Firm-Level Dispersion in Productivity - Is the Devil in the Details?\footnote{Any opinions and conclusions expressed herein are those of the authors and do not necessarily represent the views of the U.S. Census Bureau. All results have been reviewed to ensure that no confidential information is disclosed.}

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Background

- Important finding in empirical literature: productivity differences among establishments are large, even in narrowly defined industries (Syverson (2011), FGHW (2015)).
  - Dispersion is important as a measure of heterogeneity and because it is relevant for business dynamism and growth.
- This conclusion holds for both revenue-based and quantity-based productivity measures.
  - However, micro datasets rarely contain information on prices or quantities. Most of the evidence is based on revenue productivity.
- High dispersion robust to alternative estimation methods. Estimation methods viewed as not critical for this and other core findings (Syverson (2011)).
- But as we show, the alternative methods yield conceptually different measures. Moreover, this is potentially important since one specific measure has become important as an indicator of misallocation (Hsieh-Klenow (2009)).
Their insight is that dispersion in a particular revenue productivity measure reflects dispersion in distortions - under certain assumptions about production and demand.

Widely used in analyses of misallocation [keyword search in title on ideas.repec.org returns 70 records in 2014-2015].

This paper investigates the generality of this insight:

- we show that the conclusions in Hsieh-Klenow (2009) don’t necessarily hold under alternative assumptions about returns to scale (relevant because evidence suggests NCRS);
- we show that alternative revenue productivity measures have different implications even under the assumptions made by Hsieh-Klenow (2009);
- present a framework that can be used to make inferences about the properties of distortions and frictions.
**TFPR conceptual measure is critical**

- Conceptual measure of revenue per composite input, useful to consider (Foster-Haltiwanger-Syverson (2008), in logs):

\[ tfpr_i = p_i + tfpq_i = p_i + q_i - \sum_j \alpha_j x_{ij} \]

- \( \alpha_j \) are factor elasticities from Cobb-Douglas production function

- **Insight in Hsieh-Klenow (2009):**
  1. Downward sloping demand \( \Rightarrow \) negative relationship between physical productivity and product prices.
  2. Add CRS technology and iso-elastic demand \( \Rightarrow \) **TFPR** is equalized across plants in the absence of distortions or frictions because high-productivity plants experience an exactly offsetting price decline.
  3. The implication is that observed **TFPR**-dispersion must reflect distortions.
What do we measure?

*TFPR* vs. commonly used revenue productivity measures

1. Cost-share-based methods: cost min. with CRS yields factor elasticities and, by definition, *TFPR*:

   \[
   tfpr_{i}^{cs} = p_i + q_i - \sum_j \alpha_j x_{ij}
   \]

   \[\Rightarrow tfpr_{i}^{cs} = tfpr_i\]

2. Regression-based methods in general yield revenue elasticities

   \[
   tfpr_{i}^{rr} = p_i + q_i - \sum_j \beta_j x_{ij}
   \]

   \[\Rightarrow tfpr_{i}^{rr} \neq tfpr_{i}^{cs}\]

   \[\Rightarrow tfpr_{i}^{rr} \neq tfpr_i\]

Revenue elasticities will, in general, be a function of factor elasticities and demand parameters. Revenue residual will be a function of technical efficiency and demand shocks.
We show in the paper that assuming Log-linear technology, Iso-elastic demand, and arbitrary RTS...
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...TFPR dispersion ($\delta_{tfpr}$) depends on:
- Demand elasticity ($\rho$)
- RTS ($\gamma$)
We show in the paper that assuming

- Log-linear technology,
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- And arbitrary RTS...

...TFPR dispersion ($\delta_{tfpr}$) depends on:

- Demand elasticity ($\rho$)
- RTS ($\gamma$)
- Dispersion in demand shocks ($\delta_\xi$), TFPQ ($\delta_{tfpq}$) and distortions ($\delta_\kappa$).
Relationship Between Revenue Productivity Measures

(Conceptual) \( TFPR \)

\[
\delta_{tfpr} = \frac{1}{1 - \rho \gamma} \left( (1 - \gamma) \left( \delta_\xi + \rho \delta_{tfpq} \right) + (1 - \rho) \sum_j \alpha_j \delta_{\kappa_j} \right)
\]

Implication: RTS is crucial for the result on dispersion.
Relationship Between Revenue Productivity Measures

(Conceptual) TFPR

\[
\delta_{tfpr} = \frac{1}{1 - \rho \gamma} \left( (1 - \gamma) (\delta_{\xi} + \rho \delta_{tftpq}) + (1 - \rho) \sum_{j} \alpha_j \delta_{\kappa_j} \right)
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Implication: RTS is crucial for the result on dispersion.
Relationship Between Revenue Productivity Measures
(Conceptual) $TFPR$

$$\delta_{tfpr} = \frac{1}{1 - \rho \gamma} \left( (1 - \gamma) \left( \delta_\xi + \rho \delta_{tpq} \right) + (1 - \rho) \sum_j \alpha_j \delta_{\kappa_j} \right)$$

Implication: RTS is crucial for the result on dispersion.

