Conditional probabilities for euro area sovereign default risk

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Disclaimer: Not necessarily the views of ECB or ESCB.
Contributions

We propose a **novel modeling framework** to infer **conditional** and **joint probabilities** for sovereign default risk from observed CDS.

**Novel framework?** Based on a *dynamic GH skewed–t* multivariate density/copula with time-varying volatility and correlations.

Multivariate model is sufficiently flexible to be **calibrated daily** to credit market expectations. Not an "official opinion".

Analysis is based on **Euro area** CDS data from Jan 2008 to June 2011. **Event study**: SMP/EFSF announcement & initial impact on risk.
Literature

1. **Sovereign credit risk:** e.g. Pan and Singleton (2008), Longstaff, Pan, Pedersen, and Singleton (2011), Ang and Longstaff (2011).

2. **Contagion,** see e.g. Forbes and Rigobon (2002), Caporin, Pelizzon, Ravazzolo, Rigobon (2012).


4. **Non-Gaussian dependence/copula/credit modeling,** see e.g. Demarta and McNeil (2005), Patton and Oh (2011).
Empirical questions

(Q1) Financial stability information: Based on credit market expectations, what is ... 

\[ \Pr(\text{two or more credit events in Euro area})? \]
\[ \Pr(i|j) - \Pr(i), \text{ for any } i,j? \]
\[ \text{Spillovers, e.g. } \Pr(\text{PT|GR}) - \Pr(\text{PT|not GR})? \]
\[ \text{Corr}_t(i,j) \text{ at time } t? \]

(Q2) Model risk: For answering (a), how important are parametric assumptions? Normal vs Student-t vs GH skewed-t.

(Q3) Event study: did the May 09, 2010 Euro area rescue package change risk dependence? How?
Data: skewed, fat tailed, tv vol’s and correlation
The copula idea/road-map

**Step 1:**
Conceptualize Euro Area sovereign risk as a portfolio of CDS protection against correlated sovereign default.

**Step 2:**
Estimate individual countries' failure probabilities from single-name CDS

**Step 3:**
Estimate the portfolio's multivariate density from changes in CDS spreads as a dynamic GH skewed-t

**Step 4:**
Estimate/simulate measures of joint and conditional failure
Copula framework

Sovereign defaults iff benefits \( (v_{it}) \) exceed a cost \( (c_{it}) \), where

\[
v_{it} = (\zeta_t - \mu_\zeta) \tilde{L}_{it} \gamma + \sqrt{\zeta_t} \tilde{L}_{it} \epsilon_t, \quad i = 1, \ldots, n,
\]

\( \epsilon_t \sim N(0, I_n) \) is a vector of risk factors,
\( \tilde{L}_{it} \) contains risk factor loadings,
\( \gamma \in \mathbb{R}^n \) determines skewness,
\( \zeta_t \sim IG \) is an additional scalar risk factor for, say, interconnectedness.

A default occurs with probability \( p_{it} \), where

\[
p_{it} = \Pr[v_{it} > c_{it}] = 1 - F_i(c_{it}) \iff c_{it} = F_i^{-1}(1 - p_{it}),
\]

where \( F_i \) is the CDF of \( v_{it} \).

Focus on conditional probability \( \Pr[v_{it} > c_{it} | v_{jt} > c_{jt}], \ i \neq j \).
**GH skewed-t dependence**

\[ y_t = \mu + L_t e_t, \quad t = 1, \ldots, T, \ e_t \sim \text{GHST}, \ E[e_t e_t'] = I_n, \]

\[
p(y_t; \cdot) = \frac{v^\frac{v}{2} 2^{1-\frac{v+n}{2}}}{\Gamma(\frac{v}{2}) \pi^{\frac{n}{2}} \vert \tilde{\Sigma}_t \vert^{\frac{1}{2}}} \cdot \frac{K_{\frac{v+n}{2}} \left(\sqrt{d(y_t) \cdot (\gamma' \gamma)}\right) e^{\gamma' \tilde{L}_t^{-1} (y_t - \tilde{\mu}_t)}}{(d(y_t) \cdot (\gamma' \gamma))^{\frac{v+n}{4}} d(y_t)^{\frac{v+n}{2}}},
\]

where

\[
d(y_t) = v + (y_t - \tilde{\mu}_t)' \tilde{\Sigma}_t^{-1} (y_t - \tilde{\mu}_t),
\]

\[
\tilde{\mu}_t = -v / (v - 2) \tilde{L}_t \gamma,
\]

\[
\tilde{\Sigma}_t = \tilde{L}_t \tilde{L}_t' \quad \text{is scale matrix}
\]

If \( \gamma = 0 \), then GH skewed-\( t \) simplifies to Student’s \( t \) density.
If in addition \( v^{-1} \to 0 \), then multivariate Gaussian density.
\( \tilde{\Sigma}_t(f_t) = \tilde{L}_t(f_t) \tilde{L}_t(f_t)' \) is driven by 1st and 2nd derivative of the pdf.
Marginal pd’s from CDS
Average correlation
The probability of two or more credit events
The probability of $k=0,1,2,...$ failures
Conditional pds: \( \Pr(\text{country } i \mid \text{GR}) \)
Conditional pds: $\text{Pr}(\text{all } i | \text{ all } j)$
## The May 09, 2010 package

### Joint risk, Pr(i \cap j)

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Avg| 33% | 16% | 71% | 40% | 18% | 64% |

Bottom line: joint risks ↓↓↓, but dependence ↑. "Firewall"-analogy?
Conclusion

We propose a **novel modeling framework** to infer **conditional** and **joint probabilities** for sovereign default risk from observed CDS.

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Thank you