Transactions, Credit, and Central Banking in a Model of Segmented Markets

Stephen D. Williamson
Department of Economics
Washington University in St. Louis
St. Louis, MO 63130
Federal Reserve Bank of Richmond
Federal Reserve Bank of St. Louis
swilliam@artsci.wustl.edu

March 2007

Abstract

A segmented markets model is constructed with payments systems credit and a rich array of monetary policy instruments. Goods market segmentation plays an important role, in addition to the role played by conventional segmentation of asset markets. The diffusion of a money injection by the central bank depends on the interaction of agents in exchanging money for goods, and on the arrangements for clearing and settlement of payments system credit. Simple monetary policy rules are not efficient, in general.

1. INTRODUCTION

In this paper, we explore the implications of a tractable segmented markets model with credit and cash transactions, and with a rich array of possible central banking arrangements. As in traditional segmented markets models, this model has limited participation in particular asset markets, but a key element of the model is the segmentation of goods markets. The model permits open market operations, consumer credit transactions, daylight overdrafts, reserve-holding, overnight lending and borrowing, and clearing and settlement of consumer credit transactions. In the model, a central bank money injection is initially received only by some economic agents, and this injection is spread indirectly to other agents - that is, diffusion occurs over time - through goods market transactions and the clearing and settlement of payments system credit.

We interpret this model as a very short run framework for analyzing monetary and financial arrangements. Consistent with that, output is exogenous, and the velocity of money fluctuates due to shocks to payments arrangements This is meant to capture the disturbances that central banks respond to at a daily, weekly, or monthly frequency. We obtain a closed-form solution for the model, and then use it for two purposes. First, we study the operating characteristics of the model in response to monetary shocks and velocity shocks. Second, we explore the implications of alternative monetary policy rules for the behavior of consumption, prices, interest rates, and welfare.

This model builds on Williamson (2006b), which is a pure-currency framework where injections of outside money into the economy occur by way of lump-sum transfers. An important feature of that model is that there are two kinds of households, those who are *connected*, and those who are *unconnected*. Connected households can trade on asset markets, while unconnected households cannot. Further, and this is a novelty in that model, connected and unconnected households have "proximity" to different sets of goods markets, and this is critical to how monetary policy works. A Friedman rule for monetary policy is suboptimal, and an anticipated inflation effect on nominal interest rates tends to reinforce the liquidity effect, so that nominal interest rates are more volatile than in conventional segmented markets models. Monetary shocks have small effects on aggregate real quantities, but can have quantitatively important distributional effects.

This paper is related to the literature on asset market segmentation and monetary policy. One branch of the market segmentation literature is concerned with the development of general equilibrium versions of Tobin (1956) and Baumol (1952). In these models, some fraction of the population is engaged in asset transactions at any point in time, and thus central bank actions in asset markets will initially directly affect only this "participating" population. A monetary injection by the central bank causes a redistribution of wealth, which will in general result in short run changes in asset prices, employment, output, and the distribution of consumption across the population. The first models of this type were constructed by Grossman and Weiss (1983) and Rotemberg (1984). Later contributions include Alvarez and Atkeson (1997), and Alvarez, Atkeson, and Kehoe (2002), Alvarez, Atkeson and Edmond (2003), Chiu (2004) and Khan and Thomas (2007). Asset market segmentation is critical to the short-run nonneutralities of money that exist in these models, as is the case here. However, the propagation of monetary policy shocks is very different in our model. An important feature of Alvarez, Atkeson and Edmond (2003), Chiu (2004) and Khan and Thomas (2007), in particular, is that it is costly or impossible for households to exchange bonds and money frequently, and so when a central bank money injection occurs, households spend the money slowly over time. This yields persistence in liquidity effects and a sluggish response of prices. In our model, persistence results from goods market segmentation, and the responses of relative prices to monetary shocks is key to how these shocks are propagated.

The model of Alvarez, Lucas, and Weber (2001) is closely related to the one constructed here, particularly as we assume that there is a group of unconnected households that are permanently excluded from asset markets. An important difference is that Alvarez, Lucas, and Weber assume that all households interact in the same goods market. In their model, this implies that there is no propagation of monetary shocks.

Another related class of models deals with market segmentation in a representative household framework, and includes work by Lucas (1990), Fuerst (1992), and Christiano and Eichenbaum (1995). Fuerst's model, from which Christiano and Eichenbuam's is developed, features a nonneutrality of money working through a cash-in-advance constraint faced by firms that applies to the purchase of labor services. This is quite different from what holds in Tobin-Baumol-type models with endogenous labor supply.

Recent research in monetary theory is aimed at developing models of monetary economies that capture heterogeneity and the distribution of wealth in a manner that is tractable for analytical and quantitative work. One approach is to use a quasi-linear utility function as in Lagos and Wright (2005), an approach that, under some circumstances, leads to degeneracy in the distribution of money balances across the population. Another approach is to use a representative household with many agents, as in Shi (1997), in which (also see Lucas 1990) there can be redistributions of wealth within the household during the period, but these distribution effects do not persist. Work by Williamson (2006a) and Shi (2004) uses the quasi-linear-utility and representative-household approaches, respectively, to study some implications of limited participation for optimal monetary policy, interest rates, and output. Other related work is Head and Shi (2003), and Head and Lapham (2005).

In the model constructed here, each household consists of a producer and a continuum of consumers. The consumers purchase goods in different markets, but are more likely to buy from households of their own type (connected or unconnected). Goods are purchased with credit, and these debts must be settled within the period. However, some debts are settled more quickly than others. If debt is settled quickly, then sales of goods can be used to finance other purchases by the household within the period. Otherwise, if goods are sold by a household in exchange for credit instruments that do not settle quickly, then the receipts from these sales cannot be spent until the following period.

A connected household can borrow and lend on bond markets (one-period and within-period), and the central bank also borrows and lends in these markets. If a period is interpreted as one day, then the central bank can engage in actions that can be interpreted as the extension of daylight overdrafts and intervention in the overnight credit market. Connected households hold outside money as reserve accounts with the central bank, and the central bank has the option of paying interest on these reserves. Unconnected households cannot borrow and lend in bond markets, and they hold outside money in the form of currency.

In general, connected and unconnected households sell goods at different prices in equilibrium. Further, a consumer pays a premium in a goods purchase where the debt exchanged for the goods takes longer to clear. A monetary shock not only produces a liquidity effect, but it affects relative prices. That is, a positive money shock tends to reduce the nominal interest rate, increase the relative price of goods sold by connected households, and reduce the relative price of goods exchanged for debt that takes longer to clear.

In the model, the velocity of money fluctuates because of random shocks to payments arrangements. In particular, velocity rises when the average length of time required to clear debt falls. Fluctuations in velocity cause nominal interest rates, the price level, and relative prices to fluctuate. As well, there are effects on the distribution of consumption across the population.

In exploring the performance of alternative policy rules under velocity shocks, we first study optimal monetary policy in the special case where all households are connected. In this instance, a Friedman rule is optimal, and this policy rule can be implemented in several ways. The first we consider is a daylight-overdraft policy where the central bank lends households whatever transactions balances they desire each period at a zero nominal interest rate. Under this policy households hold no outside money balances between periods. The second is an interest-on-reserves policy under which interest is paid on outside money balances at the interest rate on one-period bonds. The third policy is a zero-nominal-interest-rate policy, implemented through daylight overdrafts or open market operations. All of these policies yield the same Pareto optimal equilibrium allocation if all households are connected.

Now, in the general case where there are both connected and unconnected households, we show that there exists no monetary policy that supports a Pareto optimal equilibrium allocation. Essentially, because monetary policy affects a fraction of the population only indirectly, it is a blunt tool that cannot correct all the distortions that exist in this economy. Further, the three policies that implement the Friedman rule when all households are connected imply different equilibrium allocations in the general case. Each policy rule acts to correct distortions in the financial sector of the economy, but has difficulty with the non-financial unconnected sector. For example, an optimal policy would involve paying interest on both reserves and currency, but paying interest on currency is technologically infeasible.

In Section 2 we set up the model, while in Section 3 we solve the optimization problems of households and show how to construct an equilibrium. In Section 4 we obtain the closed-form equilibrium solution of the model. Section 5 contains an examination of the operating characteristics of the economy under monetary shocks and velocity shocks, while the performance of alternative monetary policy rules is studied in Section 6. Finally, Section 7 is a conclusion.

2. THE MODEL

There is a continuum of infinitely-lived households with unit mass indexed by $i \in [0,1]$. Each household consists of a seller and a continuum of consumers with unit mass, with a consumer indexed by (i,j), and j uniformly distributed on the interval $[0,1]$. The preferences of household i are given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \int_0^1 \log(c_t^i(j))dj, \tag{1}$$

where t indexes time, $0 < \beta < 1$ and $c_t^i(j)$ is the consumption of consumer j who is a member household i.

Each household resides at a separate location. There is a fraction α of *connected* households, where $0 < \alpha < 1$. Connected households hold outside money as reserve accounts with the central bank, and also can trade on bond markets. Each connected household has M_0^1 units of outside money at the beginning of period 0. The remaining fraction $1-\alpha$ of households are *unconnected*, in that they hold outside money in the form of divisible fiat currency and do not trade on bond markets. Each unconnected household has M_0^2 units of outside money in period 0.

There is an absence-of-double-coincidence problem in this economy. At the beginning of each period, a household receives an endowment of y units of its own distinct good. This good is not consumed by any members of the household. At the beginning of the period, each consumer in the household receives a preference shock, which determines the good the consumer wishes to consume during the period. Each consumer then travels to the location of the household that produces his or her desired good, purchases some quantity of that good from the seller at that location, consumes, and then returns home. A given consumer cannot visit more than one location, in addition to his or her home location, during a period.

For a consumer in a connected household, the probability that the consumer's desired good during the current period will be one sold by a connected household is $1-(1-\alpha)\pi$ and the probability that the desired good will be sold by an unconnected household is $(1-\alpha)\pi$. For a consumer from an unconnected household, the probability that the consumer's desired good during the period is sold by an unconnected household is $1-\alpha\pi$, and the probability is $\alpha\pi$ that the consumer's desired good is sold by a connected household. For any consumer, the probability distribution for desired goods, conditional on the desired good being sold by a connected or unconnected household, is uniform. These meeting probabilities guarantee that the flows of consumers going from connected to unconnected households, and from unconnected to connected households, are equal each period. The parameter π governs the interaction between connected households and unconnected households as groups. That is, if $\pi < 1$ then the population of consumers arriving at a connected location will have a greater proportion of consumers from connected locations than would be observed arriving at an unconnected location, and similarly for unconnected locations. If $\pi = 1$, then the population of consumers is identical in composition across locations when consumers go shopping.

At the beginning of period t, a consumer receives his or her preference shock, and then visits only one other household, which will be the one selling the good that he or she desires. All goods are purchased with credit. That is, consumers exchange IOUs for goods, and the IOUs are settled during the period. Each household is a member of a clearinghouse, and clearinghouses permit the clearing and settlement of IOUs. We assume that in any meeting between a seller and a consumer, that there is a probability γ_t that the seller and the consumer belong (through their respective households) to the same clearinghouse. Note that this probability does not depend on whether the respective households of the seller and the consumer are connected or unconnected. We will assume that γ_t is stochastic. As we will see, this aggregate shock is essentially a disturbance to the technology of clearing and settlement, and is intended to capture short run shocks that occur within the payments system.

When consumers arrive to purchase goods from a seller in a household, the seller can observe whether the consumer's and seller's clearinghouses are the same or different. The law of large numbers implies that each household will be selling to a fraction γ_t of consumers who have the same clearinghouse membership, and to a fraction $1-\gamma_t$ whose clearinghouse membership is different. We will call the first type of transaction an *early-settlement transaction*, and the second a *late-settlement transaction*. In general, goods will be sold at different prices in early and late-settlement transactions, so that there are effectively two different markets for goods on which each individual household sells.

After households receive IOUs in exchange for the goods they have produced, the IOUs are sent to the appropriate clearinghouse; that is, an IOU issued by a particular household goes to the clearinghouse of which that household is a member. Thus, all of the IOUs issued by a household's consumers will at this point find their way back to the household's clearinghouse, and will represent debits on the household's account with its own clearinghouse, and the household will have received some IOUs from other households in early-settlement transactions, and these IOUs will constitute credits on the household's account with its clearinghouse. As well, the household will have credits in its accounts with other clearinghouses, due to the IOUs it has received in late-settlement transactions. Now, if all of the clearinghouses could communicate, then all within-period debts in this economy could be settled on net simultaneously, and there would be no need for outside money to intermediate transactions. However, we assume that there is no communication among clearinghouses, so that an individual clearinghouse has knowledge only of the IOUs issued that period by its own members. Also, we assume a sequence of meetings among households and clearinghouses that assures that there cannot be net clearing of IOUs across the whole economy within the period.

