Unemployment Fluctuations with Staggered Nash Wage Bargaining*

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Abstract

A number of authors have recently emphasized that the conventional model of unemployment dynamics due to Mortensen and Pissarides has difficulty accounting for the relatively volatile behavior of labor market activity over the business cycle. We address this issue by modifying the MP framework to allow for staggered multiperiod wage contracting. What emerges is a tractable relation for wage dynamics that is a natural generalization of the period-by-period Nash bargaining outcome in the conventional formulation. An interesting side-product is the emergence of spillover effects of average wages on the bargaining process. We then show that a reasonable calibration of the model can account well for the cyclical behavior of wages and labor market activity observed in the data. The spillover effects turn out to be important in this respect.

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1 Introduction

A long standing challenge in macroeconomics is accounting for the relatively smooth behavior of real wages over the business cycle along with the relatively volatile behavior of employment. A recent body of research, beginning with Shimer (2005a), Hall (2005a) and Costain and Reiter (2003), has re-ignited interest in addressing this challenge. These authors show that the conventional model of unemployment dynamics due to Mortensen and Pissarides (hereafter "MP") cannot account for the key cyclical movements in labor market activity, at least for standard calibrations of parameters. The basic problem is that the mechanism for wage determination within this framework, period-by-period Nash bargaining between firms and workers, induces too much volatility in wages. This exaggerated procyclical movement in wages, in turn, dampens the cyclical movement in firms' incentives to hire. Shimer (2005) and Hall (2005a) proceed to show that with the introduction of ad hoc wage stickiness, the framework can account for employment volatility. Of course, this begs the question of what are the primitive forces that might underlie this wage rigidity.

A rapidly growing literature has emerged to take on this puzzle. Much of this work attempts to provide an axiomatic foundation for wage rigidity, explicitly building up from assumptions about the information structure, and so on.[1] To date, due to complexity, this work has focused mainly on qualitative findings and has addressed quantitative issues only in a limited way.[2]

In this paper we take a pragmatic approach to modelling wage rigidity, with the aim of developing a framework that is tractable for quantitative analysis. In particular, we retain the empirically appealing feature of Nash bargaining, but modify the conventional MP model to allow for staggered multi-period wage contracting. Each period, only a subset of firms and workers negotiate a wage contract. Each wage bargain, further, is between a firm and its existing workforce: Workers hired in-between contract settlements receive the existing wage. We restrict the form of the wage contract to call for a fixed wage per period over an exogenously given horizon. Though it would be undoubtedly preferable to completely endogenize the contract structure, these restrictions are reasonable from an empirical standpoint. The payoff is a simple empirically appealing wage equation that is an intuitive generalization of the standard Nash bargaining outcome. The gain over a simple ad hoc wage adjustment mechanism is that the key primitive parameter of the model is the average frequency of wage adjustment, as opposed to an arbitrary partial adjustment coefficient in a wage equation. In this way, the staggered contracting structure provides more discipline in evaluating the model than do simple ad hoc adjustment mechanisms.

[1] Examples include Menzio (2005), Kennan (2006) and Shimer and Wright (2004). Others have pursued "on-the-job search" as an explanation, though both Mortensen and Nagypal (2005) and Hall (2005c) express some skepticism that this approach in isolation can solve the puzzle.

[2] An exception is Menzio (2005) who presents a calibrated model with endogenous wage rigidity. His model does well except for wages, which are too smooth. We instead focus on explaining the joint dynamics of labor market activity and wages.

The use of time dependent staggered price and wage setting, of course, is widespread in macroeconomic modelling, beginning with Taylor (1980) and Calvo (1983). More recently, Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2003) have found that staggered wage contracting is critical to the empirical performance of the recent vintage of dynamic general equilibrium macroeconomic frameworks (i.e., sticky prices alone are not sufficient). There are, however, some important distinguishing features of our approach. First, macroeconomic models with staggered wage setting typically have employment adjusting along the intensive margin. That is, wage stickiness enhances fluctuations in hours worked as opposed to total employment. As a consequence, these frameworks are susceptible to Barro's (1977) argument that wages may not be allocational in this kind of environment, given that firm's and workers have an on-going relationship. If wages are not allocational, of course, then wage rigidity does not influence model dynamics. By contrast, in the model we present, wages affect employment at the extensive margin: They influence the rate at which firms add new workers to their respective labor forces. As emphasized by Hall (2005c), in this kind of setting the Barro critique does not apply.

A second key difference involves the nature of the wage contracting process. In the conventional macroeconomic models, monopolistically competitive workers set wages. Here, firms and workers bargain over wages in a setting with search and matching frictions. As a consequence, some interesting "spillover" effects emerge of the average market wage on the contract wage. These spillover effects are a product of the staggered contract/bargaining environment. They introduce additional stickiness in the movement of real wages, much the same way that real rigidities enhance nominal price stickiness in models of staggered price setting (e.g., Kimball (1995), Woodford (2003)).

As we noted, the wage/unemployment volatility puzzle arises with standard calibrations of the MP model. An interesting recent paper by Hagedorn and Manovskii (2005) considers an alternative parameterization. In particular, these authors find parameters that allow the model to match the low elasticity of wages with respect to productivity present in the data. By generating smooth wages in this fashion, the model is then able to capture unemployment volatility. At issue, however, is that some of the key parameters required to permit the model to capture the volatility puzzle are quite different than conventional analyses suggest may be reasonable. In effect, HM make labor supply high elastic, much more so than do standard calibrations. In addition, despite calibrating to match wage data, their model does not account well for either the cyclical co-movement or volatility of wages, as we discuss below.

We differ by using a more conventional model parametrization. In our framework, accordingly, it is the overlapping multi-period wage contracts that accounts for the low elasticity of wages with respect to productivity. Further, rather than picking parameters to match this elasticity, we choose them to be consistent with the available micro evidence on the duration of wage adjustments. In this regard, we add a degree of discipline on the calibration. We then investigate how well the model captures wage dynamics, as well as the volatility of unemployment and the other key variables of the model.

In section 2 we characterize the basic features of the model. In section 3 we derive a set of simple dynamic equations for wages and the hiring rate, obtained by considering a local approximation of the model about the steady state. We also exposit the spillover effects that influence the wage bargaining process, contributing to overall wage stickiness. One additional distinguishing feature of the setup is that a "horizon effect" emerges that influences the bargaining process, since firms care about the implications of the contract wage for future hires, while workers do not. While the horizon effect is interesting from a theoretical perspective, it turns out to not be quantitatively important in our baseline calibration. In section 4 we examine the empirical performance of the model and show that the framework does a good job of accounting for the basic features of the U.S. data, including wage dynamics. In section 5, we verify that under our calibration the model satisfies the important technical condition that the wage always lies within the bargaining set over the life of the contract. Concluding remarks are in section 6. Finally, the appendix provides an explicit derivation of all the key results, including the steady state of the model. It also presents the complete loglinearized model.

2 The Model

The framework is a variation of the Mortensen and Pissarides search and matching model (Mortensen and Pissarides, 1994, Pissarides, 2000). The main difference is that we allow for staggered multi-period wage contracting. Within the standard framework, workers and firms negotiate wages based on period-by-period Nash bargaining. We keep the Nash bargaining framework, but in the spirit of Taylor (1980) and Calvo (1983), only a fraction of firms and workers re-set wages in any given period. As well, they strike a bargain that lasts for multiple periods. Workers hired in between contracting periods receive the existing contract wage. Here the idea is that due to scale economies in bargaining, employment terms are negotiated only periodically: Firms do not negotiate separate terms for the relatively small percentage of workers who enter in between contracting periods.[3]

For technical reasons, there are two other differences from MP. First, because it will turn out to be important for us to distinguish between existing and newly hired workers at a firm, we drop the assumption of one worker per firm and instead allow firms to hire a continuum of workers. We assume constant returns to scale, however, which greatly simplifies the bargaining problem. Second, we drop the conventional assumption of a fixed cost per vacancy opened and instead assume that firms face quadratic adjustment costs of adjusting employment size. The reason is as follows: With staggered wage setting, there will arise a dispersion of wages across firms in equilibrium. Quadratic costs of adjusting employment ensures a determinate equilibrium in the presence of wage dispersion. To be clear, however, while this assumption is necessary for technical reasons, it does not drive our results, as we show below.

[3] Our scenario applies to situations where workers are relatively homogenous: e.g. clerical workers as opposed to professional basketball players.

Finally, we embed our search and matching framework within a simple intertemporal general equilibrium framework in order to study the dynamics of unemployment and wages. Following Merz (1995), we adopt the representative family construct, which effectively involves introducing complete consumption insurance.

2.1 Unemployment, Vacancies and Matching

Let us now be more precise about the details: There is a continuum of infinitely lived workers and a continuum of infinitely lived firms, each of measure one. We index firms by i and workers according to the identity of their employer. Each firm i employs $n_t(i)$ workers at time t. It also posts $v_t(i)$ vacancies in order to attract new workers for the next period of operation. The total number of vacancies and employed workers are $v_t = \int_0^1 v_t(i)di$ and $n_t = \int_0^1 n_t(i)di$. The total number of unemployed workers, u_t, is given by

$$u_t = 1 - n_t. \tag{1}$$

Following convention, we assume that the number of new hires or "matches", m_t, is a function of unemployed workers and vacancies, as follows:

$$m_t = \sigma_m u_t^{\sigma} v_t^{1-\sigma}. \tag{2}$$

The probability a firm fills a vacancy in period t, q_t, is given by

$$q_t = \frac{m_t}{v_t}. \tag{3}$$

Similarly, the probability an unemployed worker finds a job, s_t, is given by

$$s_t = \frac{m_t}{u_t}. \tag{4}$$

Both firms and workers take q_t and s_t as given.

Finally, each firm exogenously separates from a fraction $1 - \rho$ of its workers each period, where ρ is the probability a worker "survives" with the firm until the next period. Accordingly, within our framework fluctuations in unemployment will be due to cyclical variation in hiring as opposed to separations. Both Hall (2005) and Shimer (2005) argue that this characterization is consistent with recent U.S. evidence.

2.2 Firms

Each period, firms produce output, $y_t(i)$, using capital, $k_t(i)$, and labor, $n_t(i)$, according to the following Cobb-Douglas technology:

$$y_t(i) = a_t k_t(i)^{\alpha} n_t(i)^{1-\alpha}, \tag{5}$$

where a_t is a common productivity factor. As we noted earlier, because we will have wage dispersion across firms, we replace the standard assumption of fixed costs of posting a vacancy with quadratic labor adjustment costs. For simplicity, we assume capital is perfectly mobile across firms and that there is a competitive rental market in capital.

It is convenient to define the hiring rate, $x_t(i)$, as the ratio of new hires, $q_t v_t(i)$, to the existing workforce, $n_t(i)$:

$$x_t(i) = \frac{q_t v_t(i)}{n_t(i)}. \tag{6}$$

Note that the firm knows the hiring rate with certainty at time t, since it knows that likelihood q_t that each vacancy it posts will be filled. The total workforce, in turn, is the sum of the number of surviving workers, $\rho n_t(i)$, and new hires, $q_t v_t(i)$:

$$n_{t+1}(i) = \rho n_t(i) + q_t v_t(i). \tag{7}$$

Let $w_t(i)$ be the the wage rate, z_t the rental rate of capital, and $\beta E_t \Lambda_{t,t+1}$ be the firm's discount rate, where the parameter β is the household's subjective discount factor and where $\Lambda_{t,t+1} = u'(c_{t+1})/u'(c_t)$. Then given quadratic costs of adjusting the workforce, the value of the firm $F_t(i)$, may be expressed as:

$$F_t(i) = y_t(i) - w_t(i) n_t(i) - \frac{\kappa}{2} x_t(i)^2 n_t(i) - z_t k_t(i) + \beta E_t \Lambda_{t,t+1} F_{t+1}(i). \tag{8}$$

At any time, the firm maximizes its value by choosing the hiring rate (by posting vacancies) and its capital stock, given its existing employment stock, the probability of filling a vacancy, the rental rate on capital and the current and expected path of wages. If it is a firm that is able to renegotiate the wage, it bargains with its workforce over a new contract. If it is not renegotiating, it takes as given the wage at the previous period's level, as well the likelihood it will be renegotiating in the future.

We next consider the firm's hiring and capital rental decisions, and defer a bit the description of the wage bargain. Let $J_t(i)$ be the value to the firm of adding another worker at time t:

$$J_t(i) = (1-\alpha)\frac{y_t(i)}{n_t(i)} - w_t(i) + \frac{\kappa}{2} x_t(i)^2 + \rho\beta E_t \Lambda_{t,t+1} J_{t+1}(i). \tag{9}$$

Then the first order condition for vacancy posting equates the marginal cost of adding a worker with the discounted marginal benefit:

$$\kappa x_t(i) = \beta E_t \Lambda_{t,t+1} J_{t+1}(i). \tag{10}$$

In turn, the first order condition for capital is simply:

$$z_t = \alpha \frac{y_t(i)}{k_t(i)} = \alpha \frac{y_t}{k_t}. \tag{11}$$

With Cobb-Douglas production and perfectly mobile capital, output/capital ratios are equalized across firms. It follows that capital/labor ratios and output/labor ratios are also equalized.

Let f_{nt} denote the firm's marginal product of labor at t (i.e., $f_{nt} = (1-\alpha)y_t/n_t$). Then, combining equations yields the following forward looking difference equation for the hiring rate:

$$\kappa x_t(i) = \beta E_t \Lambda_{t,t+1}\left[f_{nt+1} - w_{t+1}(i) + \frac{\kappa}{2}x_{t+1}(i)^2 + \rho\kappa x_{t+1}(i)\right]. \tag{12}$$

The hiring rate thus depends on a discounted stream of the firm's expected future surplus from the marginal worker: the sum of net earnings at the margin, $f_{nt+1} - w_{t+1}(i)$, and saving on adjustment costs, $\frac{\kappa}{2}x_{t+1}(i)^2$.

2.3 Workers

In this sub-section we develop an expression for a worker's surplus from employment, which becomes a critical determinant of the outcome of the wage bargain.

Let $V_t(i)$ be the value to a worker of employment at firm i and let U_t be the value of unemployment. $V_t(i)$ is given by

$$V_t(i) = w_t(i) + \beta E_t \Lambda_{t,t+1}\left[\rho V_{t+1}(i) + (1-\rho)U_{t+1}\right]. \tag{13}$$

Note that this value depends on the wage specific to firm i, $w_t(i)$, as well as the likelihood the worker will remain employed in the subsequent period.

