

Fiscal Stimulus and Distortionary Taxation*

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PRELIMINARY
COMMENTS WELCOME

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Abstract

We quantify the size, uncertainty and sensitivity of fiscal multipliers in response to the American Recovery and Reinvestment Act (ARRA) of 2009. To that end, we extend the benchmark Smets-Wouters (Smets and Wouters, 2007) New Keynesian model, allowing for credit-constrained households, a central bank constrained by the zero lower bound, government capital and a government raising taxes with distortionary taxation. We distinguish between short-run and long-run multipliers. For a benchmark parameterization, we find modestly positive short-run multipliers with a posterior mean of 0.65 and modestly negative long-run multipliers centered around -0.31. The multiplier is particularly sensitive to the fraction of transfers given to credit-constrained households, is sensitive to the anticipated length of the zero lower bound, is sensitive to the capital share and is nonlinear in the degree of price and wage stickiness. Reasonable specifications are consistent with substantially negative short-run multipliers within a short time frame. Furthermore, monetary policy is remarkably powerful: increasing the zero lower bound from 8 to 12 quarters improves output far more than does the ARRA, without raising inflation. We interpret this to cast doubt on the ability of New Keynesian models to predict the effects of monetary policy.

Keywords: Fiscal Stimulus, New Keynesian model, liquidity trap, zero lower bound, fiscal multiplier

JEL codes:

1 Introduction

“Fiscal Stimulus”, the size of “fiscal multipliers” and the impact of discretionary fiscal spending on GNP and unemployment, has once again become central to policy debates in wake of the financial crisis of 2008 and fiscal policy responses in a number of countries. In this paper, we therefore seek to quantify the size, uncertainty and sensitivity of fiscal multipliers in response to a “fiscal stimulus” as in the American Recovery and Reinvestment Act (ARRA) of 2009 in the United States, using an extension of a benchmark New Keynesian model.

From a purists’ perspective, this may be the wrong question to ask. Policy should care about welfare, rather than derivate measures such as GNP or unemployment. Moreover, it should seek to solve a Mirrlees-Ramsey problem, and use the best combinations of available tools and taxes to maximize welfare, subject to constraints imposed by markets and the asymmetry of information. We do not disagree. Indeed, there is a considerable literature on these topics. We address welfare issues in section 4.8, but they are not the main focus of this paper.

Indeed, many public debates focus on the effects of fiscal spending on GNP and unemployment. Economists have the tools to answer these questions, and therefore, perhaps they should. Several recent papers have addressed these issues. This paper seeks to make a contribution to this emerging literature. In essence, we seek to understand how much of the rather negative perspective on long-run multipliers Uhlig (2010b), due to distortionary taxation in a neoclassical growth model, survives in a model that takes a very Keynesian perspective. In a nutshell, the answer is: while the benchmark long-run multiplier is now modestly negative rather than substantially negative and while the precise answer is sensitive to some key assumptions and uncertain parameters, much survives indeed.

We view the following elements as important. First, “fiscal stimulus” takes time in practice, despite calls for immediate actions as in e.g. Blanchard (CITE). The ARRA (2009) therefore serves as a useful benchmark

and example for the speed at which fiscal policy tools can be deployed, as emphasized by Cogan et al. (2010). Second, government expenditures are financed eventually with distortionary taxes, creating costly disincentive effects, a point emphasized by Uhlig (2010b). Third, monetary policy and its restrictions due to the zero lower bound on interest rates can matter substantially for the effectiveness of “fiscal stimulus”, as emphasized by Eggertsson (2010) as well as Christiano et al. (2009), in particular if there are sticky prices and wages. Fourth, transfers are a substantial part of the ARRA and similar programs: the degree to which they are given to credit-constrained households may matter considerably, see Coenen et al. (2010). Finally, model coefficients are uncertain and results are sensitive to specific assumptions. For that reason, we use a reasonably tractable “small-scale” model rather than a larger “black box”, employing Bayesian estimation techniques as well as sensitivity analysis to quantify the uncertainty in our answers.

The analysis here has much in common and is inspired by Cogan et al. (2010), but there are a number of important differences. Like them, we start from the benchmark Smets-Wouters (Smets and Wouters, 2007) New Keynesian model and analyze the impact of the ARRA. In contrast to these authors, we allow for a government raising revenues with distortionary taxation, and we introduce credit-constrained consumers. These two features break the Ricardian equivalence of their baseline model, and move the analysis of transfers to center stage. Indeed, transfers comprise 59% of the ARRA, but their analysis has (necessarily) been absent in Cogan et al. (2010). A sizeable part of the ARRA takes the form of government investment: we therefore felt it necessary to extend the model to feature government capital. Finally, we study the interaction with monetary policy, when constrained by a zero lower bound.

We distinguish between short-run and long-run multipliers. For a benchmark parameterization, we find modestly positive short-run multipliers with a posterior mean of 0.65 and modestly negative long-run multipliers centered around -0.31. The multiplier is particularly sensitive to the fraction of transfers given to rule-of-thumb consumers, is sensitive to the anticipated length

of the zero lower bound, is sensitive to the capital share and is nonlinear in the degree of price and wage stickiness. Reasonable specifications are consistent with substantially negative short-run multipliers within a short time frame.

We find furthermore that monetary policy is remarkably powerful. Increasing the zero lower bound from 8 to 12 quarters improves output far more than does the ARRA, without raising inflation. We interpret this to cast doubt on the ability of New Keynesian models to predict the effects of monetary policy.

These models have also been criticized considerably for the lack of a financial sector, a feature likely for understanding the events of 2008, see Uhlig (2010a) (**KRUGMAN, BUTER**). We agree with this critique and therefore feature a financial friction per the “short cut” of allowing for time-varying wedges between the central bank interest rate, government bond rates and the return to private capital, following Hall (**CITE**). Our estimates show that these wedges are indeed the key to understanding the recession of 2007 to 2009. Understanding their nature more deeply should therefore be high on the research agenda, but is not the focus of this paper and beyond its scope.

Aside from the contributions cited above, the analysis here is related to a number of additional important contributions, notably Erceg and Linde (2010) as well as Leeper and various co-authors (Davig and Leeper, 2009; Leeper et al., 2010, 2009). Erceg and Linde (2010) point out that the marginal multiplier differs from the average multiplier: If the stimulus is successful, the economy leaves the binding ZLB earlier and the effect of additional spending is reduced. We address this issue by endogenizing the duration of the ZLB in robustness tests. Leeper et al. (2010) allow future government consumption and transfers to adjust in order to rebalance the government budget, and find larger long-run multipliers as a result than we do. Leeper et al. (2009) consider productive government investment and government capital, and point to several interesting implications on the fiscal multiplier. Including a quantitatively reasonable role government capital, as

we do, can therefore be of considerable importance for the analysis. Davig and Leeper (2009) allow for fiscal policy to switch between passive and active regimes in a New Keynesian model. Interestingly, they find the largest difference in multipliers due to switches in the monetary policy regime, which is related to our finding of a perhaps too potent monetary policy in these model.

The paper is structured as follows. Section 2 gives an overview of the model. Section 3 discusses the estimation and calibration procedure. It provides a decomposition of the shocks driving the 2007-2009 recession, and shows that financial frictions have been key, in stark contrast the full-sample variance decomposition. Both sections are complemented by a detailed technical appendix which provides all model details as well as code for replicating our results or calculating other fiscal experiments. Section 4 presents the main results on the fiscal multiplier. It provides a sensitivity analysis which highlights the main driving forces behind our results. In addition, it provides preliminary discussion of the welfare effects of the stimulus package. Section ?? concludes.

2 The model

The model is an extension of Smets and Wouters (2007), and we shall refer the reader to that paper as well as to the technical appendix for the complete details. Here we shall provide a brief overview as well as describe the extensions.

The Smets and Wouters (2007) model is a New Keynesian model, set in discrete time. There is a continuum of households. Workers supply homogeneous labor in monopolistic competition. Unions differentiate the labor supplied by households and set wages for each type of labor. Wages are Calvo-sticky. There is a continuum of intermediate good firms. They supply intermediate goods in monopolistic competition. They set prices. Prices are Calvo-sticky. Final goods use intermediate goods. Final goods are produced in perfect competition. Household have preferences for final goods, allowing

for habit formation, as well as leisure. Capital is produced with investment in the form of the final good, but there are adjustment costs to investment: given installed capital and previous-period investment, the marginal product of investment for producing new capital is decreasing. There is variable capital utilization.

We extend the model with several features. Briefly, we constrain the interest rate set by the central bank to be nonnegative. We let the government raise revenues with distortionary taxation. We introduce credit-constrained consumers. We feature government capital. We introduce a wedge between various returns, as a stand-in for financial frictions. There is a balanced real growth factor μ . We adopt the notation convention that variables indexed t are known in period t .

2.1 The zero lower bound

More precisely, the monetary authority follows a Taylor-type rule, but interest rates are bounded below by a constant slightly above zero. It is easier to describe it in its log-linearized form: for the original version, the reader is referred to the technical appendix. The central bank sets the log-deviation \hat{R}_t of the central bank return from its steady state to

$$\hat{R}_t^{FFR} = \max\{-(1 - \bar{R}^{FFR}) + \bar{\epsilon}, \hat{R}_t^{TR}\}$$

where \bar{R}^{FFR} is the steady state nominal return, $\bar{\epsilon} > 0$ is a constant set slightly above zero for technical reasons (and set to $\bar{\epsilon} = \frac{0.25}{400}$ in the numerical calculations, implying a lower bound of 25 basis points for the central bank interest rate), and \hat{R}_t^{TR} is the log-deviation for the shadow Taylor rule return,

$$\begin{aligned} \hat{R}_t^{TR} = & \psi_1(1 - \rho_R)\hat{\pi}_t + \psi_2(1 - \rho_r)(\hat{y}_t - \hat{y}_t^f) \\ & + \psi_3\Delta(\hat{y}_t - \hat{y}_t^f) + \rho_R\hat{R}_{t-1}^{TR} + ms_t \end{aligned}$$

where $\hat{\pi}_t$ is the log-deviation for inflation, \hat{y}_t is the log-deviation for output, \hat{y}_t^f is the log-deviation in the flexible-price version of the economy and ms_t

is a shock to the interest rate set by the central bank.

2.2 Households, distortionary taxation and financial frictions.

A fraction $1 - \phi$ of the household is unconstrained and solves an infinite-horizon maximization problem. The preferences of such a household j are given by

$$U = \mathbb{E} \left[\sum_{t=0}^{\infty} \beta^s \left(\frac{1}{1-\sigma} (c_t(j) - h c_{t-1}^{aggr})^{1-\sigma} \right) \exp \left(\frac{\sigma-1}{1+\nu} n_t(j)^{1+\nu} \right) \right] \quad (2.1)$$

where $c_t(j)$ is consumption of household j , $n_t(j)$ is its labor supply and c_t^{aggr} is aggregate consumption. $h \in [0, 1)$ captures external habit formation, σ denotes the inverse of the intertemporal elasticity of substitution, and ν equals the inverse of the labor supply elasticity. Households discount the future by $\beta \in (0, 1)$.

Following Trabandt and Uhlig (2010), we assume that the government provides transfers and collects linear taxes on labor income, capital income net of depreciation as well as consumption, adapted to the model here. The budget constraint of household j is therefore given by

$$\begin{aligned} & (1 + \tau^c) c_t(j) + x_t(j) + \frac{B_t^n(j)}{R_t^{gov} P_t} \\ & \leq s_t^{\text{unconstr}} + \frac{B_{t-1}^n(j)}{P_t} + (1 - \tau_t^n) \frac{W_t}{P_t} \left(n_t(j) + \lambda_{w,t} n_t^{(\text{aggr})} \right) + \\ & \quad + \left[(1 - \tau^k) \left(\frac{R_t^k u_t(j)}{P_t} - a(u_t(j)) \right) + \delta \tau^k \right] [(1 - \omega_{t-1}^k) k_{t-1}^p(j) + \omega_{t-1}^k k_{t-1}^{p,aggr}] + \frac{\Pi_t^p}{P_t}, \end{aligned}$$

and the capital accumulation constraint is given by

$$k_t^p(j) = \frac{(1 - \delta)}{\mu} k_{t-1}^p(j) + q_{t+s}^x \left(1 - \xi \left(\frac{x_t(j)}{x_{t-1}(j)} \right) \right) x_t(j),$$

where $c_t(j)$ is consumption, $x_t(j)$ is investment, $B_{t-1}^n(j)$ are nominal govern-

ment bond holdings, $n_t(j)$ is labor, $k_{t-1}^p(j)$ is private capital, and $u_t(j)$ is capacity utilization, all of household j and chosen by household j . R_t^{gov} is the nominal return for the one-period government bond from t to $t+1$ set at date t , $n_t^{(aggr)}$ is aggregate labor, P_t is the aggregate price level, W_t is aggregate wages, $\lambda_{w,t}$ is the aggregate mark-up from union-determined wages, R_t^k is the undistorted return on capital and ω_t^k is a friction or wedge on private capital markets. In the budget constraint, note that it enters as a variable known at date $t-1$, so that the distortions to future capital returns impacts on investment in the current period. Also note that the individual losses due to this wedge are redistributed in the aggregate, so that the wedge distorts investment decisions, but does not destroy aggregate resources directly. Π_t^p are nominal firm profits, q_{t+s}^x is an investment-specific technology parameter, $\xi(\cdot)$ are adjustment costs, satisfying $\xi(\mu) = \xi'(\mu) = 0$, $\xi'' > 0$, τ^c , τ^n , τ^k are taxes and s_t^{unconstr} are real transfers to unconstrained households, all taken as given by household j , and $a(\cdot)$ represents the strictly increasing and strictly convex cost function of varying capacity utilization. In particular, note taxing capital net of depreciation implies deducting a depreciation rate that depends on capacity utilization. Furthermore, the household receives labor income both directly from working as well as indirectly from the surplus that unions charge on labor: both sources of labor income are taxed.

The interest rate R_t^{gov} on government bonds, which unconstrained households can freely trade, equals the federal funds rate R_t^{FFR} up to an exogenous friction or wedge ω_t^{gov} :

$$R_t^{gov} = (1 + \omega_t^{gov})R_t^{FFR}.$$

In difference to Smets and Wouters (2007), the discount factor β of the households is not subject to shocks. Rather, we focus on the wedges ω_t^k and ω_t^{gov} on financial markets as part of the household budget constraint.

We assume that a fraction $\phi \in (0, 0.5)$ of the households is credit-constrained. In their version of the budget constraint, $B_{t-1}^n(j) = 0$, $x_t(j) = 0$ and $k_{t-1}^p(j) = 0$, i.e. these households do not save or borrow. They do re-

ceive profit income from intermediate producers (which equals zero in the steady state). Put differently, the budget constraint of a credit-constrained household j is

$$(1 + \tau^c)c_t(j) \leq s_t^{\text{constr}} + (1 - \tau_t^n)\frac{W_t}{P_t} \left(n_t(j) + \lambda_{w,t}n_t^{\text{(aggr)}} \right) + \frac{\Pi_t^p}{P_t}$$

where s_t^{constr} are the transfers to credit-constrained agents. As a justification, one may suppose that credit-constrained discount the future substantially more steeply, and are thus uninterested in accumulating government bonds or private capital, unless their returns are extraordinarily high. Conversely, these households find it easy to default on any loans, and are therefore not able to borrow. We hold the identity of credit-constrained households and thereby their fraction of the total population constant. Note that we allow the transfers s_t^{constr} to constrained households to differ from the transfers s_t^{unconstr} to the unconstrained households.

Wages are set by unions on behalf of the households, recognizing that each differentiated wage is Calvo-sticky. Since workers of the unconstrained households represent the majority in these unions, wages are set according to their preferences. Firms hire workers randomly from both types of households, so that labor supplied by both types of households is the same in equilibrium.

2.3 Government capital and policy feedback rules

Government capital K_{t-1}^g enters private production as an externality for the individual intermediate-goods firm, similar to the model in Barro and Sala-i Martin (1992). The technology of intermediate firm i is given by

$$Y_t(i) = \tilde{\epsilon}_t^a \left(\frac{K_{t-1}^g}{\int_0^1 Y_t(\ell) d\ell + \Phi \mu^t} \right)^{\frac{\zeta}{1-\zeta}} (K_t^{\text{eff.}(i)})^\alpha (\mu^t n_t(i))^{1-\alpha} - \mu^t \Phi,$$

where Φ are fixed costs, $K_t^{\text{eff.}}$ is effective capital used by firm i , created from aggregate private capital,

$$K_t^{\text{eff.}} = u_t k_{t-1}^p (1 - \phi)$$

(assuming symmetric choices for the unconstrained households), where ϵ_t^a is an exogenous, stochastic component of TFP, and where the services of government capital K_{t-1}^g are subject to congestion: what matters is the ratio of government capital to average gross output, i.e. inclusive of the fixed costs. As a result, the aggregate production function in the absence of price dispersion is given by

$$Y_t = \epsilon_t^a K_{t-1}^g{}^\zeta K_t^{s\alpha(1-\zeta)} (\mu^t n_t)^{(1-\alpha)(1-\zeta)} - \mu^t \Phi, \quad \epsilon_t^a \equiv (\tilde{\epsilon}_t^a)^{1-\zeta}.$$

where TFP in terms of the private factors of production is

$$TFP = \epsilon_t^a K_{t-1}^g{}^\zeta \mu^{(1-\alpha)(1-\zeta)t}$$

We assume that the accumulation of government capital is symmetric to the accumulation of private capital, i.e., is subject to a similar technology,

$$k_t^g = \frac{(1 - \delta)}{\mu} k_{t-1}^g + q_t^g \left(1 - \xi_g \left(\frac{x_t^g}{x_{t-1}^g} \right) \right) x_t^g$$

where $\xi_g(\mu) = \xi_g'(\mu) = 0$, $\xi_g''(\cdot) > 0$ represent adjustment costs, $q_t^{x,g}$ is a shock to the government-investment-specific technology parameter, and $\epsilon_t^{x,g}$ is additional, exogenous government investment. We assume that the capacity utilization of government capital and therefore its depreciation is constant. We assume that the government chooses investment to maximize the present discounted value of output net of investment costs, except for a discretionary fiscal stimulus, denoted by $\epsilon_t^{x,g}$ and set to zero at steady state. Put differently, the first-order condition of the government determines optimal government investment, while actual government investment may be higher by some amount chosen along the stimulus path. To enforce the expansion of

government investment, we stipulate that the government cannot undo the stimulus investment for the first twelve periods, but has to provide at least replacement for the depreciated ARRA investment – otherwise, the deviation from the optimality condition would imply complete crowding out.

We assume a feedback rule for labor tax rates as follows (for the full detail, see the technical appendix), following Uhlig (2010b). Break the period-by-period government budget constraint in two parts. On the “right side”, there is a “deficit” d_t , prior to new debt and labor taxes

$$d_t = \text{gov.spend.} + \text{subs.}_t + \text{old debt repaym.}_t \\ - \text{cons.tax rev.} - \text{cap.tax rev.}_t - \bar{\tau}^l \text{lab.income}_t$$

which needs to be financed on the “left side” with labor tax revenues and new debt,

$$\tau_t^l \text{lab.income}_t + \text{new debt}_t = d_t$$

Along the balanced growth path, there is a path for the debt level as well as the deficit \bar{d}_t . The labor tax rate is then assumed to solve

$$(\tau_t^l - \bar{\tau}^l) \text{lab.income}_t = \psi_\tau(d_t - \bar{d}_t) + \epsilon_{\tau,l}$$

where $\epsilon_{\tau,l}$ is a labor tax shock.

2.4 Shocks

We assume that there are ten stochastic processes driving the economy. Unless stated otherwise, the processes follow independent AR(1)’s in logs:

1. Technology $\tilde{\epsilon}_t^a$
2. Gov.bond wedge ω_t^{gov} : financial friction wedge between FFR and gov’t bonds.
3. Priv. bond wedge ω_t^k : financial friction wedge between gov’t bond returns and a component of the returns to private capital.

4. Gov. spending plus net export. Co-varies with technology.
5. Investment specific technology q_t^x (rel. price).
6. Gov. investment specific technology q_t^g (rel. price).
7. Monetary policy ms_t .
8. Labor tax rates $\epsilon_{\tau,l}$.
9. Mark-up for prices: ARMA(1,1).
10. Mark-up: wages: ARMA(1,1).

For the stimulus plan, we use three series, capturing the changes in transfers, government consumption and government investment. We followed the strategy of Cogan et al. (2010), but our decomposition of government spending into consumption and investment as well as the need to pay particular attention to transfers meant that we needed to reclassify the various spending categories, according to the American Recovery and Reinvestment Act (ARRA). As source, we have used the estimates by the CBO (2009) for the effects of the ARRA by budget title. The annual time path for these expenditures is directly taken from the CBO, whereas the distribution within each year is proportional to the Cogan et al. (2010) path within each year. The details on the components are contained in appendix 7.1. A graphical overview on the time path is presented in figure 1. Essentially, we decomposed their government spending path into a separate consumption and investment path, and furthermore included transfers. Most importantly, much of the transfers are “front-loaded”, i.e. occur earlier than government spending, while the “stimulus” government investment occurs later.

Furthermore, we assume that the central bank will leave the federal funds rate unchanged at near zero for eight quarters, and that this is fully anticipated, as of the first quarter of 2009. For the numerical calculations, the relaxation algorithm proposed by Juillard (1996) and implemented in Dynare is particularly convenient for the type of forward-simulation (rather than estimation) performed here. By solving a potentially time-varying system of

equations backward from terminal conditions, it allows to incorporate anticipated shocks even when they interact coefficients for example to “switch off” the interest rate rule temporarily.

3 Estimation and Analysis

3.1 Data and Estimation

We solve the model, using a log-linear approximation and Dynare. The first-order conditions and their log-linearized versions are in a technical appendix, available up on request. We estimate the model, using the following ten time series.

1. Output: Chained 2005 real GDP, growth rates.
2. Consumption: Private consumption expenditure, growth rates.
3. Investment: private fixed investment, growth rates.
4. Government investment: growth rates.
5. Hours worked: Civilian employment index \times average nonfarm business weekly hours worked index. Demeaned log.
6. Inflation: GDP deflator, quarterly growth rates.
7. Wages: Nonfarm Business, hourly compensation index. Growth rates.
8. FFR: Converted to quarterly rates.
9. Corporate-Treasury bond yield spread: Moody’s Baa index – 10 yr Treasury bond at quarterly rates, demeaned.
10. Dallas Fed gross federal debt series at par value. Demeaned log.

