A New Comparative Approach to Macroeconomic Modeling and Policy Analysis

MTM Conference

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Our Objective

- This paper provides a new comparative approach to model-based research and policy analysis enabling individual researchers to conduct model comparisons easily, frequently, at low cost and on a large scale.
- Using this approach a model archive is built that includes many well-known empirically estimated models that may be used for quantitative analysis of monetary and fiscal stabilization policies.

Introduction

- Today many models are available that aim to explain the behavior of the main aggregates of the world's economies.
- However, those models differ in terms of economic structure, estimation methodology and parameter estimates.
- Hence, a given policy is likely to have different effects depending on which particular model is used for its evaluation.

$\longrightarrow$ McCallum (1999) recommends:

to search for a policy rule that possesses robustness in the sense of yielding reasonably desirable outcomes in policy simulation experiments in a wide variety of models.

Introduction c’td

- Yet, systematic comparisons of the empirical implications of a large variety of available models and evaluations of the performance of different policies across many models have been infrequent and costly.
- The 6 comparison projects reported in
  - Bryant, Henderson, Holtham, Hooper, Symansky (1988); Bryant, Currie, Frenkel, Masson and Portes (1989); Klein (1991); Bryant, Hooper, Mann (1993); Taylor (1999) and Hughes, Hallett, Wallis (2004)

  have involved multiple teams of researchers, each team working only with one or a small subset of available models.

Outline

1. A Formal exposition of the approach to model comparison
2. The Model Archive
3. Example: Monetary and fiscal impulse responses and serial correlations of U.S. output and inflation

1. A Formal Approach to Model Comparison

- Consider a particular model $m \in M$
- Model output is usually not directly comparable
  - Different variables
  - Different structural assumptions
  - Different notation and definitions
- Therefore it is necessary to augment models with a set of common, comparable variables, parameters, equations and shocks.

Notation

Table: Model-Specific Variables, Parameters, Shocks and Equations

Notation	Description
x_t^m	endogenous variables in model *m*
$x_t^{m,g}$	policy variables in model *m* (also incl. in x_t^m)
η_t^m	policy shocks in model *m*
ϵ_t^m	other economic shocks in model *m*
$g_m(.)$	policy rules in model *m*
$f_m(.)$	other model equations in model *m*
γ^m	policy rule parameters in model *m*
β^m	other economic parameters in model *m*
Σ^m	covariance matrix of shocks in model *m*

A General Nonlinear Model

Define a particular model m as follows:

$$E_t[g_m(x_t^m, x_{t+1}^m, x_{t-1}^m, \eta_t^m, \gamma^m)] = 0 \quad (1)$$

$$E_t[f_m(x_t^m, x_{t+1}^m, x_{t-1}^m, \epsilon_t^m, \beta^m)] = 0, \quad (2)$$

where

$$E([(\eta_t^m)'(\epsilon_t^m)']') = 0 \quad (3)$$

$$E([(\eta_t^m)'(\epsilon_t^m)']'[(\eta_t^m)'(\epsilon_t^m)']) = \Sigma^m = \begin{pmatrix} \Sigma_\eta^m & \Sigma_{\eta\epsilon}^m \\ \Sigma_{\eta\epsilon}^m & \Sigma_\epsilon^m \end{pmatrix} \quad (4)$$

Augmented with ...

Table: Comparable Common Variables, Parameters, Shocks and Equations

Notation	Description
z_t	common variables in all models
z_t^g	common policy variables in all models (also incl. in z_t)
η_t	common policy shocks in all models
$g(.)$	common policy rules
γ	common policy rule parameters

The Augmented Model

The augmented model consists of three components:

- The common policy rules $g(.)$ expressed in terms of common variables z_t, policy shocks η_t and policy rule parameters γ.
- A set of new model-specific equations that define the common variables in terms of original model-specific endogenous variables, $h_m(.)$ with parameters θ^m.
- The original set of model equations $f_m(.)$ determining endogenous variables, excluding the model-specific policy rule $g_m(.)$.