To construct the value of unemployment, we first define $V_{x,t}$ as the average value of employment conditional on being a new worker at t. The subscript x is meant to denote that we are averaging $V_t(i)$ across new workers, i.e., workers at t who were hired in period $t-1$. $V_{x,t}$ is given by

$$V_{x,t} = \int_0^1 V_t(i)\frac{x_{t-1}(i)n_{t-1}(i)}{x_{t-1}n_{t-1}}di, \tag{14}$$

where $x_{t-1}(i)n_{t-1}(i)$ is total new workers at firm i at time t (i.e., hires from the previous period) and $x_{t-1}n_{t-1}$ is total new workers at t.[4] Next, let b be the flow value from unemployment, taken to be unemployment benefits. Then, U_t may be expressed as

$$U_t = b + \beta E_t \Lambda_{t,t+1}\left[s_t V_{x,t+1} + (1-s_t)U_{t+1}\right], \tag{15}$$

where, as before, s_t is the probability of finding a job for the subsequent period. The value of unemployment thus depends on the current flow value b and the likelihood of being employed versus unemployed next period. Note that the value of finding a job next period for a worker that

[4] $V_{x,t}$ is thus distinct from the unconditional average value of employment $V_t = \int_0^1 V_t(i)\frac{n_t(i)}{n_t}di$. However, since in the steady state, hiring rates are identical across firms and employment shares are constant, $V_{x,t}$ and V_t are identical in the steady state and have similar dynamics outside the steady state, up to a first order approximation. The appendix elaborates.

is currently unemployed is $V_{x,t+1}$, the average value of working next period conditional on being a new worker. That is, unemployed workers do not have a priori knowledge of which firms might be paying higher wages next period. They instead just randomly flock to firms posting vacancies.[5]

The worker surplus at firm i, $H_t(i)$, and the average worker surplus conditional on being a new hire, $H_{x,t}$, are given by:

$$H_t(i) = V_t(i) - U_t, \tag{16}$$

and

$$H_{x,t} = V_{x,t} - U_t. \tag{17}$$

It follows that:

$$H_t(i) = w_t(i) - b + \beta E_t \Lambda_{t,t+1} \left[\rho H_{t+1}(i) - s_t H_{x,t+1} \right]. \tag{18}$$

2.4 Consumption and Saving

Following Merz and others, we use the representative family construct, which gives rise to perfect consumption insurance. In particular, the family has employed workers at all firms and unemployed workers, representative of the population at large. The family pools their incomes before choosing per capita consumption and asset holdings. In addition to wage income and unemployment income, the family has a diversified ownership stake in firms, which pay out profits Π_t. Finally, households may either consume c_t, or save in the form of capital, which they rent to firms at the rate z_t. Let Ω_t be the value function for the representative household. Then the maximization problem may be expressed as

$$\Omega_t = \max_{\{c_t, k_{t+1}\}} \left[\log(c_t) + \beta E_t \Omega_{t+1} \right] \tag{19}$$

subject to

$$c_t + k_{t+1} = w_t n_t + (1 - n_t) b + (z_t + 1 - \delta) k_t + \Pi_t + T_t, \tag{20}$$

where T_t are transfers from the government.[6]

Let $\lambda_t \equiv c_t^{-1}$ be the marginal utility of consumption. Then the first necessary conditions for consumption/saving yields:

$$\lambda_t = \beta E_t \lambda_{t+1} (z_{t+1} + 1 - \delta). \tag{21}$$

[5] There is accordingly no directed search. Note, however, that wage differentials across firms are only due to the differential timing of contracts, which is transitory. Thus, because a worker who arrives at a firm in the midst of an existing contract may expect a new one reasonably soon, the payoff from directed search may not be large.

[6] The government simply collects lump-sum taxes (negative transfers) and uses them to pay unemployment benefits.

2.5 Nash Bargaining and Wage Dynamics

We restrict the form of the wage contract to call for a fixed wage per period over an exogenously given length of time. Though it would be undoubtedly preferable to completely endogenize the contract structure, these restrictions are reasonable from an empirical standpoint. The payoff will be a simple empirically appealing wage equation that is an intuitive generalization of the standard Nash bargaining outcome. In particular, given these restrictions on the form of the contract, workers and firms determine the contract wage through Nash bargaining.

We introduce staggered multiperiod wage contracting in a way that simplifies aggregation. In particular, each period a firm has a fixed probability $1 - \lambda$ that it may re-negotiate the wage. This adjustment probability is independent of its history. Thus, while how long an individual wage contract lasts is uncertain, the average duration is fixed at $1/(1 - \lambda)$. The coefficient λ is thus a measure of the degree of wage stickiness that can be calibrated to match the data[7]. This simple Poisson adjustment process, further, implies that it is not necessary to keep track of individual firms' wage histories, which makes aggregation simple. In the end, the model will deliver a simple relation for the evolution of wages that is the product of Nash bargaining in conjunction with staggered wage setting.

Firms that enter a new wage agreement at t negotiate with the existing workforce, including the recent new hires. Due to constant returns, all workers are the same at the margin. The wage is chosen so that the negotiating firm and the marginal worker share the surplus from the marginal match. Given the symmetry to which we just alluded, all workers employed at the firm receive the same newly-negotiated wage[8]. When firms are not allowed to renegotiate the wage, all existing and newly hired workers employed at the firm receive the wage paid the previous period. As we discussed earlier, we appeal to scale economies in bargaining to rule out separate negotiations for worker who arrive in between contracting periods.[9] Of course, the newly hired workers recognize that they will be able to re-negotiate wage at the next round of contracting. In the benchmark case where the contract length corresponds to just one period, wage dynamics are just as in the conventional model and behave counterfactually as recently argued.

Let w_t^* denote the wage of a firm that renegotiates at t. Given constant returns, all sets of renegotiating firms and workers at time t face the same problem, and thus set the same wage. As we noted earlier, the firm negotiates with the marginal worker over the surplus from the marginal match. We assume Nash bargaining, which implies that the contract wage w_t^* is chosen to solve

[7] This kind of Poisson adjustment process is widely used in macroeconomic models with staggered price setting, beginning with Calvo (1983).

[8] To be clear, with constant returns, one could either think of the firm bargaining with each marginal worker individually or bargaining with a union that wishes to maximixe average worker surplus.

[9] Bewley (1999) presents some evidence consistent with our assumption that, in between contracting periods, newly hired workers received existing wages. In particular, he shows that wages of new workers are often linked to the existing internal pay structure.

$$\max H_t(r)^{\eta} J_t(r)^{1-\eta}, \tag{22}$$

where $H_t(r)$ and $J_t(r)$ are the values of J and H for renegotiating workers and firms.

Because the contract is multi-period, we need to take into account the impact of the contract wage on the expected future path of firm and worker surplus. Let $W_t^f(r)$ denote the firm's discounted sum of expected future wage payments over both the existing contract and subsequent contracts and let $W_t^h(r)$ be the corresponding value of the worker's expected wage receipts. Note that the two values will differ in general because the firm has a longer horizon than the worker: The firm cares about the impact of the current wage contract on payments not only to the existing workforce, but also to new workers who enter under the terms of the existing contract. A worker, on the other hand, only cares about wages during his or her tenure at the firm. Accordingly, let $\Sigma_t(r)$ be the firm's cumulative discount factor and Δ_t the worker's cumulative discount factor. Then:

$$W_t^f(r) = \Sigma_t(r) w_t^* + (1-\lambda) E_t \sum_{s=1}^{\infty} \frac{n_{t+s}}{n_t}(r)\beta^s \Lambda_{t,t+s}\Sigma_{t+s}(r) w_{t+s}^*, \tag{23}$$

$$W_t^w(r) = \Delta_t w_t^* + (1-\lambda) E_t \sum_{s=1}^{\infty} (\rho\beta)^s \Lambda_{t,t+s}\Delta_{t+s} w_{t+s}^*, \tag{24}$$

with

$$\Sigma_t(r) = E_t \sum_{s=0}^{\infty} \frac{n_{t+s}}{n_t}(r)(\lambda\beta)^s \Lambda_{t,t+s}, \tag{25}$$

$$\Delta_t = \sum_{s=0}^{\infty} (\rho\lambda\beta)^s \Lambda_{t,t+s}. \tag{26}$$

Observe that each term s in the firm's cumulative discount factor depends on the expectation of the product of three factors: the employment size at firm $t+s$ relative to time t, $\frac{n_{t+s}}{n_t}(r)$, the probability the contract survives to $t+s$, λ^s, and the households' discount factor, $\beta^s \Lambda_{t,t+s}$. It is similar for the worker, except the survival probability ρ^s replaces the relative employment size. Since on average $\frac{n_{t+s}}{n_t}(r)$ exceeds ρ^s, the firm places relatively more weight on the future than does the worker. This simply reflects that, unlike the worker, the firm cares about the implications of the contract for new workers as well as existing ones.

The appendix shows that for renegotiating firms and workers we can write

$$J_t(r) = E_t \sum_{s=0}^{\infty} \frac{n_{t+s}}{n_t}(r)\beta^s \Lambda_{t,t+s}\left[f_{nt+s} - \frac{\kappa}{2} x_{t+s}(r)^2\right] - W_t^f(r), \tag{27}$$

and

$$H_t(r) = W_t^w(r) - E_t \sum_{s=0}^{\infty} (\rho\beta)^s \Lambda_{t,t+s}\left[b + s_{t+s}\beta\Lambda_{t+s,t+s+1} H_{x,t+s+1}\right]. \tag{28}$$

Equation (27) is obtained by combining the hiring rate condition with the expression for the shadow value of a worker to the firm $J_t(r)$ given by equation (9). Intuitively, given constant returns and given the hiring rate is chosen optimally, the surplus of the marginal worker at t may be expressed as discounted profits per worker at t, where the term $\frac{n_{t+s}}{n_t}(r)$ enters the discount factor to adjust for relative changes in firm size in the future. In turn, the marginal worker's surplus, $H_t(r)$, depends on the expected discounted value of wage payments, net the discounted sum of flow value of unemployment, b, plus expected discounted surplus of moving from unemployment to employment, $s_{t+s}\beta\Lambda_{t,t+s+1}H_{x,t+s+1}$.

The solution to the Nash bargaining problem, then, is

$$\eta\Delta_t J_t(r) = (1-\eta)\,\Sigma_t(r)\,H_t(r), \tag{29}$$

where $\Delta_t = \partial H_t(r)/\partial w_t^*$ is the effect of a rise in the contract wage on worker surplus, while $\Sigma_t(r) = -\partial J_t(r)/\partial w_t^*$ is minus the effect of a rise in the contract wage on firm surplus. Since on average $\Sigma_t(r) > \Delta_t$, shifts in the contract wage have a larger impact in absolute value on firms surplus than on worker surplus. This contrasts with the conventional case of period-by-period bargaining, where the two effects are of identical absolute values (since the future is irrelevant in this case.)

It is possible to rewrite equation (29) as

$$\chi_t(r)\,J_t(r) = (1-\chi_t(r))\,H_t(r), \tag{30}$$

with

$$\chi_t(r) = \frac{\eta}{\eta + (1-\eta)\,\Sigma_t(r)/\Delta_t}. \tag{31}$$

Equation (31) is a variation on the conventional sharing rule, where the relative weight $\chi_t(r)$ depends not only on the worker's bargaining power η, but also on the differential firm/worker horizon, reflected by the term $\Sigma_t(r)/\Delta_t$. Note that in the limiting case of $\lambda = 0$, $\Sigma_t(r)/\Delta_t = 1$ and $\chi_t(r) = \eta$, as in the conventional case of period-by-period wage bargaining. With $\lambda > 0$, however, $\chi_t(r)$ is less than η on average (since $\Sigma_t(r)/\Delta_t$ exceeds unity on average). Intuitively, since movements in the contract wage have a larger impact on discounted firm surplus than on worker surplus, the "horizon effect" works to raise the effective bargaining power of firms from $1-\eta$ to $1-\chi_t(r)$.[10]

As the appendix shows, combining equations yields the following first order forward looking difference equation for the contract wage:

$$\Delta_t w_t^* = w_t^o(r) + \rho\lambda\beta E_t\Lambda_{t,t+1}\Delta_{t+1}w_{t+1}^*, \tag{32}$$

where the forcing variable $w_t^o(r)$ can be thought of as the "target" wage and is given by

[10] We thank Larry Christiano for pointing out to us that in an earlier version of the paper we had not properly taken into account the impact of the "horizon effect" on the bargaining problem.

$$w_t^o(r) = \chi_t(r)\left(f_{nt} + \frac{\kappa}{2}x_t(r)^2\right) + (1 - \chi_t(r))(b + s_t\beta E_t\Lambda_{t,t+1}H_{x,t+1}). \tag{33}$$

Observe that the target wage has the same form as the wage that would emerge under period-by-period Nash bargaining, though with an adjustment for the horizon effect. In particular, it is a convex combination of what a worker contributes to the match and what the worker loses by accepting a job, where the weights depend on the worker's relative horizon-adjusted bargaining power $\chi_t(r)$. The worker's contribution is the marginal product of labor plus the saving on adjustment costs. With our quadratic cost formulation, this saving is measured by $\frac{\kappa}{2}x_t(r)^2$. The foregone benefit from unemployment, in turn, is the flow value of unemployment, b, plus expected discounted gain of moving from unemployment this period to employment next period, $s_t\beta\Lambda_{t,t+1}H_{x,t+1}$.

As in the conventional literature on time-dependent wage and price contracting (Taylor, 1980 and Calvo, 1983), the contract wage depends on an expected discounted sum of the target under perfectly flexible adjustment, in this case $w_t^o(r)$. Iterating equation (32) yields

$$w_t^* = E_t\sum_{s=0}^{\infty}\phi_{t,t+s}w_{t+s}^o(r) \tag{34}$$

with

$$\phi_{t,t+s} = \frac{(\rho\lambda\beta)^s\Lambda_{t,t+s}}{E_t\sum_{s=0}^{\infty}(\rho\lambda\beta)^s\Lambda_{t,t+s}} \tag{35}$$

Observe that in the limiting case of period by period wage negotiations, i.e., when $\lambda = 0$, w_t^* converges to $w_t^o(r)$.

A significant difference from the traditional literature on wage contracting, however, is that spillover effects emerge directly from the bargaining problem that have the contract wage depend positively on the economy-wide average wage. As we show in section 3, these spillover effects emerge because the average wage affects the two key determinates of the target wage, $w_t^o(r)$: the expected discounted surplus of moving from unemployment to employment, $s_t\beta\Lambda_{t,t+1}H_{x,t+1}$, and the hiring rate, $x_t(r)$. Through both these channels, the spillover works to enhance wage rigidity.