Sources and details for the data are described in appendix 7.1. We use an updated version of the Smets-Wouters dataset, for the range 1947:2-2009:4, using quarterly data and four periods for the start-up. In difference to the

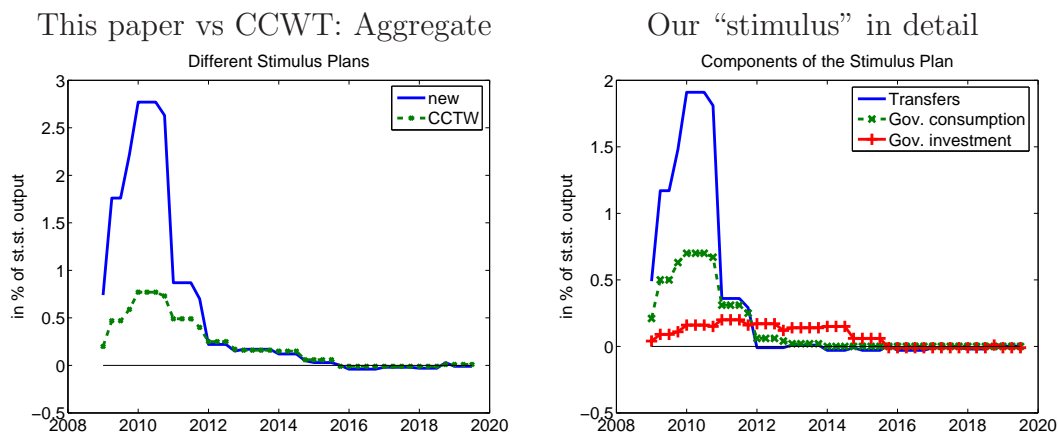
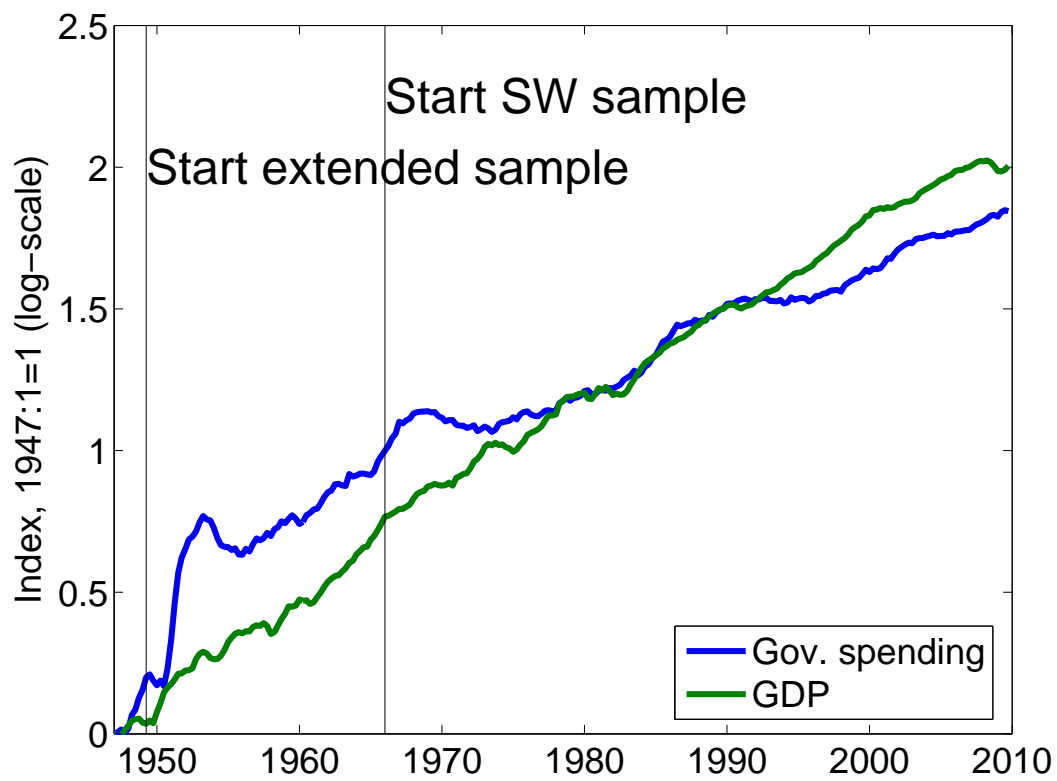


Figure 1: *Our three stimulus components and their comparison to Cogan et al. (2010). Essentially, we decomposed their government spending path into a separate consumption and investment path, and furthermore included transfers.*

original dataset, we classify consumer durables as investment expenditure. The estimation of the model uses data from 1948:2 up to 2008:4, with the additional four quarters for comparison of the model prediction to the actual evolution and the first four quarters used to presample. We choose the longer sample, as it includes the Korean war as well as the Vietnam war build-up, in contrast to the shorter Smets-Wouters sample from 1967 onwards. Figure 2 shows the additional evidence from the larger fluctuation in fiscal expenditures available in this larger sample.

We fixed (“calibrated”) several parameters a priori. For tax rates and the debt-GDP ratio, we relied on Trabandt and Uhlig (2010). Time averages of government spending components were obtained from the NIPA, Table 3.1 (quarterly), lines 35 (investment), 16 (consumption), transfers (17). Government consumption includes net exports (line 2 minus line 14 in Table 4.1). To obtain ratios relative to GDP, GDP data from line 1, Table 1.1.5 was used. Following Smets and Wouters (2007), the Kimball curvature parameter is taken from Eichenbaum and Fisher (2007), who set it to roughly match it to their data on the empirical frequency of price adjustment. Following Cooley and Prescott (1995), the depreciation rate is derived from the law of motion

Figure 2: Comparing our extended sample to the original Smets-Wouters data set. Notice the additional variation in government spending in the larger sample.



for capital and their observation of $\frac{\bar{x}}{k} = 0.0076$ at quarterly frequency. The complete list of calibrated parameters, and their comparison to the corresponding parameters in Smets and Wouters (2007), if available, is in table 1.

We estimate our model, using Dynare and a fairly standard Bayesian prior. Details on the estimation can be found in appendix 7.2. The estimates largely agree with those found by Smets and Wouters (2007), leaning somewhat more to more endogenous persistence: the habit parameter is slightly higher, as are estimates of price and wage stickiness, for example. Like these authors, our estimates also yield a rather small capital share: our posterior mean is 0.24, while they found 0.19. This is at odds with calibrated values in the literature, see e.g. Cooley and Prescott (1995), and may play a substantial role in calculating the long-horizon impact of distortionary taxation. We shall investigate this issue in our sensitivity analysis. The calibrated government investment-to-GDP ratio as well as the estimated growth trend $\mu \approx 1.005$ implies a government share in production of $\zeta = 2.30\%$.

3.2 Decomposing the 2007-2009 recession

The model allows the decomposition of movements in our ten macroeconomic time series into the ten shocks that caused them. Note that the first-order conditions of the households imply:

$$\begin{aligned} 1 &= \beta E_t \left[\frac{u_{c,t+1}}{u_{c,t}} \frac{R_t^{gov}}{\pi_{t+1}} \right] = \beta E_t \left[\frac{u_{c,t+1}}{u_{c,t}} (1 + \omega_t^{gov}) \frac{R_t^{FFR}}{\pi_{t+1}} \right] \\ &= \beta E_t \left[\frac{u_{c,t+1}}{u_{c,t}} \left((1 - \omega_t^k) [(1 - \tau^k)(r_{t+1}^k u_{t+1} - a(u_{t+1})) + \delta \tau^k] + (1 - \delta) \frac{Q_{t+1}}{Q_t} \right) \right] \end{aligned}$$

where ω_t^{gov} is due to government bond shocks and creates a wedge between between the FFR and government bonds, while ω_t^k is due to private bond shocks, creating a wedge between government bonds and private capital. Q_t is the price of capital. It is instructive to simplify the above expression by assuming a constant price of capital Q_t and constant capacity utilization as

	SW 66:1–04:4	This paper 48:2–08:4
Depreciation δ	0.025	0.0145
Wage mark-up λ_w	0.5	0.5
Kimball curvature goods mkt. $\hat{\eta}_p$	10	10
Kimball curvature labor mkt. $\hat{\eta}_w$	10	10
Capital tax τ^k	n/a	0.36
Consumption tax τ^c	n/a	0.05
Labor tax τ^n	n/a	0.28
Share credit constrained ϕ	n/a	0.25
Gov. spending, net exports-GDP $\frac{g}{y}$	0.18	0.153
Gov. investment-GDP $\frac{\bar{x}^g}{y}$	n/a	0.04
Debt-GDP $\frac{\bar{b}}{y}$	n/a	4×0.63

Table 1: *Calibrated parameters.*

well as ignoring uncertainty. Then the first line can be substituted in the second to yield:

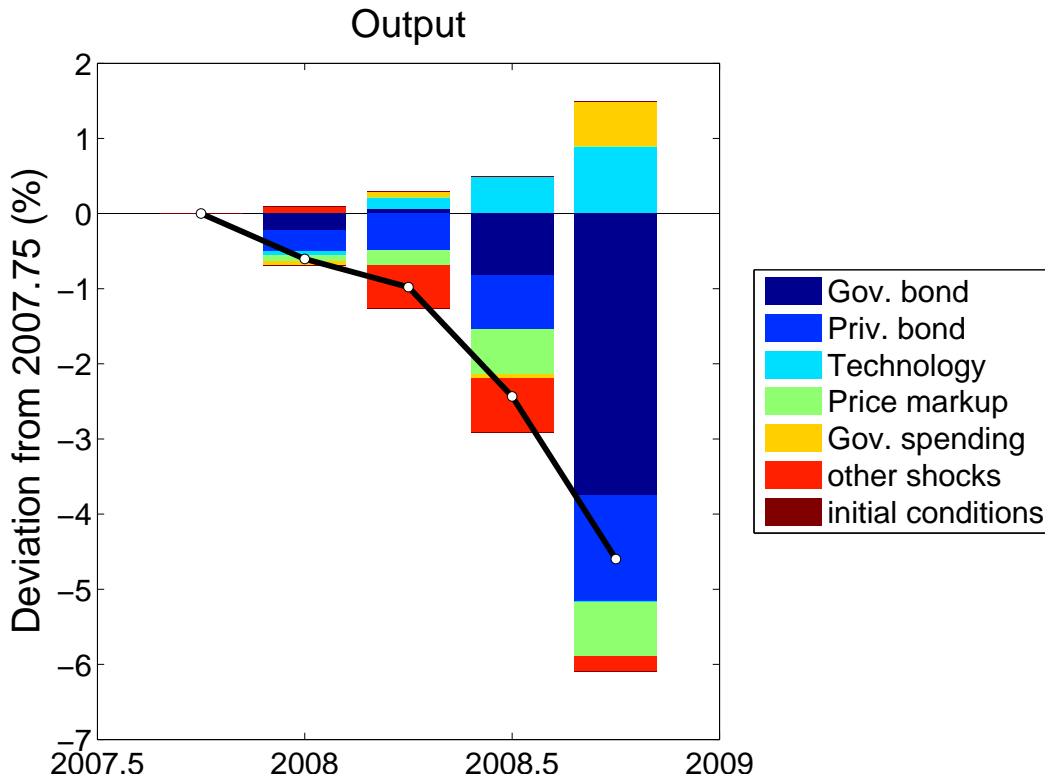
$$1 = \frac{1}{(1 + \omega_t^{gov})} \frac{\pi_{t+1}}{R_t^{FFR}} \left((1 - \omega_t^k) [r_{t+1}^k - \tau^k (r_{t+1}^k - \delta)] + (1 - \delta) \right).$$

This equation shows that, up to a first order approximation, the wedges ω_t^k (after re-scaling) and ω_t^b both add up to the total wedge between the return on private capital net of taxes and the Federal Funds Rate R_t^{FFR} . These wedges are stand-ins for financial frictions. It is therefore interesting to examine their role for the 2007-2009 recession.

As figure 3 as well as table 2 document, shocks to these wedges indeed played a large role in understanding the recent recession, in stark contrast to their small contribution to the full-sample variance of output as well as other included time series. Figure 4 provides the impulse response to a one-standard deviation shock to these two wedges. As one can see, the government bond shock depresses output, consumption and private as well as government investment, whereas the shock to the spread between private bonds and government bonds leads to a decline in consumption only with some delay and actually increases government investments. These shocks

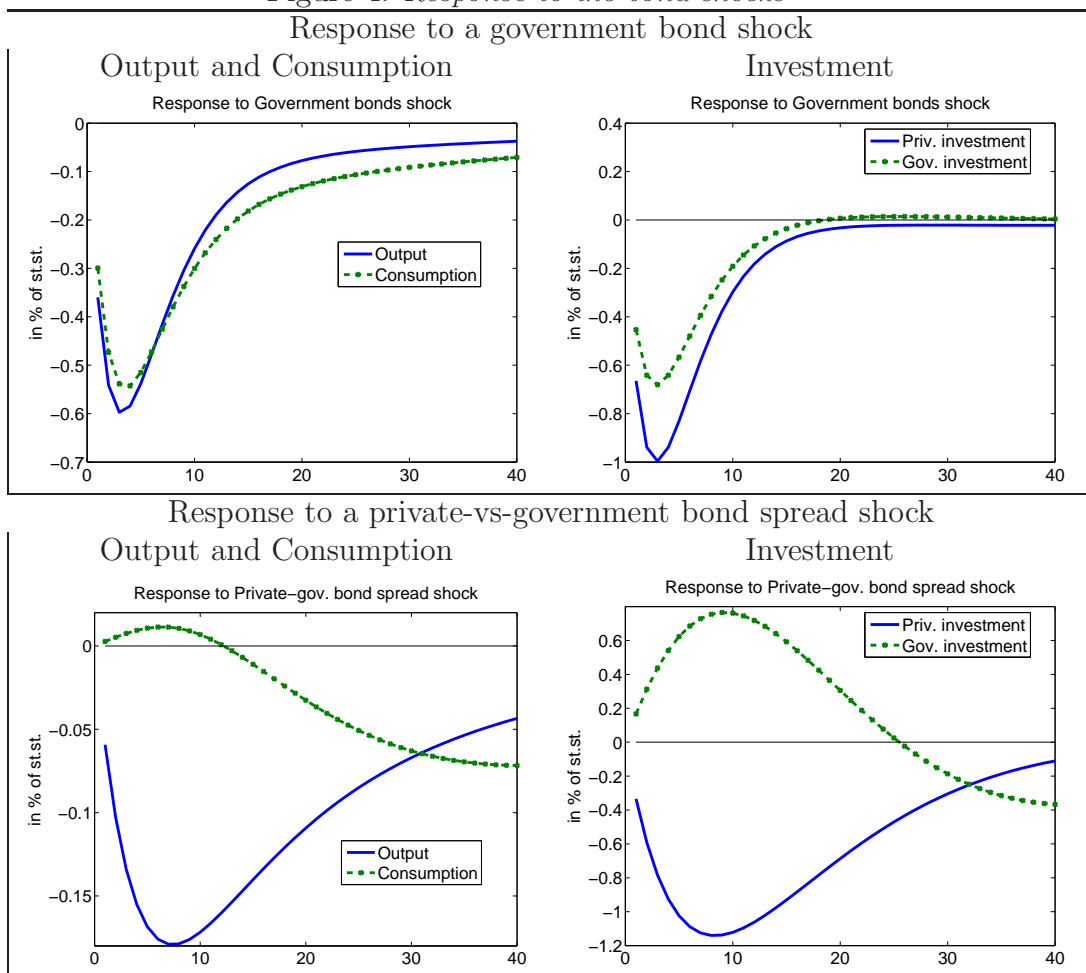
furthermore result in a modest decline in the federal funds rate (not shown).

Figure 3: *Historical Shock Decomposition: Output. Results are at the posterior median. 2007:4 is the NBER recession date.*



Since not only GDP growth but also unemployment is at the center of many public debates, we back out a predicted change in the unemployment rate from the model. To that end we regress the quarterly unemployment rate on the hours worked measure used to estimate model and use the implied OLS estimate to infer the effect on the unemployment rate. We neglect the additional parameter uncertainty introduced because of the uncertain estimates of the regression coefficients. Table 14 summarizes the estimated model and figure 17 shows the actual vs. predicted time series. The fit is reasonable with an R^2 of 0.77.

Figure 4: *Response to the bond shocks*



Shock	2008:4 vs. 2007:4		Total Sample
	Historical decomposition		
	%	%	
Gov. bond	-3.75	81.52	6.50
Priv. bond	-1.42	30.81	1.63
Technology	0.90	-19.53	19.21
Price markup	-0.73	15.86	8.59
Gov. spending	0.60	-12.98	4.14
Priv. inv.	-0.30	6.53	16.78
Labor tax	-0.27	5.91	9.20
Monetary pol.	0.20	-4.44	20.88
Wage Markup	0.15	-3.18	8.16
Gov. inv.	0.03	-0.73	4.92
Initial Values	-0.01	0.22	n/a
Sum	-4.60	100.00	100.01

Table 2: *Historical decomposition of recent recession and overall variance decomposition for output. All numbers are at the Bayesian posterior median.*

4 Results

Armed with our posterior estimates as well as the specification of the stimulus path, we shall now proceed to calculate the implied effects. We provide confidence bands, covering 90% or 67% of the posterior probability.

Our main focus is on the fiscal multiplier, i.e. the ratio of output changes to the total stimulus-planned change in spending and transfers. Note that due to the eventual balancing of the government budget, there will also be an induced movement in tax rates as a “secondary” effect. As is customary, we shall not include these secondary movements in the denominator, i.e. in quantifying the stimulus-planned changes. As this is a dynamic model, the horizon plays a role. Following Uhlig (2010b), we use the net present value fiscal multiplier φ_t , dividing the net present value of output changes up to some horizon t by the change in government spending and transfers until the same time. I.e., we shall use

$$\varphi_t = \sum_{s=1}^t \left(\mu^s \prod_{j=1}^s R_{j,ARRA}^{-1} \right) \hat{y}_s / \sum_{s=1}^t \left(\mu^s \prod_{j=1}^s R_{j,ARRA}^{-1} \right) \hat{g}_s \quad (4.1)$$

where φ_t : horizon- t multiplier, $R_{j,ARRA}$ is the government bond return, from $j - 1$ to j , \hat{y}_s is the output change at date s due to ARRA in percent of the balanced-growth GDP path and \hat{g}_s : ARRA spending at date s in percent of the balanced-growth GDP path.

4.1 Benchmark results

Figure 5 contains our benchmark results for output, the unemployment rate, the federal funds rate, inflation, and government debt. These graphs are perhaps reminiscent of the information shown in the official White House piece by Bernstein and Romer (2009). However, we include an important piece of information, which is missing there. The short-run debt dynamics shown here induce a long-run debt-and-tax dynamics, shown in figure 6. The increase in labor tax rates long after the fiscal stimulus phase has finished induces the decline of output for many years to come.

The resulting fiscal multiplier will therefore decline with the horizon. The fiscal multipliers for the shorter horizon, shown in the left panel of figure 7 can therefore be quite misleading in terms of assessing the long-term costs of fiscal stimulus. Indeed, the long-run multipliers are considerably smaller or negative, compared to the short-run multipliers, as show in the right panel of figure 7. These results are qualitatively in line with Uhlig (2010b), though the results are quantitatively rather different: the long-run fiscal multipliers are negative there and here, but considerably more negative there. One may be tempted to read the difference as “relief” compared to the pessimistic scenario in Uhlig (2010b). Note, however, that the model here is heavily tilted towards a model in which fiscal stimulus is often thought to work well: we therefore believe that the negative long-run effects of fiscal stimulus should give pause to arguments in its favor.

4.2 Sensitivity analysis: overview

Which features of the model contribute to the size of the fiscal multipliers, which are particularly important? Where does the difference to Uhlig (2010b)

Figure 5: Benchmark impact of ARRA.

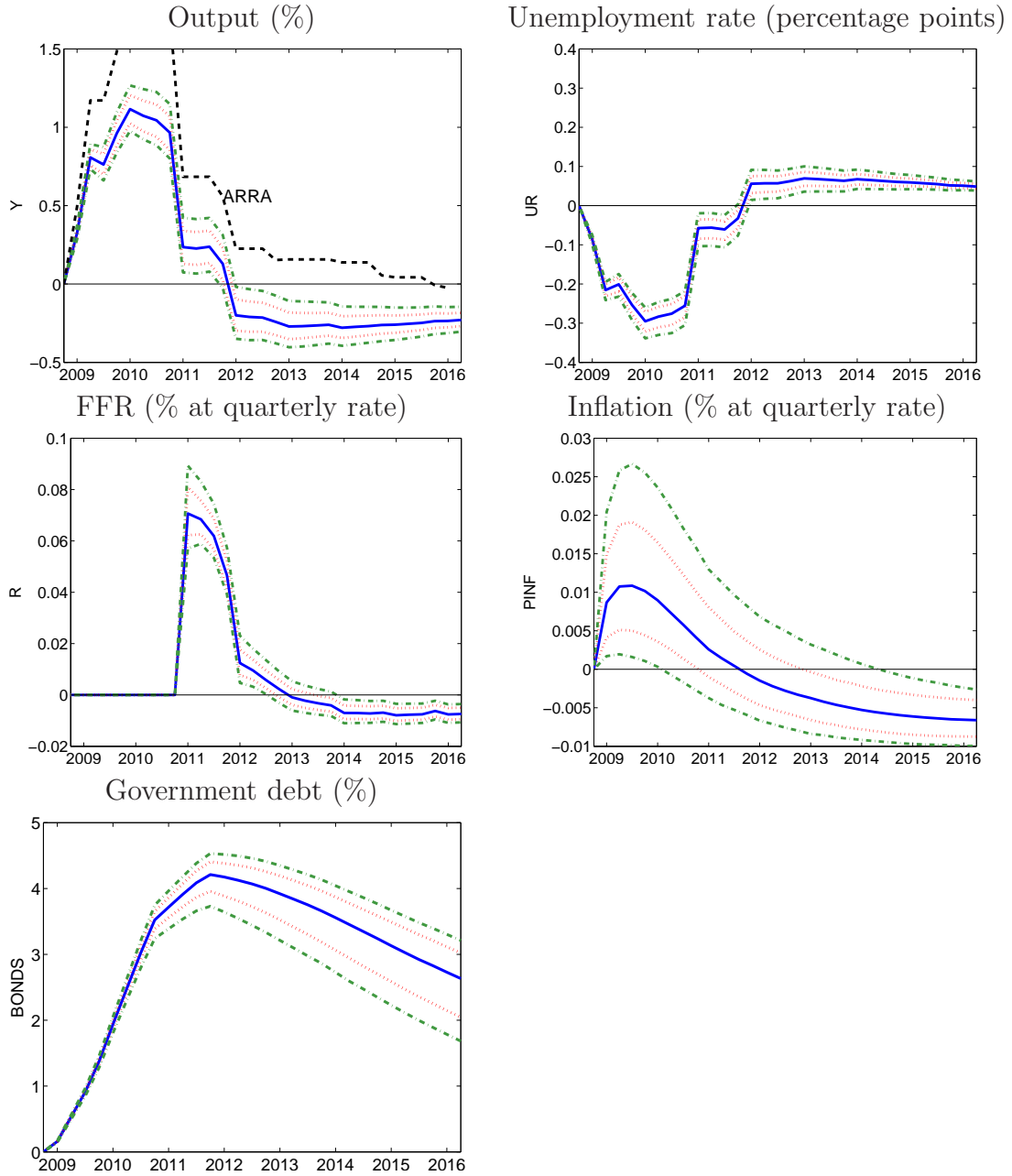


Figure 6: Short- and long-run impact of ARRA.

short-run:

long-run:

