Expenditure Switching vs. Real Exchange Rate Stabilization:

Competing Objectives for Exchange Rate Policy

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Abstract

This paper develops a view of exchange rate policy as a trade-off between the desire to smooth fluctuations in real exchange rates so as to reduce distortions in consumption allocations, and the need to allow flexibility in the nominal exchange rate so as to facilitate terms of trade adjustment. We show that optimal nominal exchange rate volatility will reflect these competing objectives. The key determinants of how much the exchange rate should respond to shocks will depend on the extent and source of price stickiness, as well as the elasticity of substitution between home and foreign goods. Quantitatively, we find the optimal exchange rate volatility should be significantly less than would be inferred based solely on terms of trade considerations. Moreover, we find that the relationship between price stickiness and optimal exchange rate volatility may be non-monotonic.

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This paper develops a novel view of exchange rate policy as a trade-off between the desire to smooth fluctuations in real exchange rates in order to achieve smaller cross-country deviations in consumer prices on the one hand, and the need to allow flexibility in the nominal exchange rate so as to facilitate terms of trade adjustment on the other hand.

There is a substantial body of empirical evidence establishing that the link between movements in exchange rates and changes in national consumer prices is weak.[1] One explanation for this weak link is that prices of all goods are sticky in local currencies (LCP, or local currency pricing), and do not respond to movements in the exchange rate. In this case, nominal exchange rate fluctuations lead to inefficient movements in real exchange rates because they alter relative prices of identical or similar goods across countries. From this perspective, it is desirable to avoid movements in exchange rates because they lead to differences in prices across countries for goods that have similar resource costs.[2]

But there is separate evidence that relative traded goods prices are linked to movements in exchange rates. Obstfeld and Rogoff (2000) show that exchange rates are highly correlated with the terms of trade, measured as the relative price of imports to exports. This suggests that exported goods tend to have prices set in the producer's currency (PCP, or producer's currency pricing), and a depreciation raises the relative price of foreign to home export goods. In this case, the exchange rate may play a role in facilitating relative price adjustment in face of country specific shocks when nominal prices of traded goods are slow to adjust to the shocks.

We present an analysis of exchange rate policy when there is a conflict between the objectives of stabilizing consumption based real exchange rates and allowing terms of trade adjustment. We build a model consistent with both the evidence of weak exchange rate pass-

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[1] See Engel (1993, 1999), Rogers and Jenkins (1996), Engel and Rogers (1996, 2001), Obstfeld and Taylor (1997), and Parsley and Wei (2001, 2003). Mussa's (1986) classic paper stimulated much of this research.

[2] Devereux and Engel (2003) find that if exporters set prices according to LCP, a fixed exchange rate regime is the optimal monetary policy. Similar results are found in Corsetti and Pesenti (2002).

through to consumer goods prices, but high pass-through to imported goods prices. In the model, imports and exports are intermediate goods. The law of one price holds for these traded products, so nominal price stickiness of these goods is of the PCP variety. Intermediate goods are used to produce final consumer goods, whose prices are sticky in the consumers' currency. Consistent with the evidence, consumer prices are unresponsive to nominal exchange rate changes. In general, optimal exchange rate movements in this setting do not deliver full terms of trade adjustment. There is a trade-off. Nominal exchange rate movement changes the terms of trade in the desired direction when there is a real shock, as the literature has suggested, but mimicking the optimal terms of trade change may imply undesirable changes in the consumption real exchange rate.

In our model, the optimal real exchange rate is constant. Although consumer goods are non-traded in the model, final goods are produced using traded inputs for which the law of one price holds. Under LCP for final goods, nominal exchange rate changes induce movements in real exchange rates that lead to inefficient consumption allocations. Stabilization of the consumption real exchange rate is a legitimate goal of exchange-rate policy, but it conflicts with the objective of achieving terms of trade adjustment.

The models are of course a simplified version of reality, and realistically optimal real exchange rates may not be constant. If there are changes in the prices of pure non-traded goods, for example, real exchange rates optimally should respond. By using a model in which optimal real exchange rates are constant, we highlight the role of monetary policy in eliminating inefficient real exchange rate movements that occur when fluctuating nominal exchange rates and LCP induce deviations in prices of consumer goods across locations[3].

[3] Empirically, Engel (1999) has found that variation in the relative price of pure non-traded goods can account for very little of the short-run real exchange rate movements in advanced countries.

Evidence that the law of one price holds relatively well for traded intermediate goods is consistent with PCP, but is also consistent with nominal price flexibility for these goods. The evidence is not refined enough to distinguish between the two possibilities. Markets for intermediate inputs are not the standard "customer" markets to which models of nominal price stickiness are typically applied. To the extent that traded intermediate prices are flexible, exchange rate adjustment is not needed to adjust the terms of trade because the nominal prices themselves can adjust.

Additionally, domestically produced products might generally be poor substitutes for imported intermediate goods. For example, Burstein, Neves, and Rebelo (2003), Burstein, Eichenbaum, and Rebelo (2002, 2003), and Corsetti and Dedola (2003) all model final traded consumption goods as being produced using a Leontief production technology that combines the imported intermediate with a domestic distribution service in fixed proportions. If the substitutability of imported intermediates with domestic goods and services is low, the expenditure-switching role of exchange rates may be secondary. It is the short-run elasticity of substitution that is relevant for exchange-rate policy: when nominal prices have had time to adjust, the real effects of nominal exchange rate changes dissipate. It is well known that the short-run elasticity of substitution for imports is quite low. Even if prices are sticky and set according to PCP, so that nominal exchange rate movements do change the relative price of imported goods, there will be little expenditure switching when substitutability is low.

We first present a series of special cases where monetary policy can achieve a first-best outcome – stabilizing the consumption real exchange rate as well as supporting efficient terms of trade adjustment. In our first specification, nominal prices of consumer goods are set in advance of the realization of shocks, while prices of intermediate goods are taken to be perfectly flexible. We find that an optimal monetary policy should maintain a fixed exchange rate. The only goal of policy is to achieve real exchange rate stability since nominal price movement of intermediate exports allows the terms of trade to adjust optimally.

We then reverse the assumptions on stickiness – final goods prices are flexible, but intermediate goods prices are set in advance in the producer's currency. Here we find that optimal exchange rate policy is aimed purely at achieving the desired terms of trade adjustment, since flexible final goods prices will ensure a stable real exchange rate. This specification is, of course, at odds with the evidence of non-responsiveness of consumer prices to exchange rate movements.

The model we consider is based very closely on that of Obstfeld (2001). Obstfeld's model has PCP for intermediate export prices and LCP for consumer prices. Surprisingly, he finds that optimal exchange rate policy should be aimed only at achieving the terms of trade goal. Stabilization of the real exchange rate is not a consideration. But that conclusion, we show, arises because of two knife-edge assumptions that nullify the distortion caused by real exchange rate fluctuations.

We also solve a version of the model in which the home and foreign inputs must be combined in fixed proportions. We show – in stark contrast to the Obstfeld (2001) result -- that fixed exchange rates are optimal when both intermediate and final goods prices are fixed in advance (with PCP for intermediates and LCP for final goods.) There is no expenditure-switching role for exchange rates when there is no substitutability between imports and domestically-produced goods.

In general, however, monetary policy will not be able simultaneously to attain fully consumption allocations as well as optimal terms of trade adjustment. In particular, when both final goods prices and intermediate goods prices are partially sticky, this will be the case (except when there is zero substitution between home and foreign inputs in production). We go on to present a quantitative analysis of the more general case where there is a real trade-off between these goals. Our analysis finds that when consumer price indices are unresponsive to exchange rate changes, an optimal monetary policy will limit exchange rate volatility substantially relative to that required to achieve terms of trade volatility in a frictionless economy – even when most or all intermediate goods prices are sticky in nominal terms. We find that optimal exchange rate volatility is never more than 50 percent of terms of trade volatility in a frictionless model.

In addition, we show that the relationship between exchange rate volatility and price stickiness may not be monotonic. While intuitively one would anticipate that reducing the flexibility of intermediate prices would increase the desirability of exchange rate adjustment, this relationship does not necessarily hold when the elasticity of substitution between home and imported intermediates is relatively low. We show that reducing the flexibility of intermediate goods prices will first increase desired exchange rate volatility. But after a certain point, as a greater share of intermediate goods prices are sticky, it becomes desirable to reduce exchange rate volatility.

The paper is organized as follows. Section 1 presents the basic model structure and solves for a flexible price equilibrium. Section 2 analyzes a series of cases under alternative assumptions about price setting and substitution possibilities. Section 3 analyzes the more general case. Some brief conclusions follow.