Finally, the average wage across workers is given by

$$w_t = \int_0^1 w_t(i)\frac{n_t(i)}{n_t}di \tag{36}$$

Since the fraction of firms that re-negotiate contracts is a random draw across the population and since all firms that renegotiate at t choose the same contract wage w_t^*, by the law of large numbers we can express the wage index as

$$w_t = (1-\lambda)w_t^* + \lambda\int_0^1 w_{t-1}(i)\frac{n_t(i)}{n_t}di \tag{37}$$

where $1-\lambda$ is the fraction of firms who are re-negotiating and λ is the fraction who are not. The average wage is thus a convex combination of the contract wage, w_t^*, and the average wage across the population of firms that do not re-negotiate, given by the integral in the second term. As we show below, up to a first order, this integral can be approximated by last period's average wage, w_{t-1}.[11]

2.6 Resource Constraint

We complete the model with the following resource constraint, which divides output between consumption, investment and adjustment costs:

$$y_t = c_t + k_{t+1} - (1-\delta)\,k_t + \frac{\kappa}{2} x_t^2 n_t. \tag{38}$$

This completes the description of the model.

3 Wage/Hiring Dynamics and Spillover Effects

To gain some intuition for the model, we next derive loglinear equations for wages and hiring. In doing so, we identify the spillover effects that make the wage bargain sensitive to the average wage in a way that works to enhance wage rigidity. We also clarify how the horizon effect that emerges because firms and workers weight the future differently affects the bargaining outcome.

We begin by deriving an expression for the target wage, $w_t^o(r)$, the forcing variable in the difference equation for wages. Loglinearizing the target wage equation (33) gives

$$\widehat{w}_t^o(r) = \left(\varphi_{fn}\widehat{f}_{nt} + \varphi_x\widehat{x}_t(r)\right) + \varphi_s\left(\widehat{s}_t + \widehat{H}_{t+1} + \widehat{\Lambda}_{t,t+1}\right) + \varphi_\chi\widehat{\chi}_t(r), \tag{39}$$

where $\varphi_{fn} = \chi f_n w^{-1}$, $\varphi_x = \chi\kappa x^2 w^{-1}$, $\varphi_s = \chi s\kappa x w^{-1}$, $\varphi_\chi = \chi\left[f_n + (\kappa/2)\,x^2 - b - s\beta H\right] w^{-1}$. Note that $\widehat{z}$ denotes the percent deviation of variable z from its steady state value and the coefficients are either parameters or steady state values of variables.[12]

The first two terms in parentheses in equation (39) reflect factors that move the target wage in the case of conventional period-by-period bargaining. The first captures the variation in the marginal worker's contribution to firm value. The second captures the variation in the worker's forgone benefit from unemployment. In addition to these conventional factors, however, the target wage is also influenced by the horizon effect on bargaining that arises with multi-period contracting.

The horizon effect influences $\widehat{w}_t^o(r)$ in two ways. First, movements in the conventional factors that cause $\widehat{w}_t^o(r)$ to vary are multiplied by the steady state horizon-adjusted bargaining weight, $\chi = \eta/[\eta + (1-\eta)\Sigma/\Delta]$, as opposed to the pure weight η. As we noted in the previous section,

[11] Since each $w_{t-1}(i)$ is weighted by $n_t(i)/n_t$ and not $n_{t-1}(i)/n_{t-1}$, $\int_0^1 w_{t-1}(i)\frac{n_t(i)}{n_t}di$ is not quite identical to w_{t-1}. However, given that the employment shares are constant in the steady state, the dynamics of this integral are equivalent to the dynamics of w_{t-1}, up to a first order.

[12] Since up to a first order approximation $\widehat{H}_{x,t+1} = \widehat{H}_{t+1}$, we drop the subscript x (see the appendix).

this adjustment in effect raises the relative weight assigned to firms, leading to workers grabbing a smaller share of the variation in the surplus. (Note $\chi < \eta$). Second, the horizon adjusted bargaining weight $\chi_t(r)$ may vary, leading to a direct influence on $\widehat{w}_t^o(r)$, as captured by the third term in equation (39). In particular, loglinearizing equations (25), (26) and (31), yields:

$$\widehat{\chi}_t(r) = -(1-\chi)\left(\widehat{\Sigma}_t(r) - \widehat{\Delta}_t\right), \tag{40}$$

with

$$\widehat{\Sigma}_t(r) = x\lambda\beta\widehat{x}_t(r) + \lambda\beta E_t\left(\widehat{\Lambda}_{t,t+1} + \widehat{\Sigma}_{t+1}(r)\right), \tag{41}$$

$$\widehat{\Delta}_t = \rho\lambda\beta E_t\left(\widehat{\Lambda}_{t,t+1} + \widehat{\Delta}_{t+1}\right). \tag{42}$$

In particular, $\chi_t(r)$ may vary due to relative movement in the firm and worker cumulative discount factors, $\Sigma_t(r)$ and Δ_t, respectively. Note that $\widehat{\Sigma}_t(r)$ depends on the hiring rate $\widehat{x}_t(r)$, since the latter influences the firm's subsequent employment relative to current employment, one of the determinants of its cumulative discount factor. (See equations (6), (7) and (25)).

Observe that as we move to the limiting case of flexible wages ($\lambda = 0$), the horizon effect disappears: $\chi_t(r)$ becomes a constant equal to η. The variation in the target wage then corresponds to the conventional outcome for period-by-period Nash bargaining.

While the horizon effect adds a new dimension to the bargaining problem, it is important to stress that it is unlikely to be quantitatively important. As we show in the next section, given a monthly job survival probability ρ that is consistent with the evidence, the difference between the firm and worker cumulative discount factors is not sufficiently large for the horizon effect to have a significant effect on the outcome. In this instance, the steady state horizon-adjusted bargaining weight χ does not differ much from the primitive bargaining weight η. Nor does $\chi_t(r)$ vary much.

We next turn to analyzing the spillover effect of market wages on the wage bargain. It turns out that there is both a direct and indirect spillover effect. The direct effect arises because the worker's outside option depends on the wage he or she can expect to earn elsewhere. As appendix shows, by making use of the Nash bargaining condition at $t+1$ and the period t vacancy posting condition, the discounted surplus of moving from unemployment today to employment next period, conditional on finding a job, may be expressed in loglinear form as

$$E_t\left(\widehat{H}_{t+1} + \widehat{\Lambda}_{t,t+1}\right) = \widehat{x}_t + (1-\chi)^{-1} E_t\widehat{\chi}_{t+1} + \Gamma E_t\left(\widehat{w}_{t+1} - \widehat{w}_{t+1}^*\right), \tag{43}$$

where

$$\Gamma = (1 - x\eta\Psi)\,\eta^{-1}\epsilon\Sigma w,$$

with $\Psi = \beta\lambda^2/\left(1-\beta\lambda^2\right)$, $\Sigma = (1-\beta\lambda)^{-1}$, and $\epsilon = \beta(\kappa x)^{-1}$. Note that since the steady state hiring rate x is a number close to zero, under any reasonable calibration, the slope coefficient Γ is positive.

Intuitively, $E_t\left(\widehat{H}_{t+1} + \widehat{\Lambda}_{t,t+1}\right)$ depends positively on the current hiring rate $\widehat{x}_t$ since the latter varies positively with the expected marginal surplus from labor at $t+1$. It also depend positively on a worker's expected bargaining power next period $E_t\widehat{\chi}_{t+1}$. The spillover effect, however, emerges in the third term, which depends positively on the difference between the expected average market wage $E_t\widehat{w}_{t+1}$ and the contract wage $E_t\widehat{w}^*_{t+1}$. If, everything else equal, $E_t\widehat{w}_{t+1}$ exceeds $E_t\widehat{w}^*_{t+1}$, opportunities are unusually good for workers expecting to move into employment next period, and vice-versa if $E_t\widehat{w}_{t+1}$ is below $E_t\widehat{w}^*_{t+1}$. By influencing the worker's outside option in this way, the expected average market wage at $t+1$ induces a direct spillover effect on the wage bargain.

The indirect spillover emerges because the hiring rate of the renegotiating firm affects the bargaining outcome. It does so by influencing both the firm's saving in adjustment costs and the horizon-adjusted bargaining weight (because it affects the firm's cumulative discount factor.) The difference between hiring rate $\widehat{x}_t(r)$ and average hiring rate $\widehat{x}_t$ depends positively on the difference between the average market wage $\widehat{w}_t$ and the contract wage $\widehat{w}^*_t$:

$$\widehat{x}_t(r) = \widehat{x}_t + \lambda\epsilon\Sigma w\left(\widehat{w}_t - \widehat{w}^*_t\right). \tag{44}$$

The dependency of the hiring rate on the wage gap thus introduces an indirect spillover of market wages on the bargaining problem.

Let $\widehat{w}^o_t$ be the target wage absent the spillover effects. Then combining equations (39), (43), and (44), we can express $\widehat{w}^o_t(r)$ as the sum of $\widehat{w}^o_t$ and the direct and indirect spillovers:

$$\widehat{w}^o_t(r) = \widehat{w}^o_t + \frac{\tau_1}{1-\rho\lambda\beta}E_t\left(\widehat{w}_{t+1} - \widehat{w}^*_{t+1}\right) + \frac{\tau_2}{1-\rho\lambda\beta}\left(\widehat{w}_t - \widehat{w}^*_t\right), \tag{45}$$

with

$$\widehat{w}^o_t = \varphi_{fn}\widehat{f}_{nt} + (\varphi_x + \varphi_s)\widehat{x}_t + \varphi_s\widehat{s}_t + \varphi_\chi\widehat{\chi}_t + (1-\chi)^{-1}\varphi_s E_t\widehat{\chi}_{t+1}, \tag{46}$$

and where τ_1 and τ_2 capture the direct and indirect spillovers, respectively

$$\begin{aligned}\tau_1 &= \varphi_s\Gamma(1-\rho\lambda\beta), \\ \tau_2 &= \left[\varphi_x\lambda - \varphi_\chi(1-\chi)x\Psi\right]\epsilon\Sigma w(1-\rho\lambda\beta).\end{aligned} \tag{47}$$

Note that absent the adjustment for the horizon effect on bargaining (captured by χ, $\widehat{\chi}_t$ and $E_t\widehat{\chi}_{t+1}$), $\widehat{w}^o_t$ would be precisely the target wage under period-by-period Nash bargaining. It is also worth emphasizing now that, given our calibration, the direct spillover effect captured by τ_1 is quantitatively more important than the indirect effect captured by τ_2. We show this explicitly in the appendix.

Next, loglinearizing the equation for the contract wage and combining with the equation above yields the contract wage as a first order forward looking difference equation, with the target wage as the forcing variable:

$$\widehat{w}^*_t = (1-\rho\lambda\beta)\,\widehat{w}^o_t(r) + \rho\lambda\beta E_t\widehat{w}^*_{t+1}. \tag{48}$$

The loglinearized wage index is in turn given by

$$\widehat{w}_t = (1-\lambda)\,\widehat{w}^*_t + \lambda\widehat{w}_{t-1}. \tag{49}$$

Combining these equations along with the relation for $\widehat{w}_t^o(r)$ (equation (45)) then yields the following second order difference equation which governs the evolution of the wage:

$$\widehat{w}_t = \gamma_b \widehat{w}_{t-1} + \gamma \widehat{w}_t^o + \gamma_f E_t \widehat{w}_{t+1}, \tag{50}$$

where

$$\begin{aligned} \gamma_b &= (1+\tau_2)\,\phi^{-1} \\ \gamma &= \varsigma \phi^{-1} \\ \gamma_f &= (\rho\beta - \tau_1)\,\phi^{-1} \\ \phi &= 1 + \tau_2 + \varsigma + \rho\beta - \tau_1 \\ \varsigma &= (1-\lambda)(1-\rho\lambda\beta)\,\lambda^{-1} \end{aligned} \tag{51}$$

with $\gamma_b + \gamma + \gamma_f = 1$. Note the forcing variable in the difference equation is the "spillover free" target wage $\widehat{w}_t^o$ (see equation (46)).

Due to staggered contracting, $\widehat{w}_t$ depends on the lagged wage $\widehat{w}_{t-1}$ as well as the expected future wage $E_t \widehat{w}_{t+1}$. Solving out for the reduced form of equation (50) will yield an expression that relates the wage to the lagged wage and a discounted stream of expected future values of $\widehat{w}_t^o$. Note that the spillover effects, measured by τ_1 and τ_2 work to raise the relative importance of the lagged wage (by raising γ_b) and reduce the importance of the expected future wage (by reducing γ_f). In this way, the spillovers work to raise the inertia in the evolution of the wage. In this respect, the spillover effects work in a similar (though not identical) way as to how real relative price rigidities enhance nominal price stickiness in monetary models with time-dependent pricing (see, for example, Woodford, 2003).

Note also that as we converge to $\lambda = 0$ (the case of period by period wage bargaining), both γ_b and γ_f go to zero), implying that $\widehat{w}_t$ simply tracks $\widehat{w}_t^o$ in this instance. Further, as we noted earlier, $\widehat{w}_t^o$, becomes identical to the target in the flexible wage case. The model thus nests the conventional period-by-period wage bargaining setup.

Finally, loglinearizing the difference equation for the hiring rate (12) and aggregating economy-wide yields:

$$\widehat{x}_t = E_t \widehat{\Lambda}_{t,t+1} + \epsilon \left(f_n \widehat{f}_{nt+1} - w E_t \widehat{w}_{t+1} \right) + \beta E_t \widehat{x}_{t+1}. \tag{52}$$

The hiring rate thus depends on current and expected movements of the marginal product of labor relative to the wage. The stickiness in the wage due to staggered contracting, everything else equal, implies that current and expected movement in the marginal product of labor will have a greater impact on the hiring rate than would have been the case otherwise.

We defer to the appendix a complete presentation of the loglinear equations of the model.