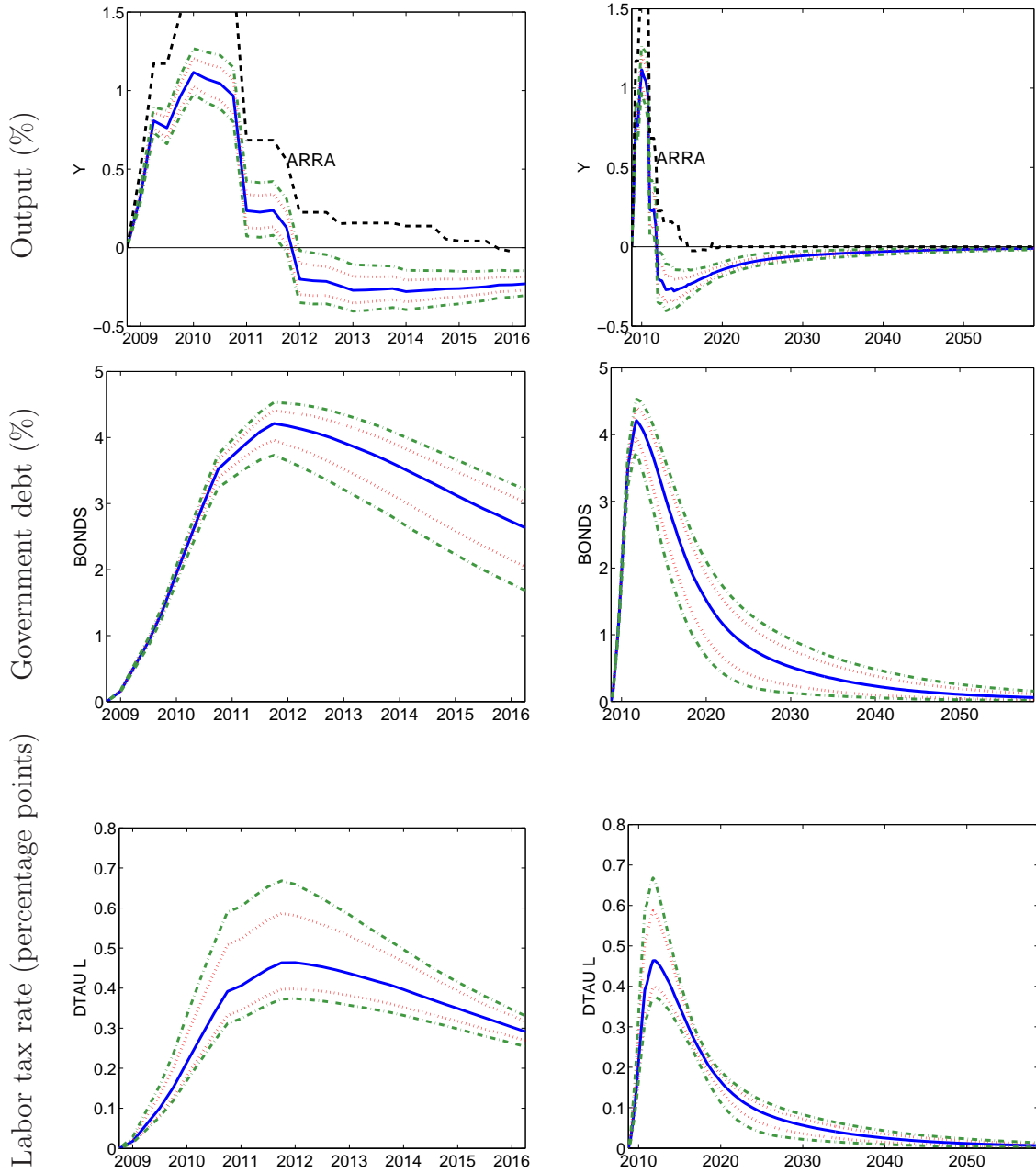
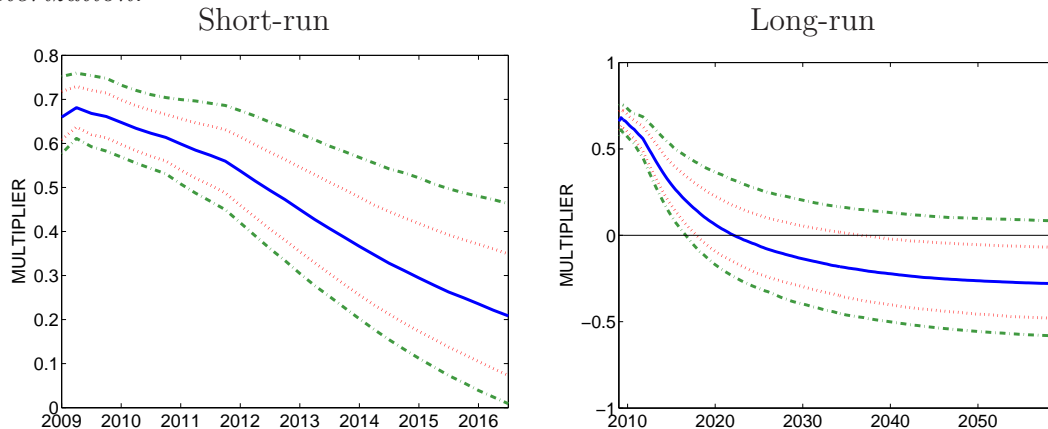


Figure 7: *Short-run and long-run fiscal multipliers in the benchmark parameterization.*



come from? Understanding the differences and understanding the sensitivity of the benchmark results to key assumptions is important. Figure 8 as well as table 3 and table 4 provide an overview of our sensitivity analysis. Let us proceed to the details.

4.3 Sensitivity to distortionary taxation

A number of models in the literature use models designed first for the purpose of analyzing monetary policy, and which therefore abstract from fiscal considerations, such as distortionary taxation. This can make a considerable difference, in particular at longer horizons. Figure 9 compares the long-run multiplier dynamics for distortionary taxes vs lump-sum taxes. As should be clear, distortionary rather than lump-sum taxation makes a considerable difference and creates considerably lower long-run multipliers.

4.4 Sensitivity to the length zero lower bound

The literature has emphasized the sensitivity of fiscal multipliers to the zero lower bound, and to generating “fiscal stimulus”, while the central bank is not changing its interest rates, see Eggertsson (2010) as well as Christiano et al. (2009). Our benchmark has been set to 8 quarters, implying that at the

Figure 8: *Comparison of long-run multipliers: medians of posterior distributions.*

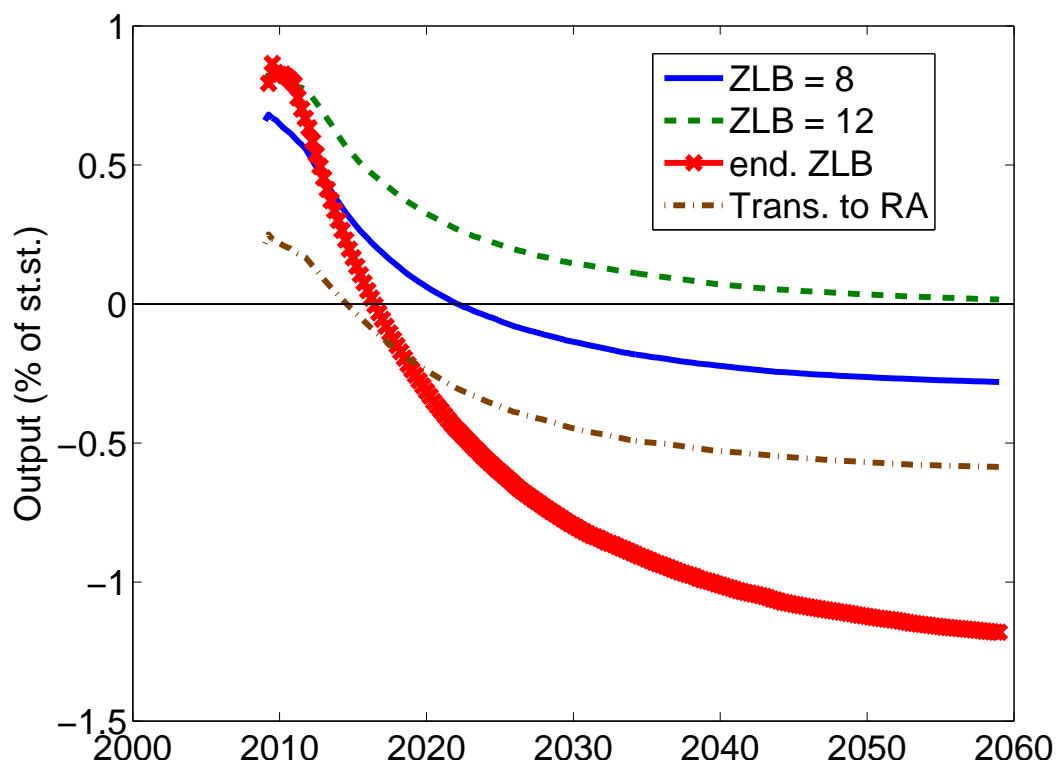


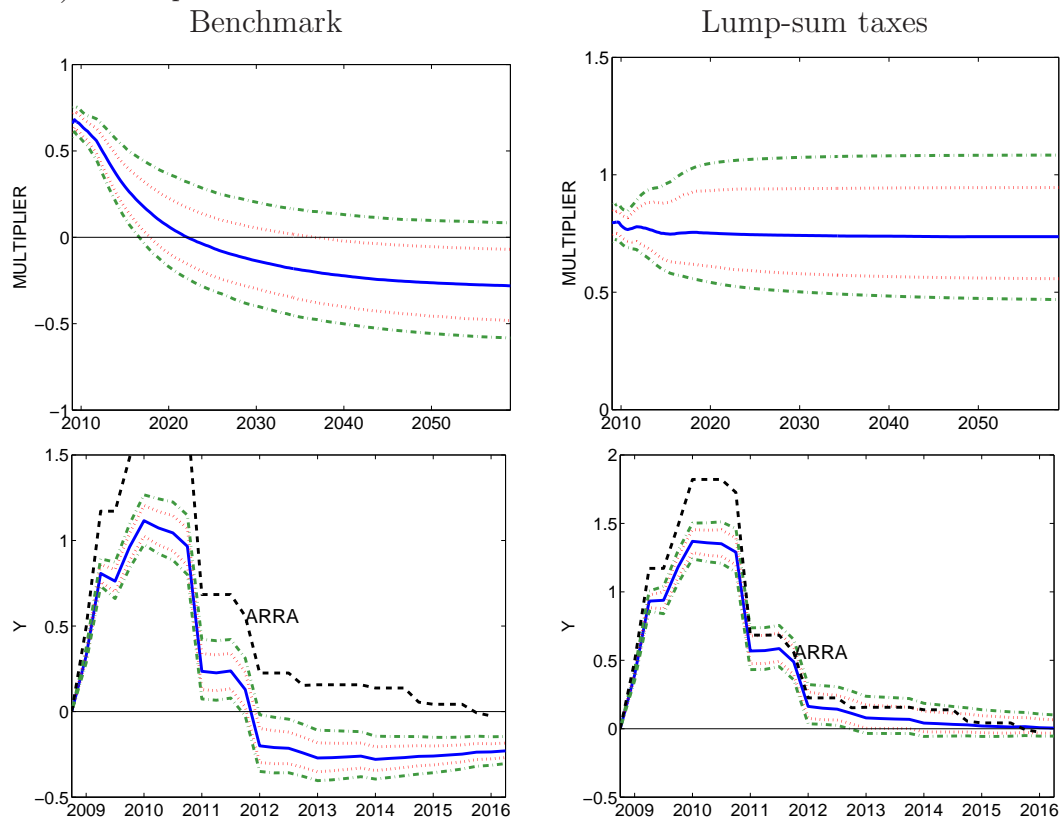
Table 3: Long run fiscal multipliers as $t \rightarrow \infty$: sensitivity

Scenario	5%	16.5%	median	83.5%	95%
Benchmark	-0.64	-0.52	-0.31	-0.09	0.06
lump-sum taxes	0.46	0.56	0.74	0.94	1.09
ZLB: 0 Quart.	-0.94	-0.82	-0.65	-0.49	-0.37
ZLB: 12 Quart.	-0.41	-0.27	-0.01	0.28	0.54
ZLB: Endogenous	-1.78	-1.53	-1.18	-0.75	-0.49
RoT=15%	-0.81	-0.68	-0.47	-0.26	-0.08
RoT=40%	-0.40	-0.31	-0.11	0.16	0.37
Share transfers to RoT=12.5%	-0.81	-0.69	-0.51	-0.33	-0.22
Share transfers to RoT=50%	-0.69	-0.56	-0.26	0.03	0.25
Share transfers to RoT=100%	-0.67	-0.39	0.06	0.54	0.93
Priv. capital share=35%	-0.98	-0.77	-0.54	-0.26	-0.13
price/wage-stickiness=10% of estim.	-0.90	-0.80	-0.66	-0.57	-0.52
price/wage-stickiness=40% of estim.	-0.74	-0.64	-0.49	-0.39	-0.34
price/wage-stickiness=115% of estim.	-0.60	-0.47	-0.23	-0.06	0.08

Table 4: One-year fiscal multipliers: sensitivity

Scenario	5%	16.5%	median	83.5%	95%
Benchmark	0.57	0.60	0.65	0.70	0.73
lump-sum taxes	0.71	0.74	0.78	0.83	0.86
ZLB: 0 Quart.	0.30	0.32	0.37	0.42	0.44
ZLB: 12 Quart.	0.71	0.74	0.80	0.88	0.94
ZLB: Endogenous	0.58	0.67	0.83	0.98	1.09
RoT=15%	0.50	0.53	0.58	0.62	0.66
RoT=40%	0.68	0.72	0.77	0.83	0.88
Share transfers to RoT=12.5%	0.36	0.37	0.41	0.44	0.47
Share transfers to RoT=50%	0.64	0.68	0.74	0.81	0.85
Share transfers to RoT=100%	1.01	1.09	1.19	1.30	1.37
Priv. capital share=35%	0.56	0.61	0.66	0.72	0.75
price/wage-stickiness=10% of estim.	0.18	0.20	0.25	0.31	0.33
price/wage-stickiness=40% of estim.	0.43	0.45	0.51	0.60	0.64
price/wage-stickiness=115% of estim.	0.62	0.64	0.67	0.71	0.73

Figure 9: Long-run fiscal multipliers. Comparing distortionary taxes (benchmark) to lump-sum taxation.



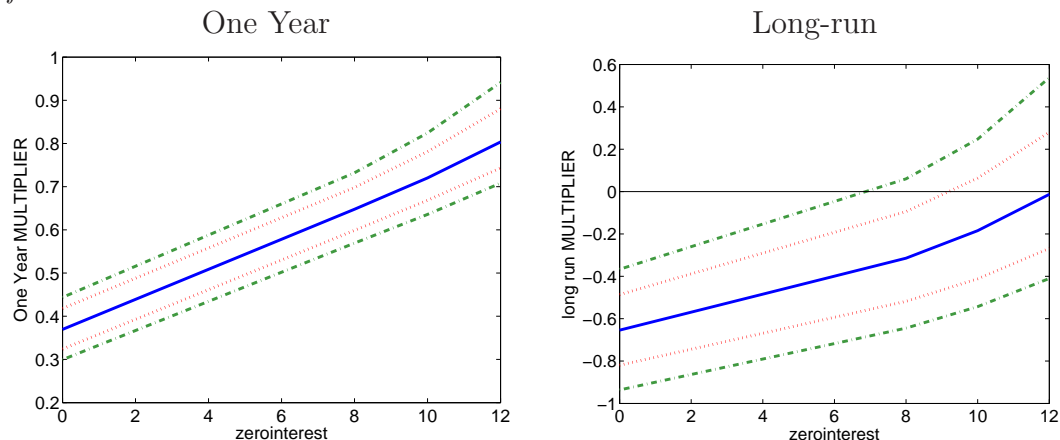
beginning of 2009, households anticipated the zero lower bound constraint to no longer bind at the beginning of 2011. That time horizon seems to have been extended meanwhile. However, it is hard to argue that this was anticipated two years ago. Figure 18 in the appendix shows that with an endogenous ZLB, two years are an upper bound for the expected duration. Nonetheless, we provide some experimentation here. Figure 10 provides that sensitivity analysis. It shows that when we endogenize the ZLB the resulting multipliers are significantly smaller since a successful stimulus shortens the ZLB and thereby reduces its effectiveness. By fixing the ZLB duration, we therefore overstate the fiscal multiplier to some extent.

The differences are quite large indeed. It may be important to remark, however, that varying the length of maintaining the Federal Funds Rate at zero has quite a dramatic effect on output all by itself, and all apparently without much effect on inflation. Starting at the historically given situation as of 2008:Q4¹ and given the fiscal stimulus as calculated above, table 5 and table 6 compare the effects on GDP and inflation, when moving the a priori imposed length of the zero lower bound away from the benchmark value of eight quarters to a range between 0 and 16 quarters. When we choose this value to be zero, the central bank will set the nominal interest rate according to the maximum of zero and the shadow Taylor rule. I.e., the difference in these scenarios is for how long the central bank chooses to hold the nominal interest rate near zero, compared to the shadow Taylor rule, shown in figure 11 for the benchmark of eight quarters ZLB. The probability of exit from the zero lower bound in a period earlier than 8 quarters is with more than 70% actually considerable.

The numbers shown in table 5 and table 6 are large, and perhaps too large: Comparing the effect of the stimulus as the ZLB in table 5 with the pure effect of imposing a ZLB without stimulus in table 6 shows that monetary policy has a dramatic influence on output. These numbers may indicate that this model, originally intended for within-central-bank policy debates,

¹The starting point of 2008:Q4 is important in this experiment in order to have the zero lower bound be “reasonable” in the first place

Figure 10: *Short-run and long-run fiscal multipliers: sensitivity to the length of the zero lower bound.*



assigns too much power to monetary policy. The model furthermore implies that the impact on inflation is remarkably small. Essentially, the forward-looking price-setting firms and workers take into account, that the central bank will “snuff out” any extra inflation, once it returns to its standard Taylor rule. In line with these results, figure 12 shows that the Taylor rule coefficient of the output gap has a large influence on the multiplier, whereas the inflationary consequences of the stimulus are so low that the reaction coefficient matters little. These results, taken at face value, indicate that the central bank can dramatically stimulate the economy, by keeping interest rates constant for, say, two or three years even under normal circumstances, with little consequences for inflation, provided it will credibly return to a Taylor rule later on. One may have doubts that this is truly a reasonable prediction of this model, and raising skepticism regarding the suitability for its originally intended use. Discussing this issue in detail is beyond the scope of this paper, however.

4.5 Sensitivity to credit-constrained households

The “credit-constrained” or “rule-of-thumb” households are important in two respects. First, there is a sizeable portion of the population, which violates

Table 5: *Extra output and inflation due to stimulus and ZLB, when holding nominal interest rates at zero for k quarters, rather than eight quarters. Results at the posterior median.*

ZLB imposed for ...	Output change (in %)			Inflation change (in %)		
	1 yr	5 yr's	NPV	1 yr	5 yr's	10 yr's
$k = 0$ quarters	0.504	-0.271	-11.917	0.010	-0.001	-0.005
$k = 4$ quarters	0.588	-0.263	-10.725	0.010	-0.002	-0.005
$k = 8$ quarters	0.960	-0.217	-5.293	0.014	-0.003	-0.007
$k = 12$ quarters	1.260	-0.154	0.962	0.021	-0.004	-0.009
$k = 16$ quarters	1.407	-0.090	5.552	0.027	-0.005	-0.011

Table 6: *Extra output and inflation due to ZLB, when holding nominal interest rates at zero for k quarters, rather than having no ZLB. Results at the posterior median. No stimulus.*

ZLB imposed for ...	Output change (in %)			Inflation change (in %)		
	1 yr	5 yr's	NPV	1 yr	5 yr's	10 yr's
$k = 4$ quarters	0.083	0.008	0.998	0.000	-0.000	-0.000
$k = 8$ quarters	0.456	0.054	6.392	0.004	-0.002	-0.002
$k = 12$ quarters	0.756	0.117	12.882	0.011	-0.003	-0.004
$k = 16$ quarters	0.902	0.181	18.294	0.017	-0.004	-0.006

Figure 11: *Shadow Taylor rule*

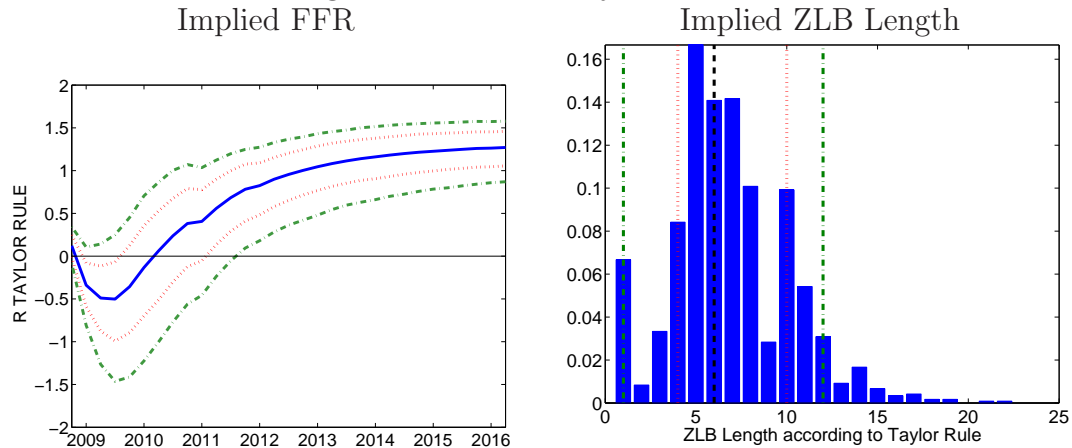
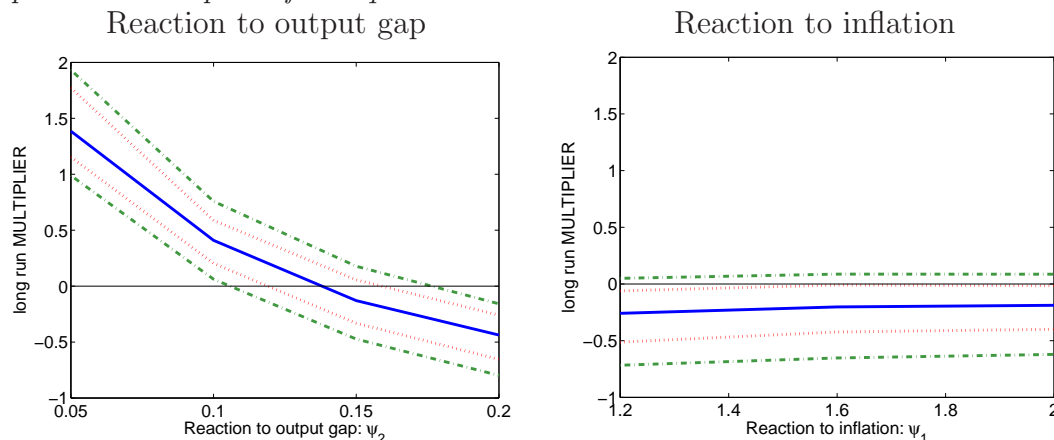


Figure 12: *Impact of changing Taylor rule coefficients on the long-run multiplier. ZLB imposed for 8 quarters.*



Ricardian equivalence. Second, the split of transfers between these households and the unconstrained households leads to distributional and thereby aggregate consequences. It turns out that the second effect is more important than the first.

The first row of table 7 shows the change in the fiscal multipliers, when we change the share of the population which is credit-constrained. In this experiment, the transfers are equally distributed across the population, i.e., the share of the transfers to the credit-constrained population equals the share of that population. This confounds two effects, however. The first is the mere rise in the share of credit-constrained households, but leaving their share of transfer receipts the same: this is shown in the second row of table 7. The second is the share of transfers received by the credit-constrained households. The third row of table 7 therefore varies the share of transfers received by these households, but keeping their share of the population constant at the benchmark value of 25%. While the second experiment has a rather modest impact on the short-run multiplier, the last experiment has a larger impact there. The long-run multipliers move considerably for both experiments. For example and for the last experiment, the median estimate, the long-run fiscal multiplier changes from -0.42 to 0.08, as that fraction is varied from zero to 100%.

One may wish to conclude from this that “fiscal stimulus” in the form of transfers to constrained agents may be quite effective in increasing output. That may be so. However, the modeling of the credit-constrained agents is done here with the simple short-cut of assuming that these agents do not keep savings and cannot borrow. For a more sophisticated exercise, the bounds to borrowing and savings should be endogenized, and may actually depend on the size of the government transfers. Furthermore, micro data can potentially be informative about the degree to which households are credit-constrained or refrain from saving. A deeper investigation into the details is called for, if “fiscal stimulus” programs in the future are to focus on this particular group.

4.6 Sensitivity to the capital share

The estimated capital share is around 0.24 rather than 0.36, as often used in the calibration literature, see Cooley and Prescott (1995). The comparisons in figure 13 figure 14 reveals, that the results are quite sensitive to this parameter.

4.7 Sensitivity to price and wage stickiness

Finally, it may be interesting to document the impact of the price and wage stickiness on the fiscal multipliers: this is done in figure 16. Note that the estimates are $\zeta_p = 0.81$ and $\zeta_w = 0.84$ for the Calvo parameter for prices and wages. While the figure mostly shows an increase in the multiplier with increasingly sticky prices and wages, this is no longer true when prices and wages get very sticky. Essentially, at that point, future inflation due to the zero lower bound no longer induces upward pressure on prices and wages, thereby lessening the impact of fiscal stimulus.

