Adaptive Learning and the Use of Forecasts in Monetary Policy*

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Abstract

This paper investigates monetary policy design when central bank and private sector expectations differ. Private agents learn adaptively; the central bank has a one of a number of models of the economy. Successful implementation of optimal policy using inflation targeting rules requires the central to have complete knowledge of private agents' learning behavior. If the central bank mistakenly assumes private agents to have rational expectations when in fact they are learning, policy rules frequently lead to divergent learning dynamics. However, if the central bank does not correctly understand agents' behavior, stabilization policy is best implemented by controlling the path of the price level rather than the inflation rate.

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1 Introduction

How should forecasts be incorporated into optimal monetary policy design? The recent literature on implementing optimal monetary policy (see Svensson and Woodford (2002), Svensson (2003), Giannoni and Woodford (2002a, 2002b), Woodford (2003, Chap. 7)) characterizes the central bank's decision procedure in terms of specific targeting rules: such rules specify a relationship between one or more target variables that must be checked each time an interest-rate decision is made. The instrument setting is deemed appropriate if the specified "target criterion" is satisfied. Since the target variables that appear in the criterion are usually not directly observable, to determine the instrument setting in any period, the central bank requires a completely specified model of the economy to solve for the equilibrium path of endogenous variables. The targeting rule approach appears to be an effective way to implement optimal monetary policy and is argued to be robust to a range of assumptions concerning the nature of economic disturbances that affect the economy.

To date, the literature on specific targeting rules rests on the assumption that the central bank is able to exploit the true structure of the economy – that it understands the true structural relations, and therefore the expectations held by private agents, when determining the instrument setting that is consistent with implementing its objectives.[1] Furthermore, the literature typically rests on the assumptions of rational expectations and common information on the part of private agents and the central bank. This implies that both these economic actors necessarily hold common expectations about future macroeconomic conditions. But suppose the central bank does not know the true model of the economy. Or that the central bank and private agents hold differing beliefs about the future evolution of the economy – does this hinder the usefulness of specific targeting rules? And given uncertainty as to the true model, should optimal monetary policy be conditioned on private agents' expectations and if so in what way? Or is it sufficient for the central bank's instrument choice to be conditioned

[1] A simple example of this type of monetary policy arrangement is inflation forecast-targeting (see Svensson (1999)): the monetary authority is charged with maintaining its inflation forecast over some horizon equal to a fixed inflation objective. If the inflation forecast deviates from the target, the central bank must adjust its instrument setting to ensure its projected evolution of the economy is consistent with the forecast target.

solely on internally constructed forecasts (using whatever model it may have at its disposal)?

This paper addresses these questions in a simple New-Keynesian model of output gap and inflation determination in which private agents must learn about the probability laws governing the evolution of state variables exogenous to their decision problems. Rational expectations are a nested special case of the proposed beliefs, and the analysis is centrally concerned with the conditions under which agents' beliefs converge to those predicted by a rational expectations equilibrium analysis of the model. Introducing learning in such a way permits the central bank and private agents to have differing beliefs about the evolution of the macroeconomy and allows examination of its implications for the design of optimal monetary policy. Because all economic actors will only hold identical believes when and if the learning process converges, the framework serves to coherently analyze robustness of rational expectations policy prescriptions to departures in underlying model assumptions; and specifically expectations formation.

Following Giannoni and Woodford (2002a), candidate targeting rules are variants of the consolidated first-order condition to the solution to the optimal commitment problem under the so-called "timeless perspective" of Woodford (1999) of a standard linear-quadratic policy problem. Two representations are analyzed. The first is a particular linear restriction on the inflation rate and the *change* in the output gap. The second, is an equivalent restriction on the price level and the contemporaneous output gap.[2] If the central bank can arrange for either of these relations to be met in all periods it will successfully implement the optimal monetary policy. The former will be referred to as the *inflation targeting rule* and the latter the *price-level targeting rule*. A policy is robust if agents' beliefs converge to the rational expectations equilibrium associated with the policy. To implement such targeting criteria, the central bank requires a model of the economy. It follows that the central bank's knowledge of the economy will have consequences for its projection of the future path of economic variables and therefore the implementation and efficacy of any given targeting rule.

Three decision procedures are considered that are equivalent in terms of the rational expec-

[2] This equivalence is in terms of the rational expectations equilibrium each policy implies, and will be formally defined in Section 3.

tations equilibria they imply. Each represents the central bank's beliefs about the evolution of the economy. Of particular interest is whether learning dynamics provide ground for choosing among alternative approaches to implementing optimal monetary policy. First, the central bank implements the target criterion incorrectly assuming agents to have rational expectations and observing only lagged aggregate variables and fundamental disturbances. Thus the evolution of the economy is projected using a rational expectations model – that would obtain if agents' solved their decision problems under rational expectations – and this implies a reaction function for the nominal interest rate that is a function only of the model's state variables and therefore independent of agents' learning behavior.[3] Second, the central bank implements the target criterion correctly understanding agents' behavior. In contrast to the former decision procedure, this approach to monetary policy induces a strong dependence on agents' forecasts – indeed agents' long-horizon forecasts of macroeconomic conditions matter for the implementation of policy.[4] Finally, since the above two decision procedures represent extreme informational assumptions on the part of the central bank, instrument rules that depend only on observed private one-period-ahead expectations are considered. If some dependency on private forecasts is desirable to implement optimal policy under learning dynamics, the use of one-period-ahead forecasts may be more feasible and effective than use of private forecasts into the indefinite future.

For the inflation targeting rule, stability under learning dynamics depends on the central bank's model of the economy. If the monetary authority correctly understands agents' behavior and projects the evolution of the economy on the basis of this model, then inflation targeting rules are always able to implement the optimal commitment equilibrium in the presence of learning dynamics. In contrast, if the central bank mistakenly assumes agents to

[3]Here arises the question of robustness – do monetary policies that have desirable stabilization properties under rational expectations succeed in bringing about the optimal equilibrium if the central bank's projections are made assuming agents have rational expectations when in fact agents must learn about the economy over time?

[4]While correct knowledge of agents' behavior is likely to be unrealistic, if it is found in the case of the first decision procedure that a targeting rule is *not* robust to small deviations from the rational expectations assumption we ought to be interested in whether acquiring additional information about the true model matters to the successful stabilization of the economy under learning dynamics. Moreover, if a given policy is prone to propagating self-fulfilling expectations under either conception of the central bank's decision problem, there will be little to recommend it.

have rational expectations, and projects the evolution of the economy under this assumption, the inflation targeting rule leads to instability for many empirically reasonable parameter values. The economy is, therefore, prone to self-fulfilling expectations of the type conjectured by Friedman (1968). A decision procedure that depends on one-period-ahead forecasts is similarly afflicted. Thus, successful implementation of optimal policy depends crucially on the central bank's forecasting procedure.

For the price-level targeting rule, the central bank can also always successfully implement the optimal equilibrium given correct knowledge of agents' behavior. However, in contrast to the inflation targeting rule, the price-level target criterion displays a degree of robustness to the model used by the central bank to construct projections. Even if the central bank mistakenly assumes agents to have rational expectations, the price-level targeting rule leads to stability under learning dynamics for many empirically reasonable parameters values. This result is of considerable interest since, for appropriately chosen initial conditions, the two proposed targeting rules imply the same state-contingent evolution of model variables under rational expectations. The difference between these two rules, in the case of learning dynamics, is that the price-level targeting rule specifies a different kind of subsequent behavior when one finds that (because the private sector does not behave as they were projected to do) one has failed to achieve the target criterion precisely. Thus the difference between the two rules is a different commitment as to how one will react to seeing that one has missed one's target. The price-level targeting rule is more robust to learning dynamics and suggest optimal monetary policy might best be implemented by explicit reference to the path of the price level rather than the inflation rate.

This paper builds on Preston (2003) which proposes a framework for modeling learning in which agents face multi-period decision problems as in the microfoundations used in Bernanke and Woodford (1997), Clarida, Gali, and Gertler (1999) and Woodford (1999) and is related to Preston (2004) which examines the efficacy of using forecasts in the design of simple instrument rules. However, the analysis of this paper is closest in spirit to recent work by Evans and Honkapohja (2002) and Honkapohja and Mitra (2003). These papers assume a log-linear model of the monetary transmission mechanism where agents need only forecast aggregate income and inflation one period in advance. The analysis of Honkapohja and Mitra (2003) examines a range of forecasting procedures (based on recursive learning algorithms) for the monetary authority and their implications for economic stability for a variety of common monetary policy rules. They show that internal forecast decision procedures of this kind often require stronger conditions for learnability than if the central bank responds to observed private sector forecasts (when these forecasts are themselves formed under a least-squares learning algorithm). Evans and Honkapohja (2002) consider the question of implementing optimal monetary policy by use of a inflation targeting rule. This approach is shown to implement successfully the optimal equilibrium and therefore to eliminate instability due to self-fulfilling expectations.

The present analysis revisits the question of implementing optimal monetary policy using a different approach to modeling learning than adopted in the papers by Evans and Honkapohja (2002) and Honkapohja and Mitra (2003). The current paper assumes agents face multi-period decision problems and use econometric models to learn about state variables that are beyond their control, so that long-horizon forecasts matter to the current determination of the economy's aggregate variables. This approach to modeling learning has the advantage that agents take proper account of their wealth and act optimally given their beliefs – ensuring that when private agents do learn the true probability laws predicted by a rational expectations analysis of the model, agents' consumption decisions necessarily satisfy their intertemporal budget constraints. Importantly, this approach provides stronger evidence that a price-level targeting rule provides more robust learnability than does an analysis of learning in the economy where only one-period-ahead forecasts matter.

