(Un)Conventional Monetary Policy and the Yield Curve

Preliminary Draft

Marcel Priebsch*

August 28, 2014

Abstract

This paper estimates a shadow-rate term structure model with unspanned macro variables based on a sample of U.S. Treasury yields, unemployment, and inflation from 1990 to 2013. According to the model, the unconditional probability that the zero lower bound (ZLB) on nominal short-term interest rates is binding at any given point in time is over ten percent. The model further implies a predictive relationship between the *shadow* short rate, unemployment, and inflation that closely resembles the Taylor (1993) rule. Similar (but nonlinear) reduced-form relationships with the macro variables can be derived for the observed short rate, long-term yields, and expectations of future short rates. Throughout the recent ZLB period in the U.S., the observed short rate, and short rate expectations one year ahead, were close to what the model would predict based on contemporaneous macro variables. On the other hand, the ten-year yield was unusually low, especially in 2011 and 2012. The model can also be used to derive a time-varying "neutral" short rate based on yield curve and macro information. The model-implied neutral rate matches the broad patterns of the natural policy rate measure proposed by Laubach and Williams (2003), but displays more time variation. Consistent with the FOMC trading

*Board of Governors of the Federal Reserve System, Washington D.C., marcel.a.priebsch@frb.gov. The analysis and conclusions set forth in this paper are those of the author and do not indicate concurrence by other members of the research staff or the Board of Governors of the Federal Reserve System.

off unemployment and inflation in its decision to first raise short-term rates, the model implies a positive correlation between inflation and unemployment at the time of short rate liftoff from the ZLB. Conditional on remaining at the ZLB, the model projects slow convergence of the economy to a state of secular stagnation reminiscent of the Japanese experience in the 1990s. Finally, the model implies that about half of the interest rate uncertainty over long forecast horizons can be attributed to uncertainty about the future state of the macroeconomy. On the other hand, interest rate uncertainty over short forecast horizons seems to be mostly unrelated to macro uncertainty.

1 Introduction

The monetary policy rule proposed by Taylor (1993), and the many variants of it that have been studied in the literature, posit a linear relationship between the short-term interest rate and macroeconomic variables (usually the output gap—sometimes translated into the unemployment gap using Okun's law—and inflation). Empirically, estimated versions of these simple rules seem to fit observed policy rates well, both in the U.S. and internationally, at least after allowing for structural breaks corresponding to shifts in monetary policy regimes (Judd and Rudebusch, 1998; Clarida et al., 1998; Huston and Spencer, 2005). However, if the short rate in a standard arbitrage-free term structure model is specified in terms of such a monetary policy rule, the model will imply a deterministic relationship between long-term yields and macro variables, such that the macro variables are spanned by the cross section of yields (as in the model of Smith and Taylor, 2009). The model-implied spanning of macro variables by yields remains true even when the model accommodates monetary policy shocks that are unrelated to the macro variables (Ang and Piazzesi, 2003; Ang et al., 2007; Bikbov and Chernov, 2010). In any such model, macro variables do not contain any information that is not already embedded in the yield curve. However, it is now well documented that macro variables help forecast future yields—and are thus relevant for the determination of term premiums and expected excess bond returns—, even in the presence of rich information on the contemporaneous cross section of yields (Cooper and Priestley, 2009; Ludvigson and Ng, 2009; Joslin et al., 2014). This empirical evidence refutes the theoretical implication of macro spanning. In addition, linear monetary policy rules for the short-term interest rate may imply a negative nominal policy rate under certain economic conditions.[1] In practice, the nominal policy rate is generally assumed to be constrained by a lower bound at or around zero (see, for example, Bernanke et al., 2004).

In this paper, I show that both concerns can be addressed by a shadow-rate term structure model with unspanned macro variables. By representing the shadow short rate as a function of latent yield factors, and allowing these factors to interact flexibly with macro variables under the data-generating distribution, a reduced-form linear relationship between the expected shadow rate and macro variables can be derived as an *implication* of the model, even as macro variables are unspanned by the yield factors. The model furthermore implies a (necessarily nonlinear) relationship between the *observed* short-term rate and macro variables that obeys the zero lower bound (ZLB). Dynamic versions of these relationships can be derived by conditioning on current yield factors and macro variables. These findings furthermore extend to longer-term yields. In addition, the model can be used to quantify the contribution of uncertainty about future macro variables to uncertainty *around* these interest rate expectations.