4.8 Welfare effects

Both the long-run and short-run multiplier are silent on welfare implications of the stimulus package: if the output increase is driven by a disproportionate

Table 7: *Short-run and long-run fiscal multipliers: sensitivity to credit-constrained fraction of the population and their share of transfers. First line: all households receive the same amount of transfers, i.e. fraction of constrained households and total transfers rise together. Second line, only the fraction of constrained household rises. Third line: only the share of transfers going to constrained households rises.*

	one year multiplier			long-run multiplier		
	10%	25%	40%	10%	25%	40%
Transfers = RoT fraction = Const. transfers/household:	0.4546	0.6499	0.9138	-0.5788	-0.2956	0.0965
Transfers =25%, RoT fraction = Fixed absolute transfers	0.5426	0.6499	0.7929	-0.5214	-0.2956	-0.0015
RoT Share =25%, Transfers = Fixed population share	0.4800	0.6499	1.1600	-0.4212	-0.2956	0.0802

Figure 13: *Long-run fiscal multipliers: comparing two capital shares.*
 Estimated private capital share ≈ 0.24 Fixed private capital share ≈ 0.36

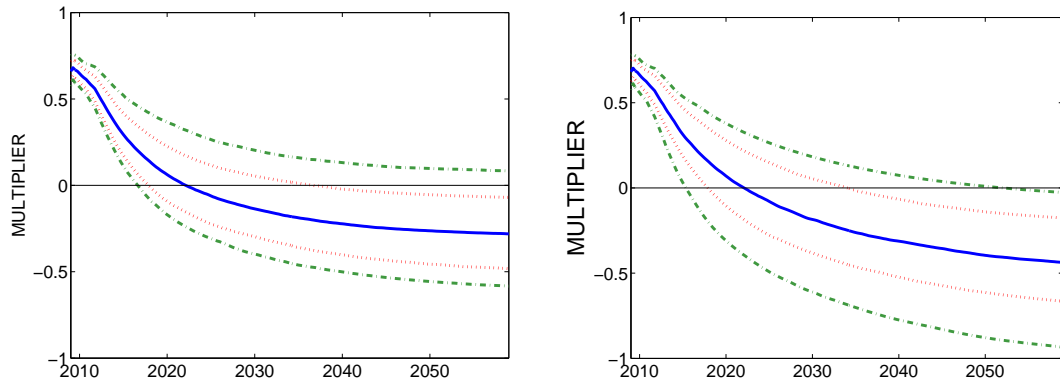


Figure 14: *Short-run and long-run fiscal multipliers: sensitivity to the capital share.*

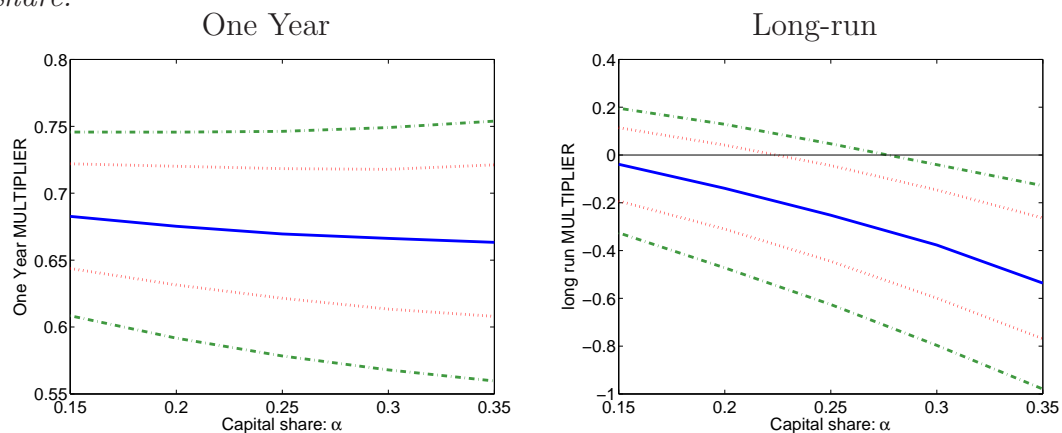


Figure 15: *Long-run fiscal multipliers: sensitivity to the stickiness of prices and wages. Results are at the posterior median.*

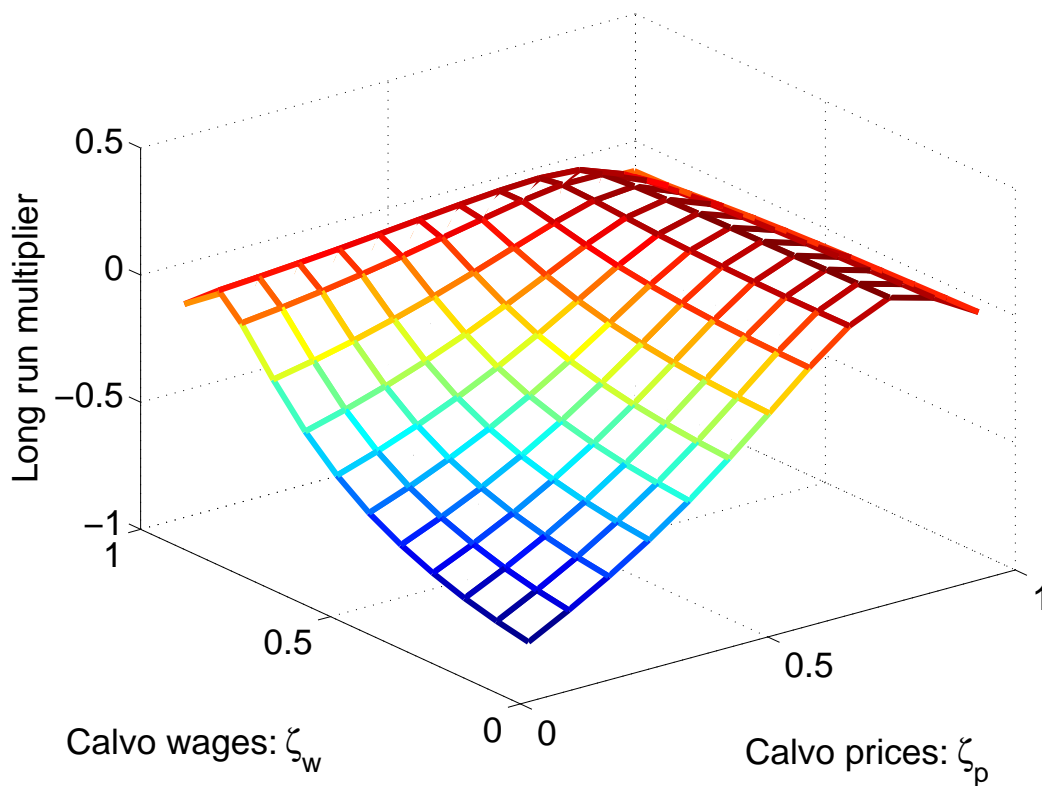
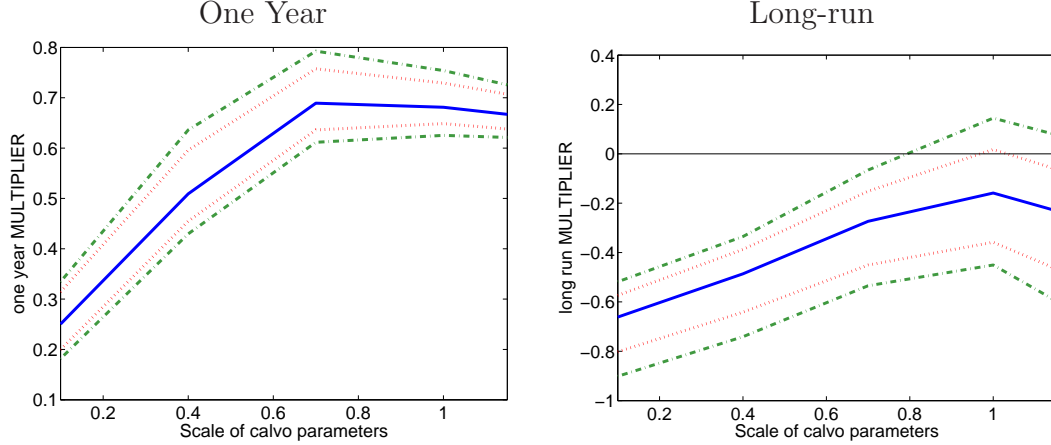


Figure 16: *Short-run and long-run fiscal multipliers: sensitivity to price and wage stickiness.*



increase in hours worked, consumers are likely to be worse off even if the multiplier is large and positive. Given perfect foresight of the stimulus plan, we can calculate the compensating variation in lifetime consumption along the balanced growth path for each consumers are indifferent between ARRA and the modified growth path. In the appendix we derive that $\Gamma \times 100$ is the percentage of lifetime consumption consumers would be willing to give up to have the ARRA in place.

$$\Gamma(\beta) = \left[\frac{1 - \beta\mu^{1-\sigma}}{(1 - h/\mu)^{1-\sigma}} \sum_{s=0}^{\infty} [\beta\mu^{1-\sigma}]^s \left[\left(e^{\hat{c}_{t+s}} - \frac{h}{\mu} e^{\hat{c}_{t+s-1}} \right) e^{-\hat{n}_{t+s}\bar{n}^{1+\nu}} \right]^{1-\sigma} \right]^{\frac{1}{1-\sigma}} - 1. \quad (4.2)$$

Our parameter estimates are only directly applicable to unconstrained households, since constrained households discount the future at a different rate. We shall therefore first measure welfare only for the unconstrained households. Our (preliminary) calculations in Table 8 show that there are no significant welfare gains from to the stimulus, and the mean effect tends to be negative.

Two caveats make the welfare calculation challenging. First, the calculation is numerically challenging because at our estimates the effective discount factor $\beta\mu^{1-\sigma}$ is close to unity so that convergence is slow. Numerical error is

important to address because we are relating the cost of an intervention over about ten years to lifetime consumption so that errors of a small magnitude might be important for the results. Second, social welfare depends on both types of households. If constrained households are sufficiently impatient and receive a high weight in the social welfare function, the suggestive results presented above could be overturned because constrained agent might value the initial consumption increase enough. The calibration of the discount rate for the constrained households is a challenge, however. Lawrance (1991) finds that rates of time preference vary by 7% on an annual basis across rich and poor households. Using data on individual choices between lump-sum and annuity payments, Warner and Pleeter (2001) find differences in annual rates of time preference of up to 30%, varying by various characteristics.

5 Conclusions

We have quantified the size, uncertainty and sensitivity of fiscal multipliers in response to the American Recovery and Reinvestment Act (ARRA) of 2009. To that end, we have extended the benchmark Smets and Wouters (2007) New Keynesian model, allowing for credit-constrained households, a central bank constrained by the zero lower bound, government capital and a government raising taxes with distortionary taxation. We have distinguished between short-run and long-run multipliers. For a benchmark parameterization, we found modestly positive short-run multipliers with a posterior mean of 0.65 and modestly negative long-run multipliers centered around -0.31. The multiplier is particularly sensitive to the fraction of transfers given to

Table 8: Welfare effects of stimulus on unconstrained agents: Lifetime-consumption equivalent of compensating variation for stimulus. Posterior mean and standard deviation. Preliminary results.

Scenario	8 quarters ZLB	0 quarters ZLB	12 quarters ZLB
$\Gamma \times 100$ (std)	-0.039 (0.065)	-0.001 (0.097)	-0.003 (0.094)

credit-constrained households, is sensitive to the anticipated length of the zero lower bound, is sensitive to the capital share and is nonlinear in the degree of price and wage stickiness. Reasonable specifications are consistent with substantially negative short-run multipliers within a short time frame. Furthermore, monetary policy is remarkably powerful: increasing the zero lower bound from 8 to 12 quarters improves output far more than does the ARRA, without raising inflation. We interpret this to cast doubt on the ability of New Keynesian models to predict the effects of monetary policy.

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7 Appendix

7.1 Data

- Sources: NIPA, FRED 2, BLS.
- Nominal series for wages, consumption, government and private investment deflated with general GDP deflator.
- Differences to Smets-Wouters dataset: Use civilian non-institutionalized population throughout, although not seasonally adjusted before 1976. Base year for real GDP: 2005 instead of 1996.
- Using the same definition, all series but real wages have a correlation of 100% across the two datasets. For the change in real wages, the correlation is 0.9. Including durables consumption in investment causes the correlation for the investment series to drop to 0.70 and for consumption to 0.78.
- No data for the Corporate-Treasury bond yield spread before 1953:1. Set to zero.
- No data on FFR before 1954:3. Use secondary market rate for 3-month TBill before.
- Dallas Fed federal debt data.

The categorization of the various stimulus components is shown in detail in tables 9, 10 and 11. As source, we have used CBO (2009), specifically “Table 2: Estimated cost of the conference agreement for H.R. 1, the American Recovery and Reinvestment Act of 2009, as posted on the website of the House Committee on Rules.” The annual time path for these expenditures is taken from CBO (2009) and the annual sum for each component is split across quarters in proportion to the aggregate series in Cogan et al. (2010).

Table 9: Categorizing the stimulus – Government Consumption

Item	Amount (bn USD)	Share
Dept. of Defense	4.53	0.59
Employment and Training	4.31	0.56
Legislative Branch	0.03	0
National Coordinator for Health Information Technology	1.98	0.26
National Institute of Health	9.74	1.26
Other Agriculture, Food, FDA	3.94	0.51
Other Commerce, Justice, Science	5.36	0.69
Other Dpt. of Education	2.12	0.28
Other Dpt. of Health and Human Services	9.81	1.27
Other Financial Services and gen. Govt	1.31	0.17
Other Interior and Environment	4.76	0.62
Special education	12.2	1.58
State and local law enforcement	2.77	0.36
State Fiscal Relief	90.04	11.68
State fiscal stabilization fund	53.6	6.95
State, foreign operations, and related programs	0.6	0.08
Other	2.55	0.33
Consumption	209.64	27.2

Table 10: Categorizing the stimulus – Government Investment

Item	Amount (bn USD)	Share
Broadband Technology opportunities program	4.7	0.61
Clean Water and Drinking Water State Revolving Fund	5.79	0.75
Corps of Engineers	4.6	0.6
Distance Learning, Telemedicine, and Broadband Program	1.93	0.25
Energy Efficiency and Renewable Energy	16.7	2.17
Federal Buildings Fund	5.4	0.7
Health Information Technology	17.56	2.28
Highway construction	27.5	3.57
Innovative Technology Loan Guarantee	6	0.78
NSF	2.99	0.39
Other Energy	22.38	2.9
Other transportation	20.56	2.67
Investment	136.09	17.66

Table 11: Categorizing the stimulus – Transfers

Item	Amount (bn USD)	Share
Assistance for the unemployed	0.88	0.11
Economic Recovery Programs, TANF, Child support	18.04	2.34
Health Insurance Assistance	25.07	3.25
Health Insurance Assistance	-0.39	-0.05
Low Income Housing Program	0.14	0.02
Military Construction and Veteran Affairs	4.25	0.55
Other housing assistance	9	1.17
Other Tax Provisions	4.81	0.62
Public housing capital fund	4	0.52
Refundable Tax Credits	68.96	8.95
Student financial assistance	16.56	2.15
Supplemental Nutrition Assistance Program	19.99	2.59
Tax Provisions	214.56	27.84
Unemployment Compensation	39.23	5.09
Transfers and Tax cuts	425.09	55.15

7.2 Estimation

Tables 12 and 13 contain the results from estimating our model, using Dynare and a Bayesian prior.

	Prior	Prior mean (s.d.)	SW Model 66:1-08:4	Our Model 49:2-08:4
Adj. cost $S''(\mu)$	norm	4.000 (1.500)	5.93 (1.1)	4.57 (0.82)
Risk aversion σ	norm	1.500 (0.375)	1.42 (0.11)	1.18 (0.07)
Habit h	beta	0.700 (0.100)	0.7 (0.04)	0.85 (0.02)
Calvo wage ζ_w	beta	0.500 (0.100)	0.77 (0.05)	0.84 (0.03)
Inv. labor sup. ela. ν	norm	2.000 (0.750)	1.96 (0.54)	2.33 (0.56)
Calvo prices ζ_p	beta	0.500 (0.100)	0.69 (0.05)	0.81 (0.04)
Wage indexation ι_w	beta	0.500 (0.150)	0.62 (0.1)	0.44 (0.09)
Price indexation ι_p	beta	0.500 (0.150)	0.26 (0.08)	0.3 (0.09)
Capacity util.	beta	0.500 (0.150)	0.59 (0.1)	0.45 (0.08)
$1 + \frac{\text{Fix. cost}}{Y} = 1 + \lambda_p$	norm	1.250 (0.125)	1.64 (0.08)	1.93 (0.06)
Taylor rule infl. ψ_1	norm	1.500 (0.250)	2 (0.17)	1.64 (0.19)
same, smoothing ρ_R	beta	0.750 (0.100)	0.82 (0.02)	0.92 (0.01)
same, LR gap ψ_2	norm	0.125 (0.050)	0.09 (0.02)	0.13 (0.03)
same, SR gap ψ_3	norm	0.125 (0.050)	0.24 (0.03)	0.2 (0.02)
Mean inflation (data)	gamm	0.625 (0.100)	0.76 (0.09)	0.56 (0.08)
100×time pref.	gamm	0.250 (0.100)	0.16 (0.05)	0.11 (0.04)
Mean hours (data)	norm	0.000 (2.000)	1.07 (0.95)	-0.25 (0.67)
Trend $(\mu - 1) * 100$	norm	0.400 (0.100)	0.43 (0.02)	0.48 (0.01)
Capital share α	norm	0.300 (0.050)	0.19 (0.02)	0.24 (0.01)
Gov. adj. cost $S''_g(\mu)$	norm	0.000 (0.500)	n/a	6.85 (1.03)
Budget bal speed $\frac{\psi_\tau - 0.025}{0.175}$	beta	0.30 (0.20)	n/a	0.07 (0.05)
Implied ψ_τ	n/a	0.078 (0.035)	n/a	0.0373 (0.01)
Mean gov. debt	norm	0.000 (0.500)	n/a	0 (0.49)
Mean bond spread	gamm	0.500 (0.100)	n/a	0.45 (0.05)

Table 12: *Estimation, part 1. The calibrated government investment-to-GDP ratio as well as the estimated growth trend μ implies a government share in production of $\zeta = 2.30\%$.*

	Prior	Prior mean (s.d.)	SW Model 66:1-08:4	Our Model 49:2-08:4
s.d. tech.	invg	0.100 (2.000)	0.46 (0.03)	0.46 (0.02)
AR(1) tech.	beta	0.500 (0.200)	0.95 (0.01)	0.94 (0.01)
s.d. bond	invg	0.100 (2.000)	0.24 (0.03)	0.97 (0.05)
AR(1) bond ρ_q	beta	0.500 (0.200)	0.27 (0.1)	0.68 (0.03)
s.d. gov't	invg	0.100 (2.000)	0.54 (0.03)	0.35 (0.02)
AR(1) gov't	beta	0.500 (0.200)	0.98 (0.01)	0.98 (0.01)
Cov(gov't, tech.)	norm	0.500 (0.250)	0.53 (0.09)	0.3 (0.05)
s.d. inv. price	invg	0.100 (2.000)	0.43 (0.04)	1.26 (0.11)
AR(1) inv. price	beta	0.500 (0.200)	0.73 (0.06)	0.55 (0.06)
s.d. mon. pol.	invg	0.100 (2.000)	0.24 (0.02)	0.23 (0.01)
AR(1) mon. pol.	beta	0.500 (0.200)	0.16 (0.07)	0.22 (0.06)
s.d. goods m-up	invg	0.100 (2.000)	0.14 (0.01)	0.31 (0.02)
AR(1) goods m-up	beta	0.500 (0.200)	0.89 (0.04)	0.91 (0.05)
MA(1) goods m-up	beta	0.500 (0.200)	0.73 (0.08)	0.96 (0.02)
s.d. wage m-up	invg	0.100 (2.000)	0.26 (0.02)	0.23 (0.02)
AR(1) wage m-up	beta	0.500 (0.200)	0.97 (0.01)	0.96 (0.02)
MA(1) wage m-up	beta	0.500 (0.200)	0.91 (0.03)	0.91 (0.04)
s.d. Tax shock	invg	0.100 (2.000)	n/a	1.42 (0.07)
AR(1) tax shock	beta	0.500 (0.200)	n/a	0.97 (0.01)
s.d. gov. inv. price	invg	0.100 (2.000)	n/a	0.79 (0.09)
AR(1) gov. inv. price	beta	0.500 (0.200)	n/a	0.97 (0.01)
s.d. bond spread	invg	0.100 (2.000)	n/a	0.08 (0)
AR(1) bond spread	beta	0.500 (0.200)	n/a	0.91 (0.02)

Table 13: *Estimation, part 2*

Figure 17: *Regression of quarterly unemployment rate on the model-implied employment measure: Actual vs. predicted unemployment rate.*

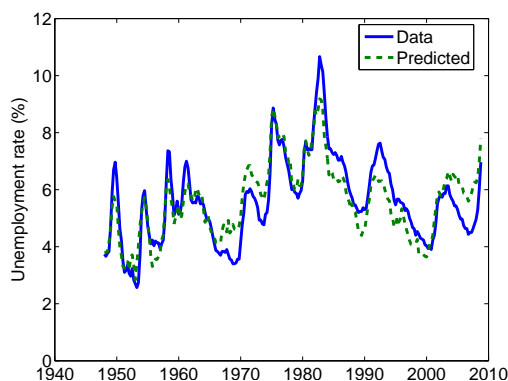
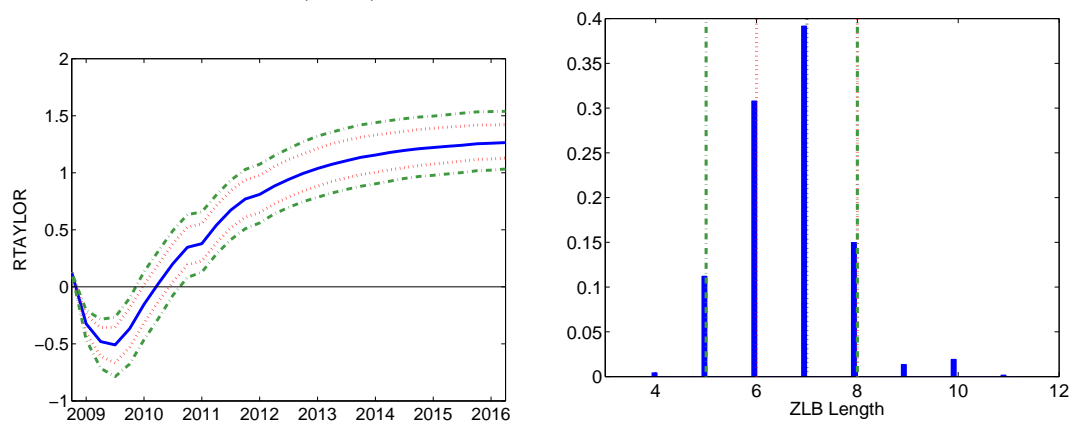


Table 14: *OLS regression estimates of unemployment rate on the model-implied employment measure.*

	Constant	Employment (lab_t)	R^2
Unemployment Rate (UR_t)	5.60 (5.51, 5.69)	-0.46 (-0.49, -0.43)	0.77

Sample period: 1948:1 – 2008:4. Unemployment rate is the arithmetic mean over the quarter. 95% confidence intervals in parentheses. Labor input in the model is measured as $lab_t \equiv \log \frac{\text{Avg. hours}_t \times \text{Employment}_t}{\text{Population}_t}$ – mean. 95% OLS confidence intervals in parentheses.

