

# Expectations Driven Fluctuations and Stabilization Policy\*

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## PRELIMINARY

### Abstract

This paper explores the constraints imposed by expectations formation on the effectiveness of stabilization policy. Agents have incomplete information about the economic environment and form beliefs by extrapolating from observed patterns in historical data. Regimes with Ricardian fiscal policy (as in the standard account of monetary policy design) and also non-Ricardian fiscal policy are considered. In both cases macroeconomic stabilization requires tighter coordination of fiscal and monetary policy than under a rational expectations analysis. However, the latter is demonstrated to be more robust in the sense that under learning dynamics they are less prone to self-fulfilling expectations. Furthermore, economies with Ricardian fiscal policies are shown to be more stable the higher the degree of nominal rigidities in price setting. For non-Ricardian regimes, the converse is true. In all regimes, instability is mitigated by central bank communication of the monetary policy rule. However, regardless of the fiscal policy regime, economy's with higher average debt to output ratios tend to be prone to expectations driven instability. Hence, even in Ricardian regimes the precise choice of fiscal policy — in terms of its steady state implications — will be relevant to expectations stabilization. These findings are all in direct contrast to the predictions of a rational expectations equilibrium analysis of the model.

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\*The views expressed in the paper are those of the authors and are not necessarily reflective of views at the Federal Reserve Bank of New York or the Federal Reserve System. The usual caveat applies.

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# 1 Introduction

In the standard account of monetary policy design, nominal interest rates are determined to actively stabilize inflation and output. Less emphasized, but no less important, is the accompanying assumption that fiscal policy is Ricardian in nature — taxes are assumed to adjust in such a way as to ensure intertemporal solvency of the government budget. Under these assumptions, a central recommendation is that monetary policy should satisfy the Taylor principle: nominal interest rates should be adjusted more than one for one with variations in inflation. Yet there are clearly historical examples in which this account of desirable policy interaction might be thought inaccurate. In particular, periods of war time finance are likely better characterized as having a more dominate role for fiscal policy in which government expenditures and taxation evolve largely independently of the state of the economy. In such cases of regime change, it is no longer obvious that economic actors ought to expect future taxation to adjust in the necessary way to ensure solvency of the government’s intertemporal budget constraint.

What are the consequences of monetary and fiscal policy configurations of this kind? A recent literature under the rubric *fiscal theory of the price level* — see, inter alia, Leeper (1991), Leeper and Yun (2005), Sims (1994) and Woodford (1996, 2001) — seeks to answer this question. It demonstrates that certain choices of fiscal policy, for those that might be associated with war time finance where government primary surpluses may be determined independently of the state of the economy, can constrain a central bank’s ability to control the price level. In contrast to the standard account of policy design, fiscal policy can have monetary consequences, and therefore affect the evolution of inflation and output. Models of regime change that permit such equilibria have also garnered some empirical support — see Davig and Leeper (2005a).

This paper seeks to further understand the interaction of fiscal and monetary policy, giving particular emphasis to the role of expectations formation, relaxing the maintained assumption in the aforementioned literature that agents have rational expectations. The analysis is concerned with the constraints that a flexible form of beliefs impose on choices of fiscal and monetary policy for macroeconomic stabilization. Beliefs are consistent with regime

change but nonetheless nest the beliefs associated with rational expectations equilibrium of any given policy regime.

Under rational expectations agents are assumed to fully understand the economic model including the prevailing policy regime and to hold beliefs that accurately reflect the likelihood of any regime holding in future dates. Yet periods of regime change may involve policy decisions which households and firms have little experience with, leaving them unable to determine the objective probability laws implied by the economic model. The analysis takes this observation seriously. Agents are not assumed to have a complete economic model of the determination of macroeconomic aggregates. They understand only their *own objectives and constraints* and use atheoretical regressions to form forecasts of those aggregate state variables relevant to their decision problems. As additional data becomes available, agents update their beliefs. Subjective expectations may therefore deviate from the optimal expectations given the actual data generating process implied by the model. The constraints that such belief formation impose on policy design anchor this study, with particular interest in the conditions under which subjective beliefs converge to the objective probability laws implied by the model.

Assuming adaptive learning implies agents can only make inferences about the true data generating process of macroeconomic aggregates, and, therefore, the policy regime, through the observation of historic data. And while a model of regime switching will not formally be developed, in such an economic environment the adopted beliefs have the advantage that expectations about future regimes need not be specified. Only past outcomes matter to belief formation. As patterns in the historical data change, beliefs adjust accordingly. Modelling beliefs this way is consistent with the existence of possibly non-recurring policy regimes, which in our opinion reflects the historical account of US monetary policy.

We adopt a simple model of inflation and output gap determination of the kind used in many recent analyses of monetary policy — see, for instance, Bernanke and Woodford (1997), Clarida, Gali, and Gertler (1999) and Woodford (1999). Monetary and fiscal policy are described by simple one parameter classes of rule. Nominal interest rates are determined by a Taylor type rule that responds to inflation expectations. Taxes are lump-sum and the government's expenditure and tax plans are assumed to be well captured by a rule describing

the evolution of primary surpluses as a function of the current real value of outstanding debt.

Concerning central bank and fiscal authorities, it is often assumed that policy decisions are taken with full knowledge of current aggregate economic conditions, such as the level of output, potential output and the growth rate of prices. In this paper we consider the more realistic case where policy authorities form expectations about the current and future states of the economy. As a result, they are likely to respond with a delay to changing economic conditions, making the task of active macroeconomic stabilization more challenging. See Friedman (1968), Orphanides (2003a ,2003b) and McCallum (1999) for discussions of informational constraints on policy decision making.

In considering the consequences of various fixed choices of monetary and fiscal policy for the paths of inflation and debt, we are interested in determining the conditions under which policy choices and learning generate macroeconomic instability through self-fulfilling expectations. Indeed, an early critique of monetary policy when implemented through nominal interest rate rules due to Friedman (1968) was founded on expectations driven instability arising from households and firms extrapolating from observed patterns in data.<sup>1</sup>

The central findings of the paper can be summarized as follows. First, if central banks are informationally constrained, in sense that monetary policy decisions are conditioned on expectations of inflation rather than the current inflation rate, then regimes with non-Ricardian fiscal policies are less prone to instability generated by self-fulfilling expectations. For Ricardian policies, stability requires the nominal interest rate to be increased by implausible magnitudes when inflation expectations rise. Moreover, this instability arises regardless of the choice of fiscal policy within the Ricardian class. In contrast, for the associated rational expectations version of the model, the Taylor principle holds. For non-Ricardian regimes stability depends on the precise choice of fiscal and monetary policies within the family of rules being considered. However, for a given choice of monetary policy there always exists a

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<sup>1</sup>A further ground for being interested in models of non-rational expectations is that even if one is only concerned with policy regimes that are Ricardian in nature, such fiscal policies can nonetheless have Keynesian expenditure effects from changes in lump-sum taxes. This does not require myopic rule-of-thumb households as in the analysis of Gali, Lopez-Salido, and Valles (2006). Our analysis assumes agents optimize over an infinite horizon subject to current beliefs. Non-Ricardian effects of Ricardian fiscal policy emerge because agents imperfectly forecast future tax changes. This is shown to have a number of implications for policy design.

choice of fiscal policy that ensures stability under learning dynamics. An implication is that a much greater degree of coordination is required between monetary and fiscal policy under learning dynamics than under rational expectations.<sup>2</sup>

Second, there exists an important interaction between the degree of nominal rigidities and policy. In contrast to a rational expectations equilibrium analysis of our model in which determinacy conditions are independent of the degree of price stickiness, convergence under learning dynamics depends critically on these rigidities. The dependence is different across Ricardian and non-Ricardian fiscal policies. Economies with higher degrees of price stickiness tend to be more stable under Ricardian regimes and less stable under non-Ricardian regimes. The difference is due to the role of the price level in equilibrium determination in each case. Regimes with non-Ricardian fiscal policies require a specific kind of adjustment in goods prices to achieve equilibrium in any period. The greater the degree of nominal rigidity the more difficult is this adjustment.

Third, the existence of instability in the benchmark model raises the question of how macroeconomic stabilization might better be achieved. Central bank communication is here modeled as households and firms knowing the precise rule being used to implement monetary policy. The announced rule is perfectly credible and market participants assume the rule will remain in place for the indefinite future. Central bank communication unambiguously improves stabilization of expectations under both Ricardian and non-Ricardian fiscal policies: self-fulfilling expectations arise under a smaller region of the model parameter space. However, there are still important differences in stability conditions relative to the rational expectations case. The nature of fiscal policy affects conditions for convergence in both types of policy regime as an economy's average debt-to-output ratio can critically affect convergence to rational expectations equilibrium. Specifically, we determine conditions for which, if an economy's average ratio of debt to output is sufficiently high, then instability will always occur under learning dynamics. Under rational expectations, determinacy/stability condi-

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<sup>2</sup>This paper views the informationally constrained central bank as a realistic benchmark. However, a corollary of these results is that if the central bank is able to respond to an accurate estimate of the current inflation rate, then the conditions for determinacy of rational expectations equilibrium and convergence under learning dynamics coincide.

tions are independent of this quantity as are the reduced form dynamics. In the special case that a government pursues a zero debt policy, the stability conditions coincide under rational expectations and learning.

Fourth, as corollary, is that non-Ricardian policies have Ricardian effects not only out of rational expectations equilibrium — because agents may fail to accurately forecast future tax changes — but also in terms of determining the likelihood of convergence to rational expectations equilibrium. Because convergence depends on the ratio of debt to output, the specific choice of “steady state” policy will be relevant. Again this is not a property of the model under rational expectations. It is a property that emerges from consideration of non-rational expectations under optimizing behavior.

This paper builds directly on the analysis of Leeper (1991). In contrast to that paper, we consider a model with sticky prices and non-rational expectations. Besides this, the paper is similar in spirit. Evans and Honkapohja (2006) also consider Leeper’s model under learning dynamics. The analysis here advances their findings by considering a model in which agents are optimizing conditional on their beliefs. This has the advantage that intertemporal budget constraints and transversality conditions are accounted for. This is particularly relevant to analyzing the fiscal theory of the price level since this theory is explicitly grounded on implications of shifting expectations of households’ intertemporal budget constraints. We also consider economies with a degree of nominal rigidity which is shown to have a richer set of implications for inflation and debt dynamics and therefore stabilization policy. In contrast to our results, Evans and Honkapohja (2006) for the most part conclude that learning dynamics impose similar constraints on monetary and fiscal policy as does a rational expectations analysis.

Aside from the immediate connections to the literature on the fiscal theory of the price level, this paper also builds on various papers exploring economic environments that question the desirability of the Taylor principle as a foundation of monetary policy design. In particular Benhabib, Schmitt-Grohe, and Uribe (2001) show that incorporating money in household and firm decisions leads to indeterminacy in the Ricardian regime. Building on Edge and Rudd (2002), Leith and von Thadden (2006) show in a Leeper (1991) style model with capital that

conditions for determinacy of rational expectations equilibrium depend on the debt-to-output ratio as in results presented here.

The paper proceeds as follows. Section 2 lays out a simple model of output gap and inflation determination. Section 3 outlines belief formation of private agents. Section 4 then discusses implications of non-Ricardian fiscal policies and hence the fiscal theory of the price level under both the rational expectations and learning assumptions. Section 5 turns to our benchmark model in which policy making is subject to informational constraints. Section 6 considers the role of communication and the use of alternative information in macroeconomic aggregates to enhance the stability properties of a given choice of fiscal and monetary policy. Section 7 concludes.

