Money Growth Monitoring and the Taylor Rule*

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Abstract

Using a series of examples, we review various ways in which a monetary policy characterized by the Taylor rule can inject volatility into the economy. In the examples, the incorporation of an escape clause into the Taylor rule can reduce or even entirely eliminate the problems. Under the escape clause, the central bank monitors the money growth rate and commits to abandoning the Taylor rule in favor of a money growth rule in case money growth passes outside a particular monitoring range.

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1. Introduction

In recent years considerable thought has gone into designing strategies for the conduct of monetary policy. An important objective is to develop policies that avoid bad macroeconomic outcomes, such as the high and volatile inflation of the 1970s. There is an emerging consensus that an effective way to achieve this is to sharply raise short term interest rates when short term inflation expectations rise above a (low) target value, and reduce short term rates when inflation expectations fall below target. We refer to this strategy as a Taylor rule, since it is similar to the strategy advocated by Taylor (1993). Money plays an active role in this strategy only to the extent that it is helpful in constructing the inflation forecast. However, even here the role of money is quite limited since in practice money is not particularly useful in forecasting inflation in the short run. In this way, the role of money in monetary policy has been greatly deemphasized.[1]

We argue that there may be an important role for money in monetary policy after all. Although there are well-defined theoretical models in which the Taylor rule leads to good outcomes, there is a growing awareness of other models in which it may lead to bad outcomes.[2] Moreover, these models do not seem implausible on empirical grounds. A central bank concerned with robustness must adopt a monetary policy strategy that would also be effective in achieving its objectives in case the economy actually were better characterized by one of these other models.

Fundamentally, in these models the Taylor rule leaves the real economy without an anchor. This is because a given interest rate policy can, in these models, be supported by various money growth rates. And, each of these money growth rates are associated with different real outcomes for the economy. We argue that a policy of monitoring the money growth rate can, in effect, provide the economy with an anchor. The strategy we study implements a Taylor rule as long as money growth falls within a specified target range, and then abandons the Taylor rule for a constant money growth rule in the event that the target range is violated.[3] If the economy is the one in which a Taylor rule is not associated with bad outcomes, then the policy of monitoring money is redundant. In this case, the policy is benign and does not interfere

[1] See Gali (2001) and the references he cites for an elaboration on the view that the role of money in a monetary policy that seeks to stabilize inflation ought to be small.

[2] See, for example, Benhabib, Schmitt-Grohe and Uribe (2000, 2001a,b,c), Carlstrom and Fuerst (2000a), and Dupor (2000,2001), and the papers that they cite.

[3] In the policies that we study, the commitment is to switch to a constant money growth rule forever. This is done to simplify the analysis only. For the monetary monitoring strategy to eliminate bad equilibria requires only that that switch be temporary.

with the operation of the Taylor rule. However, if the economy is one in which a Taylor rule can produce pathological outcomes, then a money monitoring policy may improve economic performance substantially. It does so by eliminating undesired equilibria.

In our analysis, monetary monitoring works very much like the textbook analysis of a bank run. The government's commitment to supply liquidity in the event of a bank run eliminates the occurrence of bank runs in the first place. In this analysis, the government never actually has to act on its commiment. Similarly, in our analysis the government's commitment to monitor the money growth rate and reign it in if necessary implies that money growth never gets out of line in the first place. The monetary authority is never observed reacting to money growth figures, so the strategy leaves no evidence in time series data.

We illustrate the various ways in which the Taylor rule can be a source of real economic instability. Each example is designed to highlight a different sort of pathology. These include sunspot equilibria, cycling equilibria, and high or low inflation equilibria. These equilibria exist, even if a so-called aggressive Taylor rule is pursued. This is the one advocated by Taylor himself, and corresponds to the case in which a rise of one percentage points in short-term anticipated inflation leads to a rise of 1.5 percentage points in the short-term nominal rate of interest.[4] The examples echo the message of several recent papers by Benhabib, Schmitt-Grohe and Uribe (2000, 2001a,b,c) (BSGU), who show that pathologies can even occur when the steady state associated with the inflation target implicit in an aggressive Taylor rule is determinate. Our emphasis on monetary monitoring is in the spirit of the analysis in BSGU. They show that a policy which includes a commitment to switch to a money growth target in the event that the economy slips into a deflation may eliminate the deflation equilibrium.[5] In our analysis we consider other pathologies that can occur under a Taylor rule, and show how monetary monitoring can be helpful in these cases too.

Our analysis requires that we study the global set of equilibria. This requirement is highlighted in the analysis of BSGU. They show that the standard practice of studying the properties of monetary models in a small neighborhood of steady state can generate a misleading impression about the set of possible equilibrium outcomes. Our strategy for studying the global set of equilibria requires that that set be characterizable by a first order difference equation. In practice, this means having to drop investment from the analysis, which we do.[6]

[4]What we call the 'aggressive Taylor rule' satisfies what is often called the 'Taylor principle'.

[5]Benhabib, Schmitt-Grohe and Uribe (2000) emphasize that for their money growth strategy to effectively eliminate deflation equilibria requires that fiscal policy must be non-Ricardian. Throughout our analysis we adopt (and defend) a particular non-Ricardian formulation of fiscal policy.

[6]One can construct examples with capital in which the equilibrium dynamics are character-

The rest of this paper is divided into four parts. The next section describes the Lucas and Stokey (1983), cash-credit good model. Section 3 analyzes the set of equilibria in a particular parametric version of that model. It displays the set of equilibria under a Taylor rule, and the impact on that set of monetary monitoring. Section 4 describes a cash in advance model and presents two additional examples of how monetary monitoring can improve the performance of the Taylor rule. Section 5 describes a potential problem associated with our monetary monitoring strategy. The strategy assumes that a constant money growth policy does not itself inject instability. We present two examples to show that this is not always true. These examples are presented as a caveat to the overall theme of this paper. The final section includes concluding remarks and a discussion of the limitations of our analysis.

2. The Lucas-Stokey Cash-Credit Good Model

This section describes the agents in a version of the Lucas and Stokey cash-credit good model. Analysis of the set of equilibria for this model is deferred to the next section.

2.1. Households and Firms

Household preferences are given by:

$$\sum_{t=0}^{\infty} \beta^t u(c_{1t}, c_{2t}, l_t),$$

where $c_{1t} \geq 0$ and $c_{2t} \geq 0$ denote cash and credit goods, and $0 \leq l_t \leq L$ denotes employment and $0 < \beta < 1$. The variable, L, denotes the household's total time endowment. The period is divided into two parts, an asset trading period and a goods trading period. At the end of the asset trading period, the household has total financial assets A_t^d. It divides these into money, M_t^d, and bonds, B_t^d, as follows:

$$A_t^d \geq M_t^d + B_t^d, \tag{2.1}$$

where the superscript 'd' indicates household demand. The household allocates part of its assets to non-interest bearing money in order to satisfy its cash in advance constraint:

$$P_t c_{1t} \leq M_t^d, \tag{2.2}$$

ized by a first order difference equation. Christiano and Harrison (1999) display an example like this. However, that example incorporates counterfactually high increasing returns to scale in production.

where P_t denotes the price of goods (cash or credit). The household's assets, A_t^d, reflect a net influx of funds arising from activities in the previous trading period and in the previous goods market period:

$$A_t^d = M_{t-1}^d - P_{t-1}\left(c_{1,t-1} + c_{2,t-1}\right) + W_{t-1}l_{t-1} + B_{t-1}^d(1 + R_{t-1}) + D_{t-1} - P_{t-1}\tau_{t-1}. \tag{2.3}$$

Here, D_t denotes profits, τ_t denotes real lump sum taxes, and R_t denotes the net nominal rate of interest.

To represent the necessary and sufficient conditions for household optimization, we adopt a slight change in notation. Our presentation follows the one in Woodford (1994, 1999). Let the household's non-interest income be defined as follows:

$$I_t \equiv W_t L - P_t\tau_t + D_t. \tag{2.4}$$

Note that under this definition, a household's income is beyond its control. Denote the household's spending as

$$S_t \equiv R_t M_t^d + P_t\left(c_{1t} + c_{2t}\right) + W_t(L - l_t). \tag{2.5}$$

Substituting (2.1), (2.4), and (2.5) into (2.3), we obtain the following simplified representation of the household's flow budget constraint:

$$A_t^d \leq (1 + R_{t-1})A_{t-1}^d + I_{t-1} - S_{t-1}. \tag{2.6}$$

The weak inequality here reflects the weak inequality in (2.1).

We place restrictions on the prices faced by the household that we know must hold in equilibrium:

$$P_t,\ W_t > 0,\ R_t \geq 0,\ lim_{t\to\infty} \sum_{j=0}^{t} q_{j+1} I_j \text{ finite}, \tag{2.7}$$

for all t, where

$$q_t = \prod_{j=0}^{t-1} \frac{1}{1 + R_j},\ q_0 \equiv 1. \tag{2.8}$$

If these conditions did not hold, then the household's consumption opportunity set would be unbounded above. Given non-satiation in preferences, this would give rise to unbounded consumption demand, something that is incompatible with equilibrium in an economy with bounded resources.

We impose a particular solvency condition on the household. In particular, at each date its total assets must never fall so far into the negative region that it cannot repay them from the proceeds of its future income. The solvency condition that we impose is:

$$A_t^d \geq -\frac{1}{q_t}\sum_{j=0}^{\infty} q_{t+j+1} I_{t+j}, \tag{2.9}$$

This condition may have direct intuitive appeal. To understand it better, it is useful to know that (2.9), in combination with (2.1) and (2.3) is equivalent with the following expression:

$$\frac{1}{q_t}\sum_{j=0}^{\infty} q_{t+j+1} S_{t+j} \leq A_t + \frac{1}{q_t}\sum_{j=0}^{\infty} q_{t+j+1} I_{t+j}. \tag{2.10}$$

According to this, the present value of spending at each date must not exceed the household's total financial assets at that date, plus the present value of its income.

For technical purposes it is useful to know that (2.9) can also be equivalently expressed as follows:

$$\lim_{T\to\infty} q_T A_T^d \geq 0. \tag{2.11}$$

See Proposition A.1 in the appendix for a proof. We take advantage of this proposition by imposing (2.11) instead of (2.9) in the definition of the household problem.

In period 0 the household takes A_0^d and $\{R_t,\, W_t,\, P_t,\, D_t,\, \tau_t;\, t \geq 0\}$ as given, and it selects $\{M_t^d,\, B_t^d,\, c_{1t},\, c_{2t},\, l_t;\, t \geq 0\}$ to maximize utility subject to, (2.1)-(2.3), (2.11), $M_t^d,\, c_{1t},\, c_{2t},\, l_t,\, L - l_t \geq 0$. This problem has a solution if, and only if, (2.7) holds. In this case, the necessary and sufficient conditions for household optimization are, for $t = 0, 1, 2, ...$ [7]

$$u_{1t} = \beta\left(1 + R_t\right)\frac{u_{1t+1}}{\pi_{t+1}}, \tag{2.12}$$

$$\frac{-u_{3t}}{u_{2t}} = \frac{W_t}{P_t}, \tag{2.13}$$

$$\frac{u_{1t}}{u_{2t}} = 1 + R_t, \tag{2.14}$$

$$R_t(M_t^d - P_t c_{1t}) = 0,\ M_t^d - P_t c_{1t} \geq 0. \tag{2.15}$$

[7] For a formal proof, see Albanesi, Chari and Christiano (2001) or Woodford (1994).

$$\lim_{T\to\infty} q_T A_T = 0. \tag{2.16}$$

According to (2.16) an optimizing household drives its assets (measured in date 0 terms) to zero asymptotically. Equivalently, (2.16) corresponds to the statement that the optimizing household satisfies (2.10) as a strict equality. That is, it exactly exhausts its budget constraint.

Firms have a linear production technology with marginal productivity equal to unity. We assume perfect competition, so that firm optimization requires:

$$\frac{W_t}{P_t} = 1. \tag{2.17}$$

The aggregate resource constraint is

$$c_{1t} + c_{2t} + g = l_t. \tag{2.18}$$

where $g \geq 0$ denotes government spending.

2.2. Monetary and Fiscal Authorities

We now describe the activities of the government. The government's sources of funds in the period t asset market are lump sum tax obligations arising in the previous goods market, $P_{t-1}\tau_{t-1}$, money earned by issuing new bonds, B_t, and additions to the aggregate stock of money, $M_t - M_{t-1}$, subject to $M_t \geq 0$. The government's uses of funds include payments for goods, g, purchased in the previous goods market, and interest on debt issued in the previous asset market:

$$M_t - M_{t-1} + B_t + P_{t-1}\tau_{t-1} = P_{t-1}g + (1 + R_{t-1})B_{t-1}.$$

Rewriting this expression, we obtain the equation governing the evolution of government debt:

$$B_t = (1 + R_{t-1})B_{t-1} - P_{t-1}s_{t-1}, \tag{2.19}$$

where s_{t-1} is the real government surplus, including seignorage revenues. That is,

$$s_t = \tau_t + \frac{M_{t+1} - M_t}{P_t} - g.$$

Government policy has a fiscal and monetary component. Fiscal policy refers to the setting of τ_t. We assume that τ_t is set in such a way that, regardless of the realization of prices or of

monetary policy, the government eventually pays off its debt, B_t. That is, at each date, t, the present value of subsequent surpluses must equal B_t :

$$\frac{1}{q_t}\sum_{j=0}^{\infty} q_{t+j+1}P_{t+j}s_{t+j} = B_t, \tag{2.20}$$

for $t = 0, 1, \ldots$. Given (2.19), (2.20) holds if, and only if,

$$\lim_{T\to\infty} q_T B_T = 0. \tag{2.21}$$

This result will be useful below. See Proposition A.2 in the Appendix for a proof.

Monetary policy refers to the government's choice about the evolution of M_t. We consider two types of monetary policy. Under a pure Taylor rule the government chooses M_t to achieve the following rate of interest:

$$R_t = \tilde{R}\left(\frac{\pi_{t+1}}{\tilde{\pi}}\right)^{\alpha\frac{\tilde{\pi}}{\tilde{R}}}, \ \alpha > 0, \ \tilde{R} = \frac{\tilde{\pi}}{\beta} - 1,$$

where $\tilde{\pi} \geq \beta$ and $0 < \alpha$ are parameters. Under this monetary policy rule, the supply of money is determined by the market. We refer to $\tilde{\pi}$ as the government's gross inflation target. The associated net nominal rate of interest is the interest rate target. We refer to a stationary equilibrium in which $\tilde{\pi}$ and $\tilde{R}$ are the inflation rate and the nominal rate of interest as the *target inflation steady state*.

Three features of this representation of the Taylor rule are worth emphasizing. First, the specification guarantees $R_t \geq 0$ in all possible equilibria (i.e., those in which $P_t > 0$). This is consistent with BSGU's insistence that one only analyze rules that satisfy this type of feasibility condition. Second, the parameter α corresponds to the coefficient on inflation in the Taylor rule literature. To see this, simply linearize the above rule about $R_t = \tilde{R}$ and $\pi_{t+1} = \tilde{\pi}$, and note that α is the coefficient on π_{t+1} in this linear relationship. For the most part, we study the operating characteristics of an aggressive Taylor rule, with $\alpha = 1.5$. Third, we follow several recent papers by not including a measure of the output gap in the Taylor rule.[8] It has been argued that this is a good representation of the monetary policy strategy implemented by several central banks whose primary commitment is to price stability.[9]

[8] See, for example, BSGU and Carlstrom and Fuerst (2000,2000a).

[9] See, for example Alesina, et. al. (2001) and Batini and Haldane (1998). See Christiano and Gust (1999) and Orphanides (1999) for arguments about why excessive weight on the output gap may lead to instability.

We also consider a linear version of the Taylor rule:

$$R_t = \max\{0, \tilde{R} + \alpha(\pi_{t+1} - \tilde{\pi})\}.$$

This has the advantage of being more directly comparable to the versions of the Taylor analyzed in the literature.

So, under the pure Taylor rule the monetary authorities are exclusively concerned with implementing either the nonlinear or the linear Taylor rule. The alternative that we consider is the Taylor rule with monetary monitoring. Under this rule there are 'trigger' money growth rates, $\mu^l < \mu^h$, such that if

$$\frac{M_{t+1}}{M_t} > \mu^h, \text{ or } \frac{M_{t+1}}{M_t} < \mu^l,$$

then the monetary authority switches to a constant money growth policy. We refer to the growth rate to which it switches as the 'post stabilization money growth rate'. We suppose that this money growth rate, μ^*, lies inside the monitoring range:

$$\mu^l < \mu^* < \mu^h.$$

For example, μ^* could be $\tilde{\pi}$. We refer to a stationary equilibrium in which the inflation rate is μ^* as the *monetarist steady state associated with money growth, μ^*.*

2.3. Deterministic Equilibrium

Following is our definition of equilibrium. It applies to either type of monetary policy.

Definition 2.1. *A Deterministic Equilibrium is a sequence,* $\{c_{1t}, c_{2t}, M_t, M_t^d, B_t, B_t^d, P_t, R_t, \tau_t, D_t, l_t; t \geq 0\}$, *such that*

(i) the household problem is solved

(ii) the firm first order condition is satisfied

(iii) the monetary policy rule is satisfied

(iv) markets clear, i.e., $M_t^d = M_t$, $B_t^d = B_t$, $l_t = c_{1t} + c_{2t} + g$.

The equations that characterize equilibrium include (2.7), (2.12)-(2.18), (2.21), and the equations that characterize monetary policy. It is useful to rewrite the conditions, (2.7), (2.16) and (2.21). We do so using the relation obtained after combining (2.12) with (2.8):

$$\frac{u_{1,0}}{P_0} q_{t+1} = \beta^{t+1} \frac{u_{1,t+1}}{P_{t+1}} = \beta^t \frac{u_{1,t}}{(1+R_t)P_t} = \beta^t \frac{u_{2,t}}{P_t}.$$

Recall that (2.16) implies (2.10) holds as an equality. As a result, (2.7) implies $\sum_{t=0}^{\infty} q_{t+1} S_t < \infty$, or,

$$\sum_{t=0}^{\infty} \beta^t \left[u_{1t} c_{1t} + u_{2t} c_{2t} - u_{3t}(L - l_t) \right] < \infty. \tag{2.22}$$

Here, we have made use of the expression for q_{t+1} derived above, as well as (2.13)-(2.15), (2.17) and $u_{1,0}, P_0 > 0$. In addition, combining (2.16) and (2.21), we obtain $\lim_{T \to \infty} q_T M_T = 0$, or,

$$\lim_{T \to \infty} \beta^T u_{1T} m_T = 0. \tag{2.23}$$

The equilibrium conditions may now be stated as follows. They consist of (2.12)-(2.15), (2.17)-(2.18) and the equations that characterize monetary policy. In addition, they include (2.22) and (2.23). We delete (2.16) from our list of equilibrium conditions because, conditional on (2.23), (2.16) is automatically satisfied by (2.21).