This paper proceeds as follows. Section 2 outlines the basic framework and recapitulates the analysis of Preston (2003). Section 3 details the optimal monetary policy problem of the central bank. Section 4 then begins the analysis of monetary policy rules under learning dynamics. It considers an inflation targeting rule that is designed to implement the optimal commitment equilibrium under alternative assumptions on the central bank's knowledge. Section 5 turns to an analogous analysis of price-level targeting rules. Section 6 considers target criteria that depend on one-period-ahead forecasts. The final section concludes.

2 A Simple Model

Preston (2003) analyzes the microfoundations used in several recent studies of monetary policy rules under a specific non-rational expectations assumption. This leads to important differences in the model's implied aggregate dynamics when agents are learning relative to the predictions of rational expectations equilibrium analysis. This section recapitulates the central assumptions and results of that paper and the reader is encouraged to consult it for details.

2.1 Primitive Assumptions and Aggregate Dynamics

The rational expectations paradigm comprises two stipulations: (i) agents optimize given their beliefs about the joint probability distribution for various state variables that are independent of their actions and that matter for their payoffs and (ii) that the probabilities that they assign coincide with the predictions of the model. Following a considerable literature on learning (see Sargent (1993) and Evans and Honkapohja (2001) for reviews), this paper retains (i) while replacing (ii) with the assumption that the joint probabilities are formed using an econometric model. By assumption, the predictions of this econometric model need not coincide with the predictions of the true model. As additional data become available, agents update their model estimates. The central question posed by the analysis is whether given sufficient data the predictions of the econometric model eventually converge to those of the true model.[5]

When private agents form expectations in this manner, aggregation of a log-linear approximation to the optimal decision rules of the households and firms described by Bernanke and Woodford (1997), Clarida, Gali, and Gertler (1999) and Woodford (1999) yields

$$x_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta) x_{T+1} - \sigma(i_T - \pi_{T+1}) + r_T \right] \tag{1}$$

[5] Agents are assumed to know what they need to know to behave according to (i): they know their own preferences and constraints, and, more generally, they correctly understand the mapping from their actions to their expected payoff, given a probability distribution for the variables that are outside their control. However, they are not assumed to know anything about how those variables outside their control are determined. For instance, they do not know that other agents have preferences just like their own and form expectations the way that they do, even if these things happen to be true within the particular model that is being studied.

and

$$\pi_t = \kappa x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} [\kappa\alpha\beta \cdot x_{T+1} + (1-\alpha)\beta \cdot \pi_{T+1} + u_T] \tag{2}$$

where x_t is the output gap, π_t the inflation rate, i_t the nominal interest rate and r_t and u_t are exogenous disturbance terms, with all variables being properly interpreted as log-deviations from steady state values. $\sigma > 0$ is the intertemporal elasticity of substitution, $0 < \beta < 1$ the discount rate, $\kappa > 0$ and $0 < 1 - \alpha < 1$ the probability that a firm will have an opportunity to change its price in any period. Finally, $\hat{E}_t$ denotes the assumed non-rational expectations operator (to be discussed); E_t will be used to denote the usual rational expectations operator.

The first equation represents the aggregation of optimal consumption decisions by households which are implications of their Euler equation and intertemporal budget constraint. It is therefore an aggregate demand relation, specifying that output is determined by the current real rate of interest and long-horizon expectations of the output gap, the real interest rate and exogenous disturbances into the indefinite future. The presence of long-horizon expectations arises from the intertemporal nature of the household's consumption decision: to optimally allocate consumption today requires the household to plan its future consumption over time and across states of nature, which in turn requires forecasts of variables such as income and real interest rates.[6]

Relation (2) is derived from the aggregation of the optimal prices chosen by firms to maximize the expected discounted flow of profits under a Calvo price-setting problem. It is therefore a generalized New-Keynesian Phillips curve, specifying current inflation as depending on the contemporaneous output gap and expectations of this variable and inflation into the indefinite future. Here the presence of long-horizon expectations arises due to the pricing frictions induced by Calvo pricing. When a firm has the opportunity to change its price in period t there is probability α^{T-t} that the firm will not get to change its price in the subsequent $T - t$ periods. The firm must therefore concern itself with macroeconomic

[6] Indeed the connection of this relation to the predictions of permanent income theory is immediate. The first term captures precisely the basic insight of the permanent income hypothesis that agents should consume a constant fraction of the expected future discounted wealth, given a constant real interest rate equal to $\beta^{-1} - 1$. The second term arises from the assumption of a time-varying real interest rate, and represents deviations from this constant real rate due to either variation in the nominal interest rate or inflation. The final term results from allowing stochastic disturbances to the economy.

conditions relevant to marginal costs into the indefinite future when deciding the current price of its output. Future profits are also discounted at the rate β which equals the inverse of the steady-state gross real interest rate.[7] [8]

Under the rational expectations assumption, the structural relations (1) and (2) simplify to yield

$$x_t = E_t x_{t+1} - \sigma(i_t - E_t \pi_{t+1}) + r_t \quad (3)$$

$$\pi_t = \kappa x_t + \beta E_t \pi_{t+1} + u_t \quad (4)$$

with all remaining notation as before and all parameters taking identical values. Importantly, these relations, with E_t replaced by $\hat{E}_t$, do not describe optimal behavior under the maintained microfoundations and non-rational expectations assumption. This is the central methodological contribution of Preston (2003): when agents face multi-period decision problems under learning dynamics, long-horizon expectations matter.[9] Unlike rational expectations, expectations under learning dynamics are not a fixed point of the equilibrium solution.

It is clear that learning has important implications for aggregate economic dynamics: with subjective expectations agents optimally require long-horizon expectations of macroeconomic conditions into the indefinite future. The presence of these expectational variables is important for the study of learning dynamics, as expectations represent an important source of instability. Indeed, the analysis shows that these additional expectation terms provide a stronger case for preferring decision procedures for monetary policy that are based on the price level than does an analysis of learning based on equations (3) and (4) with E_t replaced by $\hat{E}_t$.[10]

[7] See Woodford(2002) for details of the log-linearization and steady state.

[8] This expression has been modified slightly from the analysis of Preston (2003) by allowing for a cost-push shock. This is done to ensure a non-trivial stabilization problem for the monetary authority when the design of optimal monetary policy rules is considered.

[9] See section 6 of Preston (2003) for a detailed discussion of the advantages of a learning procedure based on long-horizon forecasts relative to the more standard approach of a procedure based on the Euler equation (3).

[10] See Bullard and Mitra (2002), Evans and Honkapohja (2002), Honkapohja and Mitra (2003) and Honkapohja and Mitra (2004) for analyses of this type.

2.2 Adaptive Learning

Following much of the recent literature on learning in macroeconomics, this paper assumes agents learn adaptively, using a recursive least-squares algorithm. This allows application of standard convergence, or E-Stability results, outlined in Evans and Honkapohja (2001). Appendix A.1 outlines the notion of E-Stability in the context of this model and the monetary policy considered below. The crucial idea of E-Stability is that it provides conditions under which, if agents make small forecasting errors relative to rational expectations, their learning behavior corrects these errors over time and ensures convergence to the rational expectations dynamics.

Agents are assumed to have identical beliefs, though they do not understand this to be true as they have no knowledge of the tastes and beliefs of other agents, and to construct forecasts using an econometric model that uses as regressors variables that appear in the minimum-state-variable solution to the associated rational expectations problem. For example, suppose that monetary policy is specified as a relation of the form

$$i_t = \psi_x x_{t-1} + \psi_u u_t + \psi_r r_t$$

where x_{t-1} is the lagged output gap.[11] It follows immediately from standard analysis that there exists a rational expectations equilibrium that is linear in the variables $\{x_{t-1}, u_t, r_t\}$. Agents therefore estimate the linear model

$$z_t = a_t + b_t \cdot z_{t-1} + c_t \cdot u_t + d_t \cdot r_t + \epsilon_t \tag{5}$$

where $z_t = (\pi_t, x_t, i_t)'$, ϵ_t is the usual error-vector term, $\{a_t, b_t, c_t, d_t\}$ are parameters to be estimated of the form

$$a_t = \begin{bmatrix} a_{\pi,t} \\ a_{x,t} \\ a_{i,t} \end{bmatrix}; \; c_t = \begin{bmatrix} c_{\pi,t} \\ c_{x,t} \\ c_{i,t} \end{bmatrix}; \; d_t = \begin{bmatrix} d_{\pi,t} \\ d_{x,t} \\ d_{i,t} \end{bmatrix}$$

[11] For the optimal policies considered in this paper, the reduced-form dynamics of the nominal interest rate will generally be of this form.

and

$$b_t \equiv \begin{bmatrix} 0 & b_{\pi,t} & 0 \\ 0 & b_{x,t} & 0 \\ 0 & b_{i,t} & 0 \end{bmatrix}.$$

The estimation procedure makes use of the entire history of available data in period t, $\{1, z_t, u_t, r_t\}_0^{t-1}$. As additional data become available, agents update their estimates of the coefficients (a_t, b_t, c_t, d_t). This is neatly represented as the recursive least squares formulation

$$\phi_t = \phi_{t-1} + t^{-1} R_t^{-1} w_{t-1} (z_t - \phi_{t-1}' w_{t-1}) \quad (6)$$

$$R_t = R_{t-1} + t^{-1} (w_{t-1} w_{t-1}' - R_{t-1}) \quad (7)$$

where the first equation describes how the forecast coefficients, $\phi_t = (a_t', b_{\pi,t}, b_{x,t}, b_{i,t}, c_t', d_t')'$, are updated with each new data point and the second the evolution of the matrix of second moments of the appropriately stacked regressors $w_t \equiv \{1, x_{t-1}, u_t, r_t\}_0^{t-1}$. For the remainder of this paper u_t and r_t are assumed to be AR(1) processes

$$u_t = \gamma u_{t-1} + \varepsilon_{u,t}$$

$$r_t = \rho r_{t-1} + \varepsilon_{r,t}$$

with known parameters $0 < \gamma < 1$ and $0 < \rho < 1$ and $\{\varepsilon_{u,t}, \varepsilon_{r,t}\}$ uncorrelated, bounded, i.i.d. disturbance processes. The assumption that the autoregressive parameters are known is made for algebraic convenience and is not important to the conclusions of this paper. Given homogeneity of beliefs, average forecasts can then be constructed by solving (5) backward and taking expectations to give

$$\begin{aligned} \hat{E}_t z_T = {} & (I_3 - b_t)^{-1} (I_3 - b_t^{T-t}) a_t + b_t^{T-t} z_t + \gamma u_t (\gamma I_3 - b_t)^{-1} \left(\gamma^{T-t} I_3 - b_t^{T-t}\right) c_t \\ & + \rho r_t (\rho I_3 - b_t)^{-1} \left(\rho^{T-t} I_3 - b_t^{T-t}\right) d_t \end{aligned} \quad (8)$$

for $T \geq t$, where I_3 is a (3×3) identity matrix.