## 2 The Model

### 2.1 Households

The economy is populated by a continuum of households which seek to maximize future expected discounted utility

$$\hat{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} [U(C_T^i + g; \xi_T) - v(h_T^i; \xi_T)] \quad (1)$$

where utility depends on a consumption index,  $C_T^i$ , of the economy's available goods, a vector of aggregate preference shocks,  $\xi_T$ , the amount of labor supplied for the production of each good  $j$ ,  $h_T^i$  for  $T \geq t$  and the quantity of government expenditures  $g > 0$ . The second term in the brackets captures the disutility of labor supply. The consumption index,  $C_t^i$ , is the Dixit-Stiglitz constant-elasticity-of-substitution aggregator of the economy's available goods and has an associated price index written, respectively, as

$$C_t^i \equiv \left[ \int_0^1 c_t^i(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \quad \text{and} \quad P_t \equiv \left[ \int_0^1 p_t(j)^{1-\theta} dj \right]^{\frac{1}{1-\theta}} \quad (2)$$

where  $\theta > 1$  is the elasticity of substitution between any two goods and  $c_t^i(j)$  and  $p_t(j)$  denote household  $i$ 's consumption and the price of good  $j$ . Given the absence of real money balances from the period utility function (1) the model is interpretable as an economy in a cashless

limit — see Woodford (2003). The discount factor is assumed to satisfy  $0 < \beta < 1$ . The function  $U(C_t; \xi_t)$  is concave in  $C_t$  for a given value of  $\xi_t$  and  $v(h_t(i); \xi_t)$  is convex in  $h_t(i)$  for a given value of  $\xi_t$ .

$\hat{E}_t^i$  denotes the subjective beliefs of household  $i$  about the probability distribution of the model's state variables: that is, variables that are beyond agents' control though relevant to their decision problems. The presence of a hat “^” denotes non-rational expectations and the special case of rational expectations will be denoted by  $E_t$ . Beliefs are assumed to be homogenous across households (though this is not understood to be the case by agents). Agents are not fully rational in their economic decisions. Following Marcet and Sargent (1989) and assume that in forming beliefs about future events agents do not take into account that they will update their own beliefs in subsequent periods leading to a decision problem that is not recursive.<sup>3</sup> When households solve their decision problem at time  $t$ , beliefs held at that time satisfy standard probability laws, so that standard solution methods apply. The specific details of beliefs and the manner in which agents update beliefs are developed in section 3.

Asset markets are assumed to be complete. The household's flow budget constraint is

$$\hat{E}_t^i Q_{t,t+1} A_{t+1}^i \leq A_t^i + P_t Y_t^i - T_t - P_t C_t^i \quad (3)$$

where  $Q_{t,t+1}$  prices a unit of income in each possible state in period  $t+1$ ,  $A_t^i$  is the household's stock of wealth,  $Y_t^i$  real income in period  $t$  and  $T_t$  denotes lump sum taxes and transfers. The household receives income in the form of wages paid,  $w_t$ , for labor supplied to a common factor market and used in the production of each good,  $j$ , and dividends from each firm. Period nominal income is therefore determined as

$$P_t Y_t^i = w_t h_t^i + \int_0^1 \Pi_t(j) dj$$

for each household  $i$ . This constraint must hold in all future dates and states of uncertainty. The only asset in non-zero net supply is government debt to be discussed below. Finally, there is a No-Ponzi constraint

$$\lim_{T \rightarrow \infty} \hat{E}_t^i Q_{t,T} A_T^i \geq 0$$

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<sup>3</sup>Cogley and Sargent (2006) adduce evidence that the welfare implications of not accounting for future revisions of beliefs is small.

where  $Q_{t,T} = \prod_{s=t}^{T-1} Q_{s,s+1}$  for  $T \geq 1$  and  $Q_{t,t} = 1$ .

Household optimization requires:

$$Q_{t,t+1} = \beta \frac{U_c(C_{t+1}^i)}{U_c(C_t^i)} \frac{P_t}{P_{t+1}} \quad (4)$$

and

$$\frac{U_c}{v_h} = \frac{w_t}{P_t}$$

for all states and dates and

$$\hat{E}_t^i \sum_{T=t}^{\infty} Q_{t,T} P_T C_T^i = A_t^i + \hat{E}_t^i \sum_{T=t}^{\infty} Q_{t,T} P_T \left[ Y_T^i - \frac{T_T}{P_T} \right] \quad (5)$$

The first relation is the household's Euler equation; the second the intratemporal condition for labor supply; while the third provides the intertemporal budget constraint.

## 2.2 Firms

A continuum of firms  $j \in [0, 1]$  face a demand curve  $y_t(j) = Y_t(p_t(j)/P_t)^{-\theta}$  for their good and take aggregate output  $Y_t$  and aggregate prices  $P_t$  as parametric in each period  $t$ . Good  $j$  is produced using a single labor input  $h(j)$  according to the relation  $y_t(j) = A_t h_t(j)$  where  $A_t > 0$  is an exogenous technology shock.<sup>4</sup>

Firms are assumed to face a price setting problem of the kind proposed by Calvo (1983) and Yun (1996). A fraction  $0 < \alpha < 1$  of goods prices are held fixed in any given period, while a fraction  $1 - \alpha$  of goods prices are adjusted. Given homogeneity of beliefs, all firms having the opportunity to change their price in period  $t$  face the same decision problem and therefore set a common price  $p_t^*$ . The Dixit-Stiglitz aggregate price index must therefore evolve according to the relation

$$P_t = \left[ \alpha P_{t-1}^{1-\theta} + (1 - \alpha) p_t^{*1-\theta} \right]^{\frac{1}{1-\theta}}. \quad (6)$$

The firm's price-setting problem in period  $t$  is to maximize the expected present discounted value of profits

$$\hat{E}_t^j \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} \left[ \Pi_T^j(p_t(j)) \right] \quad (7)$$

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<sup>4</sup>The assumption of linear production technology facilitates analytical results. Numerical analysis reveals that the central findings of the paper generalize with alternative production structures.

where

$$\Pi_T^j(p) = Y_T P_T^\theta p^{1-\theta} - w_T Y_T P_T^\theta p^{-\theta} / A_T \quad (8)$$

by choice of  $p$ . The factor  $\alpha^{T-t}$  in the firm's objective function is the probability that the firm will not be able to adjust its price for the next  $(T-t)$  periods. The existence of nominal rigidities in pricing requires firms to forecast macroeconomic conditions into the indefinite future.

### 2.3 Monetary and Fiscal Authorities

The central bank is assumed to implement monetary policy according to a one parameter family of interest rates rules of the form

$$i_t = \bar{i} \pi_t^{\phi_\pi}$$

and

$$i_t = \bar{i} \left( E_{t-i}^{cb} \pi_t \right)^{\phi_\pi}$$

where  $E_{t-i}^{cb} \pi_t$  is a measure of current inflation. The case where  $i = 0$  is the case that is most analyzed in the literature: the central bank has precise information about the current inflation rate. In the case  $i = 1$ , the central bank has to form expectations regarding the current evolution of prices.

Also,  $\bar{i} > 0$  is a constant equal to the steady state gross real interest rate and  $\phi_\pi \geq 0$ . This class of rule will later be generalized to permit the central bank to respond to measures of aggregate demand, as often argued to be desirable.

The study of optimal policy is not pursued on two grounds. Simple rules of the postulated form have been shown to be robust across model environments and, if appropriately chosen, to deliver much of the welfare gains inherent in more complex optimal policy rules — see Schmitt-Grohe and Uribe (2005). Second, optimal policy in the context of learning dynamics is not trivial. Assumptions have to be made about the precise information a central bank has about the structure of the economy. While households and firms need only know their own objectives and constraints to make decisions, for a central bank to design optimal policy, it

needs accurate information on all agents in the economy including the nature of beliefs. This is informationally demanding. We leave the study of such policies to future work.

The fiscal authority finances government purchases of  $g$  per period by issuing public debt and levying lump-sum taxes. Denoting  $W_t$  as the outstanding government debt at the beginning of any period  $t$ , and assuming for simplicity that the public debt is comprised entirely of one period riskless nominal Treasury bills, then government liabilities evolve according to

$$W_{t+1} = (1 + i_t) [W_t + gP_t - T_t].$$

For later purposes it is convenient to rewrite this constraint as

$$b_{t+1} = (1 + i_t) (b_t \pi_t^{-1} - s_t)$$

where

$$s_t = \frac{T_t}{P_t} - g$$

denotes the primary surplus and  $b_t = W_t/P_{t-1}$  a measure of the real value of the public debt. Observe that  $b_t$  is a predetermined variable since  $W_t$  is determined a period in advance.

The model is closed with an assumption on the path of primary surpluses  $\{s_t\}$ .<sup>5</sup> Analogous to the monetary authority, it is assumed that the fiscal authority adjusts the primary surplus according to the one parameter family of rules

$$s_t = \bar{s} \left( \frac{b_t}{\bar{b}} \right)^\tau$$

where  $\bar{s}, \bar{b} > 0$  are constants coinciding with the steady state level of the primary surplus and the public debt respectively.  $\tau > 0$  is a policy parameter. Similar remarks on the matter of optimal policy apply here.

## 2.4 General Equilibrium and Log-linear Approximation

Conditional on beliefs and the path of aggregate prices and output, the above sections fully characterize household and firm decisions. Determining the evolution of aggregate prices

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<sup>5</sup>This is without loss of generality. It would be straightforward to specify separate policies for the revenues and expenditures of the government accounts without altering the substantive implications of the model.

requires aggregation of these decisions and the imposition of market clearing conditions. Appendix A determines a log-linear approximation to the optimal decision rules of households and firms and their aggregation. Application of a log-linear approximation to the market clearing conditions

$$\int_0^1 C_t^i di + g = C_t + g = Y_t \quad \text{and} \quad \int_0^1 A_t^i di = W_t \quad (9)$$

for the goods and asset markets respectively delivers the following model of the macroeconomy.<sup>6</sup> The model comprises aggregate demand and supply relations of the form

$$\hat{x}_t = \delta\beta^{-1} (\hat{b}_t - \hat{\pi}_t) - \beta^{-1} \delta \hat{s}_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} [(1-\beta)(\hat{x}_{T+1} - \delta \hat{s}_{T+1}) - (\sigma - \delta)(\hat{i}_T - \hat{\pi}_{T+1}) + \sigma r_T] \quad (10)$$

and

$$\hat{\pi}_t = \kappa \hat{x}_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} [\kappa\alpha\beta \hat{x}_{T+1} + (1-\alpha)\beta \hat{\pi}_{T+1}] \quad (11)$$

where  $\hat{x}_t = \hat{y}_t - \hat{y}_t^n$  is the output gap, the deviation of actual output from the natural rate of output that obtains absent nominal rigidities,  $\hat{\pi}_t$  is the inflation rate,  $\hat{i}_t$  the nominal interest rate,  $\hat{s}_t$  the primary surplus,  $\hat{b}$  the real value of outstanding government debt and  $r_t$  the natural rate of interest — a composite of primitive model shocks — with all “ $\hat{\cdot}$ ” variables being in log deviations from steady state values.  $s_c = \bar{C}/\bar{Y}$  is the steady state consumption output ratio and  $\delta = \bar{s}/\bar{Y}$  is the steady state ratio of primary surpluses to output. Under the maintained assumptions of linear production and assuming for simplicity a linear dis-utility of labor gives

$$\kappa = \frac{(1-\alpha)(1-\alpha\beta)\sigma^{-1}}{\alpha} > 0.$$

The first relation specifies aggregate demand and is analogous to permanent income theory: current output depends on a weighted combination of wealth (holdings of the public debt) and the discounted future value of after-tax income. In contrast to permanent income theory, the model allows for time variation in real interest rates and stochastic components of demand, accounting for the final two terms. The supply relation is a direct generalization

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<sup>6</sup>See also Preston (2005b) for related discussion.

of an expectations augmented Phillips curve. Because prices cannot be adjusted each period with some positive probability, firms must forecast future marginal costs into the indefinite future. These marginal costs can be shown in a log-linear approximation to be a linear function of inflation and the output gap. The terms in  $\alpha\beta$  appear since firms discount future profits at rate  $\beta$  – the shareholder’s discount rate – while  $\alpha$  accounts for the likelihood that the firms current output price will prevail in subsequent periods.