2.4. The Importance of Fiscal Policy For Monetary Policy Outcomes

There is a growing literature which stresses that the equilibria associated with a particular monetary policy depend sensitively on the nature of fiscal policy. We briefly review this analysis as it pertains to us. Our discussion follows Woodford (1999) most closely.[10] Because the assumptions one makes about fiscal policy matter for monetary outcomes, we close this subsection with a defense of our assumption that fiscal policy satisfies (2.21).

An alternative to our assumption about fiscal policy is that government policy is what Woodford (1999) calls 'Ricardian'. Under this type of policy, fiscal policy drives the value of total government liabilities to zero eventually. That is, B_T in (2.21) is replaced by $A_T = M_T + B_T$. To understand the difference between a Ricardian specification and our specification of fiscal policy, it is useful to consider a particular candidate equilibrium with positive, constant money growth, $M_{t+1} = \mu^* M_t$, and $\mu^* > 1$. In this candidate equilibrium the price level is falling at the rate, β, so that $P_t = \beta^t P_0$; consumption and employment are constant; $q_t = 1$; and $R_t = 0$. This outcome may at first seem peculiar because prices are falling while money growth is positive. Still, it is easy to verify that this scenario is consistent with the household and firm first order conditions and the resource constraint, i.e., (2.12)-(2.15), (2.17)-(2.18). With prices falling and nominal money balances growing, the cash in advance constraint is increasingly non-binding, i.e., $m_t - c_{1t} > 0$ is growing. In addition, seignorage revenues, $(M_{t+1} - M_t)/P_t$,

[10] See also, for example, BSGU, Leeper (1991), Sims (1994, 2001).

are growing without bound:

$$\frac{M_{t+1} - M_t}{P_t} = \left(\frac{\mu^*}{\beta}\right)^t \frac{M_0}{P_0}(\mu^* - 1) \to \infty \text{ as } t \to \infty.$$

Fiscal policy in the deflation scenario is quite different, depending whether policy is Ricardian or whether it is constrained by (2.21). The difference can be understood in terms of what the government must do with the exploding seignorage revenues. Under a Ricardian policy it uses the seignorage initially to purchase government debt and when that hits zero it accumulates unbounded claims against households (i.e., it drives $B_t \to -\infty$). It does this because of the fact that M_T is growing and because of the Ricardian requirement that it drive $M_T + B_T$ to zero.[11] Since the private sector is indifferent between money and bonds in the deflation scenario, and total government liabilities are being driven to zero, private agents do not mind holding the increase in money balances as idle balances. In particular, the deflation candidate equilibrium is an actual equilibrium under a Ricardian fiscal policy because the household transversality condition, (2.16), is satisfied in this scenario.

Now consider the deflation scenario when fiscal policy is governed by (2.21). Under this type of fiscal policy the exploding seignorage revenues eventually give rise to a need to provide tax rebates to households. To see this, consider (2.19) with $R_{t-1} = 0$, $P_{t-1} = P_0\beta^{t-1}$, $M_t/P_t = (\mu^*/\beta)^t M_0/P_0$:

$$B_t = B_{t-1} - \left[P_0\beta^{t-1}\tau_{t-1} + M_0(\mu^*)^{t-1}(\mu^* - 1) - P_0\beta^{t-1}g\right].$$

One class of fiscal policies that is consistent with (2.21) sets τ_t so that the object in square brackets is equated to λB_{t-1}, $0 < \lambda < 1$.[12] Then,

$$\begin{aligned}\tau_{t-1} &= \frac{B_0}{P_0}\lambda\left(\frac{1-\lambda}{\beta}\right)^{t-1} - \frac{M_0}{P_0}\left(\frac{\mu^*}{\beta}\right)^{t-1}(\mu^* - 1) + g \\ &= \frac{M_0}{P_0}\left(\frac{\mu^*}{\beta}\right)^{t-1}\left[\frac{B_0}{M_0}\lambda\left(\frac{1-\lambda}{\mu^*}\right)^{t-1} - (\mu^* - 1)\right] + g.\end{aligned}$$

Using the facts, $(\mu^* - 1) > 0$ and $0 < (1-\lambda)/\mu^* < 1$, we conclude that $\tau_t \to -\infty$. Taxes go unboundedly negative because the second term in the square brackets, which corresponds to seignorage, eventually dominates all else.

[11] Here, we use the fact that in a deflation scenario, $q_t = 1$ for all t.

[12] It is easy to verify that we only require this to be true eventually.

The key thing to observe here is that the deflation candidate equilibrium is not consistent with household optimality. With the debt going to zero and money balances growing, total government liabilities are growing at a rate that is incompatible with (2.16). Optimizing households would not willingly accumulate so many assets. They would do better by reducing the amount of assets held, and using them to consume goods. Thus, the deflation candidate equilibrium is not an equilibrium under (2.21).[13] This discussion establishes the importance of fiscal policy for the set of equilibria associated with a constant money growth policy when $\mu^* > 1$.

Because the assumption about fiscal policy has important implications for equilibrium outcomes associated with different monetary policies, it is important for us to defend our specification that fiscal policy is governed by (2.21). We interpret the forces that come to bear on actual governments for maintaining fiscal solvency as operating primarily on the government debt, B_t, rather than on the government's total nominal liabilities. For example, the Maastricht Treaty places an explicit limit on the amount of interest-bearing debt that participating governments can issue. The political concerns observed in the US in the 1980s about government finances were focused on the increase in the stock of government debt, and not on the stock of government liabilities.

These are the considerations which lead us to favor (2.21). There are other considerations, not captured formally in the model, which make us skeptical of the Ricardian specification of policy, in which B_T in (2.21) is replaced by $M_T + B_T$. As noted above, the latter specification commits the government to acquiring unbounded claims on private agents, should it find itself in a deflation scenario. We suspect that, in practice, political and other forces would come into play to prevent the government from acquiring such a large stake in the private sector. Consistent with our solvency assumption, (2.21), we conjecture that a government which finds itself with excessive revenues - as occurs when seignorage grows in a deflation scenario - would come under pressure to provide tax cuts. Indeed, this is precisely what happened recently in the United States when strong economic growth produced a surplus of government revenues.

3. Properties of Equilibrium in the Cash-Credit Model

This section uses the model of the previous section to illustrate several pathologies associated with the Taylor rule, and how monetary monitoring might help. We present examples in which

[13] This argument that a deflation ('liquidity trap') outcome cannot be an equilibrium resembles Pigou's critique of the Keynesian liquidity trap, as summarized in standard intermediate macroeconomics textbooks.

there are equilibria in which inflation is higher or lower than the target rate, and in which inflation responds to sunspot shocks.

We work with the following specification of utility:[14]

$$u(c_1, c_2, l) = \log(c_1) + \log(c_2) - l.$$

We also set $g = 0$. We first examine the equilibria in the case of the nonlinear Taylor rule. We then consider the linear Taylor rule. In each case, we examine the equilibria under pure Taylor rule and under a Taylor rule with monetary monitoring.

We show that, generically, there are two steady state equilibria. One is determinate and the other is indeterminate. With an aggressive Taylor rule, the target inflation steady state is the indeterminate steady state. The other steady state involves lower inflation, and is determinate. Under a passive Taylor rule, the results are reversed: the target inflation steady state is determinate, while there is also a high inflation steady state that is indeterminate. Whether the Taylor rule is active or passive, there are multiple deterministic equilibria. There are also equilibria in which the endogenous variables are random, despite the absence of shocks to fundamentals.

We then consider the Taylor rule with monetary monitoring, in which the monitoring range brackets the target inflation steady state. In the case where the target inflation rate is indeterminate (i.e., the Taylor rule is active), we show that monetary monitoring shrinks the set of possible equilibria. When the target inflation steady state is determinate (i.e., the Taylor rule is passive), then that steady state is the unique equilibrium under monetary monitoring.

The discussion is organized as follows. First, we carry out the analysis in the context of the nonlinear Taylor rule. Here, we only study non-random equilibria. Second, we study the equilibria of the model under a linear Taylor rule. Here, we first consider non-random equilibria. We then study the random equilibria that can occur in the neighborhood of the target inflation steady state when the Taylor rule is active.

3.1. Nonlinear Taylor Rule

We first examine the set of equilibria under a pure Taylor rule. We then consider the case of monetary monitoring.

[14] The example would be very similar if instead we had adopted the following utility function:

$$u(c_1, c_2, l) = \log(c_1) + \log(c_2) + \gamma \log(L - l).$$

Pure Taylor Rule

Making use of our functional form assumption on u, the household and firm first order conditions, the Taylor rule and the resource constraint reduce to:

$$\frac{1}{c_{1t}} = \beta \frac{1+R_t}{\pi_{t+1}} \frac{1}{c_{1,t+1}} \tag{3.1}$$

$$c_{2t} = 1 \tag{3.2}$$

$$c_{1t} = \frac{1}{1+R_t} \tag{3.3}$$

$$\pi_{t+1} = \tilde{\pi}\left(\frac{R_t}{\tilde{R}}\right)^{\frac{R}{\alpha\tilde{\pi}}} \tag{3.4}$$

$$l_t = c_{2t} + c_{1t}, \tag{3.5}$$

$$R_t(m_t - c_{1t}) = 0,\ m_t - c_{1t} \geq 0. \tag{3.6}$$

$t = 0, 1, 2, \ldots$. Here, we have made use of (2.17). Rearranging the expressions in (3.1), we obtain:

$$c_{1,t+1} = \frac{\beta}{\tilde{\pi}\left(\frac{\frac{1}{c_{1t}}-1}{\tilde{R}}\right)^{\frac{\tilde{R}}{\alpha\tilde{\pi}}}} \equiv f(c_{1t}), \tag{3.7}$$

for $t = 0, 1, 2, \ldots$.

Making use of our functional form assumptions, (2.23) reduces to

$$\lim_{T\to\infty} \beta^T \frac{m_T}{c_{1T}} = 0. \tag{3.8}$$

The necessary and sufficient conditions for an equilibrium are (3.2)-(3.8). The equilibrium difference equation, f, is key for characterizing the set of equilibria for this model. It is therefore useful to first state the key properties of this function. Throughout this paper, we assume $\alpha > \tilde{R}/\tilde{\pi}$. Since we only consider inflation targets, $\tilde{\pi}$, that are very low, this is not a restrictive assumption.

Proposition 3.1. *Suppose $\alpha > \tilde{R}/\tilde{\pi}$. The function, f, defined in (3.7) has the following properties:*

(i) $f'(\beta/\tilde{\pi}) = 1/(\alpha\beta)$, $f(\beta/\tilde{\pi}) = \beta/\tilde{\pi}$.

(ii) When $\alpha \neq 1/\beta$, there are exactly two values of c_1, $0 < c_1 < 1$, with the property, $f(c_1) = c_1$. When $\alpha = 1/\beta$, there is exactly one such value of c_1.

(iii) $\lim_{c_1\to 0} f'(c_1) = \infty$, $\lim_{c_1\to 0} f(c_1) = 0$, $\lim_{c_1\to 1} f'(c_1) = \infty$.

Proof: See the Appendix.

A stylized representation of f is displayed in Figure 1, for $\alpha \neq 1/\beta$. For $\alpha = 1/\beta$, f is tangent to the 45 degree line at $c_1 = \beta/\tilde{\pi}$. The following proposition establishes the importance of f for understanding the set of equilibria.[15] We have:

Proposition 3.2. *Suppose $\alpha > \tilde{R}/\tilde{\pi}$. The sequence, c_{1t}, $t = 0, 1, 2, ...$ corresponds to an equilibrium if, and only if, it satisfies (3.7) and $0 \leq c_{1t} \leq 1$.*

Proof: see the Appendix.

The previous two propositions are useful for establishing the following facts about the set of equilibria in the model:

Proposition 3.3. *Consider monetary policy under a pure, nonlinear, Taylor rule. Suppose $\alpha > \tilde{R}/\tilde{\pi}$ and $\alpha \neq 1/\beta$.*

(i) there are exactly two interior steady states. In one of these, the inflation rate equals its target value, $\tilde{\pi}$, in the Taylor rule.

(ii) when $\alpha < 1/\beta$, the target inflation steady state is determinate. The other steady state is indeterminate and has inflation rate higher than $\tilde{\pi}$.

(iii) when $\alpha > 1/\beta$, the target inflation steady state is indeterminate. The other steady state is determinate and has inflation rate lower than $\tilde{\pi}$.

Proof: See the Appendix.

To understand the proposition, it is useful to take into account the shape of f, described formally in proposition 3.1 and represented in a stylized form in Figure 1. Let $\tilde{c}_1 = \beta/\tilde{\pi}$ denote the level of consumption in the target inflation steady state. When $\alpha < 1/\beta$, then the target inflation steady state corresponds to the upper intersection of f and the 45 degree line. The figure indicates that for $c_{1,0} \neq \tilde{c}_1$ but close to $\tilde{c}_1$, the $c_{1,t}$'s which solve (3.7) diverge. It follows that there are no equilibria close to the target inflation steady state, so that the latter is said

[15] In the equilibria with forward-looking rules that we study there is nominal indeterminacy. We ignore this in this paper, since nominal indeterminacy has no welfare consequences. However, in versions of our models with certain types of nominal frictions, these indeterminacies are real (see Carlstrom and Fuerst (2000).) We conjecture that this additional source of real indeterminacy is another pathology that could be ameliorated by the monetary monitoring strategy explored in this paper.

to be determinate. The other steady state equilibrium is one in which inflation is high. It is indeterminate, because there are many equilibria close to it. In particular, the sequence of c_{1t}'s associated with any $c_{1,0}$ sufficiently close to the high inflation steady state level of consumption corresponds to an equilibrium by Proposition 3.2.

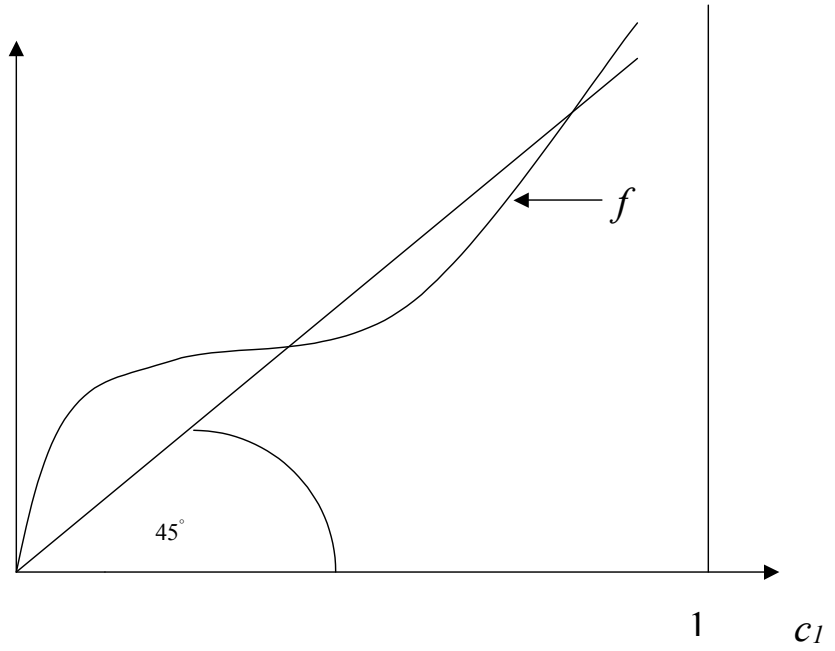

Figure 1: The Function, f, in (3.7)

When $\alpha > 1/\beta$, then the target inflation steady state corresponds to the lower interior intersection of f and the 45 degree line. In this case, the target inflation steady state is indeterminate because for $c_{1,0}$ close to $\tilde{c}_1$, the $c_{1,t}$'s which solve (3.7) converge back to $\tilde{c}_1$, and so they are close to $\tilde{c}_1$ as well. In addition, the other steady state is determinate and has a low inflation rate and interest rate.

To understand the part of the proposition dealing with inflation, combine (3.3) and the inverse of the Taylor rule in (3.4), to obtain:

$$\pi_{t+1} = \tilde{\pi} \left(\frac{\frac{1}{c_{1,t}} - 1}{\tilde{R}} \right)^{\frac{R}{\alpha \tilde{\pi}}} . \tag{3.9}$$

This shows that there is an inverse relationship between $c_{1,t}$ and anticipated inflation. As a result, the inflation rate in the high c_1 steady state equilibrium is lower than the inflation rate in the low c_1 steady state.

For understanding our results, it is useful to explain the economic intuition underlying the fact that an aggressive Taylor rule (i.e., $\alpha > 1/\beta$) produces a target inflation steady state that is indeterminate, while a passive rule makes the target inflation steady state determinate. Consider the aggressive Taylor rule case first. There is another equilibrium close by in which expected inflation is higher than it is in the target equilibrium. To see this, note that if people expect high inflation, this translates into a high real interest rate under an aggressive Taylor rule. According to the intertemporal Euler equation, this induces people to plan on a higher growth path for cash good consumption. Such a path is close to the original target steady state path if it involves low consumption, i.e., an initial drop in consumption followed by a return to the target inflation steady state level of consumption from below. This is an equilibrium if it satisfies (3.3). That indeed it does can be verified by noting that along the candidate alternative equilibrium path, the nominal rate of interest is high. This requires that real balances and, hence, cash good consumption, be low. Note that this intuition does not apply if the Taylor rule is not aggressive, i.e., if $\alpha < 1/\beta$. In this case, a candidate alternative equilibrium with high expected inflation is associated with a low real rate of interest and, hence, a low growth path for cash good consumption. Arguing by analogy with the previous example, we would posit that in the candidate alternative equilibrium the level of consumption is high and falls back toward the target steady state. This cannot be an equilibrium, however, because it does not satisfy (3.3).

This intuition can help explain why it is that when $\alpha < 1/\beta$, the high inflation equilibrium is indeterminate. With the nonlinear Taylor rule the rule becomes more aggressive at higher levels of inflation. In the high inflation equilibrium, the Taylor rule has become sufficiently aggressive that that equilibrium is indeterminate.

We now briefly summarize the implications of our analysis for the pure Taylor rule monetary policy. In all cases considered, there exist multiple equilibria. When an aggressive pure Taylor rule is pursued, then there is a continuum of equilibria near the target inflation steady state. This implies that there exist sunspot equilibria in a neighborhood of that equilibrium. We explore this further below. When a passive Taylor rule is followed, then the target inflation steady state is determinate. The kind of local analysis typically pursued in the literature would suggest equilibrium uniqueness in this case. However, this is in fact not true. We showed that there also exists a high inflation steady state equilibrium, and a continuum of non-steady state equilibrium paths which converge to it. In particular, one can construct sunspot equilibria

around the high inflation steady state. This example illustrates the important point recently emphasized in the work of BSGU, that local determinateness of equilibrium does not guarantee the absence of sunspot equilibria. At best, it guarantees the absence of sunspot equilibria in a neighborhood.

