nov. 2014

[5]Calomiris and Herring (2013) define quasi-market data as a ratio of market value of equity and book value of debt

The effectiveness of cocos hinges on bank failure being caused by banks having insufficient equity to absorb losses once they have occurred. However, the majority of bank assets is funded by demand deposits. One cannot ignore the possibility that a bank may fail because depositors run before losses actually occur, as they anticipate what may happen once the losses do occur. Jacklin and Bhattacharya (1988) and Chari and Jagannathan (1988) build on the Diamond and Dybvig (1983) model of bank runs to show that depositors who are able to update their information about the realization of bank returns act accordingly. However, early variants of the Diamond-Dybvig model have the disadvantage that runs are zero probability events, sunspot equilibria. That makes it impossible to assess the impact of fundamentals on the probability of runs and the associated bank collapse. Goldstein and Pauzner (2005) take the Diamond-Dybvig model a substantial step further by casting the standard banking problem into a global games framework, allowing them to obtain a measure for the probability of a bank run which can be linked to economic fundamentals.

In this paper, we argue that a coco conversion conveys information that will lead depositors to update their beliefs in a manner that increases the probability of bank runs. Furthermore we examine three major types of cocos and show that some designs are better than others in terms of their effect on depositor run incentives. We make a second point that is crucial for the relation between coco conversions and systemic risk. If other banks hold assets with correlated returns, depositors of other banks will interpret the coco conversion as a negative signal on their asset returns too. This updates the beliefs of the depositors of the other banks, which raises the probability of runs on said banks, even if they were non-coco-issuing. This would not happen if conversion did not occur in the coco-issuing bank. In other words, through contagion effects conversion imposes an information externality on other banks, which raises systemic risk.

The large and growing literature on contagion has by and large highlighted three contagion types: Balance sheet contagion (through firesale effects, cf Diamond and Rajan (2011)), funding squeezes whereby distress in one bank causes liquidity to dry up for another bank (Luck and Schempp (2014)), and information contagion (Ahnert and Georg (2012)). The contagion channel that plays a role in our analysis falls in the third category: A coco conversion gives out a signal, i.e. provides information about asset quality, that triggers a run not only in the bank concerned but also in banks with correlated assets.

This contagion channel is a second reason why we expect cocos to raise rather than reduce systemic risk. This is worrisome also because cocos are mentioned by Basel III as potentially useful for increasing the loss absorption capacity of SIFIs[6]. While it is true that conversion may keep the issuing banks afloat in times of distress by immediately reducing their outstanding liability, it does not reduce the liability to depositors. As such, conversion increases the risk that the converting banks, and other banks to the extent that they have correlated assets, will face a run.

While cocos have different trigger points and conversion mechanisms, many of them have a "point of nonviability" clause which effectively gives regulators control over when cocos convert. But regulators may end up having to make difficult choices in such circumstances. If conversion actually raises systemic risk, microprudential and macroprudential considerations may well be at odds, possibly leading to high pressure for regulatory forbearance.

It is sometimes argued that coco conversion encourages existing equity holders to infuse additional capital into the bank to such an extent that worries about the signaling effect of conversion is not necessary since the additional capital will stave off conversion.

However, whether this incentive exists this depends on the coco design. We show that in several currently popular coco designs, wealth transfers upon conversion actually go from junior creditors to equity holders. Clearly, this reduces the incentives for equity holders to supply capital in times of distress, possibly to the point of actually reversing the incentive. While by design equity holders cannot pull out capital, the existence of such such a peverse incentive might induce them to push for additional risk taking by bank management. We show that only cocos where existing equity holders are sufficiently strongly diluted by a conversion, wealth transfers go from equity holders to junior creditors, presumably a necessary condition for an an incentive to supply additional capital to stave off conversion.

[6]Basel Committee on Banking Supervision (2011)

## 2 Basic Model

### 2.1 Setup

As in Diamond-Dybvig (henceforth DD), there are three periods ($t = 0, 1, 2$). There is a continuum $[0, 1]$ of depositor-agents who are each born with 1 unit of wealth at $t = 0$. The agents are risk-averse with $-cu''(c)/u'(c) > 1$. for some $c > 0$. $u(0) = 0$ for all agents. A fraction $\lambda$ of the agents are early consumers who can only consume at $t = 1$ with corresponding utility $u(c_1)$. The remaining $1 - \lambda$ are late consumers who may consume at either $t = 1$ or $t = 2$, with corresponding utility $u(c_1 + c_2)$. There is no aggregate uncertainty at $t = 0$ so the proportion of early ($\lambda$) and late ($1 - \lambda$ ) consumers is known. However, the agent type will only be revealed at $t = 1$. This information is known only to the agent, so agents face idiosyncratic risk.

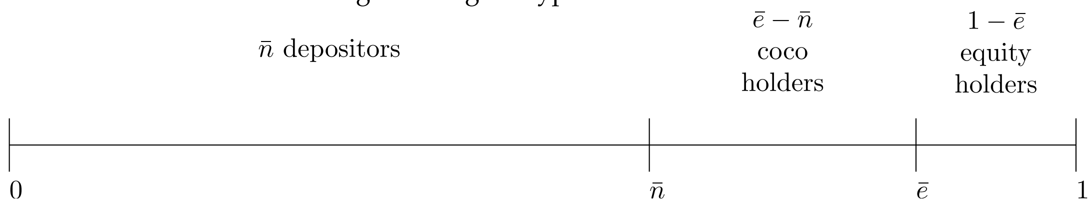
We depart from DD by introducing a risky technology that generates a return of $R > 1$ with probability $p(\theta)$ and 0 with probability $1 - p(\theta)$ after two periods, like in Goldstein and Pauzner (2005, GP henceforward). The investment may be liquidated at $t = 1$ without any costs other than the foregone yield. $\theta$ is a measure of economic fundamentals such that $p(\theta)$ is strictly increasing in $\theta$. $p(\theta) \in [0, 1]$ for any $\theta$, where $\theta \sim U[0, 1]$.

Because the risky investment can be liquidated without cost, agents are better off investing their endowment into the asset. Also, we assume that R is high enough so that $E_\theta p(\theta)u(R) > u(1)$, making it worthwhile for late consumers to wait until $t = 2$. Without any pooling of risk, the best attainable utility levels are $u(1)$ for the early consumers and $p(\theta)u(R)$ for the late consumers (for a given state of nature $\theta$).

### 2.2 Bank

If there was a social planner with perfect information about agent types, he could offer higher utility levels for the agents, because idiosyncratic risk averages out upon aggregation. The social planner would offer $r_1 > 1$ to the early consumers, and $\frac{1-\lambda r_1}{1-\lambda} R < R$ (with probability $p(\theta)$) to the

Figure 1: Agent Types and Measures




patient ones. We will refer to this set of payouts as Diamond-Dybvig contracts or DD contracts. This way, risk sharing is attained in the sense that consumers, who do not know their type yet at the moment of depositing, can transfer some income from the risky stream towards the safe stream. This is impossible in the autarkic case. Such a social planner doesn't exist, but DD show that a bank can mimic the socially optimal outcome by offering demand deposit contracts (which is to offer $r_1$ at $t = 1$ and $\frac{1-\lambda r_1}{1-\lambda} R$ with probability $p(\theta)$ at $t = 2$). Assuming $r_1$ is chosen well and incentive compatibility holds, agents will not lie about their types - only the early consumers will withdraw at $t = 1$, so that risk sharing can be implemented. But because the bank does not know the agent type, it is subject to a sequential service constraint. It will give $r_1$ to those who withdraw at $t = 1$ until the asset base is exhausted. Thus, latecomers beyond the $\frac{1}{r_1} th$ withdrawing depositor will get nothing.

We depart from DD and GP by introducing two additional types of agents that also function as additional sources of funding for the bank: coco holders and equity holders. These additional agents are not offered a demand deposit contract - instead, they are long-term investors and so cannot withdraw at $t = 1$ by design. We scale back the measure of depositors from 1 to $\bar{n} < 1$ although they are still offered DD contracts. Let $\bar{e} - \bar{n}$ be the measure of coco holders, and let $1 - \bar{e}$ be the measure of equity holders such that the total measure of agents is 1. Figure 1 shows the types of agents in the continuum.

In practice so far, issued cocos fall into one of three types: convert-to-equity (CE), principal write-downs (PWD), and principal writedowns coupled with a cash payout to the coco holders at the time the write down occurs (CASH; RABO of the Netherlands is the only bank that has issued cocos of this type). In the first part of our paper the type of coco doesn't matter because the focus is on the signal value of conversion to depositors. Also, in the first part, cocos and equity are only useful as a means of increasing the bank's capacity to serve depositors at $t = 1$. In the second part of the paper, we explicitly consider different types of cocos and the wealth transfers they imply on conversion; then equity starts playing a role too. The important thing to note is that cocos that do not convert are essentially long-term contracts which mature at $t = 2$, are illiquid at $t = 1$ and subordinated to deposits. However if the bank survives until $t = 2$, coco holders may share in the gains after the conversion, depending on the type of coco issued: CE holders do and the other two types do not.

Even with long term funding without early withdrawal possibilities, runs are still possible as long as $\frac{1}{r_1} < \overline{n}$. We assume this throughout the paper. We furthermore assume that the DD contracts offered by the banks are such that the incentive compatibility constraint $u(r_1) < p(\theta)u\left(\frac{1-\lambda r_1}{1-\lambda}R\right)$ holds: late consumers prefer to wait. Finally, there is no deposit insurance in this model[7].

### 2.3 The Regulator

There is a regulator who is interested in preserving financial stability. In accordance with the structure of many of the issued cocos, we assume conversion occurs when the regulator decides to trigger the conversion. We assume that the regulator when he/she decides to force conversion knows more than the other agents: that a regulator may discover at $t = 1$ that asset returns at $t = 2$ will be lower than what is compatible with a capital ratio above the cocos trigger value. In particular, we assume that the regulator finds out that the returns in the good state of nature will be $R_L < R$. Based on the finding, the regulator decides whether or not to convert the cocos. The regulator's decision to intervene (or for that matter collect the additional information about asset quality to begin with) is not modeled in this paper.[8] But as will be shown later, his decision to convert cocos introduces a negative signal about asset returns even though the economic

[7]Alternatively, there may be deposit insurance for retail deposits but there are also large and substantial wholesale deposits which are not insured. Or one can think of other forms of short term funding with roll over risk exposure like REPOs or commercial paper.

[8]One way to think of our set up is in line with the costly audit literature where audits, or in our case more likely an on-site regulatory inspection, are performed only infrequently and possibly randomly

fundamentals $\theta$ remain the same.

### 2.4 Timing

First, let us consider the situation prior to conversion. By assumption, at $t = 0$, a fraction $\bar{e} - \bar{n}$ of agents has invested in cocos and a fraction $1 - \bar{e}$ has invested in equity. The remaining $\bar{n}$ are depositors whose types are unknown at $t = 0$.

With these agents' endowments, the bank has a total of 1 unit of wealth. The bank invests the entire amount in the risky asset. It also promises a fixed return $r_1 > 1$ to agents who withdraw at $t = 1$, and a stochastic return that in the absence of runs by late consumers equals $\tilde{r}_2 = \max\left[\frac{\bar{n}-\lambda\bar{n}r_1}{\bar{n}-\lambda\bar{n}} R,\ 0\right]$, depending on the state of nature that materializes at $t = 2$. Note that this is similar to the DD contract since $\frac{\bar{n}-\lambda\bar{n}r_1}{\bar{n}-\lambda\bar{n}} R = \frac{1-\lambda r_1}{1-\lambda} R$ . Henceforth we use $r_D$ for $\frac{1-\lambda r_1}{1-\lambda} R$. Define $n$ as the proportion of agents who withdraw at $t = 1$. Since early consumers always withdraw at $t = 1$ , $n \geq \lambda\bar{n}$. And because the coco holders and equity holders cannot withdraw early, we also have $n \leq \bar{n}$.

At $t = 1$, before agents can act, the regulator comes in and decides whether to convert cocos or not. If conversion occurs, the return in the good state must have been found to be some $R_L < R$. In such a case, depositors' return will be scaled downwards accordingly (they will receive $\frac{1-\lambda r_1}{1-\lambda} R_L$ instead of $\frac{1-\lambda r_1}{1-\lambda} R$). Effectively, the depositors have a variable-rate contract with the bank. Without conversion, no information is revealed. This still preserves the risk-sharing feature of Diamond-Dybvig, which concerns not so much the interest rate risk as the type-related liquidity risk.

Also at $t = 1$, depositor types are revealed. The bank gives $r_1 > 1$ to depositors withdrawing at this time as long as it is able to do so. To this end, the bank must liquidate part of the amount invested in the risky asset. This means that the bank can only serve at most $n = \frac{1}{r_1}$ agents at $t = 1$. The $t = 2$ payouts to the depositors in the no-conversion case are summarized in Table 1. Depositors who wait until $t = 2$ to withdraw will receive a return which depends on how many depositors ran at $t = 1$. Coco holders and equity holders, being junior to depositors, will receive amounts only once all the depositors have been served. How that surplus is divided between them

Table 1: Time-dependent payouts to each depositor

| Withdrawal in | if $\lambda\bar{n} < n < \lambda\bar{n} + \frac{e'}{r_1}$ | if $\lambda\bar{n} + \frac{e'}{r_1} < n < \frac{1}{r_1}$ | if $n \geq \frac{1}{r_1}$ |
|---|---|---|---|
| $t = 1$ | $r_1$ | $r_1$ | $\begin{cases} r_1 & \text{w.p. } \frac{1}{nr_1} \\ 0 & \text{w.p. } 1 - \frac{1}{nr_1} \end{cases}$ |
| $t = 2$ | $r_D = \frac{1-\lambda r_1}{1-\lambda} R$ | $\begin{cases} \frac{1-nr_1}{1-(\lambda\bar{n}+\frac{e'}{r_1})r_1} r_D & \text{w.p. } p(\theta) \\ 0 & \text{w.p. } 1 - p(\theta) \end{cases}$ | 0 |

depends on coco pricing and corresponding coco returns. With probability $1 - p(\theta)$, the return (to creditors and equity holders alike) will be zero.