The evolution of the public debt is determined according to

$$\hat{b}_{t+1} = \beta^{-1} \left( \hat{b}_t - \hat{\pi}_t - (1 - \beta) \hat{s}_t \right) + \hat{i}_t \quad (12)$$

while the paths of the primary surplus and nominal interest rate are given by the policy rules

$$\hat{i}_t = \phi_\pi \hat{E}_{t-i}^{cb} \hat{\pi}_t \quad (13)$$

and

$$\hat{s}_t = \tau \hat{b}_t. \quad (14)$$

Hence the structural model comprises the five equations (10), (11) (12), (13) and (14). To close the model in the absence of the rational expectations assumption requires a set of assumptions on belief formation to which the discussion now turns.

### 3 Beliefs

Because the analysis is interested in household and firm behavior in regimes that are both unfamiliar and that may also change at some future date, it is assumed that agents learn adaptively using a recursive least squares algorithm. This assumption has the advantage that agents learn about the current policy regime only by observing historical data. Indeed in periods of significant change in the policy regime it seems hardly reasonable to suppose that households and firms are able to assign probabilities that necessarily coincide with the objective probabilities implied by the true economic model to the various objects that they must forecast in order to make decisions. And given that constraint, it is equally plausible that agents make use of historical data to form inferences about the future evolution of the economy. If there is a change in regime and therefore the underlying data generating process,

agents only learn about it through observing new data. Such an approach to modeling belief formation obviates the requirement of specifying what beliefs agents hold about future possible policy regimes, as would be the case in a rational expectations equilibrium analysis. As has been highlighted in recent discussion of determinacy of rational expectations equilibrium in regime switching models, analysis of this kind is difficult — see Davig and Leeper (2005a, 2005b) and Farmer, Waggoner and Zha (2006a, 2006b).

Convergence is assessed using E-Stability results outlined in Evans and Honkapohja (2001). E-Stability provides conditions under which, if agents make small forecasting errors relative to rational expectations, their learning behavior corrects these errors over time and ensures convergence to the underlying rational expectations dynamics.

Agents are assumed to have identical beliefs — though this is not known to each agent — and to construct forecasts using an econometric model that uses as regressors variables that appear in the minimum-state-variable solution of the associated rational expectations model. This is a strong informational assumption, but to the extent that instability arises given this knowledge, it seems unlikely that the policies under consideration would be more conducive to stability when private agents have a misspecified model that fails to nest the underlying rational expectations equilibrium — see Evans and Honkapohja (2001) for a discussion of learning dynamics with misspecified rules.

Under the policy rules specified in the previous sections, and those to be discussed in the sequel, rational expectations equilibria are linear in the variables  $\{\pi_{t-1}, b_{t-1}, r_t\}$ . Given the assumption that the natural rate of interest is determined by an exogenous i.i.d process, agents estimate the linear model

$$z_t = a_t + b_t z_{t-1} + \varepsilon_t \tag{15}$$

where  $z_t = [x_t \ \pi_t \ b_t \ i_t \ s_t]'$ ,  $\varepsilon_t$  a vector of residuals, and

$$a_t = \begin{bmatrix} a_{x,t} \\ a_{\pi,t} \\ a_{b,t} \\ a_{i,t} \\ a_{s,t} \end{bmatrix} \quad \text{and} \quad b_t = \begin{bmatrix} 0 & b_{x,t}^\pi & b_{x,t}^b & 0 & 0 \\ 0 & b_{\pi,t}^\pi & b_{\pi,t}^b & 0 & 0 \\ 0 & b_{b,t}^\pi & b_{b,t}^b & 0 & 0 \\ 0 & b_{i,t}^\pi & b_{i,t}^b & 0 & 0 \\ 0 & b_{s,t}^\pi & b_{s,t}^b & 0 & 0 \end{bmatrix}.$$

Estimation makes use of the entire history of data available in period  $t$ ,  $\{1, z_s\}_{s=0}^{s=t-1}$ . As additional data becomes available, agents update their beliefs according to the recursive least squares algorithm

$$\phi_t = \phi_{t-1} + t^{-1} R_t^{-1} w_{t-1} (z_{t-1} - \phi_{t-1}' w_{t-1})' \quad (16)$$

$$R_t = R_{t-1} + t^{-1} (w_{t-1} w_{t-1}' - R_{t-1}) \quad (17)$$

where the first equation describes how the belief coefficients,  $\phi = [a_t' \ \text{vec}(b_t)']'$ , are updated with each new data point  $w_t = \{1, z_{t-1}\}$ . Hence belief coefficients at time  $t$  are determined by data available at time  $t - 1$ .

Given homogeneity of beliefs, average forecasts can be computed as

$$\hat{E}_t z_T = (I_5 - b_t)^{-1} (I_5 - b_t^{T-t}) a_t + b_t^{T-t} z_t \quad (18)$$

for  $T > t$ , and where  $I_5$  is a  $(5 \times 5)$  identity matrix. In the flexible price version of the model, this forecasting system is truncated by the omission of the output gap,  $x_t$ , which no longer needs to be forecast. Note that expectations at time  $t$  are not predetermined. As an example consider

$$\hat{E}_t z_{t+1} = a_t + b_t z_t.$$

The belief parameters  $(a_t, b_t)$  are determined by information available at  $t - 1$ . However, given these parameters, expectations about future endogenous variables are conditioned on the state vector  $z_t$  which includes non-predetermined variables.

## 4 Foundations: Leeper Revisited

Over the past decade there has been a growing literature emphasizing the interaction of monetary and fiscal policy. A central insight is that monetary policy cannot be conducted without due regard being given to the type of policy being pursued by the fiscal authority. The fiscal theory of the price level asserts that choice of a non-Ricardian fiscal policy imposes constraints on the conduct of monetary policy to ensure intertemporal solvency of the government accounts. We here revisit the foundations of the fiscal theory of the price level and the seminal analysis of Leeper (1991) in the context of the model of section 2. The analysis is virtually identical to that paper, except for some minor differences in the adopted microfoundations, most notably the incorporation of nominal rigidities. Having established the conditions for determinacy of rational expectations equilibrium, our first major departure from Leeper (1991) considers agents whom form beliefs using adaptive learning. In this section we consider the case where the central bank can observe the current inflation rate. We then contemplate additional implications for policy design that arise from the interaction of nominal rigidities and informational limitations on the central bank when implementing policy.

### 4.1 The Fiscal Theory of the Price Level

To see directly the monetary consequences of fiscal policy, consider the household optimality conditions given by relations (4) and (5). Combining the Euler equation and intertemporal budget constraint provides

$$\hat{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \frac{U_c(C_T^i + g) P_t}{U_c(C_t^i + g)} C_T^i = A_t^i + \hat{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \frac{U_c(C_T^i + g) P_t}{U_c(C_t^i + g)} \left[ Y_T^i - \frac{T_T}{P_T} \right].$$

In a symmetric rational expectations equilibrium, the goods market equilibrium condition (9) holds in all periods  $T \geq t$  and  $C_T^i = C_T^j$ ,  $Y_T^i = Y_T^j$  and  $A_t^i = A_t^j$  for all  $i \neq j$  and in all periods  $T \geq t$ . Substituting these conditions into the above relation yields

$$\begin{aligned} \frac{W_t}{P_t} &= E_t \sum_{T=t}^{\infty} \beta^{T-t} \frac{U_c(Y_T)}{U_c(Y_t)} \left[ \frac{T_T}{P_T} - g \right] \\ &= E_t \sum_{T=t}^{\infty} \beta^{T-t} \frac{U_c(Y_T)}{U_c(Y_t)} s_T. \end{aligned} \tag{19}$$

Under the rational expectations assumption and making use of the Euler equation, this relation satisfies the log-linear approximation

$$\hat{b}_t - \hat{\pi}_t = E_t \sum_{T=t}^{\infty} \beta^{T-t} [(1 - \beta) \hat{s}_T - \beta (\hat{v}_T - \hat{\pi}_{T+1})]. \quad (20)$$

Both (19) and (20) indicate the real value of outstanding government liabilities is equal to the present discounted value of future primary surpluses. They differ insofar as the latter employs a log-linear approximation to the Euler equation to write output variations in terms of movements in real interest rates  $\hat{v}_T - \hat{\pi}_{T+1}$ .

As emphasized by Woodford (2001) and Leeper and Yun (2005), this intertemporal solvency condition is imposed on the government by household optimization. To understand the fiscal theory of the price level consider (19) — an analogous discussion applies to (20). Suppose for the sake of simplicity that the path of primary surpluses  $\{s_t\}$  is exogenously determined. In this case, under the assumption of flexible price setting, the right hand side of the intertemporal solvency condition is exogenously determined. Because the model assumes the government only to issue one period public debt, which is a predetermined variable, this intertemporal solvency condition imposes a restriction on equilibrium goods prices. This is the heart of the fiscal theory of the price level.

As an example, consider a government choosing to increase government expenditures by some constant amount each period (or equivalently a reduction in the level of taxes levied each period). This leads to a fall in the present discount value of primary surpluses. Because outstanding public debt is predetermined, equilibrium is guaranteed by an increase in the price level. This is a wealth effect. Households expect to pay a smaller present discounted value of taxes over their lifetime, implying a rise in permanent income and concomitantly expenditure in the current period. Prices must rise to clear the goods market.

Under learning dynamics, the equilibrium conditions (19) and (20) only hold if there is convergence to rational expectations equilibrium. Indeed the restriction embodied in these relations is one of the many equilibrium conditions about which agents are attempting to learn. Nonetheless, beliefs about the future state of government accounts continue to matter for the determination of the price level out of rational expectations equilibrium. To see this, recall

the aggregate demand relation (10). Because of the specification of monetary policy, and the fact that real debt  $\hat{b}_t$  is predetermined, the aggregate demand relation determines prices as a function of expectations about the future path of the natural rate of interest and also primary surpluses — even if expectations of the latter are inconsistent with intertemporal solvency of the government accounts and even if fiscal policy is Ricardian. Hence shifting expectations about future taxes lead to revised beliefs about household’s intertemporal budget constraint. The resulting wealth effects in turn alter current expenditure plans.

At a technical level the analysis is interested in learning under what conditions the economy converges to an equilibrium in which intertemporal solvency is ensured. Of course, if the intertemporal solvency condition is known by households to be satisfied, then the aggregate demand equation is given by

$$\hat{x}_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} [(1 - \beta) \hat{x}_{T+1} - \sigma (\hat{i}_T - \hat{\pi}_{T+1} - r_T)].$$

But this does not imply there are no monetary consequences of fiscal policy — though it does preclude non-Ricardian effects of Ricardian tax policies out of rational expectations equilibrium. Whether there are monetary consequences depends on the nature of equilibrium expectations which are in turn determined by whether fiscal policy is assumed to be Ricardian or not. We now turn to characterizing the equilibrium dynamics for inflation and debt under each of these assumptions on fiscal policy and the assumption of rational expectations.

## 4.2 Rational Expectations

The following characterizes the set of equilibria generated by the model under the rational expectations assumption. Following Woodford (1996), we characterize fiscal policy as being either locally Ricardian, as in the standard account of monetary policy, or locally non-Ricardian, in which case there are monetary consequences of fiscal policy. To understand the distinction recall the fiscal policy rule  $\hat{s}_t = \tau \hat{b}_t$ . If  $\tau = 0$  then the policy rule has the interpretation that the primary surplus is exogenous and completely independent of the current state of the economy. This is a non-Ricardian policy, where the adjective locally reflects the use of a log-linear approximation. More generally the degree of response to the current state

of the economy determines the stability properties of the model. If the primary surplus is sufficiently responsive to the current state then policy will be locally Ricardian and taxes are adjusted to ensure intertemporal solvency of the government accounts. The following proposition states precisely the conditions for each policy regime to obtain. All proofs are collected in the appendix.