Clearing and settlement proceeds as follows. Suppose that there are N clearinghouses, indexed by $i = 1, 2, ..., N$. A type i household is a member of clearinghouse i. The type i household first meets with its own clearinghouse, jointly with all of the other households of the same type. There is then a transfer of outside money between each household and its own clearinghouse to settle each household's own-clearinghouse account. Then, each household meets with all of the other clearinghouses sequentially. That is, a household of type i then meets with clearinghouse $i + 1$, $i + 2, ..., N$, $1, 2, ..., i - 1$, in that order. At any given time during the clearing and settlement process, all households of a given type are together at a particular clearinghouse, so that households of different types never meet during the clearing and settlement process. In each meeting between a household and a clearinghouse, a transfer is made between the clearinghouse and the household to settle the account with the clearinghouse. Note, in each meeting between a household of type i and clearinghouse j, with $j \neq i$, that the transfer is made from the clearinghouse to the household, since in these instances the household can only have credits on these clearinghouse accounts.

Money is essential in the clearing and settlement of within-period IOUs, because of the lack of communication among clearinghouses, and because of the spatial separation of the agents involved in clearing and settlement. One might imagine similar models where clearinghouses could issue circulating debt which would eliminate the need for money. However, in this context, it is impossible for credit to be used in settling accounts with the clearinghouses, and so settlement must be accomplished with outside money.

A key feature here is that a sale of goods in an early-settlement transaction results in a within-period credit that can be used to finance consumer expenditure by the household during the period. However, a sale of goods in a late-settlement transaction yields outside money balances that cannot be spent until the next period.

After goods market transactions take place, and before the clearing and settlement process begins, connected households trade assets with the central bank. In the asset market on which connected households and the central bank trade, there are three assets: reserve balances, within-period nominal bonds, and one-period nominal bonds. In period t, a within-period bond sells for one unit of reserve balances and is a claim to r_t units of reserve balances at the end of the period, while a one-period bond sells for one unit of reserve balances in period t and pays off R_{t+1} units of reserve balances in period $t+1$. One interpretation of these arrangements is that a period is one day, borrowing by a household within the period is a daylight overdraft with the central bank, and overnight borrowing and lending can be accomplished through combinations of within-period and one-period borrowing and lending.

The key consequences of these payments arrangements can be summarized in the constraints faced by households, which will differ somewhat depending on whether the household is connected or unconnected. We will consider equilibria where prices depend only on the method of payment and whether the seller of the good is a connected or unconnected household. Let p_t^1 and q_t^1 denote the prices received by a connected household for goods in early and late-settlement transactions, respectively. Similarly, p_t^2 and q_t^2 are the prices at which an unconnected household sells. A connected household faces a *finance constraint*, which is

$$\begin{aligned} & [1-(1-\alpha)\pi][\gamma_t p_t^1 c_t^{11} + (1-\gamma_t) q_t^1 d_t^{11}] \\ & +(1-\alpha)\pi[\gamma_t p_t^2 c_t^{12} + (1-\gamma_t) q_t^2 d_t^{12}] + b_{t+1} + f_t \\ \leq\ & s_t m_t^1 + p_t^1 x_t^1 + R_t b_t - \tau_{1t} \end{aligned} \tag{2}$$

In constraint (2), c_t^{11} denotes the consumption of consumers from the connected household who make early-settlement transactions with other connected households, while c_t^{12} is consumption by the consumers who buy from unconnected households in early-settlement transactions. Similarly d_t^{11} and d_t^{12} denote consumption by consumers from a connected household who buy from connected and unconnected households respectively, but who buy goods in late-settlement transactions. As well, b_t is the quantity of one-period nominal bonds acquired by the household in period $t-1$, f_t is the quantity of within-period nominal bonds purchased by the household, and m_t^1 is the household's beginning-of-period money balances. Here, s_t denotes the gross nominal interest rate on reserve balances held from the end of period $t-1$ to the beginning of period t (i.e. overnight). Finally, x_t^1 is the quantity of goods sold by the household to consumers in early-settlement transactions, and τ_{1t} is a nominal lump-sum tax paid to the government. Thus, constraint (2) states that total household expenditure on goods and nominal bonds must be financed by the money balances with which the household begins the period, plus the IOUs acquired in early-settlement transactions.

A connected household must satisfy its budget constraint, which is

$$\begin{aligned} & [1-(1-\alpha)\pi][\gamma_t p_t^1 c_t^{11} + (1-\gamma_t) q_t^1 d_t^{11}] \\ & +(1-\alpha)\pi[\gamma_t p_t^2 c_t^{12} + (1-\gamma_t) q_t^2 d_t^{12}] + b_{t+1} + f_t + m_{t+1}^1 \\ \leq \; & s_t m_t^1 + p_t^1 x_t^1 + q_t^1 (y - x_t^1) + R_t b_t + r_t f_t - \tau_{1t} - \tau_{2t} \end{aligned} \tag{3}$$

In constraint (3) $m_{t+1}^1$is the quantity of money carried by the household into period $t+1$, $y - x_t^1$ is the quantity of goods sold in late-settlement transactions, $r_t f_t$ denotes the total nominal payoff on within-period bonds, and τ_{2t} is a nominal lump-sum tax paid to the government.

As mentioned above, a key feature of the environment is that income earned by the household from late-settlement transactions cannot be spent until the following period. That is, constraint (2) looks somewhat like a cash-in-advance constraint, though this is a constraint implied by the environment and the information structure.

An unconnected household faces two constraints similar to (2) and (3). That is, an unconnected household's finance constraint is

$$\alpha\pi[\gamma_t p_t^1 c_t^{21} + (1-\gamma_t) q_t^1 d_t^{21}] + (1-\alpha\pi)[\gamma_t p_t^2 c_t^{22} + (1-\gamma_t) q_t^2 d_t^{22}] \leq m_t^2 + p_t^2 x_t^2, \quad (4)$$

and its budget constraint is

$$\alpha\pi[\gamma_t p_t^1 c_t^{21} + (1-\gamma_t) q_t^1 d_t^{21}] + (1-\alpha\pi)[\gamma_t p_t^2 c_t^{22} + (1-\gamma_t) q_t^2 d_t^{22}] + m_{t+1}^2 \leq m_t^2 + p_t^2 x_t^2 + q_t^2 (y - x_t^2). \quad (5)$$

Note that, in contrast to the connected household, the unconnected household does not trade bonds, pays no taxes, and does not receive interest on its outside money balances, which are interpreted as currency holdings for an unconnected household.

In this environment, there are important limitations on who can trade assets with whom, on what kinds of assets are traded, and on how risk can be shared. First, while unconnected households can issue within-period IOUs in exchange for goods, they cannot trade on the bond market that opens after goods market trading takes place. Further, the bonds issued in this bond market cannot be used in the clearing and settlement process. To obtain these features as outcomes, it is useful to take a legal restrictions approach, following Wallace (1983), though the way that the legal restrictions are structured here is unique to this model. As we have already specified, connected households are permitted to hold reserve balances while unconnected households are not. Bonds issued in the bond market are assumed to be bearer bonds, and the payoffs are received in terms of reserve balances. Thus, bonds cannot be traded, and an unconnected household would then not want to hold these bonds, as the payoffs would be worthless given that these households do not hold reserve balances. Further private intermediation involving the issue of circulating private money is assumed to be illegal, so that a private intermediary cannot hold bonds as assets and issue liabilities that have all of the features of outside money.

Next, households cannot trade claims contingent on γ_t or on monetary policy outcomes. As we will see in what follows, there will be important redistributive effects from γ_t shocks and from random monetary policy. Connected and unconnected households are willing to share aggregate risk, but they cannot. This is an important difference from models such as Alvarez, Atkeson, and Kehoe (2002), or Alvarez, Atkeson, and Edmond (2003), for example, which gain analytical traction at the expense of distributional effects of monetary policy by assuming complete markets. To justify the absence of contingent claims markets, we simply assume that connected and unconnected households never have the opportunity to meet and trade such claims. As we will see, in the equilibria we study connected households are identical in all respects, and therefore will not wish to trade contingent claims even if they have the opportunity. A connected and an unconnected household interact only in that consumers from one type of household may purchase goods from the other type of household. It is assumed that these meetings occur under informational circumstances that make contingent claims trading impossible.

We assume that all interest on government bonds in periods $1, 2, ..., \infty$, is financed by lump-sum taxes, so that the aggregate quantity of nominal government liabilities is fixed for all t. Our principal concern is in determining the effects of changes in the the composition of the government's debt, i.e. the effects of monetary policy. Let M_t^1 (M_t^2) denote the stock of money per household supplied to connected (unconnected) households at the beginning of period t, B_t the quantity of one-period government bonds per connected household maturing in period t, and F_t the quantity of within-period government bonds per connected household maturing in period t. The government's budget constraint is then

$$\alpha M_{t+1}^1 + (1-\alpha)M_{t+1}^2 = s_t \alpha M_t^1 + (1-\alpha)M_t^2 - \alpha B_{t+1} + R_t \alpha B_t + (r_t - 1)\alpha F_t - \alpha\tau_{1t} - \alpha\tau_{2t}, \tag{6}$$

where $B_0 = 0$. The lump-sum taxes that finance interest on the government debt are levied in such a way as to have no distributional consequences, that is

$$\tau_{1t} = (R_t - 1)B_t + (s_t - 1)M_t^1 \tag{7}$$

and

$$\tau_{2t} = (r_t - 1)F_t \tag{8}$$

The government chooses s_t, B_{t+1}, F_t, τ_{1t}, and τ_{2t} at the beginning of period t, possibly in a random fashion. The gross interest rates R_t and r_t are then market-determined, and (6), (7), and (8) then determine the total quantity of aggregate outside money in period $t+1$ on the left-hand side of (6).

3. OPTIMIZATION AND EQUILIBRIUM

In this section, our goals are to characterize the solution to the households' optimization problems, and impose equilibrium conditions.

For a connected household, given the household's objective function (1) and its constraints (2) and (3), and assuming an interior solution (which we must have in equilibrium), optimal consumption choices for the members of a connected household give

$$\frac{1}{p_t^1 c_t^{11}} = \frac{1}{q_t^1 d_t^{11}} = \frac{1}{p_t^2 c_t^{12}} = \frac{1}{q_t^2 d_t^{12}} = \lambda_t^1 + \mu_t^1, \tag{9}$$

where λ_t^1 denotes the multiplier associated with the household's finance constraint (2), and μ_t^1 is the multiplier associated with the household's budget constraint (3). In (9), log utility implies that the household will equalize expenditures across the household's consumers at the optimum. This will give us considerable mileage in the analysis. Intertemporal optimization by a connected household gives, given optimal choice of end-of-period money holdings,

$$\mu_t^1 = \beta s_{t+1} E_t \left(\lambda_{t+1}^1 + \mu_{t+1}^1 \right), \tag{10}$$

and given optimal choice of one-period bonds and within-period bonds, we have, respectively,

$$\lambda_t^1 + \mu_t^1 = \beta R_{t+1} E_t \left(\lambda_{t+1}^1 + \mu_{t+1}^1 \right), \tag{11}$$

and

$$\lambda_t^1 + \mu_t^1 = r_t \mu_t^1, \tag{12}$$

where (10) assumes that we have an interior solution where connected households hold a positive quantity of outside money at the end of each period. In what follows we will also consider a case where connected households hold zero outside money balances between periods.

As well, for a connected household to be indifferent between selling in early and late-settlement transactions, as must hold in equilibrium, from (2) and (3) we have

$$\left(\lambda_t^1 + \mu_t^1 \right) p_t^1 = \mu_t^1 q_t^1 \tag{13}$$

Similarly, for unconnected households, given (1), (4), and (5), the analogs of (9), (10), and (13) are, respectively,

$$\frac{1}{p_t^1 c_t^{21}} = \frac{1}{q_t^1 d_t^{21}} = \frac{1}{p_t^2 c_t^{22}} = \frac{1}{q_t^2 d_t^{22}} = \lambda_t^2 + \mu_t^2, \tag{14}$$

$$\mu_t^2 = \beta E_t \left(\lambda_{t+1}^2 + \mu_{t+1}^2 \right), \tag{15}$$

$$\left(\lambda_t^2 + \mu_t^2 \right) p_t^2 = \mu_t^2 q_t^2 \tag{16}$$

where λ_t^2 and μ_t^2 denote, respectively, the multipliers associated with an unconnected household's finance constraint and budget constraint.