$$E_t[g(z_t, z_{t+1}, z_{t-1}, \eta_t, \gamma)] = 0 \quad (5)$$

$$E_t[h_m(z_t, x_t^m, x_{t+1}^m, x_{t-1}^m, \theta^m)] = 0 \quad (6)$$

$$E_t[f_m(x_t^m, x_{t+1}^m, x_{t-1}^m, \epsilon_t^m, \beta^m)] = 0 \quad (7)$$

Model Solution

General model solution:

$$
\begin{aligned}
z_t &= k_z(z_{t-1}, x^m_{t-1}, \eta_t, \epsilon^m_t, \kappa_z) && (8)\\
x^m_t &= k_x(z_{t-1}, x^m_{t-1}, \eta_t, \epsilon^m_t, \kappa_x) && (9)
\end{aligned}
$$

where (κ_z, κ_x) denote the reduced-form parameters that are in turn complex functions of the structural parameters, β^m, the policy parameters, γ, and the covariance matrix Σ^m.

Model Solution c’td

Linear approximation:

$$\begin{pmatrix} z_t \\ x_t^m \end{pmatrix} = K_m(\gamma) \begin{pmatrix} z_{t-1} \\ x_{t-1}^m \end{pmatrix} + D_m(\gamma) \begin{pmatrix} \eta_t \\ \epsilon_t^m \end{pmatrix} \tag{10}$$

where $K_m(\gamma)$ and $D_m(\gamma)$ are matrices of reduced-form and depend on the policy parameters γ. We denote the dependence of the solution parameters on the other (model specific) parameters β^m with the subcript m.

Objects for Comparison

For instance impulse responses and metric for comparison:

$$IR^m_{t+j}(\gamma;\eta^i) = \begin{pmatrix} E[z_{t+j}|z_{t-1}, x^m_{t-1}, I_t] - E[z_{t+j}|z_{t-1}, x^m_{t-1}] \\ E[x^m_{t+j}|z_{t-1}, x^m_{t-1}, I_t] - E[x^m_{t+j}|z_{t-1}, x^m_{t-1}] \end{pmatrix} = K_m(\gamma)^j D_m(\gamma) I_t \tag{11}$$

where I_t is a vector of zeros that is augmented with a single entry equal to the size of the common policy shock, for which the impulse response is computed.

Consider two models $m = 1, 2$, then:

$$s(\gamma, z) = \sum_{j=0}^{\infty} (IR^1_{t+j}(\gamma;\eta^i;z) - IR^2_{t+j}(\gamma;\eta^i;z)). \tag{12}$$

Objects for Comparison c'td

Unconditional variances and serial correlation:

$$V_0^m = \sum_{j=0}^{\infty} {K_m}^j D_m \Sigma^m {D_m}' {K_m}^{j\prime} \quad (13)$$

$$V_j^m = {K_m}^j V_0^m \quad (14)$$

Metric:

$$\omega = |V_0^1(z) - V_0^2(z)| \quad (15)$$

Example: How to augment a model

Common variables:

$$z_t = [\ i_t^z \quad g_t^z \quad \pi_t^z \quad p_t^z \quad y_t^z \quad q_t^z\ ]' \tag{16}$$

Table: Comparable Common Variables

Notation	Description
i_t^z	annualized quarterly money market rate
g_t^z	discretionary government spending (share in GDP)
π_t^z	year-on-year rate of inflation
p_t^z	annualized quarter-to-quarter rate of inflation
y_t^z	quarterly real GDP
q_t^z	quarterly output gap (dev. from flex-price level)

Common Rules, Shocks, Parameters

$$i_t^z = \gamma_i i_{t-1}^z + \gamma_p p_t^z + \gamma_q q_t^z + \eta_t^i \tag{17}$$

$$g_t^z = \gamma_g \eta_t^g \tag{18}$$

$$\eta_t = \begin{bmatrix} \eta_t^i & \eta_t^g \end{bmatrix} \tag{19}$$

$$\gamma = \begin{bmatrix} \gamma_i & \gamma_p & \gamma_q & \gamma_g \end{bmatrix} \tag{20}$$