**Proposition 1** *In the model given by relations (10), (11), (12), (13) and (14) and rational expectations, if the stated conditions hold, then there exist unique bounded rational expectations equilibrium of the indicated form.*

*i) If  $\phi_\pi > 1$  and  $1 < \tau < (1 + \beta) / (1 - \beta)$  then fiscal policy is locally Ricardian and*

$$\hat{\pi}_t = \phi^R \hat{r}_t^n$$

*ii) If  $0 \leq \phi_\pi < 1$  and  $0 \leq \tau < 1$  or  $\tau > (1 + \beta) / (1 - \beta)$  then fiscal policy is locally non-Ricardian and*

$$\hat{\pi}_t = \phi_b^{NR} \hat{b}_{t-1} + \phi_\pi^{NR} \hat{\pi}_{t-1} + \phi_r^{NR} \hat{r}_t^n$$

*and*

$$b_t = \frac{(1 - \beta \phi_\pi)}{1 - (1 - \beta) \tau} \sum_{j=0}^{\infty} \left( \frac{\beta}{1 - (1 - \beta) \tau} \right)^j E_t \pi_{t+j}$$

*where the expressions for the coefficients are given in the Appendix.*

Part one of the proposition accords with the standard account of monetary policy. Fiscal policy is Ricardian and taxes are adjusted to guarantee intertemporal solvency of the government budget. Inflation is then determined independently of the path of taxes and the public debt. However, when fiscal policy is locally non-Ricardian the central bank can no longer rely on the fiscal authority to ensure intertemporal solvency. An immediate implication is that the path of real debt has consequences for the determination of inflation dynamics. Moreover, and in further contrast to the case of a locally Ricardian fiscal policy, current inflation also depends on the previous period's inflation rate. Hence, a richer set of macroeconomic dynamics obtain. We refer to the conditions for determinacy of rational expectations equilibrium in each regime as the Leeper conditions. We now begin our enquiry into whether learning dynamics require greater policy coordination for macroeconomic stability.

For the purposes of this paper, two final classes of equilibria are ignored. One concerns the case of Ricardian fiscal policy combined with a passive monetary policy satisfying  $0 < \phi_\pi < 1$ .

In this case, there is indeterminacy of rational expectations equilibrium for all parameter values. It is easily demonstrated that none of these equilibria are stable under the alternative non-rational expectations assumption being considered. The second concerns the case of non-Ricardian fiscal policy and monetary policy satisfying the Taylor principle. Under rational expectations it can be shown that there exist a class of unbounded equilibria that have explosive debt and inflation dynamics. Such a policy configuration resonates with the current US economic circumstances. And while such equilibria would be of interest to comprehensive study of regime change, it is beyond the scope of this paper.

### 4.3 Learning Dynamics

The following analysis seeks to understand the conditions under which, if agents have a sufficient amount of data, they will be able to learn the underlying rational expectations dynamics associated with any prevailing policy configuration. Because determining analytical results for non-Ricardian models is highly challenging and in order to facilitate comparisons with the Leeper (1991), we make the following simplifying assumptions for the remainder of the paper unless otherwise noted: the intertemporal elasticity of substitution is assumed to be one and the function describing the disutility of labor supply linear. Finally we consider economies in the neighborhood of flexible price equilibrium. Formally, we consider stability conditions under the limit  $\alpha \rightarrow 0$ . We emphasize that this is not equivalent to analyzing a flexible price economy and should be interpreted instead as being an economy with a small degree of nominal rigidity. The next proposition provides so-called E-Stability conditions that are analogous to conditions for determinacy of rational expectations presented in proposition 1 — see Evans and Honkapohja (2001) for further discussion.

**Proposition 2** *In the model given by relations (10), (11), (12), (13) and (14) and beliefs given by (18) if  $\alpha \rightarrow 0$  then:*

- i) If  $\phi_\pi > 1$  and  $1 < \tau < (1 + \beta) / (1 - \beta)$  then the associated rational expectations equilibrium is E-Stable.*
- ii) If  $0 \leq \phi_\pi < 1$  and  $0 \leq \tau < 1$  or  $\tau > (1 + \beta) / (1 - \beta)$  then the associated rational expectations equilibrium is E-Stable.*

The proposition states that if fiscal policy is locally Ricardian then monetary policy must be sufficiently aggressive to ensure learnability of rational expectations equilibrium. Indeed,

the requirement  $\phi_\pi > 1$  is the usual Taylor principle in the context of the one parameter family of monetary policy rules being considered in this paper. Similarly, if fiscal policy is locally non-Ricardian, then monetary policy must violate the Taylor principle: an increase in the inflation rate gives rise to falls in the real interest rate. Importantly, the conditions for stability in expectations formation are identical to those required in a rational expectations analysis of the model presented in the Proposition 1. The Leeper conditions hold. Hence learning dynamics in and of themselves do not necessarily impose any additional constraints on the choice of fiscal and monetary policy for stabilizing business cycle fluctuations. However, the next section will make clear that this conclusion rests on the central bank being able to accurately determine the current inflation rate when making its nominal interest rate decision.

The results of this proposition are analogous to Leeper (1991), though under the alternative assumption that agents are learning. The results are also identical in spirit to Evans and Honkapohja (2006). This proposition advances their analysis by considering a framework in which agents make optimal decisions conditional on their beliefs — see Preston (2005a, 2005b) for a discussion — which requires agents to forecast future paths of after tax income, debt and nominal interest rates. No such forecasts are required in the model of Evans and Honkapohja (2006) which ignores implications of intertemporal budget constraints and transversality conditions and posits that aggregate output and inflation depend only on one period ahead forecasts of these same two variables.

A more general result is available for an arbitrary degree of nominal rigidities  $0 < \alpha < 1$ .

**Proposition 3** *In the model given by relations (10), (11), (12), (13) and (14) and beliefs given by (18) if  $\alpha < 1$  then:*

- i) If  $\phi_\pi > 1$  and  $1 < \tau < (1 + \beta) / (1 - \beta)$  then the associated rational expectations equilibrium is E-Stable.*
- ii) If  $\phi_\pi = \tau = 0$  then the associated non-Ricardian rational expectations equilibrium is E-stable.*

## 5 A Framework for Policy Analysis

To permit a minimally realistic account of policy design, informational limitations on the monetary authority when implementing policy are now introduced.

## 5.1 Expectations-based Policy Rules

To account for limits on the timeliness of information that the central bank has at its disposal in setting policy we consider the more general case where the central bank needs to form expectations about the current level of inflation. Specifically we adopt the Taylor-type rule of the form

$$\hat{i}_t = \phi_\pi \hat{E}_{t-1}^{cb} \hat{\pi}_t. \quad (21)$$

The nominal interest rate is adjusted in response to expectations of the period  $t$  inflation rate conditional on time  $t - 1$  information. The superscript ‘ $cb$ ’ on the expectations operator indicates that these expectations are the internal forecasts of the monetary authority. We assume that the central bank has a regression model of the same kind as private agents but does not observe the current inflation rate. Hence

$$\hat{E}_{t-1}^{cb} \hat{\pi}_t = a_t + b_t z_{t-1}.$$

We make this assumption for two reasons. First, which is technical but of no substantive import, is under rational expectations the conditions for determinacy of rational expectations equilibrium are identical for this rule and the contemporaneous data rule (13). Second, is to explore formally consequences of a central bank reacting to information with a lag. Worth emphasizing is that none of our results rely on the precise timing of expectations in the policy rule. A rule of the form  $\hat{i}_t = \phi_\pi \hat{E}_t \hat{\pi}_{t+1}$ , which places all economic actors on the same informational footing, delivers similar, albeit more negative, conclusions. The conditions for determinacy or rational expectations equilibrium are also more stringent.

Because of the timing of the conditioning information, this class of policy rule is implementable in the sense of McCallum (1999). Moreover, many authors contend that such rules provide a decent characterization of many central bank reaction functions and, furthermore, have desirable robustness properties— see Clarida, Gali and Gertler (1998, 2000) and Hall and Mankiw (1994), Batini and Haldane (1999) and Levin, Wieland, and Williams (2003) respectively. This completes discussion of the benchmark model.

Before proceeding to the learning analysis and its implications for inflation and debt dynamics, note that the conditions for a unique bounded rational expectations equilibrium of

the model given by relations (10), (11), (21), (12) and (14) are identical to those detailed in propositions 1 for the contemporaneous data Taylor rule and flexible prices. The minimum state variable solution in each class of equilibria of proposition 1 continue to be linear functions of the previous period's debt level and inflation rate and natural rate disturbance. The precise coefficients on each of the state variables in each of the equilibria naturally differ. Nonetheless the conditions for determinacy of equilibrium are identical. The proofs are omitted and available on request. We now treat the cases of local Ricardian and locally non-Ricardian fiscal policies under learning dynamics in turn.

## 5.2 Ricardian Analysis

The analysis begins with the conventional treatment of monetary policy, asking under what conditions households and firms can learn the underlying rational expectations equilibrium when fiscal policy is assumed to be locally Ricardian. Under rational expectations there are no monetary consequences of fiscal policy.

The following propositions describe the stability analysis under those information constraints. As above, we restrict our attention to economies with very small nominal frictions, represented by the limiting case  $\alpha \rightarrow 0$ .

**Proposition 4** *If a central bank responds to inflation expectations and  $\alpha \rightarrow 0$  then the Taylor principle is not sufficient for E-stability. Moreover, the conditions for stability are independent of fiscal policy in the sense that, if  $1 < \tau < (1 + \beta) / (1 - \beta)$ , then the restrictions on the choice of monetary policy are independent of  $\tau$  and  $\delta$ .*

A necessary and sufficient condition for stability of learning dynamics is

$$\phi_\pi > \frac{1}{1 - \beta}.$$

This condition makes clear that, for a large range of monetary policy rules, there is no scope for fiscal policy to mitigate instability arising from learning dynamics in this Ricardian regime. A natural question is what is the source of this instability? Is it the presence of nominal rigidities or the existence of informational constraints on the implementation of monetary policy? To elucidate, recall the analogous result absent the information constraint in proposition 2. There the Leeper conditions obtain. Hence it is not the presence of nominal

rigidities but informational constraints that give rise to instability. By having the central bank respond with a delay to changes in the evolution of prices we introduce an additional source of instability. Policy mistakes can reinforce private sector expectational errors, by making the equilibrium more susceptible to variations in expectations and therefore opening the door to self-fulfilling expectations. Policy decisions then have the potential to reinforce private agents beliefs and lead to self-fulfilling expectations. This is analogous to rational expectations logic. Because equilibrium depends on expectations, having a central bank respond to private agent's beliefs renders the equilibrium more susceptible to variations in expectations and therefore equilibrium indeterminacy.

These results generalize the analyses of Preston (2005, 2006) which consider, under both informational assumptions, policy implemented through conventional Taylor rules that respond both to inflation and the output gap. The case of responding to one-period-ahead expectations is also considered. In those analyses the government pursued a zero debt policy — a particular case of Ricardian fiscal policy in which agents need not forecast the future path of primary surpluses.<sup>7</sup> What proposition 2 and 3 intimate, over and above the findings of Preston (2005, 2006), is that the specific choice of locally Ricardian fiscal policy, as least within a class of one parameter rules, does not materially affect stabilization objectives. However, analysis in the sequel demonstrates that this result depends critically on the assumption that the monetary policy rule is not known by agents when formulating their forecasts.