To summarize, the model of the macroeconomy comprises: an aggregate demand equation, (1), a Phillips curve, (2), and the forecasting system given by (5), (6) and (7), where the latter three will vary according to the adopted econometric model of agents.

3 Optimal Monetary Policy

The central objective of this paper is to elucidate the appropriate role of internal central bank forecasts and external private forecasts in the implementation of optimal monetary policy. This section outlines the optimal commitment problem under *rational expectations* and discusses the notion of "optimality from the timeless perspective" proposed by Woodford (1999), which serves to restrict the class of admissible policies to those that are time consistent. Subsequent sections then consider a number of decision procedures that are consistent with implementing optimal policy under rational expectations, and asks whether learning dynamics present ground to prefer one particular approach over another – that is, are any of the proposed decision procedures to be preferred from the point of view of eliminating instability from self-fulfilling expectations and therefore ensuring learnability of rational expectations equilibrium?[12]

The monetary authority is assumed to minimize the loss function

$$W = E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} L_t \tag{9}$$

where $0 < \beta < 1$ corresponds to the household's discount factor, and the period loss is given as

$$L_t = \pi_t^2 + \lambda x_t^2$$

for some weight $\lambda > 0$. Thus the central bank wishes to stabilize variation in inflation and the output gap, and λ determines the relative importance of these stabilization objectives. Woodford (2003, chap. 6) shows a quadratic loss function of this form corresponds to a second-order approximation to the private-sector utility function assumed in the microfoundations underpinning this paper and, moreover, that $\lambda = \kappa/\theta$, where θ is the is the elasticity of substitution across differentiated goods in the underlying microfoundations. The analysis of later sections will impose this restriction.

[12] Note that these policies are clearly not optimal while agents are learning. While devising such policies would clearly be of interest, the focus of the present paper is the robustness of policies prescribed under rational expectations to deviations from the usual assumptions made in a rational expectations analysis.

The central bank's optimal commitment problem is to maximize (9) the structural relation (4) implied by private-sector optimization under rational expectations.[13] This paper further restricts the class of admissible policies to those that are optimal from the so-called timeless perspective of Woodford (1999). Giannoni and Woodford (2002) and Woodford (2003, chap. 7) demonstrate that a time invariant optimal commitment can be arranged by having the central bank act subject to the additional requirement that the initial evolution of the economy coincides with the evolution associated with the policy. Consider minimizing the loss (9) subject to (4) and the additional constraint that $\pi_{t_0} = \bar{\pi}_{t_0}$ where

$$\bar{\pi}_{t_0} = (1-\mu)\frac{\lambda}{\kappa}x_{t-1} + \frac{\mu}{1-\beta\mu\gamma}u_t.$$

Thus, the central bank must bring about this initial evolution when implementing the optimal policy. The reasons for this precise form will become clear. The Lagrangian, which is to be minimized by choice of $\{\pi_t, x_t\}$, can be written as

$$E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\{\frac{1}{2}L_t - \Xi_t\left[\kappa x_t + \beta\pi_{t+1} + u_t - \pi_t\right]\} - \Xi_{t_0-1}\pi_0$$

where Ξ_{t_0-1} is the multiplier on the constraint $\pi_{t_0} = \bar{\pi}_{t_0}$. Differentiating with respect to π_t and x_t gives the first-order conditions

$$\pi_t + \Xi_t - \Xi_{t-1} = 0 \qquad (10)$$

$$\lambda x_t - \kappa\Xi_t = 0 \qquad (11)$$

for $t \geq t_0$. It is immediate that the these first-order conditions are time invariant, in the sense that they hold in all periods of the proposed commitment, and therefore characterize the optimal evolution of the economy under the timeless perspective. Absent the constraint on the initial evolution of inflation, these optimality conditions would hold with the additional requirement that $\Xi_{t_0-1} = 0$ – hence giving a time dependent policy.

[13] Since the loss function is independent of the nominal interest rate, any optimal paths for the inflation rate and output gap satisfying the Phillips curve will necessarily satisfy the aggregate demand relation as this constraint never binds. Hence we need only consider the constraint imposed by the Phillips curve. Given a solution for the optimal paths of inflation and the output gap, the optimal path for the nominal interest rate can be determined from the aggregate demand relation.

The optimal plan is then characterized by a set of bounded processes $\{\pi_t, x_t, \Xi_t\}$ for dates $t \geq t_0$ that satisfy the first-order conditions (10) and (11). Substitution of these conditions into the structural relation (4) gives the second-order difference equation

$$E_t \Xi_{t+1} - \left(\frac{\lambda + \kappa^2 + \lambda\beta}{\beta\lambda} \right) \Xi_t + \beta^{-1} \Xi_{t-1} = -\beta^{-1} u_t$$

which is easily shown to have roots satisfying $0 < \mu < 1 < 1/(\mu\beta)$ and therefore satisfies the conditions for a unique bounded rational expectations solution.[14] Standard methods show that the optimal state-contingent paths of $\{\pi_t, x_t, i_t\}$ are given by the following relations:

$$\pi_t = (1-\mu)\frac{\lambda}{\kappa} x_{t-1} + \frac{\mu}{1-\beta\mu\gamma} u_t \tag{12}$$

$$x_t = \mu x_{t-1} - \frac{\kappa}{\lambda} \cdot \frac{\mu}{1-\beta\mu\gamma} u_t \tag{13}$$

and

$$i_t = \frac{\sigma\lambda - \kappa}{\sigma\kappa}(1-\mu)\mu \cdot x_{t-1} + \frac{\sigma\lambda - \kappa}{\sigma\lambda} \cdot \frac{\mu(\mu+\gamma-1)}{1-\beta\mu\gamma} \cdot u_t + \frac{1}{\sigma} r_t^n. \tag{14}$$

These equations completely characterize the solution of the optimal monetary policy problem from the timeless perspective under the rational expectations assumption. Now note that

$$\pi_t = -\frac{\lambda}{\kappa}(x_t - x_{t-1}) \tag{15}$$

fully characterizes this optimal equilibrium in the sense that (12) – (14) hold for all $t \geq t_0$ if and only if (15) does. That the former implies the latter follows directly from manipulation of (12) and (13).[15]

There are several points to note. First, the bounded solution for the path of inflation exactly coincides with the constraint that was imposed on the initial evolution of this variable. Thus, the constraint required for optimality from the timeless perspective is characterized by a self-consistency property – it requires the central bank to ensure that the initial evolution of the economy coincides with the evolution of the economy associated with the policy. Second, the optimal solution exhibits history dependence as evidenced by the presence of the state

[14] See Blanchard and Kahn (1980) for conditions for unique bounded rational expectations solutions.

[15] See Woodford (2003, chap. 7) for a proof.

variable, x_{t-1}. This reflects the fact that the central bank, in committing to behave in a particular way in the future, optimally ties these promised actions to current decisions – subsequent actions then fulfill past promises. Finally note that the cost-push shock u_t clearly makes the stabilization problem non-trivial. Given an inflationary disturbance the central bank optimally brings about a contraction in real activity. In the absence of this shock, the optimal policy (from the timeless perspective) would be to completely stabilize both output and inflation.