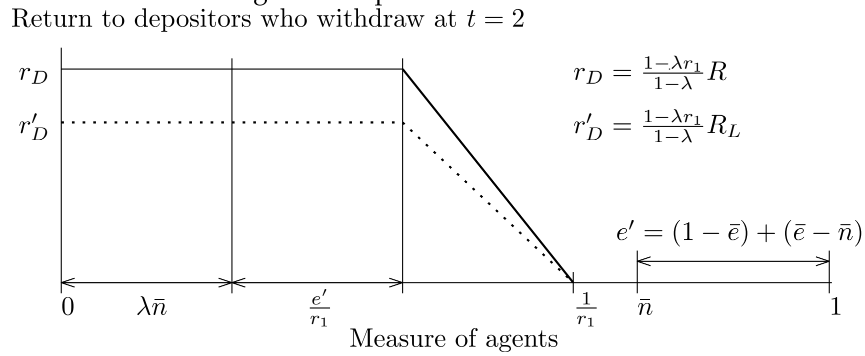
The bank will be able to pay out something at $t = 1$ as long as $n < \frac{1}{r_1}$, as in GP (see Eqn. 1 below). The bank can ensure that it pays out $r_D$ to the late consumers if $n = \lambda\bar{n}$. Our model departs from GP by having additional sources of funding from coco holders $(\bar{e} - \bar{n})$ and equity holders $(1 - \bar{n})$. Because depositors are senior to all other agents, these proceeds (call them $e' = (\bar{e} - \bar{n}) + (1 - \bar{n}) = 1 - \bar{n}$) may be exhausted in order to pay out $\frac{e'}{r_1}$ more depositors at $t = 1$, and still manage to pay out $r_D$ to the remaining depositors at $t = 2$.[9]

But for values of $n$ between $\lambda\bar{n} + \frac{e'}{r_1}$ and $\frac{1}{r_1}$, withdrawers at $t = 2$ may not obtain the entire $r_D$. As the bank's resources are finite at 1 unit, there can be only at most $n = \frac{1}{r_1}$ depositors who may be served at $t = 1$. In this case, the asset is totally liquidated - nothing is left to earn $R$ in case the good state of nature materializes at $t = 2$. Between $n = \lambda\bar{n} + \frac{e'}{r_1}$ and $n = \frac{1}{r_1}$, each additional runner requires further liquidation of the long-term asset, leaving a smaller quantity of the asset to potentially earn $R$. One can then determine the rate at which $r_D$ erodes - it depends on $n$. Figure 2 shows the payouts to depositors who wait until $t = 2$ as a function of $n$ under different levels of asset returns.

At this point it is useful to define $v(\theta, n)$ as the difference in utility of waiting versus running for given values of $\theta$ and $n$ , comparable to GP´s equation 3. This equation may be derived from Table

[9]Note that the region where money paid out to early runners eats into equity returns (because the assets generating those returns have to be liquidated) is shorter than $e'$ because runners get paid out $r_1 > 1$

Figure 2: Depositor returns at $t = 2$




1.

$$v(\theta, n) = \begin{cases} p(\theta)u\left[\frac{1-nr_1}{1-(\lambda\bar{n}+e'/r_1)r_1} r_D\right] - u(r_1) & \text{if } \lambda\bar{n} + \frac{e'}{r_1} < n < \frac{1}{r_1} \\ 0 - u(r_1)\frac{1}{nr_1} & \text{if } \frac{1}{r_1} < n < \overline{n} \end{cases}$$

If at $t = 2$ the good state of nature return on the risky asset turns out to be lower, say $R_L = R - \Delta < R$, the payout schedule to $t = 2$ withdrawers shifts down to the slotted line in Figure 2. We return to this in the next section.

Throughout we are assuming that $\bar{n} > \frac{1}{r_1}$. If there is a relatively small measure of depositors $\left(\bar{n} \leq \frac{1}{r_1}\right)$, then depositors know that if they all stage a run, all of them will receive $r_1$. But since the incentive compatibility constraint $u(r_1) < p(\theta)u\left(\frac{1-\lambda r_1}{1-\lambda} R\right)$ holds, only the early consumers will withdraw at $t = 1$, and there will be no run (in the sense that late consumers also withdraw early). This simply says that adequately-capitalized banks ($e' > 1 - \frac{1}{r_1}$) are in no danger of a run. We will not consider this case any further.

## 2.5 Probability of a Bank Run: DD in a Global Games Framework

From Table 1, one can see that $n$ is of primary importance in the payout of an agent. In the DD paper, $n$ is either only the early consumers ($\lambda\bar{n}$), or all of the depositors ($\bar{n}$). Multiplicity of equilibria arises because depositors have nothing to coordinate on except for sunspots or bad expectations.

Goldstein and Pauzner (2005) recast the DD bank run problem in a global games framework[10] and in doing so obtain unique Bayesian equilibria with well defined probabilities tied to fundamentals. We follow their approach in this paper. In the global games framework, depositors obtain private and imprecise information about the economic indicator $\theta$. In particular, at $t = 1$, each depositor obtains a private signal $\theta_i$ uniformly distributed along $[\theta - \varepsilon,\ \theta + \varepsilon]$, where the distribution is known to all. Clearly $\theta_i$ depends on the realization of $\theta$. Thus depositors know that the true value of fundamentals is at most $\varepsilon$ away from their own signal. Depositors' decisions crucially depend on their draw of $\theta_i$ and on what they can deduce from that draw on the likely signals other depositors must have received and what they are therefore likely to do.

There are two extreme regions where depositors' decisions do not depend on what other agents do. First one can define a $\theta = \underline{\theta}$ below which a late consumer always finds it optimal to run even if all other late consumers were to wait. Thus $\underline{\theta}$ solves the equation $u(r_1) = p(\underline{\theta})u\left(\frac{1-\lambda r_1}{1-\lambda}R\right)$. GP call the region $[0,\ \underline{\theta})$ the *lower dominance* region. There are always feasible values in the lower dominance region such that all signals will fall into that region if $\underline{\theta} > 2\varepsilon$; for this to obtain it is sufficient that $\underline{\theta}(1) > 2\varepsilon$ since $\underline{\theta}(r_1)$ is increasing in $r_1$ (as can be seen by differentiating the implicit equation defining $\underline{\theta}$.) $\underline{\theta}(1) > 2\varepsilon$ in turn can be rewritten as $p^{-1}\left(\frac{u(1)}{u(R)}\right) > 2\varepsilon$, which shows that $\varepsilon$ can always be chosen small enough for the lower dominance region to be non-empty.

One can similarly define a $\bar{\theta}$ above which a patient depositor finds it optimal to wait even if all other patient agents were to run (in GP's terminology the upper dominance region). GP assume that in the region $(\bar{\theta},\ 1]$, the investment is certain to yield $R$ ($p(\theta) = 1$ for $\theta > \bar{\theta}$). Then it is never optimal to run since $R > r_1$. Alternatively one can assume a Central Bank standing ready to provide liquidity in a run for high enough $\theta$ since in that case the bank is clearly solvent. Either way, we follow GP in postulating the existence of such an upper dominance region. Since $\varepsilon$ can be chosen arbitrarily small, we can also safely assume that it is possible that all draws fall into the upper dominance region, which requires $\bar{\theta} < 1 - 2\varepsilon$.

Within the region $\left[\underline{\theta},\ \bar{\theta}\right]$, depositors must rely on equilibrium behavior of other depositors receiv-

[10] Global games as used by Goldstein and Pauzner (2005) has its roots from the seminal work of Carlsson and van Damme (1993) and Morris and Shin (1998) on speculative attacks on currency.

ing nearby signals, which in turn depends on their nearby signals, and so on; continuity requires that behavior smoothly pastes to the behavior in the extreme regions. Following GP, one can prove that the unique equilibrium strategy is a switching strategy in which late consumers run if they receive a signal $\theta_i \leq \theta^*$ and wait otherwise [11]. $\theta^*$ is defined such that a depositor receiving a signal $\theta^*$ is indifferent between waiting and running at $t = 1$ over all possible outcomes of other depositors' behavior:

$$\int_{n=\lambda\bar{n}+\frac{\epsilon'}{r_1}}^{\frac{1}{r_1}} \left[ p(\theta=\theta^*)u\left(\frac{1-nr_1}{1-(\lambda\bar{n}+\frac{\epsilon'}{r_1})r_1} r_D\right) - u(r_1)\right] dn - \int_{n=\frac{1}{r_1}}^{\bar{n}} \frac{1}{nr_1} u(r_1) dn \quad = \quad 0 \qquad (1)$$

$$\text{where} \quad r_D = \frac{1-\lambda r_1}{1-\lambda} R$$

Eqn. 1 defines $\theta^*$ implicitly and is formed from the payouts described in Table 1 and the function $v(\theta,\ n)$ defined at the end of section 2.4.[12] Because the depositors obtain signals $\theta_i$ from a uniform distribution around $\theta$ and $\theta$ is itself uniformly distributed over $[0,\ 1]$, a higher $\theta^*$ means depositors run in a larger set of signals. For small $\varepsilon$, $\theta^*$ can be interpreted as the probability of a bank run. Also, each $\theta^*$ corresponds to an $n$ which is the measure of the number of runners at $t = 1$ for given value of $\theta$. This is[13]

$$n = \lambda\bar{n} + (1-\lambda)\,\bar{n}\left[\frac{1}{2} + \frac{\theta^* - \theta}{2\varepsilon}\right] \qquad (2)$$

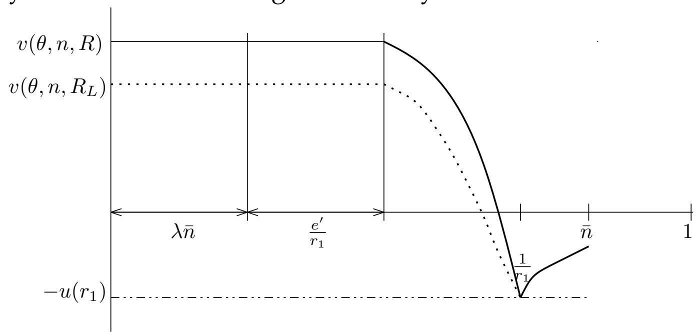
for $\theta^* - \varepsilon \leq \theta \leq \theta^* + \varepsilon$. For $\theta < \theta^* - \varepsilon$,$n = \bar{n}$ and for $\theta > \theta^* + \varepsilon$, $n = \lambda\bar{n}$.

## 3 Effect of coco conversion on the probability of a run $\theta^*$

Consider now the case when the regulator finds out that the return will be low. While $\theta^*$ depends on $r_1$, it also depends on $R$ and $\bar{n}$. As mentioned in Section 2.3, we introduce the regulator action at $t = 1$, before depositors can act. In the absence of coco conversion, depositors and other investors

[11] Since the proof in our setup is almost identical to the corresponding proof in GP, we refer to GP for a detailed proof.
[12] Eqn. 1 builds on the fact that $\theta$ is uniformly distributed. Since $n$ is linear in its arguments, $n$ must also be uniformly distributed. The expression also assumes that $p(\theta) \approx p(\theta^*)$ for $\varepsilon$ small enough, following GP.
[13] This is similar to the GP equilibrium $n$ scaled down by $\bar{n}$.

Figure 3: Utility differential of waiting versus early withdrawal for different values of $R$




believe that the return of the risky asset is $R$ with probability $p(\theta)$ and 0 with probability $1 - p(\theta)$. But when the regulator forces cocos to convert, a signal is given that the return of the risky asset is now some $R_L < R$, without an accompanying change in the state of fundamentals $\theta$. The impact of a lower $R$ on period 2 payouts can be seen in Figure 2 (the shift from the solid to the slotted line). Figure 3 recasts the payouts described in Table 1 in terms of differential utility between waiting and early withdrawal for a given $\theta$ as $n$ changes. From the diagram it should be clear that once integrated over the entire range of $n$, the utility differential shifts against waiting, so the indifference point in state space, $\theta^*$, will have to shift up to restore balance. So the threshold $\theta^*$ increases when the return of the risky asset is reduced to $R_L$.