Next, in equilibrium the market clears for goods sold in connected locations in early-settlement transactions,

$$\gamma_t \{ [1 - (1-\alpha)\pi] c_t^{11} + (1-\alpha)\pi c_t^{21} \} = x_t^1, \tag{17}$$

for goods sold in connected locations in late-settlement transactions,

$$(1-\gamma_t) \{ [1 - (1-\alpha)\pi] d_t^{11} + (1-\alpha)\pi d_t^{21} \} = y - x_t^1 \tag{18}$$

and for goods sold in unconnected locations in early and late-settlement transactions, respectively,

$$\gamma_t[\alpha\pi c_t^{12} + (1-\alpha\pi)c_t^{22}] = x_t^2, \tag{19}$$

$$(1-\gamma_t)[\alpha\pi d_t^{12} + (1-\alpha\pi)d_t^{22}] = y - x_t^2. \tag{20}$$

Finally, asset markets clear, that is

$$B_t = b_t,\ F_t = f_t,\ M_t^1 = m_t^1,\ M_t^2 = m_t^2. \tag{21}$$

4. EQUILIBRIUM SOLUTION

In this section we obtain closed-form solutions for equilibrium quantities and prices.

First, note from (9) and (14) that, $p_t^i = q_t^i$ if and only if $\lambda_t^i = 0$ and $p_t^i < q_t^i$ if and only if $\lambda_t^i > 0$, for $i = 1, 2$. Therefore, a consumer will pay a premium in a late-settlement transaction, if and only if the finance constraint binds. That is, so long as there is a binding finance constraint, then all income received by the household from the sale of goods in early-settlement transactions is spent in the current period. However, income received from selling goods in late-settlement transactions cannot be spent until the following period. Therefore, the household will demand a premium to accept an IOU in a late-settlement transaction. As well, from (12) and (13) we have

$$r_t = \frac{q_t^1}{p_t^1}, \tag{22}$$

where r_t is the gross nominal interest rate on within-period bonds. Therefore, the within-period nominal interest rate is greater than zero if and only if a consumer pays a premium when making a purchase in a late-settlement transaction , i.e. if and only if the finance constraint binds for connected households.

We will assume for now (and check this later) that conditions hold such that the finance constraints (2) and (4) always bind. Then, letting z_t^1 (z_t^2) denote nominal expenditure in period t by a connected (unconnected) household, and given (2), (4), (6), (7), (8), and (21), we get

$$z_t^1 = p_t^1 x_t^1 + M_t^1 + B_t - B_{t+1} - F_t \tag{23}$$

$$z_t^2 = p_t^2 x_t^2 + M_t^2. \tag{24}$$

From (9), (14), (17), and (19), nominal expenditure in early-settlement transactions in connected and unconnected locations, respectively, is given by

$$p_t^1 x_t^1 = \gamma_t \{[1-(1-\alpha)\pi] z_t^1 + (1-\alpha)\pi z_t^2\}, \tag{25}$$

$$p_t^2 x_t^2 = \gamma_t [\alpha\pi z_t^1 + (1-\alpha\pi) z_t^2] \tag{26}$$

Then, substituting in (23) and (24) for $p_t^1 x_t^1$ and $p_t^2 x_t^2$ using (25) and (26), and solving for z_t^1 and z_t^2, we obtain

$$z_t^1 = \frac{[1-\gamma_t(1-\alpha\pi)][M_t^1 + B_t - B_{t+1} - F_t] + (1-\alpha)\pi\gamma_t M_t^2}{(1-\gamma_t)[1-\gamma_t(1-\pi)]}, \tag{27}$$

$$z_t^2 = \frac{\alpha\pi\gamma_t[M_t^1 + B_t - B_{t+1} - F_t] + \{1-\gamma_t[1-(1-\alpha)\pi]\} M_t^2}{(1-\gamma_t)[1-\gamma_t(1-\pi)]} \tag{28}$$

In (27) and (28), note that $M_t^1 + B_t - B_{t+1} - F_t$ is the quantity of outside money available to a connected household at the beginning of period t after the the government makes asset trades, while M_t^2 is the quantity of outside money available to an unconnected household. Then, from (27) and (28), note that aggregate nominal expenditure is

$$\alpha z_t^1 + (1-\alpha) z_t^2 = \frac{\alpha[M_t^1 + B_t - B_{t+1} - F_t] + (1-\alpha)M_t^2}{1-\gamma_t}. \tag{29}$$

Since the numerator in the expression on the right-hand side of (29) is the aggregate quantity of outside money available for households to spend during the current period, therefore $\frac{1}{1-\gamma_t}$ is the velocity of money. Thus, exogenous fluctuations in γ_t imply exogenous fluctuations in velocity.

Now, from (27) and (28), nominal expenditure by each type of household is just the velocity of money multiplied by a weighted average of the quantities of money available to each type of household to spend, where the weights depend on α, π, and γ_t. The reason that the quantity of money available to one type of household helps determine nominal expenditures by the other type is that, for example, early-settlement purchases of connected-household goods by unconnected households represent income that can be spent within the period by connected households, and some of this income is spent in early-settlement purchases of goods from unconnected households by connected households. Thus, there is simultaneity in expenditures by connected and unconnected households.

It is straightforward to show that, in the expenditure expressions (27) and (28), the weight on M_t^2 is smaller for connected household nominal expenditure than for unconnected household nominal expenditure, as one might expect, since connected households tend to trade more intensively with other connected households. For both connected and unconnected households, the weight on M_t^2 in nominal expenditures is decreasing in α, as an increase in the fraction of connected households makes it more likely that any consumer will be trading with a connected household. As well, an increase in γ_t increases the weight on M_t^2 in expenditures for connected households and decreases the corresponding weight for unconnected households. That is, the more likely it is that a household sells in early-settlement transactions, the more simultaneity is created in expenditures by connected and unconnected households, implying that money available to spend by the other type of household has a greater effect on own expenditures. Finally, an increase in π has a qualitatively similar effect on the expenditures of each type of household to an increase in γ_t. This is because an increase in π implies an increase in the probability that connected and unconnected households engage in transactions with each other.

Next, we need to determine how the quantities of money per household evolve over time for connected and unconnected households. That is, we want to determine the distribution of money balances across the population in period $t+1$ given the distribution at the beginning of period t, central bank actions during period t, and private decisions during period t. Given that the finance constraints (2) and (4) bind, from (2)-(5), (6), (7), (8), and (21), we get

$$M_{t+1}^1 = q_t^1(y - x_t^1) + F_t, \tag{30}$$

$$M_{t+1}^2 = q_t^2(y - x_t^2). \tag{31}$$

Therefore, from (9), (14), (18), (20), (27), (28), (30), and (31), we obtain

$$M_{t+1}^1 = M_t^1 + B_t - B_{t+1} - \frac{(1-\alpha)\pi\left[M_t^1 + B_t - B_{t+1} - F_t - M_t^2\right]}{\left[1 - \gamma_t(1-\pi)\right]}, \tag{32}$$

$$M_{t+1}^2 = M_t^2 + \frac{\alpha\pi\left[M_t^1 + B_t - B_{t+1} - F_t - M_t^2\right]}{\left[1 - \gamma_t(1-\pi)\right]}. \tag{33}$$

Thus, given binding finance constraints, the distribution of money balances across the population evolves exogenously, in a tractable way, as a function only of exogenous monetary policy, γ_t, and the parameters α and π. Equations (32) and (33) show that, if the quantity of outside money available to spend per connected household is greater than the quantity available per unconnected household, then money will flow from connected to unconnected households, and vice-versa. Money injections can occur either through an open market purchase $(B_t - B_{t+1})$ or a daylight overdraft $(-F_t)$, so that a money injection during the current period results in an increase in $M_t^1 + B_t - B_{t+1} - F_t - M_t^2$, and in turn, from (32) and (33), in an increase in the flow of outside money balances between connected and unconnected households. Note, in equations (32) and (33) that the current-period money flows are larger the larger is π and the larger is γ_t. That is, π and γ_t determine the speed of diffusion of an outside money injection by the central bank. By diffusion, we mean the process by which a money injection, initially received only by connected households, is ultimately distributed through transactions to the whole population.

The parameter π governs the degree to which households purchase goods from other households of the same type. Note from (32) and (33) that, if $\pi = 1$, then diffusion occurs in one period, that is

$$M_{t+1}^1 = \alpha(M_t^1 + B_t - B_{t+1}) + (1-\alpha)M_t^2 + (1-\alpha)F_t = M_{t+1}^2 - F_t$$

The random variable γ_t is the fraction of goods transactions volume accounted for by early-settlement transactions (measured by the fraction of consumers in the market engaged in this type of transaction), so that an increase in γ_t also speeds diffusion. Setting $\gamma_t = 1$ in (32) and (33) gives the same result as setting $\pi = 1$, i.e. diffusion occurs in one period. However, the economy with $\gamma_t = 1$ is one where outside money is not needed as a medium of exchange.

Now, to solve for an equilibrium, first let ψ_t^1 and ψ_t^2 denote total nominal expenditure on the goods produced by a connected and unconnected household, respectively. From (27) and (28) we get

$$\begin{aligned} \psi_t^1 &= [1-(1-\alpha)\pi]z_t^1 + (1-\alpha)\pi z_t^2 \\ &= \frac{[1-\gamma_t(1-\pi)-(1-\alpha)\pi](M_t^1 + B_t - B_{t+1} - F_t) + (1-\alpha)\pi M_t^2}{(1-\gamma_t)[1-\gamma_t(1-\pi)]} \end{aligned} \tag{34}$$

$$\begin{aligned} \psi_t^2 &= \alpha\pi z_t^1 + (1-\alpha\pi)z_t^2 \\ &= \frac{\alpha\pi(M_t^1 + B_t - B_{t+1} - F_t) + [1-\gamma_t(1-\pi)-\alpha\pi]M_t^2}{(1-\gamma_t)[1-\gamma_t(1-\pi)]} \end{aligned} \tag{35}$$

Then, from (9), (14), (17), (18), (19), and (20), we get

$$p_t^1 x_t^1 = \gamma\psi_t^1, \tag{36}$$

$$q_t^1(y - x_t^1) = (1-\gamma)\psi_t^1, \tag{37}$$

$$p_t^2 x_t^2 = \gamma\psi_t^2, \tag{38}$$

$$q_t^2(y - x_t^2) = (1-\gamma)\psi_t^2. \tag{39}$$

Then, from (9), (14), (10), (15), (13), (16), and the definitions of nominal expenditure by each type of household, we obtain

$$\frac{p_t^1}{q_t^1} = \frac{\mu_t^1}{\lambda_t^1 + \mu_t^1} = \beta s_{t+1} z_t^1 E_t \left(\frac{1}{z_{t+1}^1} \right), \tag{40}$$

$$\frac{p_t^2}{q_t^2} = \frac{\mu_t^2}{\lambda_t^2 + \mu_t^2} = \beta z_t^2 E_t \left(\frac{1}{z_{t+1}^2} \right). \tag{41}$$

We can then solve for equilibrium prices and quantities from (36), (37), (38), (39), (40), and (41), obtaining

$$q_t^i = \frac{\psi_t^i}{y\omega_t^i} \left[(1-\gamma_t)\omega_t^i + \gamma_t \right], \tag{42}$$

$$p_t^i = \frac{\psi_t^i}{y} \left[(1-\gamma_t)\omega_t^i + \gamma_t \right], \tag{43}$$

$$x_t^i = \frac{\gamma_t y}{(1-\gamma_t)\omega_t^i + \gamma_t}, \tag{44}$$

for $i = 1, 2$, where

$$\omega_t^1 = \beta s_{t+1} z_t^1 E_t \left(\frac{1}{z_{t+1}^1} \right), \tag{45}$$

$$\omega_t^2 = \beta z_t^2 E_t \left(\frac{1}{z_{t+1}^2} \right). \tag{46}$$

Note, from (40), (41), (45), and (46), that ω_t^1 and ω_t^2 are the relative prices of goods sold in early-settlement transactions to those sold in late-settlement transactions, in connected and unconnected markets, respectively. As well, from(9), (10), (11), and (12), and the definition of nominal expenditure by a connected household, nominal interest rates are determined as follows.

$$R_{t+1} = s_{t+1} \left(\omega_t^1 \right)^{-1} \tag{47}$$

$$r_t = \left(\omega_t^1 \right)^{-1} \tag{48}$$

A monetary policy is a stochastic process for $\{B_{t+1}, F_t, s_{t+1}\}_{t=0}^{\infty}$ given $B_0 = 0$ and satisfying

$$M_t^1 + B_t - B_{t+1} - F_t > 0$$

and

$$1 \leq s_{t+1} \leq R_{t+1}$$

for all t, which then determines a stochastic process for $\{M_t^1, M_t^2\}$ given M_0^1 and M_0^2 from (32) and (33). Then, we can use (27), (28), (34), and (35) to determine $\{z_t^1, z_t^2, \psi_t^1, \psi_t^2\}$, which is an exogenous stochastic process. Then, equations (42)-(48) give closed-form solutions for prices and goods sold in each market. Finally, the consumption allocation is given by

$$c_t^{ij} = x_t^j \frac{z_t^i}{\gamma_t \psi_t^j}, \text{ for } i, j = 1, 2, \tag{49}$$

$$d_t^{ij} = (y - x_t^j) \frac{z_t^i}{(1-\gamma_t)\psi_t^j}, \text{ for } i, j = 1, 2. \tag{50}$$

5. MONETARY SHOCKS AND VELOCITY SHOCKS

In this section, we will take the solution from the previous section, and show what this implies for the response of this economy to central bank money injections, and to shocks to the velocity of money.