### 5.3 Non-Ricardian Analysis

The analysis of locally non-Ricardian fiscal policy is important to understanding certain episodes in U.S. economic data. The bond price support regime in the 1950s, in which the Federal Reserve pegged the price of bonds and the fiscal authority pursued expenditure policies that were largely independent of the state of the economy due to the demands of war, is one such example. Indeed, this is an example of passive monetary policy — a nominal interest rate peg violating the Taylor principle — and non-Ricardian fiscal policy. More recently,

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<sup>7</sup>In a log-linear approximation it does not matter whether the precise details of the fiscal regime are understood or not since in the special case of a zero debt fiscal policy all terms in the aggregate demand equation relating to the primary surplus drop out as the steady state primary surplus to output ratio is zero under this assumption.

wartime finance due to engagements in Afghanistan and Iraq might again be plausibly argued to be non-Ricardian in character. In contrast to the earlier episode, monetary policy has been avowedly active. This policy combination can be shown under rational expectations to give explosive dynamics in the debt level and inflation rate. While we do not explore the properties of learning dynamics in this particular class of unbounded equilibria, it is clear that having some understanding of dynamic behavior in each of these regimes requires analysis that permits non-Ricardian fiscal policies.

**Proposition 5** *If the central bank is subject to informational constraints in the implementation of policy and  $\alpha \rightarrow 0$ , then*

*i) If  $0 \leq \tau < 1$  the boundary between the E-stable and E-unstable parameter regions is defined by*

$$\tau^* = \frac{2}{[(1 - \beta\phi_\pi)^{-1} + (1 - \beta)]}.$$

*For a given  $\phi_\pi$ , every  $\tau^* < \tau < 1$  implies E-instability.*

*ii) For  $\phi_\pi = 0$  E-stability holds for all parameter values.*

*iii) For  $\tau = 0$  and  $\tau > \frac{1+\beta}{1-\beta}$  E-stability holds for all parameter values.*

Part 1 of the proposition demonstrates a tight link between the specifications of fiscal and monetary policy in the case of non-Ricardian fiscal policy. For a given choice of monetary policy,  $\phi_\pi$ , satisfying  $0 < \phi_\pi < 1$ , then fiscal policy  $\tau$  must be small enough for learnability of rational expectations equilibrium to obtain. Part 2 and 3 give special cases of this result which are also revealing. When  $\tau = 0$  and  $\tau > (1 + \beta)/(1 - \beta)$ , stability obtains regardless of the choice of monetary policy and other model parameters. Hence, given a monetary policy that fails to satisfy the Taylor principle, there is always a choice of fiscal policy that will ensure stability under learning dynamics. In this sense, the non-Ricardian regime displays greater robustness than does the Ricardian regime for these one parameter families of fiscal and monetary policy rules. This result is in our view important because a fiscal rule with  $\tau > (1 + \beta)/(1 - \beta)$  has been shown to deliver a good approximation to optimal monetary and fiscal policy, see Schmitt-Grohe and Uribe (2004). Indeed, proposition 3 makes clear that for empirically plausible monetary policy rules satisfying the Taylor principle, there does not exist a choice of fiscal policy within the class of rules considered that prevents self-fulfilling expectations.

The second special case is  $\phi_\pi = 0$ . Here stability obtains irrespective of the choice of fiscal policy. Such a rule corresponds to a form of interest rate peg which is similarly a special case of the contemporaneous data rule  $i_t = \phi_\pi \pi_t$ .

To assist the interpretation of these results further, it is again useful to recall the results of proposition 2. If the central bank responds to contemporaneous inflation rates the Leeper conditions hold. In contrast, when nominal interest rates are conditioned on inflation expectations, policy choice is constrained more tightly.

Of course, the stability results for both the contemporaneous and expectations based policy rules coincide for the special case  $\phi_\pi = 0$ . While this is unsurprising, what is interesting is that more generally the stability conditions are rather different. For  $0 < \phi_\pi < 1$ , as  $\alpha \rightarrow 0$ , so that we are considering economies in the neighborhood of a flexible price economy, the stability properties of each economy are fundamentally different. For the contemporaneous data rule, stability conditions are identical to those of a rational expectations economy — the Leeper stability conditions obtain. In contrast, in the more realistic case of a monetary policy based on inflation expectations, the conditions for stability local to the flexible price economy are different in character. We conclude, that coordination of policy choice becomes more important.

## 5.4 Nominal Rigidities and Expectation Stabilization

A natural question is to what degree nominal rigidities affect macroeconomic stabilization under learning dynamics. Under rational expectations the conditions for unique bounded rational expectations are independent of the degree of price stickiness. But is this true under learning?

First note, as a particular example, that if fiscal policy is locally Ricardian and  $\delta = 0$  then a generalization of proposition 4 implies a necessary condition for stability is

$$\phi_\pi > \frac{1}{1 - \beta} - \frac{\alpha(2 - \beta - \alpha\beta)}{(1 - \alpha)(1 - \alpha\beta)^2}.$$

Hence, economies with high degrees of nominal rigidities are less prone to instability under learning dynamics. Indeed, taking the limit  $\alpha \rightarrow 1$ , so that prices are almost never reoptimized, implies that the right hand side of the condition tends to negative infinity. Hence

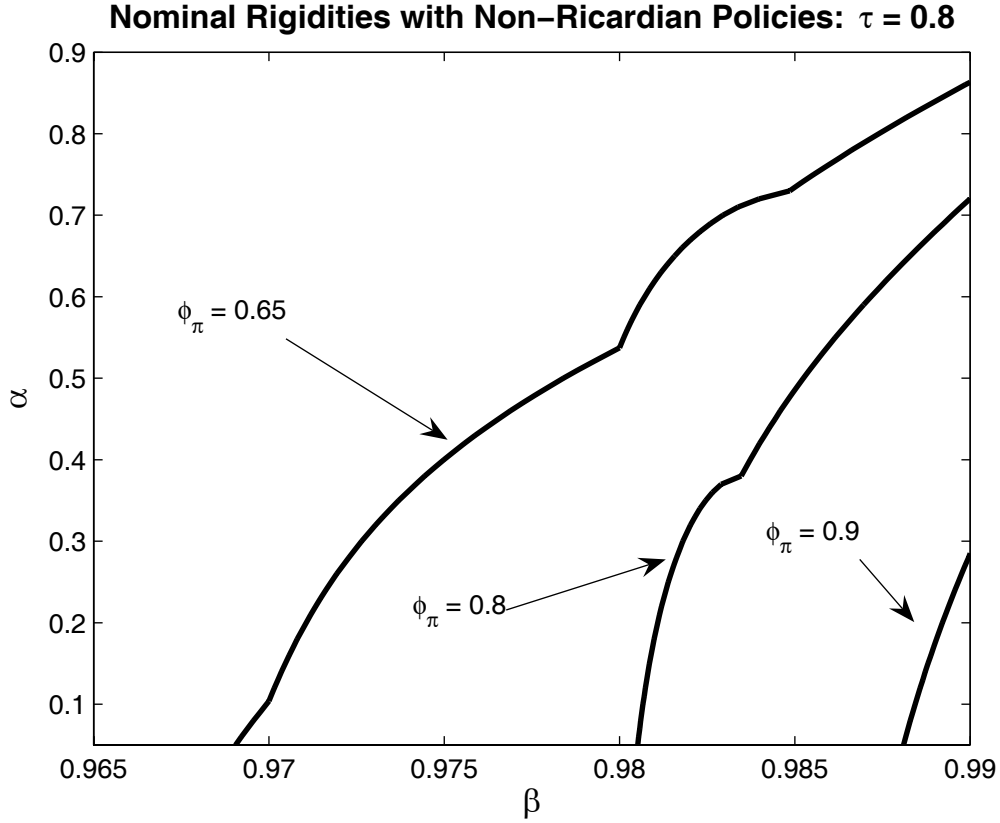


Figure 1:

for any plausible choice of monetary policy satisfying  $\phi_\pi \geq 0$  convergence is guaranteed. In contrast, as  $\alpha \rightarrow 0$  and prices become more flexible, ever more active monetary policy is required. Hence, price stickiness tends to facilitate anchoring private sector expectations, and protects against expectations driven instability. For general  $\delta$  a similar dependency can be established numerically.

Now consider the case of locally non-Ricardian fiscal policy. Because analytic results are not readily obtainable, we resort to a calibration study. Figure 1 plots regions of E-stability as a function of the parameter pairs  $(\alpha, \beta)$  assuming that  $\tau = 0.8$ .

The black contours are each associated with a particular choice of monetary policy. The region to the left of any given contour marks pairs of  $(\alpha, \beta)$  generating instability under learning; to the right, stability. Two points can be made. First, for a given household discount rate, increasing degrees of price stickiness lead to greater instability. Second, as

monetary policy becomes more active, the instability region becomes larger.

Why is price stickiness conducive to learning in the Ricardian regime and not in the non-Ricardian regime?

Take the Ricardian case first and consider a positive shock to inflation expectations. From the aggregate demand relation, forecasted real interest rates have declined leading to an intertemporal substitution towards greater current consumption. In turn, higher aggregate demand leads to inflation. However, the magnitude of the resulting inflation depends on the degree of nominal rigidities. The greater is price stickiness, the smaller the increase in inflation. Hence, economies characterized by inflexible product markets are conducive to anchoring expectations.

In contrast, in non-Ricardian fiscal regimes, intertemporal solvency of the government accounts relies on adjustments in the price level. A positive shock to inflation expectations again leads to an expansion in aggregate demand which in turn leads to increases in firm prices. While the same channel operates on inflation expectations as in the Ricardian case, there is now a countervailing effect: the more flexible are prices, the larger is the decline in the real value of outstanding government liabilities. Moreover, the larger the wealth effect, the smaller will be subsequent increases in demand. Because non-Ricardian policy requires price adjustment of a particular kind to ensure intertemporal solvency, the latter effect dominates.

## 6 Mitigating Instability

Informational constraints in the implementation of monetary policy has important consequences for macroeconomic stabilization. That the central bank responds to expectations formed by least squares learning renders the economy prone to self-fulfilling expectations, particularly under Ricardian fiscal policy. Two approaches to mitigating instability under this class of monetary policy rule are now explored. The first investigates the possible advantages of communicating the monetary policy rule to households and firms, and asks whether such information helps anchor expectations that are relevant to their spending and pricing decisions. The second considers modifications of the types of policy rules available to the central bank and fiscal authority, in particular allowing for nominal interest rates and primary

surplus decisions to be conditioned on information about the state of aggregate demand as reflected in the output gap.

## 6.1 Advantages of Communication

To date, a maintained assumption of this analysis is that agents do not know the policy rule adopted by the monetary authority, and therefore must forecast nominal interest rates independently of other macroeconomic variables. Yet an important component of recent discussions on monetary policy design has been the notion of transparency and communication. Indeed, Faust and Svensson (2001) present a model in which the central bank has an idiosyncratic employment target which is imperfectly observed by the public. Fluctuations in this target leads to central bank temptation to deviate from pre-announce inflation goals. However, increased transparency allows the private sector to observe the employment target with greater precision and therefore raises the costs to the central bank of deviating from its announced objectives. Transparency is therefore desirable as it provides a commitment mechanism. Svensson (1999) further argues on the ground of this result that for inflation targeting central banks it is generally desirable for detailed information on policy objectives, including forecasts, to be published. Such transparency enhances the public's understanding of the monetary policy process and raises the costs to a central bank from deviating from its stated objectives.