To prove this formally, we compute the threshold $\theta^*$ from the function that implicitly defines it . This function was introduced as Eqn. 1. For convenience let us call this function as $\hat{f}\,(\theta^*,\, r_1,\, R)$

$$\hat{f}\,(\theta^*,\, r_1,\, R) = \int_{n=\lambda\bar{n}+\frac{e'}{r_1}}^{\frac{1}{r_1}} \left[ p(\theta(\theta^*, n))u\left( \frac{1-nr_1}{1-(\lambda\bar{n}+\frac{e'}{r_1})r_1} \frac{1-\lambda r_1}{1-\lambda} R \right) - u(r_1) \right] dn - \int_{n=\frac{1}{r_1}}^{\bar{n}} \frac{1}{nr_1} u(r_1) dn = 0,$$

where $\theta$ was written as a function of $n$'s intermediate value (away from $n = \lambda\bar{n}$ or $n = \bar{n}$), and $\theta$ is assumed to be within $\varepsilon-$distance of $\theta^*$. That is, $\theta = \theta^* + \varepsilon \left[ 1 - \frac{2}{1-\lambda} \left( \frac{n}{\bar{n}} - \lambda \right) \right]$ (see Eqn. 2). At $\theta = \theta^*$, a patient agent is indifferent between waiting or running, by definition of $\theta^*$.

Note that since $\hat{f}\,(\cdot)$ is increasing in both $R$ and $\theta$, so in order to keep $\hat{f}\,(\cdot) = 0$, a decrease in $R$

must be compensated by an increase in $\theta$. From the implicit function theorem,

$$\frac{\partial\theta^*}{\partial R} = -\left(\frac{\partial\hat{f}}{\partial R}\right) \div \left(\frac{\partial\hat{f}}{\partial\theta^*}\right)$$

It is easy to see that $\frac{\partial\hat{f}}{\partial\theta^*} > 0$ : the dependence runs completely through $\theta(n,\theta^*)$,while $\theta(n,\theta^*)$ rises in $\theta^*$ and $\hat{f}$ rises in $\theta$ because $p'(\cdot) > 0$ by construction. Now

$$\begin{aligned}
\frac{\partial\hat{f}}{\partial R} &= \int_{n=\lambda\bar{n}+\frac{e'}{r_1}}^{\frac{1}{r_1}} \left[ p(\theta(\theta^*,n)) \frac{\partial u\left(\frac{1-nr_1}{1-(\lambda\bar{n}+\frac{e'}{r_1})r_1}\frac{1-\lambda r_1}{1-\lambda}R\right)}{\partial R} \right] dn \\
&= \int_{n=\lambda\bar{n}+\frac{e'}{r_1}}^{\frac{1}{r_1}} \left[ p(\theta(\theta^*,n))u'(\cdot)\left(\frac{1-nr_1}{1-(\lambda\bar{n}+\frac{e'}{r_1})r_1}\frac{1-\lambda r_1}{1-\lambda}\right)\right] dn \\
&> 0
\end{aligned}$$

since $\left[\frac{1-nr_1}{1-(\lambda\bar{n}+\frac{e'}{r_1})r_1}\right]\left(\frac{1-\lambda r_1}{1-\lambda}\right)$ is positive over the entire interval of integration. And with $\hat{f}_R > 0$, $\hat{f}_{\theta*} > 0$ and $\frac{\partial\theta^*}{\partial R} = -\hat{f}_R/\hat{f}_{\theta^*}$, Proposition 1 below follows:

**Proposition 1.** *$\theta^*$ is decreasing in R: $\frac{\partial\theta^*}{\partial R} < 0$ for all values of R*

As a consequence, any negative signal about asset returns that is obtained by depositors will lead to a higher run probability . Coco conversion delivers one such signal because the conversion in this model implies that the return in the good state of $t = 2$ is $R_L < R$. Upon learning this news, each depositor will expect a lower differential payout than its value before conversion, since $R_L < R$ (see Figure 2). If for return $R$ a depositor is just indifferent between running and waiting for a given $\theta^*$, then for return $R_L < R$ it must be that the same depositor prefers to run for the same value of $\theta^*$. In order to restore the depositor's indifference between running and waiting for return $R_L < R$, they must obtain a higher signal about the fundamentals, that threshold value will go up: $\theta_L^* > \theta^*$ at the point of indifference, which is what Proposition 1 says. But since depositors' $\theta_i$ are uniformly distributed between $[0, 1]$, a greater measure of them will have $\theta_i < \theta_L^*$, which implies a higher probability of a run. Note that the increase in $\theta^*$ also results in an increase in $n$ for given value of $\varepsilon$ and $\theta$.

Proposition 1 also has an important corollary on the impact of the trigger level of a coco. Let us compare the effect of two different trigger levels for two otherwise identical cocos. Consider two trigger levels defined on a bank's capital-to-asset ratio (CAR) $\tau_H$ and $\tau_L$ such that $\tau_H > \tau_L$. A coco with trigger level $\tau_L$ converts when the issuing bank's CAR falls below $\tau_L$. As $\tau_H > \tau_L$, the conversion of a $\tau_L$ coco implies the conversion of a $\tau_H$ coco. On the other hand, the conversion of a $\tau_H$ coco does not necessarily lead to the conversion of a $\tau_L$ coco. In other words, if the trigger level is low, the implied asset quality signal is more negative than the signal transmitted by a coco with a higher trigger level. Corollary 2 then follows immediately from Proposition 1:

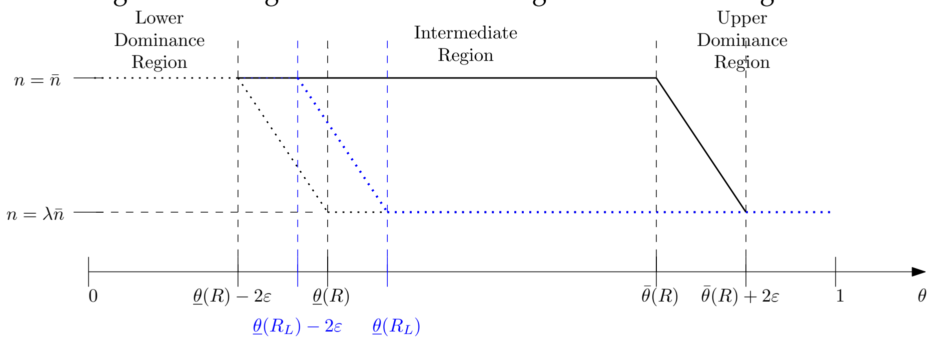
**Corollary 2.** *Conversion of a coco with a high trigger level will lead to a smaller increase in run probability than conversion of an otherwise identical coco but with a lower trigger level.*

Formally, let $CAR_0$ be the CAR thought to apply before a regulator's inspection reveals an equity shortfall. The term $CAR_0 \cdot (\tau_H - \tau_L)$ represents the difference in asset quality that is given by the conversion of both the $\tau_H$ and the $\tau_L$ cocos. Define $\theta_H^*$ ($\theta_L^*$) as the run probability that will obtain after conversion of a $\tau_H$ ($\tau_L$) coco. Direct application of Proposition 1 with the definitions just introduced shows that the following holds (exactly, since the derivative is positive for all $R$ so we can apply the Mean Value theorem):

$$\theta_L^* - \theta_H^* = -\left(\frac{\partial \theta^*}{\partial R}\right) \cdot (\tau_H - \tau_L) \cdot CAR_0 > 0$$

This result suggests that the BIS is right to insist on sufficiently high trigger levels before cocos are accepted as part of Tier 1 (T1). According to the Basel committee (BCBS 2011), cocos will be either Tier 2 (T2) or Additional Tier 1 (AT1) capital, depending on their trigger ratio: a trigger above 5.125% satisfies the *going concern* requirement for AT1 and thus allows classification as AT1. Lower triggers lead to a classification as *gone concern instruments* and consequently to a T2 status. A conversion lowers the issuing bank's leverage ratio, and increases its common equity tier 1 (CET1)

Figure 4: Change in the dominance regions due to a change in $R$




capitalization. If the coco design did not satisfy Tier 1 (T1) requirements (for example because of a trigger ratio that is too low to satisfy the going concern requirement), conversion will increase the bank's overall T1 capital requirement also.[14]

It is also worth noting that a change from $R$ to $R_L$ alters the dominance regions. Because the supremum for the lower dominance region is determined by the equation $u(r_1) = p(\underline{\theta})u\left(\frac{1-\lambda r_1}{1-\lambda}R\right)$, a change from $R$ to $R_L$ necessarily increases $\underline{\theta}$. Also, the infimum of the upper dominance region should not increase but may decline because if a minimum of $\bar{\theta}$ ensures that $R$ will be obtained with certainty, then there must at least as many $\theta$-values that will ensure $R_L$ will be obtained with certainty. This means that the post-conversion $\bar{\theta}$ must be no lower than the pre-conversion one. Figure 4 shows the shift in the dominance regions and the effect on the upper and lower bounds of $n$.

## 4 Coco design and run probabilities after conversion

Until now we have left unspecified what specifically happens after conversion. What happens after the issuing bank's capital falls below the trigger value depends on the type of coco issued. For cocos to qualify as capital at all, they need to include a so called Point of Non Viability trigger,

[14] There is one possible exception to this observation. Under some some coco designs. the coco does not convert into equity; instead the principal is partially written off with the remainder converting into unsecured debt. If such a partial write down coco is converted, the T1 capital asset ratio actually falls.

i.e. the possibility for the regulator to enforce conversion if the regulator decided that viability is threatened[15]. Currently used coco designs fall into three distinct types[16].

First are convert-to-equity (CE) cocos such as those issued for example by Standard Chartered recently[17]. These cocos completely convert to equity at some conversion rate $\psi$ or, equivalently, at a price $P = \psi^{-1}$. Most commentators and academics (cf Martynova and Perotti (2012)) have this type of coco design in mind when discussing cocos in general.

Next are principal writedown (PWD) cocos. Upon breaching the trigger value, these cocos are partially or entirely written down. In case of partial writedown, the remaining part effectively turns into subordinated debt. Credit Suisse has issued this type of coco.

Finally there are also principal writedown cocos with cash outlays (CASH). Similar to the PWD cocos with partial write off, CASH cocos are also partially written off upon the bank's breach of the trigger value. The remaining value is paid out in cash. Notably, Rabobank of the Netherlands has issued this type of coco.

In general, none of all these complicated consequences of breaching the trigger value matter for the depositors, since conversion merely redistributes between junior claimants. The one exception is the CASH coco, because there conversion implies less cash available for distribution to depositors in distress situations. In the remainder of this section, we examine the impact of coco design on the probability of a bank run after conversion and on the equity position of the bank if partial runs do occur.

## 4.1 Baseline Case: Regulatory Forbearance (RFB)

As a benchmark, we consider the case where the regulator finds out that returns will be low but decides not to publicize this finding such that the cocos do not convert. Depositors base their

[15]Obtained from http://www.bis.org/bcbs/basel3/b3summarytable.pdf

[16]cf van den Berg et al. (2014) and Avdjiev et al. (2013)for an extensive description of all cocos issued up until 2013 and 2014 respectively

[17]http://www.reuters.com/article/2015/03/26/stanchart-bonds-idUSL6N0WS3O420150326

Table 2: Depositor payouts: regulatory forbearance (undisclosed low returns $R_L$)

| | If $n < \frac{1}{r_1}$ | If $n \geq \frac{1}{r_1}$ |
|---|---|---|
| $t = 1$ | $r_1$ | $\begin{cases} r_1 & \text{w.p. } \frac{1}{nr_1} \\ 0 & \text{w.p. } 1 - \frac{1}{nr_1} \end{cases}$ |
| $t = 2$ | $\begin{cases} \left(\frac{1-nr_1}{1-(\lambda\bar{n}+\frac{e'}{r_1})r_1}\right)\left(\frac{1-\lambda r_1}{1-\lambda}\right) R_L & \text{w.p. } p \\ 0 & \text{w.p. } 1-p \end{cases}$ | 0 |

behavior on the belief that in the good state of nature returns are $R$, not knowing that in fact they will be $R_L$. Table 2 shows the payouts to depositors.

If depositors do not know that returns will be low, the differential payout function remains the same as in Eqn. 3 .

$$v_{rfb} = \begin{cases} p(\theta)u\left(\left[\frac{1-nr_1}{1-(\lambda\bar{n}+\frac{e'}{r_1})r_1}\right]\left(\frac{1-\lambda r_1}{1-\lambda}\right)R\right) - u(r_1) & \text{if } \lambda\bar{n} + \frac{e'}{r_1} \leq n \leq \frac{1}{r_1} \\ 0 - \frac{u(r_1)}{nr_1} & \text{if } \frac{1}{r_1} \leq n \leq \bar{n} \end{cases} \tag{3}$$

The corresponding implicit function that determines $\theta^*_{rfb}$ is given by Eqn. 4.

$$\begin{aligned} \hat{f}\left(\theta^*_{rfb}, r_1, R\right) &= \int_{n=\lambda\bar{n}+\frac{e'}{r_1}}^{\frac{1}{r_1}} \left[p(\theta(\theta^*_{rfb}, n))u\left(\left[\frac{1-nr_1}{1-(\lambda\bar{n}+\frac{e'}{r_1})r_1}\right]\left(\frac{1-\lambda r_1}{1-\lambda}\right)R\right) - u(r_1)\right] dn \\ &\quad - \int_{n=\frac{1}{r_1}}^{\bar{n}} \frac{1}{nr_1} u(r_1) dn = 0 \end{aligned} \tag{4}$$

Obviously, since the derivations of $\theta^*_{rfb}$ are based on the same set of beliefs as in our base case without bad news, $\theta^*_{rfb} = \theta^*$. Depositors do not know that $R$ has fallen to $R_L$, so the run probability $\theta^*$ is not affected. In the event that $n < \frac{1}{r_1}$ at $t = 1$, then

$$(\bar{n} - n)\left[\frac{1-nr_1}{1-(\lambda\bar{n}+\frac{e'}{r_1})r_1}\right]\left(\frac{1-\lambda r_1}{1-\lambda}\right)R_L$$

will be given to the remaining depositors who didn't run at $t = 1$ (this amounts to $\bar{n} - n$ depositors), while what remains of the asset base

$$R_L\left[(1-nr_1)-\frac{(\bar{n}-n)(1-nr_1)(1-\lambda r_1)}{\left(1-(\lambda\bar{n}+\frac{e'}{r_1})r_1\right)(1-\lambda)}\right]$$

will be used to first pay out the junior coco holders (who collectively have $\bar{e}-\bar{n}$ worth of claims that earn a return $r_{coco}$ per unit[18]). Finally, anything that remains after that will go to equity holders.