Taylor Rule with Monetary Monitoring

To understand the set of equilibria under this type of policy, it is convenient to first consider the equilibria under a constant money growth rule. The equations that characterize an equilibrium in this case are (3.1)-(3.3), (3.5), (3.6), (3.8) and:

$$\frac{M_{t+1}}{M_t} = \mu^*.$$

Here, $\mu^* > 1$ is the constant money growth rate. Multiplying (3.1) by M_t, making use of (3.2)-(3.3) and rearranging, we obtain:

$$m_t = \frac{\beta}{\mu^*} \frac{m_{t+1}}{c_{1,t+1}}. \tag{3.10}$$

We have the following characterization result:

Proposition 3.4. *A sequence of m_t's and $c_{1,t}$'s corresponds to an equilibrium if, and only if, for $t = 0, 1, 2, ...,$*

(i) it satisfies (3.10),
(ii) $m_t \geq c_{1t}$, $c_{1t} < 1 \Longrightarrow m_t = c_{1,t}$,
(iii) $0 \leq c_{1t} \leq 1$
(iv) it satisfies (3.8).

Proof: Suppose a sequence of m_t's and c_{1t}'s satisfies the conditions of the proposition. We need to verify that R_t, l_t, c_{2t} can be found which, together with the m_t's and c_{1t}'s, satisfy (3.1)-(3.3), (3.5)-(3.6). The interest rate, R_t, can be obtained from (3.3), and it is easily confirmed that $R_t \geq 0$. Multiply both sides of (3.10) by $1/c_{1t}$ and use (3.3) to obtain:

$$\frac{m_t}{c_{1t}} = \frac{\beta}{\mu^*} (1 + R_t) \frac{m_{t+1}}{c_{1,t+1}}.$$

The intertemporal Euler equation, (3.1), is verified once we take into account $m_t = M_t/P_t$ and the fact, $M_{t+1} = \mu^* M_t$. Finally, set $c_{2t} = 1$ and $l_t = c_{1t} + c_{2t} + g$, so that (3.2) and (3.5) are satisfied. We conclude that the sequence of m_t's and c_{1t}'s correspond to an equilibrium. Now suppose we have an equilibrium. It is trivial to verify that the conditions of the proposition are satisfied.

We now use the proposition to construct the entire set of equilibria. For this, we need to study the properties of the equilibrium difference equation, (3.10). Consider first the sequence of m_t's and c_{1t}'s satisfying $m_t = c_{1t} = \beta/\mu^*$. It is easy to verify that conditions (i)-(iv) in Proposition 3.4 are satisfied, since $\beta/\mu^* < 1$. In the appendix, it is established that this is the globally unique equilibrium. In particular, we show that there are no equilibria in which $0 \leq m_0 < \beta/\mu^*$, or $m_0 > \beta/\mu^*$. The latter is ruled out by the fact that the sequence, m_t, $t = 1, 2, ...$ which satisfies (i)-(iii) of Proposition 3.4 violates (iv). Our assumption about fiscal policy, (2.21), plays a key role here. If instead we had assumed a version of (2.21) with B_T replaced by $M_T + B_T$, then the candidate equilibria associated with $m_0 > \beta/\mu^*$ would be valid equilibria. We have:

Proposition 3.5. *With constant money growth, $\mu^* > 1$, the monetarist steady state is the globally unique equilibrium, with $m_t = c_{1t} = \beta/\mu^*$ $t = 0, 1, 2, ...$.*

We now examine the set of equilibria when monetary policy is given by the Taylor rule with monetary monitoring. We first consider the case, $\alpha < 1/\beta$. We construct the boundaries of the money growth monitoring range, μ^l, μ^h, so that they bracket the target equilibrium and exclude the non-target equilibrium. Thus, $\mu^h > \mu^l$ and μ^h is smaller than the growth rate of money in the high inflation equilibrium, while μ^l is less than the money growth rate in the target inflation equilibrium. In addition, the money monitoring range also brackets the post-stabilization money growth rate, so that $\mu^h > \mu^* > \mu^l$. Finally, we impose $\mu^l \geq \beta$, since μ^l would be irrelevant otherwise.

Also, let c_1^h and c_1^l denote the levels of consumption in a model with a pure Taylor rule, that obtain when money growth is μ^h and μ^l, respectively. These are uniquely defined since there is an inverse monotone relationship between c_{1t} and M_{t+1}/M_t in the model with a pure Taylor rule. Consequently, $c_1^h < c_1^* < c_1^l$. The equilibrium difference equation, f, is graphed in Figure 2.

We argue by contradiction that there cannot be a Taylor rule with monitoring equilibrium with $c_{1,0} \neq \tilde{c}_1$. Suppose that such an equilibrium does exist, with $\hat{c}_1 < c_{1,0} < \tilde{c}_1$, where $\hat{c}_1$

denotes the level of cash good consumption in the high inflation equilibrium. Let $c_{1,1}$, $c_{1,2}, ...$ denote the solutions to (3.7). Given the shape of f, this sequence is monotonically declining. Let

$$c_{1T} = \min \left\{ c_{1t} | c_{1t} > c_1^h \right\},$$

so that $c_{1,T+1} < c_{1T}$ and

$$c_{1,T+1} \leq c_1^h < c_1^* \tag{3.11}$$

Corresponding to c_{1t}, $t = 0, 1, 2, ..., T$, there is a monotonically increasing sequence of money growth rates, M_{t+1}/M_t. To see this, note that c_{1t}, $t = 0, 1, 2, ..., T+1$ being strictly less than unity implies $R_t > 0$, so that $m_t = c_{1t}$ for $t = 0, 1, 2, ..., T+1$. Using this, combining (3.1) and (3.2), and making use of the fact, $m_{t+1}/m_t = (M_{t+1}/M_t)/\pi_{t+1}$, we obtain

$$\frac{M_{t+1}}{M_t} = \frac{\beta}{c_{1t}}, \ t = 0, 1, ..., T.$$

Thus, T is the last period when the pure Taylor rule is followed. In period $T+1$, the pure Taylor rule is abandoned because not to do so would result in $M_{T+2}/M_{T+1} = \beta/c_{1t} > \mu^h$. Period $T+1$ is the time when the switch to the constant money growth rate, μ^*, occurs. The situation

is depicted in Figure 2.

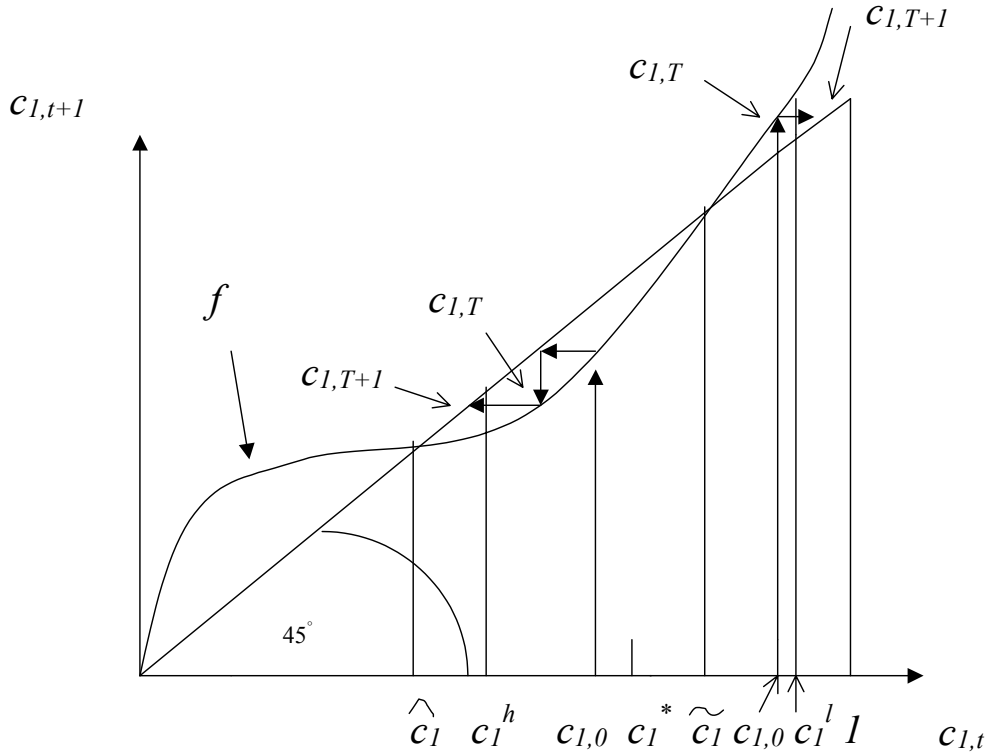

Figure 2: Taylor Rule With Monetary Monitoring, $\alpha < 1/\beta$

Equilibrium in $T+1$ requires:

$$\begin{aligned}
\frac{1}{c_{1,T+1}} &= \beta \frac{1+R_{T+1}}{\pi_{T+2}} \frac{1}{c_{1,T+2}} \\
c_{1,T+1} &= \frac{1}{1+R_{T+1}} \\
\frac{M_{T+2}}{M_{T+1}} &= \mu^*.
\end{aligned}$$

Since $c_{1,T+1} < \tilde{c}_1 < 1$ we know that $R_{T+1} > 0$, so that $c_{1,T+1} = m_{T+1}$. Substituting this into the intertemporal Euler equation, one obtains:

$$\frac{m_{T+2}}{m_{T+1}} = \frac{M_{T+2}}{M_{T+1}} \frac{1}{\pi_{T+2}} = \beta \frac{1+R_{T+1}}{\pi_{T+2}}.$$

After substituting out for R_{T+1} using (3.3) with t replaced by $T+1$:

$$\mu^* = \frac{\beta}{c_{1,T+1}},$$

so that $c_{1,T+1} = c_1^*$. This contradicts (3.11), establishing the result sought. A similar argument establishes that there can be no equilibrium with $0 < c_{1,0} \leq \hat{c}_1$. Also, $c_{1,0} = 0$ cannot be part of an equilibrium, since there exists no finite value of R_0 that satisfies (3.3) in this case.

Now consider $\tilde{c}_1 < c_{1,0} < 1$. The arrows in Figure 2 indicate a candidate equilibrium sequence of $c_{1,t}$'s associated with $c_{1,0}$. Note how eventually the $c_{1,t}$'s exceed c_1^l, which triggers a switch to a constant money growth policy rule. Because $c_{1,T+1} \geq c_1^l$ and $c_1^* < c_1^l$, it follows that there does not exist a continuation sequence of $c_{1,t}$'s that satisfy the Euler equations after the switch to constant money growth. This argument, which is essentially identical to the one used for the $\hat{c}_1 < c_{1,0} < \tilde{c}_1$ case above, establishes that there does not exist an equilibrium with $\tilde{c}_1 < c_{1,0} < 1$. Now consider $c_{1,0} = 1$. In this case, (3.7) implies $c_{1,1} = \infty$, violating non-negativity of R_t by (3.3). We have established:

Proposition 3.6. *Suppose $\alpha > \tilde{R}/\tilde{\pi}$ and $\alpha < 1/\beta$. Suppose the inflation money growth monitoring range, $\mu^l < \mu^h$, and the post-stabilization money growth rate, μ^*, satisfy:*

(i) $\mu^h > \tilde{\pi}, \mu^* > \mu^l > \beta$, where $\tilde{\pi}$ is the target inflation rate,

(ii) μ^h is less than money growth in the high inflation steady state.

Then, the target inflation steady state is the unique equilibrium of the model under a Taylor rule with monetary monitoring.

We turn to the case, $\alpha > 1/\beta$. According to Proposition 3.3, the inflation rate target equilibrium is now the lower intersection of f with the 45 degree line. The associated level of cash good consumption is $\tilde{c}_1$, and is indicated in Figure 3. That figure also indicates c_1^l and c_1^h, the levels of consumption associated with the upper and lower bounds of the money growth monitoring range, respectively. Unlike the previous case, money growth monitoring does not narrow the set of equilibria down to the target inflation equilibrium. Nevertheless, it does restrict the set of equilibria. With a pure Taylor rule the set of equilibria corresponds to $0 < c_{1,0} \leq \hat{c}_1$, where $\hat{c}_1$ is the level of consumption associated with the upper intersection of f and the 45 degree line. With monetary monitoring the set of equilibria is restricted to those with $c_1^l < c_{1,0} < c_1^h$. The set of equilibria associated with monetary monitoring is strictly

interior to the set of equilibria under a pure Taylor rule.

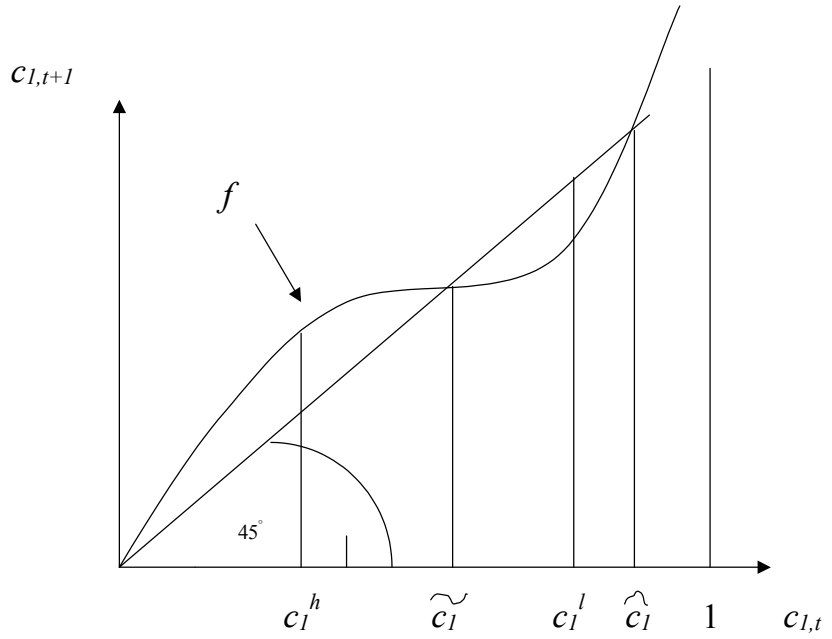

Figure 3: Taylor Rule with Monetary Monitoring and $\alpha > 1/\beta$

We summarize these observations with the following proposition:

Proposition 3.7. *Suppose $\alpha > \tilde{R}/\tilde{\pi}$ and $\alpha > 1/\beta$. Suppose the inflation money growth monitoring range, $\mu^l < \mu^h$, and the post-stabilization money growth rate, μ^*, satisfy:*

(i) $\mu^h > \tilde{\pi}, \mu^* > \mu^l > \beta$, where $\tilde{\pi}$ is the target inflation rate,

(ii) μ^l is greater than money growth in the low inflation steady state.

Then, the set of equilibria is smaller than what it is under a pure Taylor rule.

3.2. Linear Taylor Rules

We first discuss the non-random equilibria of our model under linear Taylor rules. We then consider the random equilibria that can occur in a neighborhood of the inflation target steady state with an aggressive Taylor rule.

Non-Random Equilibria

Our results for linear Taylor rules are qualitatively similar to those obtained for non-linear Taylor rules in the previous section. When the rule is aggressive, then the target inflation steady state is indeterminate and there is another steady state with low inflation. In this steady state, the gross rate of inflation is β and the rate of interest is zero. This corresponds to the liquidity trap equilibrium discussed in BSGU. When the Taylor rule is passive, then the target inflation steady state is determinate, and there is a continuum of other equilibria in which the rate of inflation explodes to plus infinity.[16]

Although the non-target inflation equilibrium under a linear Taylor rule is qualitatively similar to what it is under a non-linear Taylor rule, there are quantitative differences. In particular, when the Taylor rule is aggressive this equilibrium involves positive but finite inflation under the nonlinear Taylor rule, and infinite inflation under the linear Taylor rule. When the Taylor rule is passive then the non-target inflation equilibrium is small but greater than β under the nonlinear Taylor rule, and equal to β under the linear Taylor rule.

The effect of monetary monitoring is similar to what it is under a nonlinear Taylor rule. When that rule is aggressive, then it can limit the range of equilibria that can occur to a prescribed range in the neighborhood of the target inflation steady state. When the Taylor rule is passive, it can shrink the set to a singleton. We now establish these results.

Inverting the linear Taylor rule, we obtain:

$$\pi_{t+1} = \begin{cases} \tilde{\pi} + \frac{1}{\alpha}\left(R_t - \tilde{R}\right) & R_t > 0 \\ [0, \tilde{\pi} - \frac{1}{\alpha}\tilde{R}] & R_t = 0 \end{cases}, \ \pi_t = \frac{P_t}{P_{t-1}}. \tag{3.12}$$

This is a function for $R_t > 0$, but a correspondence for $R_t = 0$. Replacing the Taylor rule in (3.4) with (3.12) and repeating the calculations that produced (3.7), we obtain:

$$c_{1t+1} = \begin{cases} \frac{\beta}{\tilde{\pi} + \frac{1}{\alpha}\left(\frac{1}{c_{1t}} - 1 - \tilde{R}\right)} & c_{1t} < 1 \\ \left[\frac{\beta}{\tilde{\pi} - \frac{1}{\alpha}\tilde{R}}, +\infty\right] & c_{1t} = 1 \end{cases} \equiv f(c_{1t}) \tag{3.13}$$

Evidently,

$$\begin{aligned} f &\rightarrow 0 \text{ as } c_{1t} \rightarrow 0 \\ \tilde{c}_1 &= f(\tilde{c}_1), \ \tilde{c}_1 = \beta/\tilde{\pi}. \end{aligned}$$

[16] These resemble the speculative hyperinflations that Brock (1974) and Obstfeld and Rogoff (1983) found in the context of constant money growth rate rules.

As before, we refer to the steady state associated with $\tilde{c}_1$ as the target inflation steady state. We can learn more about f by differentiating it for $c_1 < 1$:

$$f'(c_1) = \frac{\beta/\alpha}{\left[\tilde{\pi}c_1(1-\frac{1}{\alpha\beta})+\frac{1}{\alpha}\right]^2}.$$

We have:

$$\begin{aligned} f' &\to \beta\alpha \text{ as } c_1 \to 0 \\ f' &\to 1/(\alpha\beta) \text{ as } c_1 \to \beta/\tilde{\pi} \\ f' &\to \frac{1/(\alpha\beta)}{\left[1+\tilde{R}(1-\frac{1}{\beta\alpha})\right]^2} \text{ as } c_1 \to 1 \end{aligned}$$

Also, f' is monotone increasing or decreasing, depending on whether $\alpha\beta < 1$ or $\alpha\beta > 1$.

