Inflation Differentials in a Currency Area: Facts, Explanations and Policy

Filippo Altissimo
European Central Bank

Pierpaolo Benigno
New York University

Diego Rodriguez Palenzuela *
European Central Bank

First draft: May 2004
Current Draft: December 2004

*This paper does not reflect views or opinions of the European Central Bank but solely those of its authors. The authors are grateful to Kosuke Aoki, Philip Lane and Roger Farmer for very helpful dicussions and to Gianni Lombardo, Klaus Masuch, Roberto Motto and Sergio Nicoletti-Altimari as well as participants at the *XVII Simposium Moneda y Crédito* in Madrid and CEPR conference *Designing a Macroeconomic Policy Framework for Europe* in Barcelona for helpful comments. Any errors are the responsibility solely of the authors.

1 Introduction

The issue of analyzing and explaining consumer price and inflation differentials in a currency area has attracted the attention of a considerable number of researchers in the recent past years.[1] Recent research was to a large extent spurred by the experience with the major change in the exchange rate regime and the implications for price formation that were triggered by the formation of the European Monetary Union. Indeed, the initial years of EMU coincided with a change in the trend of inflation dispersion across European countries. In the run-up to monetary union, inflation rates across member countries had largely converged. Since the year 2000 a sizeable degree of dispersion across national inflation rates in euro area has been at times observed. The initial interpretation in the literature of such change in trend was to appeal to the Balassa-Samuelson effect. A explanation based Balassa-Samuelson effect seemed valid, taking into account in particular that national inflation rates were highest in the fastest-growing economies such as Ireland, Portugal and Spain. However, this narrow focus on a single explanatory factors was gradually superseded by broader explanations[2] in which the observed inflation differentials were accounted for by more complex interactions among three main factors: heterogeneity in structures, common and idiosyncratic shocks from both supply and demand sides, and the role of monetary policy.

This paper aims at enhancing our theoretical and quantitative understanding of how these three dimensions interact to generate persistent differences in national inflation rates inside a monetary union and to draw possible implications for policy. Specifically, this work aims to contribute to the literature along both an empirical and a theoretical dimension.

First we present evidence on the presence on dispersion in overall inflation rates and changes in CPI components based on the experience of European countries in the EMU. Our descriptive exercise shows that there is still a sizeable dispersion of HICP inflation rates across euro area countries. In a sectorial decomposition of this dispersion, we find that most of it occurs in the Service category in the HICP, even if also the Energy category has been a relevant source in some historical periods. This suggests that most of the sources of dispersion are in the components of the HICP that are more intensely based on non-traded goods. In addition, we use a dynamic factor model to decompose the aggregate and sectorial dispersions between a common component driven by common factors and an idiosyncratic component. The heterogeneous response across countries to a change in the common factor can account for large fraction of the

[1] Our bibliography collects most of the recent and past works.

[2] See Blanchard (2001) for an early dicussion in this respect.

dispersion in national inflation rates. This is particularly the case for the Industrial good categories, even thought the cross-country dispersion of the inflation rates of items in this category is quite low. However, it also applies to the Services categories, which is instead characterized by a high level of dispersion.

Along these lines our interpretation of the data generating process is the following. Countries in a currency area might have structural differences: price-setting mechanism, degree of competition in the goods and labour markets, preferences and technologies and others. Moreover, policy-makers might interact in this environment by setting their policy instruments; countries might experience idiosyncratic or common shocks. The interrelation between shocks, structures and policies drive the data generating process and help to explain which factors can be treated as common or idiosyncratic.

Second, we build a stylized model of a currency area to aim at disentangling which shocks, structures and policies are important in explaining the qualitatively features of the data. The model assumes two productive sectors, a traded and a non-traded goods sector, in each economy, with the price of traded goods across countries assumed equal from the law of one price. Thus, consumer price and inflation differentials arise in the model as a consequence of movements in the relative prices of non-traded goods.

Some distinct aspects of the theoretical model are described. One important feature is that the model allows for the possibility of imperfect labour mobility across sectors. For this, imperfect substitutability between different types of sector-specific labour is introduced. This labour market friction generates wage differentials across sectors, which is a desirable feature from an empirical perspective (e.g. Ortega (2003)) and introduces an amplifying effect on the relative price movements in response to sectorial shocks. However, regional mobility is not allowed in the model, even over the lung run.

The model also incorporates market power in the sector of traded goods. As has been recently pointed out in the literature (Benigno and Thoenissen (2003)), a positive productivity gain in the tradable good sector has two counteracting effects on the overall price level, with upward pressure on the relative price of non-tradables offset by a decline in the terms of trade. Which effect dominates depends on the relative values of the elasticities of substitution between home and foreign tradables and between tradables and non-tradables and the strength of nominal rigidities.

In addition to supply disturbances, the model considers various fiscal shocks as well as shocks to the level of sectorial competition (as expressed by a markup, which in turn depends on the elasticity of substitution across varieties). An additional type of wedge between costs and prices is also incorporated into the model with sector-specific ad valorem indirect taxes.

In this set up distinct analysis are conducted for the flexible and rigid prices cases.

The flexible-price version of the model by construction cannot capture any dynamic beyond the one implied by the shocks. However we can describe the long-run properties of the model. Contrary to the Balassa-Samuelson argument, we show analytically that an unbalanced country-specific productivity shock in the traded sector is almost irrelevant for explaining consumer price differential. Instead, a balanced increase in productivity in one country has a negative and sizeable impact on the consumer price of the country relative to the rest of the area; it also reduces the overall mark-up in all the sectors of the country where the shock occurred. Asymmetric demand shocks, driven by government purchases, can be also an important source of consumer price dispersion. There is otherwise no role for monetary policy and for area-wide shocks in explaining price dispersion.

These features are introduced and analyzed in the version of the model with nominal rigidities. Here we address a broader set of questions.

First we explore the role of common shocks (in the form of a monetary policy shock) in generating inflation differential. We show that indeed, a discretionary shock to monetary policy can itself create persistent deviations in consumer prices if there are considerable structural asymmetries across countries.

Second, we analyze the response of the inflation differential to the different structural shocks in the system. We find that a modest degree of sticky prices can be sufficient to mute the short-run response to the shocks and to generate persistence well beyond the one implied by the stochastic process followed by the driving force. We find that most often sticky prices induce non-trivial dynamics and responses that contrast the ones of the flexible-price model.

Third, we show that the design of the monetary policy reaction function matters in shaping the dynamic of the inflation differential. Indeed, the objectives and the rules followed by monetary policy are critical in explaining the response and dynamic of price dispersion and other variables, following both common and idiosyncratic shocks. We consider in particular the case of optimal monetary policy. We find that also under optimal policy sizeable and persistent inflation differentials occur and that they are similar to those observed under conventional monetary policy rules based on aggregate inflation and output. Interestingly, we find that the response of inflation differentials to various shocks under optimal policy are indeed very close to the respective responses when monetary policy has an exclusive focus on maintaining price stability.

A fourth exercise performed in the model with nominal rigidities is the variance decomposition of endogenous variables with respect to the variability in the structural shocks. We find that the presence of different nominal rigidities in the two economies, notwithstanding the complete symmetry otherwise in the model and the shock structure, gives rise to interesting asymmetric responses in the variability of endogenous variables. Notably, we find that the dynamic of the inflation differentials is largely driven by the variability of a single shock in a single region, namely in the productivity of the non-tradable sector in the more flexible economy. Conversely, the output differentials is largely driven by the variability in the productivity of the tradable sector, also of the more flexible economy. Interestingly, we find that one effect of having optimal policy instead of a standard Taylor rule is to dampen somewhat the previous effects, creating a broader link between the variability in structural shocks and that in inflation and output differential.

The final exercise we perform is to compare the responses of the economy to case where the countries form a monetary union using an exchange rate parity which does not correspond to long-run equilibrium. This case is analyzed under three different policy regimes: the case of a currency area in which monetary policy is conducted optimally; a currency area in which monetary policy targets the overall area average CPI inflation; and finally the case where the two economies keep independent currencies and in which policy is conducted optimally by the two separate monetary authorities, i.e. in a way to maximize joint welfare. We evaluate the welfare differences across these three regimes. Our results suggest that the costs of having an incorrect real exchange rate parity by 1% leads to a loss slightly below one tenth of percentage point of steady-state consumption in the case where policy is conducted optimally and to a loss slightly above one tenth of percentage point of steady-state consumption when policy follows the strict CPI inflation targeting procedure. These costs are two times higher than the ones usually found for business cycle movements.

The paper is structured as follows: Section 2 presents the descriptive statistics on euro-area inflation differentials and proposes a dynamic factor model decomposition of it. Section 3 presents the structure of the model. Section 4 discusses the flexible-price version of the model while Section 5 analyzes the implications of the model under sticky prices. Section 6 analyzes the role of policy rules, including an approximation to the monetary policy optimal rule, in shaping inflation differentials. Section 7 reports the variance decomposition exercise. Section 8 addresses the question of what are the consequences of a country entering a monetary union with an incorrect exchange rate parity for overall inflation and inflation differentials, under both optimal and non-optimal monetary policy. The paper concludes with Section 9.

2 Euro area inflation differentials

This section addresses three empirical questions concerning the euro area economy. How sizable are the inflation differentials in the euro area in comparison to other currency areas? Are the inflation differentials in the euro area a sectorial phenomena, i.e. tradable versus non-tradable? Are the inflation differentials in the euro area the result of differentiated responses to common area-wide factors, or are they due to idiosyncratic (sector/country specific) factors?

A number of papers have recently provided empirical analysis of inflation differentials in the euro area. Alberola (2000) is one of the first analysis of post-EMU data specifically focusing on cross-country inflation differentials. Recently a more extensive review of empirical evidence and literature has been conducted by the ECB, (ECB, 2003). The latter work surveys a variety of measures of price and cost developments at the national level in EU-12 during the 1999-2002 period and explores different possible macroeconomic determinants. Here we maintain a narrower focus on which are the stylized statistical facts that are more relevant for our analysis. For this, we provide descriptive statistics as well as perform econometric analysis based on dynamic factor models.

The data used is based on the harmonized index of consumption prices (HICP). These data are harmonized indices of consumer prices for a basket of all goods and services consumed by a country, expenditure-weighted. We analyze seasonally adjusted data for the area as a whole and for ten individual countries (all euro countries except for Greece and Luxemburg), for the period from January 1990 to February 2004. Excluding Greece and Luxembourg reflects the fact that HICP data series for these countries are shorter than for the other countries in the sample. Both for the euro area and the individual countries HICP we considered the overall HICP inflation index as well as the five main subcomponents in the HICP, namely Services, Industrial Good excluding Energy, Energy, Processed Food and Unprocessed Food.[3]

Notation: To set notation, $\pi^i_{j,t}$ is the year on year growth rate of price in subindex j of country i and the aggregate euro area counterpart is $\pi^{euro}_{j,t}$. While π^i_t denotes the overall inflation rate in country i and π^{euro}_t is the overall euro area inflation rate. The inflation differential between country i and the euro area is denoted as:

$$\delta_{i,t} = (\pi^i_t - \pi^{euro}_t);$$

while the dispersion of inflation is measured by the root mean squared around the euro

[3] The weights of the five subindexes in the euro area HICP index in 2004 are 0.41, 0.31, 0.08, 0.12 and 0.08, respectively.

area counterpart, as :

$$\Delta_t = \left(\frac{\sum_{i=1}^{10} \delta_{i,t}^2}{10} \right)^{1/2}.$$

The inflation differential between subindex j in country i and the euro area one is denoted as:

$$\delta_{j,i,t} = (\pi_{j,t}^{i} - \pi_{j,t}^{euro});$$

while the dispersion of inflation in the sub-component j is measured as:

$$\Delta_{j,t} = \left(\frac{\sum_{i=1}^{10} \delta_{j,i,t}^2}{10} \right)^{1/2}.$$

The overall dispersion Δ_t is not the weighted average of the sectorial dispersion given the nonlinearity of the transformation. We decompose the overall dispersion into the relative contribution of the different subcomponents. To this end, first, we assume that the sectorial country weights inside the HICP, $w_{j,i,t}$, are equal across countries, i.e. $w_{j,i,t} = w_{j,t}\ \forall i$, then we define the contribution of sector j to the overall inflation dispersion as:

$$\frac{w_j \times \Delta_{j,t}}{\sum_j w_j \times \Delta_{j,t}}.$$

Empirical Findings: The cross-country dispersion in inflation has been declining during the 90s across euro area as documented in Figures 1-2. Dispersion was above 4% in the early 90s while it reached its minimum at 0.26% at the end of 1997. After this point, the previous trend was inverted, starting to edge up again to around 1% between 1998 and 2002. In early 2004 it stood at around 0.64%.[4]

Compared with the degree of dispersion observed within some individual euro area countries, inflation dispersion within the euro area remains relatively high. In particular, the recent degree of dispersion within the euro area is around twice the comparable measures computed across the German *Länder*, the Spanish *Comunidades Autónomas* and the Italian cities. On the contrary, the recent euro area inflation dispersion is quite comparable to the one measured among the 14 US metropolitan areas. The inflation divergence among US cities stayed remarkably constant at around 1 percent for many years. However the analogy is probably misleading, given that US cities are much smaller than EU nations, and their price indices tend to be more volatile.

Despite the possible similarities in the level of inflation dispersion between US and euro area, the high persistence of inflation differential is a characterizing feature of the

[4]This finding is slightly different from the one reported in ECB (2003), due to the fact that we considered the dispersion of the 10 indicated countries vis-à-vis the euro area while ECB (2003) considered also Greece and Luxemburg.

euro area economy, as noted by Cecchetti, Mark and Sonora (2000). Indeed, in our data set the measured persistence of the inflation differential, $\delta_{j,i,t}$, is very close to the one of a unit root process on average across countries and sectors.[5]

The importance of sectorial pattern in explaining the overall dispersion is addressed in Figures 3-6. Figure 3 shows the year-on-year inflation rates in the euro area countries in the *Services*, *Industrial Good Excluding Energy* and *Energy* sectors, respectively. Figure 4 and 5 present the dispersion for each of the five sectors, $\Delta_{j,t}$, plotted together with the overall dispersion, Δ_t. Finally, Figure 6 presents the contribution of the sectors to the overall inflation dispersion.

Several conclusions can be drawn. First, all the components of inflation contributed to the very low dispersion observed in 1997-1998. Second, the increase on dispersion between 2000 and 2002 can mainly attributed to the dispersion in Services and Energy sectors. Third, Figures 4 shows that the dispersion in the Service sector has been almost always higher than the overall dispersion and its contribution has been increasing over time as indicated in Figure 6, also in line with the increase of the weight of this subcomponent inside the HICP index. Fourth, differently to the Services, the Industrial Good excluding energy index presents a low degree of dispersion and its overall importance is decreasing. Finally, differently from the common wisdom, the dynamic of the Energy subindex is a major source of overall dispersion. This is due both to the large volatility of this subindex but also to considerable heterogeneity in the countries' response to shocks.

Interpreting those results some cautionary remarks should be made. The HIPC is an index of final consumer prices for a basket of all goods and services consumed; so Industrial Good excluding energy price includes also the prices of (tradable) imported goods and a share of final sale services, such as the prices of any non-tradable marketing and other final consumption services; the services component included in the final Industrial good price might induce to overestimate the dispersion of this sector. On the contrary, this is not the case in the Services sector, where the value added deflator of Services almost entirely accounts for the final price of Services, as indicated by the input-output evidence produced by Sondergaard (2003) for some euro countries.

Finally, we address the last of the three questions stated at the outset - namely, whether the observed dispersion in inflation across euro area countries is due to different reaction to area-wide factors or the result of country/sectorial developments. To tackle this question we follow a similar approach to the one employed by Forni and Reichlin (2001) to decompose euro countries GDP growth into a European, a national, and a

[5] If we measure the persistence of the differential process by the largest autoregressive roots of a fitted ARMA model, it turns out that the largest autoregressive root is 0.981 on average across sectors and countries.

residual component. We estimate an approximate dynamic factor model[6] on the inflation differential which allows us to decompose the differential in each countries and sectors as:

$$\delta_{j,i,t} = c_{j,i} + \Phi_{j,i}(L) \times u_t + \xi_{j,i,t} = \chi_{j,i,t} + \xi_{j,i,t} \tag{2.1}$$

where the first term of the right hand side is the average dispersion over the sample (ideally nil), the second term captures the effect of common area wide shocks, u_t, which is allowed to propagate across countries and sectors with a differentiate dynamic, and the last term captures an idiosyncratic dynamic, mainly associated to country or sectorial specific developments. We grouped the first and second term into $\chi_{j,i,t}$.

The approximate dynamic factor models exploits the cross-section dimension of a large panel of time series to identify and estimates the part of the time series driven by few common shocks, $\chi_{j,i,t}$. To this end, in order to have proper estimates of the euro area common components in (2.1), we augmented the sixty time series on differential, $\delta_{j,i,t}$, with 193 monthly macro-economic time series related to the ten euro area countries considered.[7] The estimation has been performed on the sample period 1993.01-2003.06. In line with the finding of similar exercises, the results points to the presence of strong commonalities among the 253 variables and the estimated common factors account on average of around 50% of the variance of the 253 variables.[8]

Having an estimate of the two components in (2.1), we can first ask how much of the historical dynamic of differentials is accounted by the identified area wide factors. The table below reports the average across countries of the share of variance of the differentials, $\delta_{j,i,t}$, accounted by common shocks, both for the overall index and for the individual subindexes.

Overall	Services	Industrial	Energy	Proc. Food	Unproc. Food
0.66	0.58	0.67	0.52	0.65	0.42

It turns out that the Energy and the Services sector are the ones with the large idiosyncratic components, while the Industrial Good excluding Energy and the Processed Food sectors are the most common ones.

The importance of the common factors is also clear from Figure 7-10, where we show how the dispersion in inflation can be decomposed into the part attributed to the

[6] See Stock and Watson (1999) and Forni, Hallin, Lippi and Reichlin (2000).

[7] The data considered are, inter alia, main and sectoral industrial production indexes, consumer and producer surveys, producer prices index, financial quantities, interest rates, trade statistics. The data has been seasonally adjusted and transformed to be stationary.

[8] The estimation procedure is based on the principal componet decomposition of the variance-covariance matrix of the data. The estimation indicates the presence of five static factors in the models.

common factor, $\chi_{j,i,t}$, and a remaining one, associated to the idiosyncratic part. Figure 7 presents the cross-country dispersion of the overall HICP and its decomposition. The common part is clearly responsible for the large part of the observed dispersion. The idiosyncratic part has a nil contribution to overall dispersion from 1994 to 2000, but it contributes positively to dispersion from 2000 onward. The Service sector presents a behavior similar to the overall index with large part of its dispersion explained by countries-specific reaction to common shocks. The increase in dispersion since 2000 is mainly associated to the common factor; even if idiosyncratic elements contributed positively both during 2000 and 2002. In the Industrial sector the commonality is even more striking, however it should be noted that the dispersion in this sector is relative low. By contrast, in the case of Energy prices most of the dispersion is due to presence of idiosyncratic factors, see Figure 10.

To conclude, the inflation differentials in the euro area are relevant and persistent. Our proxy of the non-tradable sector, namely the Services sector seems to make a particularly strong contribution to dispersion; this related to both the effects of common as well as the idiosyncratic factors behind the Services component. By contrast, the dispersion in our proxy of the price index of tradable goods - namely industrial goods excluding energy - is relatively limited. Finally, a tentative overall conclusion from our attempt to understand the source of inflation differentials would indicate that they are largely associated to different responses of the ten euro area economies to common, area-wide shocks.

3 Model

We propose a stylized model of a currency area to investigate which shocks can be most important to drive the price dispersion and which structural features can rationalize the observed persistence. We model a currency area as composed by two countries of equal population size. Consumption preferences depend on non-traded and traded goods. In particular, each country is specialized in the production of a bundle of traded goods. Financial markets are complete within and across countries. Labor is immobile across countries but imperfectly mobile within sectors of a country. There can be price rigidities in all sectors of the economy. The law of one price holds in the traded sector and the price dispersion at the consumer level depends on the movements of the relative prices of non-traded goods.

Our model is closely related to open-economy models, like Obstfeld and Rogoff (2000) who introduces non-traded goods in stochastic models with sticky prices. Benigno and Thoenissen (2003) use a similar model to study the real exchange rate behavior of UK with respect to the Euro area with the purpose of analyzing whether supply shocks can account for the real exchange rate appreciation in the late nineties. In reference to the recent literature on monetary policy in a currency area our model is closely related to Duarte and Wolman (2002, 2003). They further allow for price discrimination in goods that are tradeable. Moreover, they analyze the role of fiscal policy rules in reducing or amplifying inflation differentials. Indeed, in most of their work they assume that lump-sum forms of taxation are unavailable to the government. Our focus here is instead limited to the case in which lump-sum taxes are available. In Duarte and Wolman (2002) they find that their model can deliver more inflation dispersion than in the data following productivity shocks, while negligible dispersion following government spending shocks. Andrés et al. (2003) instead analyze a model with only traded goods, but they allow for price discrimination across countries due to different degrees of market competition. They show that their model can account for sizeable inflation differentials. In particular their findings are that the driving force of the price dispersion in the traded sector originates from the mechanism of price discrimination more than on price rigidity. They also find that the degree of openness of countries can play an important role. Angeloni and Ehrmann (2004) present a more stylized 12-country model of the euro area and in particular they focus on the role of past inflation in the aggregate supply equation. They find that this additional source of inflation persistence is important in driving up inflation dispersion in the currency area. There are other papers that have analyzed monetary models of currency areas, as Benigno (2004), Beetsma and Jensen (2004) and Lombardo (2004).[9] However these models do not allow for price dispersion at the consumer level and mainly focus on the role of the terms of trade and price stickiness in stabilizing asymmetric shocks.

3.1 Households

We consider a model of a currency area composed by two countries, Home (H) and Foreign (F). Each country is populated by a measure one of households. A generic household j belonging to either country H or F maximizes the following utility function:

$$U_t^j \equiv \mathrm{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \left[U(C_s^j) - V(L_s^j) \right],$$

where E_t denotes the expectation conditional on the information set at date t and β is the intertemporal discount factor, with $0 < \beta < 1$. Households derive utility from

[9] Our work is also related to model of small open economies as Natalucci and Ravenna (2002) that have investigated exchange rate policy for accession countries in the EU. See Soto (2001) for a small open-economy model with non-traded goods.

consumption and disutility from supplying hours of work.

There are two classes of goods in both economies: traded and non-traded goods. Each country produces a measure one of goods, a fraction γ (γ^* in region F)$-$ with $0 < \gamma, \gamma^* < 1$ - of which is composed by traded goods. The remaining fractions are non-traded goods. The consumption index C_t^j in region H is defined as a Dixit-Stiglitz aggregator of indexes of traded, C_T^j, and non-traded goods, C_N^j, as it follows

$$C^j \equiv \left[\omega^{\frac{1}{\varphi}}(C_T^j)^{\frac{\varphi-1}{\varphi}} + (1-\omega)^{\frac{1}{\varphi}}(C_N^j)^{\frac{\varphi-1}{\varphi}}\right]^{\frac{\varphi}{\varphi-1}},$$

where φ is the elasticity of substitution between the bundles C_T and C_N with $\varphi > 0$ while ω denotes the share of traded goods in the general consumption basket, with $0 < \omega < 1$. (This share may be different for households in region F and it will be denoted by ω^*.) The traded goods are non-homogenous and differentiated in consumption preferences. They are also produced with different technologies. In particular the index C_T^j is defined as a Dixit-Stiglitz aggregator of the bundles of home-produced traded goods, C_H^j, and foreign-produced traded goods, C_F^j,

$$C_T^j \equiv \left[n^{\frac{1}{\theta}}(C_H^j)^{\frac{\theta-1}{\theta}} + (1-n)^{\frac{1}{\theta}}(C_F^j)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}},$$

where θ, with $\theta > 0$, is the elasticity of substitution between the bundles C_H^j and C_F^j and n, with $0 < n < 1$, denotes the share of home-produced traded goods in the overall index of traded goods. In the foreign economy n^*, with $0 < n^* < 1$, denotes the share of foreign-produced traded goods in the overall basket of traded goods. The consumption bundles C_H^j and C_F^j are composed by the continuum of differentiated traded goods produced respectively in region H and F and are defined as

$$C_H^j \equiv \left[\gamma^{-\frac{1}{\sigma}} \int_0^{\gamma} c^j(h)^{\frac{\sigma-1}{\sigma}} dh\right]^{\frac{\sigma}{\sigma-1}}, \qquad C_F^j \equiv \left[\gamma^{*-\frac{1}{\sigma}} \int_0^{\gamma^*} c^j(f)^{\frac{\sigma-1}{\sigma}} df\right]^{\frac{\sigma}{\sigma-1}},$$

where $\sigma > 1$ is the elasticity of substitution among the differentiated goods. Similarly, C_N^j is the home consumption index of the continuum of differentiated non-traded goods:

$$C_N^j \equiv \left[(1-\gamma)^{-\frac{1}{\sigma}} \int_{\gamma}^{1} c_N^j(h)^{\frac{\sigma-1}{\sigma}} dh\right]^{\frac{\sigma}{\sigma-1}},$$

with the same elasticity of substitution σ. In country F, γ^* replaces γ in the consumption bundles of non-traded goods.