$CRS \implies \delta_{tfpr} = \frac{1 - \rho}{1 - \rho \gamma} \sum_j \alpha_j \delta_{\kappa_j}$
Relationship Between Revenue Productivity Measures

(Conceptual) \textit{TFPR}

\[ \delta_{tfpr} = \frac{1}{1 - \rho \gamma} \left( (1 - \gamma) \left( \delta_{\xi} + \rho \delta_{tfpq} \right) + (1 - \rho) \sum_j \alpha_j \delta_{\kappa_j} \right) \]

Implication: RTS is crucial for the result on dispersion.

\textit{CRS} \implies \delta_{tfpr} = \frac{1 - \rho}{1 - \rho \gamma} \sum_j \alpha_j \delta_{\kappa_j} \]

Conclusion: deviation from CRS yields the result that variation in \textit{TFPR} is affected also by demand shocks and \textit{TFPQ} shocks.
Under the same assumptions, we also show that empirical estimates of \( tfpr_{rr}^i \) depend on demand elasticity (\( \rho \)), demand shocks (\( \xi \)) and \( tfpq \).

\[
\begin{align*}
  tfpr_{rr}^i &= \rho tfpq_i + \ln \xi_i + p \\
  &\text{(no distortions here, RTS free)}
\end{align*}
\]
Relationship Between Revenue Productivity Measures

Empirical measures

- Under the same assumptions, we also show that empirical estimates of $tfpr_i^{rr}$ depend on demand elasticity ($\rho$), demand shocks ($\xi$) and $tfpq$.

$$tfpr_i^{rr} = \rho tfpq_i + \ln \xi_i + p$$

(no distortions here, RTS free)

- This implies we can write $tfpr_i$ as

$$tfpr_i = \lambda + \frac{1 - \gamma}{1 - \rho \gamma} tfpr_i^{rr} + \frac{1 - \rho}{1 - \rho \gamma} \sum_j \alpha_j \ln \kappa_{ij}$$
Relationship Between Revenue Productivity Measures

Empirical measures

- Under the same assumptions, we also show that empirical estimates of $tfpr_{i}^{rr}$ depend on demand elasticity ($\rho$), demand shocks ($\xi$) and $tfpq$.

$$tfpr_{i}^{rr} = \rho tfpq_{i} + \ln \xi_{i} + p$$

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$$tfpr_{i} = \lambda + \frac{1 - \gamma}{1 - \rho \gamma} tfpr_{i}^{rr} + \frac{1 - \rho}{1 - \rho \gamma} \sum_{j} \alpha_{j} \ln \kappa_{ij}$$

- Evidence (FGHW (2015)) suggests $tfpr_{i}^{cs}$ and $tfpr_{i}^{rr}$ are highly correlated.
Relationship Between Revenue Productivity Measures

Empirical measures

- Under the same assumptions, we also show that empirical estimates of $tfpr_i^{rr}$ depend on demand elasticity ($\rho$), demand shocks ($\xi$) and $tfpq$.

$$tfpr_i^{rr} = \rho tfpq_i + \ln \xi_i + p$$

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$$tfpr_i = \lambda + \frac{1 - \gamma}{1 - \rho \gamma} tfpr_i^{rr} + \frac{1 - \rho}{1 - \rho \gamma} \sum_j \alpha_j \ln \kappa_{ij}$$

- Evidence (FGHW (2015)) suggests $tfpr_i^{cs}$ and $tfpr_i^{rr}$ are highly correlated.

- Under **CRS**, this can only be possible if distortions are correlated with technology and demand shocks.
Relationship Between Revenue Productivity Measures

Empirical measures

- Under the same assumptions, we also show that empirical estimates of $\text{tfpr}_i^{rr}$ depend on demand elasticity ($\rho$), demand shocks ($\xi$) and $\text{tfpq}$.

$$
\text{tfpr}_i^{rr} = \rho \text{tfpq}_i + \ln \xi_i + p
$$

(no distortions here, RTS free)

- This implies we can write $\text{tfpr}_i$ as

$$
\text{tfpr}_i = \lambda + \frac{1 - \gamma}{1 - \rho \gamma} \text{tfpr}_i^{rr} + \frac{1 - \rho}{1 - \rho \gamma} \sum_j \alpha_j \ln \kappa_{ij}
$$

- Evidence (FGHW (2015)) suggests $\text{tfpr}_i^{cs}$ and $\text{tfpr}_i^{rr}$ are highly correlated.