Figure 18: *Shadow Taylor rule and ZLB duration with endogenous ZLB*
 Implied interest rate (level) ZLB duration



TECHNICAL APPENDIX – NOT FOR PUBLICATION

8 Model Appendix

Apart from the model extensions due to the introduction of government capital, rule of thumb consumers, and distortionary taxation, the following model appendix follows mostly the appendix of Smets and Wouters (2007), with minor changes to unify the notation.

8.1 Production

Final goods are produced in a competitive final goods sector which uses differentiated intermediate inputs, supplied by monopolistic intermediate producers.

8.1.1 Final goods producers

The representative final goods producer maximizes profits by choosing intermediate inputs $Y_t(i), i \in [0, 1]$, subject to a production technology which generalizes a CES production function: Objective:

$$\max_{Y_t, Y_t(i)} P_t Y_t - \int_0^1 P_t(i) Y_t(i) di \quad \text{s.t.} \quad \int_0^1 G\left(\frac{Y_t(i)}{Y_t}; \tilde{\epsilon}_t^{\lambda, p}\right) di = 1. \quad (8.1)$$

$G(\cdot)$ is the ? aggregator, which generalizes CES demand by allowing the elasticity of demand to increase with relative prices: $G' > 0$, $G'' < 0$, $G(1; \tilde{\epsilon}_t^{\lambda, p}) = 1$. $\tilde{\epsilon}_t^{\lambda, p}$ is a shock to the production technology which changes the elasticity of substitution.

Denote the Lagrange multiplier on the constraint by Ξ_t^f . If a positive

solution to equation (8.1) exists it satisfies the following conditions

$$\begin{aligned} [Y_t] \quad P_t &= \Xi_t^f \frac{1}{Y_t} \int_0^1 G' \left(\frac{Y_t(i)}{Y_t}; \tilde{\epsilon}_t^{\lambda,p} \right) \frac{Y_t(i)}{Y_t} di, \\ [Y_t(i)] \quad P_t(i) &= \Xi_t^f \frac{1}{Y_t} G' \left(\frac{Y_t(i)}{Y_t}; \tilde{\epsilon}_t^{\lambda,p} \right). \end{aligned}$$

From these two equations, we obtain an expression for the aggregate price index and intermediate inputs. The price index is given by:

$$P_t = \int_0^1 \frac{Y_t(i)}{Y_t} P_t(i) di. \quad (8.2)$$

Solving for intermediate input demands:

$$Y_t(i) = Y_t G'^{-1} \left(\frac{P_t(i) Y_t}{\Xi_t^f} \right) = Y_t G'^{-1} \left(\frac{P_t(i)}{P_t} \int_0^1 G' \left(\frac{Y_t(j)}{Y_t}; \tilde{\epsilon}_t^{\lambda,p} \right) \frac{Y_t(j)}{Y_t} dj \right). \quad (8.3)$$

For future reference, note that the relative demand curves $y_t(i) \equiv \frac{Y_t(i)}{Y_t}$ are downward-sloping in the relative price $\frac{P_t(i)}{P_t}$ with an decreasing elasticity as the relative quantity increases. For simplicity, the dependence of the $G(\cdot)$ aggregator on the shock $\tilde{\epsilon}_t^{\lambda,p}$ is suppressed:

$$\begin{aligned} \eta_p(y_t(i)) &\equiv - \frac{P_t(i)}{Y_t(i)} \frac{dy_t(i)}{dP_t(i)} \Big|_{dY_t = d\Xi_t^f = 0} = - \frac{G'(y_t(i))}{y_t(i) G''(y_t(i))} \quad (8.4) \\ \hat{\eta}_p(y_t(i)) &\equiv \frac{P_t(i)}{\eta_p(y_t(i))} \frac{d\eta_p(y_t(i))}{dP_t(i)} = 1 + \eta_p + \eta_p \frac{G'''(y_t(i))}{G''(y_t(i))} y_t(i) \\ &= 1 + \eta_p(y_t(i)) \left(2 + \frac{G'''(y_t(i))}{G''(y_t(i))} y_t(i) - 1 \right) \\ &= 1 + \eta_p(y_t(i)) \left(\frac{2 + \frac{G'''(y_t(i))}{G''(y_t(i))} y_t(i)}{1 - \eta_p(y_t(i))^{-1}} (1 - \eta_p(y_t(i))^{-1}) - 1 \right) \\ &\equiv 1 + \frac{1 + \lambda^p(y_t(i))}{\lambda^p(y_t(i))} \left(\frac{1}{[1 + \lambda^p(y_t(i))] A_p(y_t(i))} - 1 \right), \quad (8.5) \end{aligned}$$

where the last line defines the mark-up $\lambda^p(y_t(i)) \equiv \frac{1}{\eta_p(y_t(i))^{-1} - 1}$ and $A_p(y_t(i)) \equiv \frac{\lambda^p(y_t(i))}{2 + \frac{G'''(y_t(i))}{G''(y_t(i))} y_t(i)}$. The model will be parameterized in terms of $\hat{\epsilon}(1)$, the change in the own price elasticity of demand along the balanced growth path. To

that end, it is convenient to solve for A_p in terms of the mark-up and the $\hat{\epsilon}$:

$$A_p(y) = \frac{1}{\lambda^p(y)\hat{\eta}_p(y) + 1}. \quad (8.6)$$

Finally, note that in the Dixit-Stiglitz case $G(y) = y^{\frac{1}{1+\lambda^p}}$ so that the elasticity of demand is constant at $\eta_p(y) = \frac{1}{\lambda^p} + 1 \forall y$ and consequently $\hat{\eta}_p = 0$.

8.1.2 Intermediate goods producers

There is a unit mass of intermediate producers, indexed by $i \in [0, 1]$. Each producer is the monopolistic supplier of good i . They rent capital services K_t^{eff} and hire labor n_t to maximize profits intertemporally, taking as given rental rates R_t^k and wages W_t . Given a Calvo-style pricing friction, their profit-maximization problem is dynamic.

Production is subject to a fixed cost and the gross product is produced using a Cobb-Douglas technology at the firm level. Government capital K_t^g increases total factor productivity in each firm, but is subject to a congestion effect as overall production increases, similar to the congestion effects in the AK model in ?. Firms fail to internalize the effect of their decisions on public sector productivity. Net output is therefore given by:

$$Y_t(i) = \tilde{\epsilon}_t^a \left(\frac{K_{t-1}^g}{\int_0^1 Y_t(j) dj + \Phi \mu^t} \right)^{\frac{\zeta}{1-\zeta}} K_t^{eff}(i)^\alpha [\mu^t n_t(i)]^{1-\alpha} - \mu^t \Phi, \quad (8.7)$$

where $\Phi \mu^t$ represent fixed costs which grow at the rate of labor augmenting technical progress and $K_t(i)^{eff}$ denotes the capital services rented by firm i . $\tilde{\epsilon}_t^a$ denotes a stationary TFP process.

To see the implications of the congestion costs, consider the symmetric case that $Y_t(i) = Y_t$, $K_t^{eff}(i) = K_t^{eff} \forall i$, which is the case along the symmetric balanced growth path and in the flexible economy. We then obtain the following aggregate production function:

$$Y_t = \epsilon_t^a K_{t-1}^g{}^\zeta K_t^{eff\alpha(1-\zeta)} [\mu^t n_t]^{(1-\alpha)(1-\zeta)} - \mu^t \Phi, \quad \epsilon_t^a \equiv (\tilde{\epsilon}_t^a)^{1-\zeta}. \quad (8.8)$$

Choose units such that $\bar{\epsilon}^a \equiv 1$.

To solve a firm's profit maximization problem, note that it is equivalent to minimizing costs (conditional on operating) and then choosing the quantity optimally. Consider the cost-minimization problem first:

$$\min_{K_t(i), n_t(i)} W_t n_t(i) + R_t^k K_t(i) \text{ s.t. } (8.7).$$

Denote the Lagrange multiplier on the production function by MC_t – producing a marginal unit more raises costs marginally by MC_t . The static FOC are necessary and sufficient, given $Y_t(i)$:

$$\begin{aligned} [n_t(i)] \quad MC_t(i)(1 - \alpha) \frac{Y_t(i) + \mu^t \Phi}{n_t(i)} &= W_t, \\ [K_t(i)] \quad MC_t(i) \alpha \frac{Y_t(i) + \mu^t \Phi}{K_t(i)} &= R_t^k. \end{aligned}$$

The FOC can be used to solve for the optimal capital-labor ratio in production and marginal costs:

$$\frac{k_t(i)}{n_t(i)} = \frac{\alpha}{1 - \alpha} \frac{w_t}{r_t^k}, \quad (8.9)$$

$$MC_t = \alpha^{-\alpha} (1 - \alpha)^{-(1-\alpha)} \frac{W_t^{1-\alpha} (R_t^k)^\alpha \mu^{-(1-\alpha)t}}{\left(\frac{K_{t-1}^g}{Y_t + \mu^t \Phi} \right)^{\frac{\zeta}{1-\zeta}} \epsilon_t^a}, \quad (8.10)$$

$$mc_t = \alpha^{-\alpha} (1 - \alpha)^{-(1-\alpha)} \frac{w_t^{1-\alpha} (r_t^k)^\alpha}{\left(\frac{\mu k_{t-1}^g}{y_t + \Phi} \right)^{\frac{\zeta}{1-\zeta}} \epsilon_t^a},$$

$$mc_t = \alpha^{-\alpha} (1 - \alpha)^{-(1-\alpha)} \frac{w_t^{1-\alpha} (r_t^k)^\alpha}{\left(\frac{\mu k_{t-1}^g}{y_t + \Phi} \right)^{\frac{\zeta}{1-\zeta}} \epsilon_t^a}, \quad (8.11)$$

where lower case letters denote detrended, real variables as applicable:

$$k_t \equiv K_t \mu^{-t}, y_t \equiv Y_t \mu^{-t}, w_t \equiv \frac{W_t}{\mu^t P_t}, r_t^k \equiv \frac{R_t^k}{P_t}, mc_t \equiv \frac{MC_t}{P_t}.$$

For future reference, it is useful to detrend the FOC:

$$w_t = mc_t(i)(1 - \alpha) \frac{y_t(i) + \Phi}{n_t(i)}, \quad (8.12a)$$

$$r_t^k = mc_t(i) \alpha \frac{y_t(i) + \Phi}{k_t(i)}. \quad (8.12b)$$

Given the solution to the static cost-minimization problem, the firm maximizes the present discounted value of its profits by choosing quantities optimally, taking as given its demand function (8.3), the marginal costs of production (8.10), and respecting the Calvo-style price setting friction. The Calvo-friction implies that a firm can re-set its price in each period with probability $1 - \zeta_p$ and otherwise indexes its price to an average of current and past inflation $\prod_{l=1}^s \pi_{t+l-1}^{\zeta_p} \bar{\pi}^{1-\zeta_p}$. In each period t that the firm can change its prices it chooses:

$$P_t^*(i) = \arg \max_{\tilde{P}_t(i)} \mathbb{E}_t \sum_{s=0}^{\infty} \zeta_p^s \frac{\bar{\beta}^s \xi_{t+s} P_t}{\xi_t P_{t+s}} \left[\tilde{P}_t(i) \left(\prod_{l=1}^s \pi_{t+l-1}^{\zeta_p} \bar{\pi}^{1-\zeta_p} \right) - MC_{t+s}(i) \right] Y_{t+s}(i),$$

subject to (8.3) and (8.10). $\frac{\bar{\beta}^s \xi_{t+s}}{\xi_t}$ denotes the (non-credit constrained) representative household's stochastic discount factor and $\pi_t \equiv \frac{P_t}{P_{t-1}}$ denotes period t inflation.

To solve the problem, it is useful to define $\chi_{t,t+s}$ such that in the absence of further price adjustments prices evolve as $P_{t+s}(i) = \chi_{t,t+s} P_t^*(i)$:

$$\chi_{t,t+s} = \begin{cases} 1 & s = 0, \\ \prod_{l=1}^s \pi_{t+l-1}^{\zeta_p} \bar{\pi}^{1-\zeta_p} & s = 1, \dots, \infty. \end{cases}$$

Therefore and using the definition $y_{t+s}(i) = \frac{Y_{t+s}(i)}{Y_{t+s}}$:

$$\frac{d(Y_{t+s}(i)[P_{t+s}(i) - MC_{t+s}(i)])}{d\tilde{P}_t(i)} = y_{t+s}(i) Y_{t+s} \left(\chi_{t,t+s} [1 - \eta_p(y_{t+s}(i))] + \eta_p \frac{MC_{t+s}(i)}{P_t(i)} \right).$$

The first order condition is then given by:

$$\mathbb{E}_t \sum_{s=0}^{\infty} \zeta_p^s \frac{\bar{\beta}^s \xi_{t+s} P_t}{\xi_t P_{t+s}} y_{t+s}(i) Y_{t+s} \left([1 - \eta_p(y_{t+s}(i))] \chi_{t,t+s} + \eta_p \frac{MC_{t+s}(i)}{P_t(i)} \right) = 0 \quad (8.13)$$

For future reference, it is useful to re-write the FOC as follows:

$$\frac{P_t^*(i)}{P_t} = \frac{\mathbb{E}_t \sum_{s=0}^{\infty} (\mu \bar{\beta} \zeta_p)^s \frac{\xi_{t+s}}{\lambda_p(y_{t,t+s}(i)) \xi_t} y_{t,t+s}(i) \frac{\eta_p(y_{t,t+s}(i))}{\eta_p(y_{t,t+s}(i))-1} mc_{t+s}(i)}{\mathbb{E}_t \sum_{s=0}^{\infty} (\mu \bar{\beta} \zeta_p)^s \frac{\xi_{t+s}}{\lambda_p(y_{t,t+s}(i)) \xi_t} \prod_{l=1}^s \frac{\chi_{t,t+l}}{\pi_{t+l}} y_{t,t+s}(i)} \quad (8.14)$$

where $y_{t,t+s}(i) = G'^{-1} \left(\frac{P_t^* \chi_{t,t+s} Y_{t+s}}{\Xi_{t+s}^f} \right)$, $Y_{t,t+s}(i) = y_{t,t+s}(i) Y_{t+s}$.

Noting that measure $1 - \zeta_p$ of firms changes prices in each period and each firm faces a symmetric problem, the expression for the aggregate price index (8.2) can be expressed recursively as a weighted average of adjusted and indexed prices:

$$P_t = (1 - \zeta_p) P_t^* G'^{-1} \left(\frac{P_t^* Y_t}{\Xi_t^f} \right) + \zeta_p \pi_{t-1}^{\iota_p} \bar{\pi}^{1-\iota_p} P_{t-1} G'^{-1} \left(\frac{\pi_{t-1}^{\iota_p} \bar{\pi}^{1-\iota_p} P_{t-1} Y_t}{\Xi_t^f} \right), \quad (8.15)$$

using that price distribution of non-adjusting firms at t is the same as that of all firms at time $t-1$, adjusted for the shrinking mass due price adjustments. Along the deterministic balanced growth path the optimal price equals the average price, which is normalized to unity:

$$\bar{P}^* = \bar{P} = 1.$$

Similarly, along the deterministic growth path the price is a constant mark-up over marginal cost:

$$\frac{\bar{P}^*}{\bar{P}} = \frac{\eta_p}{\eta_p - 1} \bar{m}c = (1 + \bar{\lambda}_p) \bar{m}c = 1 \quad (8.16)$$

Finally, the assumption of monopolistic competition in the presence of free entry requires zero profits along the balanced growth path. Real and

detrended profits of intermediate producer i are given by:

$$\Pi_t^p(i) = \frac{P_t(i)}{P_t} y_t(i) - w_t n_t(i) - r_t^k k_t(i) = \frac{P_t(i)}{P_t} y_t(i) - mc_t(i) [y_t(i) + \mu^t \Phi]$$

Integrating over all $i \in [0, 1]$ and using the definition of the price index (8.2) yields:

$$\Pi_t^p = y_t - w_t \int_0^1 n_t(i) di - r_t^k \int_0^1 k_t(i) di \quad (8.17a)$$

$$= y_t - mc_t \left(\int_0^1 y_t(i) di + \Phi \right) = y_t - mc_t \left(y_t \int_0^1 \frac{P_t(i)}{P_t} di + \Phi \right) \quad (8.17b)$$

Using the expression for the steady state markup, equation (8.16), the zero profit condition (8.17b) implies that along the symmetric balanced growth path:

$$0 = \bar{\Pi}^p = \bar{y} - \frac{\bar{y} \int_0^1 \frac{P(i)}{P} di + \Phi}{1 + \bar{\lambda}_p} = \bar{y} - \frac{\bar{y} + \Phi}{1 + \bar{\lambda}_p} \Rightarrow \frac{\Phi}{\bar{y}} = \bar{\lambda}_p. \quad (8.18)$$

8.1.3 Labor packers

Intermediate producers use a bundle of differentiated labor inputs, $\ell \in [0, 1]$, purchased from labor packers. Labor packers aggregate, or pack, differentiated labor which they purchase from unions. They are perfectly competitive and face an analogous problem to final goods producers:

$$\max_{n_t, n_t(\ell)} W_t n_t - \int_0^1 W_t(\ell) n_t(\ell) d\ell \quad \text{s.t.} \quad \int_0^1 H \left(\frac{n_t(\ell)}{n_t}; \tilde{\epsilon}_t^{\lambda, w} \right) d\ell = 1, \quad (8.19)$$

where $H(\cdot)$ has the same properties as $G(\cdot)$: $H' > 0$, $H'' < 0$, $H(1) = 1$.

The FOC yield differentiated labor demand, analogous to intermediate goods demand (8.3):

$$n_t(\ell) = n_t H'^{-1} \left(\frac{W_t(\ell) n_t}{\Xi_t^n} \right) = n_t H'^{-1} \left(\frac{W_t(\ell)}{W_t} \int_0^1 H' \left(\frac{n_t(l)}{n_t}; \tilde{\epsilon}_t^{\lambda, w} \right) \frac{n_t(l)}{n_t} dl \right). \quad (8.20)$$

Given the aggregate nominal wage $W_t = \int_0^1 \frac{n_t(\ell)}{n_t} w_t(\ell) d\ell$, labor packers are

willing to supply any amount of packed labor n_t . Labor demand elasticity behaves analogously to the intermediate goods elasticity:

$$\eta_w(n_t(\ell)) \equiv - \frac{W_t(\ell)}{n_t(\ell)} \frac{dn_t(\ell)}{dW_t(\ell)} \Big|_{dn_t=d\varepsilon_t^l=0} = - \frac{H'(n_t(\ell))}{n_t(\ell)H''(n_t(\ell))} \quad (8.21)$$

$$\hat{\eta}_w(n_t(\ell)) \equiv \frac{W_t(\ell)}{\eta_w(n_t(\ell))} \frac{d\eta_w(n_t(\ell))}{dW_t(\ell)} = 1 + \frac{1 + \lambda^w(n_t(\ell))}{\lambda^w(n_t(\ell))} \left(\frac{1}{[1 + \lambda^w(n_t(\ell))]A_w(n_t(\ell))} - 1 \right), \quad (8.22)$$

where $n_t(\ell) \equiv \frac{n_t(\ell)}{n_t}$ and the mark-up is defined as $\lambda_t^n(n_t(\ell)) \equiv \frac{1}{\eta_w(n_t(\ell))-1}$. $A_w(n_t(\ell)) \equiv \frac{\lambda^w(n_t(\ell))}{2 + \frac{H'''(n_t(\ell))}{H''(n_t(\ell))}n_t(\ell)}$ can be equivalently expressed as:

$$A_w(n) = \frac{1}{\lambda^w(n)\hat{\eta}_w(n) + 1}. \quad (8.23)$$

8.2 Households

There is a measure one of households in the economy, indexed by $j \in [0, 1]$, endowed with a unit of labor each. Households are distributed uniformly over the real line, i.e. the measure of households is the Lebesgue measure Λ . We distinguish two types of households – intertemporally optimizing households $j \in [0, 1 - \phi]$ and “rule-of-thumb” households $j \in (1 - \phi, 1]$, so that they have measures $\Lambda([0, 1 - \phi]) = 1 - \phi$ and $\Lambda([0, \phi]) = \phi$, respectively.

Households’ preferences over consumption and hours worked streams $\{C_{t+s}(j), n_{t+s}(j)\}_{s=0}^{\infty}$ are represented by the life-time utility function U_t :

$$U_t = \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left[\frac{1}{1 - \sigma} (C_{t+s}(j) - hC_{t+s-1})^{1-\sigma} \right] \exp \left[\frac{\sigma - 1}{1 + \nu} n_{t+s}(j)^{1+\nu} \right]. \quad (8.24)$$

Here $h \in [0, 1]$ captures external habit formation, σ denotes the inverse of the intertemporal elasticity of substitution, and ν equals the inverse of the labor supply elasticity. Households discount the future by $\beta \in (0, 1)$, where β varies by household type.

The fraction $1 - \phi$ of the labor force who are not credit constrained, maximizes their life-time utility subject to a lifetime budget constraint and

a capital accumulation technology. The remainder of the labor force, i.e. a fraction ϕ is credit constrained (or “rule-of-thumb”): they cannot save or borrow.

8.2.1 Intertemporally optimizing households

The intertemporally optimizing households choose consumption $\{C_{t+s}(j)\}$, investment in physical capital $\{X_{t+s}(j)\}$, physical capital $\{K_{t+s}^p(j)\}$, a capacity utilization rate $\{u_{t+s}(j)\}$, nominal government bond holdings $B_{t+s}^n(j)$, and labor supply $\{n_{t+s}(j)\}$ to maximize (8.24) subject to a sequence of budget constraints (8.25), the law of motion for physical capital (8.26), and a no-Ponzi constraint. Households take prices $\{P_{t+s}\}$, nominal returns on government bonds $\{q_{t+s}^b R_{t+s}\}$, the nominal rental rate of capital $\{R_{t+s}^k\}$, and nominal wages $\{W_{t+s}\}$ as given.

The budget constraint for period $t + s$ is given by:

$$\begin{aligned}
(1 + \tau^c)C_{t+s}(j) + X_{t+s}(j) + \frac{B_{t+s}^n(j)}{R_{t+s}^{gov}P_{t+s}} \leq \\
S_{t+s} + \frac{B_{t+s-1}^n(j)}{P_{t+s}} + (1 - \tau_{t+s}^n) \frac{[W_{t+s}^h n_{t+s}(j) + \lambda_{w,t+s} n_{t+s} W_{t+s}^h]}{P_{t+s}} + \\
+ \left[(1 - \tau^k) \left(\frac{R_{t+s}^k u_{t+s}(j)}{P_{t+s}} - a(u_{t+s}(j)) \right) + \delta \tau^k \right] [(1 - \omega_{t+s-1}^k) K_{t+s-1}^p(j) + \omega_{t+s-1}^k K_{t+s-1}^{p,agg}] + \frac{\Pi_{t+s}^p \mu^{t+s}}{P_{t+s}},
\end{aligned} \tag{8.25}$$

where $(\tau^c, \tau^k, \tau_{t+s}^n)$ represent taxes on consumption expenditure, capital income, and labor income, respectively. The wage received by households differs from the one charged to labor packers because of union profits – union profits $\lambda_{w,t+s} n_{t+s} W_{t+s}^h$ are taken as given by households. Households also receive nominal lump-sum transfers $\{S_{t+s}\}$. $a(\cdot)$ represents the strictly increasing and strictly convex cost function of varying capacity utilization, whose first derivative in the case of unit capacity utilization is normalized as $a'(1) = \bar{r}^k$.² At unit capacity utilization, there is no additional cost: $a(1) = 0$. $\Pi_{t+s}^p \mu^{t+s}$ are nominal profits which households also take as given.

² \bar{r}^k represents the real steady state return on capital services.

There is a financial market frictions present in the budget constraint. $\omega_{t+s}^k \neq 0$ represents a wedge between the returns on private and government bonds and is a pure financial market friction – if $\omega_{t+s}^k > 0$ then households obtain less than one dollar for each dollar of after tax capital income they receive, representing agency costs. Agency costs are reimbursed directly to unconstrained households, so that the friction has no effect on aggregate resources. This financial market friction is similar to a shock in Smets and Wouters (2003) who introduce it ad hoc in the investment Euler equation and motivate it as a short-cut to model informational frictions which disappear at the steady state.