The preceding observations imply the following. As before, the target inflation steady state is determinate if $\alpha < 1/\beta$ and indeterminate if $\alpha > 1/\beta$. For α in the determinate region, f lies below the 45 degree line near zero and cuts the 45 degree line exactly once, at the target inflation steady state, from below. We know it only cuts once because f' is monotone increasing in c_1. The function, f, is displayed as the darkened line in Figure 4. Note how the function becomes set-valued at $c_1 = 1$. Evidently, the globally unique equilibrium is $\tilde{c}_1$. In this case, monetary monitoring has no impact on the set of equilibria. The inflation target equilibrium is the globally unique equilibrium whether there is monetary targeting or not.

Now consider the case, $\alpha > 1/\beta$. In this case, the equilibrium difference equation, f, function is displayed in Figure 5. Again, that function is set-valued at $c_1 = 1$. Now there are two steady state equilibria, one at $\tilde{c}_1$, and the other at $c_1 = 1$.[17] Each $c_{1,0}$ in the interval $0 < c_{1,0} \leq 1$ corresponds to an equilibrium under the pure Taylor rule. All of these, except $c_{1,0} = 1$ have the property that $c_{1,t} \to \tilde{c}_1$. In this case, monetary monitoring does not restrict the set of equilibria to a singleton. It does restrict the range, however, as in the nonlinear Taylor rule analysis in the previous subsection.

[17] It is easy to verify that $c_{1,t} = c_{2,t} = 1$, $R_t = 0$, $m_t = c_{1,t}$, $M_{t+1}/M_t = P_{t+1}/P_t = \beta$ for $t \geq 0$ satisfy all the equilibrium conditions.

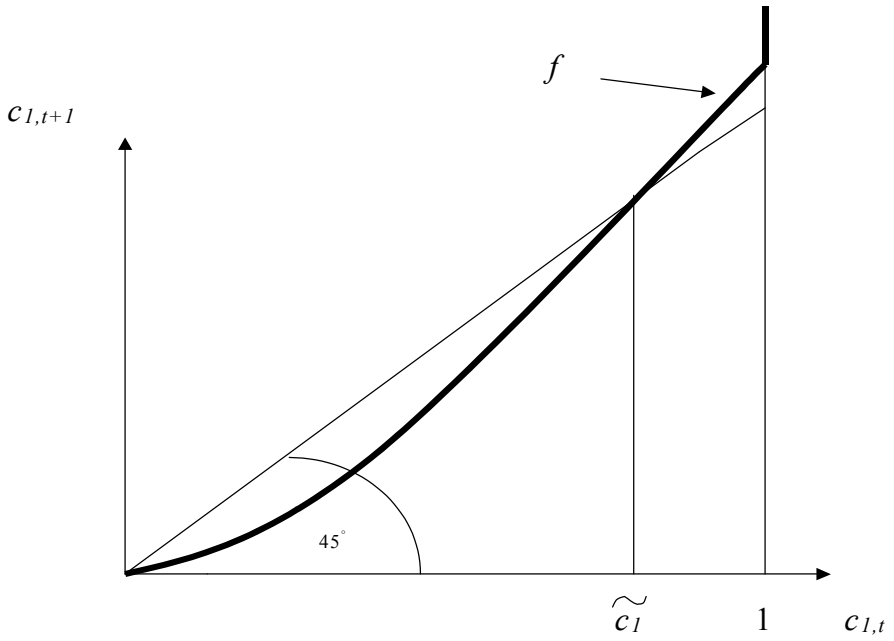

Figure 4: Equilibrium Under Pure, Linear Taylor Rule and $\alpha < 1/\beta$

These observations, in combination with the observations about the uniqueness of the money growth rule, lead naturally to conclusions about the Taylor rule with monetary monitoring that resemble those in Propositions 3.6 and 3.7:

Proposition 3.8. *Consider monetary policy under a pure, linear, Taylor rule. Then:*

(i) there always exists a steady state target inflation equilibrium in which $\pi_t = \tilde{\pi}$ for all t

(ii) when $\alpha < 1/\beta$, the target inflation steady state is determinate. It is the globally unique interior equilibrium under a pure Taylor rule or under a pure Taylor rule with monetary monitoring.

(iii) when $\alpha > 1/\beta$, the target inflation steady state is indeterminate. In this case, monetary monitoring restricts the set of equilibria that can occur.

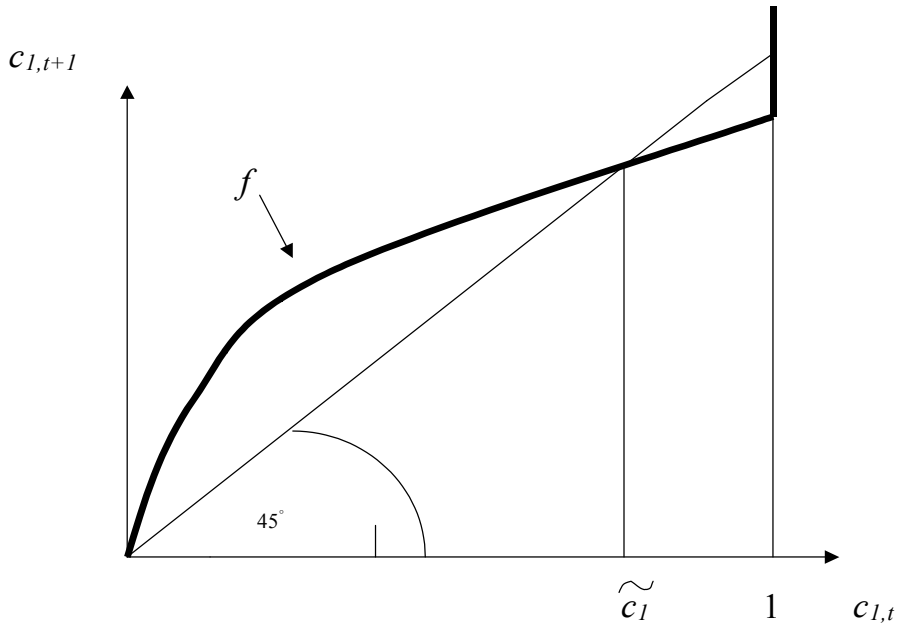

Figure 5: Equilibrium Under Pure, Linear Taylor Rule and $\alpha > 1/\beta$

Random Equilibria

It is well-known that when an equilibrium is indeterminate, then there exist equilibria in which prices, rates of return and quantity allocations are random, even when technology and preferences are deterministic. This type of equilibrium is called a sunspot equilibrium. The household problem in an economy with sunspot equilibria has to be modified slightly, to accommodate the fact that the prices and rates of return that they face are random. The modification simply involves putting an expectation operator before the household preferences. The household Euler equations are now given by:

$$E_t\left[u_{1,t} - \beta\left(1+R_t\right)\frac{u_{1,t+1}}{\pi_{t+1}}\right] = 0 \tag{3.14}$$

$$u_{3,t} + u_{2,t} = 0 \tag{3.15}$$

$$\frac{u_{1t}}{u_{2t}} = 1 + R_t, \tag{3.16}$$

for $t = 0, 1, 2, \ldots$. In addition, optimality requires:

$$R_t(M_t^d - P_t c_{1t}) = 0, \; M_t^d - P_t c_{1t} \geq 0. \tag{3.17}$$

In (3.15) we have substituted out the firm first order condition, (2.17).

For later purposes, it is useful to adopt a slightly more general specification of utility:

$$u(c_1, c_2, l) = \frac{c_1^{1-\sigma}}{1-\sigma} + \gamma \frac{c_2^{1-\delta}}{1-\delta} - \frac{\psi_0}{1+\psi} l^{1+\psi}, \tag{3.18}$$

with $\gamma, \psi \geq 0$, $\sigma, \delta > 1$. We obtain the specification of utility considered in the deterministic analysis above by setting $\sigma = \delta = \gamma = 1$, $\psi = 0$.

The Taylor rule must be modified to accommodate uncertainty. We do so by replacing π_{t+1} by its expectation:

$$R_t = \max\left\{0, R + \alpha\left[E_t \pi_{t+1} - \beta(1+R)\right]\right\}.$$

In what follows we only work with this linear representation of the Taylor rule.

Following is our definition of equilibrium. It applies to either type of monetary policy, whether a pure Taylor rule or a Taylor rule with monetary monitoring.

Definition 3.9. *A Stochastic Equilibrium is a collection of stochastic processes,* $\{c_{1t}, c_{2t}, M_t, M_t^d, B_t, B_t^d, P_t, R_t, \tau_t, D_t, l_t\}$, *such that*

(i) the household problem is solved

(ii) the firm first order condition is satisfied

(iii) the monetary policy rule is satisfied

(iv) markets clear, i.e., $M_t^d = M_t$, $B_t^d = B_t$, $l_t = c_{1t} + c_{2t} + g$.

The following discussion is divided into two parts. First, we discuss the pure Taylor rule. Then, we discuss the Taylor rule with monetary monitoring.

3.2.1. Pure Taylor Rule

In constructing sunspot equilibria, it is useful to note that the expression, $E_t x_{t+1} = 0$ is equivalent to the expression, $x_{t+1} = \nu_{t+1}$, where ν_{t+1} is any random variable whatsoever, having the property $E_t \nu_{t+1} = 0$.

Ultimately, we are concerned with equilibrium under monetary monitoring in which $R_t = 0$ is excluded. Consequently, there is no loss in generality in restricting the present discussion to

equilibria in which $R_t > 0$. As a result, $E_t\pi_{t+1}$ can be expressed as the following function of R_t:

$$E_t\pi_{t+1} = \frac{R_t - \tilde{R}}{\alpha} + \beta(1 + \tilde{R}). \tag{3.19}$$

Alternatively, we can write this:

$$\pi_{t+1} = \frac{R_t - \tilde{R}}{\alpha} + \beta(1 + \tilde{R}) + \eta_{t+1}, \tag{3.20}$$

where $E_t\eta_{t+1} = 0$. Substituting (3.20) into the intertemporal Euler equation, we obtain:

$$c_{1t}^{-\sigma} - \frac{\beta\left(1 + R(c_{1t})\right)}{\frac{R(c_{1t})-\tilde{R}}{\alpha} + \beta(1 + \tilde{R}) + \eta_{t+1}} c_{1,t+1}^{-\sigma} = \omega_{t+1}, \tag{3.21}$$

where $E_t\omega_{t+1} = 0$. In addition, $R(c_{1t})$ is the nominal rate of interest expressed as a strictly decreasing function of c_{1t}. This function is defined by the static equilibrium conditions, the two static Euler equations, (3.15) and (3.16), and the resource constraint, (2.18). We now briefly define this mapping in detail.

Using our functional form assumptions and combining the resource constraint with (3.15), we obtain:

$$c_{2,t}^{\delta} = \frac{\gamma}{\psi_0\left(g + c_{2t} + c_{1t}\right)^{\psi}}. \tag{3.22}$$

It is easy to verify that for given c_{1t} there is a unique value of $c_{2,t}$ that satisfies this expression. Moreover, c_2 is strictly decreasing in c_{1t}.[18] Denote this function of c_{1t} by $c_2(c_{1t})$. Now, consider the money demand equation:

$$\frac{1}{\gamma}\frac{c_2(c_{1t})^{\delta}}{c_{1t}^{\sigma}} = 1 + R_t. \tag{3.23}$$

This expresses R_t as a strictly decreasing function of c_{1t}. This is the function, $R(c_{1t})$, that appears in (3.21).

[18] For fixed c_{1t} the expression on the right is strictly decreasing in c_{2t}. It eventually asymptotes at zero. In addition, when expressed as a function with $c_{2,t}$ on the horizontal axis, it intersects the vertical axis at a positive value for $c_{2,t} = 0$. The expression on the left is a convex function of $c_{2,t}$ coming out of the origin and increasing monotonically. The two curves must intersect at a unique, positive, value of $c_{2,t}$. In addition, the curve defined by the expression on the right shifts down with a rise in c_{1t}, guaranteeing that $c_2(c_{1t})$ is a decreasing function.

Let c_1^u denote the upper bound on c_{1t} implied by the condition, $R_t \geq 0$. It is the unique solution to:

$$\frac{1}{\gamma}\frac{c_2(c_1^u)^\delta}{(c_1^u)^\sigma} = 1 \tag{3.24}$$

We can now state the relevant partial characterization result for stochastic equilibria.

Proposition 3.10. *Suppose c_{1t}, $t = 0, 1, 2, ...$ satisfy (3.21) and $0 \leq c_{1t} < c_1^u$ for some stochastic processes η_{t+1} and ω_{t+1} satisfying $E_t\eta_{t+1} = E_t\omega_{t+1} = 0$ for all t. Then, $\{c_{1t}\}$ corresponds to a Stochastic Equilibrium.*

Proof: Let $R_t = R(c_{1t}) > 0$, $c_{2,t} = c_2(c_{1t})$, $m_t = c_{1t}$, $l_t = c_{1t} + c_{2t} + g$. It is straightforward to verify that the sufficient conditions for household and firm optimization, (3.15) and (3.17), are satisfied.

The above proposition does not guarantee the existence of a sunspot equilibrium, since it is possible that the only equilibrium involves $\eta_{t+1} = \omega_{t+1} = 0$. Conditions sufficient for a sunspot equilibrium (i.e., one with either η_{t+1} or ω_{t+1} not equal to 0) are well known and easy to develop. The following discussion illustrates the result obtained more generally by Woodford (1986). Express (3.21) as follows:

$$f(c_{1t}, c_{1t+1}, \eta_{t+1}, \omega_{t+1}) = 0. \tag{3.25}$$

Linearize this about the inflation targeting steady state, $\tilde{c}_1$, and about $\eta_{t+1} = \omega_{t+1} = 0$, to obtain:

$$f_1\hat{c}_{1t} + f_2\hat{c}_{1,t+1} + f_3\eta_{t+1} + f_4\omega_{t+1} = 0,$$

where f_i is the derivative of f with respect to its i^{th} argument, evaluated at the indicated point, for $i = 3, 4$. In the case of $i = 1, 2$, f_i is the product of the derivative and $\tilde{c}_1$. Finally, $\hat{c}_{1t}$ denotes $(c_{1t} - \tilde{c}_1)/\tilde{c}_1$. Solve this for $\hat{c}_{1,t+1}$:

$$\hat{c}_{1,t+1} = -\frac{f_1}{f_2}\hat{c}_{1t} - \frac{f_3}{f_2}\eta_{t+1} - \frac{f_4}{f_2}\omega_{t+1}.$$

If $-f_1/f_2$ is less than unity in absolute value, then this is a well defined stochastic process. Indeed, if there are upper and lower bounds to the support of η_{t+1} and ω_{t+1}, then $\hat{c}_{1,t+1}$ must lie inside a particular bounded set. Thus, if a^h and a^l denote the upper and lower bounds, respectively, on the support of $-\frac{f_3}{f_2}\eta_{t+1} - \frac{f_4}{f_2}\omega_{t+1}$, then the set,

$$\left[\frac{a^l}{1 + \frac{f_1}{f_2}}, \frac{a^h}{1 + \frac{f_1}{f_2}}\right]$$

forms an ergodic set for the stochastic process, $\hat{c}_{1,t+1}$. That is, if $\hat{c}_{1t}$ belongs to this set, $\hat{c}_{1,t+j}$ must belong to this set for all $j \geq 0$. In addition, if $\hat{c}_{1t}$ does not belong to this set, then eventually it will. This reasoning suggests that if $-f_1/f_2$ is less than unity in absolute value, a sunspot equilibrium can be constructed about the steady state inflation targeting equilibrium. Strictly speaking, we can only be sure that a sunspot equilibrium with very small shocks can be found, since the linear approximation corresponds well to the actual equilibrium conditions only in a small region about the steady state. However, in practice, we find that this region is quite large.

Motivated by the discussion in the previous paragraph, we now construct a sunspot equilibrium about the target inflation steady state. We do so for the case when the Taylor rule is aggressive, so that that steady state is indeterminate. The sunspot equilibrium is constructed using the actual equilibrium conditions, and does not use approximations.

The first step is to assign a probability distribution for ω and η. The key constraint on these is $E_t\omega_{t+1} = 0$, $E_t\eta_{t+1} = 0$. Then, fix a value for $0 \leq c_{1,0} < c_1^u$ and compute $R(c_{1,0})$, $c_2(c_{1,0})$, $l(c_{1,0})$. This yields all the relevant period 0 variables. Suppose that these variables are available for period t. The period $t+1$ variables are obtained as follows. Draw ω_{t+1} and η_{t+1} from the appropriate random number generator, and solve (3.21) for $c_{1,t+1}$. That is,

$$c_{1,t+1} = \left[\frac{\frac{R(c_{1t})-\tilde{R}}{\alpha} + \beta(1+\tilde{R}) + \eta_{t+1}}{\beta\left(1+R(c_{1t})\right)} \left(c_{1t}^{-\sigma} - \omega_{t+1}\right) \right]^{\frac{-1}{\sigma}}. \tag{3.26}$$

Also, $R_{t+1} = R(c_{1,t+1})$, $c_{2,t+1} = c_2(c_{1,t+1})$, $l_{t+1} = l(c_{1,t+1})$, $\pi_{t+1} = \frac{R(c_{1t})-\tilde{R}}{\alpha} + \beta(1+\tilde{R}) + \eta_{t+1}$. Proceeding in this way, a time series of arbitrary length can be simulated. It is easy to verify that all the equilibrium conditions are satisfied by the stochastic process constructed in this

way if the variables lie in an ergodic set that is strictly interior to the bounded set, $[0, c_1^u]$.

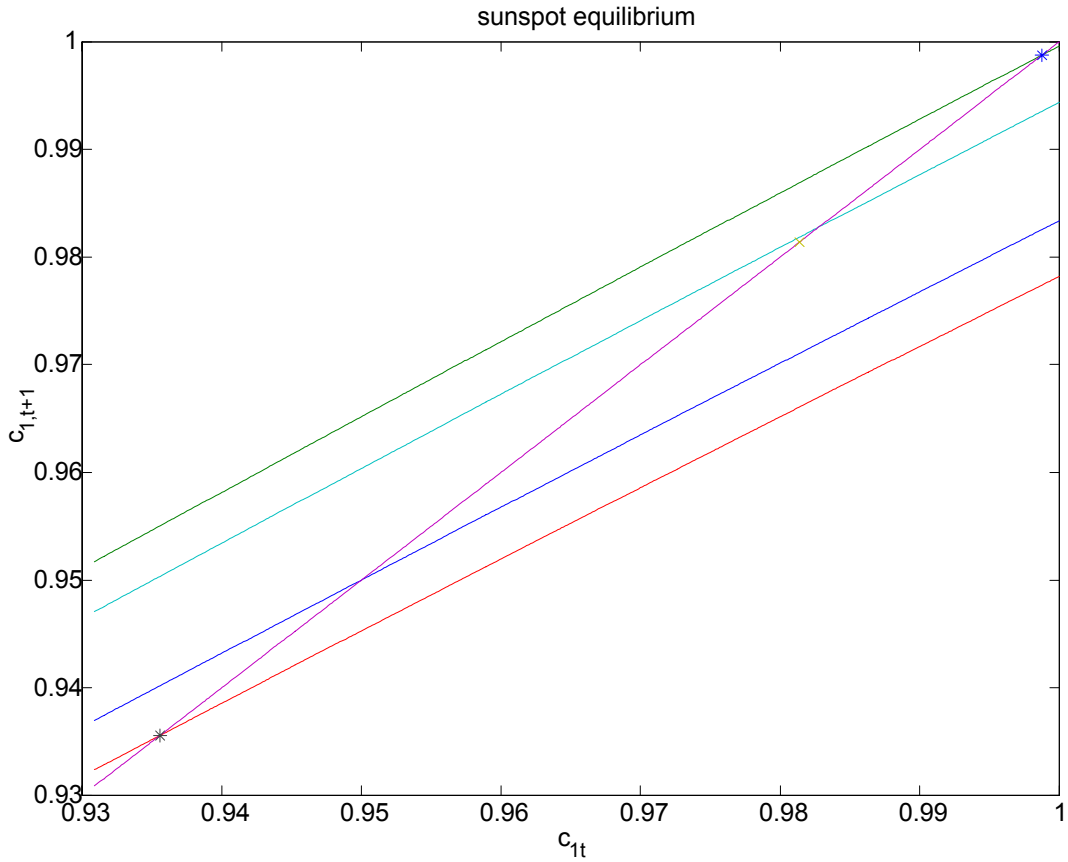

Figure 6: $f(c_{1t}, c_{1t+1}, \eta, \omega) = 0$, for Four Values of η, ω.