Suppose instead that we constrain the initial choice of the inflation rate by the condition

$$p_{t_0} - \bar{k} = \mu \left(p_{t_0-1} - \bar{k}\right) + \frac{\mu}{1-\beta\mu\gamma} u_t.$$

for some constant $\bar{k}$. Then a similar analysis implies the state-contingent paths

$$p_t - \bar{k} = \mu \left(p_{t-1} - \bar{k}\right) + \frac{\mu}{1-\beta\mu\gamma} u_t \tag{16}$$

$$x_t = -\mu \cdot \frac{\kappa}{\lambda} \left(p_{t-1} - \bar{k}\right) - \frac{\kappa}{\lambda} \cdot \frac{\mu}{1-\beta\mu\gamma} u_t \tag{17}$$

and

$$i_t = -\frac{\sigma\lambda - \kappa}{\sigma\lambda} (1-\mu)\,\mu \cdot \left(p_{t-1} - \bar{k}\right) + \frac{\sigma\lambda - \kappa}{\sigma\lambda} \cdot \frac{\mu(\mu+\gamma-1)}{1-\beta\mu\gamma} \cdot u_t + \frac{1}{\sigma} r_t^n. \tag{18}$$

It can then be shown that the relation

$$p_t = \bar{k} - \frac{\lambda}{\kappa} x_t \tag{19}$$

characterizes this alternative timelessly optimal equilibrium in the sense that (16) – (18) hold for all $t \geq t_0$ if and only if (19) does. We will interpret a commitment by the central bank to implement the restrictions (15) and (19) as inflation and price-level targeting respectively. It is important to observe that a commitment to (15) implies a different state-contingent evolution than does a commitment to (19), except in one particular choice of $\bar{k}$ that depends on the price level in period $t_0 - 1$. However, the resulting state-contingent evolutions in each case involve the same equilibrium responses to shocks that occur in period t_0 or later, and the same long-run average values of all variables.[16] To keep matters simple we set the constant, $\bar{k}$, equal to zero.[17]

4 Specific Targeting Rules

The recent literature on optimal monetary policy (see Svensson and Woodford (2002), Svensson (2003), Giannoni and Woodford (2002) and Giannoni and Woodford (2002)) proposes implementing the desired optimal equilibrium by use of specific targeting rules that emerge from the optimality conditions of the commitment problem. The above discussion shows the consolidated first-order condition is given by the relation (15), which can be equivalently stated in terms of the price level as (19). If the monetary authority can arrange for either of these relations to hold for $t \geq t_0$ then the resulting equilibrium will be consistent with implementing a timelessly optimal commitment equilibrium.

An important property of targeting rules is that their implementation requires the central bank to make use of a fully specified model of the economy. Since the inflation target rule (15) is defined as a restriction on the current values of the output gap and inflation rate, whose values themselves depend on the current instrument setting, evaluating whether the criterion is satisfied requires determining the current realizations of these variables given the instrument choice – in turn requiring the central bank to make use of a model of the economy.

However, the presence of learning dynamics raises the possibility that the central bank makes use of a number of different models of the economy, depending on its knowledge of private agents' behavior. The remainder of this paper therefore considers the implementation of the inflation-rate and price-level targeting rules under three alternative assumptions on the central bank's knowledge. First, the central bank projects the evolution of the economy under the assumption that agents have rational expectations and observing only lagged aggregate variables and primitive disturbances. Since the rational expectations solution can be expressed in terms of primitive disturbances, this gives rise to an instrument setting that is independent

[16] They differ in a transitory, deterministic component of the solution that will depend on the initial conditions – that is, on the value of $\bar{k}$.

[17] This is not important for the learning analysis.

of private-sector learning behavior. Second, the bank central projects the evolution of the economy using complete knowledge of agents' subjective expectations behavior, including accurate observations on all long-horizon forecasts relevant to aggregate dynamics. In contrast to the first decision procedure, this approach implies a reaction function for the nominal interest rate which depends on all private forecasts that are relevant for aggregate dynamics. Finally, the analysis considers a compromise on the extreme informational assumptions of these two decision procedures, and investigates an instrument rule that requires the central bank to respond to one-period-ahead private-sector expectations.

We now turn to the inflation-based targeting rule under the first two decision procedures. Section 5 provides an analogous treatment of the price-level targeting rule. Section 6 then turns to decision procedures based on instrument rules, which are considered separately as they are examples of target criteria that themselves depend on forecasts, and therefore are distinct to the inflation and price-level targeting rules laid out in the previous sections.

For the inflation targeting rule, all decision procedures are consistent with implementing the optimal equilibrium under rational expectations described in section 3. Since the equilibrium paths of $\{\pi_t, x_t, i_t\}$ are given by (12), (13) and (14) and are linear in the variables $\{x_{t-1}, u_t, r_t\}$, agents adopt an econometric model of the form (5). Given current estimates of the forecast parameters, long-horizon forecasts of the future path of inflation, the output gap and the nominal interest rate can be determined by (8).

4.1 Projecting under Rational Expectations

Suppose that the central bank does not know the true model of the economy, thinking instead that agents have rational expectations. Furthermore, assume the central bank only observes lagged aggregate variables and primitive disturbances. To implement the inflation targeting rule, the central bank projects the evolution of the economy using the structural relations (3) and (4) and decides its instrument setting to ensure satisfaction of the target criterion (15). Under the assumption of rational expectations, section 3 demonstrates that the path of the nominal interest rate that is consistent with this target criterion being satisfied is given by (14). This formulation of the central bank's decision procedure therefore determines the instrument choice in any period as a linear function of the state variables $\{x_{t-1}, u_t, r_t\}$ and is independent of agents' learning behavior.

Proposition 1 *If the monetary authority projects the evolution of the economy under the rational expectations assumption using (3) and (4), and implements the inflation targeting rule (15), the implied reaction function is given by (14) and the economy is unstable under learning dynamics if* $\theta > \sigma$*.*

Appendix A.2 sketches the learning analysis. E-Stability requires 12 restrictions on model parameters to be satisfied – three pertaining to the learning of the model's constant coefficients and three pertaining to each set of coefficients on the three state variables. Consider the restrictions arising from learning the constant dynamics.[18] The Jacobian matrix associated with the E-Stability mapping for the constant dynamics has the characteristic equation

$$P(h) = (h+1)(h^2 + A_1 h + A_0)$$

where A_i are composites of model primitives. Therefore one eigenvalue is equal to negative unity and for the remaining eigenvalues to have negative real parts, the restrictions $A_1, A_0 > 0$ must be satisfied.

The latter object can be shown to be given by

$$A_0 = -\frac{\theta\Gamma_1 + \Gamma_2(1-\beta\mu)}{(1-\beta)(1-\alpha\beta)(1-\mu)(1-\alpha\beta\mu)(\theta-\sigma)}$$

where

$$\begin{aligned}\Gamma_1 &= \mu(1-\beta)(1-\alpha\beta\mu) + \kappa\sigma \\ \Gamma_2 &= (1-\alpha\beta\mu) - \alpha\beta(1-\beta\mu)\end{aligned}$$

which are both positive under the maintained parameter assumptions. It follows that if $\theta > \sigma$ then $A_0 < 0$, violating the requirements for E-Stability. Is the restriction $\theta > \sigma$ likely to be

[18] An important property of all models considered in this paper is that the conditions for stability arising from learning the set of constants or any set of coefficients on a given state variable are independent. The stability properties can then be established by considering the dynamics of each set of coefficients in turn.

satisfied in practice? Rotemberg and Woodford (1999) estimate values for θ and σ equal to 7.88 and 6.25; and Chari, Kehoe, and McGrattan (2000) use a value for the elasticity of demand equal to 10 based on evidence from a number of papers.[19] Since the Rotemburg-Woodford estimate of the intertemporal elasticity of substitution is perhaps controversial, with much of the literature preferring somewhat lower values, it seems likely that the stated restriction will encompass the empirically relevant region of the parameter space. We therefore conclude that a central bank decision procedure based on the mistaken assumption that agents have rational expectations is likely to lead to instability under learning dynamics. For $\theta > \sigma$, the inflation targeting rule allows the propagation of self-fulfilling expectations.

4.2 Projecting with the True Model

Now suppose the central bank correctly understands agents' behavior, accurately observes all long-horizon forecasts and attempts to implement the inflation targeting rule (15). The monetary authority therefore understands the true structural relations of the economy are given by (1) and (2) and can guarantee that the target criterion is satisfied by adjusting interest rates according to the following implied reaction function

$$i_t = -\frac{1}{\sigma} \cdot \hat{x}_t + \frac{1}{\sigma} \cdot \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta) x_{T+1} - \sigma(\beta \cdot i_{T+1} - \pi_{T+1}) + r_T \right] \tag{20}$$

where

$$\hat{x}_t = \frac{\lambda}{\lambda + \kappa^2} \cdot x_{t-1} - \frac{\kappa}{\lambda + \kappa^2} \cdot \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left[\kappa\alpha\beta \cdot x_{T+1} + (1-\alpha)\beta \cdot \pi_{T+1} + u_T \right]. \tag{21}$$

The first relation is derived by solving the aggregate demand relation (1) for i_t and the second determines the value of the output gap that jointly satisfies the Phillips curve (2) and the target criterion (15). The presence of long-horizon expectations makes clear that the central bank must necessarily project the evolution of the economy to implement this rule. Moreover, the form of the implied reaction function ensures that the target criterion is satisfied regardless of expectations held by private agents. This approach to implementing

[19] Note that since the mark-up in this model is given by $\theta/(1-\theta)$, θ cannot be too small, else imply an implausibly large mark-up.

optimal monetary policy under learning dynamics was proposed by Evans and Honkapohja (2002) in the context of a model of the monetary transmission mechanism where only one-period-ahead forecasts matter.

Proposition 2 *If the monetary authority correctly understands the structural equations (1) and (2), and is charged with implementing the inflation targeting rule (15) then the economy is stable under learning dynamics for all parameter values.*

The proof is contained in the appendix. Proposition 2 demonstrates that it is possible to implement the optimal commitment equilibrium under learning dynamics using a decision procedure based on an inflation targeting rule. Commitment to this rule ensures that agents ultimately learn the rational expectations dynamics associated with this rule, and this result holds for all parameters satisfying maintained model assumptions. The inflation targeting rule under correct knowledge of the aggregate dynamics implies the reaction function (20) which has two important properties – it ensures satisfaction of (15) and it is an implication of inverting the aggregate demand relation. As such, the reaction function requires interest rates to be adjusted in response to all long-horizon forecasts of agents that are relevant determinants of aggregate dynamics. The decision procedure therefore ensures that the target criterion is satisfied each period, *whatever* the beliefs held by agents about the future evolution of the economy.