Central Bank Money Injections

In this environment, there are two ways for the central bank to increase the supply of outside money. First, the central bank can conduct an open market operation, decreasing the supply of one-period bonds outstanding. If such an open market operation occurs in period t, then the increase in the aggregate stock of money is $\alpha(-B_{t+1} + B_t)$. Second, the central bank can extend more daylight overdrafts, that is it can issue more within-period bonds, in which case the change in the aggregate money stock that is available for spending within the period is $\alpha(-F_t + F_{t-1})$. Given the way in which we have set up the the monetary/fiscal regime, a one-unit increase in the money stock accomplished in either fashion has exactly the same effects. To see this, note that monetary policy matters for the equilibrium solution only in terms of how it affects the stochastic processes for z_t^i and ψ_t^i, for $i = 1, 2$. Thus, given (27), (28), (34), and (35), either way of injecting money balances implies the same paths for z_t^i and ψ_t^i, for $i = 1, 2$, so it is irrelevant whether the stock of outside money changes through an open market operation or larger daylight overdrafts. Though trivial, this is perhaps an important result, as central bankers sometimes view the quantity of daylight overdrafts as having no consequences for monetary policy, since (the argument goes), the outside money lent during the day disappears at the end of the day. Our model tells us that this view is incorrect, as the daylight overdraft represents outside money available for transactions during the day, when it counts.

Given the structure of the model, we can learn a great deal about the effects of monetary shocks by studying a deterministic example. Suppose that, at the beginning of period T, the per capita money stock is the same for connected and unconnected households; that is, $M_T^1 = M_T^2 = M$. Also suppose that $\gamma_t = \gamma$, $B_t = 0$, and $s_{t+1} = s$ for all t, with $F_t = 0$ for $t = 1, 2, ..., T-1$.

Permanent Increase in the Stock of Outside Money.—

First, suppose that $F_t = -H$ for $t = T, T+1, T+2, ...$, so that the central bank increases nominal daylight overdrafts per connected household permanently by H beginning in period t. Note that the aggregate money stock held overnight is constant at M for all t. The effect of the increase in daylight overdrafts is to increase the quantity of outside money available to spend during each period by αH, and to do this permanently.

First, from (27), (28), $z_t^1 = z_t^2 = \frac{M}{1-\gamma}$ for $t = 1, 2, ..., T-1$. Then, when the money injection occurs, from (27) and (28) we have $z_T^1 > z_T^2 > \frac{M}{1-\gamma}$, so that nominal expenditure increases for both types of households, but by more for a connected than for an unconnected household. Further, given (34) and (35), we have $\psi_t^1 = \psi_t^2 = \frac{M}{1-\gamma}$

for $t = 1, 2, ..., T-1$, and

$$z_T^1 > \psi_T^1 > \psi_T^2 > z_T^2 > \frac{M}{1-\gamma}. \tag{51}$$

That is, since connected households sell to proportionately more connected consumers than do unconnected households, nominal expenditure on the goods sold by connected households increases more than nominal expenditure on the goods sold by unconnected households, which gives the second inequality in (51). As well, expenditure on the goods sold by a particular household is a weighted average of expenditures of each type of household, which gives the first and third inequalities in (51).

Next, from (27), (28), (32), (33), (34), and (35), we get

$$z_{t+1}^1 = \frac{[1-\gamma(1-\alpha\pi)]\psi_t^1 + (1-\alpha)\pi\gamma\psi_t^2}{[1-\gamma(1-\pi)]}, \tag{52}$$

$$z_{t+1}^2 = \frac{\alpha\pi\gamma\psi_t^1 + \{1-\gamma[1-(1-\alpha)\pi]\}\,\psi_t^2}{[1-\gamma(1-\pi)]}, \tag{53}$$

for $t = T+1, T+2, ...$ .Therefore, z_{t+1}^1 and z_{t+1}^2 are weighted averages of ψ_t^1 and ψ_t^2, and the weights in these weighted averages imply that, if $\psi_t^1 > \psi_t^2$, then $\psi_t^1 > z_{t+1}^1 > z_{t+1}^2 > \psi_t^2$. We also know from (34), and (35) that $z_t^1 > z_t^2$ implies $z_t^1 > \psi_t^1 > \psi_t^2 > z_t^2$. Therefore $z_t^1 > z_t^2$ implies $z_t^1 > z_{t+1}^1 > z_{t+1}^2 > z_t^2$. Then, by induction, we have

$$z_{T+i}^1 > z_{T+i+1}^1, \text{ for } i = 0, 1, 2, ... \tag{54}$$

$$z_{T+i}^2 < z_{T+i+1}^2, \text{ for } i = 0, 1, 2, ... \tag{55}$$

$$z_{T+i}^1 > z_{T+i}^2, \text{ for } i = 0, 1, 2, ... \ . \tag{56}$$

Therefore, as the central bank money injection that occurred in period T is distributed over time among households in this economy, nominal expenditure for each connected household falls over time, and nominal expenditure for each unconnected household rises.

Now, (34), (35), (52), and (53) imply that we can write

$$\frac{z_{t+1}^1}{z_t^1} = \theta + (1-\theta)\frac{z_t^2}{z_t^1}, \tag{57}$$

for some θ with $0 < \theta < 1$, for $t = T, T+1, \dots$. Therefore, from (54)-(57), we get

$$\frac{z_{T+i+1}^1}{z_{T+i}^1} < \frac{z_{T+i+2}^1}{z_{T+i+1}^1}, \text{ for } i = 0, 1, 2, \dots . \tag{58}$$

Similarly, it is straightforward to show that

$$\frac{z_{T+i+1}^2}{z_{T+i}^2} > \frac{z_{T+i+2}^2}{z_{T+i+1}^2}, \text{ for } i = 0, 1, 2, \dots . \tag{59}$$

Therefore, from (45) and (46), we get

$$\omega_t^1 > \beta s, \ \omega_t^2 < \beta, \text{ for } t = T, T+1, T+2, \dots$$

and

$$\omega_t^1 > \omega_{t+1}^1, \ \omega_t^2 < \omega_{t+1}^2, \text{ for } t = T, T+1, T+2, \dots$$

Therefore, from (44), (47), and (48), we obtain

$$x_t^1 < x_{t+1}^1, \ x_t^2 > x_{t+1}^2, \ R_{t+1} < R_{t+2}, \ r_t < r_{t+1}, \text{ for } t = T, T+1, T+2, \dots$$

The central bank's permanent money injection in period T therefore acts to increase (decrease) the relative price and reduce (increase) the quantity of goods sold in early-settlement transactions in connected (unconnected) markets, and to reduce nominal interest rates. All of these effects persist, though the money injection is neutral in the limit. The money injection initially redistributes outside money balances from unconnected households to connected households, which acts to increase prices in connected markets (as that is where connected consumers tend to buy) and to increase the consumption of connected consumers. After the money injection occurs, the outside money balances of connected households falls. As a result, connected households expect their consumption to decrease over time and they expect prices to fall in connected markets, following the money injection. Thus, a negative real interest rate effect and a negative Fisher effect for connected households act to reduce nominal interest rates. For connected households, the opportunity cost of selling goods in a late-settlement transaction will then decrease when the money injection occurs, and so more goods are sold in late-settlement transactions, and the relative price of such goods falls. For unconnected households, who expect inflation following the money injection, the reverse happens. In unconnected markets the opportunity cost of selling in late-settlement transactions rises, less goods are sold to such consumers, and the relative price of these goods rises.

In Figures 1 to 7, we illustrate the above responses with an example. Parameters were set arbitrarily according to $\gamma = 0.5$, $\alpha = 0.5$, $\pi = 0.2$, $\beta = 0.96$, and $y = 1$. As well, we set $M = 1$ and $H = 0.02$, so that the aggregate money stock increases by 1%, with the money injection occurring in period 1 in the figures. Figure 1 shows the time paths of beginning-of-period money balances per household. Since the money injection occurs by way of a permanent increase in daylight overdrafts, the beginning-of-period money balances of connected (unconnected) households decrease (increase) over time, with the quantity of outside money available to spend during the period (which includes daylight overdrafts) converging to the same quantity for all households in the limit. In Figure 2, nominal expenditures by connected (unconnected) households decrease (increase) over time, with nominal sales by connected (unconnected) households always less (more) than expenditures. Figure 3 shows that the prices of goods sold in early-settlement transactions are greater than the prices of goods sold in late-settlement transactions, with prices in connected (unconnected) markets falling (rising) over time. In Figure 4, the nominal interest rate falls when the money injection occurs, and then rises to its steady state value. Figure 5 shows a decrease (increase) in the quantity of goods sold in early-settlement transactions at the time of the money injection in connected (unconnected) markets, with this quantity rising (falling) over time. Finally, Figures 6 and 7 show consumption in early-settlement and late-settlement transactions, respectively. The money injection acts to increase the dispersion of consumption across individual consumers, and this dispersion falls over time as diffusion of the money injection occurs.

Velocity Shocks.—

Here, we will shut down monetary shocks so that we can focus on the effects of shocks to velocity. For this purpose, suppose that $M_t^1 = M_t^2 = M$ for all t, with $B_t = F_t = 0$ and $s_{t+1} = s$ for all t. Assume that γ_t follows a first-order Markov process. This then implies that

$$z_t^i = \psi_t^i = \frac{M}{1-\gamma_t}$$

for all t and $i = 1, 2$. Then (45) and (46) give

$$\omega_t^1 = \frac{\beta s E_t \left(1-\gamma_{t+1}\right)}{1-\gamma_t}, \tag{60}$$

$$\omega_t^2 = \frac{\beta E_t \left(1-\gamma_{t+1}\right)}{1-\gamma_t}, \tag{61}$$

and so, from (44), (47), and (48), we get

$$x_t^1 = \frac{\gamma_t y}{\beta s E_t \left(1-\gamma_{t+1}\right) + \gamma_t}, \tag{62}$$

$$x_t^2 = \frac{\gamma_t y}{\beta E_t \left(1-\gamma_{t+1}\right) + \gamma_t}, \tag{63}$$

$$R_{t+1} = \frac{1-\gamma_t}{\beta E_t \left(1-\gamma_{t+1}\right)} \tag{64}$$

$$r_t = \frac{1-\gamma_t}{\beta s E_t \left(1-\gamma_{t+1}\right)} \tag{65}$$

To obtain some intuition, consider the case where γ_t is an i.i.d. random variable. Then, from (60)-(65), the relative price of goods sold in early-settlement transactions is increasing in γ_t, goods per consumer sold in early-settlement transactions is decreasing in γ_t, and nominal interest rates are decreasing in γ_t. That is, an increase in γ_t increases the velocity of money and the current price level increases. If γ_t is i.i.d., then when γ_t is high households anticipate a low inflation rate, and therefore nominal interest rates decline through a Fisher effect. The cost of selling to consumers in late-settlement transactions then falls, so that a larger quantity of goods per consumer, is sold in late-settlement transactions. The relative price of goods sold in early-settlement transactions then must fall.

6. POLICY RULES

Now that we know something about how this model responds to monetary policy shocks and to velocity shocks separately, we can ask questions about the performance of the economy under velocity shocks when policy is endogenous and conforming to particular rules. While we cannot obtain a simple characterization of optimal policy, we can show how some typical policy rules perform.