Physical capital evolves according to the following law of motion:

$$K_{t+s}^p(j) = (1 - \delta)K_{t+s-1}^p(j) + q_{t+s}^x \left[1 - S\left(\frac{X_{t+s}(j)}{X_{t+s-1}(j)}\right) \right] X_{t+s}(j), \quad (8.26)$$

where new investment is subject to adjustment costs described by $S(\cdot)$. These costs satisfy $S(\mu) = S'(\mu) = 0, S'' > 0$. The relative price of investment changes over time, as captured by the exogenous $\{q_{t+s}^x\}$ process. Physical capital depreciates at rate δ .

For future reference, note that the effective capital stock is given by the product of capacity utilization and physical capital stock:

$$K_{t+s}^{eff}(j) = K_{t+s-1}^p(j)u_{t+s}(j). \quad (8.27)$$

To obtain the aggregate capital stock, multiply the above quantity by $(1 - \phi)$.

The solution to the household's problem is characterized completely by the law of motion for physical capital (8.26) and the following necessary and sufficient first order conditions. To derive these conditions, denote the Lagrange multipliers on the budget constraint (8.25) and the law of motion (8.26) by $\beta^t(\Xi_t, \Xi_t^k)$ – replacing the household index j by a superscript RA .

$$[C_t] \quad \Xi_t(1 + \tau^c) = \exp\left(\frac{\sigma - 1}{1 + \nu}(n_t^{RA})^{1+\nu}\right) [C_t^{RA} - hC_{t-1}^{RA}]^{-\sigma}$$

$$\begin{aligned}
[n_t] \quad \Xi_t(1 - \tau_t^n) \frac{W_t^h}{P_t} &= \exp\left(\frac{\sigma - 1}{1 + \nu} (n_t^{RA})^{1+\nu}\right) (n_t^{RA})^\nu [C_t^{RA} - hC_{t-1}^{RA}]^{1-\sigma} \\
[B_t] \quad \Xi_t &= \beta q_t^b R_t \mathbb{E}_t \left(\frac{\Xi_{t+1}}{P_{t+1}/P_t} \right) \\
[K_t^p] \quad \Xi_t^k &= \beta \mathbb{E}_t \left(\Xi_{t+1} \left[\tilde{q}_t^k \left((1 - \tau^k) \left[\frac{R_{t+1}^k}{P_{t+1}} u_{t+1} - a(u_{t+1}) + \delta \tau^k \right] + (1 - \delta) \frac{\Xi_{t+1}^k}{\Xi_{t+1}} \right] \right) \right) \\
[X_t] \quad \Xi_t &= \Xi_t^k q_t^x \left(1 - S \left(\frac{X_t^{RA}}{X_{t-1}^{RA}} \right) - S' \left(\frac{X_t^{RA}}{X_{t-1}^{RA}} \right) \left(\frac{X_t^{RA}}{X_{t-1}^{RA}} \right) \right) + \beta \mathbb{E}_t \left(\frac{\Xi_{t+1}^k}{\Xi_t} q_{t+1}^x S' \left(\frac{X_{t+1}^{RA}}{X_t^{RA}} \right) \left(\frac{X_{t+1}^{RA}}{X_t^{RA}} \right)^2 \right) \\
[u_t] \quad \frac{R_{t+1}^k}{P_t} &= a'(u_{t+1}^{RA}).
\end{aligned}$$

By setting $a'(1) \equiv \bar{r}^k$ we normalize steady state capacity utilization to unity: $\bar{u} \equiv 1$.

For what follows, it is useful to detrend these first order conditions and the law of motion for capital. To that end, use lower case letters to denote detrended and real variables as exemplified in the following definitions:

$$k_t^{RA} \equiv \frac{K_t^{RA}}{\mu^t}, w_t \equiv \frac{W_t}{P_t \mu^t}, w_t^h \equiv \frac{W_t^h}{P_t \mu^t}, r_t^k \equiv \frac{R_t^k}{P_t}, \xi_t \equiv \Xi_t \mu^{\sigma t}, Q_t \equiv \frac{\Xi_t^k}{\Xi_t}, \bar{\beta} = \beta \mu^{-\sigma}.$$

μ denotes the gross trend growth rate of the economy. For future reference, note that government expenditure is normalized differently: $g_t = \frac{G_t}{Y \mu^t}$. Substituting in for the normalized variables yields:

$$\xi_t(1 + \tau^c) = \exp\left(\frac{\sigma - 1}{1 + \nu} (n_t^{RA})^{1+\nu}\right) [c_t^{RA} - (h/\mu)c_{t-1}^{RA}]^{-\sigma} \quad (8.29a)$$

$$\xi_t(1 - \tau_t^n) w_t^h = \exp\left(\frac{\sigma - 1}{1 + \nu} (n_t^{RA})^{1+\nu}\right) (n_t^{RA})^\nu [c_t^{RA} - (h/\mu)c_{t-1}^{RA}]^{1-\sigma} \quad (8.29b)$$

$$\xi_t = \bar{\beta} R_t^{gov} \mathbb{E}_t \left(\frac{\xi_{t+1}}{P_{t+1}/P_t} \right) \quad (8.29c)$$

$$Q_t = \bar{\beta} \mathbb{E}_t \left(\frac{\xi_{t+1}}{\xi_t} \left[\tilde{q}_t^k \left((1 - \tau^k) [r_{t+1}^k u_{t+1} - a(u_{t+1})] + \delta \tau^k \right) + (1 - \delta) Q_{t+1} \right] \right) \quad (8.29d)$$

$$1 = Q_t q_t^x \left(1 - S \left(\frac{x_t^{RA} \mu}{x_{t-1}^{RA}} \right) - S' \left(\frac{x_t^{RA} \mu}{x_{t-1}^{RA}} \right) \left(\frac{x_t^{RA} \mu}{x_{t-1}^{RA}} \right) \right)$$

$$+ \bar{\beta} \mathbb{E}_t \left(\frac{\xi_{t+1}}{\xi_t} Q_{t+1} q_{t+1}^x S' \left(\frac{x_{t+1}^{RA}}{x_t^{RA}} \mu \right) \left(\frac{x_{t+1}^{RA}}{x_t^{RA}} \mu \right)^2 \right) \quad (8.29e)$$

$$r_{t+1}^k = a'(u_{t+1}^{RA}). \quad (8.29f)$$

The detrended law of motion for physical capital is given by

$$k_t^{p,RA} = \frac{(1-\delta)}{\mu} k_{t-1}^{p,RA} + q_t^x \left[1 - S \left(\frac{x_t^{RA}}{x_{t-1}^{RA}} \mu \right) \right] x_t^{RA}. \quad (8.30)$$

Combining the FOC for consumption and hours worked, gives the static optimality condition for households:

$$\frac{1 - \tau_t^n}{1 + \tau^c} w_t^h = (n_t^{RA})^\nu [c_t^{RA} - (h/\mu)c_{t-1}^{RA}]. \quad (8.31)$$

Combining (8.29a) for two consecutive periods and using (8.29c) gives the consumption Euler equation:

$$\mathbb{E}_t \left(\frac{\xi_{t+1}}{\xi_t} \right) = \mathbb{E}_t \left(\exp \left(\frac{\sigma - 1}{1 + \nu} \left(\frac{n_{t+1}^{RA}}{n_t^{RA}} \right)^{1+\nu} \right) \left[\frac{c_{t+1}^{RA} - (h/\mu)c_t^{RA}}{c_t^{RA} - (h/\mu)c_{t-1}^{RA}} \right]^{-\sigma} \right). \quad (8.32)$$

Equation (8.29d) is the investment Euler equation. The FOC for capital (8.29e) can be used to compute the shadow price of physical capital Q_t .

Using the investment Euler equation shows that along the deterministic balanced growth path the value of capital equals unity (since $S'(\mu) = S(\mu) = 0$ and $\bar{q}^x = 1$). From the consumption Euler equation and $\bar{q}^b = 1$ we obtain the interest rate paid on government bonds under balanced growth. Finally, the pricing equation for capital and the investment Euler equation pin down the rental rate on capital. Summarizing:

$$\bar{Q} = 1, \quad (8.33a)$$

$$\bar{R} = \bar{\beta}^{-1} \bar{\pi}, \quad (8.33b)$$

$$1 = \bar{\beta} [(1 - \tau^k) \bar{r}^k + \delta \tau^k + (1 - \delta)],$$

$$\Leftrightarrow \bar{r}^k = \frac{\bar{\beta}^{-1} - 1 + \delta(1 - \tau^k)}{1 - \tau^k}. \quad (8.33c)$$

The bond premium shock q_t^b differs from a discount factor shock, although it results in an observationally equivalent consumption Euler equation – if time preference was time-varying, the period utility function would become:

$$\left[\frac{1}{1-\sigma} (C_{t+s}(j) - hC_{t+s-1})^{1-\sigma} \right] \exp \left[\frac{\sigma-1}{1+\nu} n_{t+s}(j)^{1+\nu} \right] \prod_{l=1}^s \check{q}_{t+l-1}^b,$$

so that the ratio $\frac{\check{\xi}_{t+1}}{\xi_t}$ would be proportional to \check{q}_t^b , so that the consumption Euler equation conditions is unchanged. The effects differ, however, insofar that the present formulation on basis of the government discount factor also affects the investment Euler equation and the government budget constraint.

For measurement purposes, it is useful to re-write the linearized FOC for capital, after substituting out for the discount factor. It shows that the private bond shock represents the premium paid for private bonds over government bonds holding the rental rate on capital fixed:

$$\frac{\bar{r}^k(1-\tau^k)\mathbb{E}_t(\hat{r}_{t+1}^k) + (1-\delta)\mathbb{E}_t(\hat{Q}_{t+1})}{\bar{r}^k(1-\tau^k) + \delta\tau^k + 1 - \delta} - \hat{Q}_t = \left(\hat{R}_t - \mathbb{E}_t[\pi_t] \right) + \hat{q}_t^b + \hat{q}_t^k.$$

Note: the shock \tilde{q}_t^k in the budget constraint has been rescaled here. \hat{q}_t^k is the deviation of the rescaled shock from its steady state value.

8.2.2 Credit-constrained or “rule of thumb” households

A fraction $\phi \in (0, 0.5)$ of the households is assumed to be credit-constrained. As a justification, one may suppose that credit-constrained discount the future substantially more steeply, and are thus uninterested in accumulating government bonds or private capital, unless their returns are extraordinarily high. Conversely, these households find it easy to default on any loans, and are therefore not able to borrow. We hold the identity of credit-constrained households and thereby their fraction of the total population constant.

“Rule of thumb” households face a static budget constraint in each period

and are assumed to supply the same amount of labor as intertemporally optimizing households. Given

$$n_{t+s}^{RoT}(j) = n_{t+s}^{RA} = n_{t+s},$$

consumption follows from the budget constraint in each period:

$$(1+\tau^c)C_{t+s}^{RoT}(j) \leq S_{t+s}^{RoT} + (1-\tau_{t+s}^n) \frac{W_{t+s}^h n_{t+s}^{RoT}(j) + \lambda_{w,t+s} W_{t+s}^h n_{t+s}}{P_{t+s}} + \Pi_{t+s}^p \mu^{t+s}. \quad (8.34)$$

Rule-of-thumb households receive transfers, labor income including union profits, and profits made by intermediate goods producing firms.

Removing the trend from the budget constraint (8.34), omitting the j index, and solving for (detrended) consumption:

$$c_{t+s}^{RoT} = \frac{1}{(1+\tau^c)} \left(s_{t+s}^{RoT} + (1-\tau_{t+s}^n)[w_{t+s}^h n_{t+s}^{RoT} + \lambda_{w,t+s} w_{t+s}^h n_{t+s}] + \Pi_{t+s}^p \right). \quad (8.35)$$

From the budget constraint (8.34), the following steady state relationship holds:

$$\bar{c}^{RoT} = \frac{\bar{s}^{RoT} + (1-\tau^n)\bar{w}\bar{n}}{1+\tau^c}. \quad (8.36)$$

We assume that:

$$\bar{s}^{RoT} = \bar{s}. \quad (8.37)$$

8.2.3 Households: labor supply, wage setting

Households supply homogeneous labor to unions which differentiate labor into varieties indexed by $\ell \in [0, 1]$ and sell it to labor packers. In doing so, unions take aggregate quantities, i.e. households' cost of supplying labor and aggregate labor demand and wages, as given. Unions maximize the expected present discounted value of net of tax wage income earned in excess of the cost of supplying labor. In the presence of rule-of-thumb households unions act as if they were maximizing surplus for the intertemporally optimizing

households only. If the mass of rule-of-thumb households is less than the mass of intertemporally optimizing households, i.e. $\phi < 0.5$ which is satisfied in the parameterizations used, a median-voter decision rule justifies this assumption.

The labor unions problem is analogous to that of price-setting firms, with the marginal rate of substitution between consumption and leisure in the representative household taking the role of marginal costs in firms' problems. From the FOC $[C_t]$ and $[n_t]$ the marginal rate of substitution is given by $\frac{U_{n,t+s}}{\Xi_{t+s}} = (n_t^{RA})^\nu [C_t^{RA} - hC_{t-1}^{RA}](1 + \tau^c)$. Whenever a union has the chance to reset the wage it charges, it chooses $W_t^*(\ell)$:

$$W_t^*(\ell) = \arg \max_{\tilde{W}_t(\ell)} \mathbb{E}_t \sum_{s=0}^{\infty} (\zeta_w)^s \frac{\bar{\beta}^s \xi_{t+s}}{\xi_t} \left[(1 - \tau_{t+s}^n) \frac{W_{t+s}(\ell)}{P_{t+s}} + \frac{U_{n,t+s}}{\Xi_{t+s}} \right] n_{t+s}(\ell), \quad (8.38)$$

subject to the labor demand equation (8.20). $1 - \zeta_w$ denotes the probability that a union can reset its wage. If it cannot adjust, wages are adjusted according to a moving average of past and steady state inflation and labor productivity growth:

$$W_{t+s}(\ell) = W_t^*(\ell) \prod_{v=1}^s \mu(\pi_{t+v-1})^{\iota_w} \bar{\pi}^{1-\iota_w} \equiv W_t^*(\ell) \chi_{t,t+s}^w.$$

Using that $n_t = n_t^{RA}$, the first order condition is given by

$$0 = \mathbb{E}_t \sum_{s=0}^{\infty} \zeta_p^s \frac{\bar{\beta}^s \xi_{t+s}}{\xi_t \lambda^w(n_{t,t+s}(\ell))} \frac{n_{t+s}(\ell)}{W_t^*(\ell)} \left((1 - \tau_{t+s}^n) \frac{W_t^*(\ell) \chi_{t,t+s}^w(\ell)}{P_{t+s}} - [1 + \lambda^w(n_{t+s}(\ell))](1 + \tau^c) n_{t+s}^\nu [C_{t+s}^{RA} - hC_{t+s-1}^{RA}] \right) \quad (8.39)$$

and can be equivalently expressed as

$$\frac{W_t^*(\ell)}{P_t} = \frac{\mathbb{E}_t \sum_{s=0}^{\infty} \zeta_p^s \frac{\bar{\beta}^s \xi_{t+s}}{\xi_t \lambda^w(\mathbf{n}_{t,t+s}(\ell))} n_{t+s}(\ell) [1 + \lambda^w(\mathbf{n}_{t+s}(\ell))] n_{t+s}^\nu [C_{t+s}^{RA} - h C_{t+s-1}^{RA}]}{\mathbb{E}_t \sum_{s=0}^{\infty} \zeta_p^s \frac{\bar{\beta}^s \xi_{t+s}}{\xi_t \lambda^w(\mathbf{n}_{t,t+s}(\ell))} n_{t+s}(\ell) \frac{1 - \tau_{t+s}^n}{1 + \tau^c} \frac{\chi_{t,t+s}^w}{P_{t+s}/P_t}} \quad (8.40)$$

Aggregate wages evolve as

$$W_t = (1 - \zeta_w) W_t^* H'^{-1} \left(\frac{W_t^* n_t}{\Xi_t^n} \right) + \zeta_w \pi_{t-1}^{\iota_w} \bar{\pi}^{1 - \iota_w} W_{t-1} H'^{-1} \left(\frac{\pi_{t-1}^{\iota_w} \bar{\pi}^{1 - \iota_w} W_{t-1} n_t}{\Xi_t^n} \right), \quad (8.41)$$

Along the deterministic balanced growth path, the detrended desired real wage is given by a constant mark-up over the marginal rate of substitution. Given constant inflation, the symmetric deterministic growth path also implies, from equation (8.41), that the desired real wage equals the actual real wage:

$$\bar{w} = \bar{w}^* = (1 + \bar{\lambda}_w) \bar{w}^h = (1 + \bar{\lambda}_w) \frac{1 + \tau^c}{1 - \bar{\tau}^n} \bar{n}^\nu \bar{c}^{RA} [1 - h/\mu], \quad (8.42)$$

where the second equality uses (8.31).

8.3 Government

The government sets nominal interest R_t according to an interest rate rule, purchases goods and services for government consumption G_t , pays transfers S_t to households, and provides public capital for the production of intermediate goods, K_t^g . It finances its expenditures by levying taxes on capital and labor income, a tax on consumption expenditure, and one period nominal bond issues. We consider a setup in which monetary policy is active in the neighborhood of the balanced growth path.

8.3.1 Fiscal policy

In modelling the government sector, we take as given the tax structure along the balanced growth path as in ?, who used NIPA data to compute the capital

and labor income and consumption expenditure tax rates for the US. Off the balanced growth path, we follow ? in assuming that labor tax rates adjust gradually to balance the budget in the long-run, whereas in the short-run much of any additional government expenditure is tax financed.

The government flow budget constraint is given by:

$$G_t + X_t^g + S_t + \frac{B_{t-1}}{P_t} \leq \frac{B_t}{R_t^{gov} P_t} + \tau^c C_t + \tau_t^n n_t \frac{W_t}{P_t} + \tau^k \left[u_t \frac{R_t^k}{P_t} - a(u_t) - \delta \right] K_{t-1}^p. \quad (8.43)$$

Detrended, the government budget constraint is given by:

$$\bar{y}g_t + x_t^g + s_t + \frac{b_{t-1}}{\mu\pi_t} \leq \frac{b_t}{R_t^{gov}} + \tau^c c_t + \tau_t^n n_t w_t + \tau^k k_t^s r_t^k - \tau^k [a(u_t) + \delta] \frac{k_{t-1}^p}{\mu}. \quad (8.44)$$

Government consumption $g_t = \frac{G_t}{\bar{y}\mu^t}$ is given exogenously and is stochastic, driven by genuine spending shocks as well as by technology shocks.

By introducing a wedge between the federal funds rate and government bonds, we capture both short-term liquidity premia as well as changes in the term structure of government debt. Since the latter is absent with only one period bonds, in the estimation the bond premium may also reflect differences in the borrowing cost due to a more complex maturity structure.³

Labor tax rates have both a stochastic and a deterministic component. They adjust deterministically to ensure long-run budget balance at a speed governed by the parameter $\psi_\tau \in [\underline{\psi}_\tau, 1]$, where $\underline{\psi}_\tau$ is some positive number large enough to guarantee stability. To simplify notation denote the remaining detrended deficit prior to new debt and changes in labor tax rates as d_t :

$$d_t \equiv \bar{y}g_t + x_t^g + s_t + \frac{b_{t-1}}{\mu\pi_t} - \tau^c c_t - \bar{\tau}^n w_t n_t - \tau^k k_t^s r_t^k + \tau^k \delta \frac{k_{t-1}^p}{\mu}.$$

³Historical data by the Federal Reserve implies a maturity between 10 and 22 quarters with an average between 16 and 20 quarters (The Federal Reserve Board Bulletin, 1999, Figure 4).

Tax rates are adjusted according to the following rule:

$$(\tau_t^n - \bar{\tau}^n)w_t n_t + \epsilon_t^\tau = \psi_\tau(d_t - \bar{d}), \quad (8.45)$$

where ϵ_t^τ is an exogenous shock to the tax rate. Debt issues are then given by the budget constraint or equivalently as the residual from (8.45): $\frac{b_t}{R_t^{gov}} = (1 - \psi_\tau)(d_t - \bar{d}) + \epsilon_t^\tau$.

Government investment is chosen optimally for a given tax structure. Given the congestion effect of production on public infrastructure, a tax on production would be optimal (Barro and Sala-i Martin, 1992). Similarly, we neglect the potential cost of financing of productive government expenditure via distortionary taxes. To motivate this assumption note that along the balanced growth path, government capital can be completely debt-financed or privatized and financed through government bond issues, whereas other government expenditures such as transfers which are not backed by real assets have to be backed by the government's power to levy taxes.

Formally, the government chooses investment and capital stock to maximize the present discounted value of output net of investment expenditure along the balanced growth path:

$$\max_{\{K_{t+s}^g, X_{t+s}^g\}_{s=0}^\infty} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \frac{\Xi_{t+s}}{\Xi_t} [Y_{t+s} - X_{t+s}^g],$$

given K_{t-1}^g and subject to the aggregate production function (8.8) and to the capital accumulation equation

$$K_{t+s}^g = (1 - \delta)K_{t+s-1}^g + q_{t+s}^{x,g} \left[1 - S_g \left(\frac{[X_{t+s}^g + \tilde{u}_{t+s}^{x,g}]}{[X_{t+s-1}^g + \tilde{u}_{t+s-1}^{x,g}]} \right) \right] (X_{t+s}^g + \tilde{u}_{t+s}^{x,g}). \quad (8.46)$$

The government is subject to similar adjustment costs as the private sector $S_g(\mu) = S'_g(\mu) = 0, S''_g > 0$ and investment is subject to shocks to its relative efficiency $q_{t+s}^{x,g}$. We assume that government capital depreciates at the same rate as private physical capital. $\tilde{u}^{x,g}$ represents exogenous shocks to government investment spending – such as stimulus spending.

Denote the Lagrange multiplier on (8.46) at time $t + s$ as $\beta^s \frac{\Xi_t^g}{\Xi_t}$. Then the first order conditions are:

$$\begin{aligned} [X_t^g] \quad 1 &= \frac{\Xi_t^g}{\Xi_t} q_t^x \left(1 - S_g \left(\frac{[\tilde{u}_t^{x,g} + X_t^g]}{[\tilde{c}_{t-1}^{x,g} + X_{t-1}^g]} \right) - S'_g \left(\frac{[\tilde{\epsilon}_t^{x,g} + X_t^g]}{[\tilde{c}_{t-1}^{x,g} + X_{t-1}^g]} \right) \left(\frac{[\tilde{u}_t^{x,g} + X_t^g]}{[\tilde{c}_{t-1}^{x,g} + X_{t-1}^g]} \right) \right) \\ &\quad + \beta \mathbb{E}_t \left(\frac{\Xi_{t+1}^g}{\Xi_t} q_{t+1}^x S'_g \left(\frac{[\tilde{\epsilon}_{t+1}^{x,g} + X_{t+1}^g]}{[\tilde{u}_t^{x,g} + X_t^g]} \right) \left(\frac{[\tilde{\epsilon}_{t+1}^{x,g} + X_{t+1}^g]}{[\tilde{u}_t^{x,g} + X_t^g]} \right)^2 \right) \\ [K_t^g] \quad \frac{\Xi_t^g}{\Xi_t} &= \beta \mathbb{E}_t \left(\frac{\Xi_{t+1}^g}{\Xi_t} \zeta \frac{Y_t + \mu^t \Phi}{K_{t-1}^g} + (1 - \delta) \frac{\Xi_{t+1}^g}{\Xi_t} \right) \end{aligned}$$

Defining the shadow price of government capital as $Q_t^g \equiv \frac{\Xi_t^g}{\Xi_t}$ and detrending, the first order conditions can be equivalently written as:

$$\begin{aligned} 1 &= Q_t^g q_t^x \left(1 - S_g \left(\frac{[\epsilon_t^{x,g} + x_t^g] \mu}{[\tilde{\epsilon}_{t-1}^{x,g} + x_{t-1}^g]} \right) - S'_g \left(\frac{[\epsilon_t^{x,g} + x_t^g] \mu}{[\tilde{\epsilon}_{t-1}^{x,g} + x_{t-1}^g]} \right) \left(\frac{[\epsilon_t^{x,g} + x_t^g] \mu}{[\tilde{\epsilon}_{t-1}^{x,g} + x_{t-1}^g]} \right) \right) \\ &\quad + \bar{\beta} \mathbb{E}_t \left(Q_{t+1}^g \frac{\xi_{t+1}}{\xi_t} q_{t+1}^x S'_g \left(\frac{[\epsilon_{t+1}^{x,g} + x_{t+1}^g] \mu}{[\epsilon_t^{x,g} + x_t^g]} \right) \left(\frac{[\epsilon_{t+1}^{x,g} + x_{t+1}^g] \mu}{[\epsilon_t^{x,g} + x_t^g]} \right)^2 \right) \end{aligned} \quad (8.47a)$$

$$Q_t^g = \bar{\beta} \mathbb{E}_t \left(\frac{\xi_{t+1}}{\xi_t} \zeta \frac{y_t + \Phi}{k_{t-1}^g / \mu} + \frac{\xi_{t+1}}{\xi_t} (1 - \delta) Q_{t+1}^g \right), \quad (8.47b)$$

where $\epsilon_t^{x,g} \equiv \frac{1}{\mu} \tilde{\epsilon}_t^{x,g}$ denotes the detrended investment spending shock.

