Estimating the Performance and Risk Exposure of Private Equity Funds: A New Methodology*

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August 2007

Abstract

We develop a new GMM-based methodology to assess the performance and risk exposure of a non-traded asset. We apply this approach to the estimation of abnormal returns and risk exposure of private equity funds. In contrast to existing work, our methodology uses actual cash flow data and avoids the use of self-reported net asset values. Using a dataset comprising 797 mature private equity funds spanning 24 years, we find a high market beta for venture capital funds and a low beta for buyout funds, and report evidence that private equity risk-adjusted returns are surprisingly low. We also find evidence that venture capital funds load positively on SMB and negatively on HML while buyout funds load negatively on SMB and positively on HML. Finally, we shed light on previous findings that larger and more experienced funds have higher returns, by showing that this is mainly caused by higher risk exposures of those funds, and not by abnormal performance.

*We thank Frank de Jong, André Lucas, Thomas Dangl, Lubos Pástor, Marno Verbeek and seminar participants at BI Oslo, the Vienna Symposia on Asset Management, the Rotterdam Conference on Asset Managment, Netspar's Pension Workshop, and EFA 2007 Ljubljana for valuable comments. We thank Inquire-UK for financial support. Views are those of authors and not that of Inquire-UK.

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JEL classification: C51; G12; G23

Keywords: Risk; Abnormal return; Private equity

1 Introduction

Private equity funds are financial intermediaries that invest mainly in venture capital and leveraged buyouts. Most investors in private equity commit capital to these funds instead of investing directly in these assets. In 2005, a record high amount of $200 billion was invested in private equity funds for a total amount under management estimated to be above $1 trillion. The main objective of this paper is to develop a new methodology to measure the abnormal return and risk exposure faced by private equity investors.

As private equity funds are not publicly traded, fund returns are not observable. This implies that standard estimation techniques cannot be applied. The only data available are investments and dividends, which occur irregularly during the life of the fund, and net asset values (NAVs), which are self-reported accounting values for on-going investments reported each quarter. Several papers have assumed a market model with beta equal to one to assess abnormal performance of private equity funds using the "public market equivalent" or the "profitability index" (Kaplan and Schoar (2005) and Phalippou and Gottschalg (2007)). Currently, the only direct estimate of the risk faced by private equity fund investors is that of Jones and Rhodes-Kropf (2004).[1] They assume that the NAVs are stale but otherwise unbiased estimates of market values. They obtain the fund alpha and betas by regressing NAV-based returns on both contemporaneous and lagged risk factors. Their study is an important and substantial step towards estimating private equity fund risk profiles. However, we find that even when NAVs are unbiased and stale, such an approach generates large biases for estimates of both risk exposure and abnormal performance. We illustrate this claim by a simple simulation and analytical results.

To avoid the use of stale and potentially biased NAVs, we propose a new methodology. Given a factor pricing model, risk loadings and abnormal performance are estimated using the Generalized Method of Moments on a set of pricing restrictions for the cross-section of

[1] The literature also offers indirect estimates of fund risk. A first approach here consists of assuming that a fund's beta is the same as that of the publicly traded stocks in the same industry (e.g. Ljungqvist and Richardson (2003), and Phalippou and Gottschalg (2007)). An alternative approached, also used in Phalippou and Gottschalg (2007) infers the risk profile from post-IPO returns of buy-out investments and venture capital investments. Finally, Bilo et al. (2006) make inference from publicly listed companies that are involved in private equity. We refer to Phalippou (2007) for a detailed summary and discussion of the literature.

private equity funds. The pricing restriction is that the expected compounded value of all investments made by a fund should equal the expected compounded value of all dividends paid out by the fund over its finite life, where the compounding is done using the factor pricing model. By the cross-section of moment conditions, the parameters of the pricing model (alpha and factor exposures) can be estimated. The pricing restriction we apply is related to the standard Euler equation for the pricing kernel (see Cochrane (2005b) for an overview), but our setup allows for direct estimation of abnormal performance and risk exposures.

An important aspect of our methodology is that it does not require distributional assumptions on the factor returns and idiosyncratic shocks. This is an appealing feature in private equity as the returns are not directly observed, so that the return distribution cannot be estimated in standard fashion. This is why we do not use the maximum likelihood approach used by Cochrane (2005a) in a similar context. Moreover, our estimation does not rely on the NAVs for identification. This setup makes our methodology robust to both the stale pricing problem and, importantly, to systematic valuation errors in the NAVs. Grouping funds into funds-of-funds for estimation, a simulation study shows that our methodology has attractive small sample properties.

Our methodology can readily be applied to fully liquidated funds. In order to also include funds that are at the typical liquidation age of 10 years, but have not fully liquidated at the end of our sample period, we provide an extension of our methodology to deal with non-zero final NAVs. As shown in Phalippou and Gottschalg (2007), even funds that have reached the typical liquidation age report substantial final NAVs, which influences performance estimates. We thus propose an empirical model of market values at the final observation date, regressing realized net present values of cash flows on fund characteristics and the NAV at the valuation date. The net present values are calculated using the factor pricing model, so that this regression is estimated jointly with the GMM estimation of the alpha and risk exposures. We use the sample of fully liquidated funds to estimate this regression model. Using the regression estimates, we then predict the final market values of non-liquidated funds.

Our dataset comprises 20,331 cash flows from 797 private equity funds raised between 1980 to 1993. We observe cash flows until December 2003. Using our regression approach we predict the market values of funds that were not liquidated before December 2003, and find that these final market values for funds raised between 1990 to 1993 are on average only 28.7% of their self-reported NAVs. Applying the GMM estimation approach, we find that venture capital funds have a CAPM-beta of 2.18, while buyout funds have a CAPM-beta of 0.09. The low BO beta is quite puzzling given the high amount of leverage used and the fact that their portfolio companies are similar to publicly traded companies. We find a significantly alpha of -15% per year for VC funds. The alpha for BO funds is positive but not significant. However, when adding Fama-French factors (SMB and HML), the annual alpha for VC and BO funds is around -10% and -9% respectively. In general, the venture capital funds resemble small growth stocks (positive loading on SMB and negative on HML) whereas buyout funds resemble large value stocks.

We also show that using the final NAV as the final market value of the fund (as done by Kaplan and Schoar (2005)), instead of our regression-based estimate for the market value, has a significant impact on the estimated alpha and beta. Similarly, fully writing off the market values of funds that have not been liquidated, as done by Phalippou and Gottschalg (2007), also leads to different estimates for performance and risk exposure.

The flexibility of our GMM approach enables us to estimate the relation between the risk-adjusted performance, market risk and the characteristics of the funds in greater detail. We find mild evidence that the longer the duration between investments and distributions for a fund, the lower the risk-adjusted performance. We also shed light on the finding of Kaplan and Schoar (2005) that the experience of fund managers and the fund size have a positive influence on fund returns. Incorporating these fund characteristics in both the alpha and beta, we find that these higher returns are largely due to higher risk exposure and not due to abnormal performance.

Our paper is related to that of Cochrane (2005a) who assesses the alpha and beta of US venture capital investments gross-of-fees. Our study differs in several ways from Cochrane's work. As discussed above, the most important difference is that our methodology does not rely on distributional assumptions on the fund returns. Second, Cochrane's risk and return estimates are those faced by fund managers and not those faced by fund investors. As fees vary across funds, over time, and are non-linear in performance, they affect alpha and beta. Third, our dataset contains cash flows received by investors. In contrast, Cochrane needs to estimate the cash flows by inferring the fraction held by the fund manager of a each company and, in addition, has to estimate investments made in missing intermediary financing rounds (about 15% of the cases). Fourth, sample selection issues are relatively low in our dataset as cash flows are reported by *investors* and for all the investments of a given fund, including bad ones.[2] In contrast, Cochrane observes a return only if there has been a successful exit (IPO or M&A), which happens for less than half of the investments. Cochrane thus has to infer the return of the unsuccessfully exited investments.

Knowing the risk profiles of different types of private equity funds enables investors to improve their asset allocation to the private equity asset class and across private equity funds. Our methodology also permits to better mark to market private equity investments and to evaluate the relative performance of private equity funds compared with benchmarks such as public equity.

The rest of the paper proceeds as follows. Section 2 provides brief introduction to the industry and data. Section 3 presents the NAV-regression approach. Section 4 is dedicated to the GMM approach. Section 5 shows the results from the Monte Carlo simulation and discusses the bias in the NAV-regression approach. In Section 6, we propose an empirical model of the market values for the non-liquidated funds. Section 7 presents the empirical results. Section 8 briefly concludes.

[2] A sample selection bias still exists in our dataset as shown by Phalippou and Gottschalg (2007) as investors that report to Venture Economics have some fund-picking abilities. A correction for such a bias is proposed in their paper.