4 Model Evaluation

4.1 Calibration

We choose a monthly calibration in order to properly capture the high rate of job finding in U.S. data. Our parametrization is summarized in Table 1. There are ten parameters to which we need to assign values. Four are conventional in the business cycle literature: the discount factor, β, the depreciation rate, δ, the "share" parameter on capital in the Cobb-Douglas production function, α, and the autoregressive parameter for the technology shock, ρ_a. We use conventional values for all these parameters: $\beta = 0.99^{\frac{1}{3}}$, $\delta = 0.025/3$, $\alpha = 0.33$, and $\rho_a = 0.95^{\frac{1}{3}}$. Note in contrast to the frictionless labor market model, the term $1 - \alpha$ does not necessarily correspond to the labor share, since the latter will in general depend on the outcome of the bargaining process. However, here we simply follow convention by setting $\alpha = 0.33$ to facilitate comparison with the RBC literature.[13]

Table 1: Values of parameters

Production function parameter	α	0.33
Discount factor	β	0.997
Capital depreciation rate	δ	0.08
Technology autoregressive parameter	ρ_a	0.983
Survival rate	ρ	0.965
Elasticity of matches to unemployment	σ	0.5
Job finding probability	s	0.45
Bargaining power parameter	η	0.5
Relative unemployment flow value	$\overline{b}$	0.4
Renegotiation frequency	λ	0.889

There are an additional five parameters that are specific to the conventional search and matching framework: the job survival rate, ρ, the matching function parameter, σ, the bargaining power parameter, η, the steady state job finding probability, s, and the relative unemployment flow value, $\overline{b}$, equal to the ratio of the unemployment flow value, b to the steady state flow contribution of the worker to the match, $f_n + \frac{\kappa}{2}x^2$. We choose the average monthly separation rate $1 - \rho$ based on the observation that jobs last about two years and a half. Therefore, we set $\rho = 1 - 0.035$. We choose the elasticity of matches to unemployment, σ, to be equal to 0.5, the midpoint of values typically used in the literature.[14] This choice is within the range of plausible values of 0.5 to 0.7 reported by

[13] Note that while $1 - \alpha$ does not correspond to the labor share, α corresponds to the capital share.

[14] The values for σ used in the literature are: 0.24 in Hall (2005), 0.4 in Blanchard and Diamond (1989), Andolfatto (1994) and Merz (1995), 0.46 in Mortensen and Nagypal (2005), 0.5 in Hagedorn and Manovskii (2005), 0.5 in Farmer (2004), 0.72 in Shimer (2005). See also a brief discussion in Mortensen and Nagypal (2005), p. 10, comparing their value of 0.46 to Shimer's one.

Petrongolo and Pissarides (2001) in their survey of the literature on the estimation of the matching function. We then set $s = 0.45$ to match recent estimates of the U.S. average monthly job finding rate (Shimer, 2005).

To maintain comparability with much of the existing literature, we set the bargaining power parameter η to be equal to 0.5.[15] One of the few studies that provides direct estimates is Flinn (2005), who finds a point estimates of 0.4, close to the value we use. Further, while we stick with 0.5 for our baseline case, we show that our results our robust to using 0.4. An additional justification, however, is that $\eta = 0.5$ implies a steady state labor share of 0.65, which is consistent with the long run average of the labor share in the data. Finally, we note that $\eta = 0.5$ in conjunction with $\sigma = 0.5$ ensures the efficiency of the equilibrium in the flexible version of the model (Hosios, 1990).

Perhaps most controversial is the choice of $\overline{b}$. We follow much of the literature by assuming that the value of non work activities is far below what workers produce on the job (see Hall, NBER Macroannual, 2005, p. 31, for a brief discussion). In particular, we specifically follow Shimer (2005) and Hall (2005c) and set $\overline{b} = 0.4$. Under the interpretation of b as unemployment benefits, this parametrization implies a steady state replacement ratio of 0.42 (since the steady state ratio of the wage to the worker's contribution to the job is 0.956.)

We next observe that given the parameter values chosen so far, the steady state of the model pins down both the adjustment cost parameter, κ, and the steady state values of the labor share, the unemployment rate and the hiring rate (see the appendix.). Table 2 gives these values, along with the steady state consumption and investment shares. Note that as we discussed in the previous section, the horizon adjusted bargaining parameter χ does not vary much from the primitive parameter η (0.44 versus 0.50).

Table 2: Implied steady state values

Unemployment rate	u	0.07
Hiring rate	x	0.035
Horizon-adjusted bargaining power	χ	0.44
Labor share	ls	0.65
Investment/output ratio	$\frac{I}{y}$	0.24
Consumption/output ratio	$\frac{c}{y}$	0.75
Adjustment costs/output ratio	$\frac{ac}{y}$	0.01

Finally, there is one parameter that is specific to this model: the probability λ that a firm may not renegotiate the wage. We pick λ to match the average frequency of wage contract negotiations. While there is no systematic direct evidence on the frequency of wage negotiations, Taylor (1999)

[15] In the literature the bargaining power has been typically set either to satisfy the Hosios (1990) condition or to achieve symmetric Nash barganing (equally shared surplus). This has led most researchers to set values in the range 0.4 to 0.5. Shimer (2005) uses the somewhat larger value of 0.72.

argues that in most medium to large sized firms wages are typically adjusted once per year. He also argues that this pattern characterizes union workers as well as non-union workers, including in the latter workers who do not have formal employment contracts. In addition, based on microeconomic data on hourly wages, Gottschalk (2004) concludes that wage adjustments are most common a year after the last change. This evidence, of course applies primarily to base pay. There are, however, other components such as bonuses that might be adjusted more frequently over the year, though it is very unclear how important these adjustments might be in practice. Nonetheless, to be conservative, for our baseline case we set $\lambda = 1-1/9$, implying that wage contracts are renegotiated on average once every 3 quarters. We then consider the case of a 4 quarter average contract length as a robustness exercise.

4.2 Results

We judge the model against quarterly U.S. data from 1964:1-2005:1. For series that are available monthly, we take quarterly averages. Since the artificial series that the model generates are based on a monthly calibration, we also take quarterly averages of this data.

Most of the data is from the BLS. All variables are measured in logs. Output y is production in the non-farm business sector. The labor share ls and output per worker y/n are similarly from the non-farm business sector. The wage w is average hourly earnings of production workers in the private sector, deflated by the CPI. Employment n is all employees in the non-farm sector. Unemployed u is civilian unemployment 16 years old and over. Vacancies v are based on the help wanted advertising index from the Conference Board. Finally, the data are HP filtered with a conventional smoothing weight.

We examine the behavior of the model taking the technology shock as the exogenous driving force. To illustrate how the wage contracting process affects model dynamics, we first examine the impulse responses of the model economy to a unit increase in total factor productivity. The solid line in each panel of Figure 1 illustrates the response of the respective variable for our model. For comparison, the dotted line reports the response of the conventional flexible wage model with period-by-period Nash bargaining (obtained by setting $\lambda = 0$).

Observe that in the conventional case with period-by-period wage adjustment, the response of employment is relatively modest, confirming the arguments of Hall and Shimer. There is also only a modest response of other indicators of labor market activity, such as vacancies, v, unemployment u, labor market tightness, $\theta = v/u$, and the hiring rate x. Wages, by contrast, adjust quickly. The resulting small adjustment of employment leads to output dynamics that closely mimic the technology shock.

By contrast, in the model with staggered multiperiod contracting, the hiring rate jumps sharply in the wake of the technology shock along with the measures of labor market activity. A substantial rise in employment follows, certainly as compared to the conventional flexible wage case. Associated with the rise in employment, is a smooth drawn out adjustment in wages, directly a product of the staggered multiperiod contracting. The lagged rise in employment leads to a humped shaped response of output, i.e., output continues to rise for several periods before reverting to trend, in contrast to the technology shock which reverts immediately.

We next explore how well the model economy is able to account the overall volatility in the data. Table 3 reports the standard deviation, autocorrelation, and contemporaneous correlation with output for the nine key variables in the U.S. economy and in the model economy. The standard deviations are normalized relative to output.

Table 3: Aggregate Statistics

	y	w	ls	n	u	v	θ	y/n
	US Economy, 1964:1-2005:01							
Relative Standard Deviation	1.00	0.52	0.51	0.60	5.15	6.30	11.28	0.61
Autocorrelation	0.87	0.91	0.73	0.94	0.91	0.91	0.91	0.79
Correlation with y	1.00	0.56	−0.20	0.78	−0.86	0.91	0.90	0.71
	Model Economy, $\lambda \to 3Q$							
Relative Standard Deviation	1.00	0.56	0.57	0.35	4.46	5.83	9.88	0.71
Autocorrelation	0.84	0.95	0.65	0.90	0.90	0.83	0.88	0.76
Correlation with y	1.00	0.66	−0.56	0.77	−0.77	0.91	0.94	0.97
	Model Economy, $\lambda \to 4Q$							
Relative Standard Deviation	1.00	0.47	0.58	0.44	5.66	7.25	12.47	0.64
Autocorrelation	0.85	0.96	0.68	0.91	0.91	0.86	0.90	0.74
Correlation with y	1.00	0.56	−0.59	0.78	−0.78	0.94	0.95	0.95

Overall the model economy for the baseline case (3 quarters) appears to capture well most of the basic features of the data. It comes reasonably close to capturing the relative volatilities and co-movements of the key indicators of labor market activity, including unemployment u, vacancies v and the tightness measure θ. These were the variables emphasized in the Hall/Shimer analysis. The model only captures about sixty percent of the relative volatility of employment. However, here it is important to keep in mind that the framework abstracts from labor force participation, a non-trivial source of cyclical employment volatility. (We also emphasize that perhaps for this consideration, the papers in literature typically avoid reporting statistics on employment volatility.)

A distinguishing feature of our analysis is that we appear to capture wage dynamics. Note that we come very close to matching the relative volatility of wages (0.56 versus 0.52 in the data), their autocorrelation (0.95 versus 0.91 in the data) and the contemporaneous correlation of wages with output (0.66 versus 0.56 in the data).

As we noted earlier, we assumed three quarter average length wage contracts for our baseline case to error on the side of caution, even though the evidence suggests that the modal period of wage adjustments is one year. In the bottom panel of Table 3 we also report statistics based on four quarter average length wage contracts. Interestingly, the performance of the model improves overall. Not surprising, the enhanced wage rigidity raises the volatilities of the labor market variables. In the end, the model tracks the relative volatilities and co-movements of the key labor market variables, u, v, and θ as well as in the baseline case. The model, however, is also now able to capture nearly three quarters of the relative volatility of employment.

We next consider several variations of the model designed to illustrate what features are important. First, as we also discussed earlier, the inertia in wage dynamics is not simply a product of staggered multi-period contracting, but also of the spillover effect of economy-wide wages on the individual wage bargain that arises in this kind of environment. To quantify the importance of these spillovers for model dynamics, we simulate the model eliminating the spillover effects on wage dynamics. In particular, we set equal to zero the parameters τ_1 and τ_2, which govern the magnitude of the spillover effect, in equations (50) and (51).

Table 4: The Spillover Effect and Other Robustness Exercises

	Relative Standard Deviations							
	y	w	ls	n	u	v	θ	y/n
Model Economy	1.00	0.56	0.57	0.35	4.46	5.83	9.88	0.71
Model Economy - No Spillover	1.00	0.70	0.48	0.18	2.35	3.18	5.25	0.84
Model Economy - Flexible Wages	1.00	0.88	0.09	0.10	1.25	1.58	2.74	0.93
Flexible Wages - Standard Hiring Costs	1.00	0.93	0.02	0.06	0.72	1.01	1.63	0.95
Model Economy - No Horizon Effect	1.00	0.53	0.53	0.39	5.13	6.70	11.37	0.67

Table 4 reports the results. As the table makes clear, eliminating the spillovers significantly enhances wage flexibility and reduces employment volatility. When the spillovers are removed, the relative volatility of wages jumps nearly fifty percent, from 0.56 to 0.70 . Conversely, the relative volatility of employment is reduced roughly in half, from 0.35 to 0.18. The other measures of labor activity u, v and θ similarly fall by about half. Overall, the spillovers are responsible for about a half of the added rigidity in wages relative to the flexible benchmark model and for about two thirds of the added volatility in the labor market. Thus, the wage inertia and resulting employment dynamics in our model are not only a product of staggered multiperiod wage contracting, but also of the spillover effects from the Nash bargaining process.

The next two rows of the table makes clear that our assumption of quadratic adjustment costs is not responsible for the ability of our baseline model to account for the key moments of the data. The model with flexible wages and quadratic adjustment costs performs about as poorly as the conventional formulation with proportional hiring costs. It is thus the presence of staggered wage contracting in conjunction with the spillover effects that account for the results in Table 3.

The last row of Table 4 presents information on the importance of the horizon effect on the wage bargain. The row presents the model statistics for our baseline case, but with the horizon effect shut off (i.e., $\chi_t(r)$ fixed at η). The relative volatilities remain close to those arising in the baseline case. Thus, as we conjectured, the horizon effect is not that important quantitatively.

Finally, it is interesting to compare our analysis with Hagedorn and Manovskii (HM, 2005). They find from micro data that the wage elasticity with respect to labor productivity is 0.47. As we noted earlier, they choose parameters to have the model match this elasticity. To do so, they require a very low value of η, the bargaining power of workers, and a very high value of $\bar{b}$, the relative steady state flow value of unemployment, as compared to what is conventional in the literature. In particular, they require η very close to zero, well below the conventionally used value of 0.5, as well as Flinn's (2005) estimate of 0.4. In addition they require $\bar{b}$ close to unity, well above Shimer (2005) and Hall's (2005c) preferred value of 0.4. A value of $\bar{b}$ close to unity, of course implies that workers are nearly indifferent between employment and unemployment. The overall calibration effectively makes labor supply highly elastic, enabling the model to have large employment movements with moderate wage adjustments. Nonetheless, while the HM framework is able to account for labor market volatility, the resulting calibration is not without controversy.[16]

Interestingly, we find from our macro data that the wage elasticity with respect to labor productivity is 0.53, which is very close to the estimate that the authors obtained from micro evidence. However, as we suggested earlier, we stick with conventional values of η and b, and instead introduce wage sluggishness by appealing to staggered multi-period contracts. Further, as opposed to picking parameters to match the wage elasticity, we calibrate the average duration of wages contracts to match the evidence. We then ask how well the model explains the wage elasticity (along with other volatilities.) It turns out the model does very well on this accounting, generating a wage elasticity of 0.50, nearly identical to what our data suggests.

In addition, as we observed in Table 3, our model does well at explaining the overall cyclical volatility of wages, including the co-movement with aggregate activity as well as the relative

[16] In addition to having values of η and $\bar{b}$ that are at variance with the literature, Hornstein, Krusell and Violante note that the HM calibration implies suspiciously large employment effects from changes in unemployment insurance.

volatility. On the other hand, the HM model does not do well on this dimension, even though it is calibrated to match the wage elasticity with respect to productivity. How can this be? Note that since this elasticity, $el(w, p)$, is effectively a regression coefficient from the regression of log wages on log productivity it equals the product of the correlation $corr(w, p)$ and the relative standard deviations σ_w/σ_p. Since the HM calibration only fixes the product of these two moments, it needs not do well at matching them individually. This turns out to be the case, as we show next.

