Introduction to IO and WIOD

Robert Stehrer

The Vienna Institute for International Economic Studies (wiiw)

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Overview

- Input-output table and national accounts in single economy
- Input-output table and national accounts in international economy
- World Input-Output Database (WIOD)
- Fundamentals: Leontief inverse and final demand multipliers

Input-Output and National Accounts

Hypothetical example

Input-output table for 2 industries in monetary units (e.g. mn US-$)

	1	2	Final demand	Total
1	150	500	350	1000
2	200	100	1700	2000
VA	650	1400	2050	
GO	1000	2000		3000

- Interpretation of row:
  - ∗ Industry 1 delivers 150 to itself, 500 to industry 2 and 350 to final consumers.
  - ∗ Industry 2 delivers 200 to industry 1, 100 to itself and 1700 to final consumers.
- Interpretation of column:
  - ∗ Industry 1 uses 150 from itself, 200 from industry 2 and 650 from primary factors.
  - ∗ Industry 2 uses 500 from industry 1, 100 from itself and 1400 from primary factors.

- Value added
  - Payments to primary factors (labor capital) or income of factors (wages, profits)
  - $\mathrm{VA}_1 + \mathrm{VA}_2 = \mathrm{GDP}$ (=sum of factor incomes)
- Final demand
  - Consumption by households, Government expenditures, Inventories
  - $f_1 + f_2 = \mathrm{GDP}$ (=sum of final sales)

Matrix notation

- **x** … Gross output vector
- **Z** … Transactions matrix
- **f** … Final demand vector
- **w** … Value added vector

$$\mathbf{x} = \begin{bmatrix} 1000 \\ 2000 \end{bmatrix} \qquad \mathbf{Z} = \begin{bmatrix} 150 & 500 \\ 200 & 100 \end{bmatrix} \qquad \mathbf{f} = \begin{bmatrix} 350 \\ 1700 \end{bmatrix} \qquad \mathbf{w} = \begin{bmatrix} 650 \\ 1400 \end{bmatrix}$$

$$\mathbf{x} = \mathbf{Zi} + \mathbf{f} = \begin{bmatrix} 150 & 500 \\ 200 & 100 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 350 \\ 1700 \end{bmatrix} = \begin{bmatrix} 1000 \\ 2000 \end{bmatrix}$$

$$\mathbf{x}' = \mathbf{i}'\mathbf{Z} + \mathbf{w}' = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 150 & 500 \\ 200 & 100 \end{bmatrix} + \begin{bmatrix} 650 & 1400 \end{bmatrix} = \begin{bmatrix} 1000 & 2000 \end{bmatrix}$$

Notation

- n sectors
- z_{ij} ... sector i sells to sector j (interindustry sales)
- f_i ... sector i sells to final user

A row in an IO table is

$$x_i = z_{i1} + \cdots + z_{ij} + \cdots + z_{in} + f_i = \sum_{j=1}^{n} z_{ij} + f_i$$

For n sectors

$$
\begin{aligned}
x_1 &= z_{11} + \cdots + z_{1j} + \cdots + z_{1n} + f_1 \\
&\vdots \\
x_i &= z_{i1} + \cdots + z_{ij} + \cdots + z_{in} + f_i \\
&\vdots \\
x_n &= z_{n1} + \cdots + z_{nj} + \cdots + z_{nn} + f_n
\end{aligned}
$$

Define

$$\mathbf{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \qquad \mathbf{Z} = \begin{bmatrix} z_{11} & \dots & z_{1n} \\ \vdots & \ddots & \vdots \\ z_{n1} & \dots & z_{nn} \end{bmatrix} \qquad \mathbf{f} = \begin{bmatrix} f_1 \\ \vdots \\ f_n \end{bmatrix}$$

Matrix notation:

$$\mathbf{x} = \mathbf{Zi} + \mathbf{f}$$

A typical column of **Z**:

$$\begin{bmatrix} z_{1j} \\ \vdots \\ z_{ij} \\ \vdots \\ z_{nj} \end{bmatrix}$$

- Sector i sells to sector j
- Sector j demands in sector i

Multi-national models (3 countries, 2 industries)

Hypothetical example

Input-output table for 2 industries and 3 countries in monetary units (e.g. mn US-$)