1. The Model

The model is a static, two-country model with tradable intermediate goods and nontraded final consumption goods. The model's structure is very similar to that of Obstfeld (2001). We examine a static model in order to focus on the static distortions sticky prices introduce as they interfere with terms of trade adjustment and real exchange rate equilibrium. The two countries, home and foreign, are populated by a continuum of households of measure 1. Each household owns and operates a firm producing a unique variety of intermediate good, using the household's labor as input. In each country, a final goods sector assembles consumption goods using home and foreign intermediates. Final goods are not traded internationally.

1a. Model Structure

Household *i* in the home country has preferences given by:

(1) $$U(i) = \frac{1}{1-\rho} C(i)^{1-\rho} - \frac{K}{\nu} L(i)^{\nu}, \text{ with } \rho > 0, \; \nu \geq 1.$$

C is a constant-elasticity-of-substitution aggregate over a continuum of home-produced final good commodities with an elasticity of substitution of $\theta > 1$ (see the appendix for the formal definition.) L represents labor services that each household uses to produce an intermediate good. K is a stochastic preference shock to labor supply. Foreign households' preferences are identical to home households, but are defined over consumption of final goods sold in the foreign country, and foreign labor (with separate K^* preference shocks).

Each household in the home country produces an intermediate good using the technology $Y_H(i) = L(i)$. Each variety of the final consumption good in the home country is produced using domestic and foreign intermediate good aggregates. For instance, the final good variety *j* is produced using the home and foreign intermediate good aggregates, respectively $Y_H(j)$ and $Y_F(j)$, with the production function:

(2) $$Y(j) = \left(\left(\frac{1}{2}\right)^{\frac{1}{\gamma}} Y_H(j)^{\gamma-1/\gamma} + \left(\frac{1}{2}\right)^{\frac{1}{\gamma}} Y_F(j)^{\gamma-1/\gamma} \right)^{\gamma/\gamma-1},$$

where γ represents the elasticity of substitution between the home and foreign intermediate goods aggregates. The home intermediate aggregate $Y_H(j)$ is defined as an aggregator over a continuum of home-produced intermediate goods, with elasticity of substitution ϕ:

$$Y_H(j) = \left[\int_0^1 Y_H(i,j)^{\frac{\phi-1}{\phi}} di \right]^{\frac{\phi}{\phi-1}}, \text{ with } \phi > 1,$$

and $Y_F(j)$ is defined analogously. Home households consume all of each home final good variety $Y(j)$.

1.b A Flexible Price Model

We first outline a flexible-price version of the model. Since our primary interest is in asking how sticky prices influence optimal exchange rate policy, we wish to eliminate any other sources of inefficiency that are not directly related to price stickiness. One distortion arises due to monopoly pricing wedges in both intermediate and final goods sectors. To avoid these, we assume that firms receive a per unit subsidy on production so as to ensure that price would equal marginal cost at both the intermediate and final goods level if all prices were fully flexible. The subsidy is financed by lump sum profit taxes on the firms.

A second issue is the nature of international capital markets. Again, to focus exclusively on the constraints that are related to nominal rigidities, we assume that agents can engage in ex-ante cross country trade in a full set of nominal state contingent assets. This ensures that if all prices were flexible, full cross-country risk sharing would obtain. In a later section, we explore how our results would change if these assets markets did not exist.

Rather than explicitly introducing a role for money in the model, we simply define monetary policy as a rule that targets the value of nominal consumption in each country. This is consistent with a variety of alternative underlying models of money, such as money in the utility function, or a cash-in-advance specification[4].

[4] So long as money was fully neutral in the flexible price economy, our results would be unaltered by explicitly introducing the monetary side of the model.

Table 1 outlines the equations of the flexible price model, for the home economy (the model is fully derived in the Appendix). Analogous conditions hold for the foreign economy.

Table 1: Equations of the flexible price economy			
(3) Final good price	$P = c(P_H, SP_F^*)$	(6) Market Clearing	$L = c_1(P_H, SP_F^*)\left(C + C^*\right)$
(4) Intermediate price	$P_H = KL^{v-1}PC^{\rho}$	(7) Monetary Policy	$PC = \ell$
(5) Risk-sharing	$PC^{\rho} = SP^*C^{*\rho}$		

Equation (3) just says that price equals marginal cost for final goods producers in the home country, given that prices are flexible, and an optimal subsidy eliminates the monopoly price wedge. Here, P is the home currency price of the final good, $c(.,.)$ represents the unit cost function of the final goods producer[5], P_H is the home currency price index of home intermediate goods, S is the exchange rate, and P_F^* is the foreign currency price index of foreign intermediate goods. Equation (4) represents optimal pricing (again net of subsidy) for intermediate good producers. The right hand side measures the dollar cost to the intermediate firm of producing one more unit of output. Equation (5) represents a symmetric outcome of optimal ex ante trade in nominal state contingent bonds, which will equalize marginal utilities of currency across countries, in all states of the world, when evaluated in a common currency[6]. Equation (6) represents market clearing for the home intermediate good. This says that the total output of home country intermediate goods, given by L, must equal total demand, which comes from

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[5] The cost function is defined as $c(P_H, SP_F^*) = \left(\frac{1}{2}P_H^{1-\gamma} + \frac{1}{2}(SP_F^*)^{(1-\gamma)}\right)^{\frac{1}{1-\gamma}}$.

[6] The marginal utility of a dollar for any home household is $C^{-\rho}/P$. The marginal utility of home currency for a foreign household is $C^{*-\rho}/SP^*$ (i.e. the foreign currency value of a dollar, multiplied by the marginal utility of foreign currency).

demand of home and foreign final goods firms. Finally, equation (7) defines the monetary policy rule for the home economy, where ℓ represents the target nominal consumption.

Equation (3), (4), (6) and (7) have counterparts for the foreign country, determining foreign final goods prices, prices of foreign intermediate goods, foreign market clearing, and the foreign monetary policy rule. The unit cost function for the foreign final good is of identical form to that for the home firm, and may be written as $c(P_H/S, P_F^*)$. These equations for the home and foreign economy, together with equation (5), may be solved for the equilibrium values of $C, C^*, L, L^*, P_H, P_F^*, S, P$, and P^*.

From Table 1 we may derive the equilibrium of the flexible price model as follows[7]. With all prices flexible, and final goods production technologies identical across countries, from (6) and its foreign counterpart we see that purchasing power parity (PPP) always holds. The risk-sharing condition (5) them implies that consumption is equalized across countries. The flexible price equilibrium for consumption is written as:

(8) $$\tilde{C} = \tilde{C}^* = \left[\left(0.5K^{\frac{1-\gamma}{1+(v-1)\gamma}} + 0.5K^{*\frac{1-\gamma}{1+(v-1)\gamma}} \right)^{\frac{1+(v-1)\gamma}{1-\gamma}} \right]^{\frac{-1}{\rho+v-1}}$$

A decline in labor supply in either country (an increase in *K* or *K**) will reduce desired output, and reduce equilibrium consumption in both countries. The flexible price equilibrium levels of output (or employment) may be derived as:

(9) $$\tilde{L} = \left[.5 + .5\left(\frac{K}{K^*}\right)^{\frac{\gamma-1}{1+(v-1)\gamma}} \right]^{\frac{\gamma}{1-\gamma}} \tilde{C} \quad \tilde{L}^* = \left[.5 + .5\left(\frac{K^*}{K}\right)^{\frac{\gamma-1}{1+(v-1)\gamma}} \right]^{\frac{\gamma}{1-\gamma}} \tilde{C}$$

[7] Since prices are flexible and there are no other inefficiencies, the flexible price equilibrium is identical to the allocation that would be chosen by a social planner that chose consumption and employment to maximize an equally weighted world utility function.

An equal shock to labor supply in both countries will reduce consumption and output proportionally. But a country specific labor supply shock reduces a country's output more than in proportion to the fall in consumption. *Relative* output may then be written as:

$$\frac{\tilde{L}}{\tilde{L}^*} = \left[\frac{K^*}{K}\right]^{\frac{\gamma}{1+(v-1)\gamma}} . \tag{10}$$

Relative output is inversely proportional to the country specific labor supply shocks.

Finally, we may define the terms of trade $\tilde{\tau} = SP_F^* / P_H$ as:

$$\tilde{\tau} = \left[\frac{K^*}{K}\right]^{\frac{1}{1+(v-1)\gamma}} \tag{11}$$

A negative foreign labor supply shock (an increase in *K**) raises the relative price of foreign to home output.

Expressions (8)-(11) set out the goals for optimal monetary policy in environments with sticky prices. The monetary policy should attempt to equalize consumption across countries, but also tilt employment (and production) towards the country with the lowest labor supply shock. In order to do this, the monetary policy must affect the total level of world spending, and the composition of spending between home and foreign intermediate goods. To achieve the latter, policy would have to change relative prices. But the movement in relative prices may be in conflict with the desire to equalize consumption across countries, in a situation where purchasing power parity fails due to local currency pricing.