2 Model

The model proposed in this section combines the properties of the model from Joslin et al. (2014)—in which macro variables are unspanned by yields—with those of the model from Priebsch (2013)—in which nominal yields have a lower bound at $r_{\min}$. In particular, fix a complete probability space $(\Omega, \mathcal{F}, \mathbb{P})$, a continuous-time filtration $\{\mathcal{F}_t\}_{t\geq 0}$, and a probability measure $\mathbb{Q}$ on $(\Omega, \mathcal{F})$ that is equivalent to $\mathbb{P}$. Suppose the nominal short rate $\underline{r}_t$—the instantaneous risk-free rate of interest—is the greater of a "shadow rate" r_t and a lower bound $r_{\min}$; $\underline{r}_t = \max\{r_t, r_{\min}\}$. Assume further that the shadow rate can be represented as the linear combination of N_X latent yield factors, X_t,

$$r_t \equiv \rho_0 + \rho_1 \cdot X_t, \tag{1}$$

[1] The original Taylor (1993) rule as stated in (6) below, with the macro variables as defined in Section 3, does prescribe a negative policy rate during parts of the recent crisis period. For example, it implies a policy rate of −2.9 percent for September 2009. The actual target range for the federal funds rate at the time was 0 to 25 basis points.

and that the vector X_t follows a stationary multivariate Ornstein-Uhlenbeck process under $\mathbb{Q}$,

$$dX_t = K^{\mathbb{Q}}(X_t - \mu^{\mathbb{Q}})dt + \Sigma_X dW_t^{\mathbb{Q}},$$

where $W_t^{\mathbb{Q}}$ is N_X-dimensional standard Brownian motion under $\mathbb{Q}$. Suppose $\mathbb{Q}$ is a pricing measure, such that the arbitrage-free time t price of a zero-coupon bond maturing at time T is given by

$$\underline{P}_t^T = \mathbf{E}_t^{\mathbb{Q}}\left[\exp\left(-\int_t^T \max\{r_s, r_{\min}\}\, ds\right)\right]$$

with associated zero-coupon bond yield

$$\underline{y}_t^T = -\frac{\log \underline{P}_t^T}{T-t} = H_y^{T-t}(X_t).$$

The cross section of yields at time t can thus be represented in terms of a small number of factors, consistent with the findings of Litterman and Scheinkman (1991), Dai and Singleton (2000), and many others. The nonlinear mapping H_y between yield factors and yields can be accurately approximated using the second-order method developed in Priebsch (2013).

Next, suppose N_M observable variables M_t contain information on the state of the macroeconomy. Let $Z_t = (X_t; M_t)$, and suppose Z_t follows a stationary multivariate Ornstein-Uhlenbeck process under $\mathbb{P}$,

$$dZ_t = K^{\mathbb{P}}(Z_t - \mu^{\mathbb{P}})dt + \Sigma_Z dW_t^{\mathbb{P}}, \tag{2}$$

where $W_t^{\mathbb{P}}$ is $(N_X + N_M)$-dimensional standard Brownian motion under $\mathbb{P}$. By Girsanov's theorem, the upper-left block of Σ_Z coincides with Σ_X (Karatzas and Shreve, 1991). In general, specification (2) allows for flexible interaction between the yield factors X_t and macro variables M_t under $\mathbb{P}$, through $K^{\mathbb{P}}$ and Σ_Z, including the limiting cases where X_t and M_t are (conditionally and/or unconditionally) independent, and where M_t is a deterministic function of X_t (macro variables are spanned by yield factors). While, by construction, yields at time t, $\underline{y}_t$, depend on Z_t only through X_t (see the analogous discussion in Joslin et al., 2014), (2) implies that the expectation of *future* yield factors and therefore yields, $\mathbf{E}_t^{\mathbb{P}}[\underline{y}_{t+s}] = \mathbf{E}_t^{\mathbb{P}}[H_y(X_{t+s})]$,—and, by implication, term premiums and expected excess bond returns—depend on both X_t and

M_t.

3 Data

To estimate the model, I use end-of-month zero-coupon U.S. Treasury yields from January 1990 to December 2013, for maturities of 6 months, 1 to 5, 7, and 10 years. Yields are extracted from the CRSP U.S. Treasury Database using the unsmoothed Fama and Bliss (1987) methodology, as in Le and Singleton (2013).[2]

I include in M_t the unemployment gap u_t—computed as the seasonally adjusted civilian unemployment rate reported by the Bureau of Labor Statistics minus the natural rate of unemployment estimated by the Congressional Budget Office—and log annual core inflation based on the deflator for personal consumption expenditure (PCE), π_t. Both the unemployment rate and PCE inflation have been identified in FOMC statements as inputs into monetary policy decision making, and have been used explicitly in the FOMC's forward guidance on the path of the federal funds rate.