	Country 1		Country 2		Country 3					
	1	2	1	2	1	2	FD^1	FD^2	FD^3	Total
1	3	0	1.5	0	0.25	0	2.25	0	0	7
2	0	2	0	1	0	0.5	1.5	0	0	5
1	0	0	1.5	0	1.25	0	0	4.25	0	7
2	0	0	0	1	0	0.5	0	3.5	0	5
1	0	0	0	0	1.5	0	0	1.5	4	7
2	0	0	0	0	0	1	0	1	3	5
VA	4	3	4	3	4	3				
GO	7	5	7	5	7	5				

- Interpretation of row:
  - Country 1, industry 1 delivers 3 to itself
  - Country 1, industry 1 delivers 1.5 to country 1, industry 2
  - Country 1, industry 1 delivers 2.25 to its country 1 final consumers
- Interpretation of column:
  - Country 3, industry 2 uses 0.5 from country 1, industry 2
  - Country 3, industry 2 uses 1 from itself
  - Country 3, industry 2 uses 3 from its own primary inputs (payment to factors)

Macroeconomic accounts

	Country 1	Country 2	Country 3	World
Value added (GDP)	7	7	7	21
Final demand expenditures	3.75	10.75	7	21
Net saving	3.75	-3.75	0	0
Exports	3.25	1.75	2.5	7.5
Imports	0	5	2.5	7.5
Net exports	3.75	-3.75	0	0

Net trade matrix

	Country 1	Country 2	Country 3	Net imports
Country 1		2.5	0.75	3.25
Country 2	-2.5		-0.75	-3.25
Country 3	-0.75	0.75		0
Net exports	-3.25	3.25	0	0

Scheme of world input-output table

z_{11}^{11}	z_{12}^{11}	z_{11}^{12}	z_{12}^{12}	z_{11}^{13}	z_{12}^{13}	f_1^{11}	f_1^{12}	f_1^{13}	x_1^1
z_{21}^{11}	z_{22}^{11}	z_{21}^{12}	z_{22}^{12}	z_{21}^{13}	z_{22}^{13}	f_2^{11}	f_2^{12}	f_2^{13}	x_2^1
z_{11}^{21}	z_{12}^{21}	z_{11}^{22}	z_{12}^{22}	z_{11}^{23}	z_{12}^{23}	f_1^{21}	f_1^{22}	f_1^{23}	x_1^2
z_{21}^{21}	z_{22}^{21}	z_{21}^{22}	z_{22}^{22}	z_{21}^{23}	z_{22}^{23}	f_2^{21}	f_2^{22}	f_2^{23}	x_2^2
z_{11}^{31}	z_{12}^{31}	z_{11}^{32}	z_{12}^{32}	z_{11}^{33}	z_{12}^{33}	f_1^{31}	f_1^{32}	f_1^{33}	x_1^3
z_{21}^{31}	z_{22}^{31}	z_{21}^{32}	z_{22}^{32}	z_{21}^{33}	z_{22}^{33}	f_2^{31}	f_2^{32}	f_2^{33}	x_2^3
v_1^1	v_2^1	v_1^2	v_2^2	v_1^3	v_2^3				
x_1^1	x_2^1	x_1^2	x_2^2	x_1^3	x_2^3				

Scheme of world input-output table in (partitioned) matrix notation

$$\begin{aligned}
\mathbf{Z} &= \begin{bmatrix} \mathbf{Z}^{11} & \dots & \mathbf{Z}^{1,41} \\ \vdots & \ddots & \vdots \\ \mathbf{Z}^{41,1} & \dots & \mathbf{Z}^{41,41} \end{bmatrix} \begin{bmatrix} \mathbf{f}^{1,1} + \dots + \mathbf{f}^{1,41} \\ \vdots \\ \mathbf{f}^{41,1} + \dots + \mathbf{f}^{41,41} \end{bmatrix} = \mathbf{f} \\
\mathbf{w}' &= \begin{bmatrix} (\mathbf{w}^1)' & \dots & (\mathbf{w}^{41})' \end{bmatrix} \\
\mathbf{x}' &= \begin{bmatrix} (\mathbf{x}^1)' & \dots & (\mathbf{x}^{41})' \end{bmatrix}
\end{aligned}$$