2 Data

2.1 Institutional environment

The private equity funds in our study are organized as limited partnerships and have a finite life (10 years, extensible to 13 years). This structure is by far the most common in this industry. Investors, called Limited Partners (LPs), are principally institutional investors. LPs commit a certain amount of capital to private equity funds, which are run by General Partners (GPs). In general, when a GP identifies an investment opportunity, it "calls" money from its LPs up to the amount committed (undiscounted). Such "calls" mainly occur over the first 5 years of a fund's life. Each time an investment is liquidated, the GP distributes the proceeds to its LPs. The timing of these cash flows is typically unknown ex ante. Compensation from LPs to GPs consists of (i) flat fees (management fee based on the amount invested or the capital committed, various fees charged on portfolio companies) and (ii) incentive fees called carried interest (typically 20% of the profits, with profit being defined differently across funds).[3]

2.2 Data source

Data on private equity funds are from Thomson Venture Economics (TVE). TVE records the amount and date of all the cash flows as well as the aggregate quarterly book value of all unrealized investments (called Residual Values or Net Asset Values) from 1980 to 2003. Cash flows are net of fees as they are what LPs have received and paid.

Venture Economics offer the most comprehensive source of financial performance of both US and European private equity funds and has been used in previous studies (e.g., Kaplan and Schoar (2005)). It covers an estimated 66% of both venture capital funds and buyout funds (see Phalippou and Gottschalg (2007)). TVE builds and maintains this dataset based on voluntarily reported information about cash flows between GPs and LPs. The main data providers are LPs and not GPs, which reduces sample selection bias concerns. The aggregate net asset values of unrealized investments (i.e., non-exited investments) are

[3] For further details on private equity fund contracts, see Axelsson, Stromberg and Weisbach (2007), Gompers and Lerner (1999), Metrick and Yasuda (2007) and Phalippou (2007).

obtained by TVE from audited financial reports of the partnership.

2.3 Sample selection

We select our sample in the same fashion as Kaplan and Schoar (2005). A fund is included in our database if it is raised between 1980 and 1993 and is either officially liquidated as of December 2003 or has not reported any cash flow during the last 6 quarters (July 2002 to December 2003). We label the latter funds as "quasi-liquidated" funds. We exclude funds with less than $5 million of committed capital as Kaplan and Schoar (2005) did. We also exclude funds that never distribute proceeds to the LPs since we cannot have meaningful performance and risk exposure estimates without cash inflows to the LPs.

Descriptive statistics are reported in Table 1. In general, BO funds are three times larger than VC funds. In total, we have 20,331 cash flows in this sample of which 13,729 are for VC funds and 6,602 for BO funds. The cash flows come from 797 funds (573 VC funds and 224 BO funds).

3 NAV-Regression Approach

In this section, we discuss the NAV-regression approach and we also run a simulation to illustrate the magnitude and direction of the bias.

3.1 The Method

The NAV-regression approach assumes that self-reported intermediary accounting values (NAVs) are unbiased but stale assessments of fund market values. Next,[4] invoking Dimson (1979), cash flows and NAVs are cross-sectionally aggregated to obtain one time series of returns

$$R_t = \frac{(AggNAV_{t+1} + AggDiv_t - AggInv_t)}{AggNAV_t} \tag{1}$$

where $AggNAV_t$ is the sum of NAVs across funds at the beginning of quarter t, $AggDiv_t$ is aggregate dividends and $AggInv_t$ is aggregate investments during quarter t. Betas are

[4] See Woodward (2004) and Jones and Rhodes-Kropf (2004).

then estimated via a time-series OLS regression of returns on contemporaneous and lagged risk factors:

$$r_t = \alpha + \beta_0 r_{m,t} + \beta_{-1} r_{m,t-1} + ... + \beta_{-4} r_{m,t-4} + \varepsilon_t \tag{2}$$

where $r_t = R_t - r_{f,t}$, is the excess return over the T-bill rate.

The idea is that the CAPM-beta can be consistently estimated by simply summing up the betas on current and lagged factor returns. However, as we show below, the NAV-regression approach is not appropriate for the private equity industry.

3.2 Bias in simulation

To show the magnitude of the bias of the NAV-regression method, we run a simple simulation. In section 5 we present a more detailed simulation study which confirms the results presented in this section. Let us first assume that a reported value $\widehat{P}_{i,t}$ for fund i at time t equals the true value $P_{i,t}$ with probability θ and the previously reported value $\widehat{P}_{i,t-1}$ with probability $1-\theta$. We have:[5]

$$E(\widehat{P}_{i,t}) = \theta P_{i,t} + (1-\theta)\widehat{P}_{i,t-1} \tag{3}$$

The true value is assumed to follow a CAPM with alpha equal to zero and beta equal to one. The market return follows a normal distribution with mean μ and volatility of 10%. For simplicity, we assume a risk-free rate equal to zero and no idiosyncratic shocks

$$P_{i,t} = P_{i,t-1}(1 + r_{m,t}) \quad \text{where} \;\; r_m \sim N(\mu, 0.01)$$

The observed value follows equation (3) and θ is set to be $\frac{1}{8}$. We considers values for the volatility and μ that are appropriate for the quarterly frequency of NAVs (as opposed to Dimson (1979) who focused on high-frequency data).

Results are in Table 2. When the value has no positive drift ($\mu = 0$), the aggregate

[5] This specification differs slightly from Dimson (1979). NAVs is based on accounting reports, hence the staleness is not so much from slow adjustment to new information as from infrequent updating of the accounting values. Appendix 1 shows that Dimson's specification also leads to systematic biases when estimating β.

β increases monotonically with more lags to the true value of 1, but deviates far from the true β if only 4 lags are included. However, the aggregate beta with 50 lags has an upward bias of 0.35 if the drift equals 2%. The direction of the bias depends on the numbers of lags and the magnitude of the drift. In the 20 lags case, the bias switches from downward to upward as the drift increases from 0 to 1%. In practice, however, only 4 lags are included empirically in the existing literature. We would expect the aggregate beta to be downward biased in such cases.

The intuition for the bias is that the stale price process in (3) does not lead to a similar expression for returns. In appendix 2 we show this analytically in more detail and illustrate that the magnitude of the bias depends on the number of lags, the frequency of updating, and the magnitude of the drift μ. The bias is very small for daily data as in Dimson (1979) since the daily drift is close to zero. However, our results show that the bias can be large at the quarterly frequency.

4 GMM Approach

In this section, we first introduce our new approach with a simple example, and then describe the GMM setup in more detail.

4.1 Simple Example

Most private equity funds are not publicly traded. Hence, one cannot estimate the alpha and factor-exposures (the betas) directly via regression analysis. Instead, we propose a GMM approach based on pricing restrictions for the fund's cash flows.

We first illustrate our methodology with a simple example. Assume the following cash flow stream (these are cash amounts to/from LPs): a takedown of \$100 occurs at time 1, a takedown of \$200 occurs at time 2, a distribution of \$180 occurs at time 3 and a distribution of \$200 occurs at time 4. The fund is then liquidated. For simplicity, assume the market return is 10% and risk-free rate is 5% for each period. If the CAPM is the correct model to describe fund returns and if there are no idiosyncratic shocks, then the value of the fund at

$t = 4$ equals

$$\begin{aligned} V_4 &= 100[1 + 5\% + \beta(10\% - 5\%)]^3 \\ &\quad +200[1 + 5\% + \beta(10\% - 5\%)]^2 \\ &\quad -180[1 + 5\% + \beta(10\% - 5\%)] - 200 \\ &= 0 \end{aligned} \tag{4}$$

That is, the investment of \$100 grows at the CAPM rate until liquidation for 3 periods, the investment of \$200 grows at the CAPM rate until liquidation for 2 periods. There is one intermediate dividend that decreases the market value of the fund at date 3. Finally the liquidation value is \$200. Under our assumptions, the residual value of this fund should equal zero, and solving the equation then gives $\beta = 1.71$.

In reality, market returns are not constant and private equity funds exhibit considerable idiosyncratic risks. Also, funds may exhibit abnormal performance. This renders the task more difficult and a GMM approach applied to a large cross-section of funds is necessary to estimate abnormal returns and factor exposures.

4.2 GMM approach

We illustrate our approach for a one-factor market model (including a constant mispricing parameter α), but it can directly be applied to multi-factor pricing models as well, as long as the factors are portfolio returns. Each fund i invests in n_i projects. Fund i invests an amount T_{ij} in project j at date t_{ij} and liquidates this project at date d_{ij} paying a liquidation dividend D_{ij} to its LPs. The dividend is then given by

$$D_{ij} = T_{ij} \prod_{t=t_{ij}}^{d_{ij}} (1 + r_{f,t} + \alpha + \beta r_{m,t} + \varepsilon_{ij,t}) \tag{5}$$

where $r_{f,t}$ is the risk-free rate, $r_{m,t}$ is the market excess return, and $\varepsilon_{ij,t}$ is an idiosyncratic shock with mean zero and *i.i.d.* over time and across funds. As discussed below, to calculate standard errors we cluster at the fund level, so that the idiosyncratic shocks $\varepsilon_{ij,t}$ can be correlated within the fund.

In practice, we observe the cash flows at the fund level and do not know to which project a given cash flow belongs to. We then operate in three steps. First, we compound the cash flows to the final liquidation date of the fund. That is, we multiply both sides of equation (5) by the same compounding term and obtain

$$\begin{aligned} & D_{ij} \prod_{s=d_{ij}+1}^{l_i} (1 + r_{f,s} + \alpha + \beta r_{m,s}) \\ = \; & T_{ij} \prod_{t=t_{ij}}^{d_{ij}} (1 + r_{f,t} + \alpha + \beta r_{m,t} + \varepsilon_{ij,t}) \prod_{s=d_{ij}+1}^{l_i} (1 + r_{f,s} + \alpha + \beta r_{m,s}) \end{aligned} \tag{6}$$

where l_i is the liquidation date of fund i.