4 Estimation

The filtering and estimation problem is an only slightly modified version of that discussed in Priebsch (2013) and Kim and Priebsch (2014). In particular, the monthly discretized version of (2) represents the transition equation, and the observation equation is given by

$$\begin{pmatrix} \underline{y}_t^o \\ M_t^o \end{pmatrix} = \begin{pmatrix} H_y(X_t) \\ M_t \end{pmatrix} + \begin{pmatrix} e_{y,t} \\ e_{M,t} \end{pmatrix},$$

where $\underline{y}_t^o$ and M_t^o are the observed yields and macro variables described in Section 3, and $e_{y,t} \sim N(0, \sigma_y^2 I)$ and $e_{M,t} \sim N(0, \text{diag}(\sigma_u^2, \sigma_\pi^2))$ are iid measurement errors. As in Priebsch (2013), I use the unscented Kalman filter to filter the state variables and set up a quasi–maximum likelihood (QML) function. I rotate the yield factors such that the first three entries of $\mu^{\mathbb{P}}$ are zero, the upper-left 3×3 block of $K^{\mathbb{P}}$ is lower triangular, and Σ_X is equal to 0.01 times the identity matrix. Without loss of generality, I furthermore take Σ_Z to be lower triangular. Parameter estimates based on maximizing the QML function are shown in Table 1 (for the pricing distribution

[2] I am grateful to Anh Le for providing the code for this procedure.

$\mathbb{Q}$), and in Table 2 (for the data-generating distribution $\mathbb{P}$), together with asymptotic standard errors (derived following Bollerslev and Wooldridge, 1992). The observation errors on yields and inflation have estimated standard deviations of 6 and 4 basis points, respectively, while that on unemployment has an estimated standard deviation close to zero.

The estimated parameters ρ_0 and $\mu^{\mathbb{P}}$ imply that the unconditional means under $\mathbb{P}$ of the shadow short rate, the unemployment gap, and inflation are 4.07 percent, 0.03 percent, and 2.08 percent, respectively. Thus, in particular, the unemployment gap has an estimated unconditional mean that is close to zero—as it should by definition—, the estimated unconditional mean of inflation is close to the FOMC's 2 percent long-term target,[3] and the shadow short rate has an estimated unconditional mean that is close to the 4 percent "neutral" rate implied by the Taylor (1993) rule, as well as FOMC meeting participants' long-range federal funds rate projections.[4] The lower bound on the short rate (and hence on yields of all maturities) is estimated at 14 basis points, close to the mid point of the target range for the federal funds rate in effect after December 2008. Based on the stationary distribution of r_t under $\mathbb{P}$, the unconditional probability of the lower bound binding is $\mathbb{P}(r_t < r_{\min}) = 0.1285$, notably higher than common pre-crisis estimates such as the 5 percent figure reported by Reifschneider and Williams (2000), but lower than the in-sample frequency of close to 20 percent.

Figure 1 plots the model-implied shadow short rate r_t based on the filtered states $X_{t|t}$, together with the estimated lower bound $r_{\min}$. The shadow rate first falls below the lower bound in May 2009, and remains there through the end of the sample. This corresponds to the period during which the *observed* short rate was constrained by the lower bound.

Turning to the macro variables, in the estimation sample (see Section 3) the unemployment gap and inflation are negatively correlated, with a sample correlation coefficient of -0.23. The same is true under the estimated model-implied stationary distribution which implies a correlation coefficient of -0.40. Figure 2 plots the confidence contours of the model-implied joint stationary pdf of the unemployment gap u

[3] See the FOMC's "Statement on Longer-Run Goals and Monetary Policy Strategy" available at `http://www.federalreserve.gov/monetarypolicy/files/FOMC_LongerRunGoals.pdf`.

[4] A "Summary of Economic Projections" has been published regularly as part of the FOMC meeting minutes since 2011, see `http://www.federalreserve.gov/monetarypolicy`.

ρ_0	0.0407		ρ_1	0.0575	
	(0.0162)			(0.2162)	
$r_{\min}$	0.0014			0.3235	
	(0.0001)			(0.1511)	
				0.6961	
				(0.0814)	
$\mu^{\mathbb{Q}}$	0.0344	$K^{\mathbb{Q}}$	−0.1644	−0.2415	0.0737
	(0.0255)		(0.4284)	(0.1981)	(0.2210)
	0.0075		0.4992	0.0331	−0.2453
	(0.0213)		(0.3046)	(0.1060)	(0.1087)
	0.0882		1.5143	0.7126	−0.7988
	(0.0356)		(0.2046)	(0.2302)	(0.4220)

Table 1: Model parameters governing the pricing distribution $\mathbb{Q}$, estimated by quasi-maximum likelihood (asymptotic standard errors in parentheses).