Given the above consumption indices, we can derive the appropriate price indices. With respect to the general consumption index, we obtain the consumption-based price index P

$$P = \left[\omega(P_T)^{1-\varphi} + (1-\omega)(P_N)^{1-\varphi}\right]^{\frac{1}{1-\varphi}},$$

where P_T and P_N are given by

$$P_T = \left[n(P_H)^{1-\theta} + (1-n)(P_F)^{1-\theta} \right]^{\frac{1}{1-\theta}},$$

$$P_N = \left[(1-\gamma)^{-1} \int_{\gamma}^{1} p_N(h)^{1-\sigma} dh \right]^{\frac{1}{1-\sigma}},$$

while P_H and P_F are given by

$$P_H = \left[\gamma^{-1} \int_0^{\gamma} p(h)^{1-\sigma} dh \right]^{\frac{1}{1-\sigma}}, \qquad P_F = \left[\gamma^{*-1} \int_0^{\gamma^*} p(f)^{1-\sigma} df \right]^{\frac{1}{1-\sigma}},$$

where $p(h)$, $p(f)$, $p_N(h)$ are respectively the prices in the common currency faced by households in country H for a generic home-produced traded good, a foreign-produced traded good and a domestic non-traded good. Similar indices are derived for country F with the appropriate modifications of the respective shares. Prices faced by foreign consumers are denoted with asterisks.

Given the consumption-based price indexes the generic home consumer j has the following demand of each of the home-produced traded goods

$$c^j(h) = \frac{n\omega}{\gamma} \left(\frac{p(h)}{P_H} \right)^{-\sigma} \left(\frac{P_H}{P_T} \right)^{-\theta} \left(\frac{P_T}{P} \right)^{-\varphi} C^j,$$

for $0 \leq h < \gamma$; of each of the foreign-produced traded goods

$$c^j(f) = \frac{(1-n)\omega}{\gamma^*} \left(\frac{p(f)}{P_F} \right)^{-\sigma} \left(\frac{P_F}{P_T} \right)^{-\theta} \left(\frac{P_T}{P} \right)^{-\varphi} C^j,$$

for $0 \leq f < \gamma^*$ and of each of the home-produced non-traded goods

$$c_N^j(h) = \frac{1-\omega}{1-\gamma} \left(\frac{p_N(h)}{P_N} \right)^{-\sigma} \left(\frac{P_N}{P} \right)^{-\varphi} C^j,$$

for $\gamma \leq h \leq 1$. Similar demands hold in the foreign economy.

Households get disutility from supplying labor to all the firms operating in their country of residence. In particular the function $V(.)$ is increasing and convex in an index of labor L^j. Each firm uses a specific labor factor and each household can supply all the varieties of labor used in the country to produce the continuum of traded and non-traded goods. In particular each household supplies a measure one of labor varieties, of which a fraction γ will be employed in the traded sector (γ^* in the foreign economy) and the remaining fractions in the non-traded sector. In particular we assume that L^j

is a Dixit-Stiglitz aggregator of labor indices L_T and L_N in the traded and non-traded sectors, respectively, as it follows

$$L^j \equiv \left[\gamma^{\frac{1}{\phi}}(L_T^j)^{\frac{\phi-1}{\phi}} + (1-\gamma)^{\frac{1}{\phi}}(L_N^j)^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}},$$

where $\phi > 0$ is the elasticity of substitution between labor in the traded and non-traded sector and L_T^j and L_N^j are composite index of the continuum of varieties supplied in both sectors

$$L_T^j \equiv \left[\gamma^{-\frac{1}{\upsilon}} \int_0^{\gamma} l_T^j(i)^{\frac{\upsilon-1}{\upsilon}} di\right]^{\frac{\upsilon}{\upsilon-1}}, \qquad L_N^j \equiv \left[(1-\gamma)^{-\frac{1}{\upsilon}} \int_{\gamma}^{1} l_N^j(i)^{\frac{\upsilon-1}{\upsilon}} di\right]^{\frac{\upsilon}{\upsilon-1}},$$

where $\upsilon > 0$ is the elasticity of substitution between varieties of labor within a sector. In contrast with standard models of traded and non-traded production, we are not necessarily assuming that labor is perfectly substitutable and mobile across sectors and instead we allow for wage differentiation across varieties of labor. The case of perfect substitutability, and perfect labor mobility, is nested under the assumption that both ϕ, $\upsilon \to \infty$. In general, given wages $w_T(i)$ and $w_N(i)$ specific to the generic variety i in the respective traded and non-traded sector, we can write the following wage indexes associated with the above defined labor indices

$$W = \left[\gamma W_T^{1-\phi} + (1-\gamma) W_N^{1-\phi}\right]^{\frac{1}{1-\phi}},$$

$$W_T = \left[\gamma^{-1} \int_0^{\gamma} w_T^{1-\upsilon}(i) di\right]^{\frac{1}{1-\upsilon}}, \qquad W_N = \left[(1-\gamma)^{-1} \int_{\gamma}^{1} w_N^{1-\upsilon}(i) di\right]^{\frac{1}{1-\upsilon}}.$$

Given the relative wages and the choice of L^j, we can then characterize the household's labor decisions in the following way

$$l_T^j(h) = \left(\frac{w_T(h)}{W_T}\right)^{-\upsilon} \left(\frac{W_T}{W}\right)^{-\phi} L^j, \quad l_N^j(i) = \left(\frac{w_N(i)}{W_N}\right)^{-\upsilon} \left(\frac{W_N}{W}\right)^{-\phi} L^j,$$

for each variety of labor supplied for a generic firm in a traded and non-traded sector, respectively.

Each household faces the following flow budget constraint

$$B_t^j \leq A_t^j + W_t L_t^j + \Pi_t^j - P_t C_t^j + T_t^j \tag{3.2}$$

where A_t^j represents the beginning-of-period wealth that includes the bonds carried from the previous period. B_t^j is the end-of period portfolio that includes a wide selection of instruments that pay in each contingency that occurs. In particular they pay A_t in the particular contingency at date t. As of time $t-1$, A_t is a random variable whose realization depends on the state of nature at time t.

Here it is assumed that there are complete financial market which implies that there exists a unique discount factor $Q_{t,t+1}$ with the property that the price in period t of a portfolio with random value A_{t+1} is

$$B_t = E_t[Q_{t,t+1}A_{t+1}],$$

where E_t denotes the expectation conditional on the state of nature at date t. In particular we define the short-term interest rate in the following way, as the price of the portfolio that delivers one unit of currency in each contingency that occur one-period ahead, i.e.

$$\frac{1}{1+i_t} = E_t[Q_{t,t+1}].$$

In (3.2), Π_t are aggregate profits of all the firms within a country. Profits are risk shared across households. T_t^j are transfers from the government to household j. The economy is a cashless-limiting monetary economy as discussed in Woodford (2003). The flow budget constraint of the consumer can be written as

$$E_t[Q_{t,t+1}A_{t+1}^j] \leq A_t^j + W_t L_t^j + \Pi_t^j - P_t C_t^j + T_t^j$$

and the consumer's problem is further subject to the following borrowing limit condition in each contingency and date that the consumer faces.

$$A_{t+1}^j \geq -\sum_{s=t+1}^{\infty} E_{t+1}Q_{t+1,s}\{W_s L_s^j + \Pi_s^j + T_s^j\} > -\infty.$$

The borrowing limit condition together with the flow budget constraint imply the standard intertemporal budget constraint

$$\sum_{s=t}^{\infty} E_t Q_{t,s}\left[P_s C_s^j\right] \leq A_t^j + \sum_{s=t}^{\infty} E_t Q_{t,s}\left[W_s L_s^j + \Pi_s^j + T_s^j\right]. \tag{3.3}$$

Given the above decisions on how to allocate consumption and labor across all the varieties, the household j chooses the optimal path of the consumption index C_t and labor index L_t at all times and contingencies to maximize its utility under the intertemporal budget constraint. In particular the set of optimality conditions can be described by the set of Euler conditions

$$\frac{U_c(C_t^j)}{U_c(C_{t+1}^j)} = \frac{\beta}{Q_{t,t+1}}\frac{P_t}{P_{t+1}} \tag{3.4}$$

for each state of nature at time $t+1$ looking ahead from time t. By choosing appropriately the distribution of initial state-contingent wealth, complete markets assure that consumption is perfectly equalized within households belonging to a country. Moreover, across countries, the ratio of marginal utilities of consumption is proportional to the ratio of consumer prices:

$$\frac{U_c(C_t)}{U_c(C_t^*)} = \varsigma \frac{P_t}{P_t^*} \tag{3.5}$$

for a positive factor of proportionality $\varsigma > 0$ which again depends on the initial wealth distribution. As shown by Backus and Smith (1993), under the assumption of complete financial markets, consumer-price differentials directly translate into consumption differentials, so that a country which experiences an increase in its consumer price level relative to another country should experience a fall in its own consumption relative to the other country. This complete-market assumption is a convenient simplification but it comes at a cost of neglecting wealth distribution as an important channel through which price and inflation differentials can propagate into the economy and be amplified.

Real wages, computed using the general price and wage indices, are equated to the marginal rate of substitution between the labor index L and the consumption index C as

$$\frac{W_t}{P_t} = \frac{V_l(L_t)}{U_c(C_t)}. \tag{3.6}$$

Finally the last optimality condition is the exhaustion of the intertemporal budget constraint, i.e. (3.3) holds with equality at all times.

3.2 Firms

Regarding the supply side of the economy, we indeed assume that there is a continuum of firms, of measure one, which is producing the continuum of goods. In particular a fraction γ (γ^* in the foreign economy) is producing traded goods, while the remaining fractions are producing non-traded goods. Taking as representative the home economy, a generic firm producing in the traded sector is using the following technology $y^T(h) = A_T f(l(h))$, where A_T is a country- and sector-specific technological shock and $f(.)$ is a standard concave production function in the specific variety of labor used in the production of good h. In the non-traded sector the technology is given by $y^N(h) = A_N f(l(h))$ for a generic firm h in the non-traded sector. Firms in both sectors are monopolist and set their prices considering the overall demand of their goods. In the traded sector, we assume that there is no price discrimination and that all the consumers of the area face the same price for the same variety of goods. In particular in the traded sector a generic firm h faces the following demand

$$y^T(h) = \frac{n\omega}{\gamma}\left(\frac{p(h)}{P_H}\right)^{-\sigma}\left(\frac{P_H}{P_T}\right)^{-\theta}\left(\frac{P_T}{P}\right)^{-\varphi} C + \frac{n^*\omega^*}{\gamma}\left(\frac{p(h)}{P_H}\right)^{-\sigma}\left(\frac{P_H}{P_T^*}\right)^{-\theta}\left(\frac{P_T^*}{P^*}\right)^{-\varphi} C^*$$

In the non-traded sector, a generic firm faces the following demand

$$y^N(h) = \frac{1-\omega}{1-\gamma}\left(\frac{p_N(h)}{P_N}\right)^{-\sigma}\left[\left(\frac{P_N}{P}\right)^{-\varphi} C_N + G\right],$$

where in particular G is an exogenous country-specific government-purchase shock that affects only the demand of non-traded goods. In both sectors, prices are sticky and staggered as in the Calvo's style price-setting behavior.[10] In particular a mass $1-\alpha_T$ of firms in the traded sector ($1-\alpha_N$ in the non-traded sector) with $0 \leq \alpha_T, \alpha_N < 1$, is allowed in each period to reset their prices. (In the foreign economy, we have respectively α_N^* and α_T^* with $0 < \alpha_T^*, \alpha_N^* < 1$.) In this case a generic firm h in the traded sector of country H sets its price in order to maximize the present discounted value of profits, taking in consideration that the price chosen at time t will remain the same at time s, with $s \geq t$, with a probability $(\alpha_T^i)^{s-t}$. The present discounted value of profits is

$$\mathrm{E}_t \sum_{s=t}^{\infty} (\alpha_T)^{s-t} Q_{t,s}[(1-\tau_{T,s})\tilde{p}_t(h)\tilde{y}_{t,s}^T(h) - w_{T,s}(h) l_{T,s}(h)], \tag{3.7}$$

where $\tau_{T,t}$ is a sectorial country-specific time-varying proportional tax on sales in the traded sector, $\tilde{p}_t(h)$ denotes the price of the firm h chosen at date t and $\tilde{y}_{t,s}^T(h)$ is the total demand of firm h at time s conditional on the fact that the price $\tilde{p}_t(h)$ has not changed.

It can be shown that the optimal choice of the price satisfies

$$\mathrm{E}_t \sum_{s=t}^{\infty} (\alpha_T \beta)^{s-t} U_c(C_s) \tilde{y}_{t,s}^T(h) \frac{1}{\mu_{T,s}} \frac{P_{T,s}}{P_s} \left[\frac{\tilde{p}_t(h)}{P_{H,t}} \frac{P_{H,t}}{P_{H,s}} \frac{P_{H,s}}{P_{T,s}} - mc_{t,s}^T(h)\right] = 0 \tag{3.8}$$

where the real marginal cost for firm j at time s conditional on the fact that the price $\tilde{p}_t(j)$ has not changed are defined by

$$\begin{aligned} mc_{t,s}^T(h) &\equiv \mu_{T,t} \frac{P_s}{P_{T,s}} \frac{w_{T,s}(h)}{P_s} \frac{1}{f'(f^{-1}(\tilde{y}_{t,s}^T(h)/A_{T,s}))A_{T,s}} \\ &= \mu_{T,t} \frac{P_s}{P_{T,s}} \frac{W_s}{P_s} \left(\frac{W_{T,s}}{W_s}\right)^{1-\frac{\phi}{\upsilon}} \left(\frac{f^{-1}[(\tilde{y}_{t,s}^T(h)/A_{T,s}]}{L_s}\right)^{-\frac{1}{\upsilon}} \frac{1}{f'(f^{-1}(\tilde{y}_{t,s}^T(h)/A_{T,s}))A_{T,s}} \end{aligned}$$

where $1/\mu_{T,t}$ is defined as

$$\frac{1}{\mu_{T,t}} \equiv \frac{(1-\tau_{T,t})(\sigma-1)}{\sigma}.$$

[10] See Calvo (1983).

Given the Calvo's mechanism, the evolution of the price index $P_{H,t}$ is described by the following law of motion

$$P_{H,t}^{1-\sigma} = \alpha_T P_{H,t-1}^{1-\sigma} + (1-\alpha_T)\tilde{p}_t(h)^{1-\sigma}, \tag{3.9}$$

where indeed $1-\alpha_T$ is the fraction of firms that can reset their prices. Following the same reasoning we can write the first-order condition for a generic firm in the non-traded sector as

$$\mathrm{E}_t \sum_{s=t}^{\infty} (\alpha_N \beta)^{s-t} U_c(C_s) \tilde{y}_{t,s}^N(h) \frac{P_{N,s}}{P_s} \frac{1}{\mu_{N,s}} \left[\frac{\tilde{p}_{N,t}(h)}{P_{N,t}} \frac{P_{N,t}}{P_{N,s}} - mc_{t,s}^N(h) \right] = 0 \tag{3.10}$$

where

$$\begin{aligned} mc_{t,s}^N(h) &\equiv \mu_{N,s} \frac{P_s}{P_{N,s}} \frac{w_{N,s}(h)}{P_s} \frac{1}{f'(f^{-1}(\tilde{y}_{t,s}^N(h)/A_{N,s}))A_{N,s}} \\ &= \mu_{N,s} \frac{P_s}{P_{N,s}} \frac{W_s}{P_s} \left(\frac{W_{N,s}}{W_s} \right)^{1-\frac{\phi}{\upsilon}} \left(\frac{f^{-1}[(\tilde{y}_{t,s}^N(h)/A_{N,s}]}{L_s} \right)^{-\frac{1}{\upsilon}} \frac{1}{f'(f^{-1}(\tilde{y}_{t,s}^N(h)/A_{N,s}))A_{N,s}} \end{aligned}$$

and

$$\frac{1}{\mu_{N,t}} \equiv \frac{(1-\tau_{N,t})(\sigma-1)}{\sigma},$$

where τ_N is a time-varying proportional tax on sales in the non-traded sector. The evolution of the price index $P_{N,t}$ is described by the following law of motion

$$P_{N,t}^{1-\sigma} = \alpha_N P_{N,t-1}^{1-\sigma} + (1-\alpha_N)\tilde{p}_{N,t}(h)^{1-\sigma}, \tag{3.11}$$

where $1-\alpha_N$ is the mass of firms that can change their prices in the non-traded sector. Similar price conditions can be obtained in the respective sectors of the foreign country.

3.3 Monetary and Fiscal Policies

Each country has its own fiscal authority while there is a single monetary policy maker for the whole area. In each region, the government raises revenues from the distortionary sale taxes to finance the expenditure for domestic non-traded goods. Moreover, lump-sum taxes are available to balance the budget in each period. It follows that

$$\begin{aligned} 0 &= \int_{\gamma}^{1} p_{N,t}(h) g_t(h) dh + \int_0^1 T_t^j dj - \tau_{N,t} \int_{\gamma}^{1} p_{N,t}(h) y_t^N(h) dh \\ &\quad - \tau_{T,t} \int_0^{\gamma} p_t(h) y_t^T(h) dh \end{aligned}$$

for country H while

$$\begin{aligned} 0 &= \int_{\gamma^*}^{1} p_{N,t}^*(f) g_t^*(f) df + \int_0^1 T_t^{*j} dj - \tau_{N,t}^* \int_{\gamma^*}^{1} p_t^{N*}(f) y_{N,t}^*(f) df \\ & -\tau_{T,t}^* \int_0^{\gamma^*} p_t^*(f) y_t^*(f) df \end{aligned}$$

for country F, where T_t^{*j} and T_t^j are lump-sum transfers to households in countries H and F respectively. The model is closed with the policy function chosen by the common monetary authority that will be specified later.

4 Price differentials with flexible prices

We can write the ratio of the CPI prices of the two countries in the following way

$$\frac{P_t}{P_t^*} = \frac{P_{T,t}}{P_{T,t}^*} \frac{[\omega + (1-\omega) T_{N,t}^{1-\varphi}]^{\frac{1}{1-\varphi}}}{[\omega^* + (1-\omega^*) T_{N,t}^{*1-\varphi}]^{\frac{1}{1-\varphi}}}$$

where we have defined the relative price of non-traded with respect to traded goods in each country as $T_N \equiv P_N/P_T$ and $T_N^* \equiv P_N^*/P_T^*$. The ratio of the tradeable goods prices is instead given by

$$\frac{P_{T,t}}{P_{T,t}^*} = \frac{[nP_{H,t} + (1-n)P_{F,t}]^{\frac{1}{1-\theta}}}{[n^* P_{F,t}^* + (1-n^*) P_{H,t}^*]^{\frac{1}{1-\theta}}}.$$

There can be several possible ways through which differences in consumer prices can arise among countries. One obvious reason has to do with the different composition of the consumption indices, due to differences in tastes. This can happen either because there can be home bias in the consumption of traded good, so that $n^* \neq 1-n$ or because the share of traded goods in the overall consumption basket can vary across countries, $\omega \neq \omega^*$. Heterogeneity in consumption preferences can be a source of consumer-price differentials even if the prices of all the goods, traded and non-traded, are equalized across countries. Conversely, even if tastes are similar, price differentials at the single good level can produce differential at the consumer-price level. The above decomposition shows that this can happen either because of deviations from the law of one price for traded goods or because relative price of non-traded goods can vary across countries.

The issue of explaining consumer price differential in a currency area is closely related to a long-lasting puzzle in the international finance literature: the PPP puzzle and the related literature on real exchange rate behavior.[11] Using the above decomposition,

[11] See Rogoff (1996).

Engel (1999) has studied the US real exchange rate relative to several countries and shown that its variability can be accounted mostly by the variability of the deviations from the law of one price for traded goods relative to the variability of the relative price of non-traded goods. With multiple currencies, firms might set their prices sticky in the currency of the buyer, hence protecting consumers from fluctuations of the exchange rate. In this way deviations from the law of one price for traded goods reflect mostly fluctuations of the nominal exchange rate. This interpretation is naturally absent in a currency area, since there is no reason to protect prices from fluctuations of the nominal exchange rate.[12] However, a unique currency does not exclude the possibility for firms to price discriminate across different markets (countries) due to different degrees of competition in the markets or structural characteristics. Moreover, even if firms do not price discriminate and markets are characterized by similar structures and degrees of competition, there can be price deviations for traded goods that enter the consumption basket, stemming from the fact that traded goods usually carry some non-traded components (e.g. distribution costs) before reaching the consumer markets.[13] Sondergaard (2003) using input-output tables for France, Italy and Spain has shown that the traded sector relies more than others on intermediate inputs produce by other sectors in the economy; in particular total inputs from other sectors account for 60 percent of gross output in the traded sector. This suggests that the distribution sector may be an important factor in explaining inflation differentials in a currency area. Movements in the prices of non-traded goods that enter in the production or transportation of traded goods can be an important source of price dispersion for traded goods at the consumer level. Moreover, taking in consideration intermediate stages of production de-emphasizes the importance of pure traded goods prices in explaining the fluctuations of the consumer price differential. Our evidence shows that inflation dispersion in the traded sector is much lower than in the non-traded sector, which suggests that in any case differences in the relative price of non-traded goods can be much more important in explaining the consumer price

[12] A puzzling finding of Engel (1999) is that the importance of the relative price of traded goods in explaining the variability of the real exchange rate is robust across the fixed exchange rate period of Bretton Woods and the following floating exchange rate period. This decomposition holds in general also for developing economies, as in the case of Mexico studied by Engel (2001). However in this case Mendoza (2000) has shown that during periods of fixed-exchange rate regime movements in the relative price of non-traded goods are dominant in explaining real exchange rate volatility. In relation to the debate on whether deviations from the law of one price for tradeable goods originate from local currency pricing, works by Obstfeld and Rogoff (2000) and Campa and Goldberg (2002) have emphasized that the pass-through of the exchange rate at the border level can be high.

[13] The importance of distribution sectors in explaining differential at the consumer-price leve has been emphasized by Burstein et al. (2003) and Corsetti and Dedola (2003). Duarte and Wolman (2003) incorporate this feature in their currency-area model.

differential. In this work, we choose not to model any price differential in the traded sector to get more insight on what we believe can be a stronger channel.[14] We assume that there is no price discrimination and that $P_H = P_H^*$ and $P_F = P_F^*$ at all times. We also assume perfect symmetry across countries and sectors and set $\omega = \omega^* = \gamma = \gamma^*$ and $n = n^* = 1/2$. It further follows that $P_T = P_T^*$. We first focus on the flexible-price allocation. In the appendix, we solve the model by taking a log-linear approximation to the relevant structural equilibrium conditions around a deterministic steady state in which the shocks $\{\mu_{T,t},\ \mu_{T,t}^*,\ \mu_{N,t},\ \mu_{N,t}^*,\ A_{T,t},\ A_{T,t}^*,\ A_{N,t},\ A_{N,t}^*,\ G_t,\ G_t^*\}$ are such that $\bar{\mu}_T = \bar{\mu}_T^* = \bar{\mu}_N = \bar{\mu}_N^* = \bar{\mu}$ and $\bar{A}_T = \bar{A}_T^* = \bar{A}_N = \bar{A}_N^* = \bar{A}$, $\bar{G} = \bar{G}^* = 0$. In a log-linear approximation around this steady state the consumer-price differential is proportional to the non-traded goods price differential (all in logs)

$$\begin{aligned} \ln P_t - \ln P_t^* &= (1-\gamma)(\hat{T}_{N,t} - \hat{T}_{N,t}) \\ &= (1-\gamma)(\ln P_{N,t} - \ln P_{N,t}^*), \end{aligned}$$

where hat variables represent log-deviations of the respective variable from the steady state.

The most popular and often advocated reason for why there can be long-lasting departures from PPP originating from non-traded goods prices is due to Balassa (1964) and Samuelson (1964). According to this view, countries that experience higher productivity growth in the traded sector will also show higher consumer prices. The reason is that productivity growth in the traded sector will translate into an increase in the overall wage in the economy, since prices of traded goods are tied internationally and there is perfect labor mobility. In the non-traded sector, firm will increase their prices since costs have increased and there are no benefits from productivity gains. There are several assumptions needed to get this result, as discussed in Froot and Rogoff (1995): mobility of labor and capital across sectors and mobility of capital internationally, constant returns to scale in the mobile factors and exogenous world real interest rate. Indeed, we cannot expect our model to display the Balassa-Samuelson features since capital is constant in each sector and labor is not perfectly mobile. Moreover, as shown in recent works by Benigno and Thoenissen (2003), Cova (2004), Duarte and Wolman (2003), Fitzgerald (2002), MacDonald and Ricci (2002), the assumption of homogenous traded-good market is critical for the result to hold. In our model, each country is specialized in the production of a bundle of traded goods. When the traded sector of a country is subject to a productivity shock, prices of home-produced traded goods can fall with respect to foreign-produced traded goods and terms of trade worsen. In this case movements in

[14] Andres et al. (2003) aknowledge the importance of the relative price of non-traded goods but they focus on differentials that arise from price discrimination in the traded-good sector.

terms of trade can absorb the productivity shock and reduce the pressure on wages and then on non-traded goods prices.

Another feature of the Balassa-Samuelson framework is that the consumer-price differential depends only on the supply side. Instead in our framework, as in Rogoff (1992), a model with low labor mobility can also capture additional sources of price differentials driven by demand shocks.[15] This is also the case in a more modern exposition of the Balassa-Samuelson model as in Canzoneri et al. (2001). In our context, we also introduce sectorial mark-up shocks to capture if changes in taxation and competition can account for consumer-price differences.