Second, we stack all $\varepsilon_{ij,t}$ for different time periods in a vector ε_{ij} and decompose this compounded dividend into a zero-mean error term $\upsilon(\alpha, \beta, \varepsilon_{ij})$ plus an expected value conditional upon the realized risk-free rate (stacked in a vector r_f), market returns (stacked in a vector r_m) and the takedown T_{ij}

$$\begin{aligned} & D_{ij} \prod_{s=d_{ij}+1}^{l_i} (1 + r_{f,s} + \alpha + \beta r_{m,s}) \\ = \; & E[\; D_{ij} \prod_{s=d_{ij}+1}^{l_i} (1 + r_{f,s} + \alpha + \beta r_{m,s}) | r_f, \; r_m, \; T_{ij}] + \upsilon(\alpha, \beta, \varepsilon_{ij}) \end{aligned} \tag{7}$$

The error term $\upsilon(\alpha, \beta, \varepsilon_{ij})$ has mean zero by construction and depends on the parameters of the pricing model. Given the assumption that the idiosyncratic shocks $\varepsilon_{ij,t}$ are i.i.d. over time, all expectations of the cross-products of the form $\varepsilon_{ij,t}\varepsilon_{ij,s}$ are equal to zero (as well as higher-order cross-products), so that taking the condition expectation of equation (6) and combining it with equation (7) gives

$$E[D_{ij} \prod_{s=d_{ij}+1}^{s=l_i} (1 + r_{f,s} + \alpha + \beta r_{m,s}) | r_f, \; r_m, \; T_{ij}] = T_{ij} \prod_{t=t_{ij}}^{t=l_i} (1 + r_{f,t} + \alpha + \beta r_{m,t}) \tag{8}$$

and taking the unconditional expectation then gives

$$E[D_{ij} \prod_{s=d_{ij}+1}^{l_i} (1 + r_{f,s} + \alpha + \beta r_{m,s})] = E[T_{ij} \prod_{t=t_{ij}}^{l_i} (1 + r_{f,t} + \alpha + \beta r_{m,t})] \tag{9}$$

The key step to the parameter identification is the moment restriction in (9). It says that the expected value of the compounded distribution should be equal to the expected value of the compounded investment. Investments are thus fairly priced up to a (monthly) error α.

We estimate the expectation in equation (9) by averaging across projects within a fund and by averaging across funds, effectively constructing funds-of-funds or portfolios of funds. We construct P portfolios with N_p funds per portfolio. The LHS of (9) for portfolio $p = 1,.., P$ is then estimated by

$$\overline{V}^{D_p}(\alpha,\beta) = \frac{1}{N_p}\sum_i \frac{1}{n_i}\sum_{j=1}^{n_i}\left[D_{ij}\prod_{s=d_{ij}+1}^{l_i}(1+r_{f,s}+\alpha+\beta r_{m,s})\right], \quad p = 1,...,P \tag{10}$$

and the RHS of (9) is estimated by

$$\overline{V}^{T_p}(\alpha,\beta) = \frac{1}{N_p}\sum_i \frac{1}{n_i}\sum_{j=1}^{n_i}\left[T_{ij}\prod_{t=t_{ij}}^{l_i}(1+r_{f,t}+\alpha+\beta r_{m,t})\right], \quad p = 1,...,P \tag{11}$$

where n_i the number of projects of fund i. As the number of projects per fund n_i tends to infinity or as the number of funds in the fund-of-funds N_p tends to infinity, the average converges to the expectation asymptotically.

Note that, by averaging (or aggregating) projects at the fund level, we only need fund-level data to perform our estimation. In particular, we do not need information on which investment corresponds to which dividend. Estimation can be done by applying GMM to all moment conditions across the portfolios of funds.

The error term $\upsilon(\alpha,\beta,\varepsilon_{ij})$ contains the compounding term $\prod_{s=d_{ij}+1}^{l}(1+r_{f,s}+\alpha+\beta r_{m,s})$, and is thus affected by the parameters α and β. In particular, the absolute size of the error term is increasing in α. Since the optimization algorithm minimizes the sum of squared errors, this implies that a downward bias in α can emerge in finite samples. To reduce this problem, we take the log on both equation (10) and equation (11). This "takes out" the effect of compounding until the liquidation date to a large extent, because this compounding terms affects both the value of dividends and investments. We thus perform the following minimization to estimate the parameters:

$$\min_{\alpha,\beta} \sum_{p=1}^{P} [\ln \overline{V}^{D_p}(\alpha,\beta) - \ln \overline{V}^{T_p}(\alpha,\beta)]^2 \tag{12}$$

Because the log of the average converges to the log of the expectation asymptotically, our estimator is consistent as the number of projects per fund or the number of funds per portfolio tends to infinity. Simulation results (see next section) confirm that this "log" estimator is accurate and has good small sample properties.

To obtain standard errors, a bootstrap methodology is adopted. Effectively, we cluster our standard errors by resampling at the fund level. More precisely, we first resample the funds with replacement within each funds-of-funds, and then re-estimate the alpha and the beta. Repeating the process 1,000 times yields the bootstrap distributions of the alpha and the beta. We use bootstrap methods instead of the asymptotic standard errors, since the latter require knowing or estimating the correlations across the moment conditions.

Finally, note that our setup is slightly different from GMM estimation based on the Euler equation for the pricing kernel (see Cochrane (2005b) for an overview). In the latter case, the parameters of the pricing kernel are estimated and abnormal performance is not accounted for. In contrast, our approach renders direct estimates of risk exposures and abnormal performance.

The approach described in this section can readily be applied to a sample of private equity funds that are fully liquidated. For our empirical analysis, we will also include funds that are at the typical liquidation age but still reporting positive NAVs. In section 6, we describe how we incorporate these funds.

5 Simulation of PE Fund Cash Flows and NAVs

To evaluate and compare the performance of our GMM methodology with the NAV-regression approach, we run a Monte Carlo experiment. We simulate 700 funds over 24 years at a quarterly frequency to reflect our sample characteristics. At the beginning of each of the first 14 years (as vintage years from 1980 to 1993 in our sample), 50 funds are started. Each fund invests \$1 per project and invests in 4 projects in the first quarter for the first 5 years of its life. Hence, each fund invests in a total of 20 projects. Finally, for this simulation study we follow Cochrane (2005a) and assume that the growth in value of a private equity project j of fund i is lognormally distributed:

$$\ln(\frac{V_{ij,t+1}}{V_{ij,t}}) = \gamma + \ln R_t^f + \delta(\ln R_{t+1}^m - \ln R_t^f) + \varepsilon_{ij,t+1}; \qquad \varepsilon_{ij,t+1} \sim N(0, \sigma^2) \tag{13}$$

where $\varepsilon_{ij,t}$ is i.i.d. normal across projects and over time. We want to emphasize that consistency of our GMM method is obtained without specific distributional assumptions. We use lognormality here as an example to show the small-sample properties. For simplicity, we use the continuous limit where $\beta = \delta$ and

$$\alpha = \gamma + \frac{1}{2}\delta(\delta - 1)\sigma_m^2 + \frac{1}{2}\sigma^2 \tag{14}$$

where $\sigma_m^2 \equiv \sigma^2 \ln(R^m)$. The α and β here are the coefficients for the CAPM in simple returns (see Cochrane (2005a) for details). We simulate the NAVs of the projects as follows. With probability θ, a revelation state occurs and the NAV is set to the project's market value. With probability $(1-\theta)$, the NAV equals that of the previous quarter. Note that in reality, the interim NAVs are likely to be noisy and may be systematically biased. Therefore, our simulation setup favors the NAV-regression approach compared with our GMM method. We set θ to $\frac{1}{8}$ to reflect the claim in Woodward (2004) that individual investments are marked-to-market every 8 quarters on average. We assume that the risk-free rate is 1% per quarter and that the stock-market index volatility is 12% per quarter (matching US data over our sample period). The quarterly idiosyncratic volatility is set to 40%, which reflects the high number reported by Cochrane (2005a) for VC investments.[6]

To improve the small sample properties of the GMM approach, we group all funds with the same vintage year into funds-of-funds. We show results for three different scenarios: a benchmark (α, β) equal to $(0, 1)$, high performance (α, β) equal to $(1\%, 1.5)$ and low

[6] Since we have both VC and BO funds in our data, the 40% idiosyncratic shock volatility estimated from VC investments in Cochrane (2005a) is too high for our BO sample. But we want to show that even with high idiosyncratic shocks, our GMM method still gives satisfactory results.

performance (α, β) equal to (-1%, 1.5).

Table 3 - Panel A summarizes our calibration setup. Table 3 - Panel B shows that for the NAV-regression approach, the beta is found to be significantly biased downward and consequently the alpha is found be significantly biased upwards. The mean of the aggregate beta with 4 lags is 0.6, which is far away from the calibrated value of 1 in the benchmark case. The mean of the aggregate beta with 8 lags is closer to 1 but remains downward biased. The alpha is 0.83% higher than the true value in 4 lags case and gets somewhat closer to zero as more lagged terms are included.