5 Price-level Target Rule

The previous section considers two monetary policies that are equivalent in terms of the rational expectations equilibrium that they imply, but have different consequences under learning dynamics. Since the decision procedure that successfully implements the optimal equilibrium requires correct understanding of private behavior, and this might reasonably be criticized as an overly strong assumption, we now consider a parallel set of decision procedures based on the price-level formulation of the consolidated first order condition, (19), with a view to identifying policies that have desirable stabilization properties under learning dynamics but are less informationally demanding in their implementation.

To facilitate the analysis, the structural relations (1) and (2) can be written in terms of the price level to give

$$x_t = -\sigma \left(i_t + p_t\right) + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[\left(1-\beta\right) x_{T+1} - \sigma \beta i_{T+1} + \sigma \left(1-\beta\right) p_{T+1} + r_T\right] \tag{22}$$

and

$$p_t = \frac{1}{\nu} p_{t-1} + \frac{\kappa}{\nu} x_t + \frac{1}{\nu} \hat{E}_t \sum_{T=t}^{\infty} \left(\alpha\beta\right)^{T-t} \left[\kappa \alpha \beta x_{T+1} + \beta \left(1-\alpha\right)\left(1-\alpha\beta\right) p_{T+1} + u_T\right] \tag{23}$$

where $\nu = 1 + (1-\alpha)\beta$. Thus the evolution of the output gap and the price level are now determined by long-horizon forecasts of these same variables and also the path of the nominal interest rate and disturbance processes.

All decision procedures considered in this section imply the same rational expectations equilibrium paths for $\{p_t, x_t, i_t\}$, given by (16) – (18), which are linear in the state variables $\{p_{t-1}, u_t, r_t\}$. Agents are assumed to construct forecasts using an econometric model that uses as regressors variables that appear in the minimum-state-variable solution to the associated rational expectations problem. Therefore they estimate the model:

$$z_t = a_t + b_t \cdot z_{t-1} + c_t \cdot u_t + d_t \cdot r_t + \epsilon_t \tag{24}$$

where $z_t = (p_t, x_t, i_t)'$ (note the inclusion of the price level), ϵ_t is the usual error term, $\{a_t, c_t, d_t\}$ are (3×1) coefficient vectors to be estimated and

$$b_t \equiv \begin{bmatrix} b_{p,t} & 0 & 0 \\ b_{x,t} & 0 & 0 \\ b_{i,t} & 0 & 0 \end{bmatrix}.$$

Forecasts of the future path of the price level, the output gap and the nominal interest rate can then be constructed using (8) given the appropriate definition of the vector z_t and coefficient matrix b_t.

5.1 Projecting under rational expectations

Suppose the central bank attempts to implement the price-level target criterion (19) under the mistaken assumption that private agents have rational expectations. It therefore believes the model of the economy to be given by the relations

$$x_t = E_t x_{t+1} - \sigma \left(i_t - E_t p_{t+1} + p_t\right) + r_t \tag{25}$$

and

$$p_t = \frac{1}{1+\beta}\left[p_{t-1} + \kappa x_t + \beta E_t p_{t+1} + u_t\right] \tag{26}$$

which follow directly from (3) and (4) when written in terms of the price level. Given the target criterion, this system of equations can be solved for the implied reaction function of the central bank's instrument setting to give (18). This reaction function will have the property that if agents did in fact have rational expectations, the target criterion, and therefore the optimal equilibrium, would be successfully implemented.

Proposition 3 *If the monetary authority projects the evolution of the economy under the rational expectations assumption using (3) and (4), and implements the price-level targeting rule (19), then the implied reaction function is given by (18) and the economy is stable under learning dynamics for many empirically reasonable parameter values.*

The learning analysis for this proposition is virtually identical to the discussion in appendix A.2 for proposition 1, once the state vector z_t is appropriately defined to include the price level rather than the inflation rate. However, analytical results are not so easily obtained, so we appeal to a numerical analysis. E-Stability requires 12 restrictions on model parameters to be satisfied – three pertaining to the learning of the model's constant coefficients and three pertaining to each set of coefficients on the three state variables. Consider the restrictions arising from learning the constant dynamics. The Jacobian matrix associated with the E-Stability mapping has the characteristic equation

$$P\left(h\right) = \left(h+1\right)\left(h^2 + A_1 h + A_0\right)$$

where A_i are composites of model primitives. Therefore one eigenvalue is equal to negative unity and, for the remaining eigenvalues to have negative real parts, the restrictions $A_1, A_0 > 0$ must be satisfied.

To gain insight into these restrictions, we calibrate the benchmark parameter values using the estimates of Rotemberg and Woodford (1999): α, β, θ, γ and ρ are taken to be 0.66, 0.99, 7.88, 0.35, 0.35. We then consider the regions for instability in the $\kappa - \sigma$ plane taking (κ, σ) pairs with values $(0, 1]$ and $(0, 7]$ respectively. These intervals are chosen to encompass the range of values for which the empirical literature has provided some evidence. For these parameter configurations, the restrictions $A_1, A_0 > 0$ can be shown to be satisfied for the constant dynamics. Moreover, the E-Stability conditions that arise from learning the true rational expectations coefficients on the lagged output gap and the coefficients on the disturbance processes (u_t, r_t) are similarly satisfied for these parameter configurations. These results suggest the price-level targeting rule to display robustness to learning dynamics – the decision procedure is consistent with implementing the desired equilibrium when the central bank is not cognizant of the agents' learning behavior.

Comparing this result with proposition 1 provides ground to prefer a price-level targeting rule. If the central bank adopts a decision procedure that projects the evolution of the economy under the assumption of rational expectations, a price-level targeting rule is robust to learning dynamics over the region of the parameter space considered. In contrast, the inflation targeting rule leads to instability for all such parameter configurations, since the calibration assumes $\theta > \sigma$. By responding to the lagged price level in the former case, the central bank is better able to restrain inflationary expectations.

5.2 Projecting with the True Model

Now suppose the central bank attempts to implement the price-level target criterion given correct knowledge of the structural relations (22) and (23). Analogous reasoning to the previous section provides the implicit reaction function

$$i_t = \frac{\sigma\lambda - \kappa}{\sigma\kappa}\hat{x}_t + \frac{1}{\sigma}\hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta) x_{T+1} - \sigma\beta i_{T+1} + \sigma (1-\beta) p_{T+1} + r_T\right] \qquad (27)$$

where

$$\hat{x}_t = \Gamma \cdot p_{t-1} + \Gamma \cdot \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left[\kappa\alpha\beta x_{T+1} + \beta (1-\alpha) (1-\alpha\beta) p_{T+1} + u_T\right]$$

and

$$\Gamma = -\frac{\kappa}{\kappa^2 + \lambda[1 + (1-\alpha)\beta]}.$$

Again the first relation is derived by inverting the aggregate demand relations to solve for the current instrument setting, i_t, while the latter determines the value of the output gap that jointly satisfies the Phillips curve (23) and the target criterion (37).

Proposition 4 *If the monetary authority correctly understands the structural equations (1) and (2), and is charged with implementing the price-level targeting rule (19) then the economy is stable under learning dynamics for all parameter values.*

The proof is contained in the appendix. Proposition 4 demonstrates that it is possible to implement the optimal commitment equilibrium under learning dynamics using the target criterion (19), when the central bank correctly understands private-sector behavior. This is not particularly surprising given the close connection between the consolidated first order condition and the price-level target criterion. As in Proposition 2, the implied reaction function (27) has the property that it ensures that the target criterion (19) is satisfied, regardless of the expectations that private agents hold about the future evolution of the economy.

6 Instrument Rule-Based Decision Procedures

Propositions 2 and 4 assume that the central bank understands agents' behavior, and, therefore, the true structural relations of the economy, and requires the use of observed private forecasts of general macroeconomic conditions into the indefinite future. Since it is often argued that the central bank does not possess a significant informational advantage relative to the private sector and that reliable long-horizon forecasts of the type required to implement the inflation targeting rule may not be available, it is of interest to ask whether a less informationally demanding policy might be available that is nonetheless consistent with implementing the optimal monetary policy.

6.1 Instrument Rule with One-Period-Ahead Inflation Forecasts

Consider then an instrument rule of the form

$$i_t = \frac{1}{\sigma}\left[\hat{E}_t x_{t+1} - \frac{\lambda}{\lambda+\kappa^2}x_{t-1} + \left(\frac{\beta\kappa}{\lambda+\kappa^2}+\sigma\right)\hat{E}_t\pi_{t+1} + \frac{\kappa}{\lambda+\kappa^2}u_t + r_t^n\right]. \qquad (28)$$

This instrument rule coincides with the reaction function proposed by Evans and Honkapohja (2002) to implement optimal policy in an economy given by

$$x_t = \hat{E}_t x_{t+1} - \sigma(i_t - \hat{E}_t \pi_{t+1}) + r_t \tag{29}$$

$$\pi_t = \kappa x_t + \beta \hat{E}_t \pi_{t+1} + u_t. \tag{30}$$

To derive the posited rule, combine (15) and (30) to solve for the output gap as

$$x_t = \frac{\lambda}{\kappa^2 + \lambda} x_{t-1} - \frac{\kappa}{\kappa^2 + \lambda} \left(\beta \hat{E}_t \pi_{t+1} + u_t \right)$$

which on substitution into (29) and solving for i_t gives the desired rule.[20] The approach of Evans and Honkapohja (2002) is properly interpreted as one that ensures the target criterion (15) holds in an economy where private agents have the learning rules (29) and (30).