The remaining equity base under regulatory forbearance $\left(E_{rfb}\right)$ will be

$$E_{rfb} \quad = \quad \max\left\{R_L\left[(1-nr_1)-\frac{(\bar{n}-n)(1-nr_1)(1-\lambda r_1)}{\left(1-(\lambda\bar{n}+\frac{e'}{r_1})r_1\right)(1-\lambda)}\right]-r_{coco}\left(\bar{e}-\bar{n}\right),0\right\}$$

Under regulatory forbearance, there is no way to reduce a bank's liabilities, so any negative asset development is immediately absorbed by equity. We will come back to this point later.

## 4.2 Convert-to-equity (CE) cocos

Consider now the case where the regulator does force conversion. With convert-to-equity (CE) cocos coco holders turn into equity holders upon conversion, and therefore, forfeit the right to receive the amount up to $r_{coco}\left(\bar{e}-\bar{n}\right)$ but become entitled to a share in any residual income. Table 3 shows the resulting payouts to depositors. The differential payout function used by depositors is different now, since depositors do receive the negative signal associated with coco conversion. Eqn. 5 is different from Eqn. 3: $R$ is replaced by the lower value $R_L$:

$$v_{ce}=\begin{cases} p(\theta)u\left(\left[\frac{1-nr_1}{1-(\lambda\bar{n}+\frac{e'}{r_1})r_1}\right]\left(\frac{1-\lambda r_1}{1-\lambda}\right)R_L\right)-u(r_1) & \text{if } \lambda\bar{n}+\frac{e'}{r_1}\leq n\leq\frac{1}{r_1} \\ 0-\frac{u(r_1)}{nr_1} & \text{if } \frac{1}{r_1}\leq n\leq\bar{n}\end{cases} \tag{5}$$

[18]Here $r_{coco}$ is an arbitrary return to coco holders. In this paper we are taking this return as a given, as we do not delve into the pricing of cocos.

Table 3: Depositor payouts after CE cocos conversion

| | If $\lambda\bar{n} + \frac{e'}{r_1} < n < \frac{1}{r_1}$ | If $n \geq \frac{1}{r_1}$ |
|---|---|---|
| $t = 1$ | $r_1$ | $\begin{cases} r_1 & \text{w.p. } \frac{1}{nr_1} \\ 0 & \text{w.p. } 1 - \frac{1}{nr_1} \end{cases}$ |
| $t = 2$ | $\begin{cases} \left(\frac{1-nr_1}{1-(\lambda\bar{n}+\frac{e'}{r_1})r_1}\right)\left(\frac{1-\lambda r_1}{1-\lambda}\right) R_L & \text{w.p. } p \\ 0 & \text{w.p. } 1-p \end{cases}$ | 0 |

As before, we can compute the threshold run value of the economic fundamental for a CE coco implicitly. Denote by $\theta^*_{ce}$ the probability of a run for the CE case. As before, the equation that implicitly defines $\theta^*_{ce}$ is given by Eqn. 6.

$$\begin{aligned} \hat{f}_{ce}(\theta^*_{ce}, r_1, R_L) \quad = \quad & \int_{n=\lambda\bar{n}+\frac{e'}{r_1}}^{\frac{1}{r_1}} \left[ p(\theta(\theta^*_{ce}, n)) u \left( \left[ \frac{1-nr_1}{1-(\lambda\bar{n}+\frac{e'}{r_1})r_1} \right] \left( \frac{1-\lambda r_1}{1-\lambda} \right) R_L \right) - u(r_1) \right] dn \\ & - \int_{n=\frac{1}{r_1}}^{\bar{n}} \frac{1}{nr_1} u(r_1) dn = 0 \end{aligned} \tag{6}$$

Then application of Proposition 1 immediately shows that $\theta^*_{ce} > \theta^*_{rfb}$. This highlights the bind regulators are in when they find out they should force cocos to convert. The negative signal that conveys to depositors actually increases financial fragility.

On the other hand, converting CE cocos increases the equity at $t = 2$ relative to the RFB case. This is clear because when CE cocos are converted, the coco holders do not have to be paid out at $t = 2$ anymore. Let $n_{ce}$ denote the number of runners implied by the probability of bank run $\theta^*_{ce}$. We can actually see the beneficial effect of a coco conversion, because provided that $\theta^*_{ce}$ yields $n_{ce} < \frac{1}{r_1}$, the bank survives until $t = 2$ with more capital as coco holders are no longer creditors. We can denote by $E_{ce}$ the resulting equity upon conversion of the CE cocos.

$$E_{ce} \quad = \quad \max \left\{ R_L \left[ (1 - n_{ce} r_1) - \frac{(\bar{n} - n_{ce})(1 - n_{ce} r_1)(1 - \lambda r_1)}{\left[ 1 - (\lambda\bar{n} + \frac{e'}{r_1}) r_1 \right] (1 - \lambda)} \right], 0 \right\} \tag{7}$$

From Section 4.1, since $\theta^*_{rfb} < \theta^*_{ce}$, it must also be true that $n_{rfb} < n_{ce}$. $E_{ce}$ differs from $E_{rfb}$ by the difference between $n_{rfb}$ and $n_{ce}$, and also by the amount that must be paid to the coco holders (which disappears in the conversion of CE cocos). Note that

$$E_{ce} - E_{rfb} = r_{coco}\left(\bar{e} - \bar{n}\right) + R_L\left(n_{rfb} - n_{ce}\right) + \left[\left(n_{ce} - n_{rfb}\right)\left(1 + \bar{n}r_1\right) + \left(\bar{n}^2_{rfb} - \bar{n}^2_{ce}\right)\right]\Gamma R_L$$
$$= r_{coco}\left(\bar{e} - \bar{n}\right) + R_L\left(n_{ce} - n_{rfb}\right)\left(\Gamma\left(1 + \bar{n}r_1\right) - 1\right) + \left(\bar{n}^2_{rfb} - \bar{n}^2_{ce}\right)\Gamma R_L$$

where $\Gamma = \frac{1-\lambda r_1}{\left[1-(\lambda\bar{n}+\frac{e'}{r_1})r_1\right](1-\lambda)} > 0$. We have $n_{ce} - n_{rfb} > 0$, and $\Gamma(1+\bar{n}r_1) - 1 = \frac{(1-\lambda r_1)(1+\bar{n}r_1)}{(1-(\lambda\bar{n}+\frac{e'}{r_1})r_1)(1-\lambda)} - 1 = \frac{(1-\lambda r_1)(1+\bar{n}r_1)}{\bar{n}(1-\lambda r_1)(1-\lambda)} - 1 = \frac{(1+\bar{n}r_1)}{\bar{n}(1-\lambda)} - 1 > 0$, using the definition of $e'$, so up to a first-order approximation (ignoring the quadratic terms in $n$), the conversion indeed improves the equity base of the bank if it survives into the good state of nature.

**Proposition 3.** *If $n_{ce} < \frac{1}{r_1}$ (i.e. the bank survives period 1), conversion of CE cocos improves the bank's equity position at $t = 2$ as the bank is able to eliminate up to $r_{coco}\left(\bar{e} - \bar{n}\right)$ worth of liabilities.*

This result points to an incentive for regulators to actually force conversion once they find out about lower returns $R_L$. The regulator faces conflicting incentives upon the discovery of $R_L$. On the one hand, conversion increases the probability of a run because it conveys a negative signal about asset returns. On the other hand, conversion also ensures that if runs occur, there is a possibility that there will be a surviving equity base, and that it will be higher than when the regulator is forbearing. Regulators thus are forced to choose between keeping fragility low at the expense of worsening the consequences of a run if it does occur , and increasing the likelihood of a run but leaving the bank better equipped to deal with the aftermath of one.

## 4.3 Principal Writedown (PWD) cocos

We have previously described PWD cocos as having a fraction written down upon conversion. Let $1 - \varphi$ denote the fraction of cocos that is written off when conversion occurs, so $\varphi$ is the fraction that is left, where $0 \leq \varphi \leq 1$. Table 4 describes the payouts to depositors in the PWD case after conversion.

Table 4: Depositor payouts after PWD cocos conversion

| | If $\lambda\bar{n} + \frac{e'}{r_1} < n < \frac{1}{r_1}$ | If $n \geq \frac{1}{r_1}$ |
|---|---|---|
| $t = 1$ | $r_1$ | $\begin{cases} r_1 & \text{w.p. } \frac{1}{nr_1} \\ 0 & \text{w.p. } 1 - \frac{1}{nr_1} \end{cases}$ |
| $t = 2$ | $\begin{cases} \left(\frac{1-nr_1}{1-(\lambda\bar{n}+\frac{e'}{r_1})r_1}\right)\left(\frac{1-\lambda r_1}{1-\lambda}\right)R_L & \text{w.p. } p \\ 0 & \text{w.p. } 1-p \end{cases}$ | 0 |

As in the CE coco case, the amount that each depositor would obtain is the same as that under no conversion because depositors are senior to remaining coco holders. Therefore the differential payout function used by depositors here (Eqn. 4) is identical to what it is in the case of CE cocos.

$$v_{pwd} = \begin{cases} p(\theta)u\left(\left[\frac{1-nr_1}{1-(\lambda\bar{n}+\frac{e'}{r_1})r_1}\right]\left(\frac{1-\lambda r_1}{1-\lambda}\right)R_L\right) - u(r_1) & \text{if } \lambda\bar{n} + \frac{e'}{r_1} \leq n \leq \frac{1}{r_1} \\ 0 - \frac{u(r_1)}{nr_1} & \text{if } \frac{1}{r_1} \leq n \leq \bar{n} \end{cases} \tag{8}$$

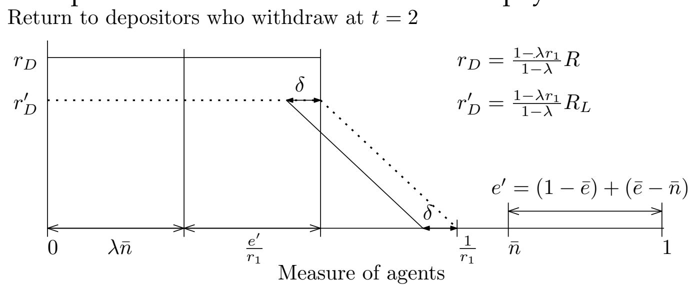
Let $\theta^*_{pwd}$ denote the threshold level of $\theta$ for the PWD case. We can again find $\theta^*_{pwd}$ from the implicit function in Eqn. 9.

$$\hat{f}_{pwd}(\theta^*_{pwd}, r_1, R_L) = \int_{n=\lambda\bar{n}+\frac{e'}{r_1}}^{\frac{1}{r_1}} \left[p(\theta(\theta^*_{pwd}, n))u\left(\left[\frac{1-nr_1}{1-(\lambda\bar{n}+\frac{e'}{r_1})r_1}\right]\left(\frac{1-\lambda r_1}{1-\lambda}\right)R_L\right) - u(r_1)\right] - \int_{n=\frac{1}{r_1}}^{\bar{n}} \frac{1}{nr_1}u(r_1)dn = 0 \tag{9}$$

Since the differential payout function is the same, it follows that $\theta^*_{pwd} = \theta^*_{ce}$. This means that PWD cocos are not an improvement over CE cocos if evaluated solely for their impact on probability of runs, because neither type of coco changes the incentives for depositors. The explanation is straightforward: while PWD and CE cocos imply different wealth transfers between coco holders and equity holders, depositors are senior to both groups of claimants, so depositors do not care how losses are allocated between the other types of agents.

**Proposition 4.** *PWD cocos have the same impact on the probability of bank runs as CE cocos:* $\theta^*_{pwd} =$

Figure 5: Depositor returns at $t = 2$ under a cash payout to coco holders




$\theta^*_{ce} > \theta^*_{rfb}$.

## 4.4 Principal Writedown cocos with Cash Outlays (CASH)

CASH cocos are a variant of PWD where in addition to writing off a fraction of coco claims, the remaining fraction is paid out in cash upon conversion. Formally, we can let $\delta r_1$ represent the total payout given to the coco holders at $t = 1$ conversion such that at most, $\frac{1}{r_1} - \delta$ running depositors at $t = 1$ can be accommodated. However, as a fraction of coco claims is written off, anything left after serving depositors at $t = 2$ will be divided among the equity holders.

Under CE and PWD cocos, each depositor who waits will receive $r_D$ as long as there are only $n = \lambda \bar{n} + \frac{e'}{r_1}$ running depositors at $t = 1$.However, under CASH, each waiting depositor will receive less compared to any of the other coco designs. Figure 5 shows what happens under that case.