We proceed now to construct a particular sunspot equilibrium. Continuing with the example studied up to now in this section, we set

$$\alpha = 1.5, \beta = 1.06^{-.25},\ \sigma = \delta = \gamma = \psi_0 = 1,\ \psi = 0,\ \tilde{\pi} = 1 + .017/4. \tag{3.27}$$

In the inflation targeting steady state, $\tilde{R} = 0.019$, $\tilde{c}_1 = 0.98$. The inflation targeting steady state is indicated by the 'x' appearing on the 45 degree line in Figure 6. We assume that the forecast errors have the following two-point distributions:

$$\eta \ = \ \begin{cases} \eta^h \text{ with probability } p \\ \eta^l \text{ with probability } 1-p \end{cases}$$

$$\omega = \begin{cases} \omega^h & \text{with probability } p \\ \omega^l & \text{with probability } 1-p \end{cases},$$

where we impose that the probabilities on the two states are the same for the two random variables. The values of p, η^h, η^l, ω^h, ω^l must satisfy:

$$\begin{aligned} p\eta^h + (1-p)\,\eta^l &= 0 \\ p\omega^h + (1-p)\,\omega^l &= 0. \end{aligned}$$

For each of the four possible values of η_{t+1}, ω_{t+1}, (3.21) defines a different mapping from c_{1t} to $c_{1,t+1}$ (see (3.26)). These are graphed for a particular set of values of η^h, η^l, ω^h, ω^l in Figure 6. The two '*' on the diagonal indicate the intersection of the highest and lowest curves with the 45 degree line. It is easy to verify that these define an ergodic set for c_{1t}, as defined above. It follows that there is an equilibrium corresponding to every $c_{1,0}$ belonging to this ergodic set.

We calibrated p, η^h, η^l, ω^h, ω^l using the following restrictions, in addition to the two mean restrictions already specified. We imposed that the lower boundary of the ergodic set on c_{1t} corresponds to roughly 15% inflation (AR) and occurs with probability roughly once every 25 periods (i.e., quarters, or 6 years). Since the lower boundary is associated with $\eta = \eta^l$ and $\omega = \omega^h$, this requires

$$(1-p)\,p = \frac{1}{25}.$$

For the lower boundary to be associated with 0.15 percent inflation we require that the value of c_1 there, $c_{1,l}$, satisfy:

$$E_t\pi_{t+1} = 1 + \frac{.15}{4} = \frac{R(c_{1,l}) - \tilde{R}}{\alpha} + \beta(1+\tilde{R}).$$

The upper boundary for c_1, $c_{1,h}$, is associated with low inflation. We imposed that the nominal rate of interest there be slightly positive:

$$R(c_{1,h}) = 0.005/4.$$

This gives us 5 restrictions for the 5 unknowns, p, η^h, η^l, ω^h, ω^l. We obtained:

$$\begin{aligned} \eta^h &= 0.0157,\ \eta^l = -6.8\times 10^{-4} \\ \omega^h &= 0.005,\ \omega^l = -2.19\times 10^{-4}, \\ p &= 0.0417. \end{aligned}$$

Figure 7 displays a simulation of 200 observations from this model:

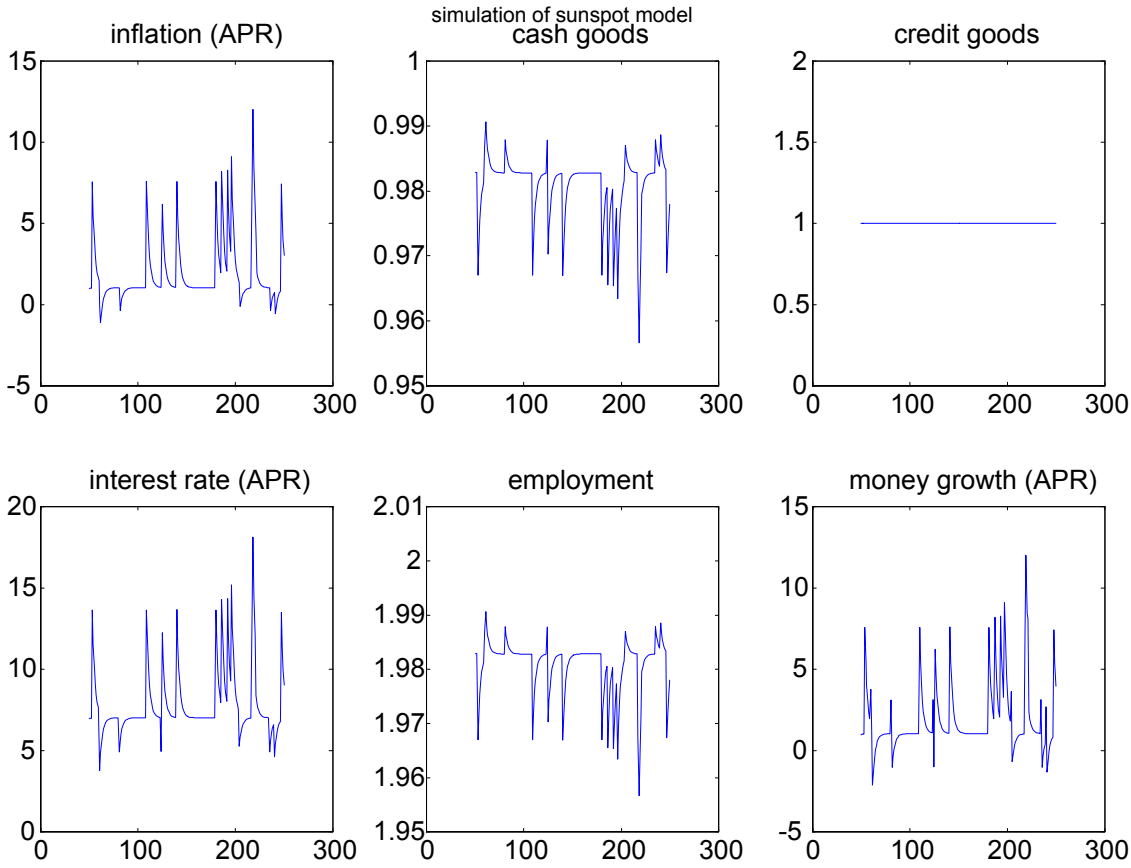

Figure 7: Simulation of Sunspot Equilibrium

Notice how most of the time inflation is quite low. This low inflation is punctuated by periodic jumps to nearly 10 percent, at an annual rate. This high inflation is associated with high money growth.

3.2.2. Taylor Rule with Money Monitoring

As before, to investigate the equilibria under the Taylor rule with monetary monitoring, it is convenient to first study the set of equilibria of the version of the model with a constant money growth rule, $M_{t+1}/M_t = \mu^*$. The results obtained in the previous section apply here too. In particular, the monetary steady state is the globally unique equilibrium of the model.

As a result, in the version of the model with monetary monitoring, the magnitude of sunspot fluctuations is limited by size of the monitoring range. For example, if the monitoring range is sufficiently narrow that the money growth rate in the sunspot equilibrium constructed above can violate it, then that equilibrium cannot occur under money growth monitoring.

3.3. A Current-Looking Taylor Rule

Some analysts study versions of the Taylor rule in which it is current inflation, π_t, that appears, rather than π_{t+1}. We briefly explore the consequences of this here. Our basic finding, that monetary monitoring is potentially helpful, survives this change. To see this, note that in the inverted Taylor rule, π_{t+1} is replaced by π_t in the nonlinear case, (3.4), and in the linear case, (3.12). This change requires replacing c_{1t+1} by c_{1t} in (3.7) and in (3.13), respectively. Note that the equilibria of this version of the model are static, and correspond to the crossings of f with the 45 degree line. Thus, with the nonlinear Taylor rule there are two equilibria: the inflation targeting equilibrium and one other one. Monetary monitoring, as long as the second equilibrium is not included inside the monitoring range, $[\mu^l, \mu^h]$, always reduces the set of equilibria to a singleton. With the linear Taylor rule, multiple equilibria only occur with an aggressive Taylor rule, $\alpha > 1/\beta$. In this case, the inflation target equilibrium is the unique equilibrium under the Taylor rule with monetary monitoring. Thus, monetary monitoring is still useful when the Taylor rule feeds back onto current inflation.

4. Low Inflation and Inflation Cycles in a Cash in Advance Model

This section describes two versions of a simple cash in advance model. One illustrates the possibility of a low inflation equilibrium under the Taylor rule, and the other illustrates the possibility of a cycling equilibrium.

Household preferences in the model are:

$$\sum_{t=0}^{\infty} \beta^t u(c_t, l_t).$$

The household begins the period holding money, M_t. It then divides M_t between deposits, D_t, with a financial intermediary and cash set aside for consumption expenditures, $M_t - D_t$. It faces the following non-negativity constraints on D_t, $0 \leq D_t \leq M_t$. The household faces the following cash constraint in the goods market:

$$P_t c_t \leq M_t - D_t + W_t l_t - T_t, \tag{4.1}$$

where T_t is lump-sum taxes paid by the household.

The household's cash evolution equation is:

$$M_{t+1} = (1 + R_t)(X_t + D_t) + M_t - D_t + W_t l_t - P_t c_t - T_t + \text{Profits}_t,$$

where X_t denotes a monetary transfer from the central bank. The household's Euler equations are:

$$\begin{aligned} u_{c,t} &= \beta u_{c,t+1} \frac{1 + R_t}{\pi_{t+1}} && (4.2) \\ \frac{-u_{l,t}}{u_{c,t}} &= \frac{W_t}{P_t}, && (4.3) \end{aligned}$$

and

$$[M_t - D_t + W_t l_t - T_t - P_t c_t] R_t = 0 \quad (4.4)$$

with $M_t - D_t + W_t l_t - P_t c_t - T_t \geq 0$. We only need to consider $R_t \geq 0$, since there is no equilibrium with $R_t < 0$.

We assume that firms must borrow cash in advance to finance their wage payments. The production technology allows firms to transform labor into output one-for-one. This leads to the following first order condition:

$$\frac{W_t}{P_t} = \frac{1}{1 + R_t}. \quad (4.5)$$

The total nominal demand for funds in financial markets is $W_t l_t$. The amount of funds that the financial intermediary has is $D_t + X_t$. When $R_t > 0$, they lend out all they have. When $R_t = 0$, the amount they lend is equal to demand. Thus, the market clearing condition in the loan market it:

$$\begin{aligned} W_t l_t &= D_t + X_t, \text{ for } R_t > 0 && (4.6) \\ W_t l_t &\leq D_t + X_t, \text{ for } R_t = 0 \end{aligned}$$

We now turn to the government. The government raises lump sump taxes, T_t, subject to $T_t = P_t g$. In addition, the government transfers an amount of cash to households (actually, taxes them if $X_t < 0$) in the amount of X_t. We rule out bonds in this discussion, although it should be clear that introducing them along the lines of our cash-credit good model discussion would change nothing. In equilibrium, the household's choice of M_t equals the amount of money outstanding. In addition,

$$M_{t+1} = M_t + X_t.$$

We assume that the monetary authority chooses the value of X_t to implement the linear version of the Taylor rule. We repeat this here for convenience:

$$R_t = \max\left\{0, \tilde{R} + \alpha\left[\pi_{t+1} - \beta(1+\tilde{R})\right]\right\}.$$

Inverting this:[19]

$$\pi_{t+1} = \begin{cases} \tilde{\pi} + \frac{1}{\alpha}\left(R_t - \tilde{R}\right) & R_t > 0 \\ [0, \pi - \frac{1}{\alpha}\tilde{R}] & R_t = 0 \end{cases}. \tag{4.7}$$

Finally, the resource constraint is:

$$c_t + g \leq l_t. \tag{4.8}$$

We now summarize the equilibrium conditions. Substituting (4.5) and (4.3) into (4.2), we obtain:

$$-u_{l,t} = \frac{\beta\lambda u_{c,t+1}}{\pi_{t+1}}. \tag{4.9}$$

Combining (4.1) and (4.6), we obtain:

$$\begin{aligned} c_t &= m_t\frac{M_{t+1}}{M_t}, \text{ if } R_t > 0 \\ c_t &\leq m_t\frac{M_{t+1}}{M_t}, \text{ if } R_t = 0 \end{aligned} \tag{4.10}$$

where $m_t = M_t/P_t$.

We can use this model to illustrate how the economy might cycle or fall into a high or low inflation trap under the pure Taylor rule. The low inflation trap illustrates a case that was stressed by BSGU. In the example where the economy cycles, the monetary authorities implement an aggressive Taylor rule (i.e., $\alpha = 1.5$) and the inflation target equilibrium is determinate. This is of interest, since this is the kind of scenario that may seem to create a 'best case' for the Taylor rule. The example complements another example recently presented by Benhabib, Schmitt-Grohe and Uribe (2001), in which the inflation process under a pure Taylor rule is chaotic. We show how our monetary monitoring policy eliminates the low inflation trap and the cycling equilibrium, as long as the monitoring range is chosen appropriately.

[19]Recall, $\pi - \frac{1}{\alpha}\tilde{R} > 0$ by an assumption we placed on α earlier.

Consider the following specification of utility:

$$u(c,l) = \log(c) - \frac{\psi_0}{1+\psi} l^{1+\psi}.$$

We now discuss the set of equilibria under a pure Taylor rule, and after that we turn to the equilibria under monetary monitoring.

Equilibria Under Pure Taylor Rule

We investigate the equilibria of the model in the same way as before. Using the assumed functional form, the Taylor rule, (4.7), the labor supply equation, (4.3), the labor demand equation, (4.5), the intertemporal Euler equation, (4.2), and the resource constraint, (4.8) can all be combined to obtain the following equilibrium difference equation:

$$l_{t+1} = \begin{cases} \frac{\beta}{\psi_0\left[\tilde{\pi}+\frac{1}{\alpha}\left(\frac{1}{\psi_0 l_t^{\psi+1}}-1-\tilde{R}\right)\right] l_t^{\psi}} & l_t \leq l^u \\ \frac{\beta}{\psi_0 \pi_{t+1} l_t^{\psi}},\ \pi_{t+1} \in \left[0, \tilde{\pi} - \frac{1}{\alpha}\tilde{R}\right] & l_t = l^u \end{cases} = f(l_t), \tag{4.11}$$

where

$$l^u = \left(\frac{1}{\psi_0}\right)^{\frac{1}{1+\psi}}. \tag{4.12}$$

Here, l^u is the highest possible equilibrium level of employment. It is the level of employment associated with $R_t = 0$. This is obtained using the following relation, which combines (4.3) and (4.5):

$$R_t = \frac{1}{\psi_0 l_t^{\psi+1}} - 1. \tag{4.13}$$

Note how f in (4.11) is a function for $0 < l_t < l^u$, while f is set-valued for $l_t = l^u$. This reflects that the inverse of the Taylor rule, (4.7), is set-valued at the level of employment, l^u.

It is easy to verify the following characterization result:

Proposition 4.1. *The sequence, l_t, $t = 0, 1, 2, \ldots$, corresponds to an equilibrium if, and only if, it satisfies:*

(i) (4.11)

(ii) $0 < l_t \leq l^u$

(iii) the transversality condition

It is straightforward to verify that the target inflation steady state is an equilibrium, with $l_t = l^*$, $m_t = l^*/\tilde{\pi}$ for all t, where

$$l^* = \left(\frac{\beta}{\psi_0 \tilde{\pi}}\right)^{\frac{1}{1+\psi}}.$$

There are other equilibria too. For example, when $\beta \leq \tilde{\pi} - \frac{1}{\alpha}\tilde{R}$ (this is guaranteed when the Taylor rule is aggressive, $\alpha\beta > 1$) the deflation equilibrium stressed by BSGU exists. This corresponds to the situation in which $l_t = l^u$, $\pi_{t+1} = \beta$ and $R_t = 0$ for all t. To see this, solve (4.11) for π_{t+1}, imposing $l_t = l_{t+1} = l^u$, to obtain

$$l^u = \frac{\beta}{\pi_{t+1}\psi_0 \left(l^u\right)^{\psi}},$$

or,

$$\pi_{t+1} = \beta.$$

From (4.10) we find that $c_t \leq m_{t+1}\pi_{t+1}$, or

$$m_{t+1} \geq \frac{l^u}{\beta},$$

so that nominal money growth must be no less than β, the rate of inflation. In addition, the transversality condition requires that the nominal stock of money shrink over time. Thus:

$$\beta \leq \frac{M_{t+1}}{M_t} < 1.$$

We have:

Proposition 4.2. *If $\beta \leq \tilde{\pi} - \frac{1}{\alpha}\tilde{R}$, then $R_t = 0$, $l_t = l^u$ corresponds to an equilibrium under the pure Taylor rule.*

We now explore another type of equilibrium in this economy, in which the rate of inflation cycles. We do this for a parameterization under which the target inflation steady state is determinate. We obtain such a parameterization by choosing a large value of ψ, so that labor supply is relatively inelastic. To understand why this should result in determinacy, we build on the intuition in section 3.1 using insights from Carlstrom and Fuerst (2000a) and Schmitt-Grohe and Uribe (2000). Suppose α is large. There is always another inflation and consumption path that is consistent with the intertemporal Euler equation, (4.2), in which consumption growth,

inflation, and the real and nominal rate of interest are transitorily high. When the target inflation steady state is indeterminate, there is a path like this close to the original steady state. This requires that employment and consumption be low during the transition. Thus, ruling out indeterminacy requires that such paths not be consistent with the labor market, as summarized by the supply and demand for labor, (4.3) and (4.5), respectively. This will be the case if the labor supply elasticity is low. For, in this case the low labor demand produced by the high nominal rate of interest during the transition will not result in a sufficiently large drop in equilibrium employment. The Frisch labor supply elasticity in this model is $1/\psi$. So, this intuition suggests setting ψ high. It is worth pointing out that doing so does not generate any obvious counterfactual implications, since the labor supply literature typically does find small

values for the labor supply elasticity.