In the current setting, the rule is better motivated as the desired instrument setting to implement the target criterion when a central bank mistakenly assumes that the economy is in a rational expectations equilibrium, though not necessarily the minimum-state-variable rational expectations equilibrium. That is, it believes that the Euler equations (29) and (30) are valid (as in a rational expectations analysis) and chooses to respond to observed forecasts. This interpretation facilitates answering the question of whether it is sufficient for learnability for the central bank to act on the assumption that some rational expectations equilibrium will be realized, as long as the decision procedure allows for the possibility of departures from the minimum-state-variable rational expectations equilibrium (note that if $\hat{E}_t x_{t+1}$ and $\hat{E}_t \pi_{t+1}$ were evaluated according to the minimum-state-variable rational expectations using the relations (12) and (13) as in proposition 1, we would obtain the reaction function (14)).[21] An analysis of the kind considered by Evans and Honkapohja (2002) based on relations (29)

[20] Note also that (28) can itself be properly interpreted as a targeting rule. While this criterion depends explicitly on observed forecasts and the interest rate, it does specify a particular relationship between the instrument setting, expectations of future macroeconomics conditions, the lagged output gap and the current disturbances. The former two variables are only implicitly defined and therefore requires the central bank to make use of a complete economic model in implementing this targeting criterion. It is also another example of a robustly optimal rule (in the sense of Giannoni and Woodford (2002)), insofar as as it implements the optimal equilibrim under rational expectations regardless of the statistical properties of the disturbances. While it does make explicit reference to the disturbances, unlike rule (15) and (19), it only refers to the extent to which the two model equations have been shifted, without making any further assumption about the kind of shocks being referred to.

[21] Since policy rules that respond only to exogenous disturbances can be shown to always result in self-

and (30) would answer in the affirmative – to avoid the instability that results from reaction functions of the type (14) it is sufficient to adopt a rule of the form (28). This is not the case in the context of the model of this paper.

Proposition 5 *Suppose the economy is given by the structural equations (1) and (2). If the central bank sets nominal interest rates according to the rule (28) the economy is not stable under learning dynamics for many reasonable parameter configurations.*

Applying the learning analysis laid out in appendix A.1 shows that for all (κ, σ) pairs, with each taking values on $(0, 1]$ and $(0, 7]$ respectively, and all other parameters taking benchmark parameter values, the conditions for E-Stability are violated. It is immediate then that the proposed specific targeting rule fails to be robust to our non-rational expectations assumption for many empirically reasonable parameter values. When the central bank attempts to implement the instrument rule (28) we find the economy to be frequently prone to self-fulfilling expectations.[22]

The critical difference between this result and proposition 2 is the manner in which monetary policy accommodates private-sector expectations. The inflation targeting rule under correct knowledge of the aggregate dynamics implies the reaction function (20) which has two important properties – it ensures satisfaction of (15) and it is an implication of inverting the aggregate demand relation. As such, the reaction function requires interest rates to be adjusted in response to all long-horizon forecasts of agents that are relevant determinants of aggregate dynamics. The decision procedure ensures that the target criterion is satisfied each period, *whatever* the beliefs held by agents about the future evolution of the economy, and interest rates are therefore adjusted to offset any instability resulting from learning dynamics. This is in direct contrast to proposition 5. Use of the instrument rule (28) cannot

fulfilling expectations in this model, one might conjecture, analogously to the determinacy of rational expectations literature (see Sargent and Wallace (1975), McCallum (1983) and Woodford (2003, chap. 4)), that making the path of the nominal interest rate more dependent on the model's endogenous variables, as proposed by this rule, might assist the central bank in behaving out-of-equilibrium in a way that eliminates the possibility of self-fulfilling expectations.

[22] This rule is also an example of the forecast-based instrument rules discussed in detail in Preston (forthcoming). This paper examines whether it is desirable as a general principle of good policy for the central bank to respond to observed private forecasts, and finds that many such rules are susceptible to instability under learning dynamics.

guarantee the target criterion is satisfied and fails to offset instability induced by agents' learning. By responding only to one-period-ahead forecasts, the decision procedure fails to mitigate instability due to long-horizon expectations and in turn validates expectations held by the private sector. (Preston forthcoming) show these concerns also cast doubt on the usefulness of recently popular forecast-based instrument rules that posit the nominal interest rate to respond to forecasts of inflation and the output gap. These observations suggest that forecast-based decision procedures must induce a particular kind of dependency on private-sector forecasts so as to ensure that self-fulfilling expectations are not propagated. shows that these considerations cast doubt on the usefulness of a range of generalized Taylor rules

6.2 Instrument Rule with One-Period-Ahead Price-level Forecasts

Thus far, we have provided evidence that a price-level targeting rule might be preferable to the inflation targeting rule to implement the optimal equilibrium, since it is better able to eliminate instability arising from self-fulfilling expectations. Under an inflation targeting rule, projecting the evolution of the economy under the mistaken presumption of rational expectations leads to instability for reasonable parameter values, while there was no instability under a price-level targeting rule for these same values. In contrast, if the central bank projects the evolution of the economy under correct understanding of the structural relations, the possibility of self-fulfilling expectations is eliminated for both targeting rules. When we considered a rule that compromised on these extreme information assumptions, and posited the instrument setting to depend on one-period-ahead expectations of the inflation rate and the output gap, instability was still found to obtain. But is this the case under an equivalent rule expressed in terms of the price level?

Consider a central bank committed to the instrument rule

$$i_t = \psi_{lp} p_{t-1} + \psi_x \hat{E}_t x_{t+1} + \psi_p \hat{E}_t p_{t+1} + \psi_u u + \psi_r r \tag{31}$$

where

$$\psi_x = \psi_r = 1/\sigma; \qquad \psi_u = \psi_{lp}$$

and

$$\psi_{lp} = \frac{\kappa - \lambda\sigma}{(\kappa^2 + \lambda + \beta\lambda)\,\sigma}; \quad \psi_p = \frac{\beta\kappa + (\kappa^2 + \lambda)\sigma}{(\kappa^2 + \lambda + \beta\lambda)\,\sigma}.$$

Again this instrument rule requires the central bank to adjust the nominal interest rate in response to agents' one-period-ahead forecasts and is derived analogously to the instrument rule (28) using (29) and (30) in conjunction with (19). Under rational expectations, it is consistent with implementing the commitment equilibrium described by (16) – (18), and is equivalent to the policy (28) in the sense that they imply the same responses to disturbances, and give the same long-run average values of variables. However, these policies lead to different conclusions about their desirability under learning dynamics.

Proposition 6 *If the central bank sets nominal interest rates according to the instrument rule (31) the economy is stable under learning dynamics for many reasonable parameter configurations for which the policy rule (28) is not.*

Applying the learning analysis of appendix A.1 we again appeal to a graphical analysis, calibrating the model at the benchmark parameter values described earlier and considering E-stability for (κ, σ) pairs taking values $(0, 1]$ and $(0, 7]$ respectively. E-stability requires four sets of restrictions on model parameters – one pertaining to learning the model's constant coefficients and three pertaining to each of the three state variables, $\{p_{t-1}, u_t, r_t\}$. The learning dynamics of each set of coefficients has an associated characteristic equation of the form

$$P(h) = h^3 + A_2 h^2 + A_1 h + A_0$$

and E-Stability requires A_0, $A_1, A_4 > 0$, where $A_4 = -A_0 + A_1 A_2$.

Figure **??** plots two regions for which either all A_0, A_2 and A_4 are positive (the white region) or at least one of these 12 objects (that is, A_0, A_1, A_4 for the learning dynamics of each of the four sets of coefficients) are negative (the black region). It is clear that there are a range of parameter configurations for which the conditions for E-Stability are violated. Thus, the policy may well permit the propagation of self-fulfilling expectations. However, the assumed range of κ is large relative to most plausible empirical estimates. Most researchers believe κ to be small, taking values of the order $0.01 - 0.05$ (see McCallum and Nelson (2000)).

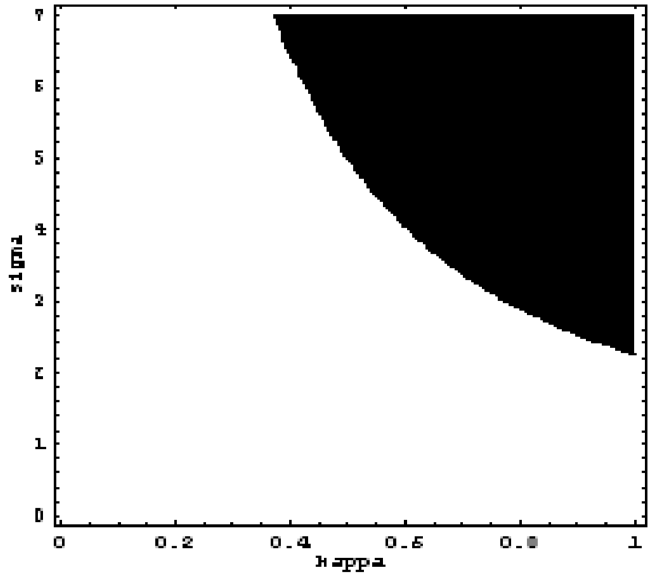

Figure 1: E-Stability Regions in $\sigma - \kappa$ space

It is clear then that if attention is restricted to this part of the parameter space, E-Stability obtains for all examined parameter configurations.