In the context of the literature on learning and monetary policy a number of papers have attempted to connect to this debate. For example, Eusepi (2005) and Preston (2006) consider in New Keynesian models of output gap and inflation determination in which agents are attempting to learn, amongst other things, the reduce form dynamics of nominal interest rates. Forecasts of this variable are relevant to spending and pricing decisions of households and firms. However, if the precise nature of policy decisions is known the future path of nominal interest rates do not need to be forecast independently of other macroeconomic aggregates, and the likelihood of instability is reduced. Clearly articulating monetary policy strategy helps anchor private sectors expectations and consequently assist in managing economic fluctuations.

Finally, in a closely related analysis, Orphanides and Williams (2005) highlight the advantages of publishing an inflation target. They show in a simple model of the output-inflation trade-off that if private agents must learn about inflation dynamics a more favorable trade-off between inflation and output can be achieved if private agents are assumed to know the central bank's long-run inflation target rather than having to learn this quantity.

Suppose then that agents know the monetary policy rule being implemented by the central bank. This implies that agents know in the equilibrium they are attempting to learn that restriction (21) is satisfied. An immediate implication is that households need no longer forecast the future path of the nominal interest rate independently from the future path of inflation — forecasting the latter is sufficient to project the former. Aggregate demand under communication is then determined as

$$\hat{x}_t = \delta\beta^{-1}(\hat{b}_t - \hat{\pi}_t) - \beta^{-1}\delta\hat{s}_t - (\sigma - \delta)\phi_\pi\hat{E}_{t-1}\pi_t + \hat{E}_t\sum_{T=t}^{\infty}\beta^{T-t}[(1 - \beta)(\hat{x}_{T+1} - \delta\hat{s}_{T+1}) - (\sigma - \delta)(\beta\phi_\pi - 1)\hat{\pi}_{T+1} + \sigma r_T].$$

The remainder of the model is unchanged as nominal interest rate forecasts only matter for households expenditure plans.<sup>8</sup> The following result obtains.

**Proposition 6** *Under central bank communication, Ricardian fiscal policy, so that  $\tau > 1$ , and  $\alpha \rightarrow 0$  stability requires*

$$\phi_\pi > \frac{1}{1 - \beta\delta}$$

*to be satisfied.*

To guard against instability from self-fulfilling expectations requires a choice of monetary policy rule that depends on two model parameters: the household's discount factor,  $\beta$ , and the steady state ratio of the primary surplus to output (or equivalently the steady state debt-to-output ratio since  $\bar{s} = (1 - \beta)\bar{b}$ ). The choice of fiscal regime, reflected in the implied average debt-to-output ratio, imposes constraints on stabilization objectives. Less fiscally responsible governments must have more aggressive monetary policy to ensure learnability of rational expectations equilibrium. In the special case that  $\delta = 0$ , as would occur if the

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<sup>8</sup>See Eusepi (2005) for analysis of a model in which firms face a cash in advance constraint and must therefore forecasts the path of nominal interest rates.

government pursued a zero debt policy each period, the Taylor principle is restored. Hence there can be Keynesian expenditure effects and monetary consequences of fiscal policy under non-rational expectations even if fiscal policy is Ricardian.

Using the fact that  $\bar{s} = (1 - \beta)\bar{b}$  the above condition can be expressed as

$$\phi_\pi \left( 1 - \beta (1 - \beta) \frac{\bar{b}}{\bar{Y}} \right) > 1.$$

Hence for  $\beta = 0$  or  $\beta = 1$  this condition coincides with the Taylor principle. For a given debt-to-output ratio, the required response coefficient in the monetary policy rule varies as the discount factor varies, with the maximum response being achieved for  $\beta = 0.5$ .

This result highlights some fundamental differences in the analysis of fiscal and monetary policy coordination under learning dynamics vis-a-vis rational expectations. Under rational expectations, so long as fiscal policy is locally Ricardian, the specific choice of policy is irrelevant to the conditions for determinacy of rational expectations equilibrium. However, under learning dynamics this need not be true. Clearly ‘out-of-equilibrium’, when households and firms are learning, Ricardian fiscal policies can have non-Ricardian effects. Because households fail to correctly anticipate future tax changes that are consistent with a Ricardian regime, current variations in taxes engender traditional Keynesian expenditure effects. What the above proposition demonstrates is that even if agents are given an infinite amount of data with which to construct beliefs, asymptotic stability hinges on the precise choice of fiscal regime not only through the chosen sensitivity of primary surpluses to outstanding public debt: economies with larger average levels of debt relative to output are more prone to instability from self-fulfilling expectations.

Communicating the monetary policy rule adopted by the central bank certainly promotes stability under the expectations based policy rule. The range of policy configurations giving rise to instability narrows relative to the case of no communication, implying communication is unambiguously good from a macroeconomic stabilization perspective. However, the benefits of communication are greater for economies that are fiscally responsible. This discussion is summarized in the following proposition.

**Proposition 7** *Communication unambiguously improves stabilization under learning dynamics. That is, the region of stability is larger. However, in the special case  $\delta = 1$  the regions of*

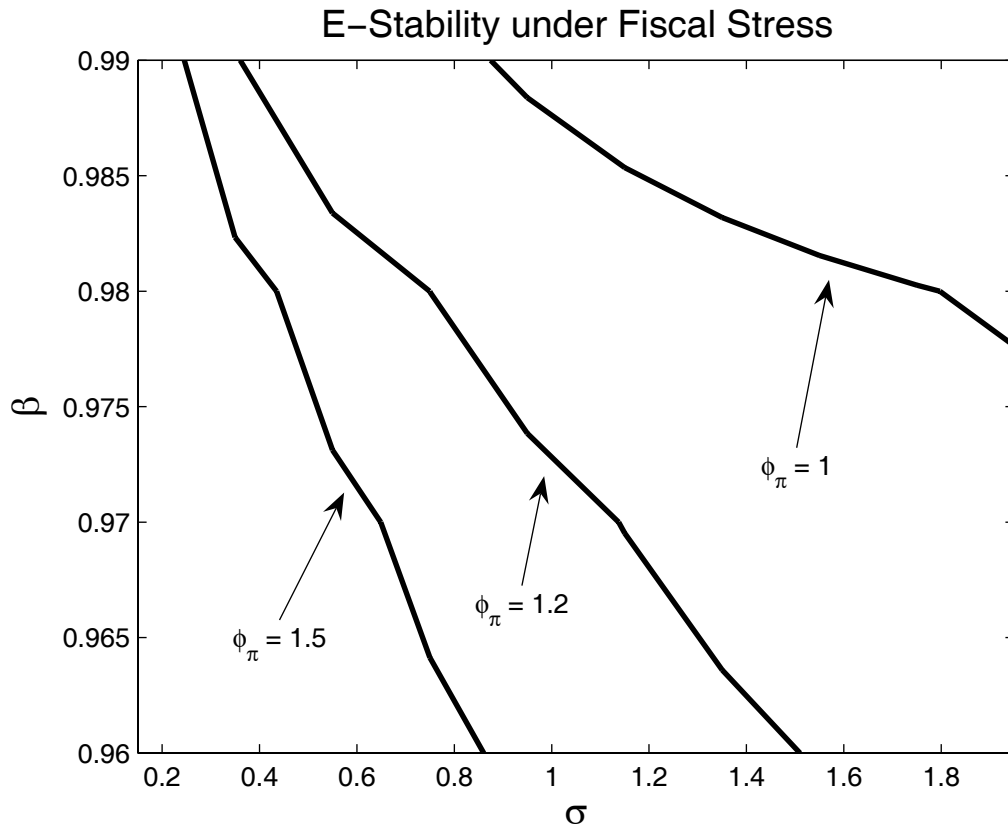


Figure 2:

*stability in the communication and no communication cases coincide.*

Why does the debt-to-output ratio matter for stabilization in Ricardian regimes?

[INTUITION]

To gain further insight, Figure 2 plots regions of instability as a function of  $(\beta, \sigma)$  pairs, relaxing the assumption that the intertemporal elasticity of substitution is unity. The steady state debt-to-output ratio is assumed to be 1.5.

Three monetary policies are considered, each partitioning the parameter space into a region of stability (north east quadrant) and instability (south east quadrant). For plausible choice of the household's discount factor, instability can result for many choices of the intertemporal elasticity of substitution. These values fall well within the range of estimates obtained from both microeconomic and macroeconomic studies. Moreover, more active monetary policies

tend to deter instability from self-fulfilling expectations, consistent with standard intuition.

By way of completeness, the following proposition considers the benefits of communication under the non-Ricardian regime.

**Proposition 8** *Under central bank communication, non-Ricardian fiscal policy such that  $0 \leq \tau < 1$  and  $\alpha \rightarrow 0$  a sufficient condition for stability is*

$$\delta < \frac{(1 - \beta + \beta^2 \phi_\pi)(1 - \phi_\pi)}{\phi_\pi \beta (1 - \beta \phi_\pi)}$$

*and communication increases the region of stability relative to the no communication case.*

Consistent with Ricardian fiscal policy regimes, even with communication the debt to output ratio constrains the choice of monetary and fiscal policy. Note that for a given choice of tax policy, represented by the pair  $(\tau, \delta)$ , there is always a choice of monetary policy that will ensure stability under learning dynamics. For this reason, the non-Ricardian regime continues to display greater robustness to non-rational expectations. Moreover, the result once more emphasizes that coordination of policy is more desirable under non-rational expectations than under rational expectations.

## 6.2 Responding to the Output Gap

McCallum (1983) showed that conditioning nominal interest rate decisions on endogenous variables rather than exogenous variables has the potential to resolve indeterminacy of rational equilibrium underscored by Sargent and Wallace (1975) — even though both rules are consistent with the same underlying rational expectations equilibrium. Furthermore, and as an example of such a rule, Taylor (1993) argues that a variant of the nominal interest rate rule

$$i_t = \phi_\pi \pi_t + \phi_x x_t$$

provides a good characterization of the U.S. monetary policy over the 80's and early 90's. In the model of this paper under rational expectations and Ricardian fiscal policy, Woodford (2003) shows this rule delivers a unique bounded rational expectations equilibrium if and only if

$$\phi_\pi + \frac{(1 - \beta)}{\kappa} \phi_x > 1.$$

Hence responding to the output gap can also help mitigate instability in macroeconomic dynamics. Furthermore, Bullard and Mitra (2002) show that this condition also ensures learnability of rational expectations equilibrium in a simple model of inflation and output gap determination in which only one period ahead forecasts matter to firm and household decisions.

Given this finding, we explore the following monetary and fiscal policy specifications:

$$\begin{aligned} i_t &= \phi_\pi E_{t-1} \pi_t + \phi_x E_{t-1} x_t \\ s_t &= \tau_b b_t + \tau_x E_{t-1} x_t \end{aligned}$$

where  $\tau_x, \phi_x > 0$ . Both rules allow for a response to the expected state of aggregate demand. These rules maintain the informational assumption that the monetary and fiscal authorities face information limitations and condition decisions on time  $t-1$  information. The monetary policy rule is otherwise identical in spirit to the conventional Taylor rule, while the tax rule is motivated by the empirical analysis of Davig and Leeper (2005a).

Figure 3 plots regions of stability and instability under learning dynamics as a function of  $(\tau_x, \beta)$ .

The remaining parameters are chosen as  $\phi_\pi = 0.9$ ,  $\tau_b = 0.5$  and  $\alpha = 0.66$  so that the average duration that any one firm's goods price remains fixed is three quarters. Two choices of monetary policy rule are considered:  $\phi_x = 0$  and  $\phi_x = 0.125$ . The policy configurations correspond to a non-Ricardian regime. Fixing the monetary policy rule, regions to the left of the marked contours correspond to parameter configurations consistent with learnability of rational expectations equilibrium.