Table 5 compares values of $el(w, p)$, $corr(w, p)$ and σ_w/σ_p against U.S. data for three models: the conventional Mortensen and Pissarides model (with capital), the framework based on the HM calibration, and our baseline model with staggered wage contracting (GT). While the HM model captures $el(w, p)$ by construction, it misses badly on the other two moments. The correlation between wages and productivity is too high (unity versus 0.62 in the data) while the relative volatility of wages is too low (0.49 versus 0.85). The former outcome is due to the period by period Nash bargaining that ties aggregate wage movements to current period productivity. The latter result arises from the low bargaining power of workers which forces their wages close to their reservation values. Thus while the HM model by construction matches $el(w, p)$, it does so by inducing offsetting errors in $corr(w, p)$ and σ_w/σ_p.

Table 5: Wages and Labor Share Statistics

	$el(w,p)$	$corr(w,p)$	σ_w/σ_p
U.S. data	0.53	0.62	0.85
MP baseline	0.98	1.00	0.98
HM	0.49	1.00	0.49
GT	0.50	0.62	0.80
	$el(ls,p)$	$corr(ls,p)$	σ_{ls}/σ_p
U.S. data	-0.50	-0.60	0.83
MP baseline	-0.02	-0.96	0.02
HM	-0.51	-1.00	0.51
GT	-0.51	-0.64	0.80

By contrast, our model does well at matching not only the wage elasticity but also the correlation of wages and productivity, as well as the relative volatility. The staggered contract structure works to dampen the correlation between productivity and wages. At the same time, because workers have more bargaining power than in the HM calibration, wages are more sensitive to productivity movements, permitting the model to match the data. Again, we stress that our model is calibrated to match the average duration of contract lengths. It is therefore not by construction that we match the wage elasticity, in contrast to HM.

Finally, the bottom part of Table 5 shows that similar conclusions apply for the volatility of the labor share. While the HM model does not explain all the relevant moments well, our framework does.

5 Bargaining Set

A key maintained hypothesis in our analysis is that workers and firms can expect that they will not want to voluntarily dissipate their relationship over the life of their relationship. This assumption simplifies how both parties form expectations when they enter relationships. Here we demonstrate that this condition holds to a reasonable approximation. Put differently, under our parametrization, wages have a negligible probability of falling outside the bargaining set. Intuitively, given our Poisson process for contract adjustment, only a very small fraction of contracts will have a duration sufficiently long for the wage to move out of the bargaining set.

Note first that the lower and upper limits of the bargaining set are given by, respectively, the reservation wage of the marginal worker and the reservation wage of the firm[17]. These limits will depend on the time elapsed since the firm has last negotiated a contract, denoted by τ. The appendix derives loglinear expressions for the worker reservation wage, denoted $R_t^w(\tau)$, and the firm reservation wage, denoted $R_t^f(\tau)$. Given these expressions we can then check whether a contract wage set τ periods earlier, $w_t^*(\tau)$, lies within the bargaining set, i.e., whether, $R_t^w(\tau) < w_t^*(\tau) < R_t^f(\tau)$.

We emphasize first that for the typical firm in the midst of a contract, the bargaining set is quite wide. Figure 2 displays a time series of the bargaining set (based on artificial time series generated by the model), for firms on a contract that has been in place for three quarters, the average length. Note that the contract lies safely within the bargaining set throughout the time series.

It is possible, however, that a small fraction of firms could stay on contracts well beyond the average length. Thus, we need to determine a threshold value for contract duration τ, where the contract wage still remains in the bargaining set. Note that the probability that a contract will last more than τ periods is given by the per period probability the contract will not be renegotiated, λ, raised to the τ power, i.e., λ^τ. Given the law of large numbers, this will also correspond to the percentage of existing contracts that have lasted more than τ periods. This percentage thus declines exponentially with τ.

We pick a threshold value for τ such that the fraction of contracts outstanding that have lasted more than τ periods is 0.0089 (a number less than one percent). We then check for the ninety nine percent plus of contracts that have lasted τ periods or less, whether the contract wage remains in the bargaining set. If this is the case, then we argue that violations of our maintained hypothesis are negligible from a quantitative standpoint.

[17] Note that all workers with the firm will have the same reservation wage, regardless of whether they are new or old.

Specifically, we set τ to satisfy $\lambda^{\tau} = (1 - 1/9)^{\tau} = 0.0089$, which leads to a value τ equal to 40 (months). We then generate artificial times series from our model and ask whether the wage lies within the bargaining set for contracts of duration 40 months or less. Figure 3 displays the results. For our cutoff, the wage is always in the bargaining set. We therefore conclude that for 99.11 percent of firms this condition is satisfied.

While the wage lies safely within the bargaining set for firms on contracts that have lasted three quarters, for (the small fraction of) firms that have been for 40 months, the boundaries of the bargaining set vary much closer to the wage. What this suggests is that if one wishes to add idiosyncratic shocks, it may be necessary to alter the simple binomial process for contract adjustment, where a small fraction of firms can remain indefinitely on the contract, by imposing a terminal length that any contract can survive.

6 Concluding Remarks

We have modified the Mortensen and Pissarides model of unemployment dynamics to allow for staggered multiperiod wage contracting. What emerges is a tractable relation for wage dynamics that is a natural generalization of the period-by-period Nash bargaining outcome in the conventional formulation. An interesting side-product is the emergence of spillover effects of aggregate wages that influence the bargaining process. We then show that a reasonable calibration of the model can account reasonably well for the cyclical behavior of wages and labor market activity observed in the data. The spillover effects turn out to be important in this respect.

As we noted earlier, in addition to the presence of the spillover effects, another important difference from existing macroeconomic models that rely on staggered multiperiod wage setting (e.g. Christiano, Eichenbaum and Evans (2005), Smets and Wouters (2005)) is that in our framework wages affect the adjustment of employment along the extensive margin, as opposed to the intensive margin. As Hall has recently emphasized, for adjustment on the intensive margin, wages may not be allocational, as originally argued by Barro (1977). The same criticism, however, does not apply to adjustment on the extensive margin. For this reason it may be interesting to consider our approach with employment adjustment along the extensive margin as a way to shore up a potential weakness of these conventional macroeconomic models. Trigari (2004) and Walsh (2005), for example, have integrated the search and matching framework within a monetary model that has many of the same features as these models, including nominal price stickiness. In their framework, though, there is period-by-period wage bargaining. We think it may be straightforward to extend their analysis by incorporating our model of staggered wage contracting. We expect that doing so will improve the overall empirical performance.

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APPENDIX A

A1. Sum of expected future wages for a worker at a firm renegotiating at t, $W_t^w(r)$

- Let $W_t^w(r)$ denote the discounted sum of expected future wages to be received by a worker over the life of the relationship at a firm renegotiating at t:

$$\begin{aligned} W_t^w(r) &= E_t \sum_{s=0}^{\infty} (\rho\beta)^s \Lambda_{t,t+s} w_{t+s}(r) \\ &= w_t(r) + (\rho\beta) E_t \Lambda_{t,t+1} w_{t+1}(r) + (\rho\beta)^2 E_t \Lambda_{t,t+2} w_{t+2}(r) + ... \end{aligned}$$

- At a firm renegotiating at time t, the current and future expected wages are given by:

$$w_t(r) = w_t^*$$

$$E_t w_{t+1}(r) = \lambda w_t^* + (1-\lambda) E_t w_{t+1}^*$$

$$\begin{aligned} E_t w_{t+2}(r) &= \lambda \left[\lambda w_t^* + (1-\lambda) E_t w_{t+1}^*\right] + (1-\lambda) E_t w_{t+2}^* \\ &= \lambda^2 w_t^* + \lambda (1-\lambda) E_t w_{t+1}^* + (1-\lambda) E_t w_{t+2}^* \end{aligned}$$

and so on....

- Using these expressions, we can write:

$$\begin{aligned} W_t^w(r) = {} & w_t^* \\ & + (\rho\beta) E_t \Lambda_{t,t+1} \left[\lambda w_t^* + (1-\lambda) w_{t+1}^*\right] \\ & + (\rho\beta)^2 E_t \Lambda_{t,t+2} \left[\lambda^2 w_t^* + \lambda (1-\lambda) w_{t+1}^* + (1-\lambda) w_{t+2}^*\right] \\ & + ... \end{aligned}$$

- Collecting terms:

$$\begin{aligned} W_t^w(r) = {} & E_t \left[1 + (\rho\lambda\beta) \Lambda_{t,t+1} + (\rho\beta\lambda)^2 \Lambda_{t,t+2} + ...\right] w_t^* \\ & + (1-\lambda)(\rho\beta) E_t \Lambda_{t,t+1} \left[1 + (\rho\lambda\beta) \Lambda_{t+1,t+2} + (\rho\lambda\beta)^2 \Lambda_{t+1,t+3} + ...\right] w_{t+1}^* \\ & + (1-\lambda)(\rho\beta)^2 E_t \Lambda_{t,t+2} \left[1 + (\rho\lambda\beta) \Lambda_{t+2,t+3} + (\rho\lambda\beta)^2 \Lambda_{t+2,t+4} + ...\right] w_{t+2}^* \\ & + ... \end{aligned}$$

- Letting

$$\Delta_t = E_t \sum_{s=0}^{\infty} (\rho\lambda\beta)^s \Lambda_{t,t+s}$$

we have

$$
\begin{aligned}
W_t^w(r) &= \Delta_t w_t^* \\
&+ (1-\lambda)(\rho\beta) E_t \Lambda_{t,t+1} \Delta_{t+1} w_{t+1}^* \\
&+ (1-\lambda)(\rho\beta)^2 E_t \Lambda_{t,t+2} \Delta_{t+2} w_{t+2}^* \\
&+ ...
\end{aligned}
$$

- Finally, rearranging:

$$
W_t^w(r) = \Delta_t w_t^* + (1-\lambda) E_t \sum_{s=1}^{\infty} (\rho\beta)^s \Lambda_{t,t+s} \Delta_{t+s} w_{t+s}^*
$$

A2. Sum of expected future wages for a firm renegotiating at t, $W_t^f(r)$

- Let $W_t^f(r)$ denote the discounted sum of expected future wage payments by a firm renegotiating at t over both the existing contract and subsequent contracts:

$$
\begin{aligned}
W_t^f(r) &= E_t \sum_{s=0}^{\infty} \frac{n_{t+s}}{n_t}(r) \beta^s \Lambda_{t,t+s} w_{t+s}(r) \\
&= w_t(r) + E_t \frac{n_{t+1}}{n_t}(r) \beta \Lambda_{t,t+1} w_{t+1}(r) + E_t \frac{n_{t+2}}{n_t}(r) \beta^2 \Lambda_{t,t+2} w_{t+2}(r) + ...
\end{aligned}
$$

- Using the expressions for the future expected wages, we can write:

$$
\begin{aligned}
W_t^f(r) &= w_t^*(r) \\
&+ E_t \frac{n_{t+1}}{n_t}(r) \beta \Lambda_{t,t+1} \left[\lambda w_t^* + (1-\lambda) w_{t+1}^*\right] \\
&+ E_t \frac{n_{t+2}}{n_t}(r) \beta^2 \Lambda_{t,t+2} \left[\lambda^2 w_t^* + \lambda(1-\lambda) w_{t+1}^* + (1-\lambda) w_{t+2}^*\right] \\
&+ ...
\end{aligned}
$$

- Collecting terms:

$$
\begin{aligned}
W_t^f(r) &= E_t \left[1 + \frac{n_{t+1}}{n_t}(r)(\lambda\beta)\Lambda_{t,t+1} + \frac{n_{t+2}}{n_t}(r)(\lambda\beta)^2 \Lambda_{t,t+2} + ...\right] w_t^* \\
&+ (1-\lambda) E_t \frac{n_{t+1}}{n_t}(r) \beta \Lambda_{t,t+1} \left[1 + \frac{n_{t+2}}{n_{t+1}}(r)(\lambda\beta)\Lambda_{t+1,t+2} + ...\right] w_{t+1}^* \\
&+ (1-\lambda) E_t \frac{n_{t+2}}{n_t}(r) \beta^2 \Lambda_{t,t+2} \left[1 + \frac{n_{t+3}}{n_{t+2}}(r)(\lambda\beta)\Lambda_{t+2,t+3} + ...\right] w_{t+2}^* \\
&+ ...
\end{aligned}
$$

- Letting

$$\Sigma_t(r) = E_t \sum_{s=0}^{\infty} \frac{n_{t+s}}{n_t}(r)(\lambda\beta)^s \Lambda_{t,t+s}$$

we have

$$\begin{aligned} W_t^f(r) &= \Sigma_t(r) w_t^* \\ &\quad + (1-\lambda) E_t \frac{n_{t+1}}{n_t}(r) \beta \Lambda_{t,t+1} \Sigma_{t+1}(r) w_{t+1}^* \\ &\quad + (1-\lambda) E_t \frac{n_{t+2}}{n_t}(r) \beta^2 \Lambda_{t,t+2} \Sigma_{t+2}(r) w_{t+2}^* \\ &\quad + \ldots \end{aligned}$$

- Finally, rearranging

$$W_t^f(r) = \Sigma_t(r) w_t^* + (1-\lambda) E_t \sum_{s=1}^{\infty} \frac{n_{t+s}}{n_t}(r) \beta^s \Lambda_{t,t+s} \Sigma_{t+s} w_{t+s}^*$$

A3. Worker surplus at a firm renegotiating at t, $H_t(r)$

- The worker surplus at a firm renegotiating at t is

$$\begin{aligned} H_t(r) &= w_t(r) - b + \rho\beta E_t \Lambda_{t,t+1} H_{t+1}(r) - s_t \beta E_t \Lambda_{t,t+1} H_{x,t+1} \\ &= W_t^w(r) - E_t \sum_{s=0}^{\infty} (\rho\beta)^s \Lambda_{t,t+s} \left[b + s_{t+s} \beta \Lambda_{t+s,t+s+1} H_{x,t+s+1}\right] \end{aligned}$$

- Substituting the expression for $W_t^w(r)$, we get

$$H_t(r) = \Delta_t w_t^* - E_t \sum_{s=0}^{\infty} (\rho\beta)^s \Lambda_{t,t+s} \left[b + s_{t+s} \beta \Lambda_{t+s,t+s+1} H_{x,t+s+1} - (1-\lambda)(\rho\beta) \Lambda_{t+s,t+s+1} \Delta_{t+s+1} w_{t+s+1}^*\right]$$