Clearly deposit returns will change after conversion of the cocos. The remaining assets at the end of $t = 1$ would now be merely $1 - \delta r_1 - n r_1$ instead of $1 - n r_1$. As a result, each depositor who waits would now only get $\left[\frac{1-\delta r_1 - n r_1}{1-(\lambda \bar{n} + \frac{e'}{r_1}) r_1}\right] \left(\frac{1-\lambda r_1}{1-\lambda}\right) R_L$ instead of the larger amount $\left[\frac{1-n r_1}{1-(\lambda \bar{n} + \frac{e'}{r_1}) r_1}\right] \left(\frac{1-\lambda r_1}{1-\lambda}\right) R_L$ Notice the impact of the cash payout $\delta r_1$ on the amounts that the depositors receive. Table 5 shows the depositor payouts under the CASH design. Notice also the change in the thresholds of $n$.

Even though equity holders absorb $\delta r_1$, depositors will still be affected: if $\delta r_1$ is paid out in cash upon conversion, there is correspondingly less cash available to pay out in case of early with-

Table 5: Payout to depositors after CASH cocos conversion

| | If $\lambda\bar{n} + \frac{e'}{r_1} - \delta < n < \frac{1}{r_1} - \delta$ | If $n \geq \frac{1}{r_1} - \delta$ |
|---|---|---|
| $t = 1$ | $r_1$ | $\begin{cases} r_1 & \text{w.p. } \frac{1}{n}\left(\frac{1}{r_1} - \delta\right) \\ 0 & \text{w.p. } 1 - \frac{1}{n}\left(\frac{1}{r_1} - \delta\right) \end{cases}$ |
| $t = 2$ | $\begin{cases} \left[\frac{1-\delta r_1 - n r_1}{1-(\lambda\bar{n}+\frac{e'}{r_1})r_1}\right]\left(\frac{1-\lambda r_1}{1-\lambda}\right) R_L & \text{w.p. } p \\ 0 & \text{w.p. } 1-p \end{cases}$ | 0 |

drawals. This will affect the differential payout function, and therefore $\theta^*$ and the corresponding expected number of runners $n$. Consider first the impact of paying out cash on $n$. The cash payout decreases the *maximum* value of $n$ from $\frac{1}{r_1}$ to $\frac{1}{r_1} - \delta$. However, because we let the equity holders and the coco holders absorb first losses, this also means that the value of $n$ where the amount $r_D$ is scaled by the number of runners is also pushed back by $\delta$ (from $\lambda\bar{n} + \frac{e'}{r_1}$ to $\lambda\bar{n} + \frac{e'}{r_1} - \delta$). This means that the bounds of $n$ change. Eqn. 10 shows the differential payout function for the CASH case.

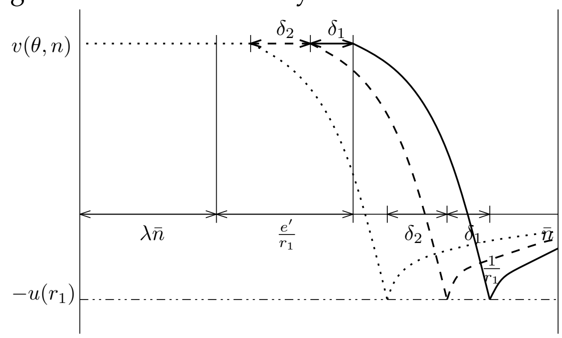
$$v_{cash} = \begin{cases} p(\theta)u\left(\left[\frac{1-\delta r_1 - n r_1}{1-(\lambda\bar{n}+\frac{e'}{r_1})r_1}\right]\left(\frac{1-\lambda r_1}{1-\lambda}\right) R_L\right) - u(r_1) & \text{if } \lambda\bar{n} + \frac{e'}{r_1} - \delta \leq n \leq \frac{1}{r_1} - \delta \\ 0 - \frac{1}{n}\left(\frac{1}{r_1} - \delta\right) u(r_1) & \text{if } \frac{1}{r_1} - \delta \leq n \leq \bar{n} \end{cases} \tag{10}$$

The equation that implicitly defines $\theta^*_{cash}$ can be formed from the differential payout equation. This is given by Eqn. 11.

$$\begin{aligned} \hat{f}_{cash}(\theta^*_{cash}, r_1, R_L) \quad = \quad & \int_{n=\lambda\bar{n}+\frac{e'}{r_1}-\delta}^{\frac{1}{r_1}-\delta} \left[p(\theta(\theta^*_{cash}, n))u\left(\left[\frac{1-\delta r_1 - n r_1}{1-(\lambda\bar{n}+\frac{e'}{r_1})r_1}\right]\left(\frac{1-\lambda r_1}{1-\lambda}\right) R_L\right) - u(r_1)\right] dn \\ & - \int_{n=\frac{1}{r_1}-\delta}^{\bar{n}} \frac{1}{n}\left(\frac{1}{r_1} - \delta\right) u(r_1) dn = 0 \end{aligned} \tag{11}$$

We can see that as $\delta \to 0$, $\theta^*_{cash} \to \theta^*_{pwd} = \theta^*_{ce}$. However, because the bounds of the integral change along with the expression within the utility function, it is difficult to be precise unless we look at the derivative of $\theta^*_{cash}$ with respect to $\delta$. A cash payout $\delta r_1$ reduces the amount that is available to depositors who wait until $t = 2$ (see the reduction in the numerator of $u(\cdot)$). However, by

Figure 6: Differential utility for different values of $\delta$




choosing to wait, depositors forgo receiving $r_1$ at $t = 1$. If $n$ falls into the range $\frac{1}{r_1} - \delta \leq n \leq \bar{n}$, a depositor's "expected opportunity loss" is only $-\frac{1}{n}\left(\frac{1}{r_1} - \delta\right) u(r_1)$ rather than $-\frac{1}{nr_1}u(r_1)$. As such, there is less to lose by waiting if $n$ happens to be large, but one must note as well that the range $\left[\frac{1}{r_1} - \delta,\, \bar{n}\right]$ rises with $\delta$. The ambiguity arises because both the gain from waiting and the loss from waiting fall at the same time. Figure 6 illustrates the differential payout functions for different values of $\delta$.

In this section, we follow the earlier procedures and calculate the derivatives of $\theta^*_{cash}$ with respect to $\delta$ explicitly using the implicit function theorem. The expressions are laborious and so relegated to the Annex, but we can unambiguously sign the derivative: $\frac{\partial \theta^*_{cash}}{\partial \delta} > 0$ . The impact of $\delta$ on the gain from waiting is higher than its impact on the expected opportunity loss from waiting. Thus, a higher $\theta^*_{cash}$ is needed to compensate for the impact of an increase in the cash component $\delta$.

**Proposition 5.** *$\theta^*_{cash}$ is increasing in $\delta$: $\frac{\partial \theta^*}{\partial \delta} > 0$*

Combining Proposition 5 with our earlier results allows us to give a definitive ranking of the types of cocos in terms of impact on probability of bank runs:

**Corollary 6.** *For $\delta > 0$, $\theta^*_{rfb} < \theta^*_{ce} = \theta^*_{pwd} < \theta^*_{cash}$*

# 5 Contagion and Systemic Risk

## 5.1 Contagion

Banks may have correlated asset returns for several reasons. The most obvious one is that banks often have cross-holdings of deposits (Allen and Gale (2000)). Another is when banks invest in the same set of industries, either by intentionally herding (like in Acharya and Yorulmazer (2008, 2007); Farhi and Tirole (2012)) or as a result of their individual diversification policies as in Wagner (2010). Banks also tend to invest in similar assets as a result of conforming to regulatory requirements by institutions such as BCBS (as in Iannotta and Pennacchi (2012)). Thus, negative information about one bank may have an adverse impact on other financial institutions. This information contagion effect has been well-documented (empirically) in the literature and is not confined to the banking sector (see Aharony and Swary (1983, 1996); Lang and Stulz (1992)). Thus, when cocos of one bank convert, they impose an information externality on the other banks that hold assets with returns correlated to those of the converting bank. In this section we show how this could happen.

To do so we consider a two-bank system. Let Bank 1 be a coco-issuing bank (as discussed in Sections 4.2, 4.3 and 4.4) (at this stage, the type of coco doesn't matter - only the conversion does) and without loss of generality, let Bank 2 be an ordinary bank without cocos. Similar to Bank 1, Bank 2 also has a continuum of depositors who obtain private signals $\theta_{2i} \sim U[\theta_2 - \varepsilon,\, \theta_2 + \varepsilon]$, and investments in risky technology with stochastic return $R_2$, and with equity but without cocos. Table 6 summarizes the setup for the two-bank case.

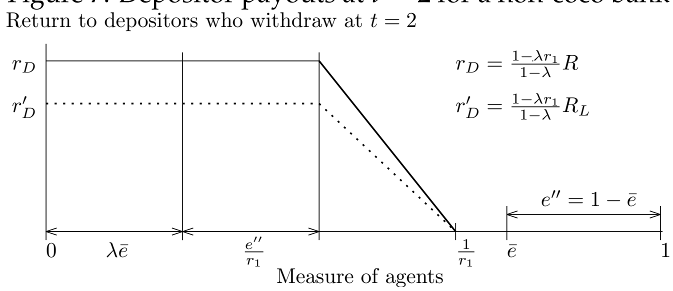
As Bank 2 depositors also obtain private signals $\theta_{2i}$, its late consumers also decide whether to wait or to run at $t = 1$ depending on their posterior assessment of $\theta_2$. The decision is made by using the differential payout function for depositors of Bank 2, shown in Eqn. 12.

$$v_2(\theta, n) = \begin{cases} p(\theta) u\left(\left(\frac{1-nr_1}{1-\left(\lambda\bar{e}+\frac{e''}{r_1}\right)r_1}\right)\left(\frac{1-\lambda r_1}{1-\lambda}\right)R_2\right) - u(r_1) & \text{if} \quad \lambda\bar{e} + \frac{e''}{r_1} \leq n \leq \frac{1}{r_1} \\ 0 - \frac{u(r_1)}{nr_1} & \text{if} \quad \frac{1}{r_1} \leq n \leq \bar{e} \end{cases} \tag{12}$$

Table 6: Summary of Bank Features: Two-Bank System

| | Bank 1 | Bank 2 |
|---|---|---|
| Bank type | coco-issuing | ordinary |
| agents | continuum from [0, 1] | continuum from [0, 1] |
| coco holders | $\bar{e} - \bar{n}$ | 0 |
| equity holders | $1 - \bar{e}$ | $1 - \bar{e}$ |
| early consumers | $\lambda \bar{n}$ | $\lambda \bar{e}$ |
| late consumers | $(1 - \lambda)\bar{n}$ | $(1 - \lambda)\,\bar{e}$ |
| probability of run | $\theta_1^*$ | $\theta_2^*$ |
| potential returns | 0 or $R_1 \in \{R_L, R\}$ | 0 or $R_2 = \{R_L, R\}$ |

Figure 7: Depositor payouts at $t = 2$ for a non-coco bank




where $e'' = 1 - \bar{e}$ (See Figure 7). As before, there is only one value of $\theta$ which makes them indifferent between waiting and running. Call this value $\theta_2^*$. As before, this can be interpreted as the probability of a run in Bank 2, and is defined implicitly by its differential payout function where now $\lambda \bar{e} \leq n \leq \bar{e}$ because Bank 2 did not issue cocos.

The function that implicitly defines Bank 2's probability of a run is given by[19]

$$\begin{aligned} \hat{f}(\theta_2^*, r_1, R) \quad = \quad & \int_{n=\lambda\bar{e}+\frac{e''}{r_1}}^{\frac{1}{r_1}} \left[ p(\theta(\theta_2^*, n))u\left(\left(\frac{1-nr_1}{1-\left(\lambda\bar{e}+\frac{e''}{r_1}\right)r_1}\right)\left(\frac{1-\lambda r_1}{1-\lambda}\right)R_2\right) - u(r_1)\right] dn \\ & - \int_{n=\frac{1}{r_1}}^{\bar{e}} \frac{1}{nr_1} u(r_1) dn = 0 \end{aligned} \qquad (13)$$

We now want to determine the impact of Bank 1's coco conversion on Bank 2's probability of a bank run. Formally, we want to determine the sign of the derivative $\frac{\partial \theta_2^*}{\partial R_1}$ at $t = 1$. This can be

[19] This, along with Eqn. 12 is the $\theta^*$ derivation in GP but scaled by $\bar{e}$ (no cocos).

written as

$$\frac{\partial \theta_2^*}{\partial R_1} = \frac{\partial \theta_2^*}{\partial R_2} \frac{\partial R_2}{\partial R_1} \tag{14}$$

where the first term is the impact of a change in Bank 2's returns on Bank 2's run probability. From Proposition 1, it is clear that $\frac{\partial \theta_2^*}{\partial R_2} < 0$. The sign of $\frac{\partial R_2}{\partial R_1}$ of course depends on the correlation of $R_2$ and $R_1$. If they are positively correlated, $\frac{\partial R_2}{\partial R_1} > 0$. If not, then $\frac{\partial R_2}{\partial R_1} = 0$. Any information about $R_1$ (and therefore $R_2$) is revealed only when cocos convert. Otherwise, no information is revealed. Thus, we have that in the event of a coco conversion and correlated asset returns, $\frac{\partial \theta_2^*}{\partial R_1} < 0$.

We have mentioned in Section 2 that when $\bar{n}$ is small $\left(\bar{n} < \frac{1}{r_1}\right)$, depositors know that they will all be served at $t = 1$ if they all withdraw. In this case only the early consumers withdraw, and all the late consumers wait until $t = 2$. However, this small $\bar{n}$ does not preclude the possibility that the regulator finds it necessary to force conversion of cocos.