World Input-Output Database: Construction

The World Input-Output Database (WIOD) project

- Large-scale EU Framework 7 project (May 2009 to April 2012)
- Inter-country Supply-Use and Input-Output tables
  - Benchmarked to National Accounts data
  - 35 sectors (NACE Rev. 1) and 59 products (CPA)
  - Linked together with bilateral trade data for goods and services
- Period: 1995-2009/2011 (current and previous year prices)
- 40 countries (85% of world GDP) + estimate for RoW
  - 27 EU countries
  - Australia, Brazil, Canada, China, India, Indonesia, Japan, Russia, Mexico, South Korea, Taiwan, Turkey, USA
- Satellite accounts
  - Socio-economic: Capital and labour (HS, MS, LS) in physical inputs and factor incomes
  - Environmental accounts (emissions, energy use, resource use)

Construction of IO tables

- Supply-Use system
  - Supply table: Value of products produced by each industry
  - Use table: Products used by each industry and final demand
- From Supply-Use System to IO tables
  - Requires particular technology assumptions

    e.g. product X is produced with same technology in each industry
  - Results in either Industry x Industry or Product x Product tables

Supply and Use System: 2 industries, 3 products, in monetary units (e.g. mn US-$)

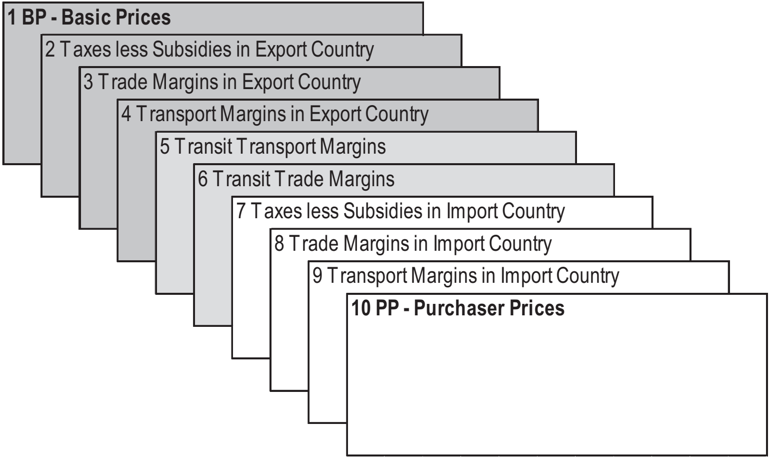
USE table (in purchaser prices)

	Industry				
	1	2	Final demand	Exports	Total
A	2	1	2	1	6
B	1	1	2	1	5
C	2	3	3	1	9
VA	4	6			
GO	9	11			

SUPPLY table (in basic prices with transformation into purchaser prices)

	Industry				
	1	2	Imports	Margins	Total
A	5	0	1	1	6
B	4	6	1	1	5
C	0	5	2	2	9
GO	9	11			

From basic to purchaser prices in global SUTs



Source: Streicher and Stehrer, 2012

Data construction issues/challenges

- Country specific issues
- Use tables in basic prices and valuation matrices
- Import by use and import use tables
    - Comparability/consistency across countries and over time
    - (Modified) BEC correspondence at HS6-digit level not fine enough (e.g. shirts and parts of shirts)
- Size, patterns and treatment of re-exports
- Services trade data
    - Quality and availability over time
    - Correspondence of BoP codes to CPA
    - Split into end-use categories
- Product and import price deflators

From National Accounts (NA) and National SUTs to WIOTs

National accounts data
- Value added by industry
- Gross output by industry
- Total final demand by type
- Total exports and imports

SUTs (infrequent)
- Supply table (basic price)
- Use table (purchasers' price)

International trade statistics (bilateral)
- Goods trade (UN COMTRADE), HS 6-digit
- Service trade (IMF, OECD, Eurostat)

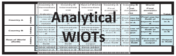
Harmonisation/estimation/benchmarked to NA
Transformation of use table into basic prices

Time series of national SUTs
- Supply tables (basic price)
- Use tables (purchasers' and basic price)
- Valuation matrices

Construction of import use tables by delivering country
- modified BEC classification
- cif/fob corrections
- etc.