A4. Firm marginal surplus for a firm renegotiating at t, $J_t(r)$

- The value of a marginal worker for a firm renegotiating at t is

$$\begin{aligned} J_t(r) &= f_{nt} - w_t(r) + \frac{\kappa}{2} x_t(r)^2 + \rho\beta E_t \Lambda_{t,t+1} J_{t+1}(r) \\ &= E_t \sum_{s=0}^{\infty} (\rho\beta)^s \Lambda_{t,t+s} \left[f_{nt+s} + \frac{\kappa}{2} x_{t+s}(r)^2\right] - W_t^w(r) \end{aligned}$$

- Substituting the expression for $W_t^w(r)$, we get

$$J_t(r) = E_t \sum_{s=0}^{\infty} (\rho\beta)^s \Lambda_{t,t+s} \left[f_{nt+s} + \frac{\kappa}{2} x_{t+s}(r)^2 - (1-\lambda)(\rho\beta) \Lambda_{t+s,t+s+1} \Delta_{t+s+1} w_{t+s+1}^*\right] - \Delta_t w_t^*$$

- Using the vacancy posting condition, the value of a marginal worker can similarly be expressed as discounted profits per worker:

$$
\begin{aligned}
J_t(r) &= f_{nt} - w_t(r) - \frac{\kappa}{2} x_t(r)^2 + [\rho + x_t(r)] \beta E_t \Lambda_{t,t+1} J_{t+1}(r) \\
&= f_{nt} - w_t(r) - \frac{\kappa}{2} x_t(r)^2 + \frac{n_{t+1}}{n_t}(r) \beta E_t \Lambda_{t,t+1} J_{t+1}(r) \\
&= E_t \sum_{s=0}^{\infty} \frac{n_{t+s}}{n_t}(r) \beta^s \Lambda_{t,t+s} \left[f_{nt+s} - \frac{\kappa}{2} x_{t+s}(r)^2 \right] - W_t^f(r)
\end{aligned}
$$

A5. The contract wage

- The Nash first-order condition is

$$
\chi_t(r) J_t(r) = (1 - \chi_t(r)) H_t(r)
$$

with

$$
\chi_t(r) = \frac{\eta}{\eta + (1-\eta) \Sigma_t(r) / \Delta_t}
$$

- Substituting $J_t(r)$ and $H_t(r)$ and rearranging, we obtain:

$$
\begin{aligned}
\Delta_t w_t^* = \; & E_t \sum_{s=0}^{\infty} (\rho\beta)^s \Lambda_{t,t+s} \Big[\chi_t(r) \left(f_{nt+s} + \frac{\kappa}{2} x_{t+s}(r)^2 \right) + (1 - \chi_t(r)) \left(b + s_{t+s} \beta \Lambda_{t+s,t+s+1} H_{x,t+s+1} \right) \\
& - (1-\lambda) \rho\beta \Lambda_{t+s,t+s+1} \Delta_{t+s+1} w_{t+s+1}^* \Big]
\end{aligned}
$$

- The above equation can be written in a recursive form in the following way:

$$
\begin{aligned}
\Delta_t w_t^* &= \chi_t(r) \left(f_{nt} + \frac{\kappa}{2} x_t(r)^2 \right) + (1 - \chi_t(r)) \left(b + s_t \beta E_t \Lambda_{t,t+1} H_{x,t+1} \right) \\
&\quad - (1-\lambda) \rho\beta E_t \Lambda_{t,t+1} \Delta_{t+1} w_{t+1}^* + \rho\beta E_t \Lambda_{t,t+1} \Delta_{t+1} w_{t+1}^*
\end{aligned}
$$

- Simplifying, we obtain

$$
\Delta_t w_t^* = w_t^o(r) + \rho\lambda\beta E_t \Lambda_{t,t+1} \Delta_{t+1} w_{t+1}^*
$$

with $w_t^o(r)$ denoting the target wage:

$$
w_t^o(r) = \chi_t(r) \left(f_{nt} + \frac{\kappa}{2} x_t(r)^2 \right) + (1 - \chi_t(r)) \left(b + s_t \beta E_t \Lambda_{t,t+1} H_{x,t+1} \right)
$$

A6. The loglinearized target wage

- Let V_t be the unconditional average value of employment at t and $V_{x,t}$ the average value of employment at t conditional on being a new hire:

$$V_t = \int_0^1 V_t(i) \frac{n_t(i)}{n_t} di$$

$$V_{x,t} = \int_0^1 V_t(i) \frac{x_{t-1}(i) n_{t-1}(i)}{x_{t-1} n_{t-1}} di$$

Note that V_t and $V_{x,t}$ are identical up to a first order approximation:

$$\widehat{V}_t = \widehat{V}_{x,t}$$

This implies that $H_t = V_t - U_t$ and $H_{x,t} = V_{x,t} - U_t$ are also identical up to a first order:

$$\widehat{H}_t = \widehat{H}_{x,t}$$

- Loglinearizing the target wage then yields

$$\widehat{w}_t^o(r) = \left[\varphi_{fn} \widehat{f}_{nt} + \varphi_x \widehat{x}_t(r)\right] + \varphi_s \left[\widehat{s}_t + \widehat{H}_{t+1} + \widehat{\Lambda}_{t,t+1}\right] + \varphi_\chi \widehat{\chi}_t(r)$$

where

$$\varphi_{fn} = \chi f_n w^{-1} \quad \varphi_x = \chi \kappa x^2 w^{-1} \quad \varphi_s = \chi s \kappa x w^{-1} \quad \varphi_\chi = \chi \left[f_n + (\kappa/2) x^2 - b - s\beta H\right] w^{-1}$$

- Finally, the weight in the target wage is

$$\chi_t(r) = \frac{\eta}{\eta + (1-\eta) \Sigma_t(r) / \Delta_t}$$

with

$$\Delta_t = 1 + \rho\lambda\beta E_t \Lambda_{t,t+1} \Delta_{t+1}$$

$$\Sigma_t(r) = 1 + [\rho + x_t(r)] (\lambda\beta) E_t \Lambda_{t,t+1} \Sigma_{t+1}(r)$$

Loglinearizing yields

$$\widehat{\chi}_t(r) = -(1-\chi)\left(\widehat{\Sigma}_t(r) - \widehat{\Delta}_t\right)$$

with

$$\widehat{\Delta}_t = \rho\lambda\beta E_t\left(\widehat{\Lambda}_{t,t+1} + \widehat{\Delta}_{t+1}\right)$$

$$\widehat{\Sigma}_t(r) = x\lambda\beta\widehat{x}_t(r) + \lambda\beta E_t\left(\widehat{\Lambda}_{t,t+1} + \widehat{\Sigma}_{t+1}(r)\right)$$

A7. Hiring rate at a firm renegotiating at t, $x_t(r)$

- Let x_t be the unconditional average value of the hiring rate:

$$x_t = \int_0^1 x_t(i) \frac{n_t(i)}{n_t} di$$

- Using the job creation condition, x_t can be written as

$$\kappa x_t = \beta E_t \Lambda_{t,t+1} \left(f_{nt+1} + \frac{\kappa}{2} {x_{t+1}}^2 - w_{t+1} + \rho\kappa x_{t+1} \right) + \varsigma_t^x$$

with

$$\varsigma_t^x = \beta E_t \Lambda_{t,t+1} \left[\int_0^1 \left(\frac{\kappa}{2} x_{t+1}(i)^2 - w_{t+1}(i) + \rho\kappa x_{t+1}(i) \right) \frac{n_t(i)}{n_t} di - \left(\frac{\kappa}{2} {x_{t+1}}^2 - w_{t+1} + \rho\kappa x_{t+1} \right) \right]$$

- Loglinearizing yields:

$$\widehat{x}_t = E_t \widehat{\Lambda}_{t,t+1} + \epsilon \left(f_n E_t \widehat{f}_{nt+1} - w E_t \widehat{w}_{t+1} \right) + \beta E_t \widehat{x}_{t+1}$$

with

$$\epsilon = \beta (\kappa x)^{-1}$$

and where

$$\widehat{\varsigma}_t^x = 0$$

- Consider now a firm renegotiating at time t. We can write:

$$\widehat{x}_t - \widehat{x}_t(r) = -\epsilon w E_t (\widehat{w}_{t+1} - \widehat{w}_{t+1}(r)) + \beta E_t (\widehat{x}_{t+1} - \widehat{x}_{t+1}(r))$$

which can be iterated forward to give:

$$\begin{aligned} \widehat{x}_t - \widehat{x}_t(r) \quad = \quad & -\epsilon w E_t (\widehat{w}_{t+1} - \widehat{w}_{t+1}(r)) \\ & -\beta\epsilon w E_t (\widehat{w}_{t+2} - \widehat{w}_{t+2}(r)) \\ & -\beta^2 \epsilon w E_t (\widehat{w}_{t+2} - \widehat{w}_{t+3}(r)) \\ & -... \end{aligned}$$

- Using the loglinear version of the wage index (37):

$$\widehat{w}_t = \lambda \widehat{w}_{t-1} + (1-\lambda) \widehat{w}_t^*$$

together with the loglinear expressions for the expected future wages at a firm renegotiating at time t (see section A1), we obtain

$$E_t (\widehat{w}_{t+1} - \widehat{w}_{t+1}(r)) = \lambda (\widehat{w}_t - \widehat{w}_t^*)$$

$$E_t (\widehat{w}_{t+1} - \widehat{w}_{t+1}(r)) = \lambda^2 (\widehat{w}_t - \widehat{w}_t^*)$$

and so on....

- Substituting and rearranging yields:

$$\widehat{x}_t(r) = \widehat{x}_t + \lambda\epsilon\Sigma w\left(\widehat{w}_t - \widehat{w}_t^*\right)$$

with

$$\Sigma = \frac{1}{1-\lambda\beta}$$

A8. Expected average worker surplus at firm renegotiating at t, E_tH_{t+1}

A8a. Average worker surplus H_t and firm marginal surplus J_t

- The unconditional average value of worker surplus H_t can be written as:

$$H_t = w_t - b + (\rho - s_t)\,\beta E_t\Lambda_{t,t+1}H_{t+1} + \varsigma_t^w$$

with

$$\varsigma_t^w = \beta E_t\Lambda_{t,t+1}\left[\rho\left(\int_0^1 V_{t+1}(i)\,\frac{n_t(i)}{n_t}di - V_{t+1}\right) - s_t\left(\int_0^1 V_{t+1}(i)\,\frac{x_t(i)n_t(i)}{x_tn_t} - V_{t+1}\right)\right]$$

- The unconditional average value of firm marginal surplus J_t can be written as:

$$J_t = f_{nt} - w_t + \frac{\kappa}{2}x_t^2 + \rho\beta E_t\Lambda_{t,t+1}J_{t+1} + \varsigma_t^f$$

with

$$\varsigma_t^f = \rho\beta E_t\Lambda_{t,t+1}\left(\int_0^1 J_{t+1}(i)\,\frac{n_t(i)}{n_t}di - J_{t+1}\right)$$

- Loglinearizing H_t and J_t and rearranging

$$\widehat{H}_t = (1-\chi)\,\chi^{-1}\epsilon w\widehat{w}_t + \rho\beta E_t\left(\widehat{\Lambda}_{t,t+1} + \widehat{H}_{t+1}\right) - s\beta E_t\left(\widehat{s}_t + \widehat{\Lambda}_{t,t+1} + \widehat{H}_{t+1}\right)$$

$$\widehat{J}_t = \epsilon f_n\widehat{f}_{nt} + x\beta\widehat{x}_t - \epsilon w\widehat{w}_t + \rho\beta E_t\left(\widehat{\Lambda}_{t,t+1} + \widehat{J}_{t+1}\right)$$

where

$$\widehat{\varsigma}_t^w = \widehat{\varsigma}_t^f = 0$$

- Then we can write the following expressions[18]:

$$\widehat{H}_t - \widehat{H}_t(i) = (1-\chi)\,\chi^{-1}\epsilon w\left(\widehat{w}_t - \widehat{w}_t(i)\right) + \rho\beta E_t\left(\widehat{H}_{t+1} - \widehat{H}_{t+1}(i)\right)$$

$$\widehat{J}_t - \widehat{J}_t(i) = x\beta\left(\widehat{x}_t - \widehat{x}_t(i)\right) - \epsilon w\left(\widehat{w}_t - \widehat{w}_t(i)\right) + \rho\beta E_t\left(\widehat{J}_{t+1} - \widehat{J}_{t+1}(i)\right)$$

[18] Note we are using $E_t\widehat{H}_{x,t+1} = E_t\widehat{H}_{t+1}$.

A8b. Expected worker surplus at a firm renegotiating at $t+1$, $E_t H_{t+1}(r')$

- Consider a firm renegotiating at time $t+1$. We can write:

$$\begin{aligned} E_t\left(\widehat{H}_{t+1} - \widehat{H}_{t+1}(r')\right) &= (1-\chi)\chi^{-1}\epsilon w E_t\left(\widehat{w}_{t+1} - \widehat{w}_{t+1}(r')\right) + \rho\beta E_t\left(\widehat{H}_{t+2} - \widehat{H}_{t+2}(r')\right) \\ &= (1-\chi)\chi^{-1}\epsilon w E_t\left[\left(\widehat{w}_{t+1} - \widehat{w}_{t+1}(r')\right) + \rho\beta\left(\widehat{w}_{t+2} - \widehat{w}_{t+2}(r')\right) + ...\right] \end{aligned}$$

- Note that, for a firm renegotiating at time $t+1$, we have:

$$E_t\left(\widehat{w}_{t+1} - \widehat{w}_{t+1}(r')\right) = E_t\left(\widehat{w}_{t+1} - \widehat{w}^*_{t+1}\right)$$

$$\begin{aligned} E_t\left(\widehat{w}_{t+2} - \widehat{w}_{t+2}(r')\right) &= E_t\left[\lambda\widehat{w}_{t+1} + (1-\lambda)\widehat{w}^*_{t+2}\right] - E_t\left[\lambda\widehat{w}_{t+1}(r') + (1-\lambda)\widehat{w}^*_{t+2}\right] \\ &= \lambda E_t\left(\widehat{w}_{t+1} - \widehat{w}_{t+1}(r')\right) \\ &= \lambda E_t\left(\widehat{w}_{t+1} - \widehat{w}^*_{t+1}\right) \end{aligned}$$

$$\begin{aligned} E_t\left(\widehat{w}_{t+3} - \widehat{w}_{t+3}(r')\right) &= E_t\left[\lambda\widehat{w}_{t+2} + (1-\lambda)\widehat{w}^*_{t+3}\right] - E_t\left[\lambda\widehat{w}_{t+2}(r') + (1-\lambda)\widehat{w}^*_{t+3}\right] \\ &= \lambda E_t\left(\widehat{w}_{t+2} - \widehat{w}_{t+2}(r')\right) \\ &= \lambda^2 E_t\left(\widehat{w}_{t+1} - \widehat{w}^*_{t+1}\right) \end{aligned}$$

and so on....