The betas of the NAV-regression approach are also downward biased in low performance cases and the alphas are all far from their true values. Using our methodology, in contrast, we find that the estimates of alpha and beta are very close to the calibration values with an alpha of -0.05% and a beta of 1.01 in the benchmark case. This shows that the GMM method can identify the alpha and beta cross-sectionally without being affected by the stale prices problem.

To see the asymptotic properties of the GMM method, we increase the project numbers within a fund from 20 to 50. Table 3 - Panel C shows that both alphas and betas are more precisely estimated and the alphas are closer to the calibration values for all cases in our GMM approach, but not for the NAV-regression method.

In sum, the Monte Carlo experiment shows that the NAV-regression approach leads to biased estimates of the alpha and beta when stale pricing exists in the context of private equity. It also shows that our GMM model, based on cash flows rather than stale NAVs, has good small sample properties.

6 Modelling the Final Market Value of Quasi-Liquidated Funds

As described in the data section, our empirical analysis uses both fully liquidated funds and so-called quasi-liquidated funds, which are at least 10 years old at the end of our sample (Dec. 2003) and have displayed no cash flow activities for the last 6 quarters. These quasi-liquidated funds report a positive NAV at the end of the sample period. Existing work has either treated these final NAVs as a final cash flow (Kaplan and Schoar (2005)) or completely written off this value (Phalippou and Gottschalg (2007)). Given the staleness and potential biases in the NAV, we propose a regression approach to predict these final market values. These predicted market values will be used as final cash flows when estimating the abnormal performance and risk exposure in the empirical analysis section.

The main idea of the regression model is to regress the value of realized cash flows of a fund on fund characteristics. To estimate our regression model, we select funds that are fully liquidated and calculate for each fund i and at each age a the present value of the subsequent net cash flows, $PVNCF_{a,i}$. The discounting model used to calculate this present value is the particular factor pricing that is estimated using our GMM approach. This means that we jointly estimate the regression coefficients on the fund characteristics and the abnormal performance and risk exposure of the pricing model.

For each age a, we estimate the following model from the cross-section of fully-liquidated funds

$$\ln(1 + PVNCF_{a,i}) = b_{a0} + b'_{a1}X_{a,i} + \varepsilon_{a,i} \tag{15}$$

The vector of explanatory variables $X_{a,i}$ includes the log of $(1 + NAV)$ at the specific fund age a, the log of the committed capital, the log of the time elapsed since the last dividend distribution, the log of the time elapsed since the last NAV update, the Profitability Index excluding the final NAV at the age a, a dummy variable for venture capital, the NAV divided by capital invested at the age a, the square of this ratio, the log of sequence number of the fund and year fixed effects.[7] The Profitability Index is also calculated using the pricing model at hand.

Since this regression is estimated for each particular pricing model (CAPM, Fama-French, etc.), the coefficient estimates vary across each pricing model. However, we find empirically that these coefficients are very similar. We therefore present regression results that we obtain with the CAPM as pricing model.

We first present descriptive statistics of the sample of funds at each age, as used to estimate the regressions in Table 4. As expected, the ratio of total NAVs reported by funds over the total amount they invest decreases with age. At 6 years old (non-tabulated) the

[7] For parsimony, we only include the year fixed effect for those vintage years with more than 10 funds.

ratio is close to 100%. This is consistent with both the fact that this is when funds typically stop investing and the practice of keeping investments at cost in the NAV calculations until a major pricing event occurs (e.g. a new financing round). The ratio then decreases quickly and goes below 10% after age 13, typically the maximum duration of funds.

For funds older than 13 years, we write off their NAVs. The written-off NAVs account for 8.64% of the capital invested in the entire sample (fully plus quasi-liquidated funds). The main reason that we write off these NAVs is that we do not have enough observations of funds that distributed dividends beyond year 13 to infer anything about their market value. Consequently, we only model the final market value of 144 non-liquidated funds raised between 1990 to 1993 (which are 10 to 13 years old at Dec. 2003). The final NAVs for these funds account for 13.09% of the capital invested in the entire sample.

In table 5, we report the results for the regressions at different fund ages. The coefficients for $ln(1 + NAV_{a,i})$ are all strongly significant. This means that NAVs do carry some information in terms of market value. However, as we will see below, the coefficient estimates imply that the NAV is typically upward biased. NAV alone generates an R^2 of 56% to 66% (non-tabulated). Adding all the control variables increases the adjusted R-square to 63% to 72%. The coefficients on fund size are positive though not all significant. Large funds might be more conservative with their accounting valuations. Both the duration between the last dividend payout month and the end of fund age t and the duration between the last NAV update month and the end of fund age a have significantly negative coefficients. This shows that the longer the period of inactivity has been, the more upward biased accounting values are. The loadings for the Profitability Index (up to age a) are positive. If a fund has performed well so far, it is more likely to contain additional good investment projects and thus distribute relatively higher cash flows. The other explanatory variables are not significant.

One may argue that the fully-liquidated sample generates different coefficient estimates than a sample with quasi-liquidated funds would have. We therefore redo our regression analysis using an alternative sample consisting of funds with more than 50% reduction of their NAVs between age a and Dec. 2003. This larger sample includes all funds in the fully-liquidated one. For robustness, we also try two other sample selection criteria of 25% and 75% reduction of their NAVs between age a and Dec. 2003.

Using the regression coefficient estimates, we then predict out-of-sample the final market values of the non-liquidated funds at Dec. 2003. Table 6 reports the ratio of the aggregated predicted present value of net cash flows over the aggregated NAVs for these non-liquidated funds raised between 1990 to 1993. This ratio decreases with fund age: the older the fund, the more upward biased the NAV is. Overall, we find a dramatic upward bias for the NAVs reported by these funds: the final NAVs for the funds raised between 1990 to 1993 are only worth 28.7% of their self-reported values. The ratios are very similar irrespective of which sample we use to estimate the regression coefficients, which indicates no severe sample selection issue. In the next section we use these predicted final market values as the final cash flow of the quasi-liquidated funds in our GMM estimation approach.

7 Risk and Return Estimates

We first illustrate how we form our funds-of-funds portfolios. The main results are reported afterwards and we also compare these with the estimates obtained from treating the NAV as fair market value or fully writing it off. Finally, we extend our model to incorporate fund characteristics in the alpha and beta.

7.1 Benchmark Results

7.1.1 Funds-of-Funds Estimation

Before proceeding with our GMM estimation, we group the private equity funds into portfolios. This way, we average out the idiosyncratic risk at the fund level to some extent. The funds are first grouped based on their vintage year and then based on their committed capital. We thus create 4 funds-of-funds for the VC funds and 3 funds-of-funds for the BO funds per vintage year, leading to 56 funds-of-funds for Venture Capital and 42 funds-of-funds for Buyouts in total. Each funds-of-funds represents one moment condition for the GMM estimation. Because there are only few funds raised in the early 80s in our sample, we also weight the moment conditions by the number of funds in each portfolio.

7.1.2 CAPM Results

We start with estimating the CAPM-beta and alpha for the 797 funds. As BO and VC funds operate in different market segments, their risk profiles might differ. We therefore separate the funds that focus on venture capital investments from those that focus on buyouts. The advantage is that we have a more homogeneous group of funds, which should help obtaining more accurate estimates but this comes at the expense of less observations and thus increased potential estimation error.

The results are in Table 7. The CAPM specification gives a beta of 2.18 and a significantly negative alpha of about -15% per year for the VC funds. Such a result is in line with the findings of low private equity fund performance of Phalippou and Gottschalg (2007), who assume $\beta = 1$. When the two Fama-French factors are added, alpha increases to -10% per year because VC funds overall are similar to small growth stocks, which have low performance over this time period.

BO funds are found to have a much lower beta of 0.09 and a slightly positive alpha. However, adding SMB and HML increases the beta to 1.04 and also leads to a significantly negative alpha of -9% per year. It might seem puzzling that betas are always much higher for VC funds than for BO funds, especially given the high amount of leverage used by BO funds and the fact that their investments are similar to publicly traded companies. One possible explanation is that the BO funds typically invest in mature companies or industries that generate steady and predictable cash flows. The lower risk exposures of these companies or industries might counterbalance the high leverage effects on their betas.

An important difference exists between VC funds and BO funds regarding their exposure to the Fama-French factors. We find that venture capital funds look like small growth stocks with an SMB loading of 0.25 and an HML loading of -0.79 whereas buyout funds look like large value stocks with an SMB loading of -1.46 and an HML loading of 1.14.

Next, we compare these results with those obtained from treating the final NAVs of non-liquidated funds as fair market value or, alternatively, writing these NAVs off completely.

Table 8 shows the difference between the betas of treating the final NAV as fair market value and writing it off is large. The VC betas are 2.92 and 1.25 under the CAPM framework for "fair market value" and "written-off" respectively. The BO betas are 0.62 and -0.22 respectively. These differences remain when adding the Fama-French factors. Hence, even though the final non-zero NAVs account for a relatively small percentage of the capital invested in our mature funds, they carry disproportionately more information about the fund's risk exposure during its life. One explanation for the low betas in the written-off case is that the managers update their NAVs in particular when market conditions are good. Hence, writing the final NAV off eliminates this exposure to high stock market returns and reduces the market risk exposure estimated by our GMM model. The low betas in the written-off case result in higher alphas even though sizeable portions of fund values are eliminated.