World SUT (without RoW)
- Data and estimation of RoW trade
- RAS for bilateral trade flows

Transformation into World IO table
- Eurostat 'Model D' ('fixed product sales structure')
- Estimation of RoW



Documents:

Timmer, M. (ed., 2012): The World Input-Output Database (WIOD): Contents, Sources and Methods, WIOD Working Paper, Nr. 10

Dietzenbacher, E., B. Los, R. Stehrer, M. Timmer and G. deVries (2013): The Construction of World Input-Output Tables in the WIOD Project, Economic Systems Research, forthcoming

Scheme of analytical WIOT

		Country A Intermediate use *Industry*	Country B Intermediate use *Industry*	Rest of World Intermediate use *Industry*	Country A Final domestic use	Country B Final domestic use	Rest of World Final domestic use	Total
Country A	*Industry*	Intermediate use of domestic output	Intermediate use by B of imports from A	Intermediate use by RoW of imports from A	Final use of domestic output	Final use by B of exports from A	Final use by RoW of exports from A	Output in A
Country B	*Industry*	Intermediate use by A of imports from B	Intermediate use of domestic output	Intermediate use by RoW of imports from B	Final use by A of exports from B	Final use of domestic output	Final use by RoW of exports from B	Output in B
Rest of World (RoW)	*Industry*	Intermediate use by A of imports from RoW	Intermediate use by B of imports from RoW	Intermediate use of domestic output	Final use by A of exports from RoW	Final use by B of exports from RoW	Final use of domestic output	Output in RoW
		Value added	Value added	Value added				
		Output in A	Output in B	Output in RoW				

Other global IO tables

- Asian IO tables (IDE-Jetro)
- GTAP project
- Eurostat consolidated EU table (Eurostat and IPTS)
- EXIOPOL
- OECD-WTO initiative
- Australia (Manfred Lentzen)
- Johnson and Noguera (2012)
- GIO tables (Yokahama University)

See Special issue of Economic Systems Research (2013)

Literature:

- Dietzenbacher, E., B. Los, R. Stehrer, M. Timmer and G. deVries (2013), The Construction of World Input-Output Tables in the WIOD Project, Economic Systems Research, forthcoming.
- Miller, R.E. and P.D. Blair (2009), Input-Output Analysis. Foundations and Extensions, CUP.
- Timmer, M. (2012, ed.), The World Input Output Database: Contents, Sources and Methods, WIOD Working Paper No. 10.
- Streicher, G. and R. Stehrer (2012), Whither Panama: Constructing a consistent and balanced world SUT including international trade and transport margins, WIOD Working Paper No. 13.

Fundamentals: Leontief matrix and final demand multipliers

Define

$$\mathbf{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \qquad \mathbf{Z} = \begin{bmatrix} z_{11} & \dots & z_{1n} \\ \vdots & \ddots & \vdots \\ z_{n1} & \dots & z_{nn} \end{bmatrix} \qquad \mathbf{f} = \begin{bmatrix} f_1 \\ \vdots \\ f_n \end{bmatrix}$$

Matrix notation:

$$\mathbf{x} = \mathbf{Zi} + \mathbf{f}$$

Production functions and IO model (in monetary terms)

- Fundamental assumption: fixed input structures
- Interindustry flow from i to j (for a given period) depends entirely on total output of sector j (in this year)

$$a_{ij} = \frac{z_{ij}}{x_j}$$

- a_{ij} ... technical coefficients:
  input of i (in \$) per unit of gross output j (in \$)
- Note: $z_{ij} = a_{ij}x_j$ (trivial)
- Implicit assumptions:
  - Constant returns to scale
  - Sectors use inputs in fixed proportions: Leontief production function

Model for closed economy with one sector only:

$$\begin{aligned} x &= z + f \\ x &= a \cdot x + f \qquad \text{as } a = z/x \\ x - a \cdot x &= f \\ (1-a) \cdot x &= f \\ x &= (1-a)^{-1} \cdot f = \frac{1}{1-a} \cdot f \end{aligned}$$

- Value added coefficient: $v = w/x$ (value added divided by gross output)
- Note that (column interpretation)

$$\begin{aligned} x &= z + w \\ x &= ax + w \\ 1 &= a + v \\ v &= 1 - a \end{aligned}$$

Therefore: Value added (GDP, total income) equals final expenditures

$$v \cdot x = v \cdot (1-a)^{-1} \cdot f = (1-a) \cdot (1-a)^{-1} \cdot f \Leftrightarrow w = f$$