NK_CGG99: The original model

Description	Equations and Definitions
Original Model	
variables	$x_t^1 = [\ i_t \quad x_t \quad \pi_t\ ], \quad x_t^{1,g} = [i_t]$
shocks	$\epsilon_t^1 = [\ g_t \quad u_t\ ]$
parameters	$\beta_1 = [\ \varphi \quad \theta \quad \phi\ ], \quad \gamma_1 = [\ \alpha \quad \gamma_\pi \quad \gamma_x\ ]$
model equations	
$g_1(.)$	$i_t = \alpha + \gamma_\pi(\pi_t - \bar{\pi}) + \gamma_x x_t$
$f_1(.)$	$x_t = -\varphi(i_t - E_t\pi_{t+1}) + \theta x_{t-1} + (1-\theta)E_t x_{t+1} + g_t$
...	$\pi_t = \lambda x_t + \phi\pi_{t-1} + (1-\phi)\beta E_t\pi_{t+1} + u_t$

NK_CGG99: The augmented model

Description	Equations and Definitions
Augmented Model	
$z_t, \eta_t, \gamma, g(.)$	as defined by equations (16-20).
$f_1(.)$	as defined above in original model.
$h_1(z_t, x_t^1, E_t x_{t+1}^1, x_{t-1}^1, \theta^1)$	$i_t^z = 4i_t$
...	$\pi_t^z = \pi_t + \pi_{t-1} + \pi_{t-2} + \pi_{t-3}$
...	$p_t^z = 4\pi_t$
...	$q_t^z = x_t$

2. Model Archive

We have created a platform, written in MATLAB, that allows straightforward comparisons of models' implications. The building blocks of this platform are:

- A set of dynamic stochastic macroeconomic models translated into the DYNARE software package.
  - Original variables, parameters, shocks and equations.
  - Common variables, parameters, shocks and equations.
- A set of objects for model comparison.

Models currently available in the data base

1. SMALL CALIBRATED MODELS

1.1	NK_RW97	Rotemberg and Woodford (1997)
1.2	NK_LWW03	Levin et al. (2003)
1.3	NK_CGG99	Clarida et al. (1999)
1.4	NK_CGG02	Clarida et al. (2002)
1.5	NK_MCN99cr	McCallum and Nelson (1999), (Calvo-Rotemberg model)
1.6	NK_MCN99pb	McCallum and Nelson (1999), (P-bar model)

Models currently available in the data base c'td

2. ESTIMATED US MODELS

2.1	US_FM95	Fuhrer and Moore (1995a)
2.2	US_OW98	Orphanides and Wieland (1998)
2.3	US_FRB03	Federal Reserve Board model lin. as in Levin et al. (2003)
2.4	US_SW07	Smets and Wouters (2007)
2.5	US_ACELm	Altig et al. (2005), (monetary policy shock)
	US_ACELt	Altig et al. (2005), (technology shocks)
	US_ACELswm	no cost channel as in Taylor and Wieland (2009) (m.p. shock)
	US_ACELswt	no cost channel as in Taylor and Wieland (2009) (tech. shocks)
2.6	US_RS99	Rudebusch and Svensson (1999)
2.7	US_OR03	Orphanides (2003)

Models currently available in the data base c'td

3. ESTIMATED EURO AREA MODELS

3.1	EA_CW05ta	Coenen and Wieland (2005), (Taylor-staggered contracts)
3.2	EA_CW05fm	Coenen and Wieland (2005), (Fuhrer-Moore-staggered contracts)
3.3	EA_AWM05	ECB's area-wide model linearized as in Dieppe et al. (2005)
3.4	EA_SW03	Smets and Wouters (2003)
3.5	EA_SR07	Sveriges Riksbank euro area model of Adolfson et al. (2007)
3.6	EA_QUEST3	Ratto et al. (2009)

Models currently available in the data base c'td

4. ESTIMATED/CALIBRATED MULTI-COUNTRY MODELS

4.1	G7_TAY93	Taylor (1993a) model of G7 economies
4.2	G3_CW03	Coenen and Wieland (2002) model of U.S.A, euro area and Japan
4.3	EACZ_GEM03	Laxton and Pesenti (2003) model calibrated to euro area and Czech republic
4.4	G2_SIGMA08	The Federal Reserve's SIGMA model from Erceg et al. (2008) calibrated to the U.S. economy and a symmetric twin.