From Proposition 7, the knowledge of Bank 1's conversion leads Bank 2 depositors to have a higher required indifference threshold $\theta_2^{**} > \theta_2^*$. This increases the proportion of depositors who obtain signals that are lower than the new threshold. Thus while conversion in Bank 1 may not cause a run in Bank 1, it raises the probability of runs in Bank 2. Moreover, it may even cause full runs in Bank 2 because $n \in [\lambda \bar{e}, \bar{e}] \supseteq [\lambda \bar{n}, \bar{n}]$ such that when $\theta_2^{**}$ is high enough, the associated $n_2$ exceeds $\frac{1}{r_1}$.

**Proposition 7.** *If bank returns are correlated, coco conversion of Bank 1 leads to a higher probability of runs in Bank 2. This is true regardless of the type of coco issued by Bank 1, and even if Bank 1 has small $\bar{n}$.*

## 5.2 Systemic Risk

From the above discussion, it is only a small step to show that coco conversion raises systemic risk. In general, systemic risk can be described as a situation where the banks fail at the same time, or if the failure of one bank spreads to other banks. While banks are not compelled under Basel III to issue cocos, an increasing number of banks have been issuing them[20]. It is therefore natural to

[20] Financial Times, May 26, 2014

examine the impact on systemic risk of having cocos in the banking system. Since systemic risk is the risk that banks fail jointly, and since in this paper, we define bank failure as being due to bank runs, we quantify systemic risk to be the product of each bank's probability of a run, as in Ahnert and Georg (2012).

It is straightforward to show that systemic risk rises when cocos convert, regardless of the type of coco issued. To do this, we still work with the two-bank case, as with the discussion on contagion. From Proposition 7, $\frac{\partial \theta_2^*}{\partial R_1} > 0$. As in Section 5.1, let us call the raised value of $\theta_2^*$ due to a drop in the returns of Bank 1 as $\theta_2^{**}$. Thus, $\theta_2^{**} > \theta_2^*$.

As a benchmark case, consider what would happen when the regulator exercises forbearance and doesn't convert cocos of Bank 1 despite knowing that returns will be low. Then, Bank 2 depositors would not obtain any signal about $R$, and they would not be affected. Systemic risk would then be $\theta^*_{rfb} \cdot \theta_2^*$ .

Next, consider what would happen when the regulator decides to convert the cocos of Bank 1. Bank 2 depositors would infer that Bank 1 returns would be $R_L$ instead of $R$ in the good state at $t = 2$. This has an impact on the returns of Bank 2, which would also be lower. As a result, Proposition 7 applies and so the probability of bank runs in Bank 2 is now $\theta_2^{**}$. Systemic risk would then be $\theta^*_{convert} \cdot \theta_2^{**}$, where we use $\theta^*_{convert}$ to refer to the heightened $\theta^*$ from any kind of converted coco. Note that we now have

$$\theta^*_{convert} \cdot \theta_2^{**} \geq \theta^*_{rfb} \cdot \theta_2^*$$

The contagion effect becomes more obvious when we increase the number of banks. Suppose we have $n$ banks, where Bank 1 is a coco bank, while the remaining $n-1$ banks are same-sized regular banks (one could think there are $n-1$ clones). Suppose also that bank asset returns are correlated throughout the financial system. Systemic risk is then the product of the probability of bank runs of all $n$ banks:

$$\theta_1^* \times \theta_2^* \times \cdots \times \theta_n^* = \theta_1^* \times \left(\theta_{n-1}^*\right)^{n-1}$$

since the $n-1$ banks are identical.

Under regulatory forbearance, $\theta_1^* = \theta_{rfb}^*$ and systemic risk would be

$$\theta_{rfb}^* \cdot (\theta_2^*)^{n-1}$$

Under a watchful regulator, $\theta_1^* = \theta_{convert}^*$ and systemic risk would be

$$\theta_{convert}^* \cdot (\theta_2^{**})^{n-1}$$

Since $\theta_{convert}^* > \theta_{rfb}^*$ and $\theta_2^{**} > \theta_2^*$,

$$\theta_{convert}^* \cdot (\theta_2^{**})^{n-1} \quad > \quad \theta_{rf}^* \cdot (\theta_2^*)^{n-1}$$

As $n \to \infty$, the difference becomes larger because $\frac{\theta_2^{**}}{\theta_2^*} > 1$, which rises with $n$.

**Proposition 8.** *When the regulator is forbearing, systemic risk due to bank runs at $t = 1$ perversely remains low. On the other hand, when the regulator is not forbearing and forces coco conversion in one bank, systemic risk rises. Moreover, systemic risk rises as the number of banks in the system increase.*

# 6 Wealth transfer effects of coco conversions

In the discussion so far we have almost completely focused on how conversion transmits a signal that affects the incentives of depositors. However, because deposits are senior claimants, they are not directly affected by a conversion - they do not care about further losses. On the other hand, the conversion affects the junior claimants, as it is essentially a reallocation of losses among them. In this section, we assess how the wealth transfers vary with the design of the coco. We compare what each original equity holder would obtain after conversion of differently-designed cocos against a benchmark case. We only consider outcomes in the good state of nature (occurring with probability $p(\theta)$) are of interest because in the bad state of nature, occurring with probability $1 - p(\theta)$), equity holders and coco holders alike will be wiped out regardless of coco design or whether conversion takes place.

## 6.1 Benchmark case

To examine whether wealth transfers occur, we need a benchmark: returns have been found out to be $R_L$ by the depositors, but no conversion has taken place. We have done this because we want to keep the threshold level of $\theta^*$constant between the different coco designs - as $\theta^*$ affects $n$, and therefore, the remaining assets to be distributed among the junior claimants. Moreover, any negative information about the returns affects depositors in the same way that a coco conversion does. Let us call the $\theta^*$ that results from the discovery of $R_L$ not arising from cocos as $\theta^*_{benchmark.}$ With the results in Section 4 on CE and PWD cocos, we have

$$\theta^*_{benchmark} = \theta^*_{ce} = \theta^*_{pwd}$$
$$n_{benchmark} = n_{ce} = n_{pwd}$$

As in this benchmark case cocos do not convert, coco holders still have claims $\bar{e} - \bar{n}$ which are senior to equity. We abstract away from coco pricing but assume cocos earn a gross return $r_{coco} > 1$ per unit invested. The total liability to coco holders would be $r_{coco}(\bar{e} - \bar{n})$. Since equity holders are junior to holders of unconverted cocos, equity holders would each receive

$$\left\{ R_L \left[ (1 - n_{benchmark} r_1) - \frac{(\bar{n} - n_{benchmark})(1 - n_{benchmark} r_1)(1 - \lambda r_1)}{\left(1 - (\lambda \bar{n} + \frac{e'}{r_1}) r_1\right)(1 - \lambda)} \right] - r_{coco}(\bar{e} - \bar{n}) \right\} \times \frac{1}{(1 - \bar{e})} \tag{15}$$

### 6.2 Convert-to-equity cocos (CE)

Suppose returns were $R_L$ and cocos converted to equity. Suppose also that the conversion rate of cocos is some $\psi > 0$. The conversion rate is such that coco claims worth $\bar{e} - \bar{n}$ will be transformed into $\psi(\bar{e} - \overline{n})$ equity claims. This means that coco holders not only lose out on the net returns $(r_{coco} - 1)$, they also have to share the residual assets with the existing measure of $1 - \bar{e}$ equity holders. Note that a high conversion rate corresponds to a low conversion price[21]. A higher conversion rate (lower conversion price) leads to more dilution of existing shareholders. Each equity holder now obtains

$$R_L \left[ (1 - n_{ce} r_1) - \frac{(\bar{n} - n_{ce})(1 - n_{ce} r_1)(1 - \lambda r_1)}{\left(1 - (\lambda \bar{n} + \frac{e'}{r_1}) r_1\right)(1 - \lambda)} \right] \left[ \frac{1}{(1 - \bar{e}) + \psi(\bar{e} - \bar{n})} \right] \tag{16}$$

We can see that the original equity holders may or may not be diluted depending on the conversion rate $\psi$. As $\psi \to 0$, the coco holders' share of the residual asset disappear, and the original equity holders gain at their expense. As $\psi \to \infty$ old equity holders are completely diluted and the coco-holders-turned-equity holders become de facto the sole residual claimants.

To determine whether a wealth transfer occurs when cocos convert, we need to compare Eqns. 15 and 16. Since $\theta^*_{benchmark} = \theta^*_{ce}$, $n_{benchmark} = n_{ce} = n'$ . Let $A = (1 - n' r_1) - \frac{(\bar{n} - n')(1 - n' r_1)(1 - \lambda r_1)}{\left(1 - (\lambda \bar{n} + \frac{e'}{r_1}) r_1\right)(1 - \lambda)}$. There will be a wealth transfer from coco holders to the original equity holders if the original equity owners' equity position after conversion is better than it is in the benchmark case:

$$\begin{aligned} R_L A \left[ \frac{1}{(1 - \bar{e}) + \psi(\bar{e} - \bar{n})} \right] &> \{R_L A - r_{coco}(\bar{e} - \bar{n})\} \times \frac{1}{(1 - \bar{e})} \\ \psi &< \frac{r_{coco}(1 - \bar{e})}{R_L A - r_{coco}(\bar{e} - \bar{n})} \end{aligned} \tag{17}$$

Eqn. 17 allows us to define a conversion rate that leaves equity holders in the same position they are in the benchmark case by defining $\psi^{benchmark}$ as the conversion rate for which Eqn. 17 holds

[21] If the coco conversion is presented as the coco holder using his coco to buy new shares, the conversion price equivalent to our conversion ratio $\psi$ is $P_{conv} = \psi^{-1}$.

with equality:

$$\psi^{benchmark} = \left( \frac{r_{coco}(1-\bar{e})}{R_L A - r_{coco}(\bar{e}-\bar{n})} \right) \tag{18}$$

If the conversion rate equals $\psi^{benchmark}$, losses due to the asset deterioration are allocated over equity holders and coco holders exactly in line with existing seniority. If $\psi > \psi^{benchmark}$, a wealth transfer is from equity holders to coco holders; if $\psi < \psi^{benchmark}$, the wealth transfers go the other way so the original equity holders actually profit from conversion.

Clearly, the direction of the wealth transfer has a major impact on the equity holders' *ex ante* incentives to issue more shares to forestall conversion or on the contrary try to force conversion (e.g. by organizing well-publicized short selling pressure). In particular, if $\psi > \psi^{benchmark}$, the CE coco is sufficiently dilutive for old equity holders to make it profitable for them to issue new shares and raise more capital that way to forestall conversion[22]. We can summarize our results in the following proposition:

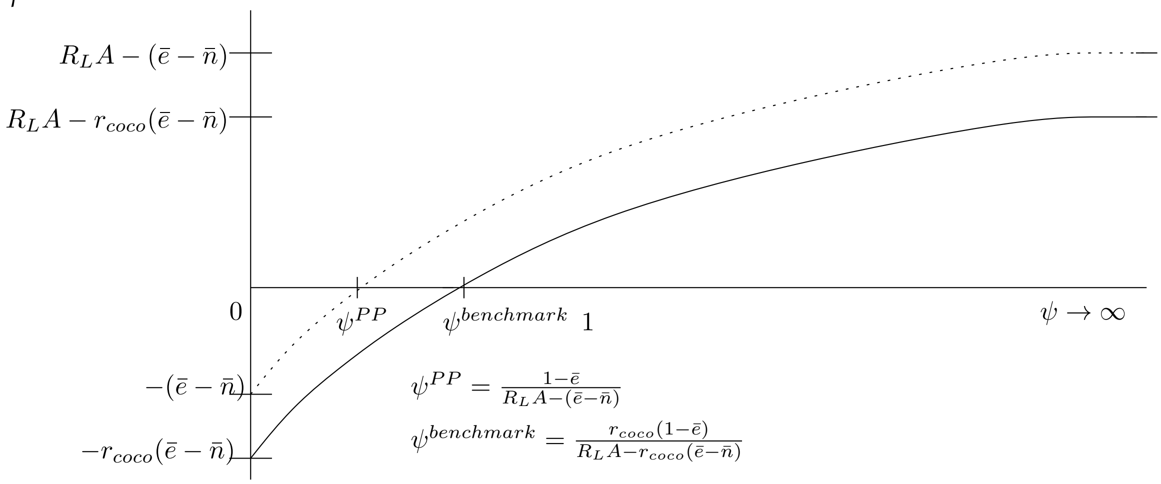
**Proposition 9.** *Cocos that convert to equity potentially effect a wealth transfer from equity holders to the coco holders or vice versa depending on the conversion rate. When*

- $\psi < \psi^{benchmark} = \frac{r_{coco}(1-\bar{e})}{R_L A - r_{coco}(\bar{e}-\bar{n})}$*, there is insufficient dilution and consequently conversion implies a wealth transfer from coco holders to original shareholders.*
- $\psi = \psi^{benchmark}$*, conversion allocates losses strictly in line with seniority, there is no wealth transfer either way.*
- $\psi > \psi^{benchmark}$*, cocos are excessively dilutive, i.e. a wealth transfer from original equity holders to coco holders takes place. As* $\psi \to \infty$*, coco holders wipe out the original shareholders.*

Fig. 8 illustrates these results graphically. Wealth transfers (a positive sign implies a transfer from equity holders to the holders of cocos) are on the vertical axis and are set out against the

[22] If there are costs of equity issuance the trigger level for $\psi$ would change commensurately.