If f increases by 1, i.e. $\Delta f = 1$, x must increase by 1; for x to increase by 1, $ax = a\Delta f$ must be produced; for $a\Delta f$ to be produced, $a(a\Delta f) = a^2\Delta f$ must be produced; ...

$$\Delta x = \underbrace{(1 + a + a^2 + a^3 + \dots)}_{\text{Geometric series}} \Delta f$$

$$\Delta x = \frac{1}{1-a}\Delta f$$

$$\Delta x = (1-a)^{-1}\Delta f$$

- As v is known, this translates into income change: $\Delta w = v \cdot \Delta x$
- Similarly, if employment or CO_2 emmissions per unit of gross output is known, one can calculate employment or CO_2 effects

Translation into multi-sector structure (in matrix notation)

Given $\mathbf{Z}$ and $\mathbf{x}$ we can derive (divide each column in $\mathbf{Z}$ by gross output)

$$\mathbf{A} = \mathbf{Z}\hat{\mathbf{x}}^{-1}$$

If technical coefficients a_{ij} are known

$$\begin{aligned} x_1 &= a_{11}x_1 + \cdots + a_{1j}x_j + \cdots + a_{1n}x_n + f_1 \\ &\vdots \\ x_i &= a_{i1}x_1 + \cdots + a_{ij}x_j + \cdots + a_{in}x_n + f_i \\ &\vdots \\ x_n &= a_{n1}x_1 + \cdots + a_{nj}x_j + \cdots + a_{nn}x_n + f_n \end{aligned}$$

In matrix notation:

$$\mathbf{x} = \mathbf{A}\mathbf{x} + \mathbf{f}$$

Assume that $\mathbf{f}$ is fixed (exogenous), then the above equation is a set of n equations in n unknowns.

$$\begin{aligned} \mathbf{x} &= \mathbf{A}\mathbf{x} + \mathbf{f} \\ \mathbf{x} - \mathbf{A}\mathbf{x} &= \mathbf{f} \\ (\mathbf{I} - \mathbf{A})\mathbf{x} &= \mathbf{f} \\ \mathbf{x} &= (\mathbf{I} - \mathbf{A})^{-1}\mathbf{f} = \mathbf{L}\mathbf{f} \end{aligned}$$

- $(\mathbf{I} - \mathbf{A})^{-1} = \mathbf{L}$ ... Leontief inverse (or total requirements matrix)
- National accounting identity (total income = final demand):

$$\mathbf{v}'\mathbf{x} = \mathbf{i}'(\mathbf{I} - \mathbf{A})(\mathbf{I} - \mathbf{A})^{-1} = \mathbf{i}'\mathbf{f}$$

The calculations

$$\mathbf{A} = \mathbf{Z}\hat{\mathbf{x}}^{-1} = \begin{bmatrix} 150 & 500 \\ 200 & 100 \end{bmatrix} \begin{bmatrix} 1000 & 0 \\ 0 & 2000 \end{bmatrix}^{-1} = \begin{bmatrix} 150 & 500 \\ 200 & 100 \end{bmatrix} \begin{bmatrix} \frac{1}{1000} & 0 \\ 0 & \frac{1}{2000} \end{bmatrix} = \begin{bmatrix} 0.15 & 0.25 \\ 0.20 & 0.05 \end{bmatrix}$$

$$\begin{aligned} \mathbf{L} = (\mathbf{I} - \mathbf{A})^{-1} &= \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.15 & 0.25 \\ 0.20 & 0.05 \end{bmatrix} \right)^{-1} = \begin{bmatrix} 1 - 0.15 & -0.50 \\ -0.20 & 1 - 0.10 \end{bmatrix}^{-1} \\ &= \begin{bmatrix} 1.2541 & 0.3300 \\ 0.2640 & 1.1221 \end{bmatrix} \end{aligned}$$

$$\mathbf{x} = \mathbf{L}\mathbf{f} = \begin{bmatrix} 1.2541 & 0.3300 \\ 0.2640 & 1.1221 \end{bmatrix} \begin{bmatrix} 350 \\ 1700 \end{bmatrix} = \begin{bmatrix} 1000 \\ 2000 \end{bmatrix}$$

What happens if final demand in sector 1 increases to 450, i.e. $\Delta f_1 = 100$?