- Under **CRS**, this can only be possible if distortions are correlated with technology and demand shocks.

- Under **NCRS**, correlation is determined by $\gamma$ and $\rho$.
To implement above decompositions exactly, need to estimate factor elasticities and demand parameters.

Absent data on prices and quantities, we follow the approach in Klette-Griliches (1996) and De Loecker (2011) to jointly identify revenue function and demand parameters. Crude approach, would be better to have data on demand.

Under those assumptions, $\alpha_j$-s can be calculated using estimates of $\beta_j$ and $\rho$, and therefore we can estimate $tfpr_i$, its dispersion and the components of the decomposition.
Digging Deeper – Exploratory empirical exercise

1) Recover quantity elasticites ($\alpha_j$) from revenue elasticities ($\beta_j$)

- Under isoelastic demand, $P_i = P(Q/Q_i)^{1-\rho} \zeta_i$ where $\zeta_i$ is a demand shifter, writing out plant-level log-revenues gives the estimating equation:

$$p_i + q_i = \rho q_i + \ln \zeta_i + (1 - \rho) q + p$$

$$= \rho \left( \sum_j \alpha_j x_i^j + \text{tfp}q_i \right) + \ln \zeta_i + (1 - \rho) q + p$$

$$= \sum_j (\rho \alpha_j) x_i^j + \rho \text{tfp}q_i + \ln \zeta_i + (1 - \rho) q + p$$

- Joint estimation of rev. elasts and demand parameter helps. $\hat{\beta}_j = \rho \hat{\alpha}_j$: rev. elasts. We can recover demand parameter using coefficient of aggregate revenues $\hat{\beta}_q = 1 - \rho$, and factor elasticities are determined by $\hat{\alpha}_j = \hat{\beta}_j / \rho$. 
2) Implement dispersion decomposition

- Revenue function estimation yields direct estimates of
  \[ \beta_j, \rho, \text{tfpr}_{rr} \]
  and we know
  \[ \text{tfpr}_{rr} = \rho \text{tfpq}_i + \ln \xi_i. \]

- Using \( \beta_j \) and \( \rho \), we can calculate
  \[ \alpha_j \]
  \[ \gamma = \sum_j \alpha_j \]
  \[ \text{tfpr}_i = p_i + q_i - \sum_j \alpha_j x_{ij} \]
  and...

- Their dispersion \( \delta \text{tfpr}, \delta \text{tfpr}_{rr} \)
  and we know
  \[ \delta \text{tfpr}_{rr} = \rho \delta \text{tfpq}_i + \delta \xi_i. \]

- So we can characterize the composite distortion term
  \[ (1 - \rho (1 - \rho \gamma \sum_j \alpha_j \ln \kappa_{ij})) \]
  and its dispersion
  \[ (1 - \rho (1 - \rho \gamma \sum_j \alpha_j \delta \kappa_j)) \]

Plant-level data: ASM, CM (1972-2010), 50 largest industries (see FGHW (2015) for details).
Revenue function estimation yields direct estimates of
1. $\beta_j$

Using $\beta_j$ and $\rho$, we can calculate
1. $\alpha_j$ and $\gamma = \sum_j \alpha_j$
2. $tfpr_i = p_i + q_i - \sum_j \alpha_j x_{ij}$ and...
3. their dispersion $\delta_{tfpr}$, $\delta_{tfpr_{rr}}$ and we know $\delta_{tfpr_{rr}} = \rho \delta_{tfpq}$

So we can characterize the composite distortion term $(1 - \rho(1 - \rho \sum_j \alpha_j \ln \kappa_{ij}))$ and its dispersion $(1 - \rho(1 - \rho \sum_j \alpha_j \delta \kappa_j))$
Digging Deeper – Exploratory empirical exercise

2) Implement dispersion decomposition

- Revenue function estimation yields direct estimates of:
  1. $\beta_j$
  2. $\rho$

- Using $\beta_j$ and $\rho$, we can calculate:
  1. $\alpha_j$ and $\gamma = \sum_j \alpha_j$
  2. $tfpr_i = p_i + q_i - \sum_j \alpha_j x_{ij}$ and...
  3. their dispersion $\delta_{tfpr}$, $\delta_{tfpr_{rr}}$ and we know $\delta_{tfpr_{rr}} = \rho \delta_{tfpq_i} + \delta_{\xi_i}$

- So we can characterize the composite distortion term \((1 - \rho)(1 - \rho \gamma \sum_j \alpha_j \ln \kappa_{ij})\) and its dispersion \((1 - \rho)(1 - \rho \gamma \sum_j \alpha_j \delta_{\kappa_j})\)

- Plant-level data: ASM, CM (1972-2010), 50 largest industries (see FGHW (2015) for details).
Revenue function estimation yields direct estimates of
1. \( \beta_j \)
2. \( \rho \)
3. \( tfpr_{i}^{rr} \) and we know \( tfpr_{i}^{rr} = \rho tfpq_i + \ln \xi_i \).