Figure 8: Wealth transfers from equity holders to coco holders as a function of the conversion rate $\psi$




Benefit obtained by CoCo holders from conversion as $\psi$ increases relative to no conversion

conversion ratio $\psi$ on the horizontal axis. The solid line corresponds to the case where coco holders are promised an *ex ante* gross return $r_{coco} > 1$. The slotted line corresponds to an *ex ante* return $r_{coco} = 1$, i.e. a zero net return. We come back to the zero *ex ante* return case below, The wealth transfer to coco holders reaches its maximum when equity holders are wiped out completely in the extreme case of complete dilution ($\psi \rightarrow \infty$). $\psi^{benchmark}$ is defined as the conversion ratio implying a zero wealth transfer either way: for $\psi = \psi^{benchmark}$ the line crosses the horizontal axis.

If we were to design a CE coco to merely preserve the coco's principal amount, we would set a conversion rate $\psi_{PP}$. The value of $\psi_{PP}$ follows from equating the principal value of cocos without a conversion $(\bar{e} - \bar{n})$ and with what the holders of CE cocos would collectively obtain under conversion, $(R_L A)\left(\frac{1}{(1-\bar{e})+\psi(\bar{e}-\bar{n})}\right)\psi\,(\bar{e}-\bar{n})$, which after some algebra yields:

$$\psi_{PP} = \frac{1-\bar{e}}{R_L A - (\bar{e}-\bar{n})} < \left(\frac{r_{coco}(1-\bar{e})}{R_L A - r_{coco}\,(\bar{e}-\bar{n})}\right) = \psi^{benchmark}$$

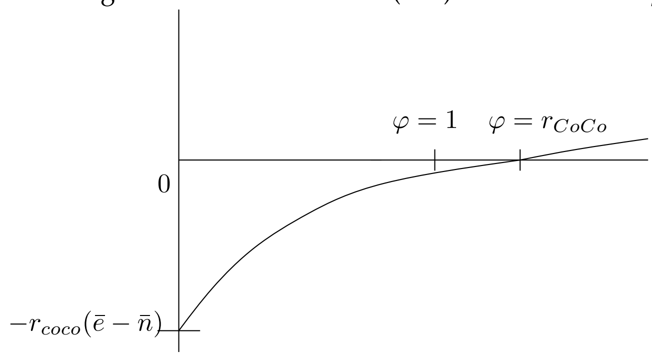
if at least cocos earn a positive return, i.e. if $r_{coco} > 1$. As Fig. 8 indicates, a conversion rate set to merely preserve the principal implies a wealth transfer from coco holders to equity holders: $\psi^{PP}$ corresponds to a value falling within the wealth transfer region in Fig. 8 is negative (below the horizontal axis). So the principal-preserving conversion rate is very similar to our benchmark conversion rate $\psi^{benchmark}$ but not exactly equal to it. If the net return on cocos is zero (i.e. $r_{coco} = 1$), the two conversion rates become the same (see the slotted line in 8). That also explains their difference: in the principal-preserving case, the original equity holders still manage to appropriate the net return of the coco holders and so still obtain some benefits compared to what strict seniority would imply. In our zero wealth transfer case where seniority is strictly imposed, coco holders receive not only their original principal value but also the statutory returns before equity holders get paid.

Calomiris and Herring (2013), presumably for that reason, propose to pay out a premium over preservation of principal to coco holders upon conversion equal to 5% of the equity value at the moment of conversion. We ignore for the moment the problem that for market-based triggers (which they prefer) the equity value may in fact reflect that possibility. But even if that is not the case, for example because the trigger is based on book values and the resulting equity values only reflect the residual value conditional on the pre-conversion situation, that rule may still not set incentives in the desired direction. The problem is that the effective return on cocos post conversion then depends on the pre-conversion ratio between equity value and coco principal. If cocos are "large" in respect to equity, 5% of equity value may not be enough compared to the statutory return on cocos $r_{coco}$. Under the Calomiris-Herring proposal, the effective net return on cocos post conversion equals: $r_{CH} - 1 = 0.05 * \frac{(1-\bar{e})}{(\bar{e}-\bar{n})}$. For the Calomiris Herring proposal to actually create *ex ante* incentives to issue more incentives in times of distress, the following needs to hold:

$$\psi_{CH} \quad > \quad \psi^{e} \Leftrightarrow 0.05 * \frac{(1-\bar{e})}{(\bar{e}-\bar{n})} > r_{coco} - 1 \tag{19}$$

A priori there is no reason to expect this inequality to hold. Note moreover that the likelihood that Eqn.19 actually holds goes down as cocos become larger. Thus there may well be a conflict between the 5% proposal of Calomiris and Herring and their other proposal, that cocos should be "large" in respect to equity. Eqn. 19 indicates that the larger cocos are in relation to equity (i.e. the smaller the ratio $\frac{(1-\bar{e})}{(\bar{e}-\bar{n})}$), the less likely it is that a 5% of total equity value premium to coco

Figure 9: Wealth Transfers (WT) as a function of $\varphi$




Benefit obtained by CoCo holders from conversion as $\varphi$ increases relative to no conversion

holders is high enough to provide equity holders with incentives to issue new capital in times of distress. In a sense larger coco holdings dilute the 5% equity value that CH want to see transferred to coco holders upon conversion. If coco holdings become too large (i.e. if $\frac{\bar{e}-\bar{n}}{1-\bar{e}} > \frac{0.05}{r_{coco}-1}$), the *ex ante* incentive for shareholders to issue more equity in times of distress turns perverse, i.e. they get an interest in doing the opposite.

### 6.3 Principal Writedown cocos (PWD)

In Section 4 we described PWD cocos in terms of how much of the original coco principal value is left after conversion into a new lower principal value debt instrument, as indicated by the remaining fraction $\varphi$. Note that if $\varphi = r_{coco}$, coco holders get exactly what they would have obtained under the benchmark case. Of course, when $\varphi = 0$, which implies a complete principal write down, equity holders completely gain at the expense of the coco holders. Graphically the possible wealth transfers in case of a PWD coco is represented in Fig. 9.

As is clear from Fig.9, any value $\varphi < r_{coco}$ implies a loss of wealth for the coco holder, so it is obvious that conversion of non-trivial PWD cocos (i.e. PWDs with $\varphi < 1$) *always* implies a wealth transfer from coco holders to the original equity holder. The existing equity holders do not have to share residual income, if any, since coco holders do not receive any equity under the PWD structure and coco holders receive less than they would in the absence of conversion[23].

**Proposition 10.** *Conversion of a PWD coco always results in a wealth transfer from the coco holders to the original equity holders.*

This has major implications for the *ex ante* incentives facing the original equity holders. Contrary to the case of sufficiently dilutive CE cocos, under PWD cocos the original equity holders always face the perverse incentive that conversion is actually in their interest. This may lead the original equity holders to attempt to extract more dividends or exert pressure on management to take on more risk in times of distress instead of facing incentives to supply additional capital as in the case of sufficiently dilutive CE cocos. The BIS accepts PWD cocos even as AT1 capital (Basel Committee on Banking Supervision (2011)), but that decision should in the light of these incentive problems arguably be reconsidered.

To see whether the original shareholders are better off under CE or PWD for arbitrary values of $\psi$ and $\varphi$, we need to compare what each equity holder would get under each case. After conversion of a PWD coco, each equity holder now obtains

$$\left\{ R_L \left[ \left(1 - n_{pwd} r_1\right) - \frac{\left(\bar{n} - n_{pwd}\right)\left(1 - n_{pwd} r_1\right)(1 - \lambda r_1)}{\left(1 - (\lambda \bar{n} + \frac{e'}{r_1}) r_1\right)(1 - \lambda)} \right] - \varphi\left(\bar{e} - \bar{n}\right) \right\} \left[ \frac{1}{(1 - \bar{e})} \right] \tag{20}$$

Comparison with an insufficiently dilutive CE coco is made easy because we have established that $n_{ce} = n_{pwd}$ by virtue of $\theta^*_{ce} = \theta^*_{pwd}$. Thus equity holders are better off under PWD relative to CE in the sense of them preferring a PWD with retention rate $\varphi$ over a CE coco with dilution parameter $\psi$ if Eqn. 20 is greater than Eqn. 16. Let $n' = n_{ce} = n_{pwd}$ and let $A =$

[23]Cocos with $\varphi > 1$ are writeup cocos instead of writedown. They do exist but do not count as part of AT1.

$R_L\left[(1-n'r_1)-\frac{(\bar{n}-n')(1-n'_{ce}r_1)(1-\lambda r_1)}{\left(1-(\lambda\bar{n}+\frac{e'}{r_1})r_1\right)(1-\lambda)}\right]$, this inequality requires:

$$\begin{aligned}\{A-\varphi(\bar{e}-\bar{n})\}\left[\frac{1}{(1-\bar{e})}\right] &> A\left[\frac{1}{(1-\bar{e})+\psi(\bar{e}-\bar{n})}\right]\\ 1-\frac{\varphi(\bar{e}-\bar{n})}{A} &> \frac{1-\bar{e}}{(1-\bar{e})+\psi(\bar{e}-\bar{n})}\\ \left[1-\frac{1-\bar{e}}{(1-\bar{e})+\psi(\bar{e}-\bar{n})}\right]\frac{A}{\bar{e}-\bar{n}} &> \varphi\end{aligned}$$

We know that $0<1-\frac{1-\bar{e}}{(1-\bar{e})+\psi(\bar{e}-\bar{n})}<1$ and that $\frac{A}{\bar{e}-\bar{n}}>1$ (if we assume that there is equity left over to distribute). Thus we can always find such a $\varphi$ for any given value of $\psi$ since $0<\varphi<1$ such that the inequality holds, because $\varphi$ can become very small if necessary. Thus as long as $\varphi<\left[1-\frac{1-\bar{e}}{(1-\bar{e})+\psi(\bar{e}-\bar{n})}\right]\frac{A}{\bar{e}-\bar{n}}$, equity holders prefer a PWD coco with retention rate $\varphi$ over a CE coco with dilution parameter $\psi$. This is very intuitive because the less of the cocos is retained, the lower total liabilities become, and the more is left for the original shareholders. This also means that if the CE coco is very dilutive, almost any PWD coco would be better for the original equity holders than the CE coco. More generally we can define a retention rate $\varphi^e$ at which an equity holder would be just indifferent between a PWD coco with that retention rate and a CE coco with dilution parameter $\psi$:

$$\varphi^e = \left[1-\frac{1-\bar{e}}{(1-\bar{e})+\psi(\bar{e}-\bar{n})}\right]\frac{A}{\bar{e}-\bar{n}} \tag{21}$$

*with* $\frac{\partial\varphi^e}{\partial\psi}=\left[\frac{(1-\bar{e})\times(\bar{e}-\bar{n})}{((1-\bar{e})+\psi(\bar{e}-\bar{n}))^2}\right]\frac{A}{\bar{e}-\bar{n}}>0$

In words, a more dilutive CE coco will lead equity holders to accept a PWD coco with higher retention rate as an alternative.

# 7 Conclusion

We have written this paper in an effort to explore the effect of (converting) cocos on systemic risk. We have done this by adding coco holders and equity holders to the agent types of an otherwise standard Diamond and Dybvig (1983) setup recast in a global games framework as in Goldstein and Pauzner (2005). Using this framework, we were able to show the impact of coco conversion on depositors, as well as on coco holders and equity holders. First we have shown that when an unanticipated decline in asset returns leads to a coco conversion, that has the immediate effect of raising the probability of a bank run. This is true regardless of the type of cocos that are converted because they all send the same kind of signal (lowering of returns) which affects depositor incentives in the same manner. Actually we show there is one exception where the coco design does matter for the impact on fragility: cocos which provide a cash payment to coco holders before writing them off (like the RABO coco does) are actually worse than straight Principal Write Down (PWD) cocos or Equity Conversion (CE) cocos in terms of raising the likelihood of a run. This is so because by paying out cash in a distress situation they reduce the amount that may be distributed to the remaining creditors of the bank after conversion occurs.

Therefore one of the main consequences of our analysis is that a regulator faces conflicting incentives when finding out about lower asset returns than expected ($R_L < R$). On the one hand, conversion increases the probability of a run because of the negative signal on asset returns that conversion conveys. But on the other hand, conversion also ensures that if runs occur, there is a higher probability that there will be a surviving equity base. Regulators thus are forced to choose between keeping fragility low at the expense of making the consequences of a run if it does occur worse, or increasing the likelihood of a run but leaving the bank better equipped to deal with the aftermath of one.

We then extend the analysis to a multibank framework to analyze the impact of coco conversion on systemic risk. When different banks hold assets that have correlated returns, a signal indicating one bank's asset quality deterioration has negative consequences for the other banks to the extent that the other banks' assets are positively correlated to those of the bank whose coco has been forced into conversion: conversion carries an information externality giving rise to contagion across banks. There are many reasons to expect positive correlation between asset returns of different banks. A very direct link leading to asset correlation establishing a channel of contagion occurs when banks hold each others' cocos. Given the obvious dangers of contagion such cross holdings give rise to, it is disturbing to see that about 50% of all cocos issued so far is in fact held by banks (Avdjiev et al. (2013)). Other mechanisms leading to asset correlation may be the predominance of a few large banks in a relatively small country, industry specialization of several banks into the same industry, or herding behavior, for example to increase the pressure on regulators to bail out banks in distress if that situation arises. We show unambiguously that in an environment of correlated risks, coco conversion, even in a single bank, leads to higher systemic risk, defined as the joint probability of failure of banks. We show that as long as bank assets are positively correlated, a coco conversion in one bank leads to an increase in the probability of a run in the other bank, regardless of coco type. This implies that systemic risk will increase when cocos convert. So when regulators consider coco conversion, microprudential and macroprudential objectives are likely to be in direct conflict.