In contrast to the policy (28) that responds to forecasts of inflation, the price-level formulation is somewhat more robust to misspecification in the assumed expectations formation mechanism. The central bank can, for a range of empirically reasonable parameter values, implement the optimal commitment equilibrium irrespective of whether it knows agents to have rational expectations or not – the policy will nonetheless ensure agents ultimately learn the rational expectations dynamics. The difference between these two rules, in the case of learning dynamics, is that the price-level targeting rule specifies a different kind of subsequent behavior when one finds that (because the private sector does not behave as they were projected to do) one has failed to achieve the target criterion precisely. Thus the difference between the two rules is a different commitment as to how one will react to seeing that one has missed one's target. This suggests the central bank ought to cast is policy rule in terms of the price level to guard against instability from learning dynamics.

A final point is worthy of comment. Comparing propositions 2 and 6 makes clear that not all uses of private forecasts improve learnability of rational expectations equilibrium. Indeed, the decision procedure underlying the latter result, which utilizes information on one-period-ahead forecasts of private agents, induces learnability of rational expectations equilibrium for only a subset of the parameter values for which learnability obtains in Proposition 2 when only information on lagged aggregate variables and primitive disturbances are used. By making central bank behavior in addition to private agents' behavior respond to such forecasts the likelihood of instability is increased.

7 Conclusions

This paper applies the framework of Preston (2003) to understand how private-sector forecasts should be incorporated into the design of optimal monetary policy rules. The analysis finds that targeting rules can guarantee the successful implementation of the optimal equilibrium from the timeless perspective under learning dynamics if the central bank has correct knowledge of the true model of the economy – that is, it understands the exact nature of private-sector behavior and its implications for aggregate dynamics. In contrast, an inflation targeting rule fails to be robust to learning dynamics, in the sense that if the central bank implements the policy under the mistaken assumption that private agents have rational expectations, the policy leads to the propagation of self-fulfilling expectations for many empirically reasonable parameter values. However, this instability appears far less severe for price-level targeting rules for the parameter values considered.

It follows that, if the central bank nonetheless attempts to implement a targeting rule under the mistaken assumption that agents have rational expectations, it is best that the decision procedure be cast in terms of arranging for a desired path for the price level rather than the inflation rate. By anchoring policy in terms of the price level, the central bank can better restrain agents' expectations, and therefore eliminate the possibility of self-fulfilling expectations for many parameter values.

Importantly, not all uses of private forecasts in the conduct of monetary policy help to improve learnability. The inflation and price-level targeting rules in this paper require the central bank to use its information about private forecasts in order to offset the effects of those forecasts on the variables that it is targeting, just as it uses its information about other disturbances in order to offset the effects of those disturbances. But forecast-based instrument rules of the kind often discussed in the literature may instead strengthen the effects of private forecasts on the economy's evolution, by making the central bank behavior as well as private behavior respond to them; and in this case the central bank's behavior makes it easier for expectations to be a source of instability.

A Appendix

This appendix first outlines the general approach to analyzing learning dynamics in the context of the model of this framework. It then turns to sketching the proofs of the central results which are all applications of this general methodology. Since the algebra underpinning these results is at times tedious, it is largely omitted. Most calculations were performed in Mathematica.

A.1 Expectational Stability

Suppose monetary policy is conducted according to the rule

$$i_t = \psi_x x_{t-1} + \psi_u u_t + \psi_r r_t$$

where x_{t-1} is the lagged output gap. Standard analysis implies there exists a rational expectations equilibrium that is linear in the variables $\{x_{t-1}, u_t, r_t\}$. If agents know the form of the minimum-state-variable solution they estimate a linear model

$$z_t = a_t + b_t \cdot z_{t-1} + c_t \cdot u_t + d_t \cdot r_t + \epsilon_t \tag{32}$$

where $z_t = (\pi_t, x_t, i_t)'$, ϵ_t is the usual error term, $\{a_t, b_t, c_t, d_t\}$ are coefficient parameter vectors to be estimated. Relation (32) is called agents perceived law of motion. Forecasts can then be constructed by solving this model forward and taking expectations to give

$$\begin{aligned} \hat{E}_t z_T &= (I_3 - b_t)(I_3 - b_t^{T-t})a_t + b_t^{T-t} z_t + \gamma u_t (\gamma I_3 - b_t)^{-1} \left(\gamma^{T-t} I_3 - b_t^{T-t}\right) c_t \\ &\quad + \rho r_t (\rho I_3 - b_t)^{-1} \left(\rho^{T-t} I_3 - b_t^{T-t}\right) d_t \end{aligned} \tag{33}$$

for $T \geq t$. To obtain the actual law of motion, substitute (33) into the system of equations (1) and (2). Collecting like terms gives a general expression of the form

$$z_t = \bar{a}_t + \bar{b}_t x_{t-1} + \bar{c}_t u_t + \bar{d}_t r_t$$

where $\{\bar{a}_t, \bar{b}_t, \bar{c}_t, \bar{d}_t\}$ are functions of the current private forecast parameters $\{a_t, b_t, c_t, d_t\}$. Comparison with (32) makes clear that agents are estimating a misspecified model of the economy – agents assume a stationary model when in fact the true model has time varying coefficients. Leading this expression one period and taking expectations (rational) provides

$$E_t z_{t+1} = \bar{a}_t + \bar{b}_t x_t + \bar{c}_t \gamma u_t + \bar{d}_t \rho r_t$$

which describes the optimal rational forecast conditional on private-sector behavior. Taken together with (33) at $T = t + 1$ it defines a mapping that determines the optimal forecast coefficients given the current private-sector forecast parameters (a_t', b_t'), written as

$$T(a_t, b_t, c_t, d_t) = (\bar{a}_t, \bar{b}_t, \bar{c}_t, \bar{d}_t). \tag{34}$$

A rational expectations equilibrium (REE) is a fixed point of this mapping. For such REE, we are then interested in asking under what conditions does an economy with learning dynamics converge to this equilibrium. Using stochastic approximation methods, Evans and Honkapohja show that the conditions for convergence of the learning algorithm (6) and (7) are neatly characterized by the local stability properties of the associated ordinary differential equation

$$\frac{d}{d\tau}(a, b, c, d) = T(a, b, c, d) - (a, b, c, d), \tag{35}$$

where τ denotes "notional" time. The REE is said to be expectationally stable, or E-Stable, if this differential equation is locally stable in the neighborhood of the REE. From standard results for ordinary differential equations, a fixed point is locally asymptotically stable if all eigenvalues of the Jacobian matrix $D\left[T(a, b, c, d) - (a, b, c, d)\right]$ have negative real parts (where D denotes the differentiation operator and the Jacobian is understood to be evaluated at the rational expectations equilibrium of interest.) See Evans and Honkapohja (2001) for further details on expectational stability.

In the context of the above model, the Jacobian takes the general form:

$$T'(\phi) - I_{12} = \begin{bmatrix} A_1 - I_3 & A_5 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & A_2 - I_3 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & A_6 & A_3 - I_3 & \mathbf{0} \\ \mathbf{0} & A_7 & \mathbf{0} & A_4 - I_3 \end{bmatrix} \tag{36}$$

where $\phi = (a', b', c', d')'$, I_3 and I_{12} are identity matrices of stated dimension, $\mathbf{0}$ a (3×3) null matrix and A_i matrices of similar dimension whose elements are composites of model primitives. It is important to note that if the Jacobian is a function of the forecast parameters, these parameters are evaluated at the rational expectations equilibrium of interest. Indeed, all models considered in the paper have a Jacobian of the above form and depend on the rational expectations values of the b_t coefficients. It is immediate that the stability properties of the Jacobian are determined by the properties of $A_1 - I_3$, $A_2 - I_3$, $A_3 - I_3$ and $A_4 - I_3$. For such matrices to have roots all having negative real parts, the associated characteristic equation

$$P(h) = \lambda^3 + \bar{A}_2 h^2 + \bar{A}_1 h + \bar{A}_0 \tag{37}$$

where $\bar{A}_i$ are composites of model primitives and stability under learning dynamics, requires $\bar{A}_2, \bar{A}_0 > 0$ and $\bar{A}_4 = -\bar{A}_0 + \bar{A}_1 \bar{A}_2 > 0$. It follows that E-Stability imposes 12 restrictions on model parameters.

A.2 Details of Proposition 1 and 3

Determining the E-Stability conditions under the rules (15) and (19), when the central bank projects using the rational expectations model (3) and (4) are a straightforward application of the above methodology in appendix A.1. Indeed, identical calculations provide the Jacobian (36). However, since the instrument rule is independent of agents' forecast parameters (it involves neither explicit dependence on private forecasts nor implicit dependence on variables that themselves depend on forecast parameters in the current period) the eigenvalues pertaining to learning each of the coefficients of the nominal interest rates equilibrium dynamics are also independent of forecast parameters and equal to negative unity. Thus, each of the matrices $A_1 - I_3$, $A_2 - I_3$, $A_3 - I_3$ and $A_4 - I_3$ has a root equal to negative unity. The associated characteristic equation for each matrix then takes the form $P(h) = (h+1)(h^2 + \bar{A}_1 h + \bar{A}_0)$. E-Stability then requires $\bar{A}_0, \bar{A}_1 > 0$ for the remaining two roots to have negative real parts.