Along the balanced growth path, $S_g(\mu) = S'_g(\mu) = 0$, $\bar{q}^{x,g} = 1$, $\bar{\epsilon}^{x,g} = 0$ ensure that the shadow price of capital equals unity. Introduce r_t^g as a shorthand for the implied rental rate on government capital:

$$r_t^g = \zeta \frac{y_t + \Phi}{k_t^g / \mu}. \quad (8.48)$$

In the steady state, from (8.47b):

$$\bar{r}^g = \bar{\beta}^{-1} - (1 - \delta) \quad (8.49)$$

Equation (8.47b) determines the optimal ratio of government capital to gross output. Importantly, the law of motion for government capital (8.46) and (8.47b) evaluated at the balanced growth path allow to back out the

share of government capital in the aggregate production function, for any given government investment to net output ratio $\frac{\bar{x}^g}{\bar{y}}$. From the law of motion along the balanced growth path:

$$\bar{x}^g = \left(1 - \frac{1 - \delta}{\mu}\right) \bar{k}^g \Leftrightarrow \frac{\bar{x}^g}{\bar{y}} = [\mu - (1 - \delta) \text{frac} \bar{k}^g] \mu \bar{y}$$

From the equation for r_t^g we have that $\frac{\bar{k}^g}{\mu \bar{y}} = \zeta \frac{\bar{y} + \Phi}{\bar{y}} \frac{1}{\bar{r}^g}$. Combined with the previous equation this allows to solve for the government capital share ζ :

$$\zeta = \frac{\bar{y}}{\bar{y} + \Phi} \frac{\bar{r}^g}{1 - (1 - \delta) \frac{\bar{x}}{\bar{y}}} \quad (8.50)$$

8.3.2 Monetary policy

The specification of the interest rate rule follows Smets and Wouters (2007). The Federal Reserve sets interest rates according to the following rule:

$$\frac{R_t^{FFR}}{\bar{R}} = \left(\frac{R_{t-1}^{FFR}}{\bar{R}}\right)^{\rho_R} \left[\left(\frac{\pi_t}{\bar{\pi}}\right)^{\psi_1} \left(\frac{Y_t}{Y_t^f}\right)^{\psi_2}\right]^{1-\rho_R} \left(\frac{Y_t/Y_{t-1}}{Y_t^f/Y_{t-1}^f}\right)^{\psi_3} \epsilon_t^r, \quad (8.51)$$

where ρ_R determines the degree of interest rate smoothing and Y_t^f denotes the level of output that would prevail in the economy in the absence of nominal frictions and with constant markups, i.e. the flexible output level. $\psi_1 > 1$ determines the reaction to inflation to deviations of inflation from its long-run average and $\psi_2, \psi_3 > 0$ determine the reaction to the deviation of actual output from the flexible economy output and to the change in the gap between actual and flexible output.

Due to financial market frictions, the return on government bonds differs from the federal funds rate:

$$R_t^{gov} = R_t^{FFR}(1 + \omega_t^b)$$

The flexible economy is the limit point of the economy characterized above with $\zeta_p = \zeta_w = 0$ and no markup shocks: $\epsilon_t^{\lambda,p} = \epsilon_t^{\lambda,w} = 0$. From the pricing

and wages setting rules this limiting solution implies:

$$\frac{P_t^f(i)}{P_t^f} = [1 + \lambda_p(y_t^f(i))]mc_t^f(i), \quad (8.52)$$

$$\frac{W_t^f(\ell)}{P_t^f} = [1 + \lambda_w(n_t^f(\ell))] \frac{1 + \tau^c}{1 - \tau_{t+s}^{n,f}} n_t^{f\nu} [C_t^f - hC_{t-1}^f], \quad (8.53)$$

where the superscript f denotes variables in the flexible economy. Given that final goods are the numeraire and given that firms are symmetric and can freely set their prices:

$$1 = P_t^f = P_t^f(i) = [1 + \lambda_p(1)]mc_t^f(i) \quad \forall t, \quad (8.54)$$

implying that marginal costs are constant for all firms.

Similarly, since all unions face a symmetric problem and can freely reset wages we have that, using that the numeraire equals unity and diving by trend growth:

$$\frac{W_t^f(\ell)}{\mu} = \frac{W_t^f}{\mu} = w_t^f = [1 + \lambda_w(1)] \frac{1 + \tau^c}{1 - \tau_{t+s}^{n,f}} n_t^{f\nu} [c_t^f - (h/\mu)c_{t-1}^f]. \quad (8.55)$$

Money does not enter explicitly in the economy: the Federal Reserve supplies the amount of money demanded at interest rate R_t .

8.4 Exogenous processes

The exogenous processes are assumed to be log-normally distributed and, with the exception of government spending shocks, to be independent. Government spending shocks are correlated with technology shocks. Shocks to the two mark-up processes follow an ARMA(1,1) process, whereas the other shocks are AR(1) processes.

$$\log \epsilon_t^a = \rho_a \log \epsilon_{t-1}^a + u_t^a, \quad u_t^a \stackrel{iid}{\sim} \mathcal{N}(0, \sigma_a^2) \quad (8.56a)$$

$$\log \epsilon_t^r = \rho_r \log \epsilon_{t-1}^r + u_t^r, \quad u_t^r \stackrel{iid}{\sim} \mathcal{N}(0, \sigma_r^2) \quad (8.56b)$$

$$\log g_t = \log g_t^a + \tilde{u}_t^g, \quad (8.56c)$$

$$\log g_t^a = (1 - \rho_g) \log \bar{g} + \rho_g \log g_{t-1}^a + \sigma_{ga} u_t^a + u_t^g, \quad u_t^a \stackrel{iid}{\sim} \mathcal{N}(0, \sigma_a^2) \quad (8.56d)$$

$$\log s_t = \tilde{u}_t^s, \quad (8.56e)$$

$$\log \epsilon_t^\tau = \rho_\tau \log \epsilon_{t-1}^\tau + u_t^\tau, \quad u_t^\tau \stackrel{iid}{\sim} \mathcal{N}(0, \sigma_\tau^2) \quad (8.56f)$$

$$\log \tilde{\epsilon}_t^{\lambda,p} = \rho_{\lambda,p} \log \tilde{\epsilon}_{t-1}^{\lambda,p} + u_t^{\lambda,p} - \theta_{\lambda,p} u_{t-1}^{\lambda,p}, \quad u_t^{\lambda,p} \stackrel{iid}{\sim} \mathcal{N}(0, \sigma_{\lambda,p}^2) \quad (8.56g)$$

$$\log \tilde{\epsilon}_t^{\lambda,w} = \rho_{\lambda,w} \log \tilde{\epsilon}_{t-1}^{\lambda,w} + u_t^{\lambda,w} - \theta_{\lambda,w} u_{t-1}^{\lambda,w}, \quad u_t^{\lambda,w} \stackrel{iid}{\sim} \mathcal{N}(0, \sigma_{\lambda,w}^2) \quad (8.56h)$$

$$\log(1 + \omega_t^b) \equiv \log q_t^b = \rho_b \log q_{t-1}^b + u_t^b, \quad u_t^b \stackrel{iid}{\sim} \mathcal{N}(0, \sigma_b^2) \quad (8.56i)$$

$$\log(1 - \omega_t^k) \equiv \log q_t^k = \rho_k \log q_{t-1}^k + u_t^k, \quad u_t^k \stackrel{iid}{\sim} \mathcal{N}(0, \sigma_k^2) \quad (8.56j)$$

$$\log q_t^x = \rho_x \log q_{t-1}^x + u_t^x, \quad u_t^x \stackrel{iid}{\sim} \mathcal{N}(0, \sigma_x^2) \quad (8.56k)$$

$$\log q_t^{x,g} = \rho_{x,g} \log q_{t-1}^{x,g} + u_t^{x,g}, \quad u_t^{x,g} \stackrel{iid}{\sim} \mathcal{N}(0, \sigma_{x,g}^2) \quad (8.56l)$$

Three shocks are deterministic and used for policy counterfactuals only:

$$\tilde{u}_t^s, \tilde{u}_t^g, \tilde{u}_t^{x,g}.$$

8.5 Equilibrium conditions

8.5.1 Aggregation

From the final goods producers' problem (8.1) and using the zero profit condition in the competitive market, net output in nominal and real terms is

given by

$$P_t Y_t = \int_0^1 P_t(i) Y_t(i) di \quad \Leftrightarrow \quad Y_t = \int_0^1 \frac{P_t(i)}{P_t} Y_t(i) di.$$

Outside the flexible economy, relative prices differ from unity, so that output is not simply the average production of intermediates. However, to a first order price dispersion is irrelevant because $y_t(i) \approx y_t - \eta_p(1)y_t \left(\frac{P_t(i)}{P_t} - 1 \right)$, so that the dispersion term averages out in the aggregate $\int_0^1 y_t(i) di \approx y_t$.

In the presence of heterogeneous labor, the measurement of labor supply faces similar issues because

$$n_t = \int_0^1 \frac{W_t(\ell)}{W_t} n_t(\ell) d\ell,$$

which, by analogy to the above argument for output, generally differs from average hours. However, to a first order:

$$\int_0^1 n_t(\ell) d\ell \approx n_t \tag{8.57}$$

Non-credit constrained households are indexed by $j \in [0, 1 - \phi]$ and there is measure $1 - \phi$ of these households in the economy. Each non-credit constraint household supplies $K_t(j) = K_t^{RA}$ units of capital services, so that total holdings of capital and government bonds per intertemporally optimizing household are given by $\frac{1}{1-\phi}$ times the aggregate quantity. Similarly, household investment is a multiple of aggregate investment. To see this, note that aggregate quantities of bond holdings B_t , investment X_t , physical capital K_t^P , and capital services K_t are computed as:

$$K_t = \int_0^{1-\phi} K_t(j) \Lambda(dj) = K_t(1 - \phi)^{-1} \Lambda([0, 1 - \phi]) = K_t.$$

Aggregate consumption is given by:

$$C_t = \int_0^1 C_t(j) \Lambda(dj) = \int_0^{1-\phi} C_t^{RA} \Lambda(dj) + \int_{1-\phi}^1 C_t^{RoT} \Lambda(dj) = (1-\phi) C_t^{RA} + \phi C_t^{RoT}. \quad (8.58)$$

Given the consumption of rule-of-thumb agents (8.36), that of intertemporally optimizing agents is given by:

$$\bar{c}^{RA} = \frac{\bar{c} - \phi \bar{c}^{RoT}}{1 - \phi}. \quad (8.59)$$

Similarly, aggregate transfers are given by

$$S_t = (1 - \phi) S_t^{RA} + \phi S_t^{RoT}, \quad (8.60)$$

where equation (8.37) implies that:

$$\bar{s} = \bar{s}^{RA} + \bar{s}^{RoT}.$$

Aggregate labor supply coincides with individual labor supply of either type of household.

8.5.2 Market Clearing

Labor market clearing requires that labor demanded by intermediaries equals labor supplied by labor packers:

$$\int_0^1 n_t(i) di = n_t = n_t \int_0^1 \frac{W_t(\ell)}{W_t} n_t(\ell) d\ell,$$

where $n_t(\ell)$ is measured in units of the differentiated labor supplies and n_t is measured in units which differs from those supplied by households.

Adding the government and the budget constraints of the two types of households, integrated over $[0, 1 - \phi]$ and $(1 - \phi, 1]$, respectively, and substituting $\int_0^1 n_t(j) W_t^h(1 + \lambda_{t,w}) dj = W_t n_t$, which results from combining the

labor packers' zero profit condition with the union problem into the household budget constraint, yields the following equation:

$$C_{t+s} + X_{t+s}(j) + G_t + X_{t+s}^g = n_t \frac{W_{t+s}}{P_{t+s}} + \left[\frac{R_{t+s}^k u_{t+s}}{P_{t+s}} - a(u_{t+s}) \right] K_{t+s-1}^p + \frac{\Pi_{t+s}^p \mu^{t+s}}{P_{t+s}},$$

Detrending and substituting in for real profits from (8.17a), using that $w_t \int_0^1 n_t(i) di = w_t n_t$:

$$c_{t+s} + x_{t+s} + \bar{y} g_{t+s} + x_{t+s}^g = y_{t+s} - a(u_{t+s}) \mu k_{t+s-1}^p, \quad (8.61)$$

which is the goods market clearing condition: Production is used for government and private consumption, government and private investment, as well as variations in capacity utilization.

8.6 Linearized equilibrium conditions

8.6.1 Firms

Log-linearizing the production function around the symmetric balanced growth path:

$$\hat{y}_t = \frac{\bar{y} + \Phi}{\bar{y}} \left(\hat{\epsilon}_t^a + \zeta \hat{k}_{t-1}^g + \alpha(1 - \zeta) \hat{k}_t + (1 - \alpha)(1 - \zeta) \hat{n}_t \right). \quad (8.62)$$

The capital-labor ratio is approximated by (8.9):

$$\hat{k}_t = \hat{n}_t + \hat{w}_t - \hat{r}_t^k, \quad (8.63)$$

where symmetry around the balanced growth path was used.

Marginal costs in (8.64) are approximated by

$$\widehat{mc}_t = (1 - \alpha) \hat{w}_t + \alpha \hat{r}_t^k - \frac{1}{1 - \zeta} \left(\zeta \hat{k}_t^g - \zeta \frac{\bar{y}}{\bar{y} + \Phi} \hat{y}_t + \hat{\epsilon}_t^a \right) \left(\frac{k_t^g}{y_t + \Phi} \right)^{\frac{\zeta}{1 - \zeta}} \tilde{\epsilon}_t^a, \quad (8.64)$$

and in the flexible economy from (8.54):

$$\widehat{mc}_t^f = 0 \quad (8.65)$$

To-log linearize the pricing FOC (8.14), note that to a first order the common terms in numerator and denominator, i.e. $\frac{\xi_{t+s} y_{t,t+s}(i)}{\lambda_p(y_{t+s}(i)) \xi_t}$, cancel out, using equation (8.16). As a preliminary step notice that in the absence of mark-up shocks:

$$\begin{aligned} \overline{mc} d \left(\frac{\eta_p(y_{t+s}(i))}{1 - \eta_p(y_{t+s}(i))} \right) \Big|_{y_{t+s}(i)=1} &= \overline{mc} \frac{\bar{\eta}_p}{1 - \bar{\eta}_p} \frac{-1}{1 - \bar{\eta}_p} \frac{d\eta_p(y_{t+s}(i)) \Big|_{y_{t+s}(i)=1}}{\bar{\eta}_p} \\ &= -\bar{\lambda}_p \hat{\eta}_p(1) d \left(\frac{P_t^*(i)}{P_{t+s}} \right) \Big|_{\frac{P_{t+s}^*(i)}{P_{t+s}}=1}, \\ d \left(\frac{P_{t+s}(i)}{P_{t+s}} \right) \Big|_{\frac{P_{t+s}^*(i)}{P_{t+s}}=1} &= d \left(\frac{\chi_{t,t+s}}{\prod_{l=1}^s \pi_{t+l}} \right) + d \left(\frac{P_t^*(i)}{P_t} \right). \end{aligned}$$

Notice that from (8.22):

$$1 + \bar{\lambda}_p \hat{\eta}_p = \frac{1}{A_p}$$

To simplify notation and to address mark-up shocks use $\bar{\epsilon}^{\lambda,p} = 1$ define

$$\begin{aligned} p_t^*(i) &\equiv \frac{P_t^*(i)}{P_t}, \\ \hat{\epsilon}_{t+s}^{\lambda,p} &\equiv \frac{\partial}{\partial \epsilon_{t+s}^{\lambda,p}} \left(\frac{\eta_p(y_{t+s}(i))}{1 - \eta_p(y_{t+s}(i))} \right) \Big|_{y_{t+s}(i)=1} \hat{\epsilon}_{t+s}^{\lambda,p} = \frac{\eta_p(1)}{[1 - \eta_p(1)]^2} \left(\frac{G'_\epsilon(1)}{G'(1)} - \frac{G''_\epsilon(1)}{G''(1)} \right). \end{aligned}$$

Now, taking a first-order approximation of (8.14) and using symmetry yields

$$\begin{aligned} 0 &= \mathbb{E}_t \sum_{s=0}^{\infty} (\mu \bar{\beta} \zeta_p)^s \left[\hat{p}_t^*(i) + \sum_{l=1}^s [l_p \hat{\pi}_{t+l-1} - \hat{\pi}_{t+l}] \right] (1 + \bar{\lambda}_p \hat{\eta}(1)) - [\widehat{mc}_{t+s} + \hat{\epsilon}_{t+s}^{\lambda,p}] \\ \Leftrightarrow \frac{1}{1 - \bar{\beta} \zeta_p \mu} \frac{1}{A_p} \hat{p}_t^* &= \mathbb{E}_t \sum_{s=0}^{\infty} (\mu \bar{\beta} \zeta_p)^s [\widehat{mc}_{t+s} + \hat{\epsilon}_{t+s}^{\lambda,p}] - \sum_{l=1}^s [l_p \hat{\pi}_{t+l-1} - \hat{\pi}_{t+l}] \frac{1}{A_p} \\ &= \widehat{mc}_t + \hat{\epsilon}_t^{\lambda,p} - \frac{\bar{\beta} \mu \zeta_p}{\beta \mu \zeta_p} \frac{1}{A_p} [l_p \hat{\pi}_t - \mathbb{E}_t \hat{\pi}_{t+1}] \end{aligned}$$

$$\begin{aligned}
& + \mu \bar{\beta} \zeta_p \mathbb{E}_t \mathbb{E}_{t+s} \sum_{s=0}^{\infty} (\mu \bar{\beta} \zeta_p)^s [\widehat{m}c_{t+1+s} + \hat{\epsilon}_{t+1+s}^{\lambda,p}] - \sum_{l=1}^s [\iota_p \hat{\pi}_{t+l} - \hat{\pi}_{t+1+l}] \frac{1}{A_p} \\
& = \widehat{m}c_t + \hat{\epsilon}_t^{\lambda,p} - \frac{\bar{\beta} \mu \zeta_p}{\bar{\beta} \mu \zeta_p} \frac{1}{A_p} [\iota_p \hat{\pi}_t - \mathbb{E}_t \hat{\pi}_{t+1}] + \mu \bar{\beta} \zeta_p \mathbb{E}_t \hat{p}_{t+1}^*.
\end{aligned}$$

Now, linearizing the evolution of the price index (8.15):

$$\hat{p}_t^* = \frac{\zeta_p}{1 - \zeta_p} [\hat{\pi}_t - \iota_p \hat{\pi}_{t-1}] \Leftrightarrow \hat{\pi}_t = \frac{1 - \zeta_p}{\zeta_p} \hat{p}_t^* + \iota_p \hat{\pi}_{t-1}.$$

Forwarding the equation once and substituting in and solving for $\hat{\pi}_t$ yields:

$$\hat{\pi}_t = \frac{\iota_p}{1 + \iota_p \bar{\beta} \mu} \hat{\pi}_{t-1} + \frac{1 - \zeta_p \bar{\beta} \mu}{1 + \iota_p \bar{\beta} \mu} \frac{1 - \zeta_p}{\zeta_p} \bar{A}_p (\widehat{m}c_t + \hat{\epsilon}_t^{\lambda,p}) + \frac{\bar{\beta} \mu}{1 + \iota_p \bar{\beta} \mu} \mathbb{E}_t \hat{\pi}_{t+1} \quad (8.66)$$

8.6.2 Households

The law of motion for capital (8.26) and the fact that individual capital holdings are proportional to aggregate capital holdings implies:

$$\hat{k}_t^p = \left(1 - \frac{\bar{x}}{k^p}\right) \hat{k}_{t-1}^p + \frac{\bar{x}}{k^p} (\hat{x}_t + \hat{q}_{t+s}^x). \quad (8.67)$$

From (8.27), capital services evolve as:

$$\hat{k}_t = \hat{u}_t + \hat{k}_{t-1}^p \quad (8.68)$$

From the static optimality condition (8.31)

$$\hat{w}_t^h = \nu \hat{n}_t + \frac{\hat{c}_t^{RA} - (h/\mu) \hat{c}_{t-1}^{RA}}{1 - h/\mu} + \frac{d\tau_t^n}{1 - \bar{\tau}^n}. \quad (8.69)$$

In the flexible economy, given the absence of mark-up shocks equation (8.55) implies:

$$\hat{w}_t^f = \nu \hat{n}_t^f + \frac{\hat{c}_t^{RA,f} - (h/\mu) \hat{c}_{t-1}^{RA,f}}{1 - h/\mu} + \frac{d\tau_t^{n,f}}{1 - \bar{\tau}^n}. \quad (8.70)$$

In the presence of rigidities, the dynamic wage setting equation (8.40) can be linearized as in the derivation of (8.66), recognizing that the analogue to

marginal costs is given by (8.69):

$$\begin{aligned}
\hat{w}_t &= \frac{\hat{w}_{t-1}}{1 + \bar{\beta}\mu} + \frac{\bar{\beta}\mu\mathbb{E}_t[\hat{w}_{t+1}]}{1 + \bar{\beta}\mu} \\
&+ \frac{(1 - \zeta_w\bar{\beta}\mu)(1 - \zeta_w)}{(1 + \bar{\beta}\mu)\zeta_w} \bar{A}_w \left[\frac{1}{1 - h/\mu} [\hat{c}_t - (h/\mu)\hat{c}_{t-1}] + \nu\hat{n}_t - \hat{w}_t + \frac{d\tau_t^n}{1 - \tau_n} \right] \\
&- \frac{1 + \bar{\beta}\mu\iota_w}{1 + \bar{\beta}\mu} \hat{\pi}_t + \frac{\iota_w}{1 + \bar{\beta}\mu} \hat{\pi}_{t-1} + \frac{\bar{\beta}\mu}{1 + \bar{\beta}\mu} \mathbb{E}_t[\hat{\pi}_{t+1}] + \frac{\hat{c}_t^{\lambda,w}}{1 + \bar{\beta}\mu}. \tag{8.71}
\end{aligned}$$