Finally, for comparison, we also apply the NAV-regression approach to our quasi-liquidated sample. The results are in Table 9. We find that VC betas are much lower than our GMM estimates. Even including 8 lagged market returns still yields a much lower aggregate beta.[8] The alphas are also very different from our estimates, as they are always positive and thus much larger than what we find. The positive alphas in the Table 9 are to some extent due to the lower aggregate betas, but may also be the result of systematically upward biased values for the NAVs.

7.2 Extensions

The flexibility of our GMM approach enables us to estimate the relation between the risk-adjusted performance, market risk and the characteristics of the funds. It helps to unveil the risk profile of the private equity funds in greater detail. We first look at the correlation between the alpha and the investment duration. Next, we examine the learning and size effects on the alpha and the beta.

E.1 Duration and risk-adjusted performance

[8] Jones and Rhodes-Kropf (2004) use the NAV-regression approach with 4 lags for a somewhat different sample consisting of 1,245 funds raised between 1980 to 1999 and report VC and BO aggregate betas of 1.8 and 0.65 respectively.

The duration between an investment and its corresponding distribution could have implications for the performance. A long-lasting project could be the product of no successful exit and the manager's reluctance to write it off. Besides, managers have incentive to exit good projects as soon as possible to meet the hurdle rate and pocket the carried interest. To test whether long duration implies low performance at the fund level, we first need to calculate the duration of a fund.

We do not have the details for investment projects, therefore, the duration of a fund is defined as the value-weighted average time at which distributions are made minus the value-weighted average time at which investments are made.[9] This definition is similar to the one in the bond literature. The alpha is thus modelled as a function of fund's log value of the duration:

$$\alpha = \alpha_0 + \alpha_{duration} * \ln(Duration) \tag{16}$$

Table 10 shows that the $\alpha_{duration}$ for both VC funds and BO funds is negative as predicted though not significant. Adding SMB and HML, the $\alpha_{duration}$ for BO funds is significant at 10% level. This means the longer the average time elapsed between the investments and the distributions, the lower the BO fund's risk-adjusted performance. We thus provide weak evidence that good BO projects exit quicker than the bad ones.

Table 10 also reports the 25th, 50th and the 75th percentiles of the alphas in terms the cross-fund distribution of the duration. Like the benchmark case, without Fama-French factors, the VC alphas are negative and economically significant where the BO ones are all positive for these three percentiles. However, after controlling for market risk and the Fama-French risk factors, BO funds also have negative outperformance.

E.2 Experience and fund size

Kaplan and Schoar (2005) find that the Profitability Index (Public Market Equivalent in their paper) is positively related to the fund size and the fund sequence. The Profitability Index (PI) assumes a beta equal to 1. Hence, a high PI could be due to positive abnormal

[9] When the value-weighted average duration for a fund is smaller than one month, which is typically due to the absence of any distributions for later investments, we treat it as the 99th percentile. We also censor the right-tail outliers at 99th percentile.

performance or due to a beta higher than 1. In this subsection we analyze which of these two effects is most prevalent.

We model the alpha as a function of an experience dummy

$$\alpha = \alpha_0 + \alpha_{Experience} * DummyExp \tag{17}$$

A first-time fund has a zero on the experience dummy where more experienced funds have a dummy equal to one. The results are in the first four columns of Table 11. Both VC and BO funds have a positive $\alpha_{Experience}$, however, only for buyout funds it is significant both economically and statistically. The VC funds, whether experienced or not, generate a large negative net-of-fee alpha. The alpha of the inexperienced BO funds is less negative, around -5% per year. However, the experienced BO funds under the CAPM framework yield positive risk-adjusted performance, around 4% per annum.

Of course, it is possible that the higher alpha for the experienced funds is proxying for higher exposure to market risk. Therefore, we next also model the beta as a function of the experience dummy.

$$\beta = \beta_0 + \beta_{Experience} * DummyExp \tag{18}$$

The 5th column of Table 11 shows that the beta of experienced VC funds exceeds the first-timers' beta by about 3. This leads to a negative VC $\alpha_{Experience}$ of -1.3% per month. The experienced BO funds are also more exposed to market risk though to a less degree than the experienced VC funds. It also leads to a less negative $\alpha_{Experience}$ of -0.25% per month. In sum, we show that the superior PI for experienced funds is mainly due to high market risk exposure rather than due to better managerial skills.

Next, we follow the same procedure to see whether the fund size plays a role in the alpha and beta. The alpha is modelled as a function of the log value of committed capital

$$\alpha = \alpha_0 + \alpha_{size} * \ln(size) \tag{19}$$

As expected, The first four columns of Table 12 show that α_{size} is positive and highly significant for both VC and BO funds. The larger the committed capital, the better the risk-adjusted performance. Table 12 also reports the 25th, 50th and 75th percentile of the alpha (given the cross-sectional distribution of fund size). The median is -1.19% per month for the VC funds and 0.59% per month for the BO funds under the CAPM. Adding Fama-French factors, the VC alpha increases but remains negative even for the 75th percentile. However, the median alpha for BO funds decreases to -0.26% per month where only the 75th percentile alpha is slightly above zero.

The size effect on the alpha of private equity funds could also be the result of heterogeneous exposure to the market risk, as was the case for fund experience. Similar to above, we incorporate a beta as a function of size into the estimation

$$\beta = \beta_0 + \beta_{size} * \ln(size) \tag{20}$$

Similar to what we found for fund experience, Table 12 indicates that the larger the fund's committed capital, the more market risk a fund takes. The β_{size} is positive and significant for both VC funds and BO funds but the magnitude is larger for the BO funds. After incorporating the size-dependent market risk, the high β_{size} for BO funds turns the α_{size} of BO funds from positive to negative. The α_{size} for VC funds also becomes insignificant and close to zero after controlling the size-varying market risk. In sum, the size effect reported by Kaplan and Schoar (2005) comes again mainly from higher market risk exposure.[10]

8 Conclusion

We develop a new econometric methodology to estimate the performance and risk exposure of private equity funds. We first show that existing NAV-based regression methods lead to substantially biased estimates for the alpha and beta. In contrast to this method, our GMM methodology does not use the stale and noisy accounting valuations (Net Asset Values), but instead uses data on fund investments and dividends. Our method generates consistent estimates and a simulation study shows it works well for our sample size. It also allows us

[10] We have also incorporated size and learning jointly in the α and β specifications. Overall, the coefficients are similar to those estimated separately, though with less precision. The main conclusion that the positive relationship between PI and the size or the experience of fund is the result of neglecting higher market risk exposure still holds.

to leave the distribution of the factor returns and idiosyncratic shocks unspecified. This is an appealing feature in private equity as the return distribution is not directly observable for these non-publicly traded funds.

We find that venture capital funds have a CAPM-beta of 2.18, while buyout funds have a CAPM-beta of 0.09. The results for the Fama-French factors (SMB and HML) show that venture capital funds look like small growth stocks (positive loading on SMB and negative on HML) whereas buyout funds look like large value stocks.

We also show that the NAVs reported near the end of the typical fund life are highly upward biased estimates of the market value of funds. Specifically, using a regression approach, we find that the final market values of funds that are 10 to 13 years old are only 28.7% of their self-reported net asset values. We incorporate the results of this regression in our estimation of abnormal performance and risk exposure.

Finally, the flexibility of our GMM model enables us to study the interaction between the characteristics of the funds and their alpha and beta. We find mild evidence that the longer the duration between investments and distributions for a fund, the lower the risk-adjusted performance. We also find that the experience of fund managers and the fund size have a positive influence on fund returns as documented in Kaplan and Schoar (2005). However, the higher performance is largely due to higher risk exposure. After controlling the beta for size and experience, we find that experience and size no longer have a significantly positive impact on the alpha.

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Appendix 1:
Dimson's aggregate beta

This appendix discusses the bias for estimating aggregate beta's (i.e., the sum of contemporaneous and lagged beta's) using the setup of Dimson (1979). He argues that because some securities are traded infrequently, an observed price $\widehat{P}_t$ may represent a transaction price P_t at time t or a price P_{t-i} established in the last trade which occurred at time $t-i$ $(i > 0)$. Observed prices therefore have an expected value which is a weighted average of a sequence of true prices, where the latter are the transaction prices which would arise if trading were continuous,

$$E(\widehat{P}_t) = \sum_{i=0}^{n} \theta_i P_{t-i}$$

and

$$E(\triangle\widehat{P}_t) = \sum_{i=0}^{n} \theta_i \triangle P_{t-i}$$

Dimson then approximates the continuously compounded return $\widehat{R}_t$, based on observed prices, using a log-transformation,

$$\begin{aligned} \Rightarrow \quad & E(\ln\widehat{P}_t - \ln\widehat{P}_{t-1}) \cong \sum_{i=0}^{n} \theta_i (\ln P_{t-i} - \ln P_{t-i-1}) \\ \Rightarrow \quad & E(\widehat{R}_t) \cong \sum_{i=0}^{n} \theta_i R_{t-i} \end{aligned}$$

Given this approximation, adding lagged market returns into the regression model can effectively solve the thin trading problem. However, we have to bear in mind two caveats when we apply this method to the stale pricing problem of PE funds. First, Dimson uses a log-approximation in his derivation to get a consistent aggregate beta. This approximation only works reasonably well when $\widehat{P}_t$ is very close to $\widehat{P}_{t-1}$ and P_t is very close to P_{t-1} in general.