2. Exchange Rate Policy under Sticky Prices

Our aim is to explore the consequences of alternative types of nominal rigidities for optimal monetary and exchange rate policy, using as a benchmark the flexible price equilibrium. We will abstract from strategic interactions between monetary policy makers. While interesting in itself, the issue of policy coordination is not directly relevant to the questions we are addressing. Implicitly, we are focusing on cooperative monetary policy rules.

In general, both the prices of intermediate goods as well as final goods may be sticky, although empirically we feel that final goods prices are much more likely to be sticky. We assume that a measure ω of final goods producers in both countries set their prices in advance (in local currency), and the measure $(1-\omega)$ adjust prices after the realization of the supply shocks. Likewise, assume that a measure κ of intermediate goods producers set prices in advance (in the producer's currency), while the measure $(1-\kappa)$ adjust prices after the shocks are known. For the rest of this section however, we will only deal with the extremes where a) all final goods prices are sticky and all intermediate prices flexible i.e. $\omega=1,\ \kappa=0$, b) all final goods prices are flexible and all intermediates prices are sticky $\omega=0,\ \kappa=1$, or c) (in a special case – see below) all prices of all goods are sticky $\omega=1,\ \kappa=1$. In each case, we will show that monetary policy can exactly attain the flexible price equilibrium. In section 2 below, we analyze more general model where the full flexible price allocation cannot be attained.

Case 1. Sticky final goods prices, flexible intermediate good prices.

When final goods prices are sticky, the home country price set by any final goods producer may be written as:

(3')
$$P=\frac{E\left(c(P_H,SP_F^*)C^{1-\rho}\right)}{EC^{1-\rho}}$$

This replaces equation (3) in Table 1. This pricing function is derived from the problem of the final good firm that maximizes discounted profits, given that an optimal production subsidy is offered. For each realization of K and K^*, equations (4), (6), and (7), along with their counterparts of the foreign economy, along with equation (5), determine $C, C^*, L, L^*, P_H, P_H^*$, and S. Given the distribution of consumption, prices and exchange rates, equations (3') and its foreign counterpart determine P and P^*.

In what way does the economy with sticky final goods prices depart from the flexible price equilibrium? The first thing to note is that the equilibrium terms of trade are identical to those of the flexible price equilibrium. To see this, use (4), (10), and the foreign counterparts, with (5), to establish that:

$$\frac{SP_F^*}{P_H} = \left(\frac{K^*}{K}\right)^{\frac{1}{1+\gamma(\nu-1)}}, \tag{12}$$

which is equivalent to (11). Hence, independent of monetary policy, relative prices adjust efficiently in an economy with sticky final goods prices but flexible intermediate goods prices.

However, output *levels* will not in general be efficient, since total demand depends on monetary policy, given sticky final goods prices. From the monetary policy rules (7), we see that final goods prices cannot in general always be at the level consistent with the flexible price equilibrium, unless ℓ and ℓ^* are designed appropriately. More formally, we can establish

Proposition 1. If monetary policies follow the rules given by:

$$\ell = \overline{\ell}\tilde{C} \qquad \ell^* = \overline{\ell}^*\tilde{C} \tag{13}$$

where $\overline{\ell}$ and $\overline{\ell}^*$ are arbitrary constant parameters, then the equilibrium with sticky final goods prices coincides with the flexible price equilibrium, with $P = \overline{\ell}$ and $P^* = \overline{\ell}^*$.

Proof: See Appendix.

The proposition ensures that PPP holds, since the monetary rules combined with (5) imply that $S = \frac{P}{P^*}\left(\frac{\overline{\ell}}{P}\frac{P^*}{\overline{\ell}^*}\right)^{\rho} = \frac{P}{P^*}$, and consumption is equalized across countries at its flexible price equilibrium level. But since final goods prices are state independent, then the exchange rate must also be state independent. These monetary policies achieve efficient consumption allocations in all states of the world because they keeps the real exchange rate fixed at unity. But with sticky final goods prices in local currency, PPP can only be achieved by fixing the nominal exchange rate.

The optimal monetary policies eliminate the distortion due to sticky final goods prices. An alternative way to see it is that the monetary rules stabilize marginal cost for final goods producers, so that equation (3) always holds, even with sticky final goods prices. Final goods firms would not wish to adjust their prices even if they could.

In principle, we might expect that exchange rate movement would be necessary to achieve efficient relative price (terms of trade) adjustment, or equivalently, to facilitate expenditure switching. But when intermediate good prices are fully flexible, the desired terms of trade adjustment is fully achieved by movements in P_H / P_F^*, without any movements in the exchange rate. This achieves efficient relative production across countries. The exchange rate is not needed to facilitate expenditure switching. Hence, in this special case where intermediate goods prices are flexible, optimal policy faces no trade off. The consumption real exchange rate can be stabilized while simultaneously achieving efficient relative price adjustment.

In fact, if any fraction of final goods prices is set in advance in consumers' currencies, Proposition 1 holds without change, and the exchange rate is fixed. The logic is simple: if monetary policy continues to stabilize marginal cost for final goods, final goods firms that are free to adjust will choose to leave their prices unchanged. That is, with these monetary policies, flexible price firms lose nothing by acting just like sticky-price firms and not adjusting prices in response to shocks. The same equilibrium obtains as in the fully sticky-price case. Hence, as long as intermediate goods prices are fully flexible, then any amount of price rigidity at the final goods level implies that a fixed exchange rate is optimal.

Case 2. Sticky intermediate goods prices, flexible final good prices.

Now we look at the polar opposite case. Say that final goods prices are fully flexible, but intermediate goods prices are sticky. We do not consider this case because of empirical relevance – in fact it is contradictory to the evidence on the responsiveness of consumer prices to exchange rates. The purpose of looking at this case is to illustrate again the dual objectives of exchange-rate policy: achieving desired terms of trade changes but avoiding undesirable real exchange rate changes.

With sticky intermediate goods prices, condition (4) becomes:

(4')
$$P_H = \frac{E(KL^{\nu})}{E\left(\dfrac{L}{PC^{\rho}}\right)}.$$

This condition says that when the intermediate producer must set their prices in advance, they trade off the expected marginal utility benefit of a price reduction in terms of greater sales, with the expected marginal utility cost in terms of greater work effort. An equilibrium of the model with sticky intermediate goods prices is defined by the values for $C, C^*, L, L^*, P, P^*,$ and S that solve (3), (6) (7) and their foreign counterparts, and (5), for each realization of K, K^*. Then P_H and P_F^* may be solved from (4') and its foreign counterpart.

Because final goods prices are flexible, PPP will always hold whatever the monetary policy rule. Hence consumption is equalized across countries. But because intermediate good prices are predetermined, in general neither the level of output nor the terms of trade will equal that of the flexible price equilibrium. Given PPP, the nominal exchange rate will equal $S = \ell / \ell^*$. Then, from (6), output in each country will be determined by:

(14)
$$L = c_1(1, \frac{SP_F^*}{P_H}) \frac{2\ell}{P_H c(1, \frac{SP_F^*}{P_H})}, \qquad L^* = c_2(1, \frac{SP_F^*}{P_H}) \frac{2\ell}{P_H c(1, \frac{SP_F^*}{P_H})}$$

Clearly, L and L^* cannot equal their efficient levels given in (9) unless the exchange rate is free to adjust. We may then establish

Proposition 2. If the monetary policies are:

(15) $$\ell = \bar{\ell} c(1, \tilde{\tau})\tilde{C} \qquad \ell = \bar{\ell}^* c(\tilde{\tau}^{-1}, 1)\tilde{C},$$

where $\bar{\ell}$ and $\bar{\ell}^*$ are arbitrary constant parameters, then the equilibrium with sticky intermediate good prices achieves the same allocation as the flexible price equilibrium, with $P_H = \bar{\ell}$ and $P_F^* = \bar{\ell}^*$.

Proof: See Appendix.

In this case, the exchange rate is equal to $S = (\bar{\ell}/\bar{\ell}^*)\tilde{\tau} = (P_H/P_F^*)\tilde{\tau}$. The exchange rate must adjust to as to ensure that the terms of trade is equal to its flexible price equilibrium level in each state of the world. The optimal monetary policy replicates the flexible price equilibrium, because it keeps the marginal cost of intermediate goods firms constant, so that (4) and its foreign counterpart hold, even with sticky intermediate goods prices. Again, this is a case where there is no trade-off between the exchange rate that is desirable for consumption allocations and that needed for terms of trade adjustment. Because final goods prices are flexible, PPP always holds, ensuring efficient consumption allocations in each state. So long as monetary policy allows the exchange rate to adjust appropriately, the efficient adjustment in the terms of trade will also be achieved. But because nominal prices of intermediate goods cannot change, the exchange rate is necessary for efficient expenditure switching.