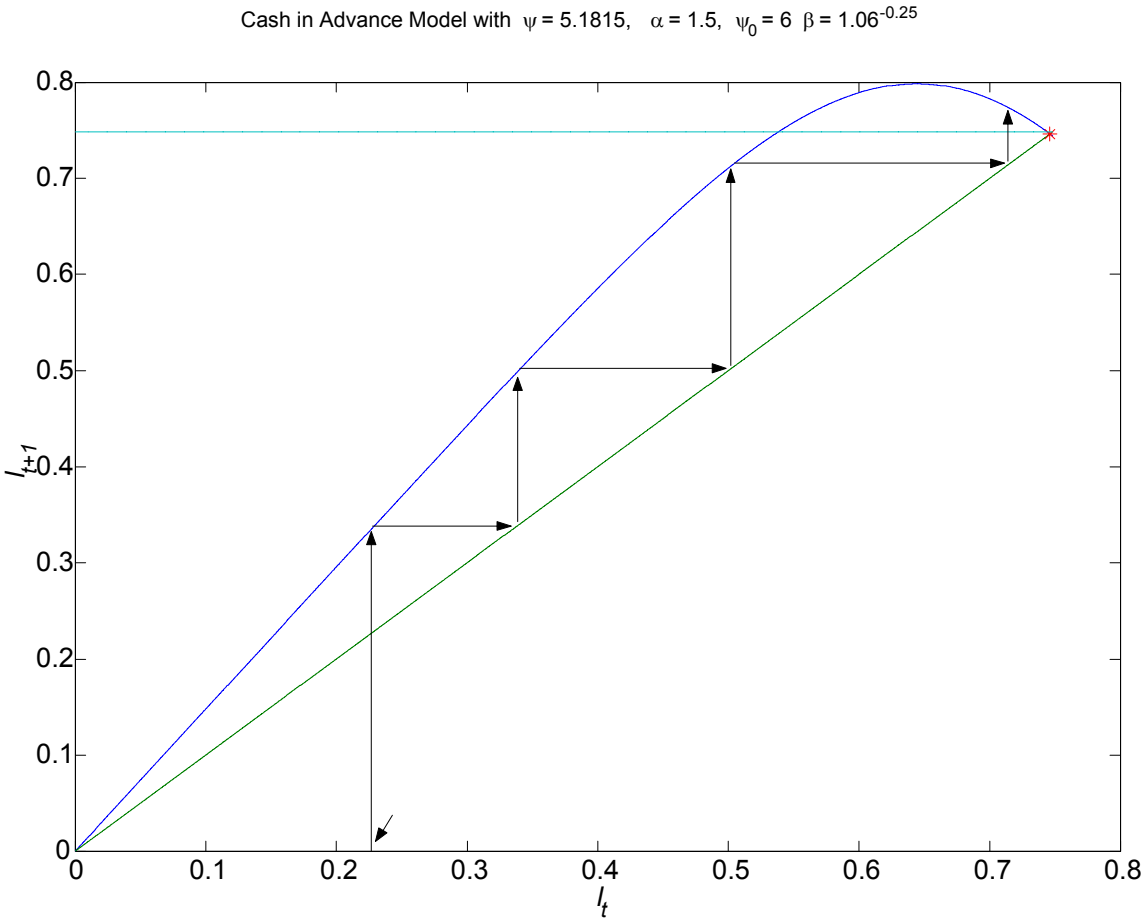

Figure 8

Consistent with this intuition, when we adopt a parameterization of the above utility function which is consistent with a relatively low labor supply elasticity, the target inflation equilibrium is determinate. In particular, we adopted the following parameter values:

$$\psi = 5.1815,\ \psi_0 = 6,\ \beta = 1.06^{-.25},\ g = 0.$$

The graph of f for $0 < l < l^u$ appears in Figure 8, with l_t on the horizontal axis and $l_{t+1} = f(l_t)$ on the vertical. In addition, the 45 degree line is reported, as well as a horizontal line at $l_{t+1} = l^u$. Note how $l = f(l)$ at two points: zero and the point indicated by a '*'. The '*'

corresponds to the inflation targeting steady state. The slope of f at the latter point exceeds -1 in absolute value, and so the inflation targeting equilibrium is determinate. A local analysis would suggest that the equilibrium for this economy is unique, and that no pathologies can occur. A casual inspection of the figure may suggest the same. For example, the arrows in the figure indicate a candidate equilibrium associated with the indicated value of l_0. That sequence approaches the inflation targeting steady state. However, it overshoots and eventually violates the non-negativity constraint on R_t (note where it breaks through the horizontal line).

The following figure (Figure 9) displays a 'close-up' of f around the inflation targeting steady state:

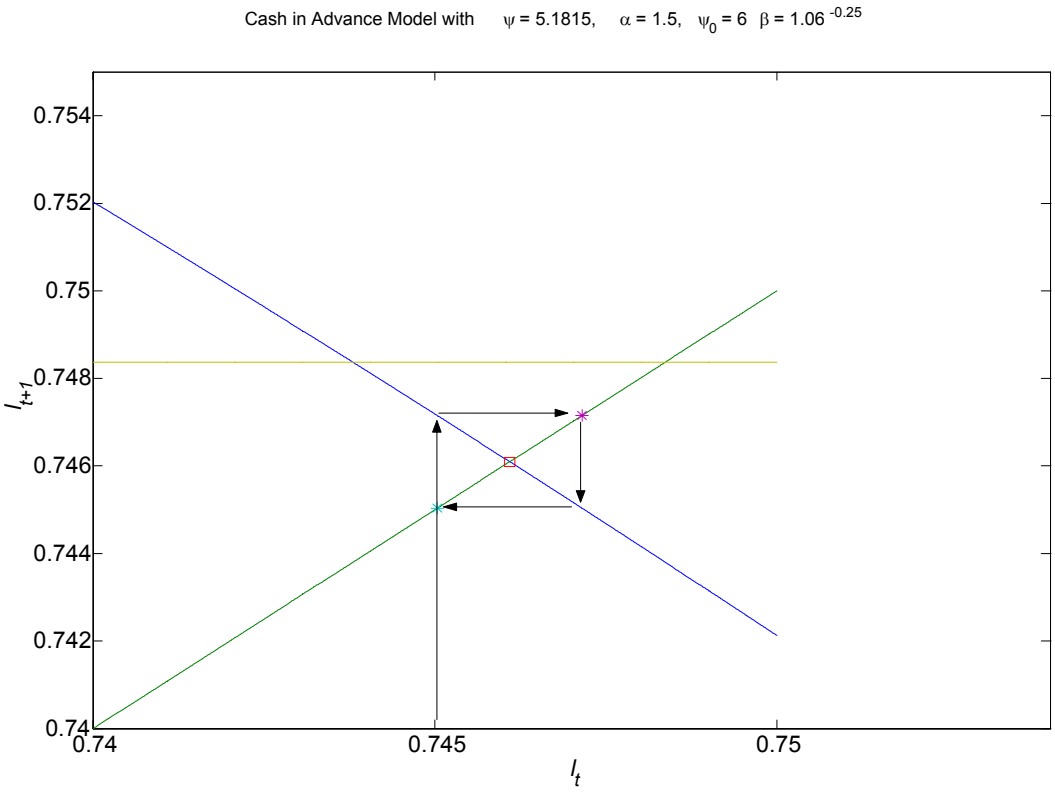

Figure 9

The horizontal line indicates l^u. The inflation-targeting steady state is indicated by a box, at the intersection of f (the downward-sloped line) and the 45 degree line. The two stars indicate the values of l associated with a two-period cycle. Thus, there are multiple equilibria after all.

There is at least the inflation-targeting steady state and the two period cycle.

	Properties of Cycling Equilibrium		
	Even t	Odd t	Inflation Targetting Steady State
$400R_t$	11.2	4.0	7.6
$400(M_{t+1}/M_t - 1)$	5.2	-1.8	1.7
$400(\pi_{t+1} - 1)$	4.1	-0.67	1.7
l_t	0.7450	0.7472	0.7461

The above table displays the properties of this cycling equilibrium. The rate of interest, inflation and money growth fluctuate substantially, by about 7 percentage points. Employment fluctuates less, about 0.15 percent from peak to trough. The relatively small movement in employment reflects the low labor supply elasticity used in the example.

It is interesting to investigate the determinacy properties of the cycling equilibrium. To do so, it is useful to study the function f^2, where

$$f^2(l) = f\left[f(l)\right].$$

Figure 10 graphs $f^2(l) - l$ against l.[20]

The figure indicates that $f^2(l) = l$ for three values of l. The middle one, indicated by the small box, is the value of l associated with the inflation targeting steady state. That is trivially a two-period cycle, one in which the variables in the two periods are identical. The other two intersections correspond to the cycle reported in the table. The left intersection corresponds to the case when l is low in even periods and high in odd, and the other intersection corresponds

[20]We do this, instead of graphing f^2 against l, because f^2 is close to l over this range and scale effects would make it impossible to differentiate f^2 and l.

to the reverse.

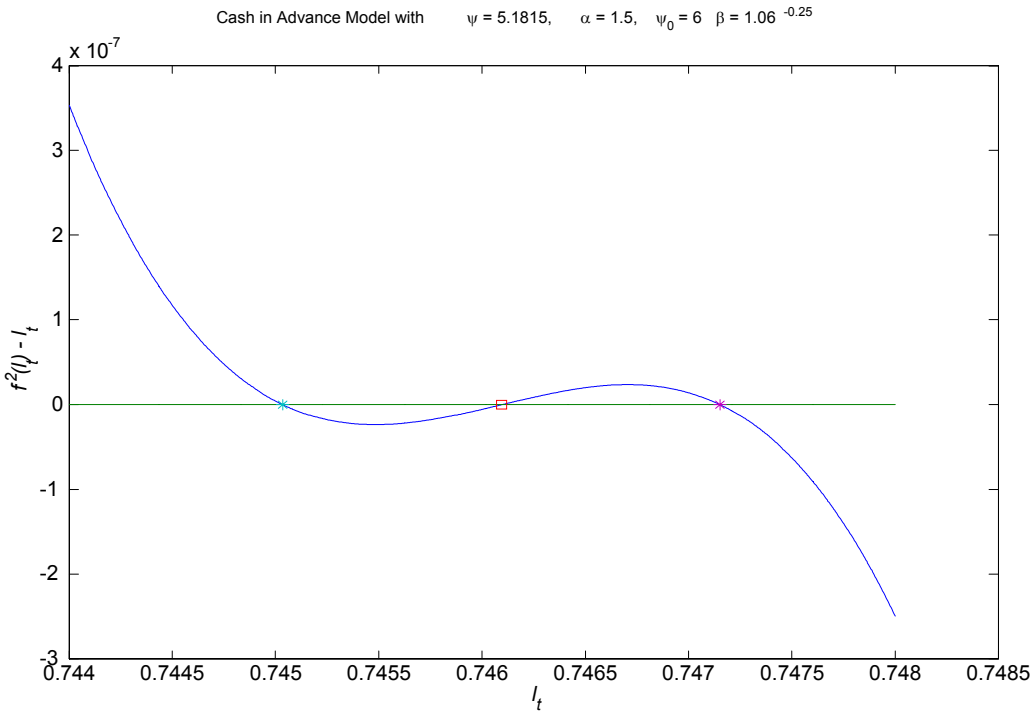

Figure 10

We can infer from Figure 10 that f^2 cuts the 45 degree line from above at the two cycling intersections and from below at the inflation targeting steady state. The latter confirms our previous finding that the inflation cycling equilibrium is determinate. The other two equilibria are indeterminate, however. This cannot easily be deduced from the figure. So, we evaluated the derivative of f^2 at the cycling steady state numerically and found that the derivative is 0.999888, i.e., less than unity. The fact that it is so very close to unity means that equilibria which converge into the cycle, do so very, very slowly.

In summary, this example shows how an aggressive Taylor rule with a determinate inflation targeting steady state may still display pathological equilibria. In this case, we exhibited a cycling equilibrium. That that equilibrium is indeterminate implies, presumably, that it is possible to construct sunspot equilibria about the cycling steady state.

Equilibria Under Monetary Monitoring

We now turn to the equilibria associated when there is money monitoring. We first establish that the set of equilibria under a constant money growth rule is unique.

Combining (4.9) with the resource constraint and $M_{t+1} = \mu M_t$:

$$\frac{l_{t+1}}{m_{t+1}} = \frac{\beta}{\mu}\frac{1}{m_t \psi_0 l_t^{\psi}} \tag{4.14}$$

Combining (4.4) and (4.6),

$$l_t \leq \mu m_t, \; l_t < l^u \rightarrow l_t = \mu m_t, \tag{4.15}$$

where l^u is the highest level of employment consistent with $R_t \geq 0$, and is given in (4.12). It is straightforward to establish the following characterization result:

Proposition 4.3. *A sequence, $\{m_t, l_t; t = 0, 1, 2, ...\}$ corresponds to an equilibrium if, and only if, it satisfies:*

(i) $0 \leq l_t \leq l^u$,

(ii) (4.14) and (4.15)

(iii) a transversality condition.

We now use the proposition to trace out the set of constant money growth equilibria. Consider first

$$l^* = \left(\frac{\beta}{\mu}\frac{1}{\psi_0}\right)^{\frac{1}{1+\psi}}.$$

It is easy to verify that, since $l^* < l^u$, the sequence, $l_t = m_t = l^*$ for all t, satisfies (i)-(iii) of the proposition. This is the monetarist equilibrium for this economy. It is easy to establish that this is the only equilibrium:

Proposition 4.4. *With constant money growth, $\mu > 1$, the monetarist steady state is the globally unique equilibrium, with $m_t = l^*$ for $t = 0, 1, 2, ...$.*

Proof: The proof is essentially identical to that of proposition 3.5, and appears in the appendix.

Appropriate choice of the monitoring range, μ^l, μ^u, has the implication that the Taylor rule with monetary monitoring has as its unique equilibrium the inflation target steady state. The monitoring range must include that steady state in its interior.

5. A Caveat on the Effectiveness of a Constant Money Growth Rule

The examples in the previous two sections indicate how monetary monitoring may help inoculate the economy against some pathologies associated with the Taylor rule. In this section we use the cash-credit model of section 2 to show that this monetary monitoring does not help in all model economies. In particular, we show that there are some model economies in which constant money growth rules have pathologies of their own.[21] It is an open question whether these examples are more plausible than the sort of example emphasized in previous sections, in which constant money growth policies produce a unique equilibrium. For now, we present them as a caveat to the main theme of this paper, namely that monetary monitoring can help improve the operating characteristics of the Taylor rule.

One type of pathology associated with a constant money growth rate, recently emphasized by BSGU, is that there is a low inflation equilibrium in which the rate of interest is zero. As discussed in section 2.4, we adopt what we argue is a very natural assumption about fiscal policy, which rules out such equilibria. Another possibility, first noticed by Brock (1975) and later analyzed by Obstfeld and Rogoff (1983), is that there are equilibria in which the economy undergoes an explosive hyperinflation and ultimately demonetizes. Brock (1975) and Obstfeld and Rogoff (1983) discuss conditions on parameter values under which explosive hyperinflation equilibria may be ruled out.[22] We may perhaps insist on adopting such parameter values on the grounds that high inflation without high money growth is never observed. However, we describe a model environment in which doing so excludes a parameterization that includes reasonable values for the interest elasticity of money demand. So, this possibility cannot be dismissed so easily after all. Finally, we illustrate two other pathologies associated with constant money growth. In one, inflation simply cycles and in the other, inflation responds to sunspot shocks.

5.1. Exploding Hyperinflation under Constant Money Growth

Consider the following specification of preferences:

$$u(c,l) = \frac{c^{1-\gamma}}{1-\gamma} - \frac{\psi_0}{1+\psi} l^{1+\psi},$$

[21] For other examples in which constant money growth results in multiple equilibria, see Benhabib and Bull (1983), Calvo (1979), de Fiore (2000), Farmer (1993), Matsuyama (1991) and Sessa (2001).

[22] Obstfeld and Rogoff (1983) describe a simple price ceiling policy that can also rule out the explosive hyperinflation equilibria.

where

$$c = [(1-\sigma)c_1^{\nu} + \sigma c_2^{\nu}]^{\frac{1}{\nu}}\ ,\ \nu < 1,$$

so that the elasticity of substitution between c_1 and c_2 is $1/(1-\nu)$. The interest elasticity of money demand is obtained from (2.14). Differentiating that expression to obtain $\eta \equiv -d\log(m)/dR$ holding $y = c_1 + c_2$ constant, and imposing $m = c_1$:

$$\eta = \left(\frac{1}{1+R}\right)\left(1 - \frac{m}{y}\right)\frac{1}{1-\nu}.$$

Empirical estimates of the annualized version of this object place it at 7 or higher (see, for example, Lucas (1988)). Note that the two objects in parentheses in the expression for η are each less than unity. Thus, for η to be greater than unity requires $\nu > 0$. Indeed, Chari, Christiano and Kehoe (1991) estimate it to be 0.83 using quarterly data. This money demand elasticity is consistent with the existence of an equilibrium with an exploding hyperinflation.