Finally we examine the impact of coco conversion on wealth transfers between coco holders and the original equity holders. This is important because these post-conversion wealth transfers have a direct impact on the pre-conversion incentives for shareholders and coco holders to either forestall or actually bring about conversion. From a systemic stability point of view, it is obviously desirable that coco design should set incentives in favor of issuing more equity in times of distress. But we show that if the coco conversion rate is too low in a well defined sense, i.e. below an explicitly derived trigger level, conversion of CE cocos actually leads to a wealth transfer from the coco holders to the equity holders. This has the perverse consequence that equity holders have an interest in forcing conversion, for example by taking on more risk or extracting cash through excessive dividends. Of course if the conversion rate is higher (more dilutive) than the trigger level, wealth transfers due to conversion go the other way, from the original equity holders to the coco holders. So if the conversion rate is sufficiently dilutive in a manner we define precisely, shareholders have an incentive to raise additional capital to forestall conversion. Finally we show that from the *ex ante* incentive point of view, Principal Write Down cocos are strictly inferior to CE cocos: their conversion always implies a wealth transfer from coco holders to the original equity holder, thereby violating seniority with consequent perverse *ex ante* incentives. These results are important as they point at clear possibly destabilizing *ex ante* incentives for equity holders under coco design characteristics that are very often observed in practice so far. Our results therefore strongly suggest that the BIS rules governing the requirements coco design needs to satisfy before cocos are allowed as capital should be reconsidered, particularly for Principal Write Down cocos.

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# 8 Annex

## Calculation of $\frac{\partial \theta^*}{\partial \delta} > 0$

By the implicit function theorem, we can show $\frac{\partial \theta^*}{\partial \delta} > 0$ if $\partial \hat{f}/\partial \theta^* > 0$, and $\partial \hat{f}/\partial \delta < 0$. We already know that $\partial \hat{f}/\partial \theta^* > 0$. We now only look at whether $\partial \hat{f}/\partial \delta > 0$ (here we drop the subscript *cash* for ease of exposition)

$$
\begin{aligned}
\hat{f}\left(\theta, r_1\, \delta\right) &= \int_{n=\lambda\bar{n}+\frac{e'}{r_1}-\delta}^{\frac{1}{r_1}-\delta} \left[ p(\theta(\theta^*, n)) u \left( \left[ \frac{1-\delta r_1 - n r_1}{1-(\lambda\bar{n}+\frac{e'}{r_1})r_1} \right] r_D^L \right) - u(r_1) \right] dn \\
&\quad - \int_{n=\frac{1}{r_1}-\delta}^{\bar{n}} u\left(r_1\right) \left( \frac{1}{r_1} - \delta \right) \left( \frac{1}{n} \right) \\
&= \int_{n=\lambda\bar{n}+\frac{e'}{r_1}-\delta}^{\frac{1}{r_1}-\delta} p\left(\theta\left(\theta^*, n\right)\right) u \left( \frac{1-\delta r_1 - n r_1}{1-\left(\lambda\bar{n}+\frac{e'}{r_1}\right) r_1} r_D^L \right) dn - \int_{n=\lambda\bar{n}+\frac{e'}{r_1}-\delta}^{\frac{1}{r_1}-\delta} u\left(r_1\right) \\
&\quad - \int_{n=\frac{1}{r_1}-\delta}^{\bar{n}} u\left(r_1\right) \left( \frac{1}{r_1} - \delta \right) \left( \frac{1}{n} \right) \\
&= \int_{n=\lambda\bar{n}+\frac{e'}{r_1}-\delta}^{\frac{1}{r_1}-\delta} p\left(\theta\left(\theta^*, n\right)\right) u \left( \frac{1-\delta r_1 - n r_1}{1-\left(\lambda\bar{n}+\frac{e'}{r_1}\right) r_1} r_D^L \right) dn \\
&\quad - u\left(r_1\right) \left[ \frac{1}{r_1} - \lambda\bar{n} - \frac{e'}{r_1} \right] - u\left(r_1\right) \left( \frac{1}{r_1} - \delta \right) \left[ \ln\bar{n} - \ln\left( \frac{1}{r_1} - \delta \right) \right]
\end{aligned}
$$

where $r_D^L = \frac{1-\lambda r_1}{1-\lambda} R_L$.

The derivative of $\hat{f}$ with respect to $\delta$ is

$$
\frac{\partial \hat{f}}{\partial \delta} = \frac{\partial}{\partial \delta} \left[ \int_{n=\lambda\bar{n}+\frac{e'}{r_1}-\delta}^{\frac{1}{r_1}-\delta} p\left(\theta\left(\theta^*, n\right)\right) u \left( \frac{1-\delta r_1 - n r_1}{1-\left(\lambda\bar{n}+\frac{e'}{r_1}\right) r_1} r_D^L \right) dn \right] - u\left(r_1\right) \left[ 1 - \left( \ln \frac{\bar{n}}{\frac{1}{r_1}-\delta} \right) \right]
$$

where the last term is negative for as long as $0 < \ln \frac{\bar{n}}{\frac{1}{r_1-\delta}} < 1$

Now consider the first term:

$$\frac{\partial}{\partial\delta}\left[\int_{n=\lambda\bar{n}+\frac{e'}{r_1}-\delta}^{\frac{1}{r_1}-\delta} p\left(\theta\left(\theta^*,\, n\right)\right) u\left(A\right) dn\right]$$

$$= \int_{n=\lambda\bar{n}+\frac{e'}{r_1}-\delta}^{\frac{1}{r_1}-\delta} p\left(\theta\left(\theta^*,\, n\right)\right) \frac{\partial u\left(A\right)}{\partial\delta} dn + p\left(\theta\left(\theta^*,\, \frac{1}{r_1}-\delta\right)\right) u\left(\frac{1-\delta r_1-\left(\frac{1}{r_1}-\delta\right) r_1}{1-\left(\lambda\bar{n}+\frac{e'}{r_1}\right) r_1} r_D^L\right)(-1)$$

$$-p\left(\theta\left(\theta^*,\, \lambda\bar{n}+\frac{e'}{r_1}-\delta\right)\right) u\left(\frac{1-\left(\lambda\bar{n}+\frac{e'}{r_1}-\delta\right) r_1}{1-\left(\lambda\bar{n}+\frac{e'}{r_1}\right) r_1} r_D^L\right)(-1)$$

$$= \int_{n=\lambda\bar{n}+\frac{e'}{r_1}-\delta r_1}^{\frac{1}{r_1}-\delta} p\left(\theta\left(\theta^*,\, n\right)\right) \frac{\partial u\left(A\right)}{\partial\delta} dn + p\left(\theta\left(\theta^*,\, \lambda\bar{n}+\frac{e'}{r_1}-\delta\right)\right) u\left(\frac{1-\left(\lambda\bar{n}+\frac{e'}{r_1}-\delta\right) r_1}{1-\left(\lambda\bar{n}+\frac{e'}{r_1}\right) r_1} r_D^L\right)$$

where $A = \frac{1-\delta r_1 - n r_1}{1-\left(\lambda\bar{n}+\frac{e'}{r_1}\right) r_1} r_D^L$. But since $\frac{\partial u(A)}{\partial\delta} = \frac{\partial u(A)}{\partial n}$, we can write $\frac{\partial}{\partial\delta}\left[\int_{n=\lambda\bar{n}+\frac{e'}{r_1}-\delta}^{\frac{1}{r_1}-\delta} p\left(\theta\left(\theta^*,\, n\right)\right) u\left(A\right) dn\right]$ in terms of $\frac{\partial u(A)}{\partial n}$:

$$\frac{\partial}{\partial\delta}\left[\int_{n=\lambda\bar{n}+\frac{e'}{r_1}-\delta}^{\frac{1}{r_1}-\delta} p\left(\theta\left(\theta^*,\, n\right)\right) u\left(A\right) dn\right]$$

$$= \int_{n=\lambda\bar{n}+\frac{e'}{r_1}-\delta}^{\frac{1}{r_1}-\delta} p\left(\theta\left(\theta^*,\, n\right)\right) \frac{\partial u}{\partial n} dn + p\left(\theta\left(\theta^*,\, \lambda\bar{n}+\frac{e'}{r_1}-\delta\right)\right) u\left(\frac{1-\left(\lambda\bar{n}+\frac{e'}{r_1}-\delta\right) r_1}{1-\left(\lambda\bar{n}+\frac{e'}{r_1}\right) r_1} r_D^L\right)$$

$$= -p\left(\theta\left(\theta^*,\, \lambda\bar{n}+\frac{e'}{r_1}-\delta\right)\right) u\left(r_D^L\right) + 2\varepsilon \int_{n=\lambda\bar{n}+\frac{e'}{r_1}-\delta}^{\frac{1}{r_1}-\delta} \left[u\left(\frac{1-\delta r_1 - n r_1}{1-\left(\lambda\bar{n}+\frac{e'}{r_1}\right) r_1} r_D^L\right)\left(\frac{p'\left(\theta\left(\theta^*,\, n\right)\right)}{\bar{n}\left(1-\lambda\right)}\right)\right] dn$$

$$+p\left(\theta\left(\theta^*,\, \lambda\bar{n}+\frac{e'}{r_1}-\delta\right)\right) u\left(\frac{1-\left(\lambda\bar{n}+\frac{e'}{r_1}-\delta\right) r_1}{1-\left(\lambda\bar{n}+\frac{e'}{r_1}\right) r_1} r_D^L\right)$$

$$= -p\left(\theta\left(\theta^*,\, \lambda\bar{n}+\frac{e'}{r_1}-\delta\right)\right)\left[u\left(r_D^L\right) - u\left(1-\frac{\delta r_1}{1-\left(\lambda\bar{n}+\frac{e'}{r_1}\right) r_1} r_D^L\right)\right]$$

$$+2\varepsilon \int_{n=\lambda\bar{n}+\frac{e'}{r_1}-\delta}^{\frac{1}{r_1}-\delta} \left[u\left(\frac{1-\delta r_1 - n r_1}{1-\left(\lambda\bar{n}+\frac{e'}{r_1}\right) r_1} r_D^L\right)\left(\frac{p'\left(\theta\left(\theta^*,\, n\right)\right)}{\bar{n}\left(1-\lambda\right)}\right)\right] dn$$

where we used integration by parts. The first term is clearly negative. The second term can be made arbitrarily small by letting $\varepsilon \to 0$.

Thus the derivative of $\hat{f}$ with respect to $\delta$ is completely given by

$$\begin{aligned}
\frac{\partial \hat{f}}{\partial \delta} &= -p\left(\theta\left(\theta^*, \lambda \bar{n} + \frac{e'}{r_1} - \delta\right)\right)\left[u\left(r_D^L\right) - u\left(1 - \frac{\delta r_1}{1 - \left(\lambda \bar{n} + \frac{e'}{r_1}\right) r_1} r_D^L\right)\right] \\
&\quad + 2\varepsilon \int_{n = \lambda \bar{n} + \frac{e'}{r_1} - \delta}^{\frac{1}{r_1} - \delta} \left[u\left(\frac{1 - \delta r_1 - n r_1}{1 - \left(\lambda \bar{n} + \frac{e'}{r_1}\right) r_1} r_D^L\right)\left(\frac{p'\left(\theta\left(\theta^*, n\right)\right)}{\bar{n}\left(1 - \lambda\right)}\right)\right] dn - u\left(r_1\right)\left[1 - \left(\ln \frac{\bar{n}}{\frac{1}{r_1} - \delta}\right)\right] \\
&< 0
\end{aligned}$$

## Calculation of the value of $\psi$from Eqn. 17

$$\begin{aligned}
R_L A \left[\frac{1}{(1 - \bar{e}) + \psi\left(\bar{e} - \bar{n}\right)}\right] &> \left\{R_L A - r_{coco}\left(\bar{e} - \bar{n}\right)\right\} \times \frac{1}{(1 - \bar{e})} \\
\frac{R_L A}{R_L A - r_{coco}\left(\bar{e} - \bar{n}\right)} &> \frac{(1 - \bar{e}) + \psi\left(\bar{e} - \bar{n}\right)}{(1 - \bar{e})} \\
\frac{R_L A}{R_L A - r_{coco}\left(\bar{e} - \bar{n}\right)} &> 1 + \frac{\psi\left(\bar{e} - \bar{n}\right)}{1 - \bar{e}} \\
\psi &< \left(\frac{R_L A}{R_L A - r_{coco}\left(\bar{e} - \bar{n}\right)} - 1\right)\left(\frac{1 - \bar{e}}{\bar{e} - \bar{n}}\right) \\
&= \left(\frac{r_{coco}\left(\bar{e} - \bar{n}\right)}{R_L A - r_{coco}\left(\bar{e} - \bar{n}\right)}\right)\left(\frac{1 - \bar{e}}{\bar{e} - \bar{n}}\right) \\
&= \frac{r_{coco}\left(1 - \bar{e}\right)}{R_L A - r_{coco}\left(\bar{e} - \bar{n}\right)}
\end{aligned}$$