Consider first a household discount factor of  $\beta = 0.99$ . It is immediate that low values of the policy parameter  $\tau_x$  are consistent with self-fulfilling expectations. However, moderate responses to the output gap have a stabilizing effect and are consistent with stability. Nevertheless, values of  $\tau_x$  that are too high again push the economy into an unstable region. Moreover, note that as the response coefficient on the monetary policy rule on output rises from 0 to 0.125 the parameter configurations consistent with stability narrows considerably. More active monetary policy tends to raise the likelihood of unstable dynamics from

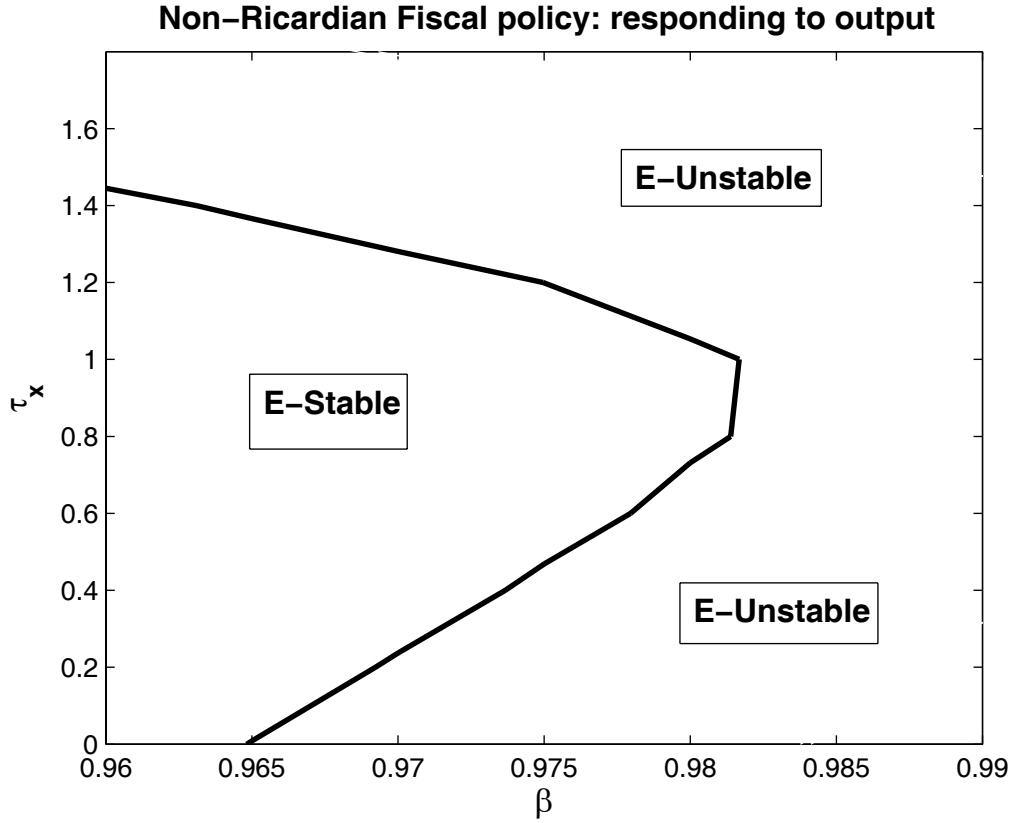


Figure 3:

expectations.

Finally, in the case of active monetary rules and passive fiscal rules, a higher response to the output gap helps stabilizing expectations. This result is interesting because the current literature (see, among the others, Schmitt-Grohe and Uribe (2004, 2005)) show that  $\phi_x = 0$  is a good approximation to the optimal Ramsey policy under the assumption of rational expectations. Under informational constraints an higher response to output proves to be an important policy recommendation. Assuming that the agents understand the policy rules and use them to forecast the path of the nominal interest rate and the surplus, Figure 4 shows that responding to expected output gap has a stabilizing effect. In fact, the “stability frontier” shifts to the left, leaving a wider set of parameters for which the REE is stable under learning (this is represented by the region to the right of the solid lines). For the simulation we chose  $\sigma = 0.5$  and  $\tau_b = 0.0014/(1 - \beta)$ , consistent with Davig and Leeper (2005a): the

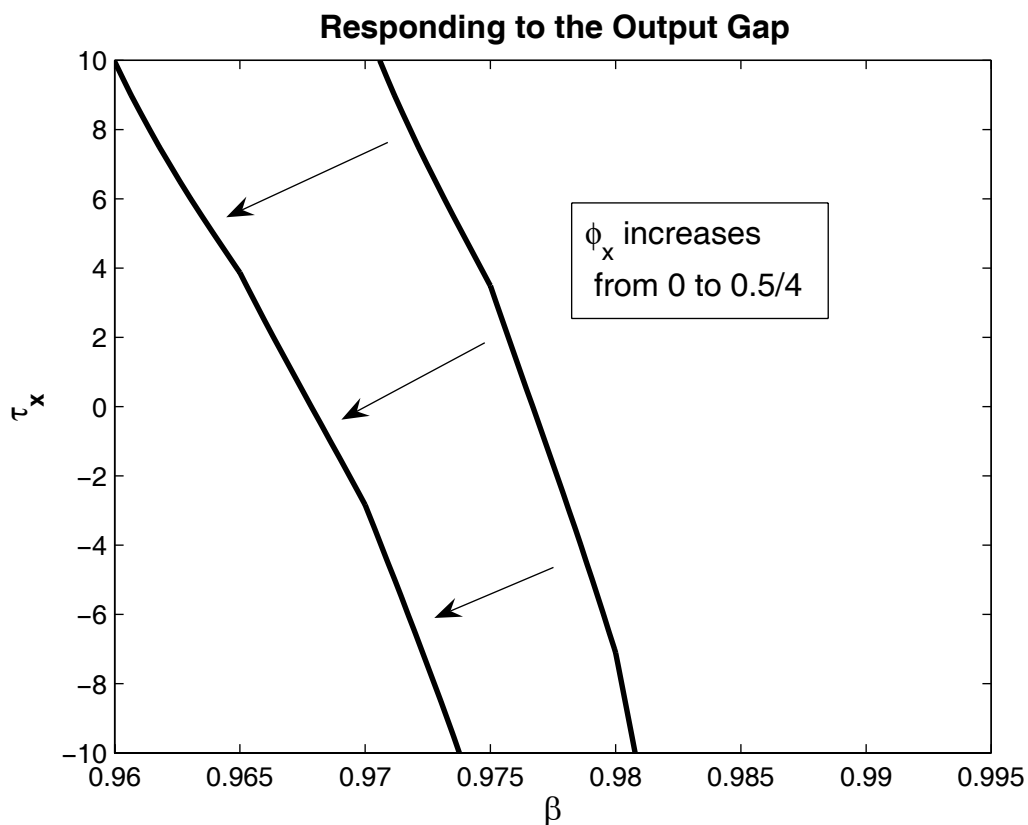


Figure 4:

latter coefficient does not show any impact on the stability conditions).

## 7 Conclusions

This paper considers the implications of a specific form of non-Rational expectations for macroeconomic stabilization policy. Of particular interest are the constraints that learning dynamics impose on the choice of monetary and fiscal policies. Desirable policies are those that rule out self-fulfilling expectations and ensure convergence to the underlying rational expectations equilibrium associated with any pair of policy choices.

The core results are as follows. First, if a central bank is informationally constrained and responds to measures of inflation expectations then non-Ricardian fiscal policies tend to be more robust to deviations from rational expectations than Ricardian fiscal policies.

Indeed, for plausible choices of monetary policy there is always a non-Ricardian fiscal policy that will ensure stability under learning dynamics. This is not true under Ricardian fiscal policies. Importantly, in either policy regime the requirements for policy coordination are more stringent relative to a rational expectations analysis of the same model. A much smaller subset of policy pairs deliver stability, with the desirable choice of one policy depending on the specification of the other.

Second, these stability results depend on the degree of nominal rigidities in price setting. Higher degrees of price stickiness tend to facilitate learnability of rational expectations equilibrium under Ricardian fiscal policies. However the converse is true for non-Ricardian policies. This difference reflects the starkly different mechanisms by which equilibrium is guaranteed under each policy regime.

Third, instability can be mitigated by central bank communication of the monetary policy rule. If agents do not need to forecast the path of nominal interest rates independently of other macroeconomic aggregates the set of policy pairs delivering stability under learning dynamics is unambiguously larger. However, the Leeper conditions are not necessarily restored. This depends on the choice of fiscal policy as reflected in the steady state debt to output ratio. Economies with greater average levels of debt to output typically have greater difficulty in guarding against expectations driven instability.

All these results are counter to a rational expectations equilibrium analysis.

# A Appendix

## A.1 The Log-linear model

## A.2 Proof 1. (PM/AF Regime) Infinite Horizon, Calvo model where $\alpha \rightarrow 0$

Here we use the fact that as  $\alpha \rightarrow 0$ ,  $\lambda_1 \rightarrow \phi_\pi$ . This allows to get analytical results. Consider first the Jacobian corresponding to the matrix of the constant coefficients  $\begin{bmatrix} a_x & a_\pi & a_b \end{bmatrix}'$ . Stability obtains if the real part of its three eigenvalue is negative. It can be shown that at  $\alpha = 0$  one eigenvalue is  $-1$  for every other values of the parameter. The remaining two eigenvalues are negative if

$$tr(J) = z_1 + z_2 - 1 < 0$$

that is if  $tr(J) + 1 < 0$  and

$$\det(J) = -z_1 z_2 > 0$$

that is  $-\det(J) > 0$ . The determinant becomes

**(Determinant\_A)**

$$-\det(J) = \frac{(1 - \tau)}{(1 - \tau + \tau\beta)}.$$

The restriction on  $\tau$  to have a non-Ricardian fiscal rule is

$$-1 < J(\tau) = \frac{\beta}{1 - (1 - \beta)\tau} < 1$$

and multiplying the determinant for  $\beta$  (which leaves its sign unchanged) we get that the condition to be satisfied is

$$\frac{\beta(1 - \tau)}{(1 - \tau + \tau\beta)} > 0. \tag{22}$$

We know that for  $\tau < 1, 0 < J(\tau) < 1$ . Substituting in (22) we get

$$J(\tau)(1 - \tau) > 0$$

which implies that the determinant is negative. For  $\tau > (1 + \beta) / (1 - \beta)$ ,  $-1 < J(\tau) < 0$ . By substituting in (22) it is easy to show that also in this case the determinant is negative. For the trace we obtain:

(Trace\_A)

$$\begin{aligned} \text{tr}(J) &= -\frac{(\tau\beta + 2 - 2\tau)}{(1 - \tau + \tau\beta)} \\ &= -\left[1 + \frac{1 - \tau}{(1 - \tau + \tau\beta)}\right] < 0 \end{aligned}$$

from the result above. Second, the Jacobian corresponding to the matrix of the coefficients on the debt variable,  $\begin{bmatrix} b_x & b_\pi & b_b \end{bmatrix}$  gives the following trace

(Trace\_B)

$$\text{tr}(J) = G^T(\phi_\pi) - \frac{(\beta^2\phi_\pi\tau - \beta\phi_\pi\tau + 2\beta\phi_\pi - 2\tau\beta + 2\tau - 2)}{(\beta\phi_\pi - 1)(1 - \tau + \tau\beta)}$$

In the case  $\phi_\pi = 0$  we get

$$G^T(0) = -\frac{2(1 - \tau + \tau\beta)}{(1 - \beta)(1 - \tau + \tau\beta)} = -\frac{2}{(1 - \beta)} < 0$$

and for  $\phi_\pi = 1$ ,

$$G^T(1) = -\frac{(\beta^2\tau - \beta\tau + 2\beta - 2\tau\beta + 2\tau - 2)}{(\beta - 1)(1 - \tau + \tau\beta)} = -\left[1 + \frac{1 - \tau}{(1 - \tau + \tau\beta)}\right] < 0$$