- Substituting these expressions and rearranging:

$$E_t\widehat{H}_{t+1}(r') = E_t\widehat{H}_{t+1} - (1-\chi)\chi^{-1}\epsilon\Delta w E_t\left(\widehat{w}_{t+1} - \widehat{w}^*_{t+1}\right)$$

with

$$\Delta = \frac{1}{1-\rho\lambda\beta}$$

A8c. Expected firm marginal surplus for a firm renegotiating at $t+1$, $E_t J_{t+1}(r')$

- We can write:

$$\begin{aligned} E_t\left(\widehat{J}_{t+1} - \widehat{J}_{t+1}(r')\right) &= x\beta E_t\left(\widehat{x}_{t+1} - \widehat{x}_{t+1}(r')\right) - \epsilon w E_t\left(\widehat{w}_{t+1} - \widehat{w}_{t+1}(r')\right) + \rho\beta E_t\left(\widehat{J}_{t+2} - \widehat{J}_{t+2}(r')\right) \\ &= x\beta E_t E_t\left[\left(\widehat{x}_{t+1} - \widehat{x}_{t+1}(r')\right) + \rho\beta\left(\widehat{x}_{t+2} - \widehat{x}_{t+2}(r')\right) + ...\right] \\ &\quad -\epsilon w E_t\left[\left(\widehat{w}_{t+1} - \widehat{w}_{t+1}(r')\right) + \rho\beta\left(\widehat{w}_{t+2} - \widehat{w}_{t+2}(r')\right) + ...\right] \end{aligned}$$

- Moreover, we have:

$$E_t\left(\widehat{x}_{t+1}\left(r'\right)-\widehat{x}_{t+1}\right)=\lambda\epsilon\Sigma wE_t\left(\widehat{w}_{t+1}-\widehat{w}^*_{t+1}\right)$$
$$E_t\left(\widehat{x}_{t+2}\left(r'\right)-\widehat{x}_{t+2}\right)=\lambda^2\epsilon\Sigma wE_t\left(\widehat{w}_{t+1}-\widehat{w}^*_{t+1}\right)$$
$$E_t\left(\widehat{x}_{t+3}\left(r'\right)-\widehat{x}_{t+3}\right)=\lambda^3\epsilon\Sigma wE_t\left(\widehat{w}_{t+1}-\widehat{w}^*_{t+1}\right)$$

and so on....

- Substituting the expressions for the expected future wages and hiring rates and rearranging:

$$E_t\widehat{J}_{t+1}\left(r'\right)=E_t\widehat{J}_{t+1}+\epsilon\Sigma wE_t\left(\widehat{w}_{t+1}-\widehat{w}^*_{t+1}\right)$$

with

$$\Sigma=\frac{1}{1-\lambda\beta}$$

A8d. Weight in the Nash bargaining first order condition

- Recall that

$$\widehat{\chi}_t\left(r\right)=-\left(1-\chi\right)\left(\widehat{\Sigma}_t\left(r\right)-\widehat{\Delta}_t\right)$$

with

$$\widehat{\Delta}_t=\rho\lambda\beta E_t\left(\widehat{\Lambda}_{t,t+1}+\widehat{\Delta}_{t+1}\right)$$
$$\widehat{\Sigma}_t\left(r\right)=x\lambda\beta\widehat{x}_t\left(r\right)+\lambda\beta E_t\left(\widehat{\Lambda}_{t,t+1}+\widehat{\Sigma}_{t+1}\left(r\right)\right)$$

- Averaging across all firms the firm cumulative discount factor yields:

$$\widehat{\Sigma}_t=x\lambda\beta\widehat{x}_t+\lambda\beta E_t\left(\widehat{\Lambda}_{t,t+1}+\widehat{\Sigma}_{t+1}\right)$$

- Taking differences and iterating forward yields

$$\begin{aligned}\widehat{\Sigma}_t\left(r\right)-\widehat{\Sigma}_t &= \left(x\lambda\beta\right)\left(\widehat{x}_t\left(r\right)-\widehat{x}_t\right)\\ &\quad+\left(\lambda\beta\right)\left(x\lambda\beta\right)E_t\left(\widehat{x}_{t+1}\left(r\right)-\widehat{x}_{t+1}\right)\\ &\quad+\left(\lambda\beta\right)^2\left(x\lambda\beta\right)E_t\left(\widehat{x}_{t+2}\left(r\right)-\widehat{x}_{t+2}\right)\\ &\quad+...\end{aligned}$$

- Substituting the expressions for the future hiring rates and collecting terms:

$$\widehat{\Sigma}_t\left(r\right)-\widehat{\Sigma}_t=x\Psi\epsilon\Sigma w\left(\widehat{w}_t-\widehat{w}^*_t\right)$$

with

$$\Psi=\frac{\lambda^2\beta}{1-\lambda^2\beta}$$

- Finally, we have

$$\widehat{\chi}_t\left(r\right) = \widehat{\chi}_t - \left(1-\chi\right)x\Psi\epsilon\Sigma w\left(\widehat{w}_t - \widehat{w}_t^*\right)$$

- Similarly, we have

$$E_t\widehat{\chi}_{t+1}\left(r'\right) = E_t\widehat{\chi}_{t+1} - \left(1-\chi\right)x\Psi\epsilon\Sigma w E_t\left(\widehat{w}_{t+1} - \widehat{w}_{t+1}^*\right)$$

A8e. Using the expected Nash condition at time $t+1$

- The expected Nash condition for firms renegotiating at time $t+1$ is

$$E_t\chi_{t+1}\left(r'\right)J_{t+1}\left(r'\right) = E_t\left(1-\chi_{t+1}\left(r'\right)\right)H_{t+1}\left(r'\right)$$

- Loglinearizing

$$E_t\widehat{J}_{t+1}\left(r'\right) + \left(1-\chi\right)^{-1}E_t\widehat{\chi}_{t+1}\left(r'\right) = E_t\widehat{H}_{t+1}\left(r'\right)$$

- Substituting the expressions found in sections A8b, A8c and A8d and rearranging yields

$$E_t\widehat{H}_{t+1} = E_t\widehat{J}_{t+1} + \Gamma E_t\left(\widehat{w}_{t+1} - \widehat{w}_{t+1}^*\right) + \left(1-\chi\right)^{-1}E_t\widehat{\chi}_{t+1}$$

with

$$\Gamma = \left(1-\eta x\Psi\right)\eta^{-1}\epsilon\Sigma w$$

- Using the loglinear expression for the hiring rate averaged across all firms:

$$\widehat{x}_t = E_t\left(\widehat{J}_{t+1} + \widehat{\Lambda}_{t,t+1}\right)$$

we finally obtain

$$E_t\left(\widehat{H}_{t+1} + \widehat{\Lambda}_{t,t+1}\right) = \widehat{x}_t + \Gamma E_t\left(\widehat{w}_{t+1} - \widehat{w}_{t+1}^*\right) + \left(1-\chi\right)^{-1}E_t\widehat{\chi}_{t+1}$$

A9. Spillover effects

- Consider the loglinear target wage

$$\widehat{w}_t^o\left(r\right) = \left(\varphi_{fn}\widehat{f}_{nt} + \varphi_x\widehat{x}_t\left(r\right)\right) + \varphi_s\left(\widehat{s}_t + \widehat{H}_{t+1} + \widehat{\Lambda}_{t,t+1}\right) + \varphi_\chi\widehat{\chi}_t\left(r\right)$$

- Substituting the following expressions in the target wage:

$$E_t\left(\widehat{H}_{t+1}+\widehat{\Lambda}_{t,t+1}\right)=\widehat{x}_t+\Gamma E_t\left(\widehat{w}_{t+1}-\widehat{w}^*_{t+1}\right)+(1-\chi)^{-1}E_t\widehat{\chi}_{t+1}$$

$$\widehat{x}_t(r)=\widehat{x}_t+\lambda\epsilon\Sigma w\left(\widehat{w}_t-\widehat{w}^*_t\right)$$

$$\widehat{\chi}_t(r)=\widehat{\chi}_t-(1-\chi)x\Psi\epsilon\Sigma w\left(\widehat{w}_t-\widehat{w}^*_t\right)$$

we obtain

$$\widehat{w}^o_t(r)=\widehat{w}^o_t+\frac{\tau_1}{1-\rho\lambda\beta}E_t\left(\widehat{w}_{t+1}-\widehat{w}^*_{t+1}\right)+\frac{\tau_2}{1-\rho\lambda\beta}\left(\widehat{w}_t-\widehat{w}^*_t\right)$$

with

$$\widehat{w}^o_t=\varphi_{fn}\widehat{f}_{nt}+(\varphi_x+\varphi_s)\widehat{x}_t+\varphi_s\widehat{s}_t+\varphi_\chi\widehat{\chi}_t+(1-\chi)^{-1}\varphi_sE_t\widehat{\chi}_{t+1}$$

and

$$\begin{aligned}\tau_1&=\varphi_s\Gamma(1-\rho\lambda\beta)\\ \tau_2&=\left[\varphi_x\lambda-\varphi_\chi(1-\chi)x\Psi\right]\epsilon\Sigma w(1-\rho\lambda\beta)\end{aligned}$$

- Note that, given our calibration described in section 4, $\tau_1=0.47$ and $\tau_2=0.015$.

A10. Reservation wages at a firm that has not renegotiated for τ periods

A10a. Worker's reservation wage, $R^w_t(\tau)$

- Consider a firm and a worker at time t who have not renegotiated for τ periods.
- The worker reservation wage, denoted with $R^w_t(\tau)$, is the wage that makes the worker surplus $H_t(\tau)$ equal to 0:

$$H_t(\tau)=R^w_t(\tau)-b+\rho\beta E_t\Lambda_{t,t+1}H_{t+1}(\tau)-s_t\beta E_t\Lambda_{t,t+1}H_{x,t+1}=0$$

- Using similar arguments as in section A8 of the appendix we can write

$$\widehat{H}_t(\tau)=\widehat{H}_t+(1-\chi)\chi^{-1}\epsilon\left(R^w\widehat{R}^w_t(\tau)-w\widehat{w}_t\right)+\rho\beta E_t\left(\widehat{H}_{t+1}(\tau)-\widehat{H}_{t+1}\right)=0$$

and

$$E_t\left(\widehat{H}_{t+1}(\tau)-\widehat{H}_{t+1}\right)=-(1-\chi)\chi^{-1}\epsilon\lambda\Delta w\left(\widehat{w}_t-\widehat{w}^*_t(\tau)\right)$$

where $w^*_t(\tau)$ is the wage that has been renegotiated in $t-\tau$.

- Combining equations yields

$$\widehat{H}_t(\tau)=\widehat{H}_t-(1-\chi)\chi^{-1}\epsilon\rho\beta\lambda\Delta w\left(\widehat{w}_t-\widehat{w}^*_t(\tau)\right)+(1-\chi)\chi^{-1}\epsilon\left(R^w\widehat{R}^w_t(\tau)-w\widehat{w}_t\right)=0$$

which gives the following expression for the worker's reservation wage:

$$R^w\widehat{R}^w_t(\tau)=w\widehat{w}_t+\rho\beta\lambda\Delta w\left(\widehat{w}_t-\widehat{w}^*_t(\tau)\right)-\chi(1-\chi)^{-1}\epsilon^{-1}\widehat{H}_t$$

- Rearranging:

$$R^w \widehat{R}_t^w(\tau) = w\Delta\widehat{w}_t - w(\Delta - 1)\widehat{w}_t^*(\tau) - \chi(1-\chi)^{-1}\epsilon^{-1}\widehat{H}_t$$

A10b. Firm's reservation wage, $R_t^f(\tau)$

- The firm reservation wage, denoted with $R_t^f(\tau)$, is the wage that makes the firm surplus $J_t(\tau)$ equal to 0:

$$J_t(\tau) = f_{nt} + \frac{\kappa}{2}x_t(\tau)^2 - R_t^f(\tau) + \rho\beta E_t\Lambda_{t,t+1}J_{t+1}(\tau) = 0$$

- Using similar arguments as in sections A7 and A8 of the appendix we can write

$$\widehat{J}_t(\tau) = \widehat{J}_t + x\beta(\widehat{x}_t(\tau) - \widehat{x}_t) - \varepsilon\left(R^f\widehat{R}_t^f(\tau) - w\widehat{w}_t\right) + \rho\beta E_t\left(\widehat{J}_{t+1}(\tau) - \widehat{J}_{t+1}\right) = 0$$

and

$$E_t\left(\widehat{J}_{t+1}(\tau) - \widehat{J}_{t+1}\right) = (\widehat{x}_t(\tau) - \widehat{x}_t) = \lambda\epsilon\Sigma w(\widehat{w}_t - \widehat{w}_t^*(\tau))$$

- Combining equations yields

$$\widehat{J}_t(\tau) = \widehat{J}_t + \beta\lambda\epsilon\Sigma w(\widehat{w}_t - \widehat{w}_t^*(\tau)) - \varepsilon\left(R^f\widehat{R}_t^f(\tau) - w\widehat{w}_t\right) = 0$$

which gives the following expression for the firm's reservation wage:

$$R^f\widehat{R}_t^f(\tau) = w\widehat{w}_t + \beta\lambda\Sigma w(\widehat{w}_t - \widehat{w}_t^*(\tau)) + \epsilon^{-1}\widehat{J}_t$$

- Rearranging:

$$R^f\widehat{R}_t^f(\tau) = w\Sigma\widehat{w}_t - w(\Sigma - 1)\widehat{w}_t^*(\tau) + \epsilon^{-1}\widehat{J}_t$$