Severe biases occurs due to the approximation of log-transformation, if the price has a trend or high variance. Since Dimson deals with daily data, it is reasonable to assume that the daily stock return has a zero mean and a small variance. But this is not the case for the NAVs of the PE funds, which are reported on quarterly basis. We find that the bias increases both with the mean and the variance of the stock in an unreported simulation. Hence, although the stale pricing problem is similar to the thin trading problem, Dimson's method cannot be directly applied to estimate the systematic risk of private equity funds.

An additional issue is that the above assumption of price revelation is ad hoc in his setup for the observed prices have an expected value equal to past true prices. This implies, for example, the observed price at day t could be the true price of $t-1$, but the observed price at day $t+1$ could be the true price of $t-3$. Hence, the observed prices can be inconsistent along the timeline by design. Such a price revelation process is not in line with the observations we have on stale NAVs. In appendix 2 we propose a more appropriate model of stale prices.

Appendix 2:
Analytical results for biases in the aggregate beta

In this appendix we shed light on the biases found in the simulation results in Table 2, for the case where.the aggregate beta (i.e., the sum of the contemporaneous and the lagged betas) is applied to estimate the true beta in a framework with stale prices and positive expected returns. In contrast to appendix 1 (where Dimson's (1979) setup was used), we use the following value revelation process for fund i:

$$E_t(\widehat{P}_{i,t}) = \theta P_{i,t} + (1-\theta)\widehat{P}_{i,t-1}$$

That is to say the realization of the observed or reported price at time t is either the true price at time t or the observed price at time $t-1$. The expectation of the observed price at t then is simply the weighted average depending on the trading frequency (in case of private equity funds, the update frequency of the NAVs), θ. The return then is as follows

$$\begin{aligned} E_t(\Delta\widehat{P}_{i,t}) &= \theta\Delta P_{i,t} + (1-\theta)\Delta\widehat{P}_{i,t-1} \\ \frac{E_t(\Delta\widehat{P}_{i,t})}{\widehat{P}_{i,t-1}}\widehat{P}_{i,t-1} &= \theta\frac{\Delta P_{i,t}}{P_{i,t-1}}P_{i,t-1} + (1-\theta)\frac{\Delta\widehat{P}_{i,t-1}}{\widehat{P}_{i,t-2}}\widehat{P}_{i,t-2} \end{aligned}$$

At time t, the observed price at time $t-1$ is a realized value, so we can put $\widehat{P}_{i,t-1}$ into the expectation, and

$$E_t(\frac{\Delta\widehat{P}_{i,t}}{\widehat{P}_{i,t-1}})\widehat{P}_{i,t-1} = \theta\frac{\Delta P_{i,t}}{P_{i,t-1}}P_{i,t-1} + (1-\theta)\frac{\Delta\widehat{P}_{i,t-1}}{\widehat{P}_{i,t-2}}\widehat{P}_{i,t-2}$$

The observed return then follows

$$\begin{aligned} E_t(\widehat{R}_{i,t})\widehat{P}_{i,t-1} &= \theta R_{i,t}P_{i,t-1} + (1-\theta)\widehat{R}_{i,t-1}\widehat{P}_{i,t-2} \\ E_t(\widehat{R}_{i,t}) &= \theta R_{i,t}\frac{P_{i,t-1}}{\widehat{P}_{i,t-1}} + (1-\theta)\widehat{R}_{i,t-1}\frac{\widehat{P}_{i,t-2}}{\widehat{P}_{i,t-1}} \\ \widehat{R}_{i,t} &= \theta R_{i,t}\frac{P_{i,t-1}}{\widehat{P}_{i,t-1}} + (1-\theta)\widehat{R}_{i,t-1}\frac{\widehat{P}_{i,t-2}}{\widehat{P}_{i,t-1}} + \nu_{i,t} \end{aligned}$$

where $\nu_{i,t}$ is a mean-zero error term. If we replace $\widehat{R}_{i,t-k}$ with $\theta R_{i,t-k}\frac{P_{i,t-1-k}}{\widehat{P}_{i,t-1-k}} + (1-\theta)\widehat{R}_{i,t-1-k}\frac{\widehat{P}_{i,t-2-k}}{\widehat{P}_{i,t-1-k}} + \nu_{i,t-k}$ recursively and assume $\widehat{R}_{i,0} = R_{i,0}$ and $\nu_{i,0} = 0$, then

$$\widehat{R}_{i,t} = \sum_{j=0}^{t}\theta(1-\theta)^j R_{i,t-j}\frac{P_{i,t-1-j}}{\widehat{P}_{i,t-1}} + \nu_{i,t} + \sum_{k=1}^{t}(1-\theta)^j \nu_{i,t-k}\frac{\widehat{P}_{i,t-1-k}}{\widehat{P}_{i,t-1}}$$

Assuming a market model, $R_{i,t-j} = \alpha + \beta R_{m,t-j} + \varepsilon_{i,t-j}$ and substituting it into the equation above, we have

$$\begin{aligned}\widehat{R}_{i,t} &= \sum_{j=0}^{t}\theta(1-\theta)^j[\alpha + \beta M_{t-j} + \varepsilon_{i,t-j}]\frac{P_{i,t-1-j}}{\widehat{P}_{i,t-1}} + \nu_{i,t} + \sum_{k=1}^{t}(1-\theta)^j \nu_{i,t-k}\frac{\widehat{P}_{i,t-1-k}}{\widehat{P}_{i,t-1}} \\ &= \sum_{j=0}^{t}\theta(1-\theta)^j\frac{P_{i,t-1-j}}{\widehat{P}_{i,t-1}}\alpha + \sum_{j=0}^{t}\theta(1-\theta)^j\frac{P_{i,t-1-j}}{\widehat{P}_{i,t-1}}\beta R_{m,t-j} + \\ &\quad \sum_{j=0}^{t}\theta(1-\theta)^j\frac{P_{i,t-1-j}}{\widehat{P}_{i,t-1}}\varepsilon_{i,t-j} + \nu_{i,t} + \sum_{k=1}^{t}(1-\theta)^j \nu_{i,t-k}\frac{\widehat{P}_{i,t-1-k}}{\widehat{P}_{i,t-1}} \qquad (21)\end{aligned}$$

Focusing on the terms multiplying $R_{m,t-j}$, equation (21) shows that aggregating lagged beta's in this setup does not lead to a consistent estimator of β. Given that $\sum_{j=0}^{\infty}\theta(1-\theta)^j = 1$, a consistent estimator is obtained if $\frac{P_{i,t-1-j}}{\widehat{P}_{i,t-1}} = 1$ for all j. However, when the price process exhibits a drift, the ratio $\frac{P_{i,t-1-j}}{\widehat{P}_{i,t-1}}$ will differ from 1 on average. For large values of j, this ratio will be smaller than 1, while for small values of j it will exceed 1, leading to biased aggregate beta's. The simulation results in Table 2 confirm these results.

Table 1: Descriptive Statistics

This table gives descriptive statistics of the sample ending December 2003. The sample consists of fully and quasi-liquidated funds raised between 1980 and 1993. A fund is considered quasi-liquidated if it has cashflow information and has no cash flow from July 2002 to December 2003. We also eliminate funds that pay no dividend at all. Statistics for venture capital and buyout funds within each sample are reported separately. We report for each sample: (i) the average and median of the amount invested by funds in millions of dollars (Invested); (ii) the proportion of first time funds; (iii) the proportion of non-US funds (in numbers). The last two rows present the number of cash flows (monthly frequency) and the number of funds.

	All	VC	BO
Mean Invested (mn. Dec 2003)	132	81	262
Median Invested (mn. Dec 2003)	59	48	119
First time (%)	37	34	43
Non-US (%)	28	21	48
Number of cash flows	20,331	13,729	6,602
Number of funds	797	573	224

Table 2: Biased Aggregate Beta: Simulation Example

This table shows that Dimson's aggregate beta method generates a bias if the stock price has a positive expected return. The simulation setup is as follows: the true price follows a CAPM with alpha equal to zero, beta equal to one, zero risk-free rate and no idiosyncratic risk. The market excess return follows a normal distribution with mean μ and volatility of 10%.

$$P_{i,t} = P_{i,t-1}(1 + r_{m,t}) \quad \text{where} \quad r_m \sim N(\mu, 0.01)$$

The price revelation process is

$$E(\widehat{P}_{i,t}) = \theta P_{i,t} + (1-\theta)\widehat{P}_{i,t-1}$$

so that the realization of the observed price at t will be either the true price at t or the observed price at $t-1$. The θ is set to be $\frac{1}{8}$ as before, which means the true price is on average revealed every 8 periods. Column 3, 4, 5, 6 and 7 report the aggregate betas with 0, 4, 8, 20 and 50 lags respectively. We run 1,000 simulations with 10,000 periods per simulation for three different levels of μ. We report both the mean and the median of the aggregate beta.

