Discussion of In-Koo Cho and Kenneth Kasa, "Learning and Model Validation"

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- Most of the action in actual economies?
- Appeal: Forecasters can "express doubt" about their models, switch models.
- Mostly thinking in terms of non-nested models.

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- The authors are interested in whether policymaker model switching might *induce* escape dynamics ...
- ... and hence the regime switching we see in many macroeconomic variables.
- Some examples maintain the possibility of escape, others do not.

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- ... *and* misspecification tests based on the Kullback-Leibler Information Criterion (KLIC) are carried out.
- Models can be discarded, replaced with alternatives, and possibly reincarnated.
- Knowledge of the true data generating process is not required. Multiple, misspecified models can be compared.

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- But "endogenous data" generated from beliefs-outcomes feedback makes application of statistical theory problematic.
- *Dominant recursive learning model* has smallest asymptotic rejection probability.
- Main result: The authors provide conditions under which validation dynamics converge to the dominant recursive learning model, which the agent then uses almost always.

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- In this paper, agents are endowed with multiple, simple, *misspecified* PLMs. Agents use endogenously generated data to try to find the "best choice" among these, the *least falsifiable*.
- Each model may induce a self-confirming equilibrium.

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- Asynchronous updating (p. 12). Agent may be unaware that an alternative model is better until rejection of current model occurs.

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- A general difficulty for validation dynamics is that models are being fit to data generated by other models. A policymaker that is learning would know this.

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- Literature is simulation-based.

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- Evolutionary dynamic not part of the story here.

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- Cogley-Sargent (2004) story about Samuelson-Solow vs. Lucas-Sargent. The decision-maker downweights the evidence because of the economic consequences of choosing the wrong model.
- Kocherlakota (2006): better fit not the same as better model.

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- Why not simply adopt today's best model?

Restricted perceptions example

- Choice between two misspecified models is made based on

$$H_1(\beta_1) = \left(\frac{2(1-\alpha)}{\eta\Sigma_1(\bar{\beta}_1)}\right)(\beta_1 - \bar{\beta}_1)^2 \tag{1}$$

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- Dominant recursive learning model is the alternative with smaller Σ_i. It fits better.
- But this may not be the better restricted perceptions equilibrium for household allocations.
- The agent may prefer to use an alternative model, experience the RPE associated with that model, and adopt that.

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- Dynamic model superior outside the SCE, which is most of the time.
- The switch to the static model is reminiscent of Brock and Hommes (1997).
- Parsimony a key ingredient in this story. Something to hang our hats on?

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- 'Minimal deviation from rational expectations' has been a valuable feature of the recursive learning literature.
- Doing more than a 'minimal deviation from rational expectations'?