Benchmark Case: $\alpha = 1$

A useful benchmark to consider is the case where $\alpha = 1$, so that all households are connected. In this case, a Pareto optimum is very easy to characterize. If the social planner weights the utility of all households equally, then each consumer in each household consumes y at the optimum, in each period. With $\alpha = 1$, there are monetary policy rules that support this Pareto optimum as a competitive equilibrium, and it is possible to interpret essentially all of these policy rules as Friedman rules. That is, any optimal policy rule will drive the nominal interest rate on within-period loans to zero at all dates and in all states of the world.

We will study three optimal monetary policy rules. Under the first policy, a *daylight-overdraft policy*, the government makes within-period loans to households in each period at a zero nominal interest rate, and eliminates the need for households to carry outside money from one period to the next. With the second policy, an *interest-on-reserves policy*, the government pays interest on outside money balances held between periods at the interest rate on one-period bonds. The third policy is a *zero-nominal-interest-rate policy*, under which the central bank conducts open market operations and extends daylight overdrafts over time in order to drive the nominal interest rates on one-period bonds and daylight overdrafts to zero in each period. Finally, we will study the performance of an *inflation rate peg*, under which monetary policy is conducted so that the inflation rate is a constant for all t. This policy is suboptimal, but the differences between the performance of the inflation rate peg with $\alpha = 1$ and in the general case with $0 < \alpha < 1$ will be interesting.

Daylight Overdraft Policy.—

First, suppose that the central bank sets the intraday gross nominal interest rate $r_t = 1$ for all t, and then accommodates whatever household demand for daylight overdrafts arises at a zero nominal interest rate. Further, lump sum taxes in period 0 are set so that the entire initial stock of outside money is taxed away and destroyed. Thus, the monetary regime is one in which, at the beginning of each period, each household borrows the money balances from the central bank that it deems necessary to carry out transactions during the period. Then, the central bank loan is repaid without interest at the end of the period, and the household carries no money balances into the next period.

The price level is indeterminate here, but the central bank can determine the price level by agreeing to exchange outside money for goods at a fixed rate at the end of the period. In any case, the finance constraint (2) is relaxed, i.e. $\lambda_t^1 = 0$, and so from (13) we have $p_t^1 = q_t^1$ for all t. This then implies, from (9), that consumption is identical for all consumers in each period, and so (17) and (18) give $c_t^{11} = d_t^{11} = y$, with $x_t^1 = \gamma_t y$.

Clearly, this is a Pareto optimal allocation. The central bank extends sufficient within-period credit at a zero nominal interest rate that delays in clearing and settlement are irrelevant, and households do not hold idle outside money balances between periods. Money is still necessary to the clearing and settlement process, but the economy essentially breaks down into a sequence of static economies, with the rate of inflation being irrelevant.

Interest on Reserves.—

Since all households are connected, all outside money balances can be interpreted as reserves. Here, we want to show that there is a monetary policy with interest on reserves that achieves the same optimal allocation as in the previous subsection.

First, set up the monetary policy regime so that $B_t = F_t = 0$ for all t, which implies $M_t^1 = M$, a constant, for all t, with $z_t^1 = \psi_t^1 = \frac{M}{1-\gamma_t}$ for all t. We then obtain an equilibrium solution identical to the one we studied under velocity shocks, i.e. equations (60)-(65). Then, the central bank sets the nominal interest rate on money balances held between periods equal to the one-period nominal bond rate, that is $s_{t+1} = R_{t+1}$, which from (64) gives

$$s_{t+1} = \frac{1-\gamma_t}{\beta E_t \left(1-\gamma_{t+1}\right)}. \tag{66}$$

Assume for simplicity that

$$\gamma_t < 1 - \beta(1 - E_t \gamma_{t+1}), \tag{67}$$

for all realizations of γ_t, so that the nominal interest rates on reserves and one-period nominal bonds are always strictly positive. This monetary policy then implies that the nominal interest rate on daylight overdrafts is 0, that is $r_t = 1$. The zero nominal interest rate on daylight overdrafts in turn implies that the finance constraint (2) does not bind, $p_t^1 = q_t^1$ for all t, $c_t^{11} = d_t^{11} = y$, and $x_t^1 = \gamma_t y$. Therefore, we obtain the same allocation as under the daylight-overdraft policy. The interest-on-reserves policy acts to eliminate the opportunity cost of selling goods in late-settlement transactions, as money balances held over to the next period bear the same rate of return as do nominal bonds.

Note from (66) that the optimal interest-on-reserves policy implies that the nominal interest rate on reserves needs to fluctuate with the one-period bond rate, which in turn reflects fluctuations in γ_t. For example, if γ_t is i.i.d., then the nominal interest rate on reserves and the one-period nominal bond rate decrease with γ_t.

Zero Nominal Interest Rate.—

Another policy which achieves a Pareto optimum when $\alpha = 1$ is one for which the nominal interest rate is zero in each period, or $R_{t+1} = 1$ for all t. In this case, the government sets $s_{t+1} = 1$ for all t, and the desired policy rule, from (45) and (47) has the property that

$$\beta z_t^1 E_t \left(\frac{1}{z_{t+1}^1} \right) = 1,$$

which requires that $z_t^1 = \beta^t H_1$ where H_1 is a constant. Then, from (27), this implies that

$$M_t^* = \beta^t H_1 (1 - \gamma_t),$$

where

$$M_t^* \equiv M_t^1 + B_t - B_{t+1} - F_t$$

is the quantity of outside money available to each household in period t. This policy not only yields $R_{t+1} = 1$ for all t; it also implies, from (45) and (48), that $r_t = 1$ for all t, so all nominal interest rates are equal to zero. As with the daylight-overdraft policy and the interest-on-reserves policy, the zero-nominal-interest rate policy yields an allocation where consumption for each consumer in each period is y.

Inflation Rate Peg.—

With $\alpha = 1$, since total nominal expenditure is z_t^1 and total real GDP is y, therefore from (27) the price level is

$$P_t = \frac{M_t^*}{(1-\gamma_t)\,y}.$$

Therefore, if the goal of the central bank is to peg the inflation rate to some constant $\hat{\rho} \geq \beta$ for all t, then this is achieved with the policy rule

$$M_t^* = \hat{\rho}^t H_2 (1-\gamma_t),$$

where H_2 is some constant. This is a standard type of policy rule which pegs the inflation rate through constant trend growth of the outside money stock available to households, compensating for fluctuations in the velocity of money. Then, if there is no interest on reserves, so that $s_{t+1} = 1$ for all t, from (45), (47), and (48), we get

$$R_{t+1} = r_t = \frac{\hat{\rho}}{\beta}.$$

Therefore, with $\alpha = 1$, the inflation rate peg implies constant nominal interest rates, and a standard Fisher relationship. Note, as long as $\hat{\rho} > \beta$, so that the nominal interest rate is strictly positive, that the equilibrium allocation is suboptimal, as consumers who purchase goods in early-settlement transactions will consume more than y, while other consumers get less.

General Case

Now, suppose that $0 < \alpha < 1$, so that there are connected and unconnected households and distributional effects of monetary policy. We will first characterize Pareto optimal allocations, and then evaluate the performance of alternative monetary policy rules.

Pareto Optimal Allocations.—

Suppose that there is a social planner who can allocate consumption goods among the consumers in each location. We will confine attention to allocations that treat all connected households and all unconnected households identically, but where consumers in the same location may consume different amounts, depending on whether they come from connected or unconnected households. Let C_t^{ij} denote the consumption of a consumer of type (i, j) in period t, where $i, j = 1, 2$. Here, $i = 1$ $(i = 2)$ denotes a consumer from a connected (unconnected) household, and $j = 1$ $(j = 2)$ denotes consumption in a connected (unconnected) market. For example, C_t^{12} denotes the consumption of a consumer from a connected household who consumes at an unconnected location. Then, letting ν denote the Pareto weight on the utility of a connected household, the social planner solves

$$\max_{\{C_t^{11}, C_t^{12}, C_t^{21}, C_t^{22}\}_{t=0}^{\infty}} \left\{ \begin{array}{c} \nu \sum_{t=0}^{\infty} \beta^t \{[1-(1-\alpha)\pi] \log C_t^{11} + (1-\alpha)\pi \log C_t^{12}\} \\ +(1-\nu) \sum_{t=0}^{\infty} \beta^t [\alpha\pi \log C_t^{21} + (1-\alpha\pi) \log C_t^{22}] \end{array} \right\}$$

subject to

$$[1-(1-\alpha)\pi]C_t^{11} + (1-\alpha)\pi C_t^{21} = y,$$

$$\alpha\pi C_t^{12} + (1-\alpha\pi)C_t^{22} = y.$$

The solution to the planner's problem has the property that

$$\frac{C_t^{11}}{C_t^{21}} = \frac{C_t^{12}}{C_t^{22}} = \frac{\nu(1-\alpha)}{(1-\nu)\alpha}, \tag{68}$$

for all t.

Now, in order to study the relationship between competitive equilibria and Pareto optima in this model, the following is useful.

Lemma 1 (1) *If a competitive equilibrium is Pareto optimal, then* $r_t = 1$ *for all* t, *and* $\beta z_t^2 E_t\left(\frac{1}{z_{t+1}^2}\right) = 1$ *for all* t.

Proof. A Pareto optimum has the property that $c_t^{ij} = d_t^{ij}$ for all (i, j), otherwise the social planner could reallocate consumption within at least one type of household and increase welfare for that type of household while leaving the other type unaffected. Therefore, from (9) and (14), if a competitive equilibrium is Pareto optimal then $p_t^i = q_t^i$ for $i = 1, 2$. Then, from (13) and (16) we must have $\lambda_t^i = 0$ for $i = 1, 2$, that is finance constraints do not bind for either connected or unconnected households. Therefore, from (12) and (15) we have $r_t = 1$ for all t, and $\beta z_t^2 E_t \left(\frac{1}{z_{t+1}^2} \right) = 1$ for all t.. ■

Next, recall that M_t^* is the quantity of outside money available to each connected household to spend in period t, with $M_t^* \equiv M_t^1 + B_t - B_{t+1} - F_t$.

Proposition 2 *There does not exist a competitive equilibrium that is Pareto optimal.*

Proof. First, suppose that there exists a competitive equilibrium that is Pareto optimal. Since (68) must be satisfied if the equilibrium allocation is Pareto optimal, from (27), (28), (34), (35), (49), and (50), this requires that $z_t^1 = z_t^2 = z_t$ for all t, so that nominal expenditures for connected and unconnected households are equal in each period. This in turn implies, from (27) and (28) that $M_t^* = M_t^2$ for all t, so that connected and unconnected households also have the same quantity of outside money available in each period. Therefore, from (33), we have $M_t^* = M_t^2 = M_0^2$ for all t. This then gives, from (27) and (28), $z_t^1 = z_t^1 = \frac{M_0^2}{1-\gamma_t}$. But then, $\beta z_t^2 E_t \left(\frac{1}{z_{t+1}^2} \right) = \frac{\beta E_t \left(1-\gamma_{t+1} \right)}{1-\gamma_t} < 1$ for all t, from (67). From Lemma 1, this is a contradiction. ■

The distributional effects of monetary policy necessarily imply that there will be a distortion from active monetary policy. That is, a positive money injection redistributes consumption from unconnected households to unconnected households, but does this in an inefficient manner, relative to what could be accomplished by a benevolent social planner. Therefore, the only chance monetary policy has of achieving a Pareto optimal allocation is if policy is inactive, that is if the aggregate outside money stock is constant and uniformly distributed among households for all t. But this inactive monetary policy implies that the finance constraint will bind for unconnected households, which implies that different consumers in general face different prices in unconnected markets, which is inefficient.

We then know that none of the policies we examined for the special case where $\alpha = 1$ will achieve a Pareto optimum in the general case. Though these policies do not support a Pareto optimum, it will be useful to examine exactly what they do achieve. We will study this in the next subsections.