To get further insight into the model, we assume that the production functions are linear in the only factor of production, labor. In the appendix, we show that under this assumption terms of trade and the log difference of non-traded goods prices are determined by the following two equations

$$\ln P_{N,t}/P^*_{N,t} = -\frac{\left(1-\frac{\theta}{\phi}\right)}{\left(1-\frac{b}{\phi}\right)}\hat{T}_t - \frac{1}{\left(1-\frac{b}{\phi}\right)\phi}\hat{G}^R_t + \frac{\left(1-\frac{1}{\phi}\right)}{\left(1-\frac{b}{\phi}\right)}(\hat{A}^R_{T,t} - \hat{A}^R_{N,t}) - \frac{1}{\left(1-\frac{b}{\phi}\right)}(\hat{\mu}^R_{T,t} - \hat{\mu}^R_{N,t}) \tag{4.12}$$

$$\begin{aligned}\left[1+\left(\eta+\frac{1}{\phi}\right)\theta\gamma-\frac{\theta}{\phi}\right]\hat{T}_t &= \left(\eta+\frac{1}{\phi}\right)b(1-\gamma)(\ln P_{N,t}/P^*_{N,t}) + \left(1-\frac{1}{\phi}\right)\hat{A}^R_T \\ &\quad -\hat{\mu}^R_T + \left(\eta+\frac{1}{\phi}\right)[\gamma\hat{A}^R_T + (1-\gamma)\hat{A}^R_N] \\ &\quad -\left(\eta+\frac{1}{\phi}\right)(1-\gamma)\hat{G}^R_t \end{aligned} \tag{4.13}$$

where $\hat{T}_t \equiv \ln P_{F,t}/P_{H,t}$ and where $b \equiv \varphi\gamma + (1-\gamma)\,\rho^{-1}$; η–the inverse of the elasticity of labor supply– is defined as $\eta \equiv \bar{V}_{ll}\bar{L}/\bar{V}_l$, ρ–the risk-aversion coefficient– is given by $\rho \equiv \bar{U}_{cc}\bar{C}/\bar{U}_c$ and an index R denotes the difference between the home and the foreign respective variable. Under perfect labor mobility, $\phi \to \infty$, we obtain, by using (4.12) and (4.13), that

$$\begin{aligned}\ln P_{N,t}/P^*_{N,t} &= \frac{(\theta-1)\gamma\eta}{(1+\theta\gamma\eta+\eta b(1-\gamma))}\hat{A}^R_{T,t} - \frac{[1+\theta\gamma\eta+\eta(1-\gamma)]}{(1+\theta\gamma\eta+\eta b(1-\gamma))}\hat{A}^R_{N,t} + \\ &\quad -\frac{\theta\gamma\eta}{(1+\theta\gamma\eta+\eta b(1-\gamma))}\hat{\mu}^R_{T,t} + \frac{\eta(1-\gamma)}{(1+\theta\gamma\eta+\eta b(1-\gamma))}\hat{G}^R_t \\ &\quad +\frac{(1+\theta\gamma\eta)}{(1+\theta\gamma\eta+\eta b(1-\gamma))}\hat{\mu}^R_{N,t},\end{aligned}$$

[15] De Gregorio et al. (1994) find evidence that demand shocks can be important in explaining deviations from PPP. As well, Froot and Rogoff (1991) find that government spending can be an important explanatory variable for real exchange rate movements of several European countries in the EMS.

which shows that when the intratemporal elasticity of substitution, θ, is close to one then an increase in the productivity in the home traded sector does not increase in a significant way the non-traded good price and thus the consumer price in the home economy. Contrary to the Balassa-Samuelson result, a balanced productivity shock in all the sectors within a country creates a major reduction in the domestic consumer price. A relative increase in government spending in the domestic economy increases price differentials across countries. The reason is that government spending falls on non-traded goods and induces an increase in their prices. Mark-up shocks in the non-traded sector increase price differential; the opposite happens if they originate in the traded sector. A balanced mark-up shock in the domestic economy increases the price differential. In the limiting case in which domestic and foreign traded goods become perfectly substitutable (as in the Balassa-Samuelson hypothesis), i.e. θ goes to infinity, then the above solution collapses to

$$\ln P_{N,t}/P^*_{N,t} = \hat{A}^R_{T,t} - \hat{A}^R_{N,t} + \hat{\mu}^R_{N,t} - \hat{\mu}^R_{T,t}$$

which captures now the Balassa-Samuelson result augmented with mark-up shocks. Note that in this limiting case, demand shocks and balanced shocks do not matter.[16] Our results on the role of the elasticity of substitution between domestic and foreign traded goods in driving the Balassa-Samuelson effect are consistent with the findings of Duarte and Wolman (2003).

Another illustrative case is when $\varphi = \theta = \rho = 1$. Under this assumption

$$\begin{aligned}\ln P_{N,t}/P^*_{N,t} \quad = \quad & -\frac{(\phi\eta+1)\,\gamma}{(\phi-1)\,(1+\eta)}\hat{\mu}^R_{T,t} - \hat{A}^R_{N,t} + \frac{(1+\phi\eta)(\phi(1-\gamma)-1)-(\phi-1)}{(\phi-1)\phi(1+\eta)}\hat{G}^R_t \\ & +\frac{[\phi+(\phi\eta+1)\,\gamma-1]}{(\phi-1)\,(1+\eta)}\hat{\mu}^R_{N,t}\end{aligned}$$

in which the Balassa-Samuelson effect is absent and where we note that values of ϕ below one can change the sign of some of the responses as well as the magnitude.

We now go back to our more general model and calibrate it in order study which of the shocks in the model can be more relevant to explain price differential when prices

[16] As discussed in Froot and Rogoff (1995), balanced productivity shocks can matter when labor intensity differs across sectors.

are flexible.

Table 1: Calibration of the parameters

$\beta = 0.99$	Intertemporal discount factor in consumer preferences
$\rho = -\bar{U}_{cc}\bar{C}/\bar{U}_c = 2$	Risk aversion coefficient in consumer preferences
$\varphi = 0.44$	Elas. of substitution between traded and non-traded goods
$\theta = 1.5$	Elas. of substitution between domestic and foreign traded goods
$\sigma = 7.88$	Elas. of substitution across goods within a sector
$\gamma = 0.5$	Share of traded goods in the consumption bundle
$\eta = \bar{V}_{ll}\bar{L}/\bar{V}_l = 0.25$	Inverse of the Frisch elasticity of labor supply
$\lambda = \bar{f}'\bar{l}/\bar{f} = 0.75$	Labor share
$1 - \tilde{\lambda} = -\bar{f}''\bar{l}/\bar{f}' = 0.25$	Curvature of the production function

Table 1 presents the calibration of the parameters. The coefficient of risk aversion in consumer preferences is set to 2 as in Stockman and Tesar (1995) to get an intertemporal elasticity of substitution equal to 0.5. From Stockman and Tesar (1995), we borrow also the elasticity between traded and non-traded goods, $\varphi = 0.44$, and the share of traded goods in the consumption basket, $\gamma = 0.5$. The intratemporal elasticity of substitution between home and foreign traded goods is set such that $\theta = 1.5$ as in Backus et al. (1995). Consistent with several microeconomic studies, the Frisch elasticity of labor supply, $1/\eta$, is set to 4 and the labor share is set such that $\lambda = 0.75$. The discount factor β is assumed to be 0.99 and the elasticity of substitution for goods within a sector, σ, is set to 7.88 to imply a 15% mark-up as in Rotemberg and Woodford (1997). In Table 2, we present the results. We are not calibrating the parameter ϕ which measures the degree of labor mobility across sectors. We discuss three possible cases, $\phi = \infty$ to capture the perfect labor mobility case, an intermediate case of $\phi = 4.5$ and a case of low elasticity below 1, $\phi = 0.5$. In our model, when prices are flexible, the real variables of the model inherits the stochastic properties of the shocks. There is no intrinsic persistence in the model.[17] Our table can then capture short- and long-run components of consumer price differentials. In particular in table 2, we analyze the response of the consumer price differential and the terms of trade to a 1% increase of the shocks that are in the model. We also describe the response of terms of trade to capture the dimension through which movements in terms of trade absorb the usual Balassa-Samuelson effect. And indeed, focusing on the perfect labor mobility case, we see that a 1% increase in the traded-sector productivity in the home country does increase the price differential only of 2 basis point, consistent with the results of Duarte and Wolman (2003). The terms of trade worsen to absorb most of the shock. Instead an increase in productivity in the non-traded sector and a balanced increase in productivity in the domestic economy lower the domestic consumer price with respect to the foreign. In the same way, terms-of-trade movements absorb mark-up shocks in the traded-sector with negligible spillovers on consumer prices, while mark-up shocks in the non-traded sector produce increases in prices. Government-purchase shocks that affect demand of non-traded goods increase the non-tradeable goods price and thus the overall consumer price level. With intermediate values of the labor-mobility parameter, the picture does no change much. On the opposite, when ϕ goes below the unitary value, most of the responses are amplified and some change sign. Most important, the impact of government-spending shocks is of larger magnitude.

[17] By assuming incomplete financial market is instead possible to introduce persistent behavior of the price differential through a unit root, as shown in Rogoff (1992).

Table 2: Price Differential and Terms of Trade under Flexible-Price

	$\phi \to \infty$		$\phi = 4.5$		$\phi = 0.5$	
1% shock to:	$\ln P_t/P_t^*$	$\hat{T}_t$	$\ln P_t/P_t^*$	$\hat{T}_t$	$\ln P_t/P_t^*$	$\hat{T}_t$
$\hat{A}_T$	0.02	0.85	0.04	0.89	-0.21	0.55
$\hat{A}_N$	-0.60	0.04	-0.55	0.09	-0.98	-0.46
$\hat{A}_T$ and $\hat{A}_N$	-0.58	0.89	-0.51	0.98	-1.19	0.09
$\hat{\mu}_T$	-0.05	-0.57	-0.13	-0.67	0.64	0.35
$\hat{\mu}_N$	0.40	0.03	0.44	0.08	0.07	-0.40
$\hat{\mu}_T$ and $\hat{\mu}_N$	0.35	-0.54	0.31	-0.59	0.71	-0.05
$\hat{G}_t$	0.19	-0.07	0.11	-0.18	0.91	0.86

5 Nominal rigidities and inflation differentials

The flexible-price model provides useful benchmark. However, it has also obvious limitations. First, price differentials under flexible prices can arise only if there are asymmetric shocks across countries. This result hinges on the fact that we have imposed symmetry on the steady state of the model. Allowing for asymmetries in the steady-state position of countries, like for example different labor shares, would imply that balanced shocks to the whole area would have an effect on the price differential. Second, under flexible prices persistence stems only from the intrinsic persistence of the exogenous shock process, although adding capital accumulation and incomplete financial markets would create additional persistence. Finally, there is no role for monetary policy to affect price differentials.

We show that introducing sticky prices in the model can account for these three deficiencies: balanced shocks in the whole area create price and inflation dispersion; the model generates persistence; and monetary policy will matter for the determination of price dispersion and its degree of persistence. These three aspects of the model are shown below, after introducing the model's log-linear equations under sticky prices.

In a log-linear approximations to the structural equilibrium conditions (in particular equations (3.8), (3.9), (3.10) and (3.11) and the respective conditions for the foreign country) we obtain four aggregate supply (AS) equations for the respective traded and non-traded sector in each economy. These AS equations are familiar forms to the standard New-Keynesian AS equations of the closed-economy models of Galí and Gertler (1999) and Sbordone (2001). The inflation rate in each sector will depend on the real marginal cost in the sector and on the discounted value of the expected future sectorial inflation rate as follows

$$\pi_{H,t} = k_T(\widehat{mc}_{T,t}) + \beta E_t \pi_{H,t+1}$$

$$\pi_{N,t} = k_N(\widehat{mc}_{N,t}) + \beta E_t \pi_{N,t+1}$$

$$\pi^*_{F,t} = k^*_T(\widehat{mc}^*_{T,t}) + \beta E_t \pi^*_{F,t+1}$$

$$\pi^*_{N,t} = k^*_N(\widehat{mc}^*_{N,t}) + \beta E_t \pi^*_{N,t+1}$$

where we have defined $\pi_{H,t} = \ln P_{H,t}/P_{H,t-1}$, $\pi_{N,t} = \ln P_{N,t}/P_{N,t-1}$, $\pi^*_{F,t} = \ln P_{F,t}/P_{F,t-1}$, $\pi^*_{N,t} = \ln P^*_{N,t}/P^*_{N,t-1}$ and $k_j \equiv (1-\alpha_j)(1-\alpha_j\beta)/(\alpha_j(1+\vartheta\sigma\lambda^{-1}))$ where α_j can assume different values across sectors and countries (generically we have α_T, α_N, α^*_T, α^*_N) and $\vartheta \equiv 1 - \tilde{\lambda} - \upsilon^{-1}$. Moreover the deviations of real marginal costs from the steady-state are given by

$$\widehat{mc}_{T,t} = \mu_{T,t} + (1-\gamma)\hat{T}_{N,t} + \frac{1}{2}\hat{T}_t + \eta\hat{L}_t + \rho\hat{C}_t - \frac{1}{\phi}(\hat{l}_{T,t} - \hat{L}_t) - \hat{A}_{T,t} + (1-\tilde{\lambda})\hat{l}_{T,t},$$

$$\widehat{mc}_{N,t} = \mu_{N,t} - \gamma\hat{T}_{N,t} + \eta\hat{L}_t + \rho\hat{C}_t - \frac{1}{\phi}(\hat{l}_{N,t} - \hat{L}_t) - \hat{A}_{N,t} + (1-\tilde{\lambda})\hat{l}_{N,t},$$

$$\widehat{mc}^*_{T,t} = \mu^*_{T,t} + (1-\gamma)\hat{T}^*_{N,t} - \frac{1}{2}\hat{T}_t + \eta\hat{L}^*_t + \rho\hat{C}^*_t - \frac{1}{\phi}(\hat{l}^*_{T,t} - \hat{L}^*_t) - \hat{A}^*_{T,t} + (1-\tilde{\lambda})\hat{l}^*_{T,t},$$

$$\widehat{mc}^*_{N,t} = \mu^*_{N,t} - \gamma\hat{T}^*_{N,t} + \eta\hat{L}^*_t + \rho\hat{C}^*_t - \frac{1}{\phi}(\hat{l}^*_{N,t} - \hat{L}^*_t) - \hat{A}^*_{N,t} + (1-\tilde{\lambda})\hat{l}^*_{N,t}.$$

We can manipulate the above aggregate supply equations to obtain the following form

$$\pi_{H,t} = k_H[\delta_{T,1}\hat{T}_{N,t} + \delta_{T,2}\hat{T}_t + \delta_{T,3}\hat{T}^R_{N,t} + (\tilde{\eta}+\rho)\hat{Y}^w_t + \delta'_{T,\xi}\xi_t] + \beta E_t\pi_{H,t+1}, \tag{5.14}$$

$$\pi_{N,t} = k_N[\delta_{N,1}\hat{T}_{N,t} + \delta_{N,2}\hat{T}_t + \delta_{N,3}\hat{T}^R_{N,t} + (\tilde{\eta}+\rho)\hat{Y}^w_t + \delta'_{N,\xi}\xi_t] + \beta E_t\pi_{N,t+1}, \tag{5.15}$$

$$\pi^*_{F,t} = k^*_F[\delta^*_{T,1}\hat{T}^*_{N,t} + \delta^*_{T,2}\hat{T}_t + \delta^*_{T,3}\hat{T}^R_{N,t} + (\tilde{\eta}+\rho)\hat{Y}^w_t + \delta^{*\prime}_{T,\xi}\xi_t] + \beta E_t\pi^*_{F,t+1}, \tag{5.16}$$

$$\pi^*_{N,t} = k^*_N[\delta^*_{N,1}\hat{T}^*_{N,t} + \delta^*_{N,2}\hat{T}_t + \delta^*_{N,3}\hat{T}^R_{N,t} + (\tilde{\eta}+\rho)\hat{Y}^w_t + \delta^{*\prime}_{N,\xi}\xi_t] + \beta E_t\pi^*_{N,t+1}, \tag{5.17}$$

where the coefficients δ are functions of the parameters of the model and ξ_t is a vector that includes the log-deviation from the steady state of all the shocks that are in the model. Moreover we note that

$$\hat{T}_{N,t} = \hat{T}_{N,t-1} + \pi_{N,t} - (\frac{1}{2}\pi_{H,t} + \frac{1}{2}\pi^*_{F,t}) \tag{5.18}$$

$$\hat{T}_t = \hat{T}_{t-1} + \pi_{F,t} - \pi_{H,t} \tag{5.19}$$

$$\hat{T}_{N,t} - \hat{T}^*_{N,t} = (\hat{T}_{N,t-1} - \hat{T}^*_{N,t-1}) + \pi_{N,t} - \pi^*_{N,t} \tag{5.20}$$

5.1 Monetary policy shocks and price dispersion

Our first objective is to explore the importance of monetary policy shock in explaining consumer-price differentials. There is some evidence in the VAR literature that monetary policy shocks can be an important source of movements in the real exchange rate as discussed in Rogers (1999).[18] We analyze the following issue. Consider the common monetary policymaker in the currency area that moves its instrument of policy in a discretionary way, how much price and inflation dispersion is going to be generated? In our context, we interpret a monetary policy shock as it is usually done in the VAR literature. We consider a one-time negative shock to the instrument of policy, the short-term nominal interest rate. This shock produces usually an hump-shaped positive response of output. We use the system of equations (5.14) to (5.20) to determine the responses of relative prices and inflation rates to this hump-shaped positive response of output.[19] To address this issue, we can then write (5.14) to (5.20) as a system of the form

$$E_t \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix} \begin{bmatrix} k_{t+1} \\ k_t \end{bmatrix} = \begin{bmatrix} B_1 & B_2 \\ B_3 & B_4 \end{bmatrix} \begin{bmatrix} k_t \\ k_{t-1} \end{bmatrix} + \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} F_t$$

where $k_t = [\hat{T}_{N,t} - \hat{T}^*_{N,t} \;\; \hat{T}_t \;\; \hat{T}_{N,t}]$ and $F_t = [\hat{Y}^w_t \;\xi_t]$ and A_j, B_j, C_j are matrices. Provided there are 3 stable eigenvalues of the above system, we can solve it obtaining the law of motion of the state variables as

$$k_t = -V_1^{-1}V_2 k_{t-1} - V_1^{-1}\Lambda^{-1}E_t \sum_{T=t}^{\infty} \Lambda^{-(T-t)} V\tilde{C}F_T \tag{5.21}$$

where $V = [V_1 \; V_2]$ is the matrix of left eigenvectors associated with the unstable eigenvalues, where V_1 and V_2 are 3 by 3 matrices, Λ is a 3 by 3 diagonal matrices which

[18] See also Benigno (2004) and Chari et al. (2002) on the issue whether a model with price rigidities and local currency pricing can account for monetary policy shocks to be important in capturing the volatility and persistence of real exchange rate

[19] This analysis is similar to Woodford (2003)'s analysis of the persistence of real wages after monetary policy shocks in a model with sticky wages and prices.

contains the unstable eigenvalues and $\tilde{C} \equiv A^{-1}C$. In particular in the vector k_t we are interested in the variable $\hat{T}_{N,t} - \hat{T}^*_{N,t}$ which is proportional to the consumer price differential $\ln P_t/P_t^*$. The eigenvalues of the matrix $V_1^{-1}V_2$ characterize the intrinsic persistence due to the stickiness of prices and intersectorial relative prices. In particular, under the assumption of symmetry in the rigidity across all the sectors in the area ($\alpha_T = \alpha_N = \alpha_T^* = \alpha_N^*$), a shock to the area output $\hat{Y}_t^w$ does not produce any consumer price dispersion. Asymmetries in the structures of the AS equations, across countries and sectors, are critical for monetary policy to have a role in creating price dispersion. This is at the same time good and bad news, since it means that a monetary policy shock, in the way we interpreted it, does not create any consumer price dispersion but on the other side monetary policy is limited in its role to correct any unwanted price dispersion that can arise following other disturbances. In our experiment, we feed in (5.21) the estimated output response to a monetary shock found by Smets and Wouters (2003).[20] As shown in the bottom chart of Figure 11, a discretionary decrease in the interest rate produces an hump-shaped response of output. Output in the union increases up to 0.4% after 5 quarters and then converges back to the steady state. To perform this experiment, we need to calibrate additional parameters on top of the ones used in Table 1. We set all the elasticities in the aggregators of labor to infinity to get perfect labor mobility across sectors within a country. As already mentioned it is crucial to calibrate the degrees of rigidities. We refer here to some micro and macro studies. On the micro side, Le Bihan and Sevestre (2004), by analyzing CPI micro-data for France, find that the average duration between price adjustment in service sector is 9.66 months while for foods and goods is around 4 to 5 months. Costa Dias and Neves (2003) analyze consumer prices for Portugal and find relatively short "live" of posted prices, around 3 times a year on average with higher duration for services. Stahl (2004) shows that for producer prices in manufacturing sector the average duration can be of 9 months. On the opposite, there are studies that estimate AS equations for European countries, as Galí et al. (2001) and Benigno and Lopez-Salido (2002). They find that reasonable estimates of α for a country can be around 0.78.

In this work, we assume that one country has high rigidity and in particular a rigidity in the traded sector as in the work of Stahl (2004) equal to 0.67 (a duration of nine months) and a correspondingly rigidity in the non-traded sector such that $\alpha_N = 0.84$ so that the overall rigidity in this country is around 0.78. In the other country, we assume that the average duration in the traded sector is in line with the work of Costa Dias and Neves, and equal to 5 months and that in the non-traded sector is as in Le Bihan and

[20] However, the model estimated by Smets and Wouters (2003) does not include the possibility of heterogeneity across countries.

Sevestre (2004) equal to 9 months. Our benchmark I calibration is then: $\alpha_T = 0.67$, $\alpha_N = 0.84$, $\alpha_T^* = 0.37$, $\alpha_N^* = 0.67$. We also propose two other possible cases: in benchmark II, we assume $\alpha_T = 0.67$, $\alpha_N = 0.84$, $\alpha_T^* = 0.6$, $\alpha_N^* = 0.75$, in benchmark III we assume $\alpha_T = 0.7$, $\alpha_N = 0.4$, $\alpha_T^* = 0.67$, $\alpha_N^* = 0.37$. In benchmark II and III with respect to I the two countries are more similar in terms of rigidities; in benchmark II the rigidities are increased in both sectors of country F, in benchmark III they are reduced in both sectors of country H.

The results are presented in the first three charts of Figure 11, where the impulse responses of the log of the consumer-price differential and of the CPI inflation rates in both countries are plotted following the output shock. The impulse responses of the inflation rates are also consistent with VAR responses to a discretionary monetary policy shock. However, CPI inflation rate increases by a larger amount in the more flexible-price economy, but it is more persistent in the rigid economy. The overall picture that emerges is that the consumer-price differential is highly persistent. In the benchmark I case, the price differential has a peak after 10 quarters (i.e. after 5 quarters the peak of output). The differential reaches 8 basis point. This differential is of much smaller magnitude in the other two benchmarks, however the persistence is not altered by the different assumption. Overall a 'discretionary' monetary shock produces persistent price and inflation differentials although only of a small magnitude.

5.2 Nominal rigidities and inflation dispersion

Under flexible prices, consumer-price differentials inherit the stochastic properties of the shocks. There is no persistence other than the one implied by the shocks. We now investigate how the introduction of sticky prices affects the response to the shocks and whether it increases persistence.

The answer depends on the policy rule used by the common monetary policymaker as we will further investigate in the next section. In this section, we assume that the policymaker sets its policy in a way to target a weighted average of the CPI inflation rates of the two countries, with weights given by the economic size of each country, in our case 1/2 for each country. The monetary policymaker then sets the average CPI inflation rate in the area equal to zero as

$$\frac{1}{2}\pi_t + \frac{1}{2}\pi_t^* = 0$$

at all times. The second important assumption in this experiment is the one on the persistence of the shocks. We assume that there are three possible degrees of persistence.

Indeed we assume that shocks are autoregressive of type AR(1) as

$$s_t = \rho s_{t-1} + \varepsilon_t$$

for a generic shock s_t where ρ can be either 0 (for a white-noise process), or 0.9 for a persistent process or 1 for a unit-root process.[21] We analyze impulse response functions to a one-time unitary increase (1% movement) in ε_t. The analysis of the response to a disturbance with zero correlation will be helpful to capture the persistence implicit in the model given by the combination of the stickiness of prices and policy rule; the analysis of a more persistent process will be useful to understand how the persistence of the shock interacts with the persistence intrinsic in the model. Finally the analysis to permanent shocks can allow to investigate whether sticky prices are adding important transitional dynamics. In particular, the long-run response to a permanent shock captures exactly the flexible-price response, with the important caveat that short- and long-run are the same under flexible prices. Having in the same graph the flexible-price response would serve as an important benchmark for comparison.