As before, we can extrapolate to the case where some but not all intermediate goods prices are sticky. So long as a fraction of prices cannot adjust, Proposition 2 still holds. If monetary policy stabilizes marginal cost for intermediate good producers, those producers that can adjust prices will not wish to, and relative price adjustment is fully achieved by nominal exchange rate adjustment.

Case 3. A special case of fixed proportions technologies

What happens if both final goods prices *and* intermediate goods prices are sticky? In general, this leaves monetary policy incapable of fully achieving the socially optimal allocation. A fixed exchange rate would ensure consumption in the two countries is equal in all states, but by eliminating relative price adjustment, it would fail to sustain the desired rate of relative price adjustment. Production patterns would not be efficient. But there is a particular case where both objectives may be met, even when all prices are sticky. This is when domestic and foreign intermediates are perfect complements in production; that is, when $\gamma = 0$. In this case, the production function (2) takes on a fixed proportions form. From (8) and (9), the flexible price equilibrium is:

$$\tilde{C}^{FP} = \tilde{C}^{*FP} = L^{FP} = L^{*FP} = \left(0.5K + 0.5K^*\right)^{\frac{-1}{\rho+\nu-1}}, \quad (16)$$

where FP stands for `fixed proportions'.

In this case, the flexible price equilibrium would equalize not only consumption across countries, but also output levels. Since relative output is independent of K shocks, we might guess that relative price adjustment is not a priority. Engel (2002) notes that the expenditure-switching role of exchange rate adjustment depends critically on the substitutability of inputs in production. When substitutability is low, then expenditure switching is not important. We have:

Proposition 3: When $\gamma = 0$, and the monetary policy rules are given by:

$$\ell = \overline{\ell}\tilde{C}^{FP} \qquad \ell^* = \overline{\ell}^*\tilde{C}^{FP}, \quad (17)$$

where $\overline{\ell}$ and $\overline{\ell}^*$ are arbitrary constant parameters, then the equilibrium where both final goods prices and intermediate goods prices are sticky coincides with the flexible price equilibrium, with $P = \overline{\ell}$ and $P^* = \overline{\ell}^*$.

Proof: See Appendix.

Since PPP is attained, the exchange rate is $S = P/P^* = \overline{\ell}/\overline{\ell}^*$, and state independent. Again, a fixed exchange rate is necessary for efficient consumption allocations. Now this holds even with intermediate goods prices sticky, because with fixed proportions technology, no relative price adjustment is necessary to facilitate expenditure switching, so there is no trade-off between consumption efficiency and terms of trade adjustment[8].

This case has much of the same flavor of the recent models by Burstein, Neves, and Rebelo (2003), Burstein, Eichenbaum, and Rebelo (2002, 2003), and Corsetti and Dedola (2003). In those models, imports are combined in fixed proportions with a local nontradable (distribution services) to produce a nontraded final consumer good. The implications of terms of trade changes are similar in this model and those: terms of trade changes do not induce any substitution between home-produced and foreign-produced goods. Even though intermediate goods prices are sticky, the exchange rate is not needed for expenditure switching because no expenditure switching is required.

In considering empirically the size of this elasticity of substitution, we should focus on the short run. We are considering in this context the role of exchange rate movements as a method of ameliorating the distortions introduced by sticky prices. The horizon for such considerations is determined by the speed of adjustment of nominal prices. But the short-run elasticity of substitution of imported intermediate inputs is likely to be quite low.

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[8] Note that equation (11) indicates that the terms of trade will still respond to shocks when $\gamma = 0$. But this is not allocative, since from (10), $\tilde{L} = \tilde{L}^*$ in this case, and therefore any monetary rule that targets overall world output can achieve the efficient outcome without any relative price change.

2b. No asset trade

So far, all our results pertain to an economy full state contingent asset trade. But perhaps a more realistic benchmark would have asset markets absent or limited. Do the same results carry over to such an environment? In this section we assume that risk sharing is limited to equity markets, and in addition, equity shares are historically given.

The budget constraint for the representative home household is:

(18) $PC = \sigma^I P_H L + \lambda\Pi + (1-\lambda)S\Pi^* + \tau$, with $\lambda \neq 1$.

τ is a lump-sum tax or transfer from the government. σ^I is a subsidy to production of the intermediate good that eliminates the distortion due to monopoly pricing, which is financed by the lump-sum tax. Π represents aggregate profits of domestic firms that sell final goods, and Π^* is the aggregate profit of foreign firms selling final goods.[9] Each domestic household holds a share λ of home firms and a share $1-\lambda$ of foreign firms. We impose the restriction that there not be complete home bias.[10] We take λ as given, so we do not model a pre-market in which portfolios are chosen. Again, we abstract from the monetary side of the economy, simply assuming a nominal consumption rule for the monetary policy as above.

We assume that the optimal subsidy on intermediate goods is set at $\sigma^I = \phi/(\phi-1) > 1$, and lump sum taxes are $\tau = (\sigma^I - 1)P_H L$. We also assume that final goods producing firms are given subsidies to production $\sigma^F = \theta/(\theta-1)$, and are levied with lump sum tax bills equal to $\tau^f = (\sigma^F - 1)PC$ (for the home firm) to finance this subsidy. As for the case with state contingent asset trade, this arrangement ensures that both intermediate good and final good pricing will be efficient in a flexible price equilibrium.

9 See the Appendix for complete definitions.

10 Technically, our models allow for *super* home bias ($\lambda > 1$) and even *super* foreign bias ($\lambda < 0$).

Do the Propositions now extend to this environment with incomplete markets? The flexible price equilibrium conditions (3), (4), (6), and (7) (and their foreign counterparts) apply exactly as before, but (5) is now replaced with (18). If (18) is consistent with PPP and full consumption risk sharing under the monetary rules (13) (for sticky final goods prices) or (15) (for sticky intermediate goods prices), then Propositions 1 and 2 will hold as before.

Taking the budget constraint (18), and rearranging, we have:

$$(19) \qquad (1-\lambda)(PC - SP^{*}C^{*}) = c(p_H, Sp_F)\left(\left(\frac{p_H c_1(p_H, Sp_F)}{c(p_H, Sp_F)} - \lambda\right)C - C^{*}\left(1-\lambda-\frac{p_H c_1(p_H, Sp_F)}{c(p_H, Sp_F)}\right)\right)$$

This equation is consistent with PPP and full consumption risk sharing only when $\frac{p_H c_1(p_H, Sp_F)}{c(p_H, Sp_F)} = \frac{1}{2}$. From the definition of $c(.,.)$, the latter expression can be written:

$$\frac{p_H c_1(p_H, Sp_F)}{c(p_H, Sp_F)} = \frac{P_H^{1-\gamma}}{P_H^{1-\gamma} + (SP_F^{*})^{(1-\gamma)}}.$$

As discussed in previous subsection, an efficient allocation requires relative price change whenever $\gamma > 0$. But this expression can only equal half and still be consistent with efficient relative price change when $\gamma = 1$. In that case, both the right and left hand sides of (19) are zero when $P = SP^{*}$ and $C = C^{*}$. Hence without full asset trade and with $\gamma > 0$, Propositions 1 and 2 will extend only for a unit of elasticity of substitution between intermediate goods in production.

The intuition for this result is easy to see. When all prices are flexible, profits net of subsidies and taxes are zero, so we may write the budget constraint (18) as $PC = P_H L$, which implies that:

$$C = \frac{P_H c_1(P_H, SP_F^{*})}{c(P_H, SP_F^{*})}(C + C^{*}),$$

where the term $\frac{P_H c_1(P_H, SP_F^*)}{c(P_H, SP_F^*)}$ denotes the home countries share in world real output. This condition is only consistent with full consumption risk sharing if this term is equal to half. But a requirement for this is that the share is independent of the terms of trade. In a mechanism that is well known in the literature[11], real income is independent of the terms of trade only with unit elasticity of substitution between home and foreign good.

In the special case when $\gamma = 0$, the expression $\frac{p_H c_1(p_H, Sp_F)}{c(p_H, Sp_F)}$ may also equal half if $P_H = SP_F^*$ in all states of the world. This will be the case if intermediate goods prices are sticky and the nominal exchange rate is fixed. In this case, no relative price change is needed in order to sustain the efficient allocation. Then the monetary rules (17) support the allocations for consumption and output (9) and (10). Thus, Proposition 3 also carries over to an economy without asset trade.