$$\Delta \mathbf{x} = \mathbf{L}\Delta \mathbf{f} = \begin{bmatrix} 1.2541 & 0.3300 \\ 0.2640 & 1.1221 \end{bmatrix} \begin{bmatrix} 100 \\ 0 \end{bmatrix} = \begin{bmatrix} 125.41 \\ 26.40 \end{bmatrix}$$

Notation:

$$\Delta \mathbf{x} = \mathbf{x}_1 - \mathbf{x}_0 \qquad \mathbf{L}_1 = \mathbf{L}_0 \qquad \Delta \mathbf{f} = \mathbf{f}_1 - \mathbf{f}_0$$

$$\mathbf{x}_1 = \mathbf{x}_0 + \Delta \mathbf{x} = \begin{bmatrix} 1000 \\ 2000 \end{bmatrix} + \begin{bmatrix} 125.41 \\ 26.40 \end{bmatrix} = \begin{bmatrix} 1125.41 \\ 2026.40 \end{bmatrix}$$

- A change in final demand in sector 1, also increases gross output (and therefore value added, employment,...) in sector 2 via inter-industry linkages
- Multiplier effects
- Analogous interpretation if 1 and 2 denote countries (rather than industries)

Final demand in each sector is

$$\text{Round 0} : \mathbf{I}\Delta\mathbf{f} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 100 \\ 0 \end{bmatrix} \begin{bmatrix} 100 \\ 0 \end{bmatrix}$$

To produce this, each sector needs input from its own and the other sector:

$$\text{Round 1} : \mathbf{A}\Delta\mathbf{f} = \begin{bmatrix} 0.15 & 0.25 \\ 0.20 & 0.05 \end{bmatrix} \begin{bmatrix} 100 \\ 0 \end{bmatrix} = \begin{bmatrix} 15 \\ 20 \end{bmatrix}$$

However, to produce the extra output (15,20) again inputs from other sectors are needed:

$$\text{Round 2} : \mathbf{A}(\mathbf{A}\Delta\mathbf{f}) = \mathbf{A}^2\Delta\mathbf{f} = \begin{bmatrix} 0.15 & 0.25 \\ 0.20 & 0.05 \end{bmatrix}^2 \begin{bmatrix} 100 \\ 0 \end{bmatrix} = \begin{bmatrix} 7.25 \\ 4.00 \end{bmatrix}$$

However, to produce the extra output (7.25,4.00) again inputs from other sectors are needed:

$$\text{Round 3} : \mathbf{A}\big(\mathbf{A}(\mathbf{A}\Delta\mathbf{f})\big) = \mathbf{A}^3\Delta f = \begin{bmatrix} 0.15 & 0.25 \\ 0.20 & 0.05 \end{bmatrix}^3 \begin{bmatrix} 100 \\ 0 \end{bmatrix} = \begin{bmatrix} 2.0875 \\ 1.6500 \end{bmatrix}$$

However, ...

The rounds-approach

	0	1	2	3	4	5	...	∞
Industry 1	100.0	15.0	7.3	2.1	0.7	0.2	...	0
Industry 2	0.0	20.0	4.0	1.7	0.5	0.2	...	0
Cumulative total								
Industry 1	100.0	115.0	122.3	124.3	125.1	125.3	...	125.4
Industry 2	0.0	20.0	24.0	25.7	25.2	26.3	...	26.4
Percent of Total Effect Captured								
Industry 1	79.7	91.7	97.5	99.1	99.7	99.9	...	100
Industry 2	0.0	75.8	90.9	97.2	99.1	99.7	...	100

The power series approximation of the Leontief inverse

Remember:

$$1 + a + a^2 + a^3 + \cdots = \frac{1}{1-a} = (1-a)^{-1} \qquad \text{for } 0 < a < 1$$

This suggests

$$\mathbf{L} = (\mathbf{I} - \mathbf{A})^{-1} = \mathbf{I} + \mathbf{A} + \mathbf{A}^2 + \mathbf{A}^3 + \ldots$$

under certain conditions:

- $\mathbf{A} \geq \mathbf{0}$ (coefficients matrix contains only non-negative terms)
- The system produces more output than it requires inputs $N(\mathbf{A}) < 1$
- $|\mathbf{I} - \mathbf{A}| > 0$
- More general: Hawkins-Simon conditions

Literature:

- Miller, R.E. and P.D. Blair (2009), Input-Output Analysis. Foundations and Extensions, CUP.