Figure 12 presents the impulse responses of the consumer-price differential to productivity shocks. In the order (from the left to the right starting from the top), we analyze a one-time shock to the productivity in the home traded sector, to the productivity of the home non-traded sector, to a balanced increase in the productivity in both sectors in the home economy, to a balanced productivity increase in all the sectors of the area. As already discussed, a first presumption is that the stickiness of prices and, in particular, of relative prices can add persistence to the shock. And indeed this is the case when we focus on a white-noise shock. Under flexible prices the effect of this shock would disappear after one period, under sticky prices this lasts for more or less 8 quarters. The second presumption is that sticky prices, while adding persistence, dampen the response to the shocks. Interestingly, this is not necessarily a feature of our model. Indeed, by inspecting the impulse response to a productivity shock in the home traded sector, we see that actually sticky prices can revert the sign of the response and sometimes magnify it. Indeed, with our parametrization, the flexible-price response (as in Table 2 and in the long-run response to a permanent shock) would require $\ln P/P^*$ to increase by 2 basis points. With sticky prices, we actually get a complete reversal of the Balassa-Samuelson effect with a relative magnification of the response which can achieve 8 basis points when shocks are more persistent. The intuition for this result is that even the terms of trade is very slow in the adjustment when sectorial prices are sticky. We note that under a more persistent shock, the response changes sign after 20 quarters. Indeed, when the shock is

[21] In the case of a unit-root process, we are allowed to use our log-linear approximation provided the shock lasts for a finite period of time.

permanent it should reach the long-run positive value of 2 basis points. The picture is different when we analyze a shock to productivity in the non-traded sector or a balanced productivity shock in the home economy. These two cases are similar since the shock in the non-traded sector dominates. Indeed, we find that the existence of sticky prices mutes the response to these shocks. In particular the short-run response is negative and below or around -0.1% compared to -0.6% of the flexible-price model. The adjustment towards the long-run equilibrium proceeds slowly as in the previous case. A further interesting question to address is whether area-wide shocks can have important consequences. We remind that under flexible price these shocks have no effects on price differentials. We find that in the case of a balanced area-wide productivity shock, the deviations from full equalizations of the consumer price level are quantitatively insignificant.[22]

Figure 13 repeats the same analysis following mark-up shocks. The analysis is parallel to the case of productivity shocks with the appropriate qualifications. We find that there is a change in the sign of the short-run response for shocks in the traded sector, a dampening effect for non-traded and balanced shocks and the non-significant effect of area-wide mark-up shock.

Finally in Figure 14, we focus on demand shocks. Namely, we consider a one-time shock to government purchase in the home country and then a common shock to both countries. The response to an asymmetric government purchase shock shows again the dampening effect of sticky prices as well as the additional source of inertia implied. Most interesting, we find that an area-wide government purchase shock can have some non-negligible effect on price differential. Given our parametrization, a symmetric government-purchase shock lowers the consumer price in the home economy relative to the foreign, since the home economy is more rigid. Indeed, a demand shock increases prices by a larger amount in the more flexible-price country. This is why consumer prices in the home economy fall below foreign. As a difference from other area-wide shock, government-purchase shocks affect only the non-traded sector, so it is not surprising that they can create larger asymmetric responses of prices, even if they are common to both countries.

6 Optimal monetary policy, alternative rules and price dispersion

In the previous section we have shown that a one-time monetary shock produces persistent but small price and inflation differentials. This section addresses the question of

[22] The symmetry imposed in the steady-state obviously plays an important role in this result.

how the systematic component of the common monetary policy affects price dispersion and its persistence in response to the various shocks. We indeed compare the outcomes under different targeting and interest rate rules with the one that would be achieved under optimal policy. In this case, we consider that policy is chosen optimally to maximize a weighted average of the welfare of the two regions with equal weights as

$$W_t = \mathrm{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \left[\frac{1}{2} U(C_s) + \frac{1}{2} U(C_s^*) - \frac{1}{2} V(L_s) - \frac{1}{2} V(L_s^*) \right] \qquad (6.22)$$

under all the relevant structural equations of the model.[23] We consider that the monetary policymaker is committed to maximize (6.22) at time t_0 under appropriate additional constraints given by a 'timeless perspective' commitment as discussed in Benigno and Woodford (2004a). These constraints are necessary to characterize the optimal policy problem in a recursive way and to obtain stationary policy rules. We aim to analyze the optimal allocation up to a first-order approximation to the solution of the non-linear problem. As shown in Benigno and Woodford (2004b), this can be seen as the solution of an equivalent problem in which a quadratic objective function is maximized under linear constraints – what is called a linear-quadratic (LQ) solution. Given the large dimension of our optimal policy problem, we use a Matlab-coded version of this LQ solution given by Altissimo et al. (2004). One of the advantage of using this LQ approach is that it will deliver a quadratic objective function through which we can make welfare comparisons.

We compare the optimal policy with other four monetary policy rules as described in the following table.

Table 3: Monetary policy rules

(1)	$\pi_t^W = \frac{1}{2}\pi_t + \frac{1}{2}\pi_t^* = 0$
(2)	$r_t = 1.5\ \pi_t^W + 0.5\ y_t^W$
(3)	$r_t = \pi_t^W + 0.5\ y_t^W$
(4)	$\pi_t = \pi_t^*$

Rule 1 stabilizes a weighted average of the CPI inflation rate of the two countries, as in the previous section. Rule 2 is the standard Taylor rule and Rule 3 is a Taylor rule with a higher relative weight on output. Finally, Rule 4 considers an extreme case in which the CPI inflation differential across the two countries is closed to zero at all times.

[23] To simplify the analysis, we consider that all the monopolistic distortions are offset in the steady-state by appropriate taxation subsidies. However, we allow mark-ups to have small perturbations around this steady state.

Figures 15-17 show the impulse responses of the CPI inflation differential, the area-wide CPI inflation (a weighted average of the CPI inflation rates of the two countries) and of the area-wide output to: *i*) a shock to the productivity of the region $H's$ traded sector (Figure 15); *ii*) a shock to the productivity of the region $H's$ non-traded sector (Figure 16); *iii*) a shock to region $H's$ government purchases (Figure 17), respectively. We assume that the shocks are $AR(1)$ with correlation coefficient equal to 0.8.

One feature that is common to all Figures 15-17 is that the optimal rule tolerates sizeable and persistent inflation differentials, in particular for the two productivity shocks. The inflation differential is of the same sign and comparable in absolute value with the one produced by other rules for both productivity shocks. It is also broadly as persistent as under all the alternative rules for the two productivity shocks. By contrast, for the case of government shocks, the optimal rule generates a differential of smaller size than for most of the other rules. Overall, the comparison of Figures 15-17 shows a high degree of alignment between the impulse responses under optimal policy and under the rule ensuring price stability, particularly as regards the responses under these two rules of inflation differentials and aggregate inflation.

It appears that permitting CPI inflation rates across the two countries to differ in a persistent way helps to stabilize overall inflation in the union as a whole. For productivity shocks specific to the traded sector the departure of inflation from the steady state under the optimal rule is relatively moderate. However the two Taylor rules and the policy that strictly closes the inflation differential are quite off in stabilizing the area inflation rate. Indeed, the short run is characterized by periods of deflation in which also the response of output is muted compared to the optimal policy. A similar conclusion can be reached for the case of the productivity shock specific to the non-traded sector, where the optimal rule performs well both in terms of inflation stabilization and allows output to increase significantly. Note that in this cases small variations in the inflation differentials across the various rules considered can produce large deviations in the area inflation and output This conclusion is shared more noticeably in the case of demand shocks, where the optimal policy allows to avoid any significant departure of inflation without producing any depressing effect on output.

7 Variance decomposition of the inflation differential

The previous section has shown that the optimal response to the shocks allows in general for persistent and sometimes sizeable inflation differentials. We are now interested in evaluating in which proportion each of the shock contributes to the overall variability of the inflation differential implied by our model. In order to perform the variance decomposition exercise, we need to specify and calibrate the stochastic properties of all the shocks considered in the model. While a fully fledged exercise would require the estimation of the whole model, here we resort to a simplifying short-cut and indeed assume that the productivity, the mark-up and the government shocks mimics the ones estimated by Smets and Wouters (2003) once aggregated across sectors and countries.

Starting from the productivity shock, we assume that each one of the four productivity shocks, $\{A_{T,t}, A_{N,t}, A^*_{T,t}, A^*_{N,t}\}$, follows independent $AR(1)$ process with lag coefficient of 0.823 and standard error of the innovation of 0.012; this ensures that the aggregate productivity, resulting from the weighted average of the countries/sectorial ones, exactly matches the process estimated by Smets and Wouters.

The shocks to government expenditure in each country are assumed to follow the same independent and very persistent $AR(1)$ process with coefficient on the lag term of 0.949 and standard deviation of the innovation of 0.0046.

Finally, for the mark-up shocks, we impose that they are common between tradable and non-tradable sectors in each economy and we also assume the same process for both economies but uncorrelated across economies. Furthermore, we impose that mark-ups are white noises with a standard deviation of 0.0066, in order to match the overall variability of the price and wage mark-up shocks of Smets and Wouters.

While the present calibration is admittedly ad hoc, it nonetheless provides some hints on the way in which different shocks contribute to the dynamic decomposition of the variance. Moreover, it allows to compare the decomposition for alternative monetary regimes. In this work, we consider the decomposition given by the optimal policy with respect to the first Taylor rule in Table 3.

Tables 4 and 5 present the variance decomposition for the cases of an optimal policy and Taylor rule, respectively. In each table, we show the forecast error variance in the short and long run – the latter coincides with the unconditional variance– for several variables of the model. In particular the first column considers the CPI inflation differential. We also present the fraction of the total variance to which each shock contributes for each of the respective variables.

Table 4: Variance decomposition under optimal policy

		$\pi - \pi^*$	$\frac{P_H}{P_F}$	$\frac{P_N}{P_N^*}$	$Y - Y^*$	$\frac{W}{P}$	$\frac{W^*}{P^*}$
Short Run	Forecast error variance	0.0016	0.0110	0.0032	0.0112	0.0071	0.0069
	$\hat{A}_T$	0.03	0.33	0.03	0.31	0.14	0.19
	$\hat{A}_N$	0.26	0.02	0.26	0.04	0.02	0.06
	$\hat{A}_T^*$	0.02	0.62	0.02	0.55	0.46	0.37
Decomposition	$\hat{A}_N^*$	0.65	0.00	0.65	0.00	0.22	0.19
(fractions)	$\hat{\mu}$	0.00	0.01	0.00	0.02	0.02	0.01
	$\hat{\mu}^*$	0.04	0.02	0.04	0.06	0.14	0.18
	$\hat{G}$	0.00	0.00	0.00	0.01	0.00	0.00
	$\hat{G}^*$	0.00	0.00	0.00	0.00	0.00	0.00
Long Run	Forecast error variance	0.0020	0.021	0.0163	0.033	0.0117	0.0117
	$\hat{A}_T$	0.03	0.40	0.01	0.39	0.24	0.19
	$\hat{A}_N$	0.30	0.02	0.42	0.05	0.22	0.04
	$\hat{A}_T^*$	0.01	0.57	0.00	0.52	0.31	0.28
Decomposition	$\hat{A}_N^*$	0.62	0.00	0.57	0.02	0.17	0.41
(fractions)	$\hat{\mu}$	0.00	0.00	0.00	0.00	0.01	0.01
	$\hat{\mu}^*$	0.03	0.01	0.00	0.01	0.05	0.06
	$\hat{G}$	0.00	0.00	0.00	0.01	0.00	0.00
	$\hat{G}^*$	0.00	0.00	0.00	0.01	0.00	0.00

Table 5: Variance decomposition under Taylor rule

		$\pi-\pi^*$	$\frac{P_H}{P_F}$	$\frac{P_N}{P_N^*}$	$Y-Y^*$	$\frac{W}{P}$	$\frac{W^*}{P^*}$
Short Run	Forecast error variance	0.0016	0.0100	0.0035	0.0117	0.0038	0.0041
	$\hat{A}_T$	0.00	0.22	0.00	0.19	0.08	0.19
	$\hat{A}_N$	0.12	0.02	0.12	0.01	0.00	0.03
	$\hat{A}_T^*$	0.00	0.73	0.00	0.66	0.72	0.53
Decomposition	$\hat{A}_N^*$	0.83	0.00	0.83	0.02	0.15	0.16
(fractions)	$\hat{\mu}$	0.00	0.00	0.00	0.01	0.01	0.00
	$\hat{\mu}^*$	0.05	0.04	0.05	0.01	0.03	0.09
	$\hat{G}$	0.00	0.00	0.00	0.01	0.00	0.00
	$\hat{G}^*$	0.00	0.00	0.00	0.01	0.00	0.00
Long Run	Forecast error variance	0.0022	0.0022	0.0182	0.0344	0.0056	0.0067
	$\hat{A}_T$	0.01	0.31	0.02	0.26	0.13	0.16
	$\hat{A}_N$	0.13	0.00	0.17	0.02	0.15	0.01
	$\hat{A}_T^*$	0.01	0.66	0.03	0.63	0.43	0.31
Decomposition	$\hat{A}_N^*$	0.81	0.01	0.78	0.06	0.24	0.49
(fractions)	$\hat{\mu}$	0.00	0.00	0.00	0.00	0.00	0.00
	$\hat{\mu}^*$	0.03	0.02	0.00	0.01	0.02	0.02
	$\hat{G}$	0.00	0.00	0.00	0.01	0.00	0.00
	$\hat{G}^*$	0.00	0.00	0.00	0.01	0.01	0.00

Focusing on the CPI inflation differential we note that the long-run variance under optimal policy is equal to 0.0020 which implies a standard deviation of 0.44% quarterly which roughly corresponds to a standard deviation of 1.38% yearly.[24] These numbers compare well with the empirical evidence for the euro area. Interestingly, under the sub-optimal Taylor rule the yearly standard deviation is slightly higher and equal to 1.45%. The important difference between the two policies is in how the shocks contribute to these variances. First we notice that productivity shocks matter for most of the total variability and in particular 90% of this variability is explained by the productivity shocks in the non-traded sectors. The striking difference between optimal and sub-optimal policies is in the importance of foreign versus domestic non-traded productivity shocks. Compared to the sub-optimal rule, optimal policy gives higher relevance to the

[24] To convert on a yearly base we use a coefficient of 3.1 that takes into account of the autocorrelation of the series.

productivity in the non-traded sector of country H and dampens the importance of the respective productivity in the other country. These results can be intuited by noting that in the current experiment the only elements of asymmetries across the sectors and countries are indeed the degrees of nominal rigidities. In particular we are assuming that the sector with lowest degree of nominal rigidity is the non-traded sector in the foreign economy, whose productivity indeed contributes mostly to the inflation differential. The optimal monetary policy corrects for this importance. Even productivity shocks in the non-traded sector of region $H-$ which is the one with highest degree of rigidity– should matter and monetary policy should aim at this outcome.

Productivity shocks that affect the traded sector in general do not contribute much to the variability of the inflation differential since they are absorbed by terms of trade movements – indeed they are responsible for most of this volatility. Most important, they are the determinants of the variability of the output differential across countries. Productivity shocks are in general responsible for the variability of real wages too. Interestingly neither mark-up nor government-purchase shocks contribute significantly to the variability of the variables displayed in the table. For government-purchase shock the result is in line with Duarte and Wolman (2003).

8 Consequences of incorrect exchange rate parity

In this section, we consider the case of a country, H, that joins the union at a nominal exchange rate S_0 and real exchange rate $RER_0 = S_0 P_0^*/P_0$. We assume that immediately after joining the union, it is understood that country H has a overall productivity level higher and incompatible with the current real exchange rate. In particular we assume that the new equilibrium real exchange rate, given by $RER_\infty = S_\infty P_\infty^*/P_\infty$, should be depreciated by 1%.[25]

We compare the responses of the economy to this shock under three policy regimes. The first is the case of a currency area in which monetary policy is conducted optimally. The second considers again a currency area but in which monetary policy targets to zero the average CPI inflation of the area, i.e. $1/2 \cdot \pi_t + 1/2 \cdot \pi_t^* = 0$. The third regime is the case of economies with independent currencies in which policy is conducted optimally by the two separate monetary authorities in a way to maximize the cooperative objective function given by (6.22). We evaluate the welfare differences across these three regimes.

Joining a currency area represents a major change in the policy of a country, with respect indeed to the conduction of monetary policy. For the Lucas's critique, parame-

[25] To achieve this result we assume a permanent increase of 1/0.5127 % in the productivity of both sectors of country H.

ters in the reduced form of a model change when policy changes. However, dramatic changes in policy can have some effects on so called 'deep' parameters of a model. In our case, a monetary union can affect price and wage rigidities in a country, its degree of sectorial labor mobility, its trade pattern of goods and assets. As a first step, we keep all the structural parameters of the model constant. This is one of the reasons why this experiment is biased toward giving an upper bound on the costs of currency areas. There are other important reasons not developed here for this bias. Indeed, with independent currencies, monetary policy in both countries might not be necessarily conducted optimally and cooperatively. It can be the case that a currency area with an optimizing monetary authority can perform better than an economy with two separate monetary authorities acting in a non-cooperative way. Finally, there can be movements of the nominal exchange rate that are not directly related to the policy chosen or to the fundamental shocks of the model which can exacerbate the welfare costs when exchange rate is free to float.

Leaving aside these considerations, it is true that in our analysis the welfare under an optimally-designed currency area will be always lower than the welfare under an optimally-designed floating exchange rate regime since in the latter case there is an extra degree of freedom over which policymakers can optimize.

In our experiment, we continue to assume that the monopolistic distortions are completely offset by appropriate subsidies in all sectors. Given this assumption and our calibration, the optimal cooperative policy under floating exchange rates coincides with the flexible price allocation. In this allocation the new equilibrium is achieved immediately after the new level of productivity is known. Table 6 discusses the decomposition of the long-run adjustment in the real exchange rate between the CPI inflation differential and the exchange rate depreciation or appreciation. In the floating exchange rate regime the 1% depreciation of the real exchange rate is brought about by a 2% depreciation of the nominal exchange rate which feeds into an increase of region H's CPI prices of 0.5% (since the share of foreign traded good in the overall CPI basket is 1/4) and a proportional fall in foreign CPI prices of 0.5%. As shown in Figure 18 the optimal cooperative policy with independent currencies requires zero producer price inflation in all sectors and in both countries. This implies that GDP inflation remains as well zero.

On the opposite, in a currency area, all the adjustment should be brought about by movements in prices. First, we look at the optimal monetary policy. Country H, the one that experiences the favorable productivity shock, experiences a deflation in both the non-traded and traded sectors. Output increases in both sectors but slowly and below potential for at least 5 quarters. Country F instead benefits of an expansion and suffers of inflation in both of its sectors. The overall impact on CPI inflation is positive for both countries, but higher in country F. This is not surprising since foreign CPI should increase more than domestic CPI to achieve the new long-run equilibrium real exchange rate. Although country H experiences deflation and output below potential, its CPI increases because imported goods are now more expensive. Indeed, most of the adjustment is brought about by an increase in the prices of foreign traded goods. Overall, average CPI inflation in the area should show a positive sign for some quarters. In this case, optimal policy can be understood more simply as a strict target of the area CPI inflation over a medium horizon (around 6 quarters).

The real exchange rate moves slowly towards the new equilibrium level. CPI indices go up in both countries and more in country F, the opposite of what occurs under floating exchange rate – the difference being explained by indeed the movements of the nominal exchange rate.

Table 6: Decomposition of the long-run adjustment in the real exchange rate
$\Delta rer = \Delta s + \pi^* - \pi$

	Δs	π^*	π
Optimal Policy under Floating Regime	2%	-0.5%	0.5%
Optimal Policy in a Currency Area	0%	1.18%	0.18%
Area-CPI target in a Currency Area	0%	0.5%	-0.5%

We now consider the policy of strict targeting of the area CPI inflation in all periods. In this case CPI inflation cannot go up in both countries and should move instead specularly across the two countries. It follows that the adjustment of price differential to the new long-run equilibrium is slower. The deflation is stronger in both sectors of region H and output in these sectors is much more depressed and for longer periods below potential than under the optimal policy.

An important observation to draw from this example is that price differential at the consumer level may naturally arise as an equilibrium adjustment. However, it is not only important the dimension of the dispersion but also its dynamic. The fact that the same dispersion can be achieved with more adjustment of consumer prices of a country instead of another can have important welfare consequences. We study this issue more deeply.

We use our quadratic objective function to evaluate welfare differences between the three regimes. In particular we convert these differences in gains or losses of percentage of steady state consumption in an alternative economy in which consumption and labor are always constant. Our calculations show that under this experiment the costs of having the incorrect nominal and real exchange rates amounts to 0.093% of steady-state consumption when policy is conducted optimally and to 0.13% when policy follows the strict CPI inflation targeting procedure. These costs are two times higher than the ones Lucas (1987) found for business cycle movements. They are much more important if we consider that the experiment is built under the assumption of a one time –though permanent– shock that involves a misalignment of the real and nominal exchange rate of just 1%. By a parallel, a misalignment of 5% would imply costs around 0.5% and 0.65% of steady state consumption under optimal policy and strict CPI inflation targeting policy, respectively.

We now analyze an alternative scenario in which the currency area involves more flexibility in the pricing decisions of firms. We assume a drastic reduction in the parameters measuring the degree of rigidities, α, and set them all equal to 0.37 implying an average duration of price contracts just above 1 quarter for all sectors. Our analysis is facilitated by the fact that the optimal cooperative monetary policy under floating exchange rate is independent of the assumption on the degrees of price rigidities. We find that under this new calibration, the costs of a 1% misaligned real exchange rate amounts to 0.036% of steady state consumption. They are reduced by more than a half with respect to the previous parametrization. Greater flexibility in the goods market can mitigate in a substantial way the costs of joining the union at the wrong nominal exchange rate. These results are in line with the high cost of nominal inertia that Canzoneri *et al.* (2004) found in closed-economy versions of this model. Our model would further suggest that the consideration of the slow adjustment in the intersectorial relative prices, because of sticky prices, can amplify more these costs. Interestingly when all the sectorial degrees of rigidity are equalized the optimal policy coincides with the strict targeting of the area CPI inflation. It is indeed the case, by inspection of Figure 18 that another simple way through which optimal policy can be understood following this particular shock can be that of targeting a rigidity-adjusted weighted average of GDP inflation rates in the area which actually coincides with targeting average CPI inflation when all the sectors have the same degree of rigidity.[26]

[26] This result is similar to Benigno (2004), with the important qualification that his model considers only traded sector. As in Benigno (2004) the appropriate inflation target involves prices of the sectors with nominal rigidities.

9 Conclusions

This paper provides and empirical and a theoretical analysis of the factors accounting for inflation differentials in an heterogeneous currency union. Some implications for policy are also considered. This concluding Section discusses sketches some possible avenues for future work and points to the main caveats and limitations of the theoretical analysis.

One additional exercise of interest that could be conducted within the current framework would be to use the model to account for the component of inflation differentials that arise as a consequence of the interaction between common external shocks and the heterogeneity among euro area members in terms of their trade linkages with non-EMU partner countries. Honohan and Lane (2003, 2004) have emphasized the relevance of this source of inflation differentials in the euro area, showing that movements in national trade-weighted multilateral exchange rates are significantly correlated with national inflation rates. Even if our model relates to a currency area with no external trade, the previous channel of inflation differentials stemming from common external shocks could be proxied in our framework by balanced productivity shocks which interact with heterogeneous country structures.

There are at least two more important limitations to the framework developed in this paper. First, the assumption of perfect international risk-sharing - introduced for analytical convenience, closes down one potentially important source of inflation differentials. Second, the model assumes away investment. This precludes the analysis of another potentially important source of differential inflation dynamics. For instance, it has sometime been argued that persistent inflation in a region within a monetary union could generate procyclical real interest rates: with a common nominal interest rate, the high-inflation countries will have lower real interest rates than their low-inflation fellow members. This could in principle amplifies the local business cycle, endogenously generating inflation persistence and possibly local asset price booms. At the same time, a protracted positive inflation differential cumulates into real appreciation, which over time would offset the previous effect.

More generally, the framework developed in the paper could be extended to tackle the previous limitations. In addition, a number of further extensions could be of interest. First, including a role for generating inflation differentials from other sources, like the presence of international price discrimination in the tradables sector and the existence of non-traded distribution services in determining the consumer prices of tradables would increase realism in the model. Another potentially interesting route would be to allow for firm-level and industry-level heterogeneity in productivity and trade costs, which could generate further sources of price dispersion and persistence, through the endogenous entry of firms and shifts in the non-traded/traded margin, alike to effects present in Bergin and Glick (2004).

A further avenue for future work is to analyze the interdependence between the optimal area-wide monetary policy rule and optimal fiscal policies at the national level. Related to the latter, an interesting extension could be to look at 'optimal' indirect tax rate: an intensely debated policy issue in a number of euro area countries has been whether price increases in various sectors should be offset by endogenous adjustment in the level of indirect taxation. The level of direct taxation on labour has also been discussed as a policy instrument: a rise in demand for local labour can in part be met by encouraging greater labour force participation and increased work effort through a reduction in labour taxes.

References

[1] Alberola, E. (2000), Interpreting inflation differentials in the euro area, Banco de España Economic Bulletin, April: 61-70.

[2] Altissimo, F., V. Curdia and D. Rodriguez-Palenzuela (2004), "Linear-Quadratic optimal policy: computation and applications" mimeo, October 2004.

[3] Andrés, J., Ortega, E. and J. Vallés (2003), "Competition and Inflation Differentials in EMU," unpublished manuscript, Bank of Spain.

[4] Angeloni, I. and M. Ehrmann (2004), "Euro Area Inflation Differntials," unpublished manuscript, European Central Bank.

[5] Backus, D., P. Kehoe and F. Kydland (1995) "International Business Cycle: Theory and Evidence, " in Thomas F. Cooley (ed.) *Frontiers of Business Cycle Research,* Princeton: Princeton University Press, pp. 331-356

[6] Backus, D. and G. W. Smith (1993), "Consumption and Real Exchange Rates in Dynamic Economies with Non-traded Goods," *Journal of International Economics* 35, pp. 297-316.