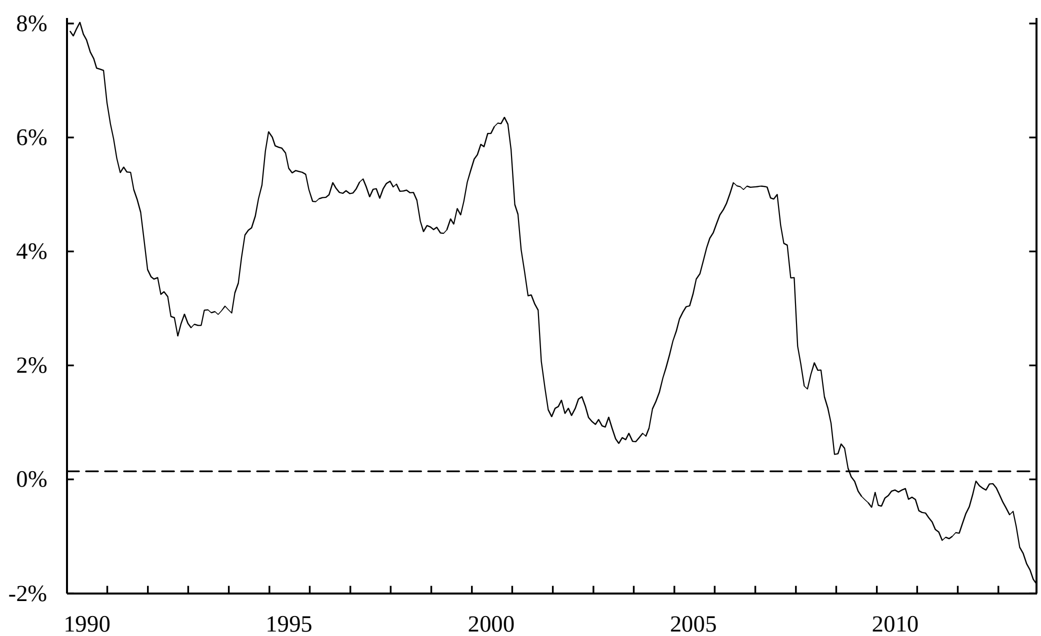


Figure 1: The model-implied shadow short rate r_t and the estimated lower bound $r_{\min}$ (the dashed line).

$(\mu^{\mathbb{P}})^\top$	0.0000	0.0000	0.0000	−0.0003	0.0208
				(0.0090)	(0.0042)
$K^{\mathbb{P}}$	−0.3265	0.0000	0.0000	−0.2394	0.1350
	(0.2672)			(0.1853)	(0.4500)
	0.3054	−0.3563	0.0000	−0.1650	−0.6330
	(0.2927)	(0.4046)		(0.3692)	(0.3597)
	1.8345	2.8984	−1.7795	0.7786	1.0671
	(0.0803)	(0.1135)	(0.4272)	(0.3074)	(0.4043)
	−0.5020	−1.1920	0.5652	−0.5295	−0.2942
	(0.1623)	(0.2847)	(0.0287)	(0.1858)	(0.2744)
	0.1829	0.0959	−0.0417	0.0204	−0.3644
	(0.0331)	(0.1591)	(0.0629)	(0.1330)	(0.1871)
Σ_M	0.0000	−0.0001	−0.0007	0.0051	0.0000
	(0.0004)	(0.0004)	(0.0005)	(0.0002)	
	−0.0002	0.0005	0.0005	−0.0000	0.0041
	(0.0003)	(0.0004)	(0.0006)	(0.0002)	(0.0007)

Table 2: Model parameters governing the data-generating distribution $\mathbb{P}$, estimated by quasi-maximum likelihood (asymptotic standard errors in parentheses).