Daylight Overdraft Policy.—

First, suppose that the central bank always extends sufficient daylight overdrafts so that the within-period nominal interest rate is zero, i.e. $r_t = 1$ for all t, and so that connected households hold zero outside money balances between periods. The central bank sets F_t so that $M^1_{t+1} = 0$, which implies, from (32), that

$$F_t = \left\{ \frac{(1-\alpha)\pi - [1-\gamma_t(1-\pi)]}{(1-\alpha)\pi} \right\} M^1_t - M^2_t. \tag{69}$$

Then, substituting in (27) and (28) for F_t, we get

$$z^1_t = \left[\frac{1-\gamma_t(1-\alpha\pi)}{(1-\gamma_t)(1-\alpha)\pi} \right] M^1_t + \frac{M^2_t}{1-\gamma_t}, \tag{70}$$

$$z^2_t = \left[\frac{\alpha\pi\gamma_t}{(1-\gamma_t)(1-\alpha)\pi} \right] M^1_t + \frac{M^2_t}{1-\gamma_t}. \tag{71}$$

Therefore, since in general $M^1_0 > 0$, in this monetary regime period 0 will look different from each succeeding period, as (70) and (71) imply that $z^1_0 \neq z^2_0$ in general. However, since $M^1_t = 0$ for $t = 1, 2, 3, ...$, therefore from (69)-(71) we have

$$z^1_t = z^2_t = \frac{\alpha M^1_0 + (1-\alpha)M^2_0}{(1-\alpha)(1-\gamma_t)}, \text{ for } t = 1, 2, 3, ... \tag{72}$$

with

$$F_t = -M^2_t = \frac{\alpha M^1_0 + (1-\alpha)M^2_0}{(1-\alpha)}, \text{ for } t = 1, 2, 3, ... \tag{73}$$

Now, in period 0, we will have $r_0 = 1$, and $p^1_0 = q^1_0$, so that there is no price distortion in connected markets, with consumers engaged in early and late-settlement transactions always paying the same prices. However, given (70) and (71), nominal spending is in general different in period 0 for connected and unconnected households, so given (49) and (50), connected and unconnected consumers will consume different amounts even if they are purchasing in the same markets. However, from (72) and (73), in periods $t = 1, 2, 3, ...$, we have

$$x_t^1 = \gamma_t y, \tag{74}$$

$$x_t^2 = \frac{\gamma_t y}{\beta E_t \left(1 - \gamma_{t+1}\right) + \gamma_t}, \tag{75}$$

$$R_{t+1} = \frac{1 - \gamma_t}{\beta E_t \left(1 - \gamma_{t+1}\right)} \tag{76}$$

$$r_t = 1 \tag{77}$$

$$c_t^{ij} = d_t^{ij} = y, \text{ for } j = 1, \ i = 1, 2, \tag{78}$$

$$c_t^{ij} = \frac{y}{\beta E_t \left(1 - \gamma_{t+1}\right) + \gamma_t}, \text{ for } j = 2, \ i = 1, 2, \tag{79}$$

$$d_t^{ij} = \frac{y \beta E_t \left(1 - \gamma_{t+1}\right)}{\left(1 - \gamma_t\right) \left[\beta E_t \left(1 - \gamma_{t+1}\right) + \gamma_t\right]}, \text{ for } j = 2, \ i = 1, 2. \tag{80}$$

The daylight overdraft policy sets a zero nominal interest rate on within-period loans from the central bank for connected households, which implies that all consumers in connected markets consume y in periods $t = 1, 2, 3, ...$. However, in contrast to the case with $\alpha = 1$, the nominal interest rate on one-period bonds is not zero, and it fluctuates, from equation (76). As well, from equations (79) and (80), in unconnected markets consumers who buy from a household in an early-settlement transaction generally consume more than do other consumers, and the difference in consumption between the two types of consumers fluctuates over time.

The reason why the daylight overdraft policy does not achieve optimality is that the central bank cannot lend to unconnected households. It is possible for the central bank to relax finance constraints for connected households and to eliminate the need for connected households to hold outside money between periods. However, unconnected households necessarily face a fluctuating intertemporal distortion, i.e. a fluctuating positive opportunity cost of holding outside money between periods.

Interest on Reserves.—

Next, consider the interest-on-reserves policy that achieved optimality when $\alpha = 1$. In the general case, if interest is paid at the one-period bond rate on the between-period money balances of connected households, we have, from (45) and (47),

$$s_{t+1} = \left[\beta z_t^1 E_t \left(\frac{1}{z_{t+1}^1}\right)\right]^{-1}$$

Then, from (45), (47), and (48), we get

$$R_{t+1} = \left[\beta z_t^1 E_t \left(\frac{1}{z_{t+1}^1}\right)\right]^{-1},$$

$$r_t = 1,$$

$$\omega_t^1 = 1,$$

for all t. Therefore, interest on reserves corrects the distortions in connected markets, though the distribution of consumption between connected and unconnected consumers will be affected by the relative quantities of outside money in the possession of connected and unconnected households, respectively. In the limit, however, as $t \to \infty$, we obtain the same allocation as specified by (74)-(80) for the daylight overdraft policy.

The key difference in the general case between the daylight-overdraft policy and the interest-on-reserves policy is that the daylight-overdraft policy eliminates, after one period, any effects from differing initial money balances on the distribution of consumption across agents. However, with the interest-on-reserves policy these distributional effects persist, but disappear in the limit. Neither policy, however, achieves an optimal allocation where all consumers consume y in each period. This is because the best either policy can do is to correct distortions in connected markets. Central bank loans cannot be extended to unconnected households, and interest cannot be paid on currency.

What is clear though, is that an interest-on-reserves policy must be welfare-improving, since the equilibrium effect of moving the nominal interest rate on reserves to zero is equivalent to reallocating consumption within households in a way that must increase household utility, as consumption is equalized within the household across types of consumers.

Zero Nominal Interest Rate.—

As in the case with $\alpha = 1$, with the zero-nominal-interest-rate policy we have $R_{t+1} = 1$ for all t and the central bank sets $s_{t+1} = 1$ for all t. The desired policy rule has the property that

$$\beta z_t^1 E_t \left(\frac{1}{z_{t+1}^1} \right) = 1,$$

which requires that $z_t^1 = \beta^t H_3$ where H_3 is a constant. Then, from (27), this implies that

$$M_t^* = \frac{\beta^t H_3 (1-\gamma_t) \left[1-\gamma_t(1-\pi)\right] - (1-\alpha)\pi\gamma_t M_t^2}{1-\gamma_t(1-\alpha\pi)}, \tag{81}$$

As when $\alpha = 1$, this policy implies that $r_t = 1$, so that distortions are eliminated in connected markets. However, here we will have distributional effects from the active monetary policy that supports zero nominal interest rates. In particular, from (28) and (81), we get

$$z_t^2 = \frac{\alpha\pi\gamma_t\beta^t H_3 + M_t^2}{1-\gamma_t(1-\alpha\pi)}, \tag{82}$$

so that $z_t^1 \neq z_t^2$, in general, which implies that consumers from connected and unconnected households consume different amounts in each market. As well, it is straightforward to show that given the policy described by (81) and (82), $\beta z_t^2 E_t \left(\frac{1}{z_{t+1}^2} \right) \neq 1$, in general, which implies from (41) that (i) the finance constraint may bind in some states of the world for unconnected households; (ii) the zero-nominal-interest rate rule may not be feasible.

Pegging nominal interest rates to zero through open market operations and/or daylight overdrafts will relax the finance constraint for connected households. However, given the distributional effects of monetary policy, the central bank cannot simultaneously relax the finance constraint for unconnected households. The zero-nominal-interest-rate policy is either not feasible, or it is feasible with the finance constraint binding for unconnected households in some states of the world. The only case where finance constraints can be relaxed for all households is when $\gamma_t = \gamma$, a constant, for all t, though we know that this will not yield a Pareto optimal allocation, as we do not have $z_t^1 = z_t^2$ for all t.

Inflation Rate Peg.—

For the case $0 < \alpha < 1$, total nominal expenditures are $\alpha z_t^1 + (1-\alpha) z_t^2$, and real GDP is y, so from (27) and (28) the price level is

$$P_t = \frac{\alpha M_t^* + (1-\alpha) M_t^2}{(1-\gamma_t) y}.$$

Therefore, if the central bank chooses to peg the gross inflation rate at $\hat{\rho}$ for all t, we must have $P_t = \hat{\rho}^t H_4$ for some constant $H_4 > 0$, and so this policy rule requires

$$M_t^* = \frac{(1-\gamma_t)\,\hat{\rho}^t H_4 - (1-\alpha) M_t^2}{\alpha}$$

Then, from (27) and (28), we get

$$z_t^1 = \frac{[1-\gamma_t(1-\alpha\pi)]\,\hat{\rho}^t H_4 - (1-\alpha) M_t^2}{\alpha\,[1-\gamma_t(1-\pi)]},$$

$$z_t^2 = \frac{\pi \gamma_t \hat{\rho}^t H_4 + M_t^2}{1-\gamma_t(1-\pi)}$$

Here, note that in general $z_t^1 \neq z_t^2$, so that consumers from connected and unconnected households consume different amounts, even when buying at the same price in the same market. As well, it is straightforward to show, from (33), (45), and (47), that the nominal interest rate fluctuates with γ_t, in contrast to what occurs under an inflation rate peg with $\alpha = 1$. This is due to the distributional effects of monetary policy that come into play when the central bank acts to counteract the effects of velocity shocks on the inflation rate.

Optimal Policy.—

For this project, determining the optimal monetary policy is too ambitious, as this would require additional detailed numerical work. However, what is clear is that, if interest could be paid on currency, then optimality could be achieved in a straightforward way. That is, if paying interest on currency is feasible, the government should conduct a transfer policy that equalizes outside money balances in connected and unconnected markets at $t = 0$, and then set $B_{t+1} = F_t = 0$ for $t = 0, 1, 2, ...$. This implies that each household will hold the same quantity of money in each period. Finally, the central bank needs to pay interest on reserves and currency at that interest rate on one-period bonds (in zero net supply in equilibrium). Given this policy, prices are the same across all markets each period, and prices will fluctuate with the velocity shock. However, all distortions in connected and unconnected markets will be removed, and each consumer will consume y in each period.

The key message here is that monetary policy is a blunt tool. In the context of segmented markets, the central bank has only indirect control over the distribution of money balances across the population and the intertemporal prices faced by a fraction of the population. As a result, it appears that there is no simple characterization of optimal monetary policy. Mainstream monetary models tell us that monetary policy is easy - efficiency can be achieved through a Friedman rule that pegs the nominal interest rate at zero in all states of the world. In this environment, monetary intervention conforming to a Friedman rule will correct one or more distortions, but may exacerbate some other distortions.

7. CONCLUSION

We have constructed a tractable model where there are alternative payments arrangements for purchasing goods, and where the central bank can use different vehicles to inject outside money into the private economy. All goods are purchased with IOUs, but some IOUs clear more quickly than others. As a result, outside money is useful in settling debts. However, only connected households can borrow and lend on bond markets and hold outside money as reserve balances with the central bank.

Because of goods market segmentation, prices are in general different in different markets, and a central bank money injection will affect relative prices in the short run across goods markets. Further, consumers pay a premium in a goods purchase if the IOU with which the good is purchased does not clear quickly. Thus, in a particular market, prices depend on the payment instrument used, and relative prices in any given market change in response to a money injection.

In response to short-run velocity shocks, standard Friedman-rule monetary policies that implement optimal equilibrium allocations without market segmentation need not have good properties when we take account of market segmentation effects. That is, monetary policy is a blunt tool, as it affects some economic agents only indirectly. In the context of market segmentation there is nothing in our model that points to a simple optimal policy rule, such as a rule stabilizing a nominal interest rate, the price level, or the rate of inflation.

REFERENCES

Alvarez, F. and Atkeson, A. 1997. "Money and Exchange Rates in the Grossman-Weiss-Rotemberg Model," *Journal of Monetary Economics* 40, 619-40.

Alvarez, F., Atkeson, A., and Edmond, C. 2003. "On the Sluggish Response of Prices to Money in an Inventory-Theoretic Model of Money Demand," working paper, New York University.

Alvarez, F., Atkeson, A., and Kehoe, P. 2002. "Money, Interest Rates, and Exchange Rates with Endogenously Segmented Markets," *Journal of Political Economy* 110, 73-112.

Alvarez, F., Lucas, R. and Weber, W. 2001. "Interest Rates and Inflation," *American Economic Review Papers and Proceedings* 91, 219-225.

Baumol, W. 1952. "The Transactions Demand for Cash: An Inventory Theoretic Approach," *Quarterly Journal of Economics* 66, 545-556.

Chiu, J. 2005. "Endogenously Segmented Asset Market in an Inventory Theoretic Model of Money Demand," working paper, University of Western Ontario.

Christiano, L. and Eichenbaum, M. 1995. "Liquidity Effects, Monetary Policy, and the Business Cycle," *Journal of Money, Credit, and Banking* 27, 1113-1136.