2c. Obstfeld's (2001) Model

Obstfeld's (2001) model also has no asset trade, and $\gamma = 1$, but assumes complete home bias in equity holdings. The budget constraint (19) with $\gamma = 1$ becomes:

(20) $$(1-\lambda)(PC - SP^*C^*) = -(\lambda - \tfrac{1}{2})(C - C^*)(P_H SP_F^*)^{1/2}.$$

It is clear that the assumption of $\lambda = 1$ is very special. For any other value of λ, obtaining the perfect risk-sharing outcome $C = C^*$ requires purchasing power parity, $P = SP^*$. But for $\lambda = 1$, we find $C = C^*$ even if PPP fails. The fact that the model generates perfect risk sharing even in the absence of PPP when $\lambda = 1$ is a double knife-edge. In the first place, it requires unit elasticity of substitution, as before. But when there are deviations from PPP for final goods (as occurs in this model when final goods prices are sticky) there is another channel for shocks to

11 See for example Cole and Obstfeld (1991) and Corsetti and Pesenti (2001).

have a differential impact on the wealth of home and foreign households. An increase in S raises the real value of profits for home households relative to foreign ones. This effect disappears, however, under the special assumption of $\lambda = 1$, in which case foreign exchange rate changes have no wealth effects because all household portfolios are fully concentrated in ownership of local firms.

As (20) shows, when $\lambda \neq 1$, then obtaining $C = C^*$ requires $P = SP^*$. When P and P^* are fixed, this condition requires a fixed exchange rate. But under the assumption of $\lambda = 1$, $C = C^*$ occurs automatically. Eliminating deviations from PPP is not an objective of monetary policy in this case. That means that exchange rate policy can be fully devoted toward the other goal: achieving desirable terms of trade changes. The key point is that optimal exchange rate behavior mimics the terms of trade in Obstfeld's set-up, and no consideration is given to eliminating deviations from the law of one price even though there is local-currency pricing of final goods, because of the special properties of the model when $\lambda = 1$.

Nominal exchange rates must in general play the role of adjusting the terms of trade, but this objective competes with the need for exchange rate stability to stabilize CPI real exchange rates. These dual roles come into play in the models of Obstfeld and Rogoff (2000, 2002), Devereux and Engel (2003), and Corsetti and Pesenti (2002), in which only final goods are produced and traded. When the final goods are priced in the producers' currencies (PCP), then the law of one price holds. Eliminating law-of-one-price deviations is not a goal, and the objective of exchange rate policy is only to achieve the desired terms of trade. When final goods are priced in consumers' currencies (LCP), exchange rate changes are completely ineffective at influencing the relative price of home and foreign goods for consumers in either country. But exchange rate stability is needed to avoid unwarranted deviations from the law of one price – so perfectly fixed exchange rates are optimal.

3. The General Trade-off between Terms of Trade adjustment and Deviations from PPP

In the previous section, we showed that if final goods prices are partially or fully pre-set but all intermediate goods prices are free to adjust, then an optimal monetary rule maintains a fixed exchange rate. On the other hand, if intermediate goods prices are partially or fully sticky but all final goods prices are flexible, then an optimal monetary rule uses the exchange rate to replicate the terms of trade adjustment that would take place in a flexible price economy. More realistically however, both final goods prices and intermediate goods prices are likely to be partially sticky. We now extend the model to allow for this. Now we find that there is a real trade-off. Except when foreign and domestic inputs are perfect complements, monetary policy cannot remove all distortions. Hence, our interest in this section is quantitative. For different degrees of price stickiness at the intermediate and final goods level, how much exchange rate volatility should be allowed so as to facilitate relative price adjustment (or expenditure switching), at the cost of weakening the efficiency of cross-country consumption allocations?

We now let ω (the measure of final goods producers who set their prices in advance, in the consumer's currency), and κ (the measure of intermediate goods producers who set prices in advance in the producer's currency) fall between zero and one. Empirically, our prior would be that $\omega > \kappa$, but we do not impose this in the simulations.

In a symmetric equilibrium, the price index for final goods is written as

$$P = \left[\omega \hat{P}^{1-\theta} + (1-\omega)\tilde{P}^{1-\theta} \right]^{\frac{1}{1-\theta}},$$

where a $\hat{P}$ indicates the price of a good that is set in advance, and $\tilde{P}$ indicates the ex-post flexible price. The flexible price $\tilde{P}$ is just equal to marginal cost, as before, whereas $\hat{P}$ is defined by the condition:

(21) $$\hat{P} = \frac{E\left[\left(\tfrac{1}{2} P_H^{1-\gamma} + \tfrac{1}{2} P_F^{*1-\gamma}\right)^{\frac{1}{1-\gamma}} C^{1-\rho} P^{\lambda-1} \right]}{E\left[C^{1-\rho} P^{\lambda-1} \right]},$$

which differs from (6) due to the fact that the aggregate price index P is now stochastic.

The intermediate good price index is:

$$p_H = \left(\kappa \hat{P}_H^{1-\phi} + (1-\kappa) \tilde{P}_H^{1-\phi} \right)^{\frac{1}{1-\phi}} ,$$

where again, $\hat{P}_H$ is the sticky price of the intermediate good, and $\hat{P}_H$ is the flexible price. The flexible price intermediate is set as:

(22) $$\tilde{P}_H = K \left(L \left(\frac{\tilde{P}_H}{P_H} \right)^{-\phi} \right)^{\nu-1} PC^{\rho}$$

where the term inside the parentheses on the right hand side indicates that the relative price of fixed to flexible-price intermediate goods affects the composition of demand facing price setters. The sticky price of the intermediate good is written as:

(23) $$\hat{P}_H = \frac{E\left[K \left(L \left(\frac{\hat{P}_H}{P_H} \right)^{-\phi} \right)^{\nu} \right]}{E\left[L \left(\frac{\hat{P}_H}{P_H} \right)^{-\phi} P^{-1} C^{-\rho} \right]}$$

The risk sharing condition (5) is written as before, while the market clearing condition for output of the intermediate good is written as:

(24) $$L = \frac{1}{2} \left(\frac{P_H}{P_I} \right)^{-\gamma} \left[\left(\omega \left(\frac{\hat{P}}{P} \right)^{-\theta} + (1-\omega) \left(\frac{\tilde{P}}{P} \right)^{-\theta} \right) C + \left(\omega \left(\frac{\hat{P}^*}{P^*} \right)^{-\theta} + (1-\omega) \left(\frac{\tilde{P}^*}{P^*} \right)^{-\theta} \right) C^* \right],$$

where we define $P_I = \left[\frac{1}{2} P_H^{1-\gamma} + \frac{1}{2} P_F^{1-\gamma} \right]^{\frac{1}{1-\gamma}}$ as the aggregate price index for intermediate goods. An equivalent set of conditions may be written for the foreign economy.

An optimal monetary policy in this model is aimed at eliminating three types of distortions. First, as before, there is an inefficiency due to the failure of PPP, which leads to distorted consumption allocations. Second, there is an inefficiency due to the lack of adjustment of the terms of trade (the relative price of the home and foreign intermediate) to the labor supply shocks. Finally, there is a new inefficiency coming from the fact that with some intermediate good prices set in advance, production levels will differ across sticky price and flexible price intermediate goods firms.

An optimal monetary rule cannot eliminate all these inefficiencies simultaneously, except in the special cases of the previous section. Moreover, it is not possible to characterize the optimal monetary policies analytically in this more general case. Rather, we solve the model numerically, choosing the monetary policy that maximizes expected utility for a given calibration of parameter values and distribution of labor supply shocks.

The model is entirely symmetric, so that home and foreign expected utility are identical when monetary policies are identically chosen across countries. As in the previous section, we abstract from issues of strategic interaction across policy makers and derive an optimal policy rule that maximizes an equal-weighted sum of home and foreign expected utilities.

As emphasized by Obstfeld and Rogoff (2002), when shocks are global there is no need for terms of trade change or exchange rate adjustment. Hence, we focus only on country specific labor supply shocks, so that $K+K^*$ is constant. Moreover, we assume a two-state distribution of K across the two countries, where K is either high or low, and normalize so that the standard deviation of the terms of trade in the flexible price economy is unity. In the benchmark version of the model, we assume that $\theta = \phi = 2$, $\rho = v = \varepsilon = 1$, and we impose unit elasticity of substitution between home and foreign intermediate goods, so that $\gamma = 1$. We also report results from alternative parameter settings below. Assuming again that the monetary instrument is the nominal value of consumption in each country, we simulate the model, searching across state contingent values of ℓ and ℓ^* that maximize utility. Given these values, we can derive the variance of the exchange rate and the cross-country correlation of consumption.