To see this, substitute (2.14) into (2.12) and use $M_{t+1} = \mu^* M_t$, to obtain:

$$u_{2t} m_t = \frac{\beta}{\mu^*} u_{1,t+1} m_{t+1}. \tag{5.1}$$

It is convenient to express u_{2t}, m_t, u_{1t}, c_{1t} in terms of $w_t = c_{1t}/c_{2t}$. We do so using (2.13), (2.14), (2.17) and (2.18):

$$\begin{aligned}
\frac{1-\sigma}{\sigma}\left(\frac{c_2}{c_1}\right)^{(1-\nu)} &= 1+R \\
\frac{\psi_0 l^{\psi} c_2^{\gamma}}{\left(\frac{c}{c_2}\right)^{1-\nu-\gamma}\sigma} &= 1 \\
c_1 + c_2 + g &= l,
\end{aligned}$$

and the definitions of u_{1t} and u_{2t}. It is immediate that the first of these expressions defines R as a monotone decreasing function of w. Now consider l and c_2. Let

$$A(w) = \frac{\psi_0}{\left(\frac{c}{c_2}\right)^{1-\nu-\gamma}\sigma}.$$

Note that A is well continuous and strictly positive for $w \geq 0$ when $\nu > 0$. Substituting the previous expression and the implication of the resource constraint, $c_2 = (l-g)/(1+w)$, into

the intratemporal Euler equation:

$$A(w)l^{\psi}\left[\frac{l-g}{1+w}\right]^{\gamma}=1. \tag{5.2}$$

Given w, this is one equation in one unknown, l. Note that the left hand side is strictly increasing in l for each w. As a result, for given w, there is a unique value of l, $l(w)$, which satisfies this equation. Note in particular that l is strictly positive and well defined and continuous at $w=0$. We obtain c_2 as follows:

$$c_2(w)=\frac{l(w)-g}{1+w},\ c_1(w)=wc_2(w).$$

The function, $c_2(w)$, is well defined, continuous and strictly positive at $w=0$. It follows that $c_1(w)$ is zero at $w=0$. Rewriting (5.1) in the new notation:

$$a(w_t)=b(w_{t+1}), \tag{5.3}$$

where

$$a(w)=u_2(w)c_2(w)w,\ b(w)=\frac{\beta}{\mu^*}u_1(w)c_2(w)w.$$

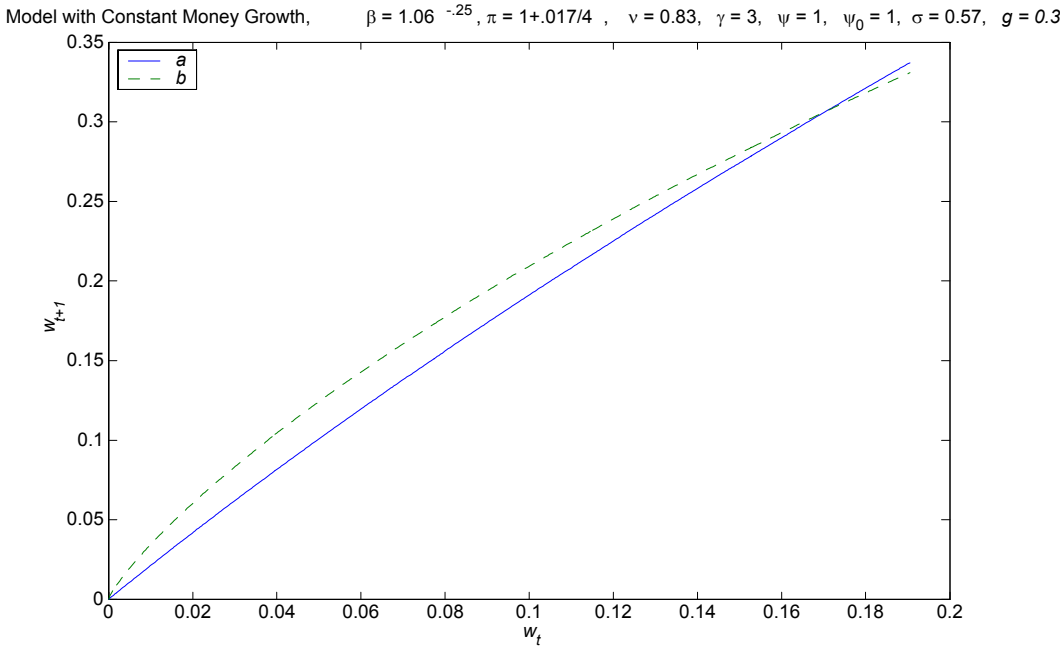

Figure 11

Consider the following parameterization:

$$\beta = 1.06^{-.25},\ \mu = 1 + .017/4,\ \nu = 0.83,\ \sigma = 0.57,\ \gamma = 3,\ \psi = 1,\ \psi_0 = 1,\ g = 0.3.$$

Of these, ν, γ, ψ, σ are taken from the empirical estimates reported in Chari, Christiano and Kehoe (1991). The a and b functions are displayed in Figure 11. To iterate on (5.3), start with a given value for w_t and evaluate $a(w_t)$. Then move horizontally to the b curve. The corresponding point along the horizontal axis is w_{t+1}. Note in the figure how it is that for any w_0 to the left of the intersection of a and b, w_1, w_2, w_3, ... converges to zero. It is easy to verify that such a path satisfies all the equilibrium conditions. These paths correspond to the exploding hyperinflations studied by Brock (1975) and Obstfeld and Rogoff (1983).[23]

[23]This example stands as a counterexample to a conjecture in Carlstrom and Fuerst (2000b). After studying several example model economies they conjecture that the Brock-Obstfeld-Rogoff hyperinflations can occur only if the interest elasticity of money demand elasticity is implausibly large. The difference in conclusions reflects differences in functional form assumption and the fact that Carlstrom and Fuerst consider endowment economies. To see why these things matter, write b in (5.3) as a function of m : $b(m) = u_1(m)c_1(m) = u_2(m)(1 + R(m))m$. (Here, we have used (2.14) and (2.15).) Since u_2 converges to a positive constant with $m \to 0$, the exploding hyperinflation scenario, in which $b \to 0$, can only occur if $R(m)m \to 0$, as $m \to 0$. In an endowment economy, when income is fixed, $R(m)$ can be interpreted as the inverse of the money demand curve. Thus, a high money demand elasticity (i.e., R' small in absolute value) is needed for an explosive hyperinflation to be possible in equilibrium. Carlstrom and Fuerst display a model in which the required money demand elasticity is implausibly high. There are several factors that limit the generality of this result. First, it is the money demand elasticity for real balances near zero that matters. We do not have observations on this because situations in which real balances are nearly zero are rarely observed. Carlstrom and Fuerst rely heavily on functional form assumptions to extrapolate money demand elasticities for m near zero from evidence on money demand elasticities for m in the empirically observed range. It is not clear how much weight we should assign to their functional form assumptions. Second, Carlstrom and Fuerst's analysis is sensitive to their specification that output is exogenous. In a production economy, R' can be small even if the interest elasticity of money demand is small. In this case, R' is to be interpreted as the convolution of the interest elasticity of money demand, the income elasticity of money demand and the general equilibrium response of output to m. Even with a small interest elasticity, if a fall in m is associated with a large fall in income and that in turn shifts money demand left substantially, then R' can still be quite small. Although we can expect both considerations to apply in general, the second one plays at best a minor role in explaining the difference in our results. Christiano and Fitzgerald (2000) show that even in an endowment economy version of the cash-credit good model in the text, Brock-Obstfeld-Rogoff hyperinflations can occur for reasonable money demand elasticities.

5.2. Cycles and Sunspots Under Constant Money Growth

This section illustrates two other types of equilibrium pathologies that can occur in the cash-credit good model with constant money growth, μ^*. We adopt the specification of utility in (3.18) and the resource constraint in (2.18). Applying our functional form assumption to the household's intertemporal Euler equation, (2.12), multiplying both sides of the result by M_t and imposing the constant money growth rule, we obtain the following equilibrium difference equation:

$$c_{1t}^{-\sigma} m_t = \frac{\beta}{\mu^*} \left(1 + R(c_{1,t})\right) c_{1,t+1}^{-\sigma} m_{t+1}. \tag{5.4}$$

Here, $R(c_{1t})$ is the nominal rate of interest expressed as a strictly decreasing function of c_{1t} immediately after (3.21).

Combining the definition of c_1^u in (3.24) with (2.15),

$$\begin{aligned} c_{1t} &< c_1^u \text{ implies } m_t = c_{1t},\ R_t = R(c_{1t}) > 0 \\ c_{1t} &= c_1^u \text{ implies } m_t \geq c_{1t},\ R_t = 0. \end{aligned} \tag{5.5}$$

There is no equilibrium with $c_{1t} > c_1^u$ or $c_{1t} = 0$. This is because the first condition implies a negative nominal rate of interest and the second implies an infinite rate of interest by (3.23). Combining these conditions with (5.4), we can define a mapping from $0 < c_{1t} \leq c_1^u$ to $c_{1,t+1}$:

$$\begin{aligned} c_{1,t+1} &= \min \left\{ c_1^u, \left[\frac{\beta}{\mu^*} \left(1 + R(c_{1,t})\right) \right]^{\frac{1}{\sigma - 1}} c_{1t} \right\}, \text{ for } 0 < c_{1t} < c_1^u, \\ c_{1,t+1} &= \left(\frac{\beta}{\mu^*} \right)^{\frac{1}{\sigma - 1}} c_1^u, \text{ for } c_{1t} = c_1^u. \end{aligned}$$

In the last expression, we have made use of our assumptions, $\sigma > 1$ and $\mu^* > \beta$. The previous two expressions define an equilibrium difference equation, f :

$$c_{1,t+1} = f(c_{1t}),\ f : [0, c_1^u] \to [0, c_1^u]. \tag{5.6}$$

For any sequence, $c_{1,0}$, $c_{1,1}, ...$ that satisfies (5.6) with $0 < c_{1,0} < c_1^u$, a sequence m_t that satisfies (5.4) and (5.5) is obtained using

$$m_{t+1} = \left(\frac{c_{1,t+1}}{c_{1t}} \right)^{\sigma} \frac{\mu^*}{\beta} \frac{m_t}{1 + R(c_{1t})}, \tag{5.7}$$

with $m_0 = c_{1,0}$. We $c_{1,0} = c_1^u$, this procedure is adjusted simply by setting m_0 arbitrarily, subject to $m_0 \geq c_{1,0}$.

We can now state the following characterization result:

Proposition 5.1. *Suppose* $\gamma, \psi \geq 0$, $\sigma, \delta > 1$, $\mu^* > \beta$, $0 < \beta < 1$. *The sequence,* $c_{1,t}$, $t \geq 0$ *corresponds to an equilibrium if, and only if, it satisfies* $0 \leq c_{1t} \leq c_1^u$ *and (5.6).*

Proof: Suppose $\{c_{1t}\}$ satisfy (5.6), so that $0 < c_{1t} \leq c_1^u$. The remaining objects in equilibrium can be computed as follows. Let $R_t = R(c_{1t})$. Solve for m_t using (5.7), with $m_0 = c_{10}$ if $c_{10} < c_1^u$ and $m_0 \geq c_{10}$ otherwise. Solve for $c_{2t} = c_2(c_{1t})$ using the function defined (3.22). Solve for l_t using the resource constraint, (2.18). Inflation can be obtained from the relation,

$$\pi_{t+1} = \mu^* \frac{m_t}{m_{t+1}},$$

for $t = 0, 1, 2, \ldots$. Given the initial stock of money, M_0, the initial price level can be solved using $P_0 = M_0/m_0$. A sequence of money stocks, $\{M_t\}$, is obtained from $M_{t+1} = \mu^* M_t$, $t \geq 0$. Finally, set $W_t = P_t$ for $t \geq 0$. It is easy to verify that all the equilibrium conditions, (2.12)-(2.18), are satisfied.

Now suppose we have an equilibrium, so that (2.12)-(2.18), are satisfied. The argument preceding the statement of the proposition establishes that the sequence, $\{c_{1t}\}$, satisfies $0 < c_{1t} \leq c_1^u$ and (5.6).

We exploit this proposition to understand the nature of the equilibria in the model. We first consider the following parameter values:

$$\alpha = 1.5, \beta = 1.06^{-.25}, \sigma = \delta = 3, \ \psi_0 = \psi = 1, \gamma = 3, \tilde{\pi} = 1 + .017/4. \tag{5.8}$$

The corresponding equilibrium difference equation is displayed in Figure 12. The derivative of f evaluated at the monetarist steady state is -0.67, so that that equilibrium is indeterminate. Thus, monetary monitoring is not helpful in this economy. The commitment to monitor will

not 'unravel' the sort of bad equilibria that occur under the pure Taylor rule.

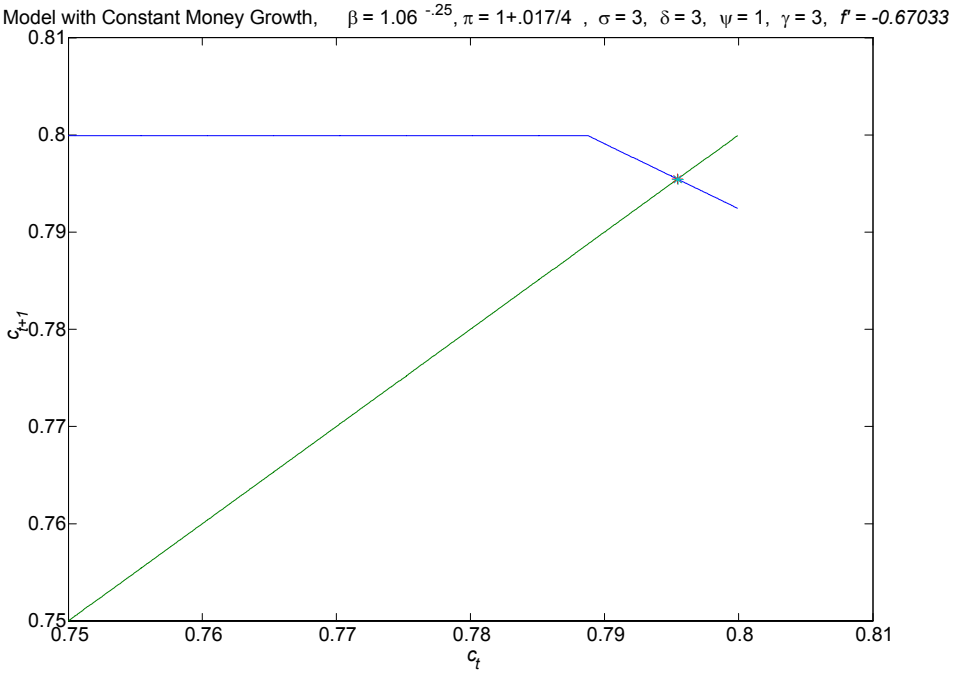

Figure 12: Indeterminacy of Steady State Under Constant Money Growth

Next, we consider a version of the model with slightly smaller σ, δ. Now, $f' = -1.27$ in the monetarist steady state, so that that equilibrium is determinate. However, there is a two-period cycle, as indicated by the arrows in Figure 13. It is evident that that equilibrium is indeterminate. Let the two values of consumption in the cycle be $c_1^a < c_1^b$, so that $c_1^a = f(c_1^b)$ and $c_1^b = f(c_1^a)$. Then, it is evident that for $c_{1,0}$ sufficiently close to c_1^a, $c_{1,1} = c_1^b$ and the continuation sequence, $c_{1,t}$, $t > 1$ follows the two period cycle exactly. The same is true for $c_{1,0} < c_1^b$ sufficiently close to c_1^b. Thus, it is possible to construct a sunspot equilibrium in the neighborhood of this cycle. When we repeated these calculations for $\psi = .5$ and 2, we obtained the same results as in Figure 13. This also happened when we went back to the parameter values in (5.8), but replaced γ with unity. We also obtained the result in Figure 13 when we returned to the parameter values in (5.8), replacing ψ with 10.

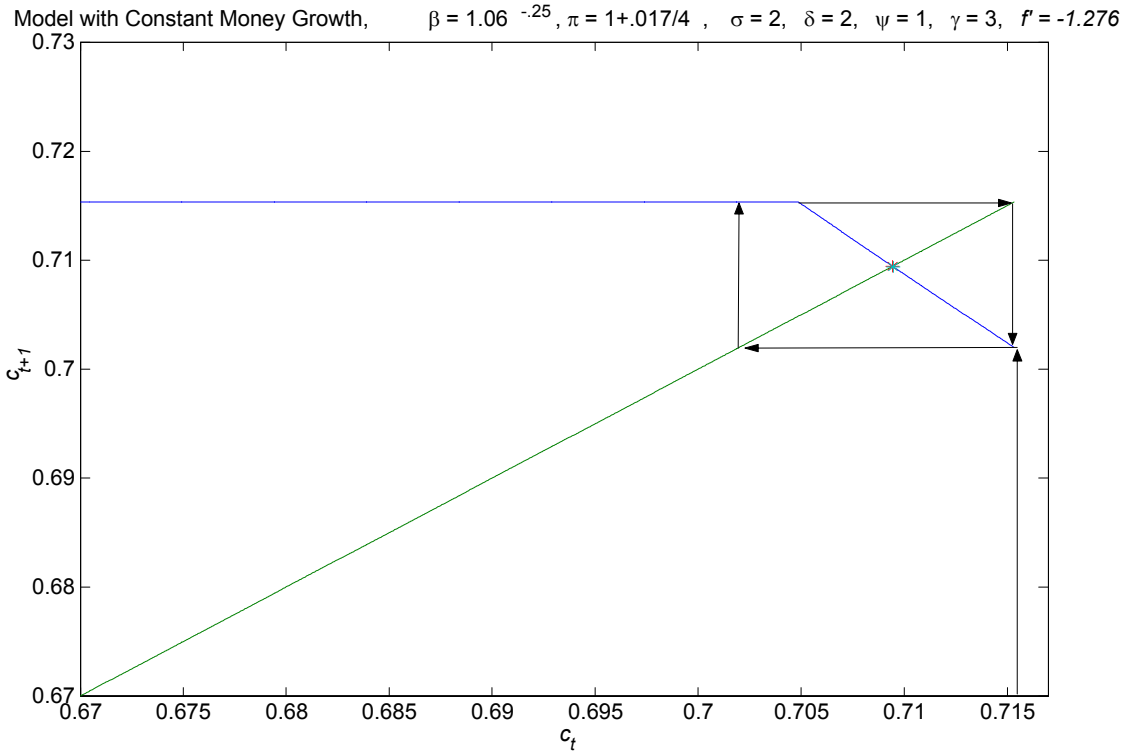

Figure 13: Two-period Cycle Under Constant Money Growth

These examples appear to contradict the impression given in section 3 that monetary monitoring can help undo the pathologies associated with the Taylor rule. The big question is which example is more 'typical'. One way to evaluate this example is to consider the money demand elasticity. As before, we obtain this by totally differentiating (2.14) with respect to m and R, holding $y = c_1 + c_2 + g$ constant, to obtain:

$$\eta \equiv -\frac{d\log m}{dR} = \frac{1-\frac{m}{y-g}}{1+R}\frac{1}{\delta\frac{m}{y-g}+\sigma\left(1-\frac{m}{y-g}\right)}.$$

Note that the first ratio is definitely less than unity. It is easy to verify that the second ratio is less than unity too, for $\delta, \sigma > 1$. So, the money demand elasticity in this example is low. Perhaps it can be dismissed on the grounds that the elasticity is too small.

6. Concluding Remarks

Conventional wisdom holds that raising short term interest rates when expected inflation is above target and lowering them otherwise is a good policy for stabilizing inflation about target. However, researchers studying the operating characteristics of this Taylor rule policy in models are finding that, ironically, it may actually be a source of economic instability. This paper has explored one particular adaptation to the Taylor rule that may eliminate or at least reduce the magnitude of the problem. Under this adaptation, the central bank specifies a target range of money growth rates. It pursues the Taylor rule as long as money growth is inside the target range, and commits to bringing the money growth rate into the interior of this range if it should wander outside. We find, in several models, that the Taylor rule with monetary monitoring performs well. This suggests that monetary monitoring may be a good practice from the point of view of robustness. If the world is of the type where the Taylor rule has no pathologies associated with it, then our modification is benign and has no consequence. If, on the other hand, the world better resembles some of the model economies studied in this paper, then monetary monitoring can improve the operating characteristics of the economy.