Cooley, T. and Hansen, G. 1989. "The Inflation Tax in a Real Business Cycle Model," *American Economic Review* 79, 733-748.

Grossman, S. and Weiss, L. 1983. "A Transactions-Based Model of the Monetary Transmission Mechanism," *American Economic Review* 73, 871-880.

Head, A. and Lapham, B. 2005. "Limited Participation, Market Power and Inflation Dynamics," working paper, Queen's University.

Head, A. and Shi, S. 2003. "A Fundamental Theory of Exchange Rates and Direct Currency Trades," *Journal of Monetary Economics* 50, 1555-1592.

Khan, A. and Thomas, J. 2007. "Inflation and Interest Rates with Endogenous Market Segmentation," working paper, Federal Reserve Bank of Philadelphia.

Lagos, R., and Wright, R. 2005. "A Unified Framework for Monetary Theory and Policy Analysis," *Journal of Political Economy* 113, 463-484.

Lucas, R. 1990. "Liquidity and Interest Rates," *Journal of Economic Theory* 50, *237-264.*

Lucas, R. 2000. "Inflation and Welfare," *Econometrica* 68, 247-274.

Rotemberg, J. 1984. "A Monetary Equilibrium Model with Transactions Costs," *Journal of Political Economy* 92, 40-58.

Shi, S. 1997. "A Divisible Model of Fiat Money," *Econometrica* 65, 75-102.

Shi, S. 2004. "Liquidity, Interest Rates, and Output," working paper, University of Toronto.

Tobin, J. 1956. "The Interest-Elasticity of Transactions Demand for Cash," *Review of Economics and Statistics* 38, 241-247.

Wallace, N. 1983. "A Legal Restrictions Theory of the Demand for 'Money' and the Role of Monetary Policy," *Federal Reserve Bank of Minneapolis Quarterly Review*, Winter, pp. 1-6.

Williamson, S. 2006a. "Search, Limited Participation, and Monetary Policy," *International Economic Review* 47, 107-128.

Williamson, S. 2006b. "Monetary Policy and Distribution," working paper, Washington University in St. Louis.