Figure 1 illustrates the implications of the optimal monetary policy for the benchmark model. The Figure shows the standard deviation of the log exchange rate under alternative degrees of price stickiness in final goods prices and intermediate goods prices. The Figure confirms the result of Proposition 1, showing that, irrespective of the degree of price rigidity in final goods prices, when intermediate goods prices are fully flexible the optimal monetary policy requires a fixed exchange rate[12]. But as the degree of price rigidity in intermediate goods prices increases, it becomes more and more desirable to use the exchange rate to achieve terms of trade adjustment. Hence, holding the fraction of final goods prices that are sticky constant, increasing the degree of price rigidity in intermediate goods will increase the optimal exchange rate volatility when $\gamma = 1$. In this case, we should allow a greater departure from full consumption risk sharing as intermediate goods prices become less capable of adjusting to labor supply shocks. Even so, the magnitude of exchange rate adjustment is far less than the terms of trade adjustment that would occur in a frictionless economy. When a quarter of all intermediate goods have prices set in advance, then the exchange rate volatility is only about a quarter of that in the flexible price model. Even when $\kappa = 1$, so that all intermediate goods prices are sticky, the standard deviation of the exchange rate under an optimal monetary policy is only 0.5.

Not surprisingly, as the proportion of flexible final goods prices increases, the standard deviation of the exchange rate under an optimal policy increases, because it is now easier to ensure the efficient terms of trade adjustment through exchange rate movement without distorting consumption allocations. But since (as discussed above) empirical evidence indicates

[12] In the knife-edge case where $\omega = \kappa = 0$, optimal monetary rules are undefined.

little responsiveness of final goods prices to exchange rates, Figure 1 suggests that an optimal monetary policy involves much less exchange rate adjustment than would be desired in an economy with full exchange rate pass-through. Hence, the main message of the previous section continues to apply in the extended model: when prices are sticky in local currency, an optimal monetary policy implies less exchange rate flexibility than would be inferred from the traditional pricing model with full pass-through to consumer prices (and PPP). Moreover, despite local currency price stickiness in final goods, this is a model where there is a substantial *expenditure-switching* role for the exchange rate in production, since there is full exchange rate pass-through at the intermediate good level.

In Figure 1, the relationship between κ and exchange rate volatility is concave. As intermediate goods prices become more and more sticky (for a given degree of price rigidity in final goods), exchange rate volatility increases, but at a diminishing rate. As κ tends to unity, the gain from terms of trade adjustment in response to exchange rate changes is offset by the costs in terms of reduced consumption risk sharing. In fact, the numerical solution shows that the optimal monetary rules are effectively independent of movements in κ, for values of κ greater than 0.5. This is true for all values of ω. Hence, in the benchmark model, the benefit of further exchange rate adjustment in facilitating terms of trade adjustment falls to zero as intermediate good prices become more and more rigid.

Figure 1 suggests that as intermediate goods prices become less and less flexible, exchange rate adjustment becomes more important, so that an optimal policy trades off one distortion against another. But this property does not hold generally. Figure 2 illustrates the effect of reducing the elasticity of substitution, γ, on the relationship between price stickiness and optimal exchange rate volatility. In this Figure, we set $\omega = 1$, and vary κ, as in Figure 1, but for a smaller γ. The dark schedule in Figure 1 is identical to the baseline case of Figure 1, where the elasticity of substitution between home and foreign intermediates is set at unity. The light schedule represents the case where there is a low elasticity of substitution between home and foreign intermediates, i.e. $\gamma = 0.25$. The Figure shows that optimal exchange rate volatility is reduced as the elasticity of substitution between intermediates is lowered. This is what we would expect given Proposition 3, since when $\gamma = 0$, exchange rate volatility should be equal to zero for any degree of intermediate good price rigidity. With perfect complementarity, there is no need for relative price adjustment. But Figure 3 also demonstrates another intriguing result. The relationship between κ and exchange rate volatility is *non*-monotonic. As κ increases, beginning at $\kappa = 0$, exchange rate volatility initially rises. But above $\kappa = 0.6$, exchange rate volatility falls with increasing stickiness in intermediate goods prices. There is a *hump-shaped* relationship between price stickiness and exchange rate volatility.

What is the intuition behind the hump-shaped pattern? The answer comes when we recall the third source of distortion in the model discussed above. When some prices are sticky and some are flexible (i.e. $0 < \kappa < 1$), then the *K* shocks will affect the relative production levels of sticky price and flexible price firms. For instance, a positive home *K* shock will reduce the output of flexible price intermediate goods firms relative to sticky price firms, in the home country. This creates a welfare loss[13]. It therefore becomes desirable to reduce aggregate demand for the home intermediate good, through a tight monetary policy – reducing the output of the sticky price firms and moving towards a more uniform pattern of production across home intermediate firms. But this requires a home country currency appreciation. If by contrast, $\kappa = 1$, a shock will affect all home intermediate firms in the same way. The policymaker then has to worry only about consumption allocations and terms of trade adjustment. For a very low

[13] This is well known in the closed economy literature on price stickiness – see Rotemberg and Woodford (1997, 1999), King and Wolman (1999), and Woodford (2003), for instance.

elasticity of substitution, the latter consideration is much less important. Hence, overall, moving from an economy where half of all intermediate goods prices are fixed to one where all prices are fixed can *reduce* the optimal exchange rate volatility.

Figures 3 and 4 show other properties of the optimal policy. Figure 3 maps out the degree to which exchange rate volatility under an optimal policy is sensitive to the coefficient of relative risk aversion ρ. As ρ increases, consumption differentials become less sensitive to real exchange rates. As a result, optimal exchange rate volatility increases: more terms of trade adjustment can be achieved for a given degree of consumption risk sharing.

Figure 4 gives a different perspective. It shows the trade-off between consumption correlation and terms of trade adjustment, under alternative degrees of risk aversion, for progressively higher degrees of price stickiness in the intermediate goods sector, for the benchmark calibration. Each locus illustrates the moment pair $\sqrt{E((C/C^*)-1)^2}$ (measuring deviation from consumption risk sharing), and $\sqrt{E(\frac{P_H}{SP_F}-\frac{K}{K^*})^2}$ (measuring deviation from the flexible price terms of trade) for increasing values of κ, under the optimal monetary policy, in the case where $\omega = 1$. Hence, when $\kappa = 0$, the monetary rule achieves the optimal consumption allocation and efficient terms of trade adjustment, whatever the value of ρ. As κ increases, the deviations from perfect consumption correlation and efficient terms of trade adjustment both increase. Interestingly, the Figure shows a convex trade-off between consumption risk sharing and terms of trade adjustment. As κ increases, the optimal rule allows more and more deviations of the terms of trade form its flexible price equilibrium in return for a given deviation in consumption risk sharing. This mirrors the fact that the optimal exchange rate volatility in Figure 1 is a concave function of κ. As the fraction of intermediate goods prices that are pre-set increases more and more, the monetary authority is less and less willing to allow an increase in exchange rate volatility. Of course, when the intermediate goods production elasticity of substitution is very small, these loci would be backward bending, for the same reasons that we obtain the hump-shaped relationship in Figure 2.

4. Conclusions

There is a large body of evidence establishing that pass-through from changes in exchange rates to consumer goods prices is weak or non-existent. When this is the case, exchange rate fluctuations automatically move around consumption-based real exchange rates. This means that consumer prices do not allocate goods efficiently across countries, and builds an a priori case for exchange rate stability. On the other hand, exchange rates may have a high pass-through to prices at the intermediate good level, and at this level, exchange rate movements may have a significant allocational role to play through expenditure switching among foreign and domestic intermediate goods. This opens up a trade-off. Exchange rate adjustment is desirable for expenditure switching, but costly because it moves around real exchange rates. This paper has identified this trade-off and explored its nature, both qualitatively and quantitatively. In some cases, we show that a welfare evaluation of the trade-off gives a significant emphasis on exchange rate stability. Quantitatively, we find that exchange rate volatility should be significantly less than that which would be inferred based on models that focus exclusively on the expenditure-switching role of exchange rates.

Appendix A. Derivation of conditions in Table 1

The consumption index, C, from equation (1) is given by:

(A1) $$C = \left[\int_0^1 C(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}, \ \theta > 1,$$

where $C(j)$ is the consumption of variety j. The aggregate price index is then given by:

$$P = \left[\int_0^1 P(j)^{1-\theta} dj \right]^{\frac{1}{1-\theta}}.$$

Minimizing $\int_0^1 P(j)Y(j)dj$ subject to (A1), and using the equilibrium condition $C(j) = Y(j)$ gives demand for the firm's product:

(A2) $$Y(j) = \left[\frac{P(j)}{P} \right]^{-\theta} C.$$

Aggregate profits of home final goods firms are given by $\Pi = \int_0^1 \Pi(j)dj$, where $\Pi(j)$ is defined by $\Pi(j) = \sigma^F P(j)Y(j) - c(P_H, SP_F^*)Y(j) - \tau^F$, and the subsidy and taxes satisfy $\sigma^F = \dfrac{\theta}{\theta-1}$, and $\tau^F = (\sigma^F - 1)PC$.

Profit maximization for the final goods firm, subject to (A2), in face of the optimal subsidy, gives equation (3) of Table 1.