[7] Balassa, B., (1964), "The Purchasing Power Parity Doctrine: A Reappraisal," *Journal of Political Economy,* 72, pp. 584-596.

[8] Beetsma, R. and H. Jensen (2004), "Monetary and Fiscal Policy Interactions in a Micro-founded Model of a Monetary Union," unpublished manuscript.

[9] Benigno, G. and C. Thoenissen (2003), "Equilibrium Exchange Rates and Supply-Side Performance," *The Economic Journal*, vol. 113, iss. 486, pp. 103-124(1).

[10] Benigno, G. (2004), "Real Exchange Rate Persistence and Monetary Policy Rules", *Journal of Monetary Economics*, Volume 51, Issue 3, pp. 473-502.

[11] Benigno, P. (2004), "Optimal Monetary Policy in a Currency Area," *Journal of International Economics,* forthcoming.

[12] Benigno, P. and D. Lopez-Salido (2003), "Inflation Persistence and Optimal Monetary Policy in the Euro Area," ECB working paper no. 178.

[13] Benigno, P., and M. Woodford [2004a], "Inflation Stabilization and Welfare: The Case of Large Distortions," NBER working paper No. .

[14] Benigno, P., and M. Woodford [2004a], "Linear-Quadratic Approximation of Optimal Policy Problems" in preparation, New York University.

[15] Bergin, P. and R. Glick (2004), "Tradability, Productivity and Understanding International Integration," mimeo, UC-Davis and Federal Reserve Bank of San Francisco.

[16] Blanchard, O. (2001), "Country Adjustments within Euroland. Lessons after Two Years", CEPR annual report on the European Central Bank, March 2001.

[17] Burstein, A., J. Neves and S. Rebelo (2004), "Distribution Costs and Real Exchange Rate Dynamics During Exchange-Rate-Based-Stabilizations," *Journal of Monetary Economics*, Volume 50, Issue 6, Pages 1189-1214.

[18] Campa, J. M. and L. Goldberg (2002), "Exchange Rate Pass Through into Import Prices," unpublished manuscript.

[19] Canzoneri, M., R. Cumby, B. Diba and G. Eudey (2001), "Productivity Trends in Europe: Implications for Real Exchange Rates, Real Interest Rates and Inflation," unpublished manuscript, Georgetown University.

[20] Canzoneri, M., R. Cumby, B. Diba, L. Behzad (2004), "The Cost of Nominal Inertia in NNS Model, " NBER Working Paper No. 10889.

[21] Calvo, G. A. (1983), "Staggered Prices in a Utility Maximizing Framework", *Journal of Monetary Economics*, 12, 383-398.

[22] Cecchetti, S., N. C. Mark, and R. Sonora (2002), "Price Level Convergence among United States Cities: Lessons for the European Central Bank," *International Economic Review*, Vol. 43, No. 4, Pages 1081-1099.

[23] Chari, V.V, P. Kehoe, and E. McGrattan (2002), "Can Sticky Price Models Generate Volatile and Persisten Real Exchange Rates?," *Review of Economic Studies,* Vol. 69, No. 3.

[24] Corsetti, G, and L. Dedola (2003), "Macroeconomics of International Price Discrimination," unpublished manuscript.

[25] Costa Dias, M. and P. Neves (2003), "Stylised Features of Price Setting Behavior," unpublished manuscript, Banco de Portugal.

[26] Cova, P. (2003), "Demand Shocks and Equilibrium Relative Prices in the Euro Area," unpublished manuscript, Georgetown university.

[27] De Gregorio, J., A. Giovannini and H. Wolf (1994), "International Evidence on Tradables and Nontradables Inflation," *European Economic Review* 38, pp. 1225-1244.

[28] Duarte, M. (2003), "The Euro and Inflation Divergence in Europe," Federal Reserve Bank of Richmond, *Economic Quarterly,* Volume 89/3.

[29] Duarte, M. and A. Wolman (2002), "Regional Inflation in a Currency Union: Fiscal Policy Vs. Fundamentals," ECB Working Paper Series No. 180.

[30] Duarte, M. and A. Wolman (2003), "Fiscal Policy and Regional Inflation in a Currency Union," Federal Reserve Bank of Richmond, Working Paper No. 3-11.

[31] ECB (2003), "Inflation Differentials in the Euro Area: Potential Causes and Policy Implications".

[32] Engel, C. (1999), "Accounting for U.S. Real Exchange Rate Changes," *Journal of Political Economy,* vol. 107, no. 3, pp. 507-538.

[33] Engel, C. (2001), "Optimal Exchange Rate Policy: The Infleunce of Price Setting and Asset Markets," *Journal of Money, Credit, and Banking,* vol. 33, No. 2, pp. 518-541.

[34] Engel, C. and J. Rogers, (2001), "Deviations from Puchasing Power Parity: Causes and Welfare Costs," *Journal of International Economics* 55, pp. 29-57.

[35] Engel, C. and J. Rogers, (2004), "European Product Market Integration after the Euro," unpublished manuscript, University of Wisconsin.

[36] Fitzgerald, D. (2003), "Terms-of-Trade Effects, Interdependence and Cross-Country Differences in Price Levels," unpublished mansucript, UC-Santa Cruz.

[37] Forni, M. and L. Reichlin, (2001), "Federal policies and local economies:Europe and the US," *European Economic Review* 45, pp. 109-134.

[38] Forni, M., M. Hallin, M. Lippi, and L. Reichlin, (2000) "The Generalized Factor Model:Identification and Estimation", *The Review of Economic and Statistics*, 82(4), 540-554.

[39] Froot, K. and K. Rogoff (1991), "The EMS, the EMU, and the Transition to a Common Currency", in Olivier Blanchard and Stanley Fisher (eds.) *NBER MAcroeconomics Annual 1991*, Cambridge MA: The Mit Press.

[40] Froot, K. and K. Rogoff (1995), "Perspectives on PPP and Long-Run Real Exchange Rates," in Handbook of International Economics vol. 3, Gene Grossman and Kenneth Rogoff (eds.), (Amsterdam: Elsevier Science Publishers B.V., 1995): 1647-88.

[41] Galí, J. and M. Gertler (1999), "Inflation Dynamics: A Structural Econometric Analysis," *Journal of Monetary Economics,* 44, 195-222.

[42] Galí, J., M. Gertler and J. David López-Salido (2001), "European Inflation Dynamics," *European Economic Review,* 45, 1237-1270.

[43] Honohan, P. and P. R. Lane (2003), "Divergent Inflation Rates under EMU," Economic Policy, 37, 358-394.

[44] Honohan, P. and P. R. Lane (2004), "Exchange Rates and Inflation under EMU: An Update," Economic PolicyWeb Essay, July 2004. Available at: http://www.economic-policy.org/responses.asp.

[45] Le Bihan, H. and P. Sevestre (2004), "Analyzing French CPI Micro-Data: Some Preliminary Notations and Results," unpublished manuscript, Banque de France and IPN.

[46] Lombardo, G. (2004), "Inflation Targeting Rules and Welfare in an Asymmetric Currency Area," Discussion Paper, No. 04/2004, Deutsche Bundesbank.

[47] Lucas, R. [1987], *Models of Business Cycles,* Blackwell, Oxford.

[48] MacDonald R. and L. Ricci (2002), "Purchasing Power Parity and New Trade Theory," IMF working paper 02-32.

[49] Mendoza, E. (2000), "On the Instability of Variance Decompositions of the Real Exchange Rate Across Exchange-Rate-Regimes: Evidence from Mexico and the United States," NBER Working Paper 7768.

[50] Natalucci, F. and F. Ravenna (2002), "The Road to Adopting the Euro: Monetary Policy and Exchange Rate Regimes in EU Candidate Countries," International Finance Discussion Paper, Numer 741, Board of Governors of the Federal REserve System.

[51] Obstfeld, M., Rogoff, K. (2000), "New Directions for Stochastic Open Economy Models," *Journal of International Economics* 50, 117–153.

[52] Ortega, E. (2003), "Persistent Inflation Differentials in Europe," Banco De Espana, *Economic Bulletin*, January 2003.

[53] Rogers, J. (1999), "Monetary Shocks and Real Exchange Rates," *Journal of International Economics* 49, pp. 269-288.

[54] Rogers, J. (2002), "Monetary Union, Price Level Convergence, and Inflation: How Close is Europe to the United States?" International Finance Discussion Papers Number 740, Board of Governors of the Federal Reserve System.

[55] Rogoff, K. (1992), "Traded Goods Consumption Smoothing and the Random Walk Behavior of the Real Exchange Rate," *Bank of Japan Monetary and Economic Studies* 10, November 1992, 1-29.

[56] Rogoff, K. (1996), "The Purchasing Power Parity Puzzle," *Jounal of Economic Literature* Vol. XXXIV, pp. 647-668.

[57] Rotemberg, J.J., Woodford, M., 1997. An optimization-based econometric framework for the evaluation of monetary policy, in Bernanke, B.S., Rotemberg, J.J, (Eds.), *NBER Macroeconomic Annual 1997*, MIT Press, Cambridge, pp. 297–346.

[58] Samuelson, P. A., (1964), "Theoretical Notes on Trade Problems," *Review of Economics and Statistics* 46, pp. 145-164.

[59] Sbordone, A. (2001), "An Optimizing Model of U.S. Wage and Price Dynamics", unpublished manuscript, Rutgers University.

[60] Smets, F. and R. Wouters (2004), "An Estimated Stochastic Dynamic General Equilibrium Model of the Euro Area," *Journal of the European Economic Association* vol. 1, Issue 5, pp. 1123-1175.

[61] Sondergaard, L. (2003), "Inflation Dynamics in the Traded Sectors of France, Italy and Spain," unpublished manuscript, Georgetown University.

[62] Soto, C. (2003), "Non-traded Goods and Monetary Policy Trade-offs in a Small Open Economy," unpublished manuscript, New York University.

[63] Stahl, H. (2003), "Price Rigidity in German Manufacturing," unpublished manuscript.

[64] Stockman, A. and L. Tesar, (1994), "Tastes in a Two-Country Model of the Business Cycle: Explaining International Comovement," *American Economic Review,* vol. 85 No. 1, pp. 168-185.

[65] Stock, J. and M. Watson, (1999), "Diffusion Index ", NBER working paper 6702.

[66] Woodford, M. (2003), *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton University Press.

Appendix

When prices are fully flexible, all firms within a sector set the same price. In the traded sector of country H, the first-order conditions imply

$$1 = \mu_{T,t} \frac{P_t}{P_{T,t}} \frac{P_{T,t}}{P_{H,t}} \frac{V_l(L_t)}{U_c(C_t)} \left(\frac{l_{T,t}}{L_t} \right)^{-\frac{1}{\phi}} \frac{1}{f'(l_{T,t})A_{T,t}}, \tag{9.23}$$

in the traded sector of country F, we obtain that

$$1 = \mu^*_{T,t} \frac{P^*_t}{P^*_{T,t}} \frac{P^*_{T,t}}{P_{F,t}} \frac{V_l(L^*_t)}{U_c(C^*_t)} \left(\frac{l^*_{T,t}}{L^*_t} \right)^{-\frac{1}{\phi}} \frac{1}{f'(l^*_{T,t})A_{T,t}}. \tag{9.24}$$

In the non-traded sectors, respectively in country H and F, we obtain that

$$1 = \mu_{N,t} \frac{P_t}{P_{N,t}} \frac{V_l(L_t)}{U_c(C_t)} \left(\frac{l_{N,t}}{L_t} \right)^{-\frac{1}{\phi}} \frac{1}{f'(l_{N,t})A_{N,t}}, \tag{9.25}$$

$$1 = \mu^*_{N,t} \frac{P^*_t}{P^*_{N,t}} \frac{V_l(L^*_t)}{U_c(C^*_t)} \left(\frac{l^*_{N,t}}{L^*_t} \right)^{-\frac{1}{\phi}} \frac{1}{f'(l^*_{N,t})A^*_{N,t}}. \tag{9.26}$$

We define $T \equiv P_F/P_H$, $T_N \equiv P_N/P_T$ and $T^*_N \equiv P^*_N/P^*_T$. Note that since the law of one price holds for traded goods it follows that $P_F = P^*_F$ and $P_H = P^*_H$. Using the definition of the price indexes, we obtain that

$$\left(\frac{P_{H,t}}{P_{T,t}} \right)^{\theta-1} = n + (1-n)T_t^{1-\theta}, \tag{9.27}$$

$$\left(\frac{P_{F,t}}{P_{T,t}} \right)^{\theta-1} = nT_t^{\theta-1} + (1-n), \tag{9.28}$$

$$\left(\frac{P_{H,t}}{P^*_{T,t}} \right)^{\theta-1} = n^* + (1-n^*)T_t^{1-\theta}, \tag{9.29}$$

$$\left(\frac{P_{F,t}}{P^*_{T,t}} \right)^{\theta-1} = n^*T_t^{\theta-1} + (1-n^*), \tag{9.30}$$

$$\left(\frac{P_{N,t}}{P_t} \right)^{\varphi-1} = \omega T_{N,t}^{\varphi-1} + (1-\omega), \tag{9.31}$$

$$\left(\frac{P_{T,t}}{P_t} \right)^{\varphi-1} = \omega + (1-\omega)T_{N,t}^{1-\varphi}, \tag{9.32}$$

$$\left(\frac{P^*_{N,t}}{P^*_t} \right)^{\varphi-1} = \omega^* T_{N,t}^{*\varphi-1} + (1-\omega^*), \tag{9.33}$$

$$\left(\frac{P^*_{T,t}}{P^*_t} \right)^{\varphi-1} = \omega^* + (1-\omega^*)T_{N,t}^{*1-\varphi}. \tag{9.34}$$

We finally note that (9.32) and (9.34) imply

$$\frac{P_t}{P_t^*} = \frac{P_{T,t}}{P_{T,t}^*} \frac{[\omega + (1-\omega)T_{N,t}^{1-\varphi}]^{\frac{1}{1-\varphi}}}{[\omega^* + (1-\omega^*)T_{N,t}^{*1-\varphi}]^{\frac{1}{1-\varphi}}}$$

moreover (9.27) and (9.29) imply

$$\frac{P_{T,t}}{P_{T,t}^*} = \frac{[n + (1-n)T_t^{1-\theta}]^{\frac{1}{1-\theta}}}{[n^* + (1-n^*)T_t^{1-\theta}]^{\frac{1}{1-\theta}}}. \tag{9.35}$$

We can finally write that

$$\frac{P_t}{P_t^*} = \frac{[n + (1-n)T_t^{1-\theta}]^{\frac{1}{1-\theta}}}{[n^* + (1-n^*)T_t^{1-\theta}]^{\frac{1}{1-\theta}}} \frac{[\omega + (1-\omega)T_{N,t}^{1-\varphi}]^{\frac{1}{1-\varphi}}}{[\omega^* + (1-\omega^*)T_{N,t}^{*1-\varphi}]^{\frac{1}{1-\varphi}}}$$

which can be substituted in (3.5) to obtain

$$\frac{U_C(C_t)}{U_C(C_t^*)} = \frac{[n + (1-n)T_t^{1-\theta}]^{\frac{1}{1-\theta}}}{[n^* + (1-n^*)T_t^{1-\theta}]^{\frac{1}{1-\theta}}} \frac{[\omega + (1-\omega)T_{N,t}^{1-\varphi}]^{\frac{1}{1-\varphi}}}{[\omega^* + (1-\omega^*)T_{N,t}^{*1-\varphi}]^{\frac{1}{1-\varphi}}}. \tag{9.36}$$

Under flexible prices, we note that $L_{T,t} = \gamma l_{T,t}$ and $L_{T,t}^* = \gamma^* l_{T,t}^*$ and $L_{N,t} = (1-\gamma) l_{N,t}$ and $L_{N,t}^* = (1-\gamma^*) l_{N,t}^*$ so that

$$L_t = \left[\gamma (l_{T,t})^{\frac{\phi-1}{\phi}} + (1-\gamma)(l_{N,t})^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}} \tag{9.37}$$

$$L_t^* = \left[\gamma (l_{T,t}^*)^{\frac{\phi-1}{\phi}} + (1-\gamma)(l_{N,t}^*)^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}} \tag{9.38}$$

The sectorial demand of goods are given by

$$y_{T,t} = \frac{n\omega}{\gamma} \left(\frac{P_H}{P_T}\right)^{-\theta} \left(\frac{P_T}{P}\right)^{-\varphi} C + \frac{n^*\omega^*}{\gamma} \left(\frac{P_H}{P_T^*}\right)^{-\theta} \left(\frac{P_T^*}{P^*}\right)^{-\varphi} C^* \tag{9.39}$$

$$y_{T,t}^* = \frac{(1-n)\omega}{\gamma^*} \left(\frac{P_F}{P_T}\right)^{-\theta} \left(\frac{P_T}{P}\right)^{-\varphi} C + \frac{(1-n^*)\omega^*}{\gamma^*} \left(\frac{P_F}{P_T^*}\right)^{-\theta} \left(\frac{P_T^*}{P^*}\right)^{-\varphi} C^* \tag{9.40}$$

$$y_{N,t} = \frac{1-\omega}{1-\gamma} \left(\frac{P_{N,t}}{P_t}\right)^{-\varphi} C_t + G_t \tag{9.41}$$

$$y_{N,t}^* = \frac{1-\omega^*}{1-\gamma^*} \left(\frac{P_{N,t}^*}{P_t^*}\right)^{-\varphi} C_t^* + G_t^*. \tag{9.42}$$

while

$$l_{T,t} = f^{-1}\left(\frac{y_{T,t}}{A_{T,t}}\right) \tag{9.43}$$

$$l_{T,t}^* = f^{-1}\left(\frac{y_{T,t}^*}{A_{T,t}^*}\right) \tag{9.44}$$

$$l_{N,t} = f^{-1}\left(\frac{y_{N,t}}{A_{N,t}}\right) \tag{9.45}$$

$$l_{N,t}^* = f^{-1}\left(\frac{y_{N,t}^*}{A_{N,t}^*}\right) \tag{9.46}$$

Equations (9.23), (9.24), (9.25), (9.26), (9.27), (9.28), (9.29), (9.30), (9.31), (9.32), (9.33), (9.34), (9.36), (9.37), (9.38), (9.39), (9.40), (9.41), (9.42), (9.43), (9.44), (9.45), (9.46) should be solved for the variables $\{\frac{P_t}{P_{T,t}}, \frac{P_{T,t}}{P_{H,t}}, L_t, C_t, l_{T,t}, \frac{P_t^*}{P_{T,t}^*}, \frac{P_{T,t}^*}{P_{F,t}}, L_t^*, C_t^*, l_{T,t}^*, \frac{P_t}{P_{N,t}}, \frac{P_t^*}{P_{N,t}^*}, l_{N,t}^*, l_{N,t}, T_t, T_{N,t}, T_{N,t}^*, \frac{P_{F,t}}{P_{T,t}}, \frac{P_{F,t}}{P_{T,t}^*}, y_{T,t}, y_{T,t}^*, y_{N,t}, y_{N,t}^*\}$ given the processes $\{\mu_{T,t}, \mu_{T,t}^*, \mu_{N,t}, \mu_{N,t}^*, A_{T,t}, A_{T,t}^*, A_{N,t}, A_{N,t}^*, G_t, G_t^*\}$.

We approximate the previous solution around a deterministic steady state in which the process $\{\mu_{T,t}, \mu_{T,t}^*, \mu_{N,t}, \mu_{N,t}^*, A_{T,t}, A_{T,t}^*, A_{N,t}, A_{N,t}^*, G_t, G_t^*\}$ are such that $\bar{\mu}_H = \bar{\mu}_H^* = \bar{\mu}_F = \bar{\mu}_F^* = \bar{\mu}_N = \bar{\mu}_N^* = \bar{\mu}$ and $\bar{A}_T = \bar{A}_T^* = \bar{A}_N = \bar{A}_N^* = \bar{A}$, $\bar{G} = \bar{G}^* = 0$. We assume for simplicity that $\gamma = \gamma^*$, $n = n^* = 1/2$, $\omega = \omega^* = \gamma$. We show that there is a steady-state in which $\frac{P_t}{P_{T,t}} = \frac{P_{T,t}}{P_{H,t}} = \frac{P_t^*}{P_{T,t}^*} = \frac{P_{T,t}^*}{P_{F,t}} = \frac{P_t}{P_{N,t}} = \frac{P_t^*}{P_{N,t}^*} = \frac{P_{F,t}}{P_{T,t}} = \frac{P_{F,t}}{P_{T,t}^*} = T_t = T_{N,t} = T_{N,t}^* = 1$, $L_t = l_{T,t} = l_{N,t} = \bar{L}$ and $L_t^* = l_{T,t}^* = l_{N,t}^* = \bar{L}^*$, $y_{T,t} = y_{T,t}^* = y_{N,t} = y_{N,t}^* = C_t = C_t^* = \bar{C}$. It is clear that (9.27), (9.28), (9.29), (9.30), (9.31), (9.32), (9.33), (9.34), (9.36), (9.37), (9.38), (9.39), (9.40), (9.41), (9.42) are satisfied by the above steady-state conditions. Moreover (9.43), (9.44), (9.45), (9.46) imply that

$$\bar{L} = f^{-1}\left(\frac{\bar{C}}{\bar{A}}\right) = \bar{L}^*, \tag{9.47}$$

while (9.23), (9.24), (9.25), (9.26) imply

$$1 = \bar{\mu}\frac{V_l(\bar{L})}{U_c(\bar{C})}\frac{1}{f'(\bar{L})\bar{A}}. \tag{9.48}$$

Equations (9.47) and (9.48) can be solved for $\bar{L}$ and $\bar{C}$ given $\bar{A}$. Under standard preference specifications the values $\bar{L}$ and $\bar{C}$ exist uniquely.

Maintaining the assumption that $\gamma = \gamma^*$, $n = n^* = 1/2$, $\omega = \omega^* = \gamma$ we study the solution of the model in a log-linear approximation of the structural equations for small perturbations of the exogenous shocks around the steady state outlined above. We start by log-linearizing equations (9.27) to (9.34). We obtain that

$$\widehat{\frac{P_{H,t}}{P_{T,t}}} = -\frac{1}{2}\hat{T}_t = -\widehat{\frac{P_{F,t}}{P_{T,t}}}$$

$$\frac{\widehat{P_{H,t}}}{P^*_{T,t}} = -\frac{1}{2}\hat{T}_t = -\frac{\widehat{P_{F,t}}}{P^*_{T,t}}$$

$$\frac{\widehat{P_{N,t}}}{P_t} = \gamma\hat{T}_{N,t} = -\frac{\gamma}{1-\gamma}\frac{\widehat{P_{T,t}}}{P_t}$$

$$\frac{\widehat{P^*_{N,t}}}{P^*_t} = \gamma\hat{T}^*_{N,t} = -\frac{\gamma}{1-\gamma}\frac{\widehat{P^*_{T,t}}}{P^*_t}$$

Equation (9.36) yields to

$$-\rho(\hat{C}_t - \hat{C}^*_t) = (1-\gamma)(\hat{T}_{N,t} - \hat{T}^*_{N,t}). \tag{9.49}$$

A log-linear approximation to (9.23), (9.24), (9.25), (9.26) yields to

$$\hat{\mu}_{T,t} + (1-\gamma)\hat{T}_{N,t} + \frac{1}{2}\hat{T}_t + \eta\hat{L}_t + \rho\hat{C}_t - \frac{1}{\phi}(\hat{l}_{T,t} - \hat{L}_t) - \hat{A}_{T,t} + (1-\tilde{\lambda})\hat{l}_{T,t} = 0 \tag{9.50}$$

$$\hat{\mu}^*_{T,t} + (1-\gamma)\hat{T}^*_{N,t} - \frac{1}{2}\hat{T}^*_t + \eta\hat{L}^*_t + \rho\hat{C}^*_t - \frac{1}{\phi}(\hat{l}^*_{T,t} - \hat{L}^*_t) - \hat{A}^*_{T,t} + (1-\tilde{\lambda})\hat{l}^*_{T,t} = 0 \tag{9.51}$$

$$\hat{\mu}_{N,t} - \gamma\hat{T}_{N,t} + \eta\hat{L}_t + \rho\hat{C}_t - \frac{1}{\phi}(\hat{l}_{N,t} - \hat{L}_t) - \hat{A}_{N,t} + (1-\tilde{\lambda})\hat{l}_{N,t} = 0 \tag{9.52}$$

$$\hat{\mu}^*_{N,t} - \gamma\hat{T}^*_{N,t} + \eta\hat{L}^*_t + \rho\hat{C}^*_t - \frac{1}{\phi}(\hat{l}^*_{N,t} - \hat{L}^*_t) - \hat{A}^*_{N,t} + (1-\tilde{\lambda})\hat{l}^*_{N,t} = 0 \tag{9.53}$$

where $\eta \equiv \bar{V}_{ll}\bar{l}/\bar{V}_l$, $\rho \equiv -\bar{U}_{cc}\bar{C}/\bar{U}_c$, $(1-\tilde{\lambda}) \equiv -\bar{f}''\bar{l}/\bar{f}'$ with $\tilde{\lambda} \leq 1$. A log-linear approximation to equations (9.37) and (9.38) yields to

$$\hat{L}_t = \gamma\hat{l}_{T,t} + (1-\gamma)\hat{l}_{N,t}, \tag{9.54}$$

$$\hat{L}^*_t = \gamma\hat{l}^*_{T,t} + (1-\gamma)\hat{l}^*_{N,t}. \tag{9.55}$$

A log-linear approximation to equations (9.39), (9.40), (9.41), (9.42) yields

$$\hat{y}_{T,t} = \frac{\theta}{2}\hat{T}_t + \frac{\varphi}{2}(1-\gamma)\hat{T}_{N,t} + \frac{1}{2}\hat{C}_t + \frac{\varphi}{2}(1-\gamma)\hat{T}^*_{N,t} + \frac{1}{2}\hat{C}^*_t, \tag{9.56}$$

$$\hat{y}^*_{T,t} = -\frac{\theta}{2}\hat{T}_t + \frac{\varphi}{2}(1-\gamma)\hat{T}_{N,t} + \frac{1}{2}\hat{C}_t + \frac{\varphi}{2}(1-\gamma)\hat{T}^*_{N,t} + \frac{1}{2}\hat{C}^*_t, \tag{9.57}$$

$$\hat{y}_{N,t} = -\varphi\gamma\hat{T}_{N,t} + \hat{C}_t + \hat{G}_t, \tag{9.58}$$

$$\hat{y}^*_{N,t} = -\varphi\gamma\hat{T}^*_{N,t} + \hat{C}^*_t + \hat{G}^*_t, \tag{9.59}$$

where $\hat{G}_t = G_t/\bar{Y}$ and $\hat{G}^*_t = G^*_t/\bar{Y}$. Finally a log-linear approximation to (9.43), (9.44), (9.45), (9.46) imply

$$\lambda\hat{l}_{T,t} = \hat{y}_{T,t} - \hat{A}_{T,t} \tag{9.60}$$

$$\lambda \hat{l}^*_{T,t} = \hat{y}^*_{T,t} - \hat{A}^*_{T,t} \tag{9.61}$$

$$\lambda \hat{l}_{N,t} = \hat{y}_{N,t} - \hat{A}_{N,t} \tag{9.62}$$

$$\lambda \hat{l}^*_{N,t} = \hat{y}^*_{N,t} - \hat{A}^*_{N,t} \tag{9.63}$$

where $\lambda \equiv \bar{f}'\bar{l}/\bar{f}$. with $0 < \lambda \leq 1$.