Last,

$$\begin{aligned} G^{TT}(\phi_\pi) &= \frac{\beta\phi_\pi(\beta - 1)}{(1 - \tau + \tau\beta)^2(\beta\phi_\pi - 1)} \\ &= \frac{\beta\phi_\pi(1 - \beta)}{(1 - \tau + \tau\beta)^2(1 - \phi_\pi\beta)} \geq 0 \text{ for every } \phi_\pi \in [0, 1] \end{aligned}$$

which implies that the trace is negative for every value of  $\phi_\pi$  and  $\tau$  consistent with the determinate and stationary REE. Let's consider the determinant,

(Determinant\_B)

$$\begin{aligned} -\det(J) &= G^D(\phi_\pi) \\ &= \frac{(1 + \beta^3\phi_\pi^2\tau + \tau^2\beta^2 - \beta^2\phi_\pi^2\tau - 2\tau + 2\beta^2\phi_\pi\tau^2 - \beta\phi_\pi\tau^2 - 3\beta^2\phi_\pi\tau)}{(\beta^2\phi_\pi\tau - \beta\phi_\pi\tau + \beta\phi_\pi - \tau\beta - 1 + \tau)^2} \\ &\quad + \frac{3\beta\phi_\pi\tau + 2\tau\beta - 2\beta\phi_\pi + \beta^2\phi_\pi^2 - \beta^3\phi_\pi\tau^2 + \tau^2 - 2\tau^2\beta}{(\beta^2\phi_\pi\tau - \beta\phi_\pi\tau + \beta\phi_\pi - \tau\beta - 1 + \tau)^2} \end{aligned}$$

For  $\phi_\pi = 0$

$$G^D(0) = \frac{(1 + \tau^2\beta^2 - 2\tau + 2\tau\beta + \tau^2 - 2\tau^2\beta)}{(-\tau\beta - 1 + \tau)^2} = 1 > 0$$

imposing  $\phi_\pi = 1$

$$G^D(1) = \frac{(1 - \tau)}{(1 - \tau + \tau\beta)} > 0$$

Calculating

$$G^{D'}(\phi_\pi) = -\frac{\beta\phi_\pi(1 - \beta)}{(1 - \tau + \tau\beta)^2(1 - \beta\phi_\pi)} < 0$$

obtains that the determinant is negative for all parameter consisting with the equilibrium of interest. This completes the proof.

### A.3 Proof 1. (PM/AF Regime) Infinite Horizon (no knowledge about the policy rule), Calvo model where $\alpha \rightarrow 0$

The proof has the same structure as above. Again, the stability of two eigenvalues determines E-Stability. Consider the matrix of the constants: the trace is

(Trace\_A)

$$\text{tr}(J) = -\left[1 + \frac{1 - [1 - \beta\phi_\pi(1 - \beta)]\tau - \beta\phi_\pi}{1 - (1 - \beta)\tau - \beta\phi_\pi}\right]$$

**(i) If  $\phi_\pi = 0$ , the stability condition is the same as contemporaneous expectations.**

Let's first consider the case of  $\phi_\pi = 0$ . Constant coefficients. Then the trace collapses to the same as in the case of contemporaneous expectations.

$$\text{tr}(J) = -\left[1 + \frac{1 - \tau}{1 - (1 - \beta)\tau}\right] < 0$$

and for the same for the determinant

$$-\det(J) = \frac{(1 - \tau)}{(1 - (1 - \beta)\tau)} > 0.$$

Coefficients on real debt. The trace becomes

$$\begin{aligned}\text{tr}(J) &= \frac{2(1-\beta)\tau - 2}{(1-(1-\beta)\tau)} \\ &= -2\end{aligned}$$

while the determinant is equal to  $-1$  for all parameter values.

**(ii) If  $\tau = 0$  then the REE is stable for every parameter value.** Consider the constants. The trace becomes

$$\begin{aligned}\text{tr}(J) &= -1 - \frac{1 - \beta\phi_\pi}{1 - \beta\phi_\pi} \\ &= -2\end{aligned}$$

and the determinant

$$\begin{aligned}-\det(J) &= \frac{(1 - \beta\phi_\pi)}{(1 - \beta\phi_\pi)} \\ &= 1 > 0.\end{aligned}$$

The trace for the matrix of the real debt coefficients becomes

$$\begin{aligned}\text{tr}(J) &= \frac{-2(\beta\phi_\pi - 1)^2}{(1 - \beta\phi_\pi)^2} \\ &= -2 < 0\end{aligned}$$

while the determinant is equal to  $-1$ .

**(iii) Let  $\beta \rightarrow 1$ , for each  $\phi_\pi > 1/2$  there exist  $\tau^*$  such that for  $1 > \tau > \tau^*$  the REE is E-unstable.** Consider the constants. The trace as  $\beta \rightarrow 1$  becomes

$$\text{tr}(J) = \frac{-2(1 - \phi_\pi) + \tau}{1 - \phi_\pi}$$

delivering the instability result.

#### **A.4 Proof 2. (PM/AF Regime) Infinite Horizon: agents know the monetary policy rule.**

**For  $\delta < \delta^{TA}$  the REE is stable under learning.** We proceed as above, For for the stability of the coefficients we look at the trace and determinant of the Jacobian. Let's start

with the trace. We are going to show that the trace is negative. First we can define the trace of the constant coefficient as  $\Phi^{T,A}(\tau, \phi_\pi, \delta)$ . First notice that

$$\Phi_\tau^{T,A}(\tau, \phi_\pi, \delta) = \frac{(1 - \beta\phi_\pi)(\beta\phi_\pi\delta - \phi_\pi + 1)\beta}{(-1 + \tau - \tau\beta + \beta\phi_\pi)^2} > 0$$

for every admissible values of  $\delta$ ,  $\phi_\pi$ , and  $\tau$ . Second we show that for values of  $\delta < \delta^{TA}$  the trace is negative. Consider  $\tau < 1$ . It is sufficient to show that for  $\delta < \delta^{TA}$

$$\Phi^{T,A}(1, \phi_\pi, \delta) = -\frac{(\beta^2\phi_\pi - \beta^2\phi_\pi^2 + \beta^2\phi_\pi^2\delta + \beta\phi_\pi - \beta\phi_\pi\delta - \beta - \phi_\pi + 1)}{(1 - \beta)/(1 - \phi_\pi)} < 0$$

where we can calculate

$$\delta^{TA} = \frac{((1 - \beta + \phi_\pi\beta^2)(1 - \phi_\pi))}{\phi_\pi\beta(1 - \beta\phi_\pi)} > 0.$$

consider  $\tau > (1 + \beta)/(1 - \beta)$ . we use the fact that *under this assumption* it is easy to verify that  $\Phi_\delta^A(\tau, \phi_\pi, \delta) < 0$ . Thus, we impose  $\delta = 0$  which gives

$$\Phi_{\delta=0}^A(\tau, \phi_\pi) = \frac{((\beta^3\phi_\pi - 2\beta^2\phi_\pi - \beta^2 + \beta\phi_\pi + 3\beta - 2)\tau - \beta^3\phi_\pi^2 + 3\beta^2\phi_\pi - 2\beta\phi_\pi - 2\beta + 2)}{(1 - \beta)((1 - \beta)\tau - 1 + \beta\phi_\pi)}.$$

Substituting  $\tau = ((1 + \beta)/(1 - \beta))$  we get

$$(1 - \beta)((1 - \beta)\tau - 1 + \beta\phi_\pi)\Phi_{\delta=0}^A(\tau, \phi_\pi) = (\beta^2 - \beta) + (\beta^2\phi_\pi - \beta\phi_\pi) + (2\beta^2\phi_\pi - 2\beta) - \beta^3\phi_\pi - \beta^3\phi_\pi^2 < 0$$

Last, we show that the coefficient on  $\tau$  is positive, that is

$$(\beta^3\phi_\pi - \beta^2\phi_\pi) + R(\phi_\pi, \beta) < 0$$

where

$$R(\phi_\pi, \beta) = -\beta^2\phi_\pi - \beta^2 + \beta\phi_\pi + 3\beta - 2$$

$$R(0, \beta) = -(\beta - 1)^2 - 1 + \beta < 0, \quad R(1, \beta) = -2(\beta - 1)^2 < 0$$

and

$$R_{\phi_\pi}(\phi_\pi, \beta) = -\beta^2 + \beta > 0$$

Finally, the determinant of the Jacobian is

$$\det(J) = \frac{(1 - \tau)(1 - \beta\phi_\pi)}{1 - (1 - \beta)\tau - \beta\phi_\pi} > 0$$

We follow the same process for the matrix of the debt coefficients. We get

$$\delta^{TB} = \frac{(1 - \beta\phi_\pi + \phi_\pi\beta^2 + 1 - \beta\phi_\pi - \phi_\pi\beta^2(1 - \phi_\pi))(1 - \phi_\pi)}{\beta\phi_\pi^2(1 - \beta\phi_\pi)} > \delta^{TA}$$

and

$$\Phi_{\delta=0}^{T,B}(\tau, \phi_\pi) = (\beta^2\phi_\pi^2 - 2\beta\phi_\pi + 2)/(1 - \beta\phi_\pi) < 0$$

Finally, the determinant is equal to one for every parameter value. This completes the proof.

**(ii) communication implies E-Stability for a larger set of parameters.** We have that

$$\Phi_{\delta=1}^A(\tau, \phi_\pi) = -\frac{(\beta\phi_\pi\tau + \beta\tau)}{(1 - (1 - \beta)\tau - \beta\phi_\pi)} = \tilde{\Phi}^A(\tau, \phi_\pi)$$

with

$$\tilde{\Phi}^A(\tau, \phi_\pi) = -\frac{((\beta\phi_\pi + \beta)\tau)}{(1 - (1 - \beta)\tau - \beta\phi_\pi)}$$

Since we know that  $\Phi_\delta^A < 0$  for  $0 < \tau < 1$  we have that  $\Phi^A < \tilde{\Phi}^A$  for  $\delta < 1$ . This completes the proof.

## A.5 Proof 3. (PF/AM Regime) Infinite Horizon (knowledge about the policy rule), Calvo model where $\alpha \rightarrow 0$

**(i) if  $b/y \neq 0$  the Taylor Principle is not sufficient for E-Stability.** Let's consider the case of a **Ricardian** fiscal policy and active monetary policy. We can show that for  $\alpha \rightarrow 0$ , the sign of the trace of the constants' matrix depends on the following expression

$$1 + (\beta\delta - 1)\phi_\pi < 0$$

so for  $\delta = 0$ , we get the usual Taylor principle. But  $\delta$  positive might lead to instability, especially with  $\phi_\pi$  close to 1. Using  $\delta = (1 - \beta)\frac{b}{y}$  we can re-write the stability condition as

$$\phi_\pi(1 - \beta(1 - \beta)\frac{b}{y}) - 1 > 0$$

so that for high levels of debt to gdp ratios and for intermediate values of the discount factor instability is likely to arise. The determinant is negative provided  $(\phi_\pi - 1) > 1$ . Consider the matrix of b coefficients. The trace in the case of flexible prices is negative if

$$(-1 + \beta + \beta^2\phi_\pi\delta - \beta\phi_\pi\delta)\tau - (1 - \delta)\beta\phi_\pi + 1 < 0$$

which is negative provided the trace of the matrix for the constants is negative ( $\tau > 1$  in the Ricardian fiscal regime). This completes the proof.

**(ii) Central bank communication increases the E-Stability region, for every value of the model's parameters.** Consider the case of  $\delta = 1$ . Then the trace of the constants' matrix becomes

$$1 + (\beta - 1)\phi_\pi$$

which coincides with the stability condition in the case where the agents have no knowledge about the policy rule. Thus, communication is always stability enhancing. This completes the proof.

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