Minimizing $\int_0^1 P_H(i)Y_H(i,j)di$ subject to $Y_H(j) = \left[\int_0^1 Y_H(i,j)^{\frac{\phi-1}{\phi}} di \right]^{\frac{\phi}{\phi-1}}$ gives demand for the intermediate good i by firm j:

(A3) $$Y_H(i,j) = \left[\frac{P_H(i)}{P_H} \right]^{-\phi} Y_H(j).$$

Minimizing $P_H Y_H(j) + P_F Y_F(j)$ subject to (2) gives the demand function for each intermediate aggregate. For example, the demand for the home intermediate aggregate (by firm *j*) is:

(A4) $$Y_H(j) = \frac{1}{2}\left(\frac{P_H}{(\frac{1}{2}P_H^{1-\gamma} + \frac{1}{2}(SP_F^*)^{1-\gamma})^{\frac{1}{1-\gamma}}}\right)^{-\gamma} Y(j).$$

.

Households *i*'s budget constraint under complete markets is given by

(A5) $$P(z)C(i,z) = \sigma^I P_H(i,z)L(i,z) + \tau(z) + B(i,z),$$

where the *z* index refers to the state (and, as previously, *i* to the household) , and $\sigma^I = \frac{\phi}{\phi - 1} > 1$, with lump sum taxes $\tau = (\sigma^I - 1)P_H L$. $B(i,z)$ is household *i*'s holdings of bonds that pay off in state *z*. The market for state contingent bonds is open prior to the realization of shocks. Home households are endowed with ownership of all domestic final goods firms. State contingent claims are chosen subject to the constraint that the value of all claims, $\sum_z q(z)B(i,z)$, equals the initial value of firms.

The first order conditions for the household *I's* choice of state contingent consumption and price, given the demand for household *I's* good from (A3) above, are given by:

(A6) $$C(i,z)^{-\rho} = \lambda q(z)P(z)$$

(A7) $$\frac{KL(i,z)^{\nu}\phi}{P_H(i,z)} = \lambda q(z)\sigma^I L(i,z)(\phi - 1)$$

Putting these two equations together, and imposing symmetry, so that all households in the home country set identical prices of intermediates, gives equation (4) in Table 1. Equation (5) is obtained by using the identical equation to (A6) for the foreign country, along with the assumption of ex ante equality, so that the Lagrange multipliers are identical across the two countries. Finally, the market clearing equations (6) is obtained by using (A2), (A3) and (A4), aggregated across goods, again using the symmetry assumption that all home final goods prices are equal, and all home intermediate goods prices are equal. Condition (7) in Table 1 is just the assumed monetary policy rule.

Appendix B. Proof of Propositions

Proof of Proposition 1

Using the monetary policies (13), and the market clearing equation (6), and its foreign counterpart, in equation (4) and its foreign counterpart, we get

(B1) $$P_H = K\left(\left(\tfrac{1}{2}+\tfrac{1}{2}\tilde{\tau}^{1-\gamma}\right)^{\frac{\gamma}{1-\gamma}}\left(\frac{\overline{\ell}}{P}+\frac{\overline{\ell}^*}{P^*}\right)\tilde{C}\right)^{\nu-1} P\left(\frac{\overline{\ell}}{P}\tilde{C}\right)^{\rho}$$

(B2) $$P_F^* = K^*\left(\left(\tfrac{1}{2}+\tfrac{1}{2}\tilde{\tau}^{-(1-\gamma)}\right)^{\frac{\gamma}{1-\gamma}}\left(\frac{\overline{\ell}}{P}+\frac{\overline{\ell}^*}{P^*}\right)\tilde{C}\right)^{\nu-1} P^*\left(\frac{\overline{\ell}^*}{P^*}\tilde{C}\right)^{\rho}$$

Now using equations (B1) and (B2) to construct $\left(\tfrac{1}{2}P_H^{1-\gamma}+\tfrac{1}{2}(SP_F^*)^{(1-\gamma)}\right)^{\frac{1}{1-\gamma}}$ with the definitions of $\tilde{\tau}$ and $\tilde{C}$, we can establish that $\left(\tfrac{1}{2}P_H^{1-\gamma}+\tfrac{1}{2}(SP_F^*)^{(1-\gamma)}\right)^{\frac{1}{1-\gamma}}$ is state independent. This implies from (3') and its foreign counterpart that PPP holds, which implies from (5) that consumption risk sharing holds. Then from the definition of the monetary policy rules, we must have $\frac{\ell}{P}=\frac{\ell^*}{P^*}$.

From the pricing equations again, we can establish that the equilibrium pre-set prices satisfy $P=\ell, \quad P^*=\ell^*$. This implies that PPP and full consumption risk sharing hold. Then, from equations (6) and (7), both consumption and output are at their flexible price levels, given that the terms of trade is equal to its flexible price level.

Proof of Proposition 2

Since final goods prices are flexible, it must be from (5) that PPP holds, so that from (5), consumption is equalized across countries. Given PPP and (3), we have

(B3) $$S = \frac{P}{P^*} = \frac{P_H}{P_F^*}\tau \, .$$

From the monetary policy (15), in conjunction with the fact that consumption is equalized across countries, we have

(B4) $$S = \frac{P}{P^*} = \frac{\ell}{\ell^*}\frac{C^*}{C} = \frac{\overline{\ell}}{\overline{\ell}^*}\tilde{\tau} \, .$$

Hence, the equilibrium terms of trade are $\tau = \frac{P_F^*}{P_H}\frac{\overline{\ell}}{\overline{\ell}^*}\tilde{\tau}$. Again, using the monetary rules (15), equilibrium consumption and employment may be expressed as

(B5) $$C = \frac{\overline{\ell}}{P_H}\tilde{C} = \frac{\overline{\ell}^*}{P_F^*}\tilde{C}$$

(B6) $$L = \left(\tfrac{1}{2} + \tfrac{1}{2}\tilde{\tau}^{1-\gamma}\right)^{\frac{\gamma}{1-\gamma}} \frac{\overline{\ell}}{P_H}\tilde{C}$$

Using these equations, it is clear that $\frac{P_F^*}{P_H}\frac{\overline{\ell}}{\overline{\ell}^*} = 1$, so that $\tau = \tilde{\tau}$. Now using the definition of consumption and employment in equation (4), we may write

(B7) $$P_H = K\left(\left(\tfrac{1}{2} + \tfrac{1}{2}\tilde{\tau}^{1-\gamma}\right)^{\frac{\gamma}{1-\gamma}} \frac{\overline{\ell}}{P_H}\tilde{C}\right)^{\nu-1} P_H \left(\tfrac{1}{2} + \tfrac{1}{2}\tau^{1-\gamma}\right)^{\frac{1}{1-\gamma}} \left(\frac{\overline{\ell}}{P_H}\tilde{C}\right)^{\rho}$$

Then, using the definitions of $\tilde{\tau}$ and $\tilde{C}$, we may establish that a), the right hand side of (B7) is state independent, and b) $P_H = \overline{\ell}$. Hence, the monetary policy rules (15) stabilize marginal cost for intermediate good producers, and ensure that the economy with sticky intermediate goods prices attains the flexible price allocation.

Proof of Proposition 3.

From (5), with the monetary policies (17), and given that P and P^* are predetermined, the exchange rate is predetermined. With fixed proportions, and given that P_H, P_F^*, and S are predetermined, condition (3') becomes $P = \frac{1}{2}(P_H + SP_F^*)$. The market clearing condition becomes $L = C$. Hence, the monetary policies ensure that PPP holds (since all terms in the final goods pricing relationship are predetermined) and there is efficient risk sharing. Employment is also equalized across countries. The level of employment is determined from the monetary policies such that $L = C = \frac{\overline{\ell}}{P}\tilde{C}^{FP}$. From condition (4'), and the analogous condition for the foreign country, added together, we get

(B9) $$1 = \frac{P_H + SP_F^*}{2P} = \frac{E\left(\frac{(K + K^*)}{2} L^{\nu}\right)}{EL^{1-\rho}}$$

Substitute in (B9) for L and $\tilde{C}^{FP}$ as defined in (16), it follows immediately that $P = \overline{\ell}$ (and $P^* = \overline{\ell}^*$), so that consumption and employment are at the flexible price equilibrium levels.