Plant-level data: ASM, CM (1972-2010), 50 largest industries (see FGHW (2015) for details).
Digging Deeper – Exploratory empirical exercise

2) Implement dispersion decomposition

- Revenue function estimation yields direct estimates of:
  1. $\beta_j$
  2. $\rho$
  3. $tfpr_{i}^{rr}$ and we know $tfpr_{i}^{rr} = \rho tfpq_{i} + \ln \zeta_{i}$.

- Using $\beta_j$ and $\rho$, we can calculate
Revenue function estimation yields direct estimates of

1. $\beta_j$
2. $\rho$
3. $tfpr_{ij}^{rr}$ and we know $tfpr_{ij}^{rr} = \rho tfpq_i + \ln \xi_i$.

Using $\beta_j$ and $\rho$, we can calculate

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3. $tfpr_{i}^{rr}$ and we know $tfpr_{i}^{rr} = \rho tfpq_i + \ln \xi_i$. 

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Digging Deeper – Exploratory empirical exercise

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- So we can characterize the composite distortion term
  \[
  \left( \frac{1-\rho}{1-\rho \gamma} \sum_j \alpha_j \ln \kappa_{ij} \right) \text{ and its dispersion } \left( \frac{1-\rho}{1-\rho \gamma} \sum_j \alpha_j \delta_{\kappa_j} \right)
  \]
Digging Deeper – Exploratory empirical exercise

2) Implement dispersion decomposition

- Revenue function estimation yields direct estimates of
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- Using $\beta_j$ and $\rho$, we can calculate
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  \left( \frac{1-\rho}{1-\rho \gamma} \sum_j \alpha_j \ln \kappa_{ij} \right)
  \]

- Plant-level data: ASM, CM (1972-2010), 50 largest industries
  (see FGHW (2015) for details).
Exploratory empirical exercise - Findings

- Dispersion in $tfpr_{i}^{rr}$ and $\kappa$ (derived estimate of distortions) are similar, on average .2-.3.
- Correlation between $tfpr_{i}^{rr}$ and $\kappa$ is high.
- Interpretation:
  - $corr(tfpq_{i}, \kappa)$ and $corr(\xi, \kappa)$ positive ($\approx$ FGHW (2015) with much less structure)
  - In other words, empirical evidence suggests that $tfpq$ shocks and demand shocks are more likely to hit plants with higher distortions - under HK assumptions.
- Why?
Conclusions

- Empirical evidence suggests that $tfpq$ shocks and demand shocks are more likely to hit plants with higher distortions under HK assumptions.
- An alternative interpretation associates the derived distortion estimates with frictions. Establishments with high $tfpq$ have high $tfpr$ because it takes time to adjust their production factors.
- In sum, caution needs to be used interpreting dispersion in revenue productivity as reflecting distortions.
- Estimation methods matter and can be insightful in this context.
- Additional caution since alternative demand/production functions yield more wedges between $tfpr$ and distortions.
**Table:** Cross-industry moments of the estimated demand parameter ($\rho$), returns to scale ($\gamma$), and dispersion measures: $tfpr$, $tfpr^{rr}$, $tfpr^{cs}$ and distortions.

<table>
<thead>
<tr>
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<th>$\rho$</th>
<th>$\gamma$</th>
<th>$\delta_{tfpr}$</th>
<th>$\delta_{tfpr^{rr}}$</th>
<th>$\delta_{K}$</th>
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Table: Within-industry correlations of terms underlying dispersion measures.

Panel 1: 50 industries

A: Cross-industry averages

<table>
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<tr>
<th></th>
<th>OP</th>
<th>tfpr$^{rr}$</th>
<th>tfpr</th>
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<th>tfpr$^{rr}_{0}$</th>
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<td>0.76</td>
<td>0.82</td>
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Revenue productivity measures
Relationship between returns to scale and the correlation between TFPR and distortions ($r(\text{tfpr, dist})$).

$$y = -5.85x^2 + 11.63x - 4.79$$

(adjusted) $R^2 = 0.73$