A.3 Proof of Proposition 2

To study the properties of this model under learning, note that the minimum-state-variable solution now includes the lagged output gap as a state variable. The forecast functions are therefore assumed to be of the form $z_t = a_t + b_t x_{t-1} + c_t u_t + d_t r_t + \varepsilon_t$ where $z = (\pi_t,\ x_t, i_t)$. Solving this relation backwards recursively from time T to t and taking expectations at that date gives

$$\begin{aligned}\hat{E}_t z_T &= (I_3 - b_t)^{-1} (I_3 - b_t^{T-t}) a_t + b_t^{T-t} z_0 + \gamma u_t (\gamma I_3 - b_t)^{-1} \left(\gamma^{T-t} I_3 - b_t^{T-t}\right) c_t \\ &\quad + \rho r_t (\rho I_3 - b_t)^{-1} \left(\rho^{T-t} I_3 - b_t^{T-t}\right) d_t \end{aligned} \tag{38}$$

Substituting the forecasts into the structural relations (1), (2) and (15) gives the actual law of motion. Constructing the E-Stability mapping gives the associated ordinary differential equation

$$\frac{\partial \phi}{\partial \tau} = T(\phi) - I_{12}\phi = \begin{bmatrix} A_1 & A_5 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & A_2 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & A_6 & A_3 & \mathbf{0} \\ \mathbf{0} & A_7 & \mathbf{0} & A_4 \end{bmatrix} \phi$$

where $\phi = (a_z', b_\pi, b_x, b_i, c_z', d_z')$, A_i for $i \in \{1, 2, 3, 4, 5, 6, 7\}$ are (3×3) matrices whose elements are composites of model primitives, I_{12} is an identity matrix of indicated dimension and $\mathbf{0}$ a null (3×3) matrix. E-Stability requires all eigenvalues of the Jacobian matrix $T'(\phi) - I$ to have real parts for stability under learning dynamics. Recall that the Jacobian matrix must be evaluated at the rational expectations equilibrium of interest. The A_i matrices are functions of the (b_π, b_x, b_i) parameters. At the rational expectations equilibrium of interest these are given by the coefficients on x_{t-1} in (12), (13) and (14). Therefore

$$\begin{aligned} b_\pi &= (1-\mu)\frac{\lambda}{\kappa} \\ b_x &= \mu \\ b_i &= \frac{\sigma\lambda - \kappa}{\sigma\kappa}(1-\mu)\mu. \end{aligned}$$

Calculation of the eigenvalues (using Mathematica) establishes four roots to be equal to negative unity, three roots taking negative values

$$-\frac{1}{1-\beta};\ -\frac{1}{1-\beta\rho};\ -\frac{1}{1-\beta\gamma}$$

and the remainder given as

$$\begin{aligned}
\lambda_1 &= -\frac{(1-\alpha\beta)(1-\beta\mu)\lambda+\kappa^2}{(1-\alpha\beta)([1+(1-\mu-\alpha)\beta]+\kappa^2)} \\
\lambda_2 &= -\frac{(1+(1-2\mu-\alpha)\beta+R_1\beta^2)\lambda^2+R_2\lambda\kappa^2+\kappa^4}{([1+(1-\mu-\alpha)\beta]\lambda+\kappa^2)^2} \\
\lambda_3 &= -\frac{(1-\mu\beta)\kappa^2+(1+(2-2\mu-\alpha)\beta-(1-\mu)\mu\beta^2)\lambda}{(1-\mu\beta)(k^2+(1-(1-\mu-\alpha)\beta)\lambda)} \\
\lambda_4 &= -\frac{\kappa^2+\lambda(1+\mu\alpha\beta^2\rho+\beta(1-\mu-\alpha-\rho))}{(1-\alpha\beta\rho)([1+(1-\mu-\alpha)\beta]+\kappa^2)} \\
\lambda_5 &= -\frac{\kappa^2+\lambda(1+\mu\alpha\beta^2\gamma+\beta(1-\mu-\alpha-\gamma))}{(1-\alpha\beta\gamma)([1+(1-\mu-\alpha)\beta]+\kappa^2)}
\end{aligned}$$

for μ the rational expectations coefficient which was shown in Section 3 to satisfy $0<\mu<1$ and where $R_1 = 1+\mu^2+2\mu(\alpha-1)-\alpha$ and $R_2 = 2+(2-2\mu-\alpha)\beta$. By inspection, the first root is negative given the maintained parametric assumptions. To sign the latter roots takes a little more work.

To determine the sign of λ_2 consider the numerator, written as

$$N(\mu) = (1+(1-2\mu-\alpha)\beta+R_1\beta^2)\lambda^2+R_2\lambda\kappa^2+\kappa^4.$$

Evaluated at $\mu=0$ and $\mu=1$ gives:

$$N(0) = \left(1+\beta+\alpha\beta+(1-\alpha)\beta^2\right)\lambda^2+(2+(2-\alpha)\beta)\lambda\kappa^2+\kappa^4>0$$

and

$$N(1) = (1-\beta)(1-\alpha\beta)\lambda^2+(2-\alpha\beta)\lambda\kappa^2+\kappa^4>0.$$

Furthermore, differentiating the numerator with respect to μ gives

$$\frac{\partial N}{\partial\mu} = -2\beta\lambda[[1-(1-\mu-\alpha)\beta]\lambda+\kappa^2]$$

which has the same sign for all $0 < \mu < 1$. These facts combined imply $\lambda_2 < 0$. Thus for all parameter values the conditions for E-Stability under least-squares learning dynamics are satisfied.

Consider λ_3. The denominator is clearly positive under the maintained assumptions. Note that the numerator can be written as

$$(1-\mu\beta)\,\kappa^2 + (1-\alpha\beta) + \beta\,(1-\mu)\,(2-\mu\beta) > 0.$$

It follows that the root is necessarily negative, since the numerator is positive by inspection.

Consider the root, λ_4 and in particular the term in parentheses. A necessary condition for the root to be positive is this latter term being greater than zero. This implies the condition

$$\mu < -\frac{(1-\alpha\beta) + \beta\,(1-\rho)}{\beta\,(1-\alpha\beta\rho)} < 0$$

must hold, contradicting the maintained assumptions. λ_5 follows immediately on noting that it is the same expression as λ_4 with ρ replaced by γ. Since they both satisfy the same parameter restriction, λ_5 must also be negative. Therefore the conditions for E-Stability are satisfied.

A.4 Proof of Proposition 4

To study the properties of this model under learning, note that the MSV solution is linear in $\{p_{t-1}, r_t, u_t\}$. Agents therefore use an econometric model of the form (24) and long-horizon forecasts are constructed using the relation (8) given the appropriate definition of $z_t = (p_t,\, x_t, i_t)$ and the coefficient matrix b_t. Substituting the forecasts into the structural relations (1), (2) and (15) gives the actual law of motion. Constructing the E-Stability mapping gives the associated ordinary differential equation

$$\frac{\partial \phi}{\partial \tau} = T(\phi) - I_{12}\phi = \begin{bmatrix} A_1 & A_5 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & A_2 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & A_6 & A_3 & \mathbf{0} \\ \mathbf{0} & A_7 & \mathbf{0} & A_4 \end{bmatrix} \phi$$

where $\phi = (a_z', b_\pi, b_x, b_i, c_z', d_z')$, A_i for $i \in \{1, 2, 3, 4, 5, 6, 7\}$ are (3×3) matrices whose elements are composites of model primitives, I_{12} and identity matrix of indicated dimension and $\mathbf{0}$ a null (3×3) matrix. E-Stability requires the Jacobian matrix $T'(\phi) - I$ to have all eigenvalues having negative real parts for stability under learning dynamics. Recall that the Jacobian matrix must be evaluated at the rational expectations equilibrium of interest. The A_i matrices are functions of the (b_π, b_x, b_i) parameters. At the rational expectations equilibrium of interest these are given by the coefficients on $\tilde{p}_{t-1}$ in (16), (17) and (18). Therefore

$$\begin{aligned} b_p &= \mu \\ b_x &= -\frac{\kappa}{\lambda}\mu \\ b_i &= -\frac{\sigma\lambda - \kappa}{\sigma\lambda}(1-\mu)\mu. \end{aligned}$$

Calculation of the eigenvalues (using Mathematica) establishes four roots to be equal to negative unity, three roots taking negative values

$$-\frac{1}{1-\beta};\ -\frac{1}{1-\beta\rho};\ -\frac{1}{1-\beta\gamma}$$

and the remainder given as

$$\begin{aligned} \lambda_1 &= -\frac{(1-\alpha\beta)(1-\beta\mu)\lambda + \kappa^2}{(1-\alpha\beta)([1+(1-\mu-\alpha)\beta] + \kappa^2)} \\ \lambda_2 &= -\frac{(1+(1-2\mu-\alpha)\beta + R_1\beta^2)\lambda^2 + R_2\lambda\kappa^2 + \kappa^4}{([1+(1-\mu-\alpha)\beta]\lambda + \kappa^2)^2} \\ \lambda_3 &= -\frac{(1-\mu\beta)\kappa^2 + (1+(2-2\mu-\alpha)\beta - (1-\mu)\mu\beta^2)\lambda}{(1-\mu\beta)(k^2 + (1-(1-\mu-\alpha)\beta)\lambda)} \\ \lambda_4 &= -\frac{\kappa^2 + \lambda(1+\mu\alpha\beta^2\rho + \beta(1-\mu-\alpha-\rho))}{(1-\alpha\beta\rho)([1+(1-\mu-\alpha)\beta] + \kappa^2)} \\ \lambda_5 &= -\frac{\kappa^2 + \lambda(1+\mu\alpha\beta^2\gamma + \beta(1-\mu-\alpha-\gamma))}{(1-\alpha\beta\gamma)([1+(1-\mu-\alpha)\beta] + \kappa^2)} \end{aligned}$$

for μ the rational expectations coefficient which was shown in Section 3 to satisfy $0 < \mu < 1$ and where

$$\begin{aligned} R_1 &= 1 + \mu^2 + 2\mu(\alpha - 1) - \alpha \\ R_2 &= 2 + (2 - 2\mu - \alpha)\beta. \end{aligned}$$

It is immediate that these conditions are precisely those obtained in the proof of proposition 2. E-Stability follows.

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