Discussion of In-Koo Cho and Kenneth Kasa, “Learning and Model Validation”

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Main idea: Allow agents to select forecasting models in addition to recursive learning.
Parameters versus models

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- **Most of the action in actual economies?**
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- **Appeal**: Forecasters can “express doubt” about their models, switch models.
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Mostly thinking in terms of non-nested models.
Connections to escape dynamics

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Some examples maintain the possibility of escape, others do not.
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• Models can be discarded, replaced with alternatives, and possibly reincarnated.

• Knowledge of the true data generating process is not required. Multiple, misspecified models can be compared.
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*Dominant recursive learning model* has smallest asymptotic rejection probability.

Main result: The authors provide conditions under which validation dynamics converge to the dominant recursive learning model, which the agent then uses almost always.
• Any assignment of the perceived law of motion (PLM) will affect system dynamics.
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Each model may induce a self-confirming equilibrium.
Nature of the validation dynamics

- An agent retains the current model unless it is rejected by an appropriate test.
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- Asynchronous updating (p. 12). Agent may be unaware that an alternative model is better until rejection of current model occurs.
• Expectational stability may fail to hold for some misspecified PLMs.

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- A general difficulty for validation dynamics is that models are being fit to data generated by other models. A policymaker that is learning would know this.
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• Literature is simulation-based.
Comparison with artificial intelligence

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• Evolutionary dynamic not part of the story here.
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Statistical versus economic selection

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Cogley-Sargent (2004) story about Samuelson-Solow vs. Lucas-Sargent. The decision-maker downweights the evidence because of the economic consequences of choosing the wrong model.
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- Authors use a statistically-based concept for model selection.
- Natural part of the attempt to get econometricians into the model.
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- Cogley-Sargent (2004) story about Samuelson-Solow vs. Lucas-Sargent. The decision-maker downweights the evidence because of the economic consequences of choosing the wrong model.
- Kocherlakota (2006): better fit not the same as better model.
• Model $k$ remains the model of choice for an extended period.
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• What economic advantage does the agent gain from resistance to switching models?
Hypothesis testing

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- What economic advantage does the agent gain from resistance to switching models?

- Why not simply adopt today’s best model?
Discussion of Cho and Kasa, “Validation.”

J. Bullard

Discussion
Parameters versus models
Connections to escape dynamics
Model validation
Specification testing
Assignment of the PLM
Nature of the validation dynamics
Instability generated by rival models
Artificial intelligence
Comparison with artificial intelligence
Statistical versus economic selection
Hypothesis testing
Restricted perceptions example
Conquest example
Conclusions

- Choice between two misspecified models is made based on

\[ H_1 (\beta_1) = \left( \frac{2 (1 - \alpha)}{\eta \Sigma_1 (\bar{\beta}_1)} \right) (\beta_1 - \bar{\beta}_1)^2 \]  \hspace{1cm} (1)

\[ H_2 (\beta_2) = \left( \frac{2 (1 - \alpha)}{\eta \Sigma_2 (\bar{\beta}_2)} \right) (\beta_2 - \bar{\beta}_2)^2 \]  \hspace{1cm} (2)

Dominant recursive learning model is the alternative with smaller \( \Sigma \). It fits better. But this may not be the better restricted perceptions equilibrium for household allocations. The agent may prefer to use an alternative model, experience the RPE associated with that model, and adopt that.
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Restricted perceptions example

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• Parsimony a key ingredient in this story. Something to hang our hats on?
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Doing more than a ‘minimal deviation from rational expectations’?