We first solve the model for the aggregate variables and then we solve for relative prices and consumption. First, we take a weighted average of equations (9.50) and (9.52) with weights γ and $1-\gamma$ and obtain

$$\gamma\hat{\mu}_{T,t} + (1-\gamma)\hat{\mu}_{N,t} + \frac{\gamma}{2}\hat{T}_t + \left(\eta + 1 - \tilde{\lambda}\right)\hat{L}_t + \rho\hat{C}_t - \gamma\hat{A}_{T,t} - (1-\gamma)\hat{A}_{N,t} = 0$$

and take a weighted average of equations (9.51) and (9.53) to obtain

$$\gamma\hat{\mu}^*_{T,t} + (1-\gamma)\hat{\mu}^*_{N,t} - \frac{\gamma}{2}\hat{T}_t + \left(\eta + 1 - \tilde{\lambda}\right)\hat{L}^*_t + \rho\hat{C}^*_t - \gamma\hat{A}^*_{T,t} - (1-\gamma)\hat{A}^*_{N,t} = 0$$

We can define $\hat{\mu}_t \equiv \gamma\hat{\mu}_{T,t} + (1-\gamma)\hat{\mu}_{N,t}$, $\hat{\mu}^*_t \equiv \gamma\hat{\mu}^*_{T,t} + (1-\gamma)\hat{\mu}^*_{N,t}$, $\hat{A}_t \equiv \gamma\hat{A}_{T,t} + (1-\gamma)\hat{A}_{N,t}$, $\hat{A}^*_t \equiv \gamma\hat{A}^*_{T,t} + (1-\gamma)\hat{A}^*_{N,t}$ and take a weighted average of the above conditions with weights $1/2$ and obtain

$$\hat{\mu}^W_t + \left(\eta + 1 - \tilde{\lambda}\right)\hat{L}^W_t + \rho\hat{C}^W_t - \hat{A}^W_t = 0, \tag{9.64}$$

where an index W denotes a weigthed average of home and foreign variables. We can as well take a weighted average of (9.56) to (9.59), with weights $\frac{\gamma}{2}$, $\frac{\gamma}{2}$, $\frac{1-\gamma}{2}$, $\frac{1-\gamma}{2}$

$$\frac{\gamma}{2}\hat{y}_{T,t} + \frac{\gamma}{2}\hat{y}^*_{T,t} + \frac{(1-\gamma)}{2}\hat{y}^*_{N,t} + \frac{(1-\gamma)}{2}\hat{y}_{N,t} = \hat{C}^W_t + (1-\gamma)\hat{G}^W_t.$$

We can use (9.60) to (9.63) to obtain

$$\hat{C}^W_t + (1-\gamma)\hat{G}^W_t = \lambda\hat{L}^W_t + \hat{A}^W_t. \tag{9.65}$$

We can use (9.64) and (9.65) to obtain

$$\hat{L}^W_t = \frac{1-\rho}{\lambda(\tilde{\eta}+\rho)}\hat{A}^W_t + \frac{\rho(1-\gamma)}{\lambda(\tilde{\eta}+\rho)}\hat{G}^W_t - \frac{1}{\lambda(\tilde{\eta}+\rho)}\hat{\mu}^W_t$$

and

$$\hat{C}^W_t = \frac{\tilde{\eta}+1}{(\tilde{\eta}+\rho)}\hat{A}^W_t - \frac{\tilde{\eta}(1-\gamma)}{(\tilde{\eta}+\rho)}\hat{G}^W_t - \frac{1}{(\tilde{\eta}+\rho)}\hat{\mu}^W_t$$

$$\hat{Y}^W_t = \frac{\tilde{\eta}+1}{(\tilde{\eta}+\rho)}\hat{A}^W_t + \frac{\rho(1-\gamma)}{(\tilde{\eta}+\rho)}\hat{G}^W_t - \frac{1}{(\tilde{\eta}+\rho)}\hat{\mu}^W_t$$

where we have defined $\tilde{\eta} \equiv (1 - \tilde{\lambda} + \eta)/\lambda$. To solve for the difference variables in the model, we first consider equation (9.49) which can be written as

$$-\rho\hat{C}^R_t = (1-\gamma)\hat{T}^R_{N,t} \tag{9.66}$$

where a variable with a superscript R has been defined as the difference between the respective home and foreign variable.

We can take the difference between equations (9.50) and (9.51) to obtain

$$\hat{\mu}^R_{T,t} + (1-\gamma)\hat{T}^R_{N,t} + \hat{T}_t + c_1(1-\gamma)\hat{l}^R_{N,t} + \rho\hat{C}^R_t + (\gamma c_1 + c_2)\hat{l}^R_{T,t} - \hat{A}^R_{T,t} = 0$$

where we have defined

$$c_1 \equiv \left(\eta + \frac{1}{\phi}\right)$$

$$c_2 \equiv (1-\tilde{\lambda}) - \frac{1}{\phi}$$

Using (9.66) we can obtain

$$\hat{\mu}^R_{T,t} + \hat{T}_t + c_1(1-\gamma)\hat{l}^R_{N,t} + (\gamma c_1 + c_2)\hat{l}^R_{T,t} - \hat{A}^R_{T,t} = 0. \tag{9.67}$$

We can take the difference between (9.52) and (9.53) to obtain

$$\hat{\mu}^R_{N,t} - \gamma\hat{T}^R_{N,t} + \gamma c_1\hat{l}^R_{T,t} + \rho\hat{C}^R_t - \hat{A}^R_{N,t} + [(1-\gamma)c_1 + c_2]\hat{l}_{N,t} = 0,$$

which can be written as

$$\hat{\mu}^R_{N,t} - \hat{T}^R_{N,t} + \gamma c_1\hat{l}^R_{T,t} - \hat{A}^R_{N,t} + [(1-\gamma)c_1 + c_2]\hat{l}_{N,t} = 0, \tag{9.68}$$

We can use equations (9.67) and (9.68) to solve for $\hat{l}^R_{T,t}$ to obtain

$$\lambda\tilde{\eta}c_3\hat{l}^R_{T,t} = -c_4\hat{\mu}^R_{T,t} - c_4\hat{T}_t + c_4\hat{A}^R_{T,t} + c_1(1-\gamma)\hat{\mu}^R_{N,t} - c_1(1-\gamma)\hat{T}^R_{N,t} - c_1(1-\gamma)\hat{A}^R_{N,t} \tag{9.69}$$

where

$$c_3 \equiv (1-\tilde{\lambda}) - \frac{1}{\phi}$$

and

$$c_4 \equiv (1-\gamma)c_1 + c_2$$

We can also use (9.67) and (9.68) to solve for $\hat{l}^R_{N,t}$ to obtain

$$\lambda\tilde{\eta}\xi_\phi\hat{l}^R_{N,t} = \gamma c_1\hat{\mu}^R_{T,t} + \gamma c_1\hat{T}_t - \gamma c_1\hat{A}^R_{T,t} - c_5\hat{\mu}^R_{N,t} + c_5\hat{T}^R_{N,t} + c_5\hat{A}^R_{N,t} \tag{9.70}$$

where

$$c_5 \equiv \gamma c_1 + c_2.$$

We can take the difference between (9.58) and (9.59), after using (9.62) and (9.63) to obtain

$$\hat{l}^R_{N,t} = -\frac{\varphi\gamma}{\lambda}\hat{T}^R_{N,t} + \frac{1}{\lambda}\hat{C}^R_t + \frac{1}{\lambda}\hat{G}^R_t - \frac{1}{\lambda}\hat{A}^R_{N,t}$$

which can be re-written as

$$\hat{l}^R_{N,t} = -c_6\hat{T}^R_{N,t} + \frac{1}{\lambda}\hat{G}^R_t - \frac{1}{\lambda}\hat{A}^R_{N,t} \tag{9.71}$$

where c_6 =? We can now take the difference between (9.56) and (9.57) and use the difference between (9.60) and (9.61) to obtain

$$\hat{l}^R_{T,t} = \frac{\theta}{\lambda}\hat{T}_t - \frac{1}{\lambda}\hat{A}^R_{T,t} \tag{9.72}$$

Equations (9.69), (9.70), (9.71), (9.72) can be solved for $\hat{l}^R_{N,t}$, $\hat{l}^R_{T,t}$, $\hat{T}^R_{N,t}$, $\hat{T}_t$. We can write the following matrix form in relation to the vector $x_t = [\hat{l}^R_{T,t} \;\; \hat{l}^R_{N,t} \;\; \hat{T}_t \;\; \hat{T}^R_{N,t}]$

$$Ax_t = Bs_t$$

where $s_t = [\hat{\mu}^R_{T,t} \;\; \hat{\mu}^R_{N,t} \;\; \hat{A}^R_{T,t} \;\; \hat{A}^R_{N,t} \;\; \hat{G}^R_t]$

$$A = \begin{bmatrix} \lambda\tilde{\eta}c_3 & 0 & c_4 & c_1(1-\gamma) \\ 0 & \lambda\tilde{\eta}c_3 & -\gamma c_1 & -c_5 \\ 1 & 0 & -\frac{\theta}{\lambda} & 0 \\ 0 & 1 & 0 & c_6 \end{bmatrix}$$

$$B = \begin{bmatrix} -c_4 & c_1(1-\gamma) & c_4 & -c_1(1-\gamma) & 0 \\ \gamma c_1 & -c_5 & -\gamma c_1 & c_5 & 0 \\ 0 & 0 & -\frac{1}{\lambda} & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{\lambda} & \frac{1}{\lambda} \end{bmatrix}$$

We can obtain a solution

$$x_t = A^{-1}Bs_t.$$

Consider the case in which $\lambda = 1$ and $\tilde{\lambda} = 1$. The relevant equations for the determination of $(\hat{T}_t, \hat{T}^R_{N,t}, \hat{L}^R_t, \hat{l}^R_{T,t}, \hat{l}^R_{N,t})$ are

$$\hat{\mu}^R_{T,t} + \hat{T}_t + \eta\hat{L}^R_t - \frac{1}{\phi}(\hat{l}^R_{T,t} - \hat{L}^R_t) - \hat{A}^R_{T,t} = 0 \tag{9.73}$$

$$\hat{\mu}^R_{N,t} - \hat{T}^R_{N,t} + \eta\hat{L}^R_t - \frac{1}{\phi}(\hat{l}^R_{N,t} - \hat{L}^R_t) - \hat{A}^R_{N,t} = 0 \tag{9.74}$$

$$\hat{l}^R_{N,t} = -b\hat{T}^R_{N,t} + \hat{G}^R_t - \hat{A}^R_{N,t} \tag{9.75}$$

$$\hat{l}^R_{T,t} = \theta\hat{T}_t - \hat{A}^R_{T,t} \tag{9.76}$$

$$\hat{L}^R_t = \gamma\hat{l}^R_{T,t} + (1-\gamma)\hat{l}^R_{N,t} \tag{9.77}$$

where $b \equiv \varphi\gamma + (1-\gamma)\rho^{-1}$. I left ρ generic. We first solve for $\hat{T}_t, \hat{T}^R_{N,t}$. To do this we consider the difference between (9.73) and (9.74) and obtain

$$\hat{\mu}^R_{T,t} - \hat{\mu}^R_{N,t} + \hat{T}_t + \hat{T}^R_{N,t} - \frac{1}{\phi}(\hat{l}^R_{T,t} - \hat{l}^R_{N,t}) - (\hat{A}^R_{T,t} - \hat{A}^R_{N,t}) = 0 \tag{9.78}$$

as well as the difference between (9.75) and (9.76) and obtain

$$\hat{l}^R_{T,t} - \hat{l}^R_{N,t} = \theta\hat{T}_t + b\hat{T}^R_{N,t} - \hat{G}^R_t - (\hat{A}^R_{T,t} - \hat{A}^R_{N,t}) \tag{9.79}$$

We can substitute (9.79) into (9.78) to obtain

$$\hat{\mu}^R_{T,t} - \hat{\mu}^R_{N,t} + \left(1 - \frac{\theta}{\phi}\right)\hat{T}_t + \left(1 - \frac{b}{\phi}\right)\hat{T}^R_{N,t} + \frac{1}{\phi}\hat{G}^R_t - \left(1 - \frac{1}{\phi}\right)(\hat{A}^R_{T,t} - \hat{A}^R_{N,t}) = 0$$

which implies a relation between $\hat{T}^R_{N,t}$ and $\hat{T}_t$ of the form

$$\hat{T}^R_{N,t} = -\frac{\left(1-\frac{\theta}{\phi}\right)}{\left(1-\frac{b}{\phi}\right)}\hat{T}_t - \frac{1}{\left(1-\frac{b}{\phi}\right)\phi}\hat{G}^R_t + \frac{\left(1-\frac{1}{\phi}\right)}{\left(1-\frac{b}{\phi}\right)}(\hat{A}^R_{T,t} - \hat{A}^R_{N,t}) - \frac{1}{\left(1-\frac{b}{\phi}\right)}(\hat{\mu}^R_{T,t} - \hat{\mu}^R_{N,t}) \tag{9.80}$$

We need another relation to determine $\hat{T}_N$ and $\hat{T}$. Consider a weighted average of (9.75) and (9.76), we obtain

$$\hat{L}^R_t = \theta\gamma\hat{T}_t - b(1-\gamma)\hat{T}^R_{N,t} - \gamma\hat{A}^R_{T,t} - (1-\gamma)\hat{A}^R_{N,t} + (1-\gamma)\hat{G}^R_t$$

which can be substituted together with (9.76) into (9.73) to obtain

$$\begin{aligned} 0 \;&=\; \hat{\mu}^R_{T,t} + \left(1-\frac{\theta}{\phi}\right)\hat{T}_t + \left(\eta + \frac{1}{\phi}\right)\theta\gamma\hat{T}_t - \left(\eta + \frac{1}{\phi}\right)b(1-\gamma)\hat{T}^R_{N,t} + \\ &\left(\eta + \frac{1}{\phi}\right)(1-\gamma)\hat{G}^R_t - \left(\eta + \frac{1}{\phi}\right)[\gamma\hat{A}^R_T + (1-\gamma)\hat{A}^R_N] - \left(1 - \frac{1}{\phi}\right)\hat{A}^R_T \end{aligned}$$

from which we can get that

$$\begin{aligned} \left[1 + \left(\eta + \frac{1}{\phi}\right)\theta\gamma - \frac{\theta}{\phi}\right]\hat{T}_t \;&=\; \left(\eta + \frac{1}{\phi}\right)b(1-\gamma)\hat{T}^R_{N,t} + \left(1 - \frac{1}{\phi}\right)\hat{A}^R_{T,t} \\ &-\hat{\mu}^R_{T,t} + \left(\eta + \frac{1}{\phi}\right)[\gamma\hat{A}^R_{T,t} + (1-\gamma)\hat{A}^R_{N,t}] \\ &-\left(\eta + \frac{1}{\phi}\right)(1-\gamma)\hat{G}^R_t \end{aligned} \tag{9.81}$$

We can combine (9.80) and (9.81) to solve for $(\hat{T}^R_{N,t}, \hat{T}_t)$.

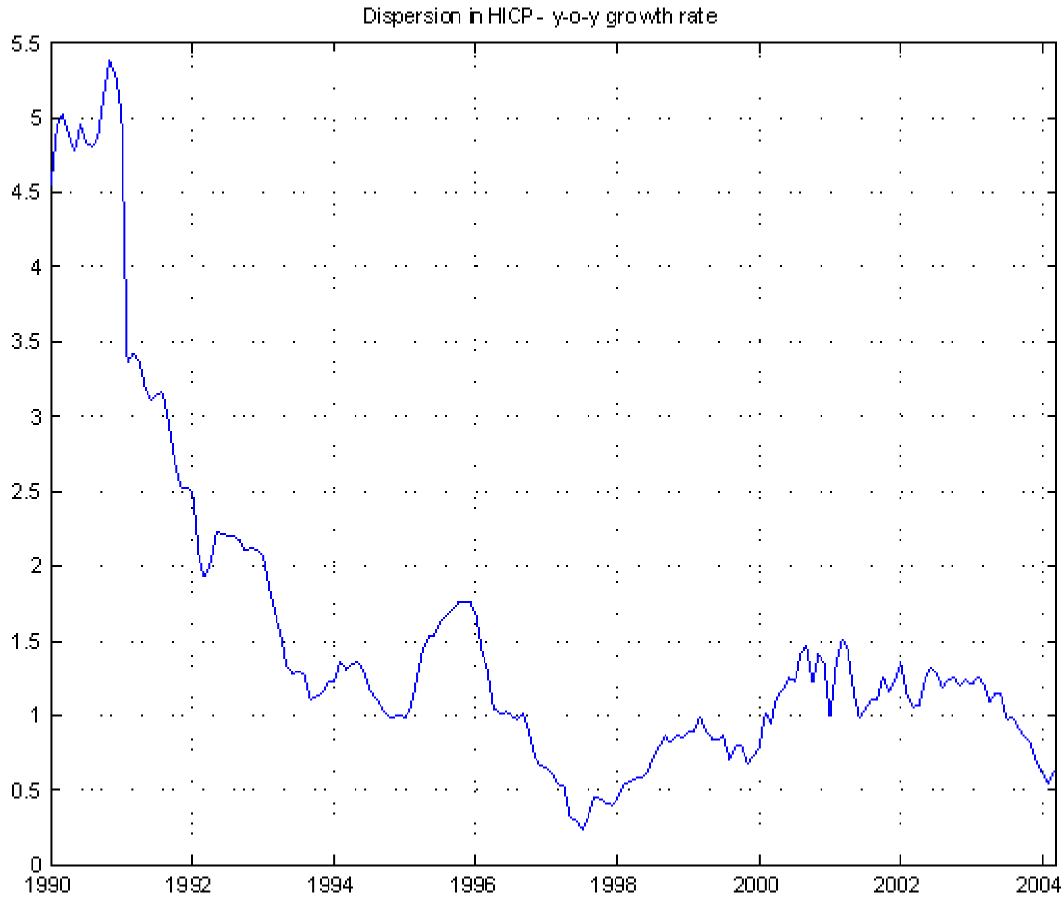

Fig. 1: Dispersion in HICP - y-o-y growth rate

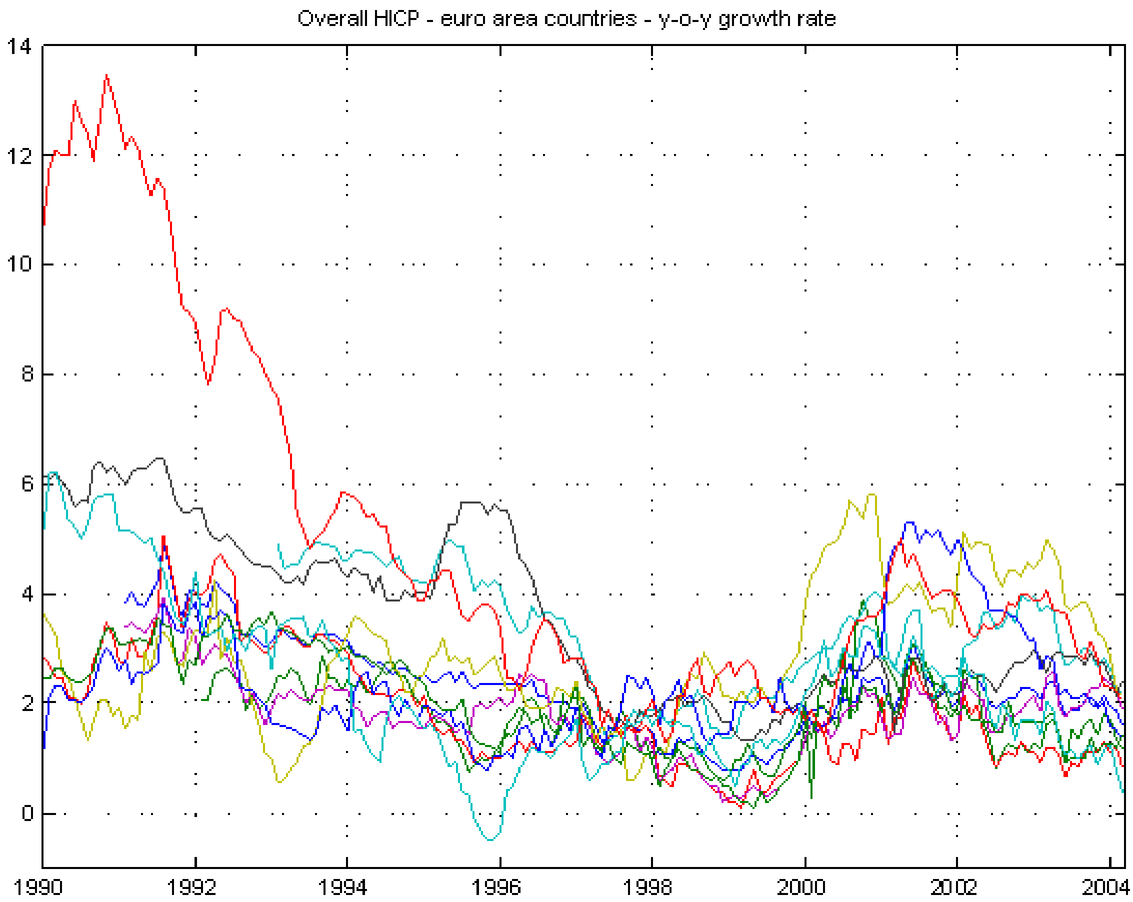

Fig. 2: Overall HICP - euro area countries - y-o-y growth rate

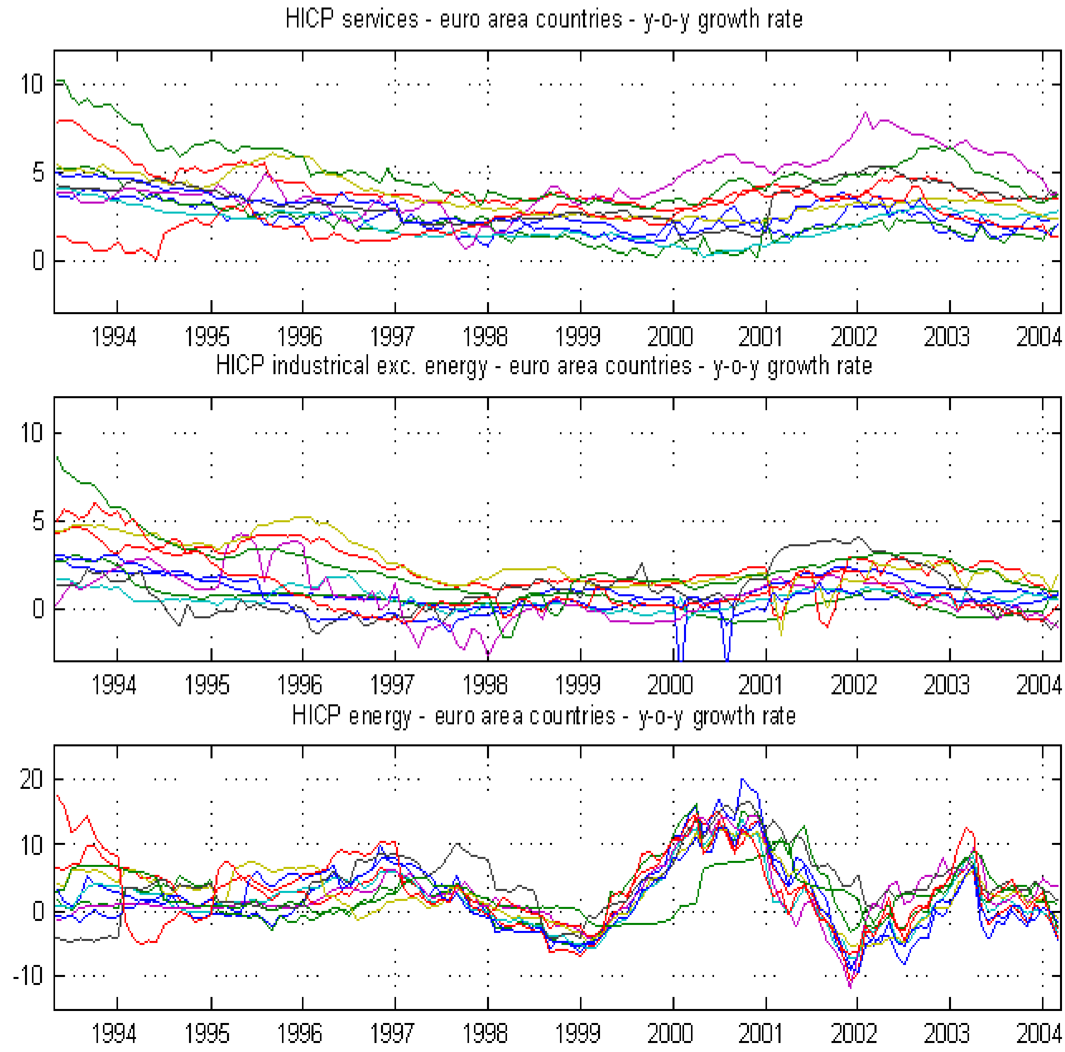

Fig. 3: HICP main sectors - euro area countries - y-o-y growth rate

Fig. 4: Dispersion by sector.