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Figure 1 Exchange Rate Volatility and Price Stickiness

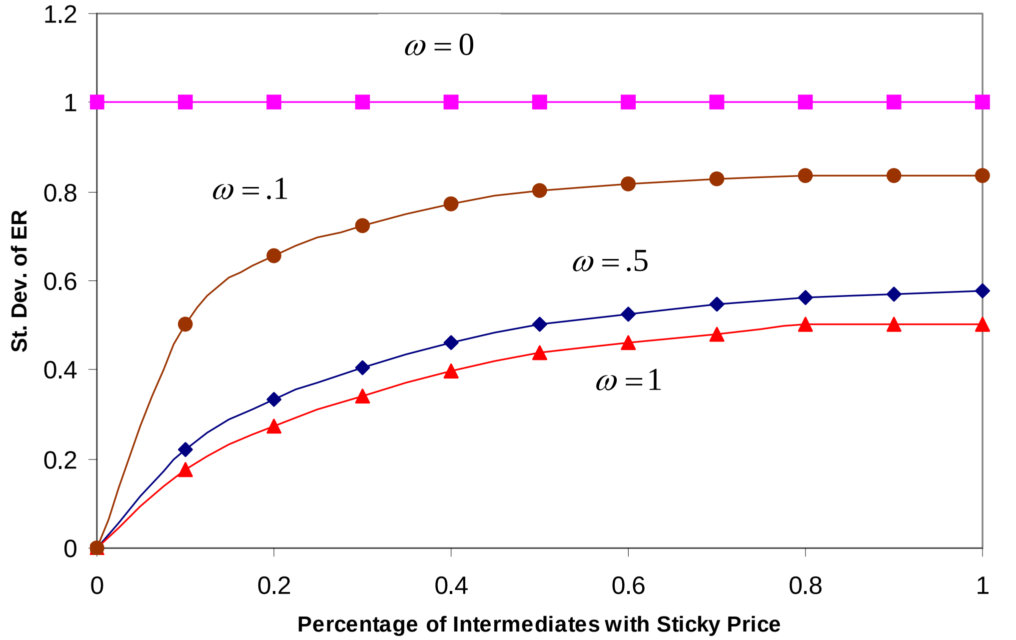
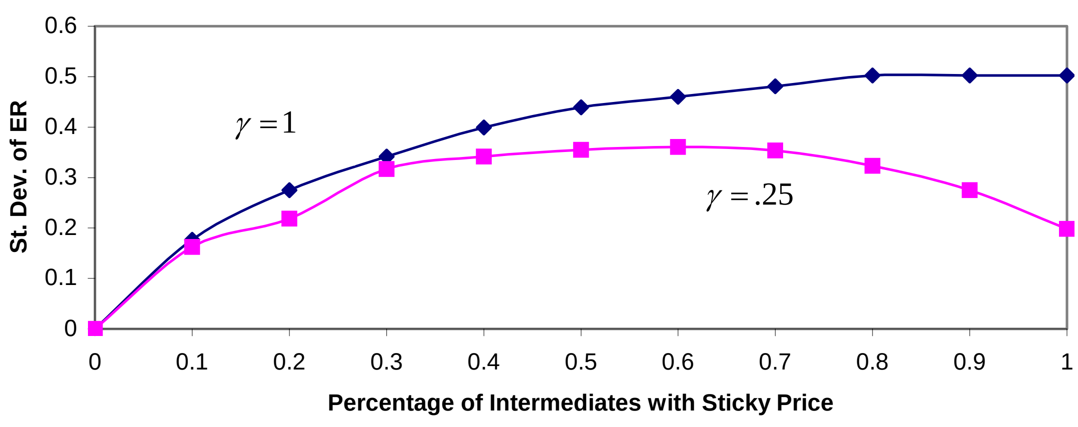


Figure 2 The Elasticity of Substitution between Home and Foreign Intermediates



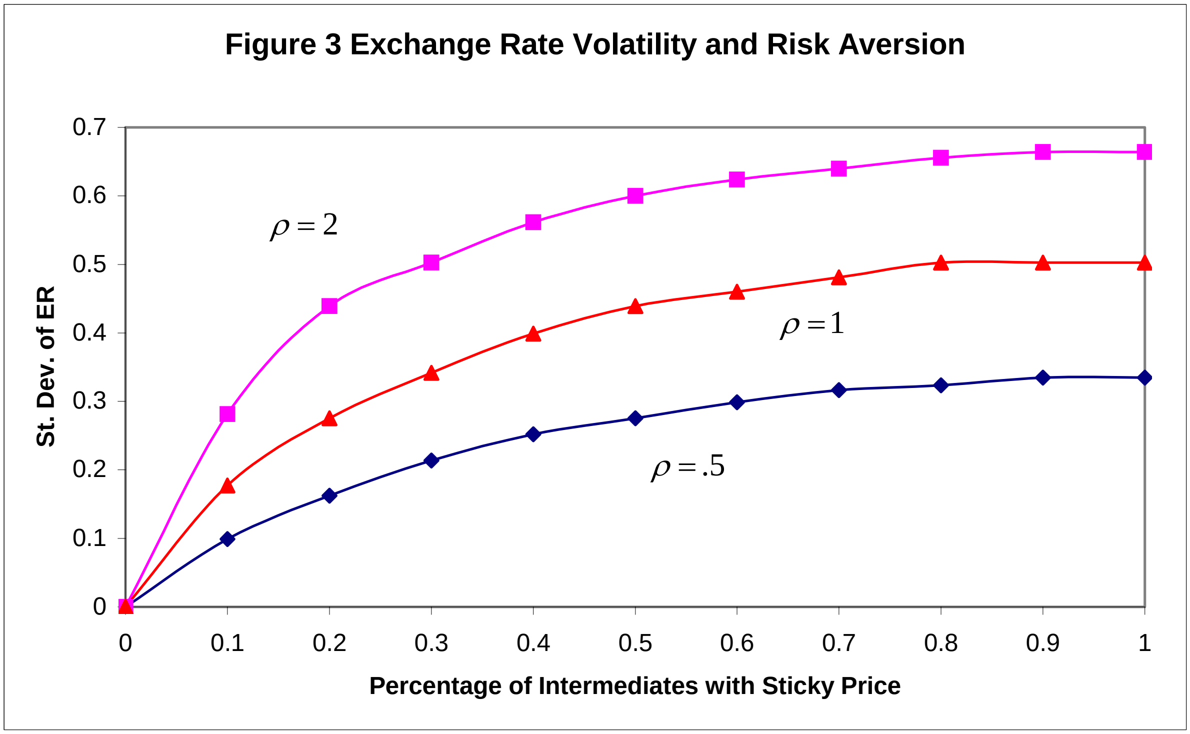


Figure 3 Exchange Rate Volatility and Risk Aversion
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
St. Dev. of ER
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Percentage of Intermediates with Sticky Price
$\rho = 2$
$\rho = 1$
$\rho = .5$

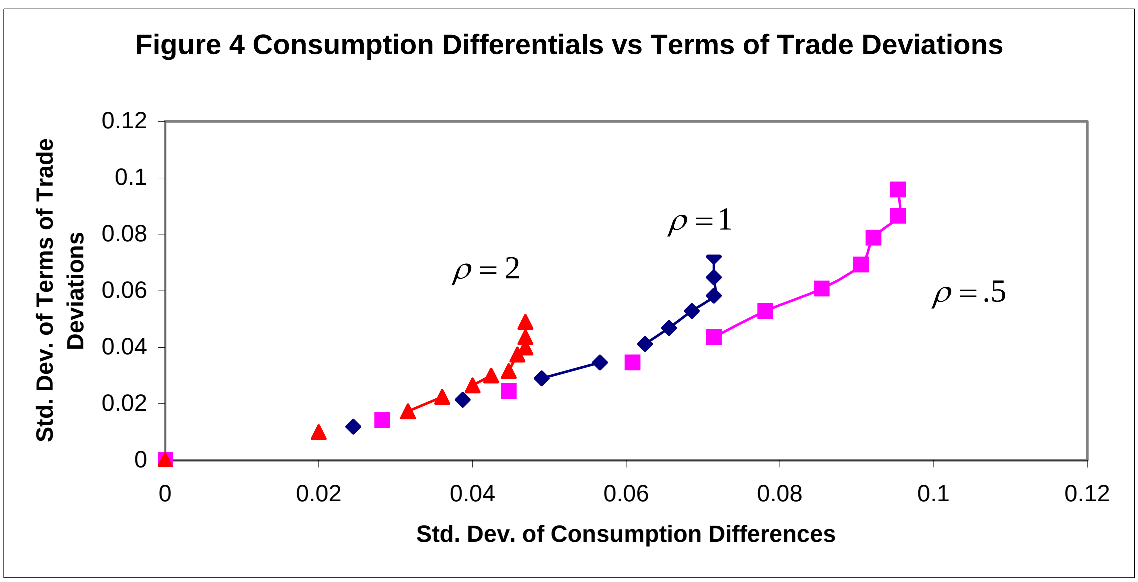


Figure 4 Consumption Differentials vs Terms of Trade Deviations
0.12
0.1
0.08
0.06
0.04
0.02
0
Std. Dev. of Terms of Trade Deviations
0
0.02
0.04
0.06
0.08
0.1
0.12
Std. Dev. of Consumption Differences
$\rho = 2$
$\rho = 1$
$\rho = .5$