Figure 1: Permanent Money Injection: Beginning−of−Period Money Stocks

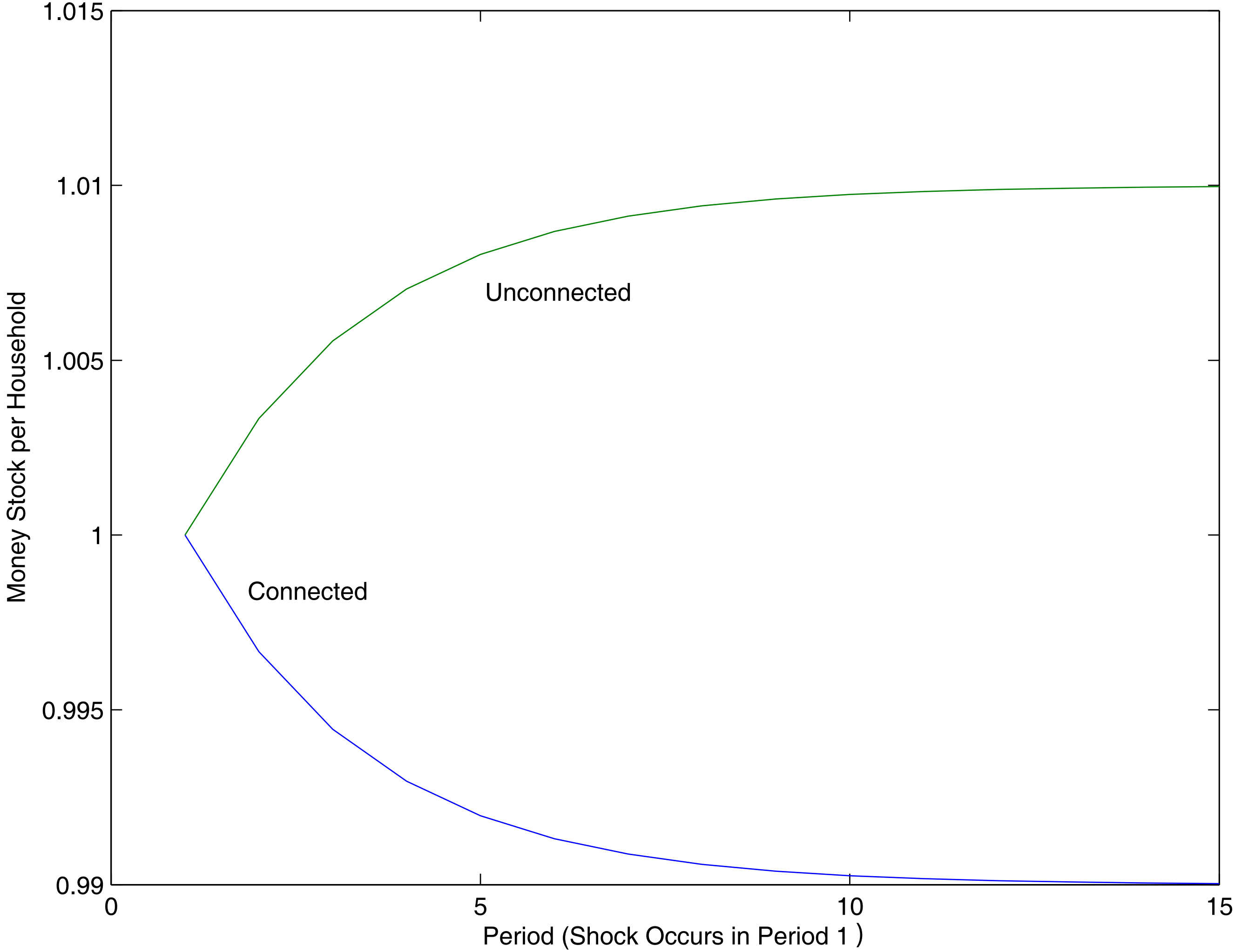

Figure 2: Permanent Money Injection: Expenditures and Sales

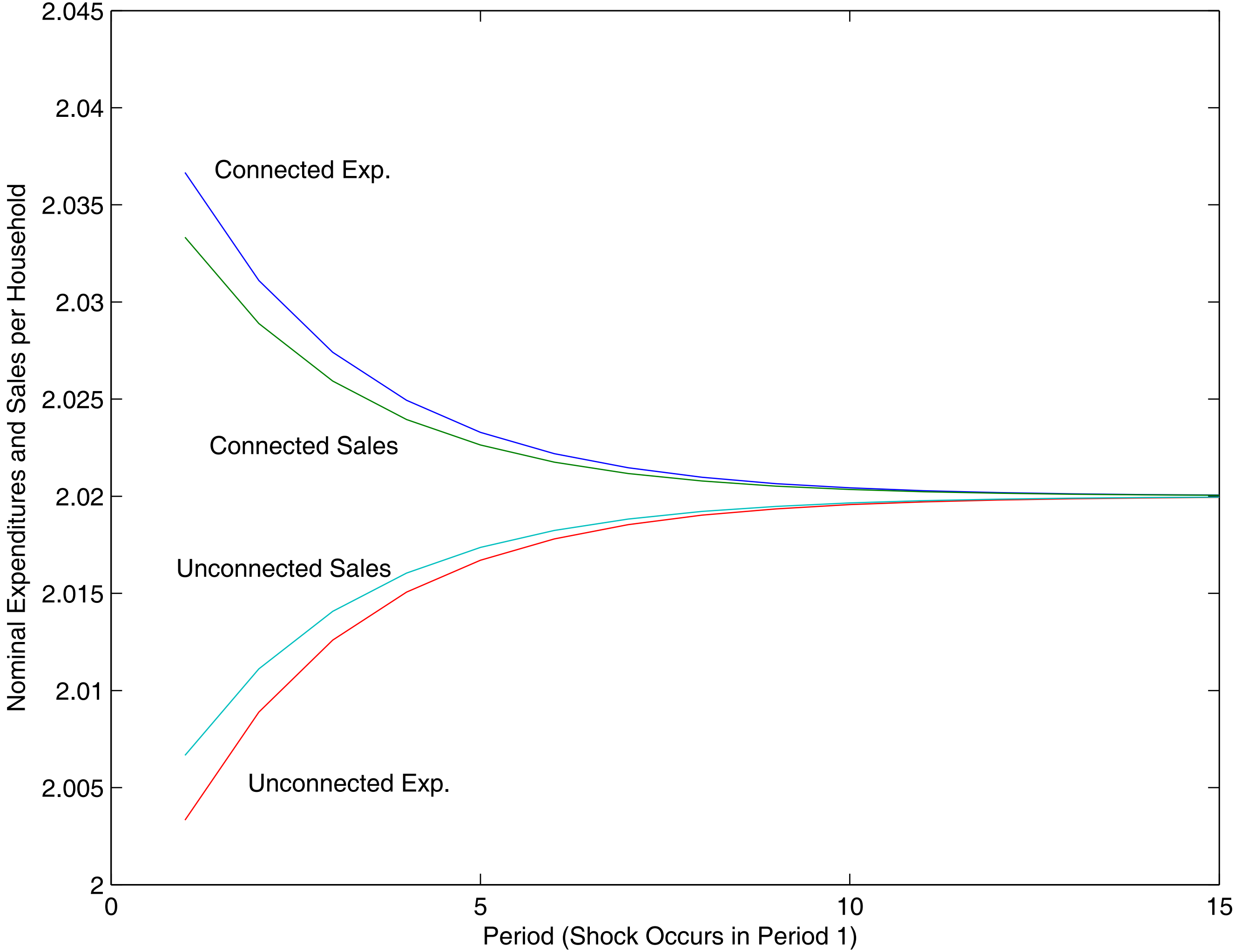

Figure 3: Permanent Money Injection: Prices

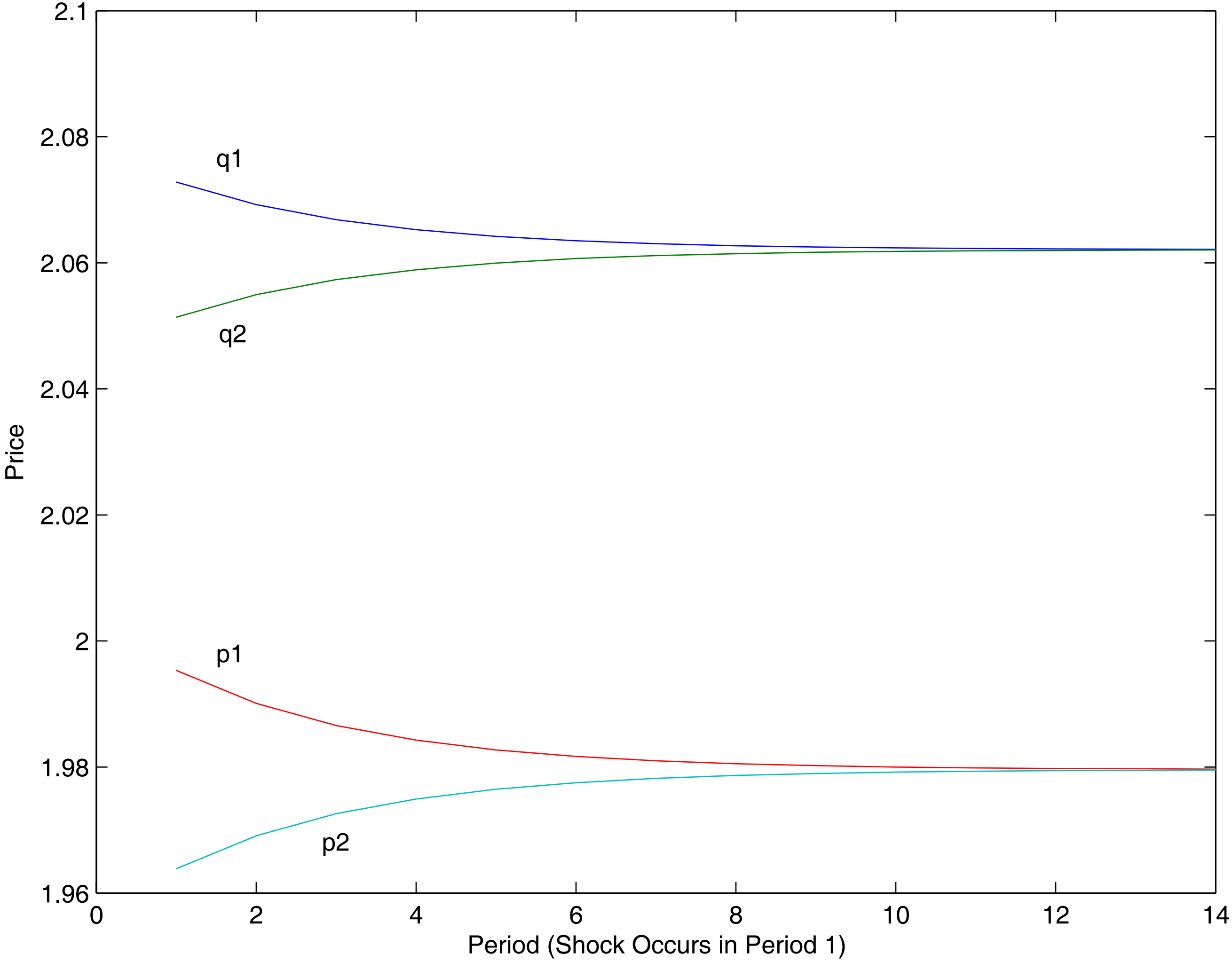

Figure 4: Permanent Money Injection: Nominal Interest Rate

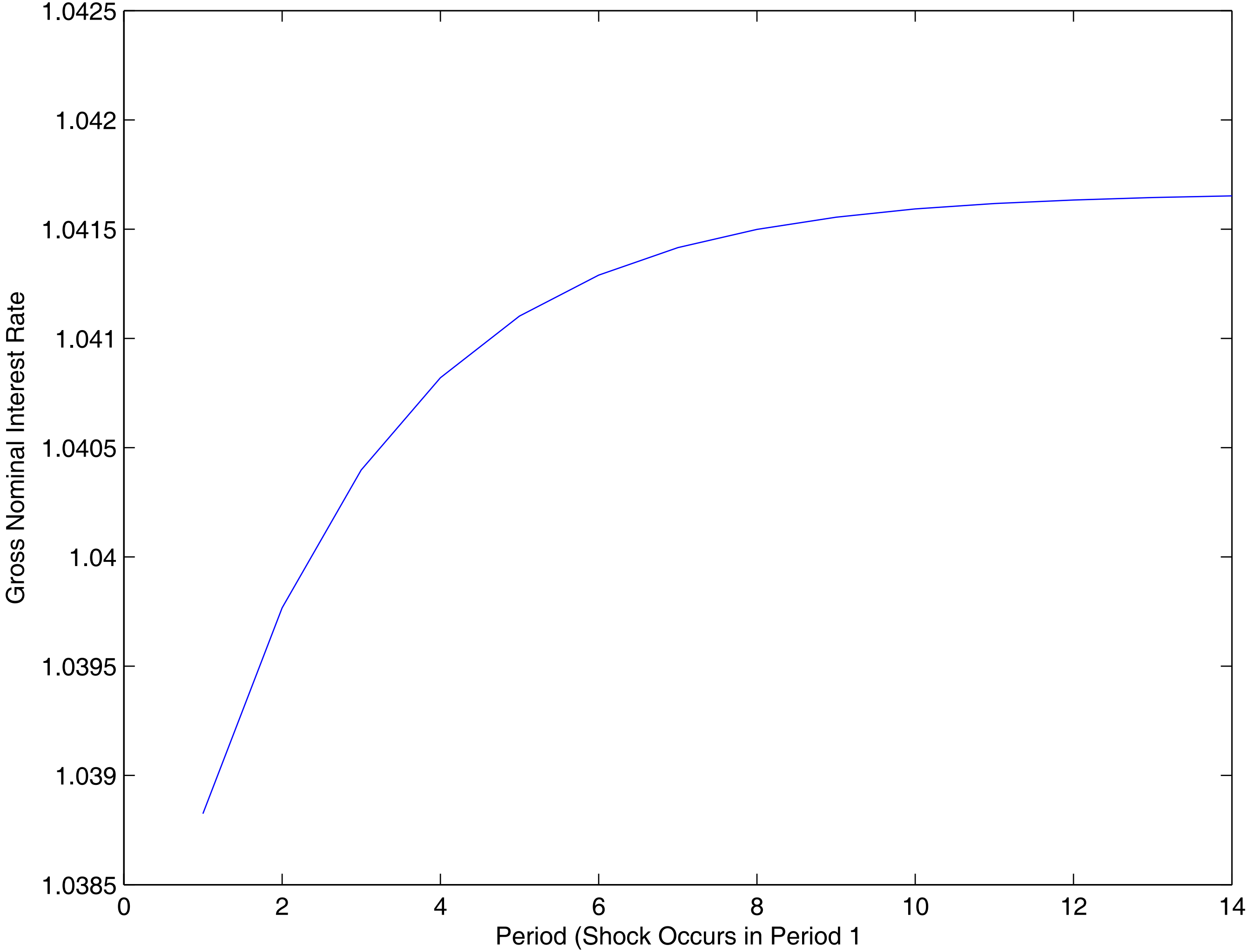

Figure 5: Permanent Money Injection: Goods Allocation

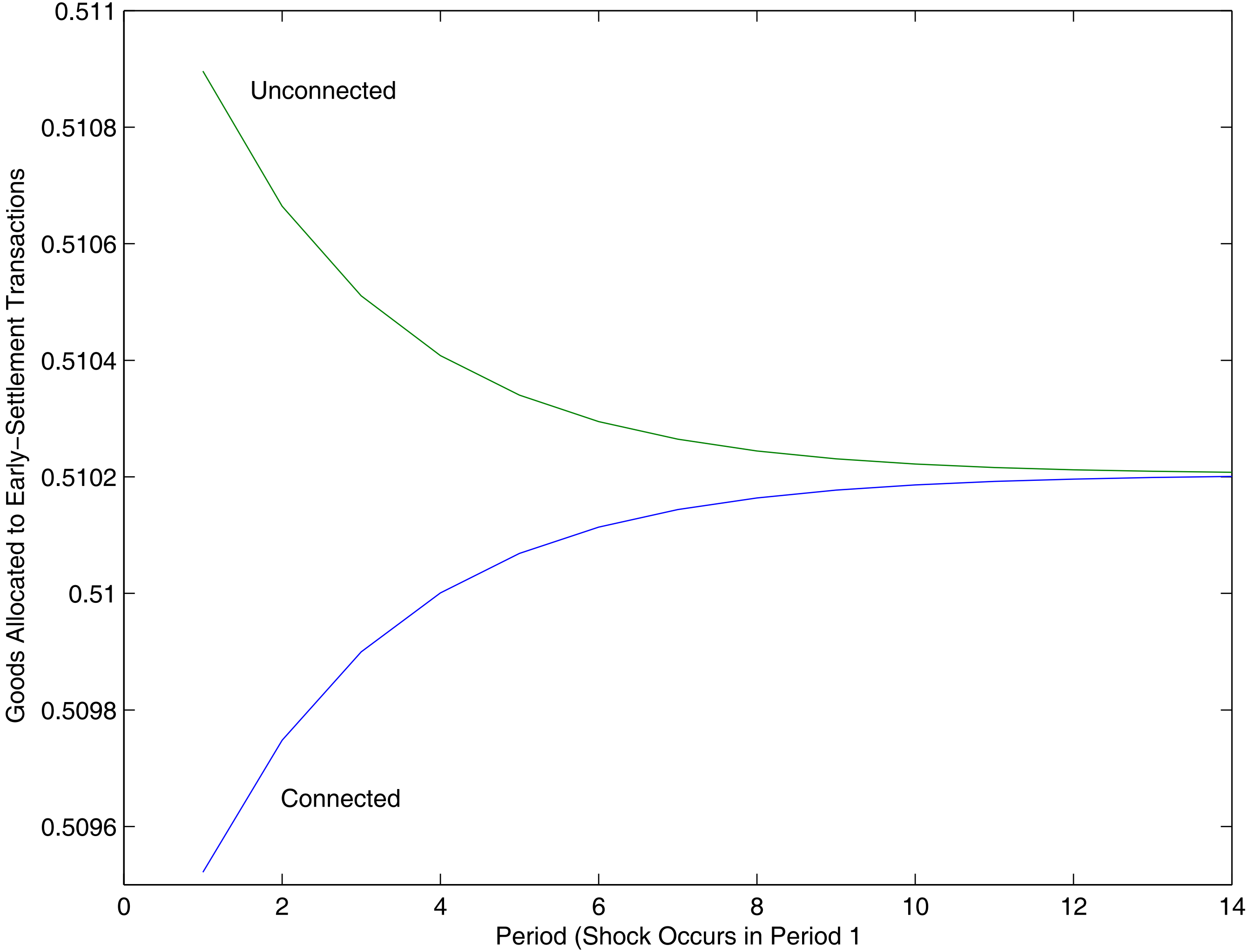

Figure 6: Permanent Money Injection: Consumption, Early Settlement

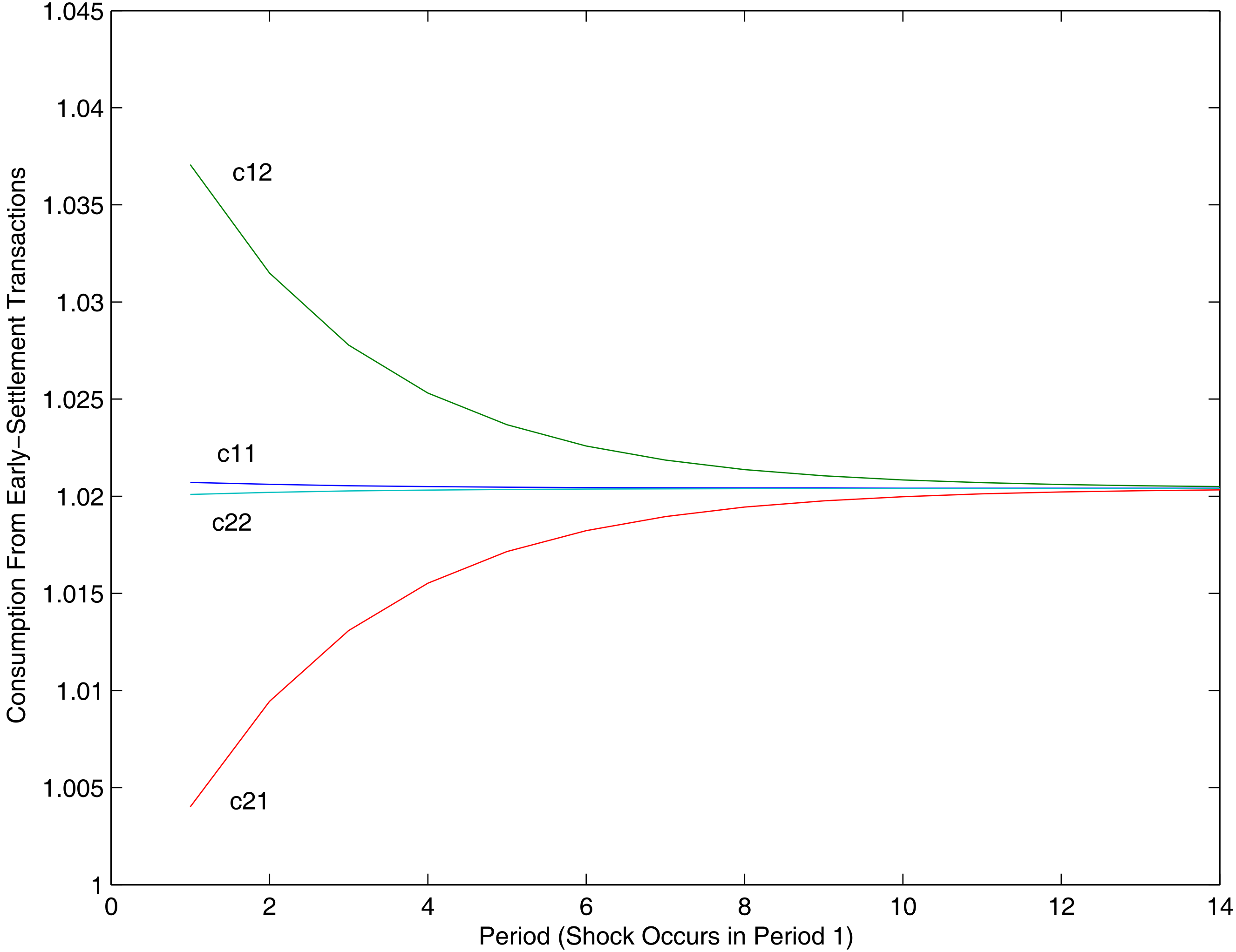

Figure 7: Permanent Money Injection: Consumption, Late Settlement