We now briefly discuss some of the limitations of our analysis. First, our analysis depends on a constant money growth regime not having pathologies of its own. But, as stressed in section 5, one can construct economies in which constant money growth results in exploding hyperinflation, deflation, cycling inflation, random inflation or even chaotic inflation. In these cases, our monetary monitoring policy may be counterproductive and exacerbate instability. Clearly, our monetary monitoring policy is compelling to the extent that these examples are mathematical oddities, and therefore not of concern to practical policy makers. Although it is our impression that this is indeed the case, to our knowledge a definitive argument for this has not yet been made.

Second, we have worked with a framework in which the rationale for adopting a pure Taylor rule, as opposed to a pure money growth rule, is unclear. Indeed, the examples that illustrate our case for monetary monitoring all have the implication that a constant money growth rule dominates the Taylor rule. We actually have in mind a model environment with velocity and other shocks in which a Taylor rule is capable of producing results that are superior to a constant money growth rule. We have not constructed such an environment in an effort to keep the analysis as simple and transparent as possible. However, such a construction is an important next step. This analysis would have to confront a trade-off. The monetary targeting range would have to be wide enough to allow the Taylor rule to do its work accommodating money demand and other shocks. At the same time, it would have to be narrow enough to ensure

that money monitoring could do its work protecting the economy from possible pathologies associated with the Taylor rule. It remains an important outstanding question how well our monitoring strategy would fare when confronted with this trade-off.

Third, we have excluded investment from our analysis. Results in the literature suggest that this would alter the details of the analysis. For example, it is known that whether investment is a cash or credit good can in some models determine whether the steady state is indeterminate or determinate (see Carlstrom and Fuerst (2000a).) We abstract from investment in order to ensure that the equilibria of the model can be characterized as the solution to a first order difference equation. This is what allowed us to study the global set of equilibria in our models. Studying versions of the model with investment is an important next step. However, we conjecture that the basic message of this paper - that monetary monitoring may improve monetary policy - would survive. Fourth, the version of the Taylor rule we have studied only relates the interest rate to inflation and does not include the output gap. In doing this, we follow several theoretical studies. Still, it is important to explore what happens when the central bank also looks at measures of the output gap when setting policy.

Fifth, we have emphasized that the monitoring range should be specified in terms of money growth rates and not some other variable. But, the logic of the model does not specifically require this. Any variable that is a monotone function of money growth - consumption, employment, inflation - would serve just as well in constructing the monitoring range. A compelling case remains to be made that the monitoring range should be specified specifically in terms of money and not one of these other variables. We believe such a case can be made, but this is a task beyond the scope of this paper.

Sixth, in analyzing Taylor rules we have abstracted from one important practical problem. To implement the rule, it is necessary to know the steady state real rate of interest, $1/\beta$. This is what allows the monetary authorities to translate a target rate of inflation into a target nominal rate of interest. In practice, the steady state real rate of interest is not known. The Taylor rule may produce bad outcomes if the policymaker misspecifies its value. We suspect that monetary monitoring can help insulate the economy from the consequences of this type of error. However, we have not explored this question here.[24]

[24]For example, suppose the policymaker believes the discount rate is $\bar{\beta}$, whereas in fact it is β. The nominal interest rate target appearing in the Taylor rule is $\bar{R} = \tilde{\pi}/\bar{\beta}$. Then, (3.13) becomes:

$$c_{1,t+1} = \frac{\beta}{\tilde{\pi} + \frac{1}{\alpha}\left(\frac{1}{c_{1t}} - 1 - \bar{R}\right)}.$$

Setting $c_{1,t+1} = c_{1t} = c_1$ and solving this expression for the steady state level of consumption,

A. Appendix

The following property of household budget sets is exploited in section 2.1:

Proposition A.1. *Suppose (2.1) and (2.3) are satisfied. Then, (2.9) holds if, and only if (2.11) holds.*

Proof We first establish that (2.11) implies (2.9). Recursively solving for assets using (2.6) from t to T yields:

$$q_T A_T \leq \sum_{j=0}^{T-t-1} q_{t+j+1} I_{t+j} + q_t A_t - \sum_{j=0}^{T-t-1} q_{t+j+1} S_{t+j}.$$

Taking into account $q_{t+j+1} S_{t+j} \geq 0$ and rewriting this expression, we obtain

$$q_t A_t \geq q_T A_T - \sum_{j=0}^{T-t-1} q_{t+j+1} I_{t+j}.$$

Fixing t, taking the limit, $T \to \infty$, and using (2.11) yields (2.9).

We now show that (2.9) implies (2.11). Note first that the limit in (2.7) being finite implies

$$\lim_{t\to\infty} \sum_{j=1}^{\infty} q_{t+j+1} I_{t+j} = 0.$$

Using this and (2.9), (2.11) follows trivially.■

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c_1 :

$$c_1 = \frac{\tilde{\pi}}{\beta}\left[\frac{1-\frac{1}{\alpha\beta}}{1-\frac{1}{\alpha\bar{\beta}}}\right].$$

The annualized interest rate in this steady state is $400(1/c_1 - 1)$ and the corresponding inflation rate is $400[\tilde{\pi} + \frac{1}{\alpha}\left(\frac{1}{c_1} - 1 - \bar{R}\right) - 1]$. We set $\beta = 1.06^{-.25}$ and $\bar{\beta} = 1.03^{-.25}$. Thus, the annualized real rate of interest is understated by three percentage points. Also, $\tilde{\pi} = 1 + .017/4$, so that the target inflation rate is 1.7 percent at an annual rate. With this error in policymaker beliefs, the actual steady state inflation and money growth rates are 7.7 percent at an annual rate. The 3 percentage point understatement in the real rate leads to a 6 percentage point overshooting in these variables, in stationary equilibrium. For additional analysis of monetary policy when the central bank is uncertain of model parameters, see Rudebusch (2001).

The following property of the government budget constraint is exploited:

Proposition A.2. *Given (2.19), (2.20) holds if, and only if, (2.21) holds.*

Proof: Consider the case, $t = 0$. Apply repeated substitution to (2.19), to obtain

$$\begin{aligned} B_t &= (1+R_{t-1})\cdots(1+R_0)B_0 - P_{t-1}s_{t-1} - (1+R_{t-1})P_{t-2}s_{t-2} \\ &\quad -... - (1+R_{t-1})\cdots(1+R_1)P_0 s_0, \end{aligned}$$

$t = 2, 3, ...$. Multiply both sides by q_T :

$$q_T B_T = B_0 - \sum_{j=0}^{T-1} q_{j+1} P_j s_j. \tag{A.1}$$

Suppose (2.21) holds. Drive $T \to \infty$ in (A.1) and impose (2.21) to obtain (2.20). Now suppose (2.20) holds. Drive $T \to \infty$ in (A.1) and (2.21) follows. The proposition is established for the cases, $t = 1, 2, 3, ...$, in a similar way.

A.1. Proof of Proposition 3.1

We study the properties of the function, f, in (3.7). Its derivative is:

$$\begin{aligned} f'(c_1) &= \frac{\beta}{\alpha\tilde{\pi}^2}\left(\frac{\frac{1}{c_1}-1}{\tilde{R}} c_1^{\frac{2}{\frac{\tilde{R}}{\alpha\tilde{\pi}}+1}}\right)^{-\frac{\tilde{R}}{\alpha\tilde{\pi}}-1} \\ &= \frac{\beta}{\alpha\tilde{\pi}^2}\left(\frac{c_1^{\frac{\alpha\tilde{\pi}-\tilde{R}}{\tilde{R}+\alpha\tilde{\pi}}} - c_1^{\frac{2\alpha\tilde{\pi}}{\tilde{R}+\alpha\tilde{\pi}}}}{\tilde{R}}\right)^{-\frac{\tilde{R}}{\alpha\tilde{\pi}}-1}. \end{aligned}$$

Our assumption on the value of α guarantees that the power on c_1 is positive in both cases in the above expression. Note that as c_1 goes to zero, the expression in parentheses in f' also goes to zero. Since the power on that expression is negative, it follows that $f'(c_1) \to \infty$ as $c_1 \to 0$. Since (3.7) indicates $f \to 0$ as $c_1 \to 0$, it follows that $f(c_1) > c_1$ for c_1 sufficiently close to zero. In addition, (3.7) indicates that $f \to \infty$ as $c_1 \to 1$, so that $f(c_1) > c_1$ for c_1 close to unity. Since $f(c_1) = c_1$ it follows that, with one exception, there is at least one other interior (i.e., $0 < c_1 < 1$) value of c_1 such that and $f(c_1) = c_1$ (see Figure 1). The exceptional case occurs when $f' = 1$, when $\alpha = 1/\beta$.

Proceeding with the case, $\alpha \neq 1/\beta$, by examining the properties of f'' we can establish that there are no more than two interior values of c_1 where $f(c_1) = c_1$. Differentiating f', we obtain:

$$f''(c_1) = -\frac{\beta\left(\frac{\tilde{R}}{\alpha\tilde{\pi}}+1\right)}{\alpha\tilde{\pi}^2\left(\tilde{R}+\alpha\tilde{\pi}\right)\tilde{R}}\left(\frac{c_1^{\frac{\alpha\tilde{\pi}-\tilde{R}}{\tilde{R}+\alpha\tilde{\pi}}}-c_1^{\frac{2\alpha\tilde{\pi}}{\tilde{R}+\alpha\tilde{\pi}}}}{\tilde{R}}\right)^{-\frac{\tilde{R}}{\alpha\tilde{\pi}}-2}\left[g_1(c_1)-g_2(c_1)\right]. \tag{A.2}$$

where

$$\begin{aligned} g_1(c_1) &= \left(\alpha\tilde{\pi}-\tilde{R}\right)c_1^{\frac{-2\tilde{R}}{\tilde{R}+\alpha\tilde{\pi}}} \\ g_2(c_1) &= 2\alpha\tilde{\pi}c_1^{\frac{\alpha\tilde{\pi}-\tilde{R}}{\tilde{R}+\alpha\tilde{\pi}}}. \end{aligned}$$

The function g_1 is strictly decreasing in c_1, with $g_1 \to \infty$ as $c_1 \to 0$, and $g_1 \to \alpha\tilde{\pi}-\tilde{R}$ as $c_1 \to 1$. Also, g_2 is strictly increasing in c_1, with $g_2 \to 0$ as $c_1 \to 0$ and $g_2 \to 2\alpha\tilde{\pi}$ as $c_1 \to 1$. It follows that $g_1 > g_2$ for c_1 sufficiently small and $g_1 < g_2$ for $c_1 = 1$. Hence, there is at least one c_1, say $\hat{c}_1$ where $g_1(\hat{c}_1) = g_2(\hat{c}_1)$. The monotonicity of g_1 and g_2 guarantees that $\hat{c}_1$ is unique. Since the sign of f'' is controlled by the sign of the term in square brackets in (A.2), we conclude that $f'' < 0$ for $c_1 < \hat{c}_1$ and $f'' > 0$ for $c_1 > \hat{c}_1$. That is, f is concave for, $c_1 < \hat{c}_1$, and convex for $c_1 > \hat{c}_1$. Since f only switches between concavity and convexity once, it can cross the 45 degree line at most twice. This establishes that, for $\alpha \neq 1/\beta$, there are exactly two interior steady states in the model. This establishes the proposition.

A.2. Proof of Proposition 3.2

Suppose we have an equilibrium. Then, it is easy to verify that (3.7) and $0 \leq c_{1t} \leq 1$ are satisfied. Now suppose we have a sequence, c_{1t}, $t = 0, 1, 2, ...$, which satisfy these two conditions. We can compute sequences, R_t, l_t and π_{t+1}, $t = 0, 1, 2, ...$ in a straightforward way from (3.1), (3.3) and (3.5). The choice of $\pi_0 > 0$ is arbitrary. It remains to compute M_t and B_t for $t \geq 0$, and to verify that (3.6) and (3.8) are satisfied.

Using the results in Proposition 3.1 (see Figure 1), we can deduce that every sequence, c_{1t}, $t = 0, 1, ...$, which satisfies (3.7) and $0 \leq c_{1t} \leq 1$ has the property, $c_{1t} < 1$ for all t. Suppose not, so that $c_{1\tilde{t}} = 1$ for some $\tilde{t}$. Then (see Figure 1), the fact that (3.7) is satisfied implies $c_{1\tilde{t}+1} > 1$. This contradicts the supposition, $0 \leq c_{1t} \leq 1$. So, it must be that $c_{1t} < 1$ for all t. By (3.3) it follows that $R_t > 0$, so that (3.6) implies $M_t = P_t c_{1t}$ for all t. We how have a sequence, M_t. Finally, (3.8) is satisfied because $\beta < 1$. Given the variables just computed, a sequence of B_t's

that satisfy the government's flow budget constraint can be computed. A sequence, τ_t, can be found to assure that the solvency condition on B_t is satisfied.

A.3. Proof of Proposition 3.3

Part (i) of the proposition is obvious from the previous two propositions. Now consider part (ii). The determinacy of the target inflation steady state is obvious from (i) in Proposition 3.1. Because f cuts the 45 degree line from below at the target inflation steady state, it follows that the other steady state occurs for a lower value of c_1. This establishes that the rate of inflation in that steady state is higher than $\tilde{\pi}$. Since $f' > 0$ and f must cross the 45 degree line from above at the high inflation steady state, if follows that the slope of f is positive and less than unity there. This establishes that this steady state is indeterminate, so that (ii) has been established.

Now consider part (iii). The indeterminacy of the target inflation steady state follows trivially from (i) in Proposition 3.1. Because f cuts the 45 degree from above at this steady state, it must be that the other steady state occurs for $c_1 > \tilde{c}_1$ and that it cuts the 45 degree line from below. It follows that that steady state must be determinate. This establishes part (iii) and, hence, the proposition.

A.4. Proof of Proposition 3.5

We have already shown that $m_0 = \beta/\mu^*$ is associated with an equilibrium. It remains to show that there are no other equilibria. Consider a candidate equilibrium with $m_0 = 0$. Since $0 \leq c_{1,0} \leq m_0$, it must be that $c_{1,0} = 0$. There is no finite value of R_0 that solves (3.3), so this cannot be an equilibrium. Now consider a candidate equilibrium with $0 < m_0 < \beta/\mu^*$. Suppose $m_1 < 1$. By (ii), $c_{1,1} = m_1$. Equation (3.10) then implies $m_0 = \beta/\mu^*$. Contradiction. Now suppose $m_1 \geq 1$ and solve (3.10) to obtain:

$$c_{1,1} = \frac{\beta}{\mu^*}\frac{m_1}{m_0}, \tag{A.3}$$

so that $c_{1,1} > 1$. This cannot be an equilibrium since (iii) of the proposition is violated (there is no $R_1 \geq 0$ that solves (3.3) when $c_{1,1} > 1$.) We conclude that there are no equilibria with $0 \leq m_0 < \beta/\mu^*$.

Now consider a candidate equilibrium with $\beta/\mu^* < m_0$. Suppose that $m_1 < 1$. Then, $c_{1,1} = m_1$ and, by (3.10), $m_0 = \beta/\mu^*$. Contradiction. Now suppose $1 \leq m_1 < (\mu^*/\beta)m_0$. Solve (A.3) to obtain $c_{1,1} < 1$. By (ii), $m_1 = c_{1,1}$, contradicting $m_1 \geq 1$. Suppose $m_1 > (\mu^*/\beta)m_0$. By (A.3), $c_{1,1} > 1$, contradicting condition (ii) of the proposition.

Finally, suppose $m_1 = (\mu^*/\beta)m_0 > 1$. In this case, $c_{1,1} = 1$ and $c_{1,1}$, m_1 are consistent with the conditions of the proposition. Applying the above reasoning to periods $t = 2, 3, ...$, we find that $m_{t+1} = (\mu^*/\beta)m_t$, $c_{1,t} = 1$, $t = 1, 2, ...$ This is a sequence of m_t's and $c_{1,t}$'s that satisfy (i)-(iii) of the proposition. This is the deflation equilibrium scenario discussed in section 2.4 above. However, it is easily verified that part (iv) of the proposition is not satisfied by this sequence. So neither the deflation scenario, nor paths that converge into it, constitute valid equilibria for the model.

A.5. Proof of Proposition 4.4

That $m_t = l_t = l^*$ corresponds to an equilibrium was discussed in the text. We establish uniqueness by contradiction. Thus, suppose there is an equilibrium with $0 \leq l_0 \leq l^u$, and $l_0 \neq l^u$. Suppose consider $l_0 = 0$. This cannot be part of an equilibrium since there is no finite rate of interest that solves (4.13).

Consider $0 < l_0 < l^*$. Since $l_0 < l^u$, it follows from (4.15) that $m_0 = l_0$. Suppose $l_1 < l^u$. Then $m_1 = l_1$ and (4.14) implies $l_0 = l^*$. This is a contradiction. So, it must be that $l_1 = l^u$ and $m_1 \geq l^u$. Solving (4.14) for m_1, we obtain:

$$m_1 = l^* \psi_0 l_0^{1+\psi} \frac{\mu}{\beta} < l^*,$$

since $\psi_0 l_0^{1+\psi} \mu/\beta < 1$. This contradicts $m_1 \geq l^u$.

Consider $l^* < l_0 < l^u$, so that $m_0 = l_0$. Suppose $l_1 < l^u$. Then $m_1 = l_1$ and (4.14) implies $l_0 = l^*$. This is a contradiction. So, it must be that $l_1 = l^u$ and $m_1 \geq l^u$. Solving (4.14), we obtain:

$$m_1 = l^* \psi_0 l_0^{1+\psi} \frac{\mu}{\beta} > l^*.$$

It must be that $m_1 \geq l^u$, for otherwise (4.15) would be violated. So, suppose $m_1 \geq l^u$. Consider l_2. It cannot be that $l_2 < l^u$, since by (4.15) that would imply $m_2 = l_2$ and then (4.14) would imply that $l_1 = l^* < l^u$. So, it must be that $l_2 = l^u$ and $m_2 \geq l^u$. By the same reasoning, $l_t = l^u$ for all t. It then follows from (4.14) that

$$m_{t+1} = \frac{\mu}{\beta} m_t \psi_0 \left(l^u\right)^{1+\psi} = \frac{\mu}{\beta} m_t,$$

for $t = 1, 2, 3, ...$. Thus,

$$\beta^t m_t = \mu^t m_0 \to \infty \text{ as } t \to \infty,$$

violating (iii).

Consider $l_0 = l^u$. This results in a contradiction using the same argument just applied. This establishes the result sought.

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