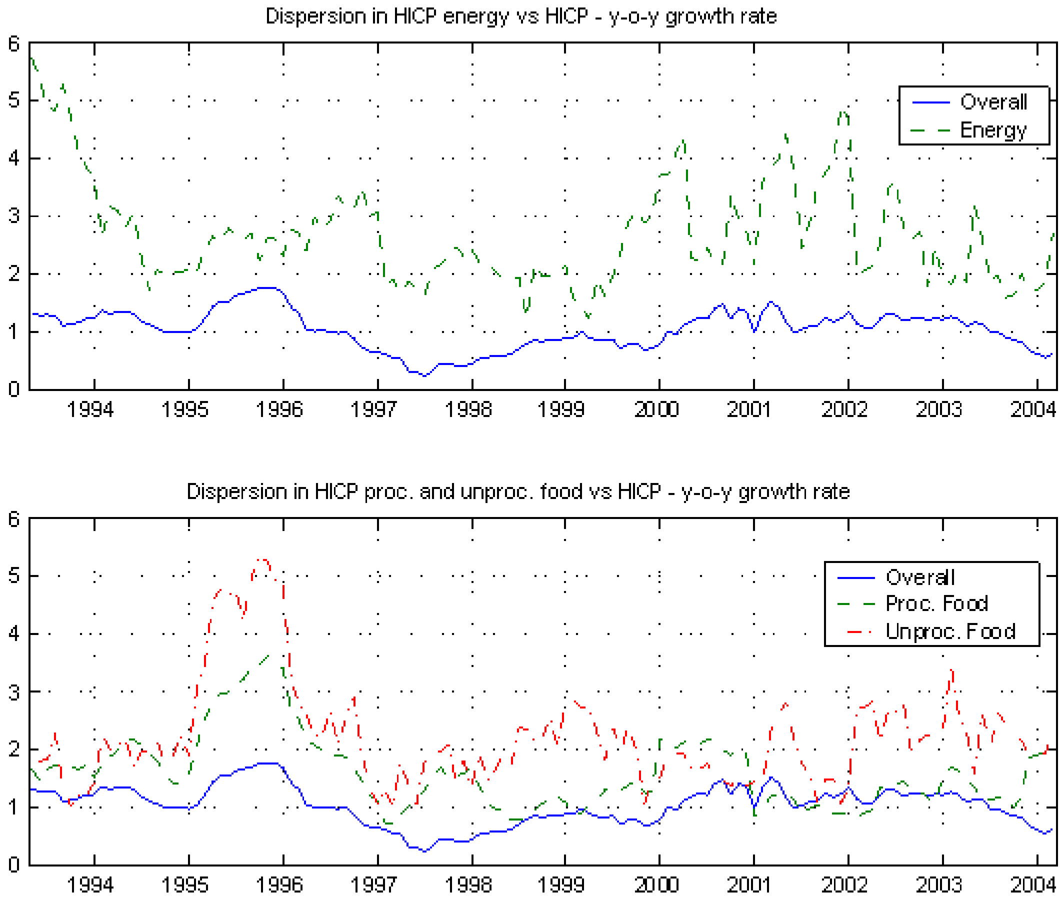

Fig. 5: Dispersion by sector.

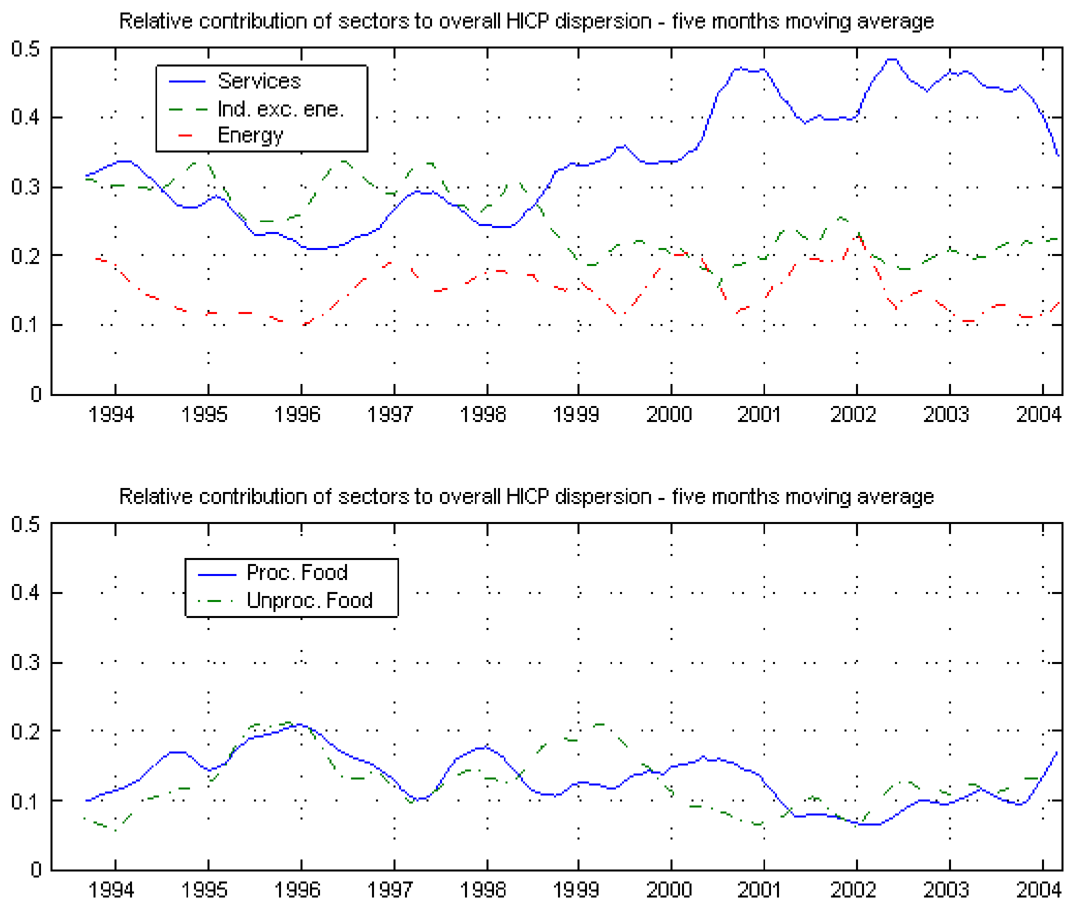

Fig. 6: Dispersion decomposition.