From the consumption Euler equation (8.32):

$$\begin{aligned}
\mathbb{E}_t[\hat{\xi}_{t+1} - \hat{\xi}_t] &= \mathbb{E}_t \left((\sigma - 1)\bar{n}^{1+\nu}[\hat{n}_{t+1} - \hat{n}_t] - \frac{\sigma}{1 - h/\mu} \left[\hat{c}_{t+1}^{RA} - \left(1 + \frac{h}{\mu}\right)c_t^{RA} + \frac{h}{\mu}\hat{c}_{t+1}^{RA} \right] \right) \\
&= \frac{1}{1 - h/\mu} \mathbb{E}_t \left((\sigma - 1) \frac{\bar{n}^{1+\nu}[\bar{c}^{RA} - h/\mu\bar{c}^{RA}]}{\bar{c}^{RA}} [\hat{n}_{t+1} - \hat{n}_t] \right. \\
&\quad \left. - \sigma \left[\hat{c}_{t+1}^{RA} - \left(1 + \frac{h}{\mu}\right)c_t^{RA} + \frac{h}{\mu}\hat{c}_{t+1}^{RA} \right] \right) \\
&= \frac{1}{1 - h/\mu} \mathbb{E}_t \left((\sigma - 1) \frac{1}{1 + \bar{\lambda}_w} \frac{1 - \bar{\tau}^n}{1 + \tau^c} \frac{\bar{w}\bar{n}}{\bar{c}^{RA}} [\hat{n}_{t+1} - \hat{n}_t] \right. \\
&\quad \left. - \sigma \left[\hat{c}_{t+1}^{RA} - \left(1 + \frac{h}{\mu}\right)c_t^{RA} + \frac{h}{\mu}\hat{c}_{t+1}^{RA} \right] \right),
\end{aligned}$$

where the last equality uses (8.42). Solving for current consumption growth:

$$\begin{aligned}
\hat{c}_t^{RA} &= \frac{1}{1 + h/\mu} \mathbb{E}_t[\hat{c}_{t+1}^{RA}] + \frac{h/\mu}{1 + h/\mu} \hat{c}_{t-1}^{RA} + \frac{1 - h/\mu}{\sigma[1 + h/\mu]} \mathbb{E}_t[\hat{\xi}_{t+1} - \hat{\xi}_t] \\
&- \frac{[\sigma - 1][\bar{w}\bar{n}/\bar{c}]}{\sigma[1 + h/\mu]} \frac{1}{1 + \bar{\lambda}_w} \frac{1 - \tau^n}{1 + \tau^c} (\mathbb{E}_t[\hat{n}_{t+1}] - \hat{n}_t). \tag{8.72}
\end{aligned}$$

The remaining households' FOC linearize as:

$$\mathbb{E}_t[\hat{\xi}_{t+1} - \hat{\xi}_t] = -\hat{q}_t^b - \hat{R}_t + \mathbb{E}_t[\hat{\pi}_{t+1}], \tag{8.73a}$$

$$\begin{aligned}
\hat{Q}_t &= -\hat{q}_t^b - (\hat{R}_t - \mathbb{E}_t[\pi_{t+1}]) + \frac{1}{\bar{r}^k(1 - \tau^k) + \delta\tau^k + 1 - \delta} \times \\
&\times [(\bar{r}^k(1 - \tau^k) + \delta\tau^k)\hat{q}_t^k + \bar{r}^k(1 - \tau^k)\mathbb{E}_t(\hat{r}_{t+1}^k) + (1 - \delta)\mathbb{E}_t(\hat{Q}_{t+1})], \tag{8.73b}
\end{aligned}$$

$$\hat{x}_t = \frac{1}{1 + \bar{\beta}\mu} \left[\hat{x}_{t-1} + \bar{\beta}\mu \mathbb{E}_t(\hat{x}_{t+1}) + \frac{1}{\mu^2 S''(\mu)} [\hat{Q}_t + \hat{q}_t^x] \right], \quad (8.73c)$$

$$\hat{u}_t = \frac{a'(1)}{a''(1)} \hat{r}_t^k \equiv \frac{1 - \psi_u}{\psi_u} \hat{r}_t^k. \quad (8.73d)$$

For the credit constrained households, (8.35) implies the following linear consumption process: consumption evolves as

$$\hat{c}_t^{RoT} = \frac{1}{1 + \tau^c} \left(\frac{\bar{s}^{RoT}}{\bar{c}^{RoT}} \hat{s}_t + \frac{\bar{w}\bar{n}}{\bar{c}^{RoT}} [(1 - \tau^n)(\hat{w}_t + \hat{n}_t) - d\tau_t^n] + \frac{\bar{y}}{\bar{c}^{RoT}} \frac{d\Pi_t^p}{\bar{y}} \right), \quad (8.74)$$

where the change in profits is given by:

$$\frac{d\Pi_t^p}{\bar{y}} = \frac{1}{1 + \lambda_p} \hat{y}_t - \widehat{m}c_t.$$

8.6.3 Government

Labor tax rates evolve according to (8.45), which is linearized as:

$$\begin{aligned} \bar{\tau}^n \frac{\bar{w}\bar{n}}{\bar{c}} \frac{\bar{c}}{\bar{y}} \left[\frac{d\tau_t^n}{\tau_n} \right] + \hat{c}_t^\tau = \frac{\psi_\tau}{\mu} \left[\mu [\hat{g}_t^a + \hat{g}_t^s] + \mu \frac{\bar{s}}{\bar{y}} \hat{s}_t + \frac{\bar{b}}{\bar{y}} \frac{\hat{b}_{t-1} - \hat{\pi}_t}{\bar{\pi}} - \mu \tau^n \frac{\bar{w}\bar{n}}{\bar{c}} \frac{\bar{c}}{\bar{y}} (\hat{w}_t + \hat{n}_t) \right. \\ \left. - \mu \tau_c \frac{\bar{c}}{\bar{y}} \hat{c}_t - \tau^k [\bar{r}^k r_t^k + (r_t^k - \delta) \hat{k}_{t-1}^p] \mu \frac{\bar{k}}{\bar{y}} \right]. \quad (8.75) \end{aligned}$$

Debt holdings are determined from the budget constraint (8.44):

$$\begin{aligned} \hat{g}_t + \frac{\bar{s}}{\bar{y}} \hat{s}_t + \frac{1}{\mu \bar{\pi}} \frac{\bar{b}}{\bar{y}} [\hat{b}_{t-1} - \hat{\pi}_t] = \frac{1}{\bar{R}} \frac{\bar{b}}{\bar{y}} [\hat{b}_t - \hat{R}_t - \hat{q}_t^b] + \tau_c \frac{\bar{c}}{\bar{y}} \hat{c}_t + \tau^n \frac{\bar{w}\bar{n}}{\bar{c}} \frac{\bar{c}}{\bar{y}} \left[\frac{d\tau_t^l}{\tau_l} + \hat{w}_t + \hat{n}_t \right] \\ + \tau^k [\bar{r}^k r_t^k + (\bar{r}^k - \delta) \hat{k}_{t-1}^p] \frac{\bar{k}}{\bar{y}}. \quad (8.76) \end{aligned}$$

The linearized counterpart to the law of motion for government capital (8.46) is given by:

$$\hat{k}_t^g = \left(1 - \frac{\bar{x}^g}{\bar{k}^g} \right) \hat{k}_{t-1}^g + \frac{\bar{x}^g}{\bar{k}^g} \hat{q}_t^{x,g} + \frac{\bar{x}^g}{\bar{k}^g} [\hat{x}_t^g + \hat{c}_t^{xg}], \quad (8.77)$$

where $u_t^{x,g} \equiv \frac{\tilde{u}_t^{x,g}}{\bar{x}^g}$.

The marginal product of government capital (8.48) is approximated by

$$\hat{r}_t^g = \frac{\bar{y}}{\bar{y} + \Phi} \hat{y}_t - \hat{k}_{t-1}^g \quad (8.78)$$

The shadow price of government capital (8.47b) has the following linear approximation:

$$\hat{Q}_t^g = -(\hat{R}_t + \hat{q}_t^b - \mathbb{E}_t[\pi_{t+1}]) + \frac{1}{\bar{r}^g + 1 - \delta} [\bar{r}^g \mathbb{E}_t(\hat{r}_{t+1}^g) + (1 - \delta) \mathbb{E}_t(\hat{Q}_{t+1}^g)], \quad (8.79)$$

The Euler equation for government investment (8.47a) is approximated as:

$$\hat{x}_t^g = \frac{1}{1 + \bar{\beta}\mu} \left[\hat{x}_{t-1} + u_{t-1}^{xg} + \bar{\beta}\mu \mathbb{E}_t([\hat{x}_{t+1}^g + u_{t+1}^{xg}]) \right] + \frac{1}{\mu^2 S_g''(\mu)} [\hat{Q}_t^g + \hat{q}_t^{x,g}] - u_t^{xg} \quad (8.80)$$

The monetary policy rule (8.51) is approximated by:

$$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1 - \rho_R) [\psi_1 \hat{\pi}_t + \psi_2 (\hat{y}_t - \hat{y}_t^f)] + \psi_3 \Delta(\hat{y}_t - \hat{y}_t^f) + \hat{\epsilon}_t^r \quad (8.81)$$

8.6.4 Exogenous processes

The shock processes (8.56) are linearized as

$$\hat{\epsilon}_t^a = \rho_a \hat{\epsilon}_{t-1}^a + u_t^a, \quad (8.82a)$$

$$\hat{\epsilon}_t^r = \rho_r \hat{\epsilon}_{t-1}^r + u_t^r, \quad (8.82b)$$

$$\hat{g}_t = \hat{g}_t^a + \tilde{u}_t^g, \quad (8.82c)$$

$$\hat{g}_t^a = \rho_g \hat{g}_{t-1}^a + \sigma_{ga} u_t^a + u_t^g, \quad (8.82d)$$

$$\hat{s}_t = \tilde{u}_t^s, \quad (8.82e)$$

$$\hat{\epsilon}_t^\tau = \rho_\tau \hat{\epsilon}_{t-1}^\tau + u_t^\tau, \quad (8.82f)$$

$$\hat{\epsilon}_t^{\lambda,p} = \rho_{\lambda,p} \hat{\epsilon}_{t-1}^{\lambda,p} + u_t^{\lambda,p} - \theta_{\lambda,p} u_{t-1}^{\lambda,p}, \quad (8.82g)$$

$$\hat{\epsilon}_t^{\lambda,w} = \rho_{\lambda,w} \hat{\epsilon}_{t-1}^{\lambda,w} + u_t^{\lambda,w} - \theta_{\lambda,w} u_{t-1}^{\lambda,w}, \quad (8.82h)$$

$$\hat{q}_t^b = \rho_b \hat{q}_{t-1}^b + u_t^b, \quad (8.82i)$$

$$\hat{q}_t^k = \rho_k \hat{q}_{t-1}^k + u_t^k, \quad (8.82j)$$

$$\hat{q}_t^x = \rho_x \hat{q}_{t-1}^x + u_t^x, \quad (8.82k)$$

$$\hat{q}_t^{x,g} = \rho_{x,g} \hat{q}_{t-1}^{x,g} + u_t^{x,g}. \quad (8.82l)$$

8.6.5 Aggregation

Aggregate consumption (8.58) and transfers (8.60) are linearized as

$$\hat{c}_t = (1 - \phi) \frac{\bar{c}^{RA}}{\bar{c}} \hat{c}_t^{RA} + \phi \frac{\bar{c}^{RoT}}{\bar{c}} \hat{c}_t^{RoT}, \quad (8.83)$$

$$\hat{s}_t = (1 - \phi) \frac{\bar{s}^{RA}}{\bar{s}} \hat{s}_t^{RA} + \phi \frac{\bar{s}^{RoT}}{\bar{s}} \hat{s}_t^{RoT}. \quad (8.84)$$

8.6.6 Market Clearing

Goods market clearing:

$$\hat{y}_t = \frac{\bar{c}}{\bar{y}} \hat{c}_t + \frac{\bar{x}}{\bar{y}} \hat{x}_t + \frac{\bar{x}^g}{\bar{y}} \hat{x}_t^g + \hat{g}_t + \frac{\bar{r}^k \bar{k}}{\bar{y}} \hat{u}_t. \quad (8.85)$$

8.6.7 Solution

In addition to the exogenous processes in (8.82), the economy with frictions is reduced to 21 variables, whereas the flexible economy is characterized by 19 variables only, given perfectly flexible prices and wages. Table 15 on the following page lists the remaining variables and the corresponding equations. For the flexible economy, all variables other than those with an “n/a” entry have an f superscript. The markup shock processes affect only the economy with frictions. Table 16 on page 77 lists the steady state relationships which enter the linearized equations.

Variable	Economy with frictions	Economy without frictions
\hat{c}	(8.83)	(8.83)
\hat{c}^{RA}	(8.72)	(8.72)
\hat{c}^{RoT}	(8.74)	(8.74)
\hat{x}	(8.73a) in (8.73c)	(8.73c), (8.73a)
\hat{k}^p	(8.67)	(8.67)
\hat{k}	(8.68)	(8.68)
\hat{u}	(8.73d)	(8.73d)
\hat{Q}	(8.73a) in (8.73b)	(8.73b), (8.73a)
\hat{r}^k	(8.63)	(8.63)
\hat{x}^g	(8.73a) in (8.80)	(8.80), (8.73a)
\hat{k}^g	(8.77)	(8.77)
\hat{Q}^g	(8.73a) in (8.79)	(8.79), (8.73a)
\hat{r}^g	(8.78)	(8.78)
$d\tau^n$	(8.75)	(8.75)
\hat{b}	(8.76)	(8.76)
\hat{R}	(8.81)	indirectly via (8.65)
$\hat{\pi}$	(8.66)	=0
\widehat{mc}	(8.64)	=0
\hat{w}	(8.71)	(8.70)
\hat{y}	(8.85)	(8.85)
\hat{n}	(8.62)	(8.62)

Table 15: Unknowns and equations

Constant	Equation	Expression
$\frac{\bar{c}}{\bar{y}}$	(8.61)	$1 - \frac{\bar{x}}{\bar{y}} - \frac{\bar{x}^g}{\bar{y}} - g$
$\frac{\bar{c}^{RA}}{\bar{c}^{RoT}}$	(8.59)	$\frac{\bar{c} - \phi \bar{c}^{RoT}}{\bar{y}(1-\phi)}$
$\frac{\bar{c}^{RoT}}{\bar{y}}$	(8.36)	$\frac{\bar{s}^{RoT} + (1-\tau^n)\bar{w}\bar{n}}{\bar{y}(1+\tau^c)}$
$\frac{\bar{x}}{k^P}$	(8.30)	$1 - \frac{1-\delta}{\mu}$
$\frac{\bar{x}}{k}$	(8.30)	$\mu - (1 - \delta)$
$\frac{\bar{k}}{\bar{y}}$	(8.8)	$\left(\frac{\bar{y}+\Phi}{\bar{y}}\right)^{\frac{1}{1-\zeta}} \left(\frac{\bar{k}^g}{\bar{y}}\right)^{\frac{-\zeta}{1-\zeta}} \left(\frac{\bar{k}}{\bar{n}}\right)^{1-\alpha}$
\bar{u}	normalization	$a'^{-1}(\bar{r}^k)$
$\bar{\beta}$	definition	$\beta\mu^{-1}$
\bar{r}^k	(8.33c)	$\frac{\bar{\beta}^{-1} - \delta\tau^k - (1-\delta)}{1-\tau^k}$
$\frac{\bar{k}^g}{\bar{y}}$	(8.46)	$\left(1 - \frac{1-\delta}{\mu}\right)^{-1} \frac{\bar{x}^g}{\bar{y}}$
ζ	(8.50)	$\frac{\bar{y}+\Phi}{\bar{y}} \frac{1-(1-\delta)/\mu}{\bar{r}^g}$
\bar{r}^g	(8.49)	$\beta^{-1} - (1 - \delta)$
\bar{R}	(8.33b)	$\bar{\beta}^{-1}\bar{\pi}$
$\bar{m}\bar{c}$	(8.16)	$(1 + \bar{\lambda}_p)^{-1}$
$\bar{\lambda}_p$	(8.18)	$\frac{\Phi}{\bar{y}}$
\bar{w}	(8.11)	$\frac{\alpha^{\frac{\alpha}{1-\alpha}}(1-\alpha)}{(1+\lambda_w)^{\frac{1}{(1-\zeta)(1-\alpha)}}} \frac{\left(\frac{\bar{k}^g}{\bar{y}}\right)^{\frac{\zeta}{(1-\zeta)(1-\alpha)}}}{\bar{r}^k \bar{k}^{\frac{\alpha}{1-\alpha}}}$
$\frac{\bar{w}\bar{n}}{\bar{y}}$	$[n_t(i)], [K_t(i)], (8.16), (8.18)$	$1 - \bar{r}^k \frac{\bar{k}}{\bar{y}}$
$\frac{\bar{k}}{\bar{n}}$	(8.9)	$\frac{\alpha}{1-\alpha} \frac{\bar{w}}{\bar{r}^k}$

Table 16: Steady state relationships

8.7 Measurement equations

For the estimation of the model, the following measurement equations are appended to the model:

$$\Delta Y_t = 100(\hat{y}_t - \hat{y}_{t-1}) + 100(\mu - 1), \quad (8.86a)$$

$$\Delta C_t = 100(\hat{c}_t - \hat{c}_{t-1}) + 100(\mu - 1), \quad (8.86b)$$

$$\Delta X_t = 100(\hat{x}_t - \hat{x}_{t-1}) + 100(\mu - 1), \quad (8.86c)$$

$$\Delta X_t^g = 100(\hat{x}_t^g - \hat{x}_{t-1}^g) + 100(\mu - 1), \quad (8.86d)$$

$$\Delta \frac{W_t}{P_t} = 100(\hat{w}_t - \hat{w}_{t-1}) + 100(\mu - 1), \quad (8.86e)$$

$$\hat{\pi}_t^{obs} = 100\hat{\pi}_t + 100(\bar{\pi} - 1), \quad (8.86f)$$

$$\hat{R}_t^{obs} = 100\hat{R}_t + 100(\bar{R} - 1), \quad (8.86g)$$

$$\hat{q}_t^{k,obs} = 100\hat{q}_t^k + \bar{q}^{k,obs}, \quad (8.86h)$$

$$\hat{n}_t^{obs} = 100\hat{n}_t + \bar{n}^{obs}, \quad (8.86i)$$

$$\hat{b}_t^{obs} = 100\hat{b}_t + \bar{b}^{obs}. \quad (8.86j)$$

The constants give the inflation rate $\bar{\pi}$ along the balanced growth path and the trend growth rates. $100(\mu - 1)$ represents the deterministic net trend growth imposed on the data. Note that apart from the trend growth rate and the constant nominal interest rate, the discount factor can be backed out of the constants:

$$\beta = \frac{\bar{\pi}}{\bar{R}}\mu^\sigma.$$

The constant terms in the measurement equation are necessary even if the data is demeaned for the particular observation sample because the allocation in the flexible economy cannot be attained in the economy with frictions. Given a non-zero output gap, also other variables will deviate from zero. To see why notice that for the allocations to be the same in both the economy with frictions and the its frictionless counterpart required that the Calvo constraints on price and wage setting were slack – otherwise the equilibrium allocations would differ from that in the flexible economy. Slack

Calvo constraints in turn required that aggregate prices and wages were constant, which implied a constant real wage. Finally, a constant real wage would be inconsistent with the allocation in the flexible economy.

8.8 Welfare implications

To evaluate welfare implications, we approximate the compensating variation in terms of quarterly consumption of each type of agent separately as well as the population weighted average.

Independent of whether a household is constrained or not, equation (8.24) gives the preferences of the household. Using the log-linearized model solution around the deterministic balanced growth path, the lifetime utility of any time-path of consumption and hours worked can be computed as:

$$\begin{aligned}
U_t(\{\hat{c}_{t+s}, \hat{n}_{t+s}\}) &= \sum_{s=0}^{\infty} \beta^s \left[\frac{(\mu^{1-\sigma})^{t+s}}{1-\sigma} (\bar{c} \exp[\hat{c}_{t+s}] - \frac{h}{\mu} \bar{c} \exp[\hat{c}_{t+s-1}])^{1-\sigma} \right] \\
&\quad \times \exp \left[\frac{\sigma-1}{1+\nu} (\bar{n} \exp[\hat{n}_{t+s}])^{1+\nu} \right] \\
&\approx (\mu^{1-\sigma})^t \sum_{s=0}^{\infty} [\beta \mu^{1-\sigma}]^s \left[\frac{\bar{c}}{1-\sigma} (\exp[\hat{c}_{t+s}] - \frac{h}{\mu} \exp[\hat{c}_{t+s-1}])^{1-\sigma} \right] \\
&\quad \times \exp \left[\frac{\sigma-1}{1+\nu} \bar{n}^{1+\nu} [1 + (1+\nu)\hat{n}_{t+s}] \right] \\
&= (\mu^{1-\sigma})^t \frac{\bar{c}^{1-\sigma}}{1-\sigma} \exp \left[\frac{\sigma-1}{1+\nu} \bar{n}^{1+\nu} \right] \sum_{s=0}^{\infty} [\beta \mu^{1-\sigma}]^s \left[\left(e^{\hat{c}_{t+s}} - \frac{h}{\mu} e^{\hat{c}_{t+s-1}} \right) e^{-\hat{n}_{t+s} \bar{n}^{1+\nu}} \right]^{1-\sigma}.
\end{aligned}$$

Lifetime utility along the balanced growth path when hours worked equal the steady state value and consumption differs in each period by $\Gamma \times 100$ percent from its steady state value is obtained by setting $\hat{c}_{t+s} = \hat{n}_{t+s} = 0$ and replacing \bar{c} by $(1 + \Gamma)\bar{c}$:

$$U_t(1 + \Gamma) = (\mu^{1-\sigma})^t (1 + \Gamma)^{1-\sigma} (1 - h/\mu)^{1-\sigma} \frac{\bar{c}^{1-\sigma}}{1-\sigma} \exp \left[\frac{\sigma-1}{1+\nu} \bar{n}^{1+\nu} \right] \frac{1}{1 - \beta \mu^{1-\sigma}}.$$

The compensating variation is therefore:

$$\Gamma = \left[\frac{1 - \beta\mu^{1-\sigma}}{(1 - h/\mu)^{1-\sigma}} \sum_{s=0}^{\infty} [\beta\mu^{1-\sigma}]^s \left[\left(e^{\hat{c}_{t+s}} - \frac{h}{\mu} e^{\hat{c}_{t+s-1}} \right) e^{-\hat{n}_{t+s}\bar{n}^{1+\nu}} \right]^{1-\sigma} \right]^{\frac{1}{1-\sigma}} - 1 \quad (8.87)$$

For large s the deviations from the balanced growth path are numerically indistinguishable from zero. However, since $\beta\mu^{1-\sigma}$ is in practice close to unity, even for $s = 1,000$, the infinite sum has not converged. We therefore approximate:

$$\begin{aligned} \sum_{s=0}^{\infty} [\beta\mu^{1-\sigma}]^s \left[\left(e^{\hat{c}_{t+s}} - \frac{h}{\mu} e^{\hat{c}_{t+s-1}} \right) e^{-\hat{n}_{t+s}} \right]^{1-\sigma} &\approx \sum_{s=0}^T [\beta\mu^{1-\sigma}]^s \left[\left(e^{\hat{c}_{t+s}} - \frac{h}{\mu} e^{\hat{c}_{t+s-1}} \right) e^{-\hat{n}_{t+s}\bar{n}^{1+\nu}} \right]^{1-\sigma} \\ &+ \frac{[\beta\mu^{1-\sigma}]^{T+1}}{1 - \beta\mu^{1-\sigma}} (1 - h/\mu)^{1-\sigma}, \end{aligned}$$

for some large T . In practice, we use $T = 1000$ but checked the results for $T = 10,000$ at the posterior median.

To obtain $\bar{n}^{1+\nu}$, multiply equation (8.42) by \bar{n} and divide by \bar{y} . This shows that $\bar{n}^{1+\nu} = \frac{\bar{w}\bar{n}}{\bar{y}} \frac{1}{(1+\lambda^w)} \frac{1}{\bar{c}^{RA/\bar{y}}} \frac{1}{1-\frac{h}{\mu}} \frac{1-\bar{\tau}^n}{1+\tau^c}$, which is in terms of the constants in table 16.