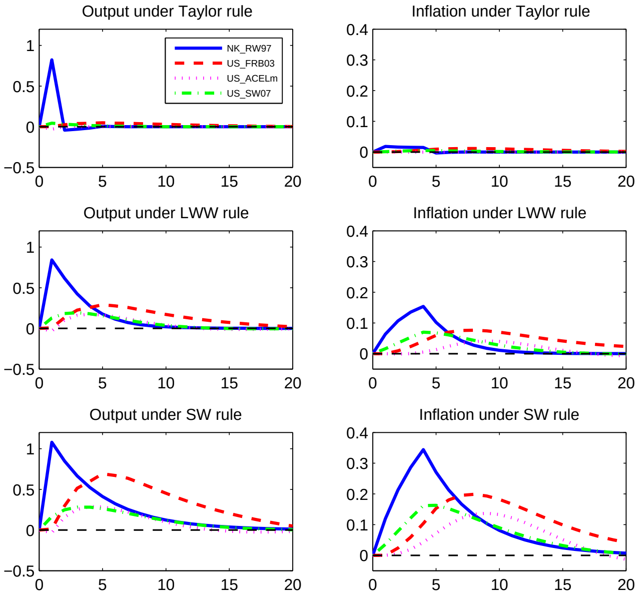
3. Example: Monetary and fiscal impulse responses and serial correlations of U.S. output and inflation

Four models: NK_RW97, US_FRB03, US_ACEL, US_SW07
Three monetary policy rules:

Taylor (1993b):	$i_t^z = \sum_{j=0}^{3} 0.38 p_{t-j}^z + 0.50 q_t^z + \eta_t^i$
Levin et al. (2003):	$i_t^z = 0.76 i_{t-1}^z + \sum_{j=0}^{3} 0.15 p_{t-j}^z + 1.18 q_t^z - 0.97 q_{t-1}^z + \eta_t^i$
Smets and Wouters (2007):	$i_t^z = 0.81 i_{t-1}^z + 0.39 p_t^z + 0.97 q_t^z - 0.90 q_{t-1}^z + \eta_t^i$

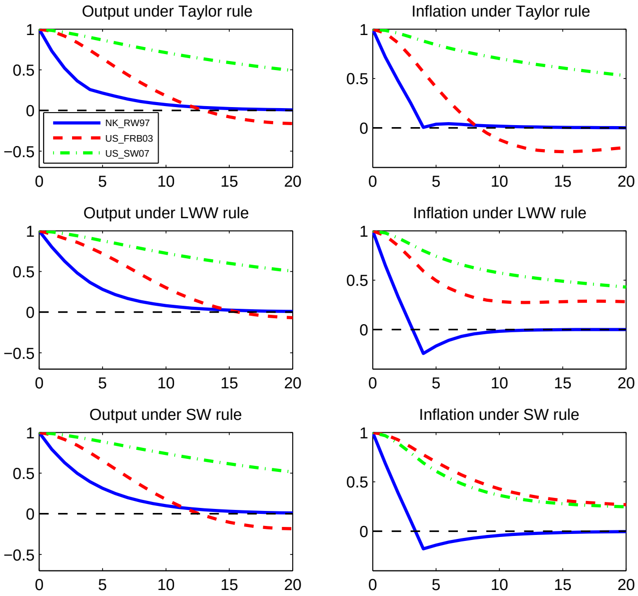
We look at:

- Impulse responses to a one time unexpected reduction of the nominal interest rate of 1 percentage point.
- Autocorrelation functions
- Impulse responses to a one time unexpected increase of government spending of 1 percent of GDP.