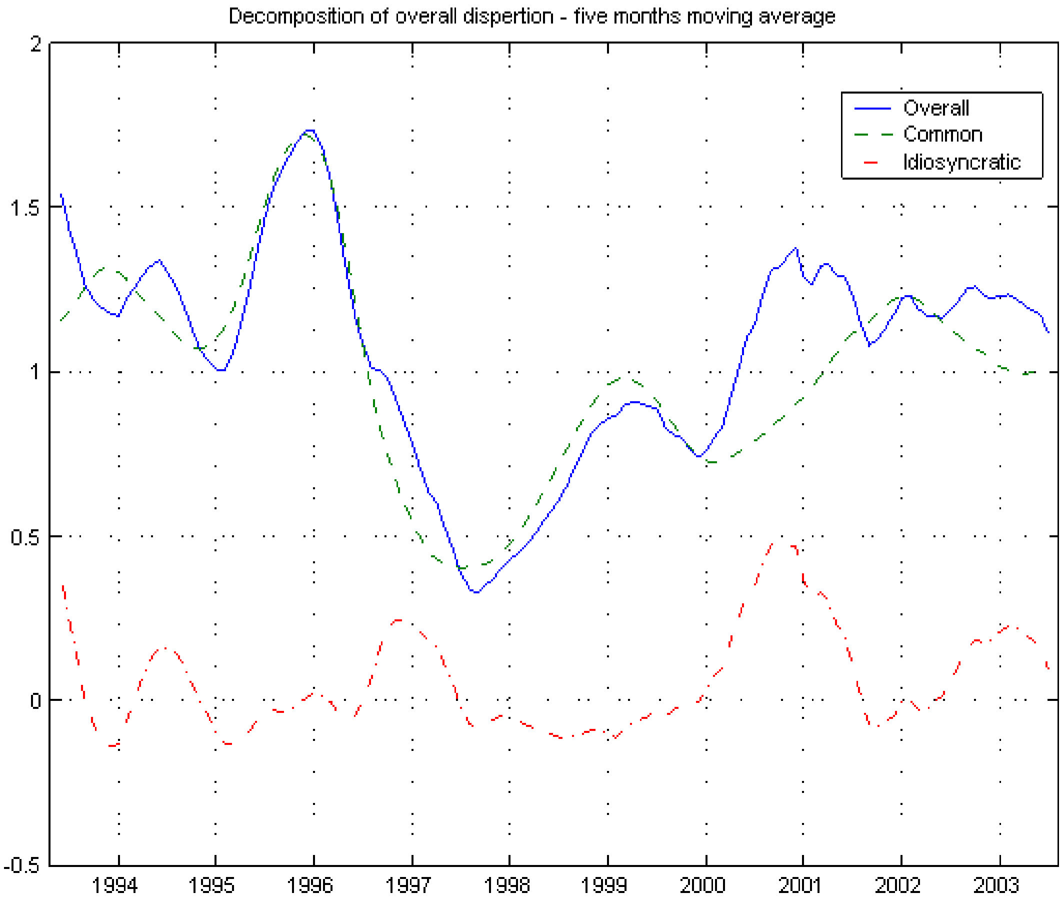

Fig. 7: Decomposition of Overall dispersion

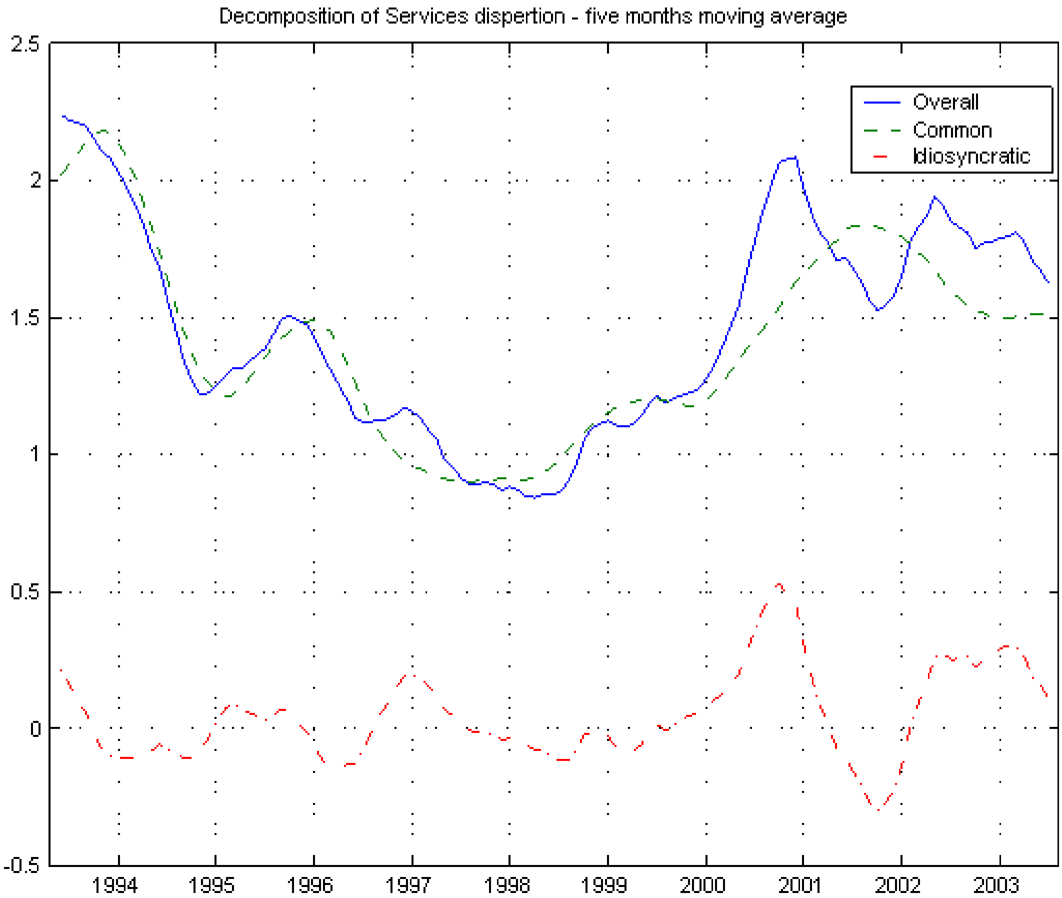

Fig. 8: Decomposition of Services dispersion

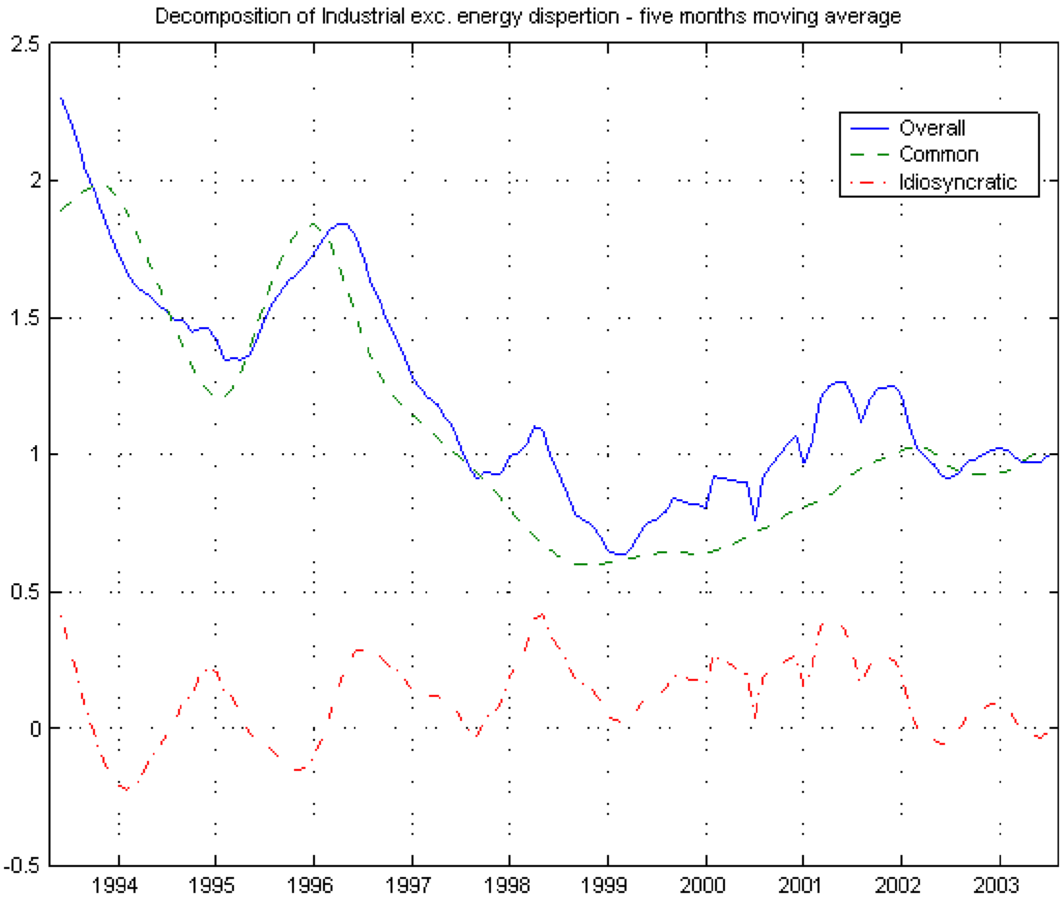

Fig. 9: Decomposition of Industrial Goods exc. energy dispersion

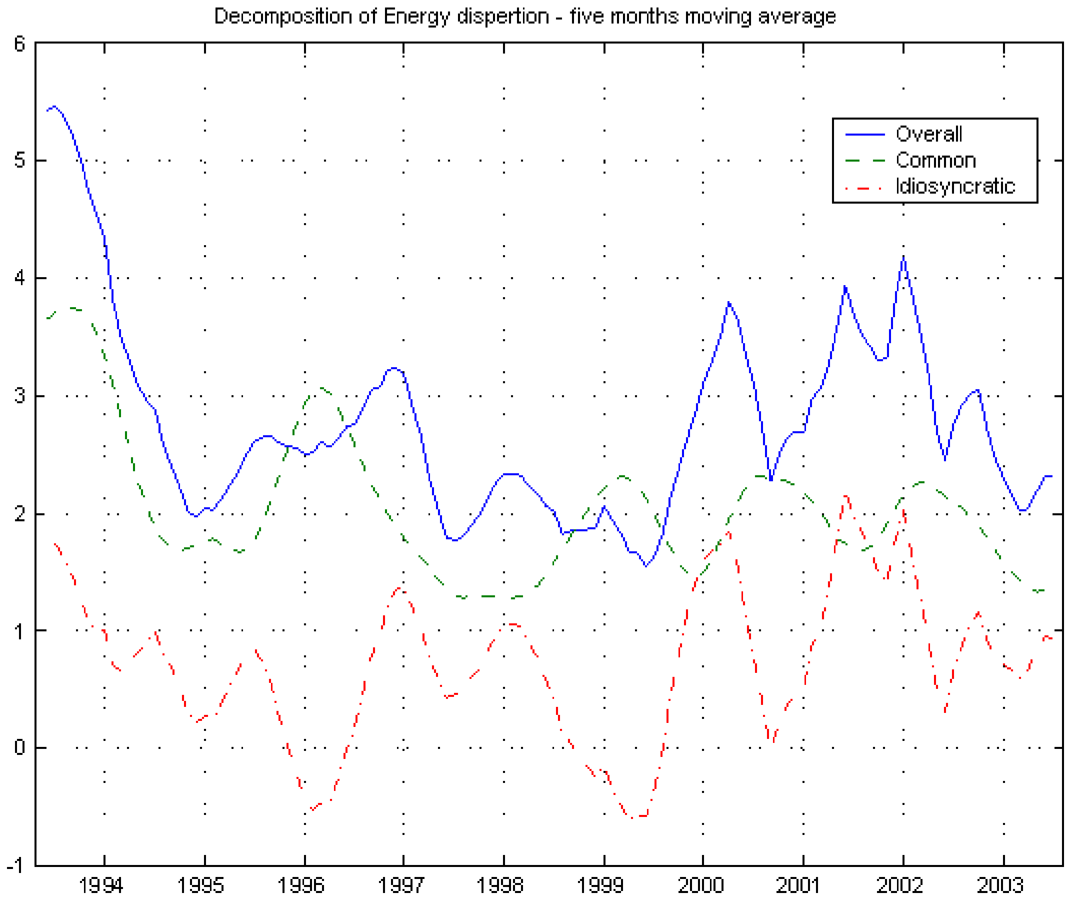

Fig. 10: Decomposition of Energy dispersion

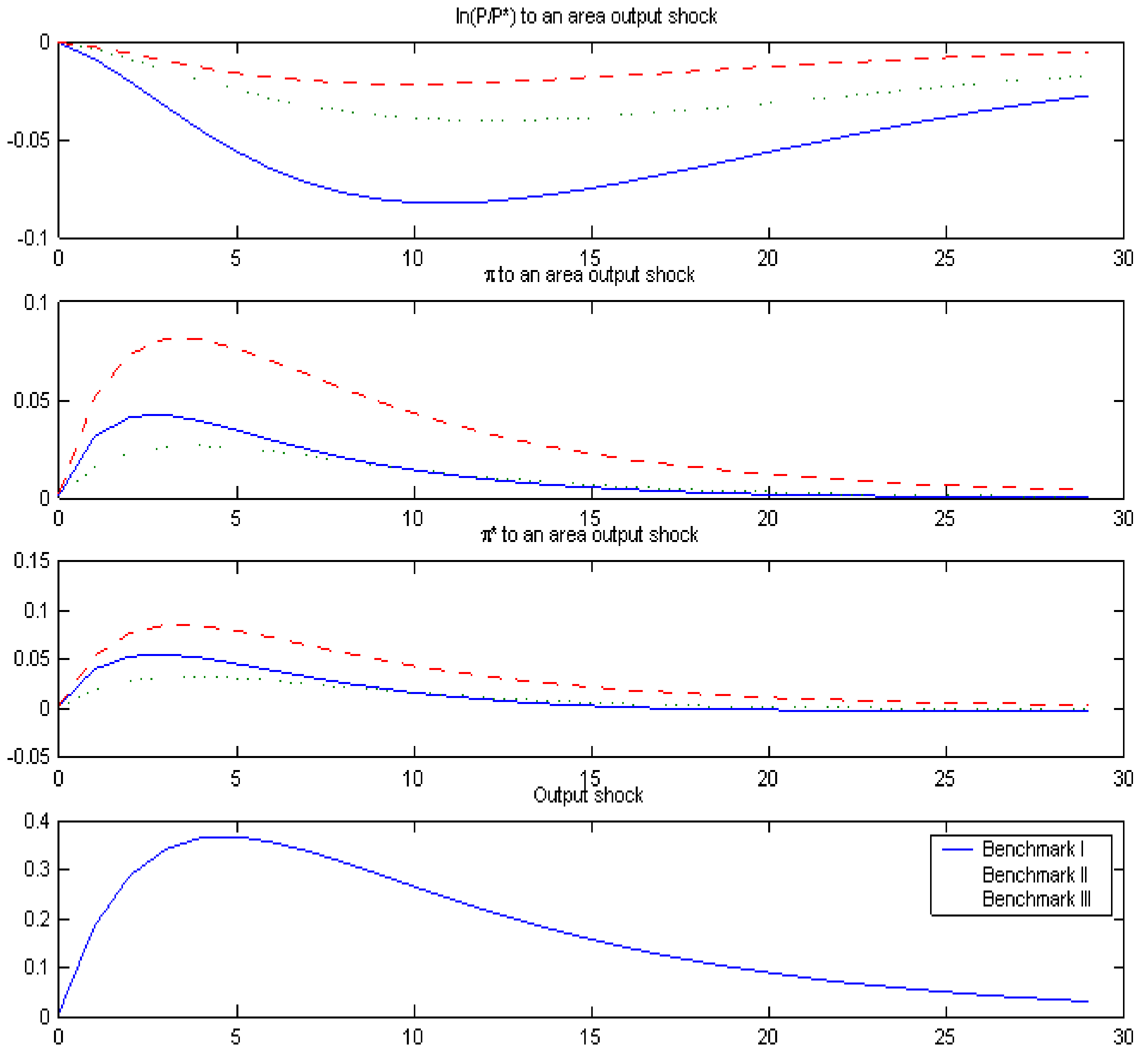

Fig. 11: monetary policy shock

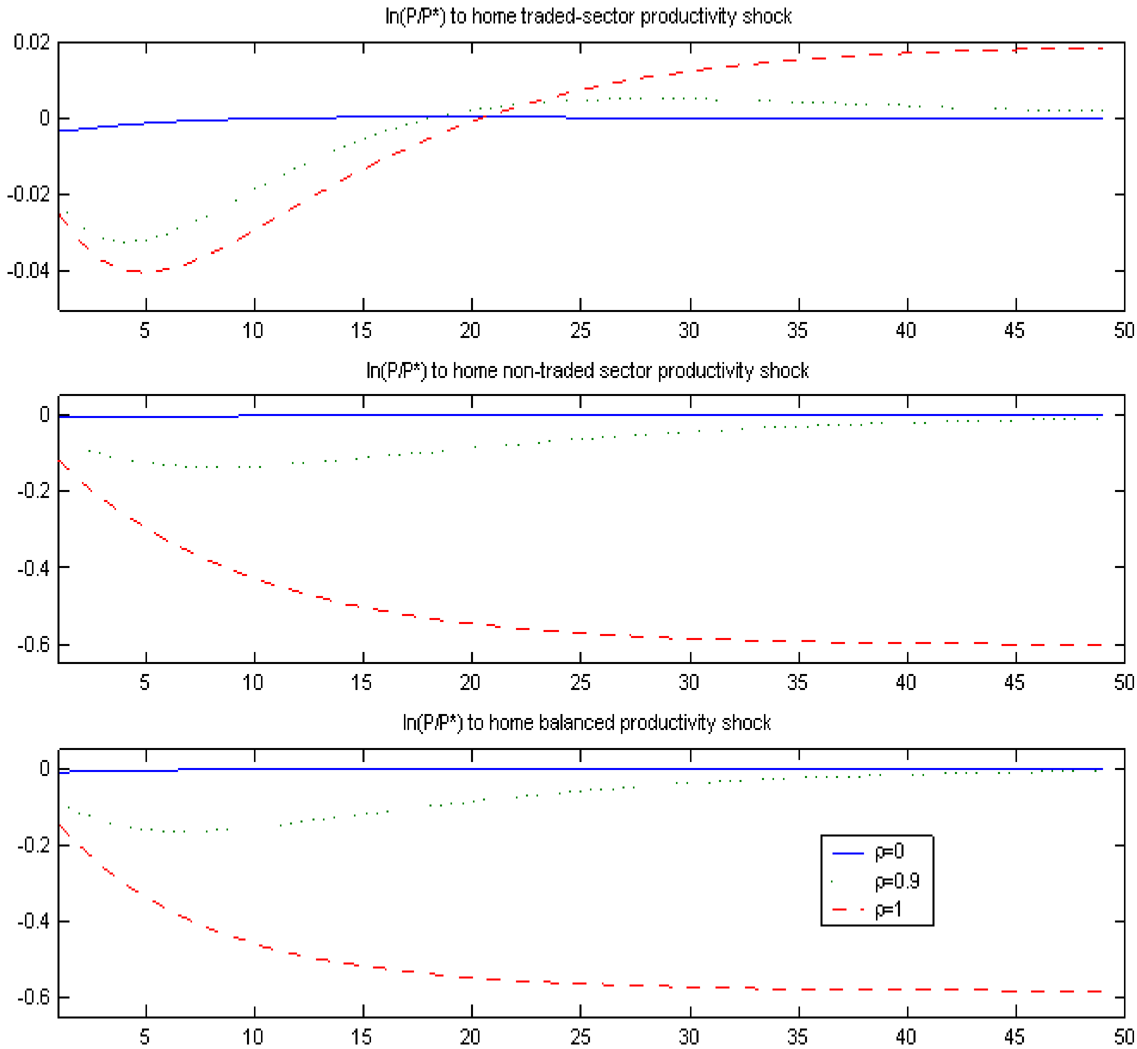

Fig. 12: Productivity shocks

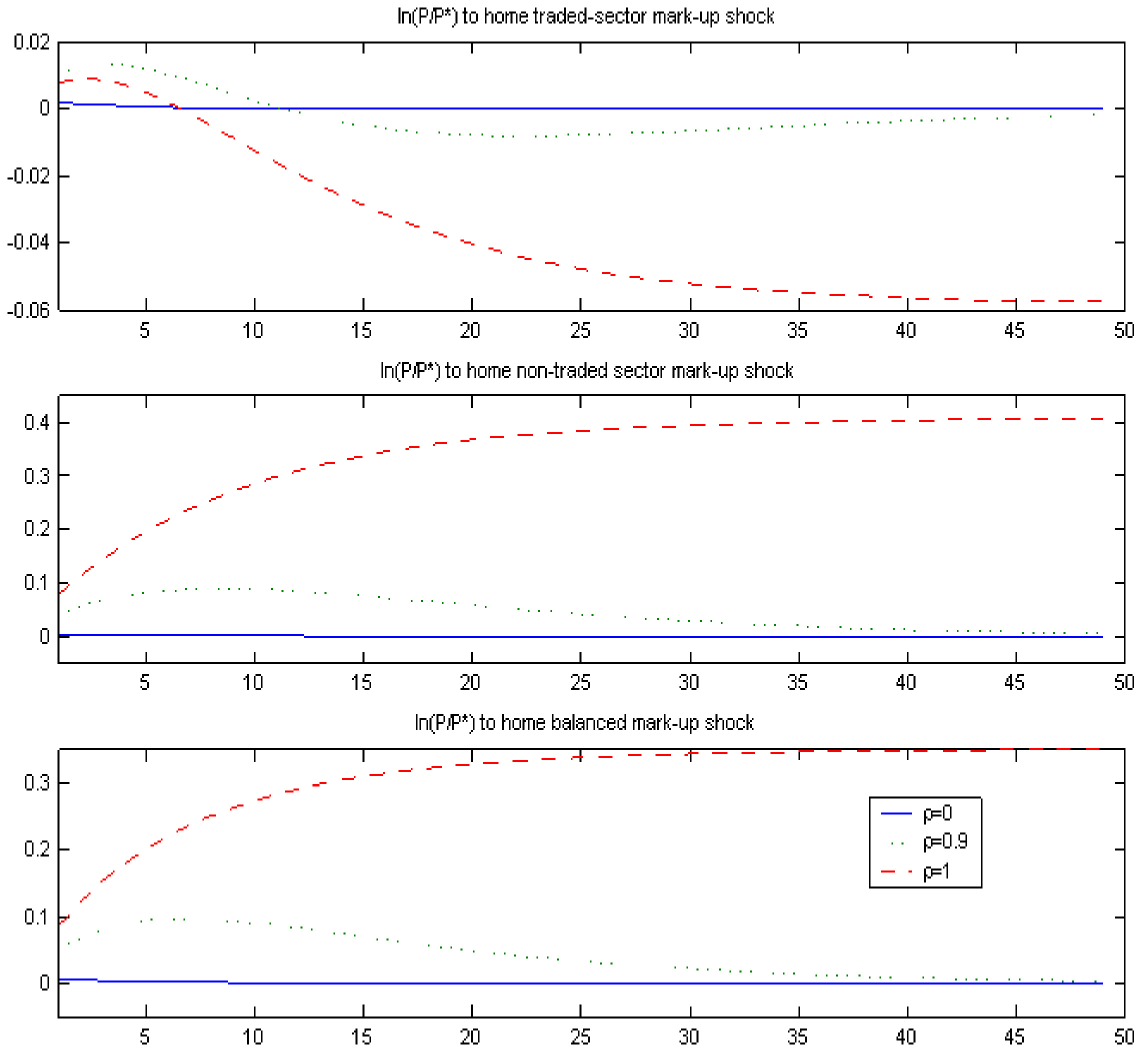

Fig.13: Mark-up shocks

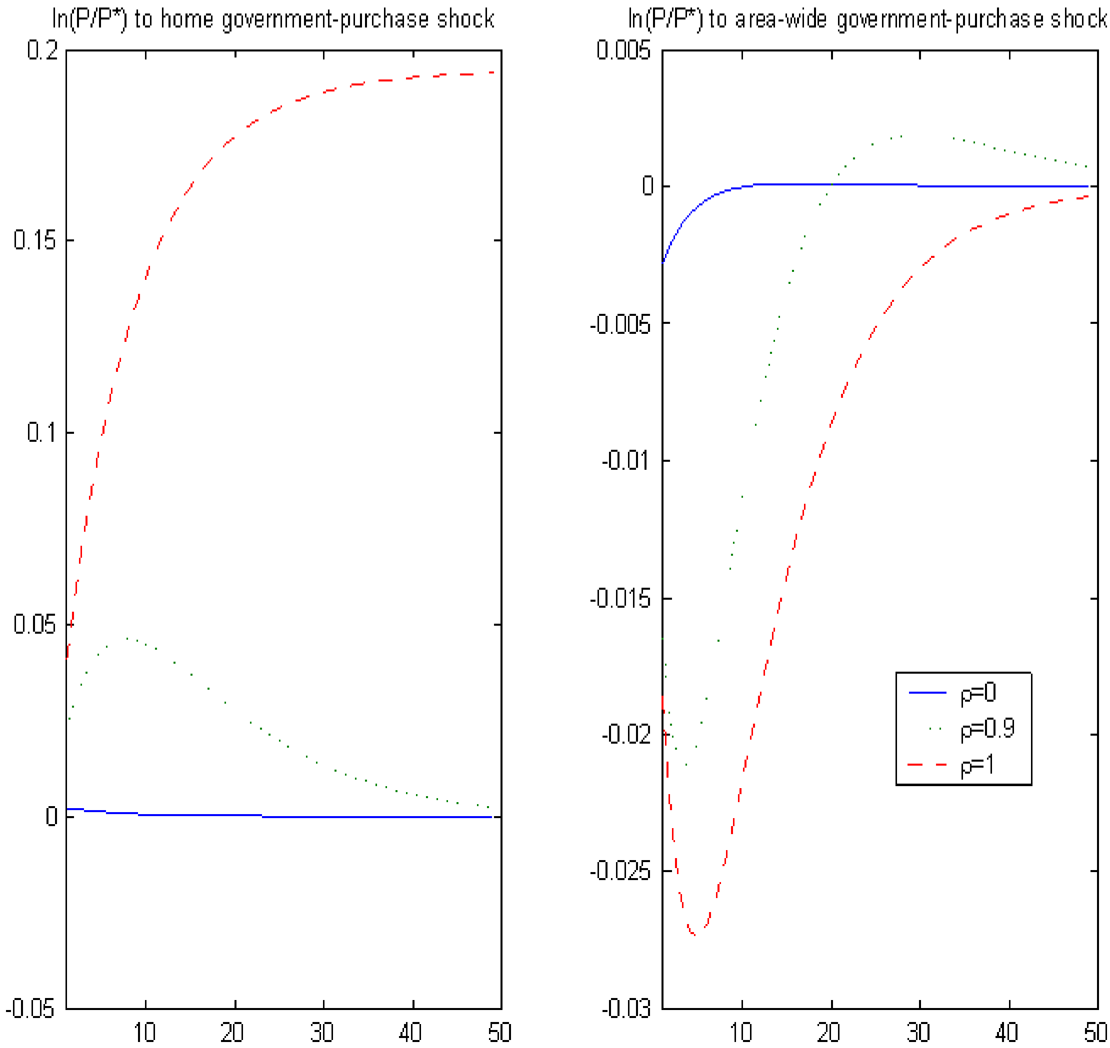

Fig. 14: Government purchase shocks

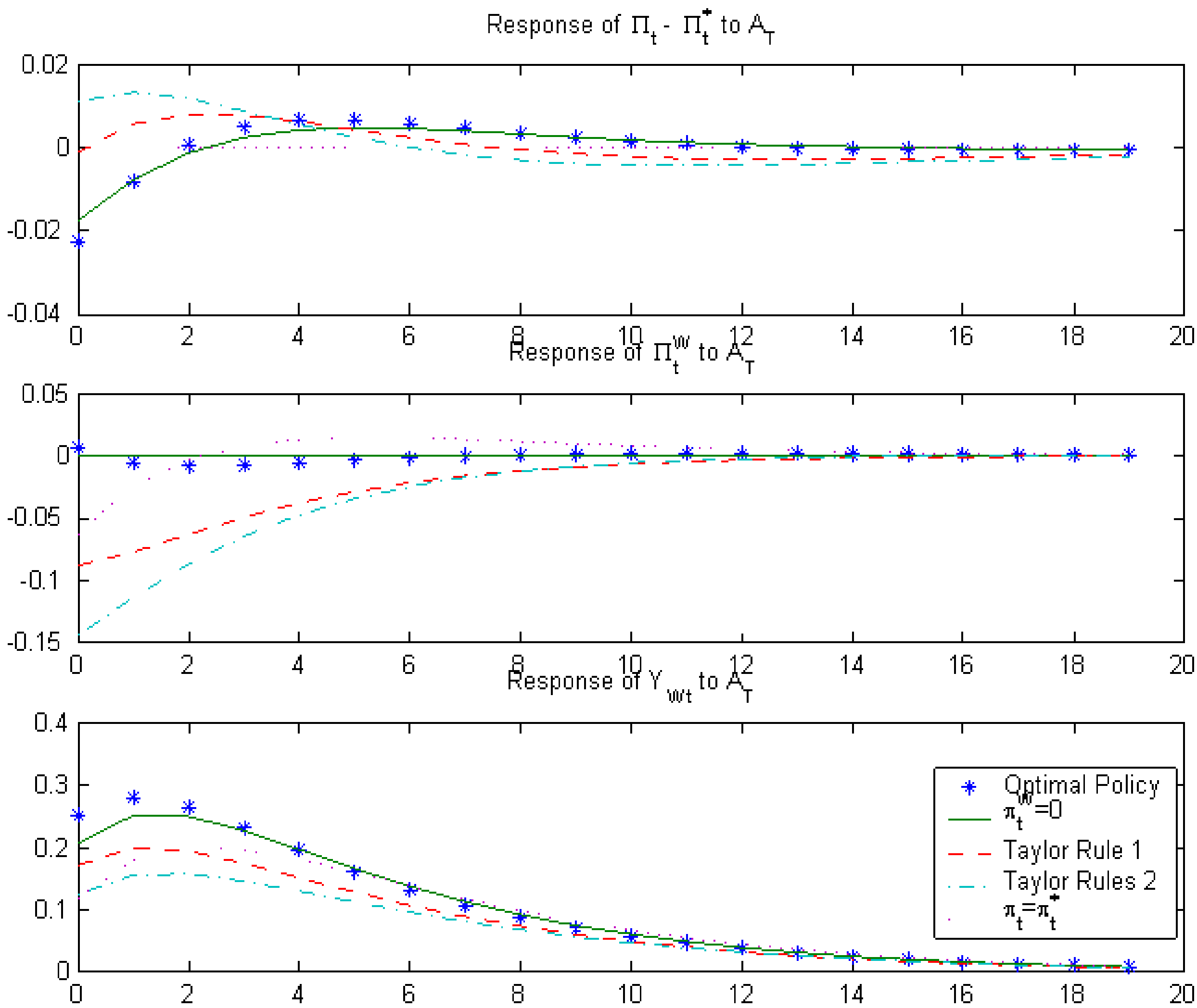

Fig. 15: Monetary policy and home trade sector productivity

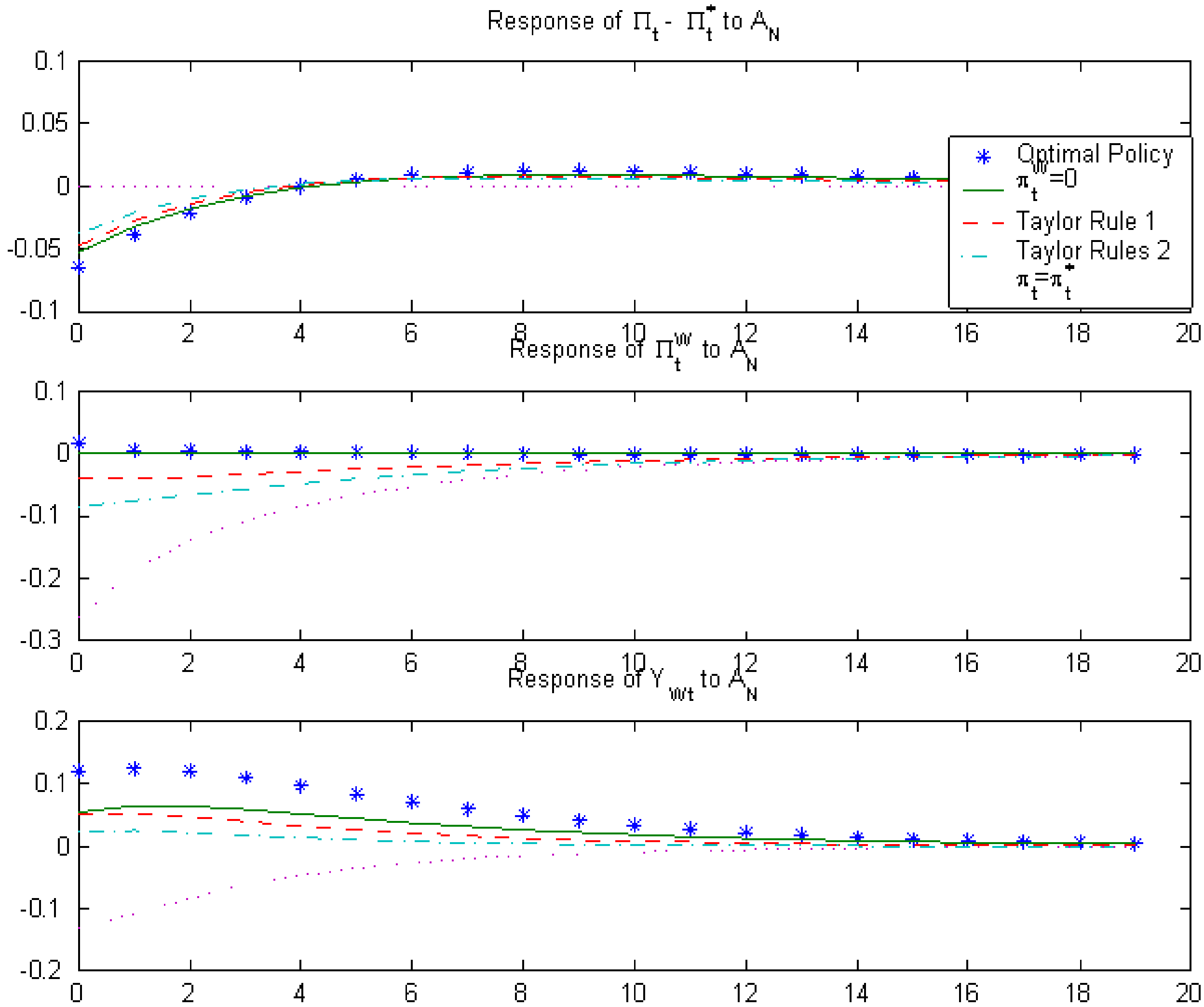

Fig. 16: Home non-trade sector productivity shock

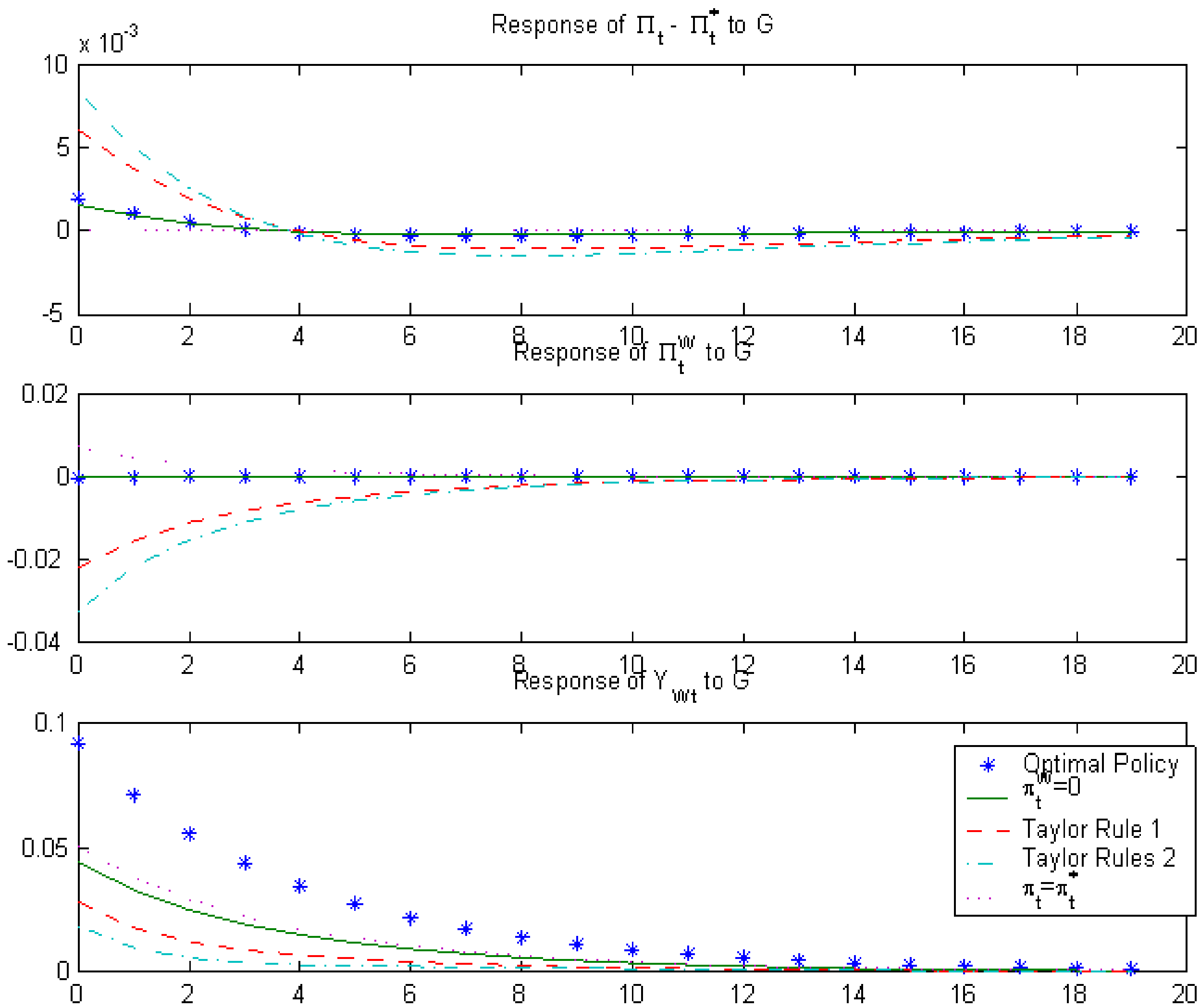

Fig. 17: Government purchase productivity shock

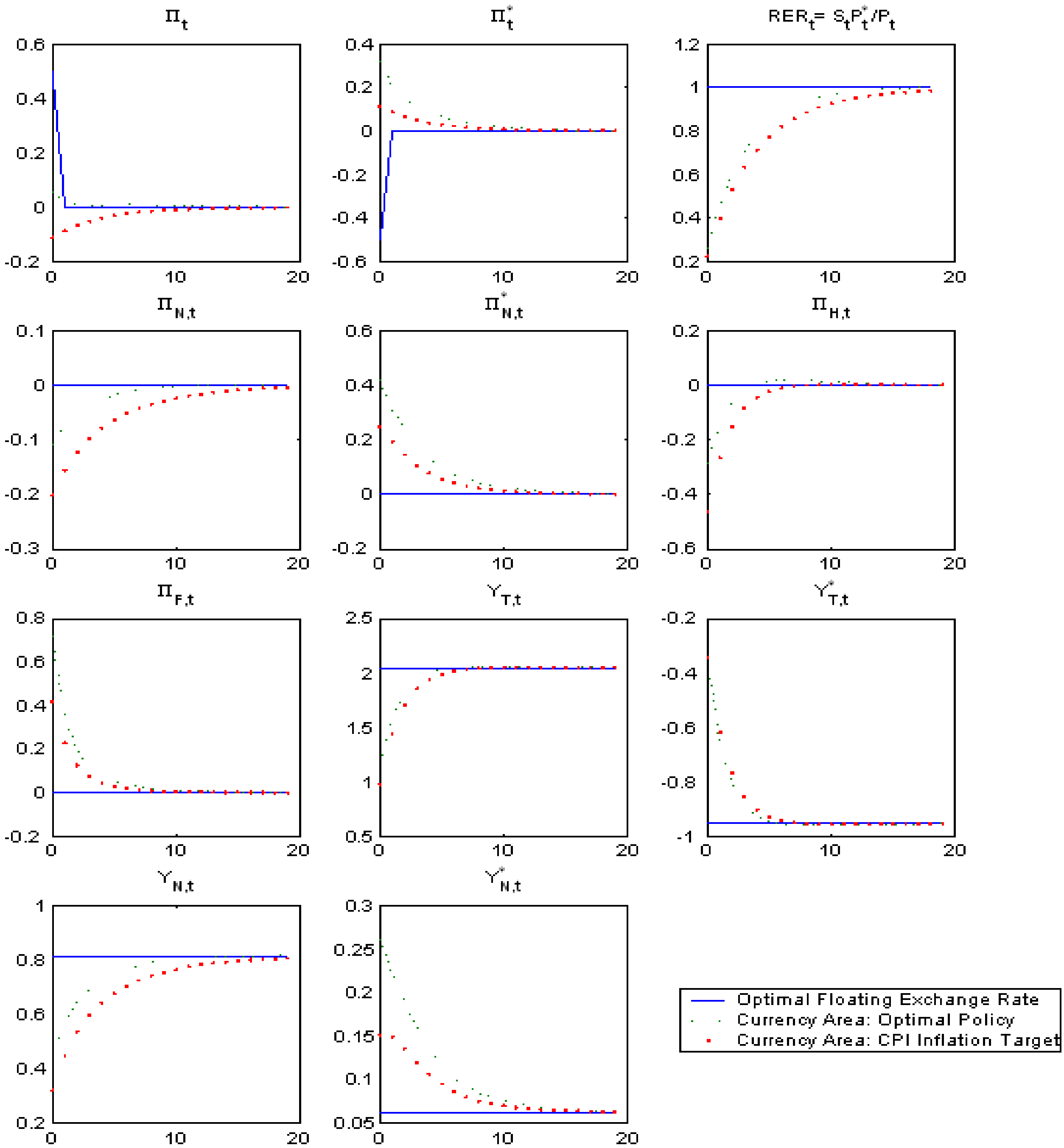

Fig. 18: Real exchange rate misalignment: responses under different monetary regimes