		β_0	$\sum_{i=0}^{4}\beta_i$	$\sum_{i=0}^{8}\beta_i$	$\sum_{i=0}^{20}\beta_i$	$\sum_{i=0}^{50}\beta_i$
$\mu = 0$	mean	0.13	0.49	0.70	0.94	1.00
	median	0.12	0.49	0.70	0.94	0.99
$\mu = 1\%$	mean	0.13	0.53	0.77	1.07	1.15
	median	0.14	0.53	0.77	1.07	1.14
$\mu = 2\%$	mean	0.14	0.58	0.86	1.23	1.35
	median	0.15	0.58	0.86	1.23	1.34

Table 3: Results for the Simulated Private Equity Economy

Panel A reports the parameter values used for the simulation of private equity fund cash flows and NAVs, as described in section 5. We simulate 700 funds, with each fund investing in 20 projects, over a period of 24 years. We run 1000 simulations and report the mean and the standard deviation of the estimates. Panel B shows the risk and return estimates with 40% quarterly idiosyncratic volatility for the NAV-regression model with 4 lags, 8 lags and the GMM method. The high quarterly idiosyncratic volatility reflects the number reported by Cochrane (2005a). Panel C reports the results when a fund invests in 50 rather than 20 projects to see the large sample properties of the GMM method.

Panel A: Calibration of the simulation

	Benchmark	High Perf.	Low Perf.
Alpha (per quarter)	0	1%	-1%
Beta	1	1.5	1.5
Risk-free rate (per quarter)	1%	1%	1%
Market expected return (per quarter)	3%	3%	3%
Market volatility (per quarter)	10%	10%	10%
Frequency of revealed value	0.125	0.125	0.125
Idiosyncratic volatility (per quarter)	40%	40%	40%

Panel B: Risk and Return Estimates: 20 projects per fund

Benchmark ($\alpha = 0, \beta = 1$)						
	Agg.β with 4 lags		Agg.β with 8 lags		GMM-approach	
	Alpha(%)	Beta	Alpha(%)	Beta	Alpha(%)	Beta
Mean	0.83	0.60	0.32	0.91	-0.05	1.01
Std	0.53	0.14	0.59	0.21	0.70	0.38

High Performance ($\alpha = 1\%, \beta = 1.5$)						
	Agg.β with 4 lags		Agg.β with 8 lags		GMM-approach	
	Alpha(%)	Beta	Alpha(%)	Beta	Alpha(%)	Beta
Mean	2.33	1.01	1.49	1.51	0.95	1.53
Std	0.78	0.21	0.84	0.31	0.70	0.39

Low Performance ($\alpha = -1\%, \beta = 1.5$)						
	Agg.β with 4 lags		Agg.β with 8 lags		GMM-approach	
	Alpha(%)	Beta	Alpha(%)	Beta	Alpha(%)	Beta
Mean	-0.05	0.86	-0.81	1.34	-1.06	1.50
Std	0.63	0.17	0.63	0.25	0.66	0.38

Panel C: Risk and Return Estimates: 50 projects per fund

Benchmark ($\alpha = 0, \beta = 1$)						
	Agg.β with 4 lags		Agg.β with 8 lags		GMM-approach	
	Alpha(%)	Beta	Alpha(%)	Beta	Alpha(%)	Beta
Mean	0.82	0.60	0.32	0.91	-0.01	0.99
Std	0.48	0.12	0.51	0.17	0.45	0.22

High Performance ($\alpha = 1\%, \beta = 1.5$)						
	Agg.β with 4 lags		Agg.β with 8 lags		GMM-approach	
	Alpha(%)	Beta	Alpha(%)	Beta	Alpha(%)	Beta
Mean	2.33	1.01	1.50	1.51	0.99	1.52
Std	0.71	0.19	0.72	0.29	0.37	0.22

Low Performance ($\alpha = -1\%, \beta = 1.5$)						
	Agg.β with 4 lags		Agg.β with 8 lags		GMM-approach	
	Alpha(%)	Beta	Alpha(%)	Beta	Alpha(%)	Beta
Mean	-0.05	0.89	-0.80	1.33	-1.01	1.48
Std	0.58	0.15	0.50	0.22	0.41	0.23

Table 4: Descriptive Statistics for the Fully-liquidated Sample

This table reports the descriptive statistics of the data used for the regressions for the final market value. The numbers are derived by averaging all funds at specific ages into one portfolio. For example, the variables in the "age 10" column are obtained by averaging the numbers reported by all funds with age 10 (1990 for vintage 1980, 1991 for vintage 1981..etc). NAV is the net asset value at age a. CI is the Capital Invested, which is the sume of all takedowns up to the age a. NAV/CI is the net asset value over the capital invested at age a. NCF is the sum of all net cashflows after age a. The discount rate for present value of net cashflows and the Profitibility Index is the S&P 500 return. We calculate the Profitability Index both with and without the final NAV. The last row is the number of funds with positive NAV for each age.

Age	10	11	12	13	14
NAV_a (mn.)	9,965	7,168	3,941	2,757	799
CI at a	31,743	28,006	28,229	25,220	21,243
NAV/CI at a (mn.)	0.31	0.26	0.14	0.11	0.04
NCF after a (mn.)	13,253	9,025	4,721	2,899	886
PVNCF after a (mn.)	9,477	6,978	3,736	2,515	760
NCF/NAV_a	1.33	1.26	1.20	1.05	1.11
PVNCF/NAV_a	0.95	0.97	0.95	0.91	0.95
PI with final NAV	1.04	0.96	0.97	1.00	1.08
PI without final NAV	0.91	0.86	0.91	0.96	1.07
# funds ($NAV_a > 0$)	282	231	187	142	89

Table 5: Results of Final Market Value Regressions

This table reports the regression results for age 6 to age 13. The regression model is as follows:

$$\ln(1 + PVNCF_{a,i}) = b_{a0} + b'_{a1}X_{a,i} + +\varepsilon_{a,i}$$

PVNCF is the present value of the net cashflows. The discount rate is based on the CAPM and simultaneously estimated within our GMM framework. The vector of explanatory variables $X_{a,i}$ includes the log value of the 1+NAV at the specific fund age a, the log of the committed capital, the log of the time elapsed since the last dividend distribution, the log of the time elapsed since the last NAV update, the Profitability Index excluding the final NAV at the age a, a dummy variable for venture capital, the NAV divided by capital invested at the age a, the square of this ratio, and the log of sequence number of the fund and year fixed effects.. For parsimony, only if more than 10 funds are raised within a vintage year a year fixed effect dummy is included into the regression model.

Age	10	11	12	13
$\ln(1 + NAV_a)$	0.88***	0.91***	0.84***	0.75***
	(0.07)	(0.07)	(0.11)	(0.10)
logsize	0.09*	0.04	0.11*	0.05
	(0.05)	(0.06)	(0.06)	(0.07)
loglastDiv	-0.07*	-0.10**	-0.05	-0.20***
	(0.04)	(0.05)	(0.05)	(0.07)
loglastNAV	-0.24***	-0.19***	-0.30***	-0.14**
	(0.04)	(0.05)	(0.08)	(0.07)
PI w/o NAV	0.01	0.22**	0.14	0.25**
	(0.05)	(0.09)	(0.09)	(0.11)
dummyVC	0.07	0.00	-0.16	0.10
	(0.11)	(0.13)	(0.13)	(0.17)
NAV/CI	0.15	-0.15	0.01	-0.35
	(0.30)	(0.27)	(0.34)	(0.40)
NAV/CI^2	0.04	0.06	0.02	0.06
	(0.08)	(0.05)	(0.06)	(0.05)
logSeq	0.07	0.05	0.00	0.16
	(0.11)	(0.13)	(0.13)	(0.18)
Adj. R^2	0.71	0.70	0.72	0.63
# obs	282	231	187	142

Table 6: The Predicted Market Values over the Final NAVs for Non-liquidated Funds

This table reports the ratio of the aggregated predicted final market values over the reported NAVs for funds raised between 1990 to 1993 which are not fully-liquidated. NAVs are written off for funds raised before 1989 (more than 14 years old). The first row reports the results of using fully-liquidated funds for the in-sample estimation (Table 5). The other rows report the results using alternative estimation-samples consisting of funds with more than 50%, 25%, 75% reduction of their NAVs between age t and Dec. 2003 respectively. The larger alternative estimation samples by construction include all fully-liquidated funds.

	Vintage 90~93	Vintage 90	Vintage 91	Vintage 92	Vintage 93
Fully-liquidated	28.7%	9.7%	24.8%	31.0%	40.0%
50% reduction	24.8%	16.0%	22.1%	20.5%	33.3%
25% reduction	24.9%	16.9%	24.1%	20.3%	31.8%
75% reduction	26.0%	16.4%	25.3%	21.3%	33.6%
# obs out-of-sample		36	31	36	41

Table 7: Benchmark Results

This table reports results of GMM estimation of abnormal returns and risk exposures, based on cash flows of fully-liquidated and quasi-liquidated funds, with final market values as predicted by the regression in Table 5. Estimation is performed by grouping funds into funds-of-funds portfolios. The funds are first grouped based on their vintage year and then based on their committed capital. There are 56 funds-of-funds for Venture Capital and 42 funds-of-funds for Buyouts in total. We bootstrap 1,000 times to obtain the standard errors. ***, **, * indicate significance levels at 1%, 5% and 10%.