Output under Taylor rule
Inflation under Taylor rule
Output under LWW rule
Inflation under LWW rule
Output under SW rule
Inflation under SW rule
NK_RW97
US_FRB03
US_ACELm
US_SW07
1
0.5
0
−0.5
0.4
0.3
0.2
0.1
0
5
10
15
20

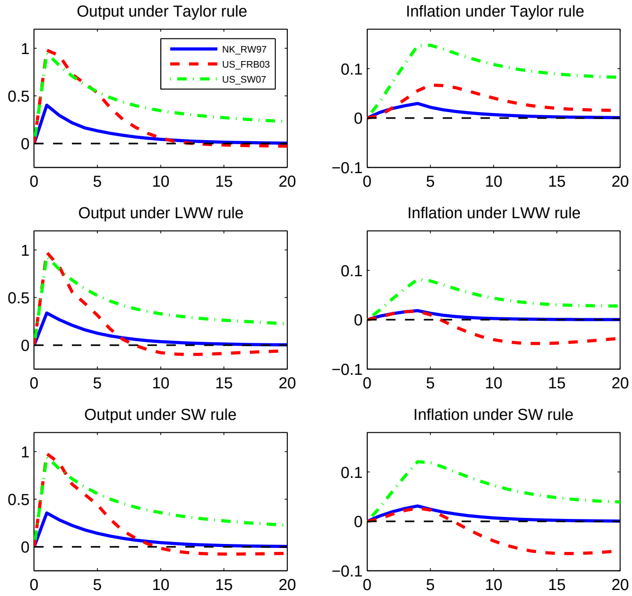
A negative monetary policy shock: Observations

- The monetary shock affects real output, since all four models exhibit nominal rigidities.
- Under the Taylor rule the output effect is short-lived.
- Under the LWW and SW rules the output response is both larger and longer-lasting.
- Only in the US_FRB03 model we observe the peak response of output in the second year after the shock. The other models exhibit no long policy lags of more than one year.
- The impulse response functions of inflation are more drawn out with the peak effect occurring later than the peak in output.


Output under Taylor rule
Inflation under Taylor rule
Output under LWW rule
Inflation under LWW rule
Output under SW rule
Inflation under SW rule
NK_RW97
US_FRB03
US_SW07
1
0.5
0
−0.5
0
5
10
15
20

Autocorrelation functions: Observations

- The small calibrated NK_RW97 model exhibits the lowest degree of output and inflation persistence.
- The US_FRB03 model comprises a larger set of dynamics and adjustment costs, translating into a larger degree of output and inflation persistence.
- Somewhat surprisingly, the strictly microfounded US_SW07 model exhibits the highest degree of output persistence under all three policy rules.


Output under Taylor rule
Inflation under Taylor rule
Output under LWW rule
Inflation under LWW rule
Output under SW rule
Inflation under SW rule
NK_RW97
US_FRB03
US_SW07
1
0.5
0
0.1
0
−0.1
0
5
10
15
20

A positive fiscal policy shock: Observations

- Initially, output increases in response to the shock under all three models, followed by a drawn-out decline over subsequent periods.
- However, the exact profile of the responses differs across models.
- The smallest impact is observed for the NK_RW97 model of around 0.4 percent of output, reflecting an immediate crowding out of private consumption.
- In the other two models the initial effect is about 1 percent of output and the subsequent decline evolves more slowly.

Conclusion

- We introduce a new comparative approach to model-based research and policy analysis that enables individual researchers to conduct model comparisons easily and on a large scale.
- For this purpose a model archive based on a common computational platform has been built that includes many well-known empirically estimated models.
- New models may easily be introduced and compared to established benchmarks.
- This approach should improve replicability of quantitative macroeconomic analysis and strengthen the robustness of policy recommendations.