APPENDIX B

Steady state calculation

- Given the calibrated parameters and target values in Table 1, we obtain implied values of $n, u, x, \chi, ls, I/y, ac/y$ and c/y from steady state calculations.
- First obtain

$$n = \frac{s}{1-\rho+s}$$
$$u = 1-n$$
$$x = \frac{su}{n}$$

and

$$\chi = \frac{\eta}{\eta + (1-\eta)\,\Sigma/\Delta}$$

- Then get

$$z = 1/\beta - 1 + \delta$$
$$\frac{k}{y} = \frac{\alpha}{z}$$
$$\frac{I}{y} = \delta \frac{k}{y}$$
$$k = \left(a\frac{k}{y}\right)^{\frac{1}{1-\alpha}} n$$
$$y = ak^{\alpha}n^{1-\alpha}$$
$$ac = \frac{\kappa}{2}x^2 n$$
$$f_n = (1-\alpha)\,a\left(\frac{k}{n}\right)^{\alpha}$$

- Then κ and w solve the following system (equations (12) and (33))

$$\begin{cases} \kappa x = \beta\left(f_n - w + \frac{\kappa}{2}x^2 + \rho\kappa x\right) \\ w = \chi\left(f_n + \frac{\kappa}{2}x^2 + s\kappa x\right) + (1-\chi)\,\overline{b}\left(f_n + \frac{\kappa}{2}x^2\right) \end{cases}$$

where

$$\overline{b} = \frac{b}{f_n + \frac{\kappa}{2}x^2}$$

- The flow value of unemployment is given by

$$b = \overline{b}\left(f_n + \frac{\kappa}{2}x^2\right)$$

- The steady state labor share is calculated from

$$ls = \frac{wn}{y} = w\frac{n}{k}\frac{k}{y}$$

- Finally

$$\frac{c}{y} = 1 - \frac{I}{y} - \frac{\kappa}{2}\frac{x^2 n}{y}$$

APPENDIX C

The complete loglinear model

- Technology

$$\widehat{y}_t = \widehat{a}_t + \alpha \widehat{k}_t + (1-\alpha)\,\widehat{n}_t \tag{E1}$$

- Resource constraint

$$\widehat{y}_t = cy\widehat{c}_t + iy\widehat{I}_t + (1 - cy - iy)\,(2\widehat{x}_t + \widehat{n}_t) \tag{E2}$$

where $cy = \frac{c}{y}$, $iy = \frac{I}{y}$ and $1 - cy - iy = \frac{\kappa}{2}\frac{x^2 n}{y}$

- Matching

$$\widehat{m}_t = \sigma \widehat{u}_t + (1-\sigma)\,\widehat{v}_t \tag{E3}$$

- Employment dynamics

$$\widehat{n}_{t+1} = \rho \widehat{n}_t + (1-\rho)\,\widehat{m}_t \tag{E4}$$

- Transition probabilities

$$\widehat{q}_t = \widehat{m}_t - \widehat{v}_t \tag{E5}$$

$$\widehat{s}_t = \widehat{m}_t - \widehat{u}_t \tag{E6}$$

- Unemployment

$$\widehat{u}_t = -\frac{n}{u}\widehat{n}_t \tag{E7}$$

- Capital dynamics

$$\widehat{k}_{t+1} = (1-\delta)\,\widehat{k}_t + \delta \widehat{I}_t \tag{E8}$$

- Aggregate vacancies

$$\widehat{x}_t = \widehat{q}_t + \widehat{v}_t - \widehat{n}_t \tag{E9}$$

- Consumption-saving

$$\widehat{\lambda}_t = E_t \widehat{\lambda}_{t+1} + \frac{1/\beta - 1 + \delta}{1/\beta} E_t \widehat{z}_{t+1} \tag{E10}$$

- Marginal utility

$$\widehat{\lambda}_t = -\widehat{c}_t \tag{E11}$$

- Aggregate hiring rate

$$\widehat{x}_t = E_t \widehat{\Lambda}_{t,t+1} + \epsilon \left(f_n \widehat{f}_{nt+1} - w E_t \widehat{w}_{t+1} \right) + \beta E_t \widehat{x}_{t+1} \tag{E12}$$

where

$$\epsilon = \beta \left(\kappa x \right)^{-1}$$

- Marginal product of labor

$$\widehat{f}_{nt} = \widehat{y}_t - \widehat{n}_t \tag{E13}$$

- Capital renting

$$\widehat{y}_t - \widehat{k}_t = \widehat{z}_t \tag{E14}$$

- Weight in Nash bargaining

$$\widehat{\chi}_t = -\left(1 - \chi\right) \left(\widehat{\Sigma}_t - \widehat{\Delta}_t \right) \tag{E15}$$

with

$$\widehat{\Delta}_t = \rho \lambda \beta E_t \left(\widehat{\Lambda}_{t,t+1} + \widehat{\Delta}_{t+1} \right) \tag{E16}$$

$$\widehat{\Sigma}_t = x \lambda \beta \widehat{x}_t + \lambda \beta E_t \left(\widehat{\Lambda}_{t,t+1} + \widehat{\Sigma}_{t+1} \right) \tag{E17}$$

- Spillover-free target wage

$$\widehat{w}_t^o = \varphi_{fn} \widehat{f}_{nt} + \left(\varphi_x + \varphi_s \right) \widehat{x}_t + \varphi_s \widehat{s}_t + \varphi_\chi \widehat{\chi}_t + \left(1 - \chi\right)^{-1} \varphi_s E_t \widehat{\chi}_{t+1} \tag{E18}$$

where

$$\varphi_{fn} = \chi f_n w^{-1} \quad \varphi_x = \chi \kappa x^2 w^{-1} \quad \varphi_s = \chi s \kappa x w^{-1} \quad \varphi_\chi = \chi \left[f_n + (\kappa/2)\, x^2 - b - s \beta H \right] w^{-1}$$

- Aggregate wage

$$\widehat{w}_t = \gamma_b \widehat{w}_{t-1} + \gamma \widehat{w}_t^o + \gamma_f E_t \widehat{w}_{t+1} \tag{E19}$$

where

$$\gamma_b = \left(1 + \tau_2\right) \phi^{-1} \quad \gamma = \varsigma \phi^{-1} \quad \gamma_f = \left(\rho \beta - \tau_1 \right) \phi^{-1}$$

$$\phi = 1 + \tau_2 + \varsigma + \rho \beta - \tau_1 \qquad \varsigma = \left(1 - \lambda\right) \left(1 - \rho \lambda \beta\right) \lambda^{-1}$$

$$\tau_1 = \varphi_s \Gamma \left(1 - \rho \lambda \beta\right)$$

$$\tau_2 = \left[\varphi_x \lambda - \varphi_\chi \left(1 - \chi\right) x \Psi \right] \epsilon \Sigma w \left(1 - \rho \lambda \beta\right)$$

$$\Gamma = \left(1 - x \eta \Psi\right) \eta^{-1} \epsilon \Sigma w \qquad \Psi = \beta \lambda^2 / \left(1 - \beta \lambda^2\right) \qquad \Sigma = \left(1 - \beta \lambda\right)^{-1}$$

- Technology process

$$\widehat{a}_t = \rho_a \widehat{a}_{t-1} + \varepsilon_t^a \tag{E20}$$

Figure 1: Impulse responses to a technology shock

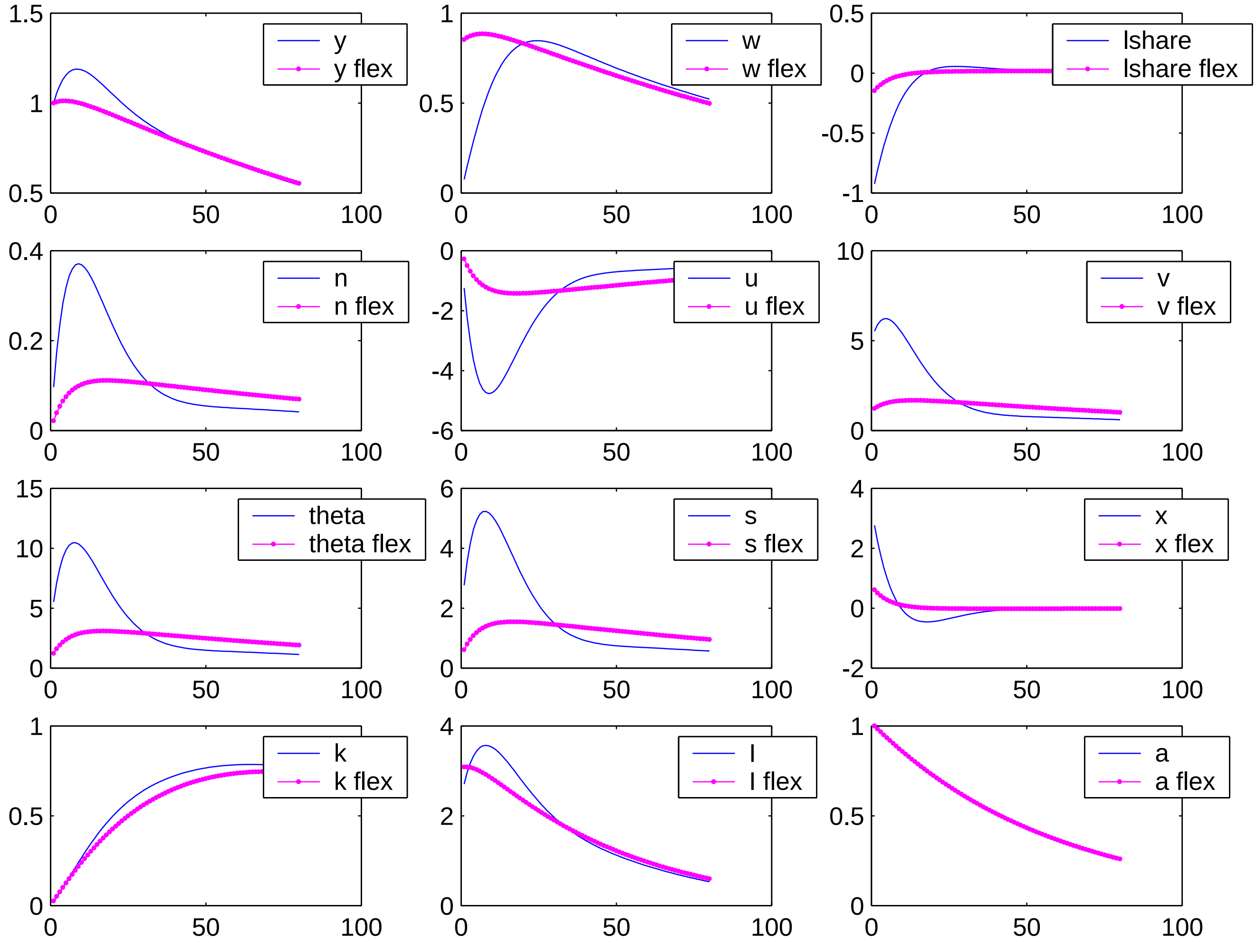

Figure 2: Bargaining set for contracts still in place after the average duration (3Q)

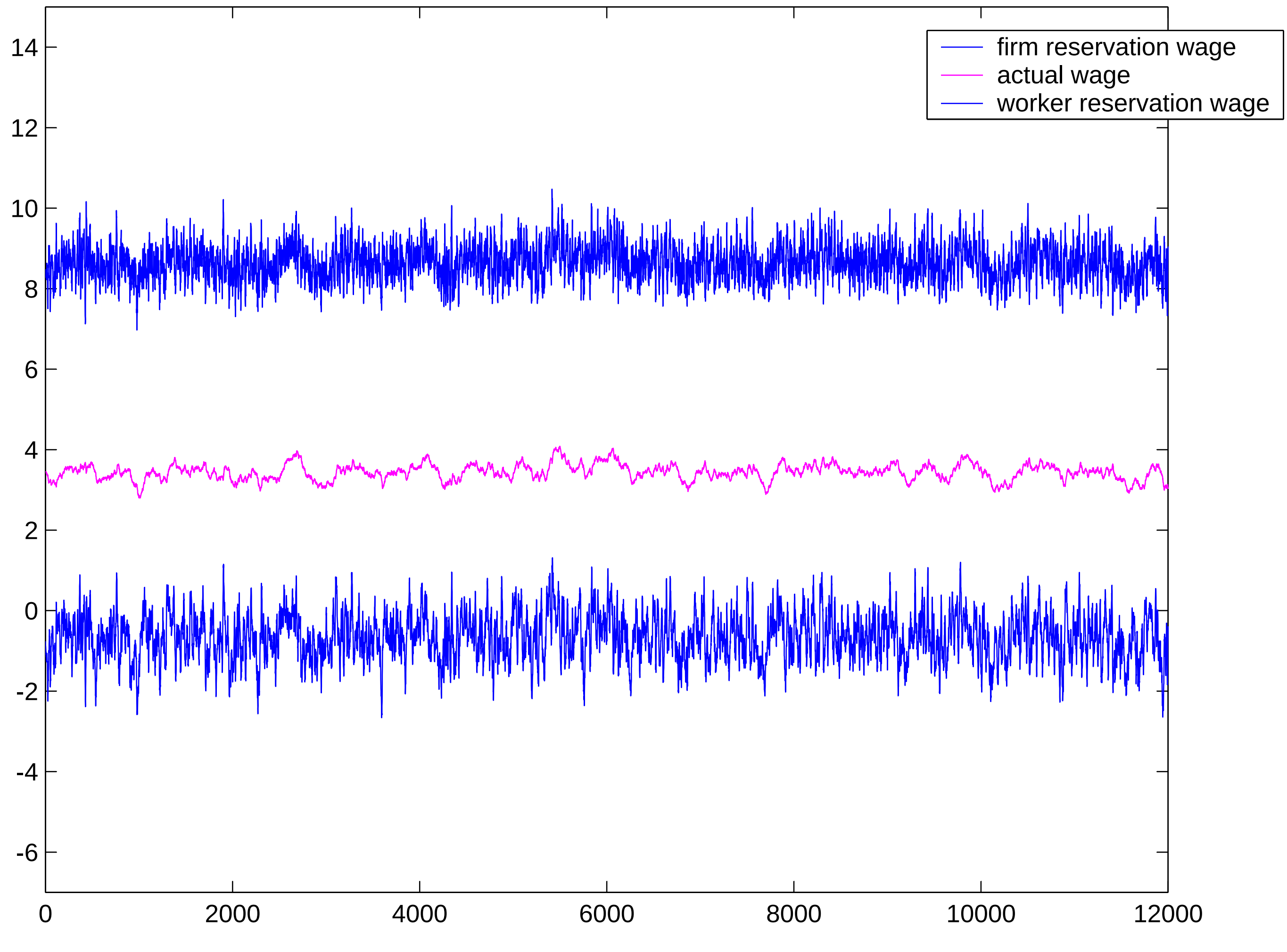

Figure 3: Bargaining set for contracts still in place after 40 months