	VC		BO	
	CAPM	FF	CAPM	FF
α (%, per month)	-1.28***	-0.83*	0.26	-0.75
	(0.20)	(0.46)	(0.38)	(0.48)
β_{market}	2.18***	1.93***	0.09	1.04*
	(0.43)	(0.60)	(0.44)	(0.63)
β_{SMB}		0.25		-1.46***
		(0.70)		(0.56)
β_{HML}		-0.79**		1.14**
		(0.37)		(0.57)

Table 8: Results for Final NAV Treated as Final Market Value / Writing Off Final NAVs.

This table reports results of GMM estimation of abnormal returns and risk exposures, based on cash flows of fully-liquidated and quasi-liquidated funds. Panel A shows the GMM estimation results when NAVs are used as final market values for quasi-liquidated funds. Panel B shows the results for the final NAVs being written off. Both panels are estimated by grouping funds into funds-of-funds portfolios. The funds are first grouped based on their vintage year and then based on their committed capital. We bootstrap 1,000 times to obtain the standard errors. ***, **, * indicate significance levels at 1%, 5% and 10%.

	Panel A: Final NAV as accurate measure			
	VC		BO	
	CAPM	FF	CAPM	FF
α (%, per month)	-1.22***	-0.64**	-0.05	-0.81**
	(0.08)	(0.28)	(0.36)	(0.38)
β	2.92***	2.13***	0.62	1.32***
	(0.36)	(0.40)	(0.47)	(0.53)
β_{SMB}		0.98**		-1.03***
		(0.42)		(0.39)
β_{HML}		-0.61**		1.42***
		(0.30)		(0.54)

	Panel B: Final NAVs are written off			
	VC		BO	
	CAPM	FF	CAPM	FF
α (%, per month)	-0.84***	-0.12	0.52	-0.03
	(0.30)	(0.36)	(0.38)	(0.45)
β	1.25***	0.85**	-0.22	0.06
	(0.47)	(0.37)	(0.40)	(0.45)
β_{SMB}		1.22**		-1.02
		(0.50)		(0.66)
β_{HML}		-0.68**		1.21**
		(0.32)		(0.57)

Table 9: Estimation Results for the NAV-Regression Method: Quasi-liquidated Funds

The table reports estimation results for the NAV-regression method for the sample of all quasi-liquidated funds, which are funds raised between 1980 and 1993 with no cash flows activities between July 2002 to December 2003. Column one and two report the alpha and the aggregate beta with 4 lagged market returns (at the quarterly level). Column three and four report the alpha and the aggregate beta with 8 lagged market returns. The NAV-regression method calculates the time-series of returns from the quarterly reported NAVs and hence the alphas in this table are at quarterly frequency. The returns in the year 1980 are excluded because no NAV data are available in the first two quarters for buyout funds.

VC Funds

α(%, per quarter)	$\sum_{i=0}^{4}\beta_i$	α(%, per quarter)	$\sum_{i=0}^{8}\beta_i$
1.68^{**}	0.34^{**}	1.07^{*}	0.62^{***}
(0.63)	(0.15)	(0.64)	(0.18)

BO Funds

α(%, per quarter)	$\sum_{i=0}^{4}\beta_i$	α(%, per quarter)	$\sum_{i=0}^{8}\beta_i$
1.82^{**}	0.16	1.47^{*}	0.31
(0.77)	(0.18)	(0.83)	(0.23)

Table 10: Duration Effect in the Alpha

This table reports GMM estimation results with alpha being a function of the duration:

$$\alpha = \alpha_0 + \alpha_{duration} * \ln(Duration) + \beta_{SMB} * SMB + \beta_{HML} * HML$$

The duration of a fund is defined as the average time at which distributions are made minus the average time at which investment are made. When the average duration for a fund is smaller than one, which is typically due to no distributions for later investments, we treat it as 99th percentile. We also censor the right-tail outliers at 99th percentile. We bootstrap 1,000 times to obtain the standard errors. ***, **, * indicate significance levels at 1%, 5% and 10%.

	VC		BO	
	CAPM	FF	CAPM	FF
α_0(%, per month)	-0.57	-0.16	2.18*	1.22
	(0.75)	(0.70)	(1.33)	(1.26)
$\alpha_{duration}$(%, per month)	-0.16	-0.16	-0.48	-0.50*
	(0.15)	(0.15)	(0.31)	(0.29)
α (%, per month)				
25*th percentile*	-1.19	-0.78	0.46	-0.56
50*th percentile*	-1.24	-0.83	0.31	-0.73
75*th percentile*	-1.28	-0.86	0.15	-0.88
β	2.07***	1.88***	0.16	1.13*
	(0.45)	(0.61)	(0.40)	(0.62)
β_{SMB}		0.19		-1.46**
		(0.72)		(0.51)
β_{HML}		-0.79**		1.23**
		(0.37)		(0.55)

Table 11: Experience Effects in the Alpha and Beta

This table reports GMM estimation results with the alpha and beta being a function of the experience of the fund managers. Experience is a dummy which equals zero when a fund is a first-timer, and equals 1 otherwise. The specification is as follows:

$$\alpha = \alpha_0 + \alpha_{Experience} * DummyExp + \beta_{SMB} * SMB + \beta_{HML} * HML$$

$$\beta = \beta_0 + \beta_{Experience} * DummyExp + \beta_{SMB} * SMB + \beta_{HML} * HML$$

First four columns show the results when only the alpha is a function of the experience. The second four columns give the results when the beta is also a function of the experience dummy. We bootstrap 1,000 times to obtain the standard errors. ***, **, * indicate significance levels at 1%, 5% and 10%.

	Experience in Alpha				Experience in Alpha and Beta			
	VC		BO		VC		BO	
	CAPM	FF	CAPM	FF	CAPM	FF	CAPM	FF
α_0 (%, per month)	-1.33***	-0.90**	-0.40	-1.17**	-0.07	0.14	0.23	-0.47
	(0.24)	(0.45)	(0.44)	(0.52)	(0.57)	(0.64)	(0.62)	(0.70)
$\alpha_{Experience}$ (%, per month)	0.08	0.15	0.73**	0.70**	-1.30**	-1.10*	-0.25	-0.40
	(0.21)	(0.21)	(0.36)	(0.35)	(0.62)	(0.61)	(0.96)	(1.08)
β_0	2.19***	1.95***	0.28	1.06*	-0.01	-0.09	-0.56	0.07
	(0.40)	(0.42)	(0.41)	(0.59)	(0.88)	(0.95)	(0.66)	(0.76)
$\beta_{Experience}$					3.00***	2.76**	1.42	1.86
					(1.07)	(1.14)	(1.12)	(1.47)
β_{SMB}		0.25		-1.14**		0.18		-1.55***
		(0.68)		(0.50)		(0.72)		(0.48)
β_{HML}		-0.87**		0.82		-0.69*		0.66
		(0.37)		(0.55)		(0.42)		(0.56)

Table 12: Fund Size Effects in the Alpha and Beta

This table reports GMM estimation results with the alpha and the beta being a function of the log value of the fund's committed capital. The specification is as follows:

$$\alpha = \alpha_0 + \alpha_{size} * \ln(size) + \beta_{SMB} * SMB + \beta_{HML} * HML$$

$$\beta = \beta_0 + \beta_{size} * \ln(size) + \beta_{SMB} * SMB + \beta_{HML} * HML$$

First four columns show the results when only the alpha is a function of the fund size. The second four columns give the results when the beta is also a function of the fund size. We bootstrap 1,000 times to obtain the standard errors. ***, **, * indicate significance levels at 1%, 5% and 10%.

	logsize in Alpha				logsize in Alpha and Beta			
	VC		BO		VC		BO	
	CAPM	FF	CAPM	FF	CAPM	FF	CAPM	FF
α_0 (%, per month)	-2.08***	-1.52***	-0.89**	-1.65***	-1.44***	-0.70	1.89	3.12
	(0.26)	(0.46)	(0.36)	(0.49)	(0.51)	(0.82)	(1.36)	(1.99)
α_{size} (%, per month)	0.23***	0.23***	0.31***	0.29***	0.06	0.03	-0.25	-0.66*
	(0.04)	(0.04)	(0.07)	(0.06)	(0.10)	(0.12)	(0.28)	(0.36)
α (%, per month)								
25th percentile	-1.33	-0.76	0.34	-0.50	-1.24	-0.61	0.91	0.49
50th percentile	-1.19	-0.62	0.59	-0.26	-1.20	-0.59	0.70	-0.05
75th percentile	-1.06	-0.49	0.87	0.01	-1.16	-0.58	0.48	-0.65
β_0	1.91***	1.48***	-0.32	0.45	0.28	-0.28	-3.29**	-5.84**
	(0.40)	(0.57)	(0.39)	(0.61)	(0.78)	(1.07)	(1.48)	(2.39)
β_{size}					0.44***	0.46**	0.61*	1.33***
					(0.17)	(0.20)	(0.31)	(0.46)
β								
25th percentile					1.72	1.21	-0.88	-0.58
50th percentile					2.00	1.49	-0.38	0.41
75th percentile					2.25	1.75	0.18	1.72
β_{SMB}		0.52		-1.56***		0.65		-1.78***
		(0.63)		(0.52)		(0.69)		(0.46)
β_{HML}		-0.74**		0.84		-0.68*		0.66
		(0.35)		(0.55)		(0.36)		(0.56)