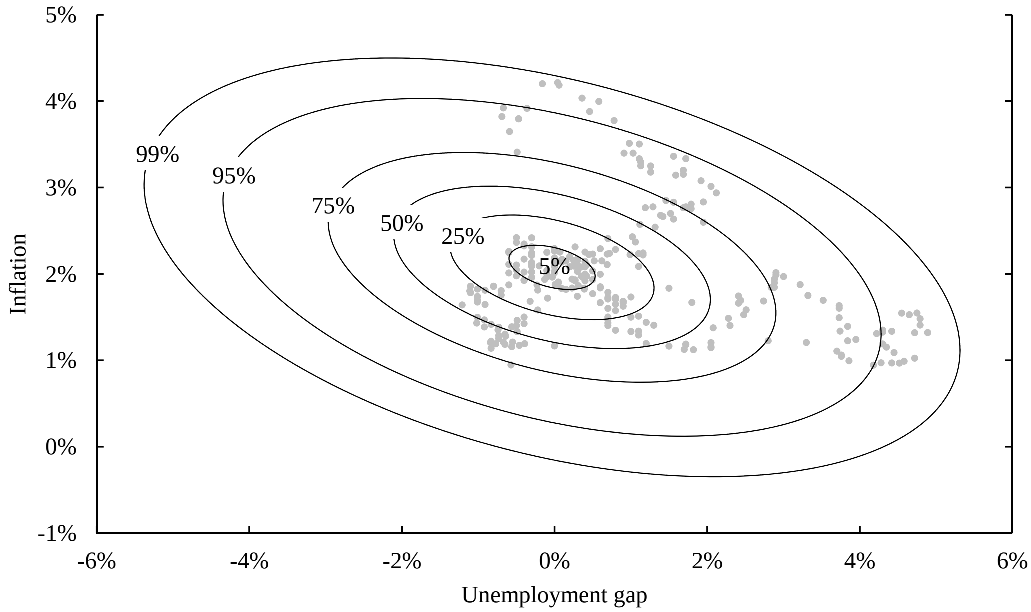


Figure 2: Confidence contours of the joint stationary pdf of the unemployment gap u and inflation π.

and inflation π, as well as the sample observations.[5,6] The inverse reduced-form relationship seen in Figure 2 reflects the usual (dis-)inflationary pressure associated with low (high) unemployment, as represented by the (expectations-augmented) Phillips curve.

4.1 Are Yields Less Informative During the ZLB Period?

When the short end of the yield curve is effectively tied down by the lower bound, the cross section of yields may be less informative about the underlying yield factors than would otherwise be the case (see Ichiue and Ueno (2013) for an argument along these lines). The shadow-rate model introduced in Section 2 and estimated in Section 4 accommodates this phenomenon: The nonlinear relationship between yield factors and yields implies that at or near the ZLB a given change in the factors translates into a smaller change in yields—particularly short-term yields—than away from the ZLB.

[5] The contours plotted in the figure correspond to the graphs of $\{(u,\pi) : f(u,\pi) = c\}$, with c chosen such that $\int_{\{(u,\pi):f(u,\pi)\geq c\}} f(u,\pi)\, d(u,\pi) = p$, where p is the indicated confidence level.

[6] Note that under the model, each observation in the sample is unconditionally identically, but not independently, distributed, so we would not expect the sample to look like i.i.d. draws from the stationary distribution.

At one extreme, the model-implied short rate is entirely unresponsive to changes in the yield factors when the shadow rate is below the lower bound. For a given magnitude of measurement error, therefore, the model-implied signal-to-noise ratio in observed yields is lower near the ZLB.

The unscented Kalman filter allows us to quantify this potential loss of information by studying the time-varying posterior uncertainty of the filtered yield factors. In particular, the ratio

$$\gamma_t = \left(\frac{||\mathrm{Var}_{t|t}(X_t)||_F}{||\mathrm{Var}(X_t)||_F} \right)^{-\frac{1}{2}}$$

expresses the precision gain of the filtered state variables based on information up to time t relative to the best guess based purely on the unconditional distribution.[7] As shown in Figure 3, in normal times the precision of filtered state variables is about 30 times higher than an unconditional guess and shows little time variation. However, the ratio drops notably during the ZLB period, to a level below 10. The solid line in the figure represents γ_t when states are filtered based only on yield information up to time t. The dashed line is based on filtering yield factors based on both yields and macro variables up to time t. Through their contemporaneous covariation with yields—in other words, through their spanned components—, macro variables allow for more precise filtering of the yield factors. However, the effect, while discernible, is quantitatively small, improving precision by no more than 15 percent. As might be expected, the improvement is most pronounced during the ZLB period, when yields themselves have relatively lower information content.

The fact that macro variables seem to contribute relatively little to identifying contemporaneous yield factors—even in the presence of measurement error on all yields—underscores the premise of Joslin et al. (2014). The finding provides complementary evidence on the importance of the unspanned components of macro variables. If macro variables were predominantly spanned by yield factors, incorporating macro information into the filtering information set should improve the precision of filtered yield factors to a more economically significant degree.

[7] The ratio γ_t is rotation invariant and can be interpreted in terms of relative standard deviation. For example, if all posterior variances and covariances are reduced by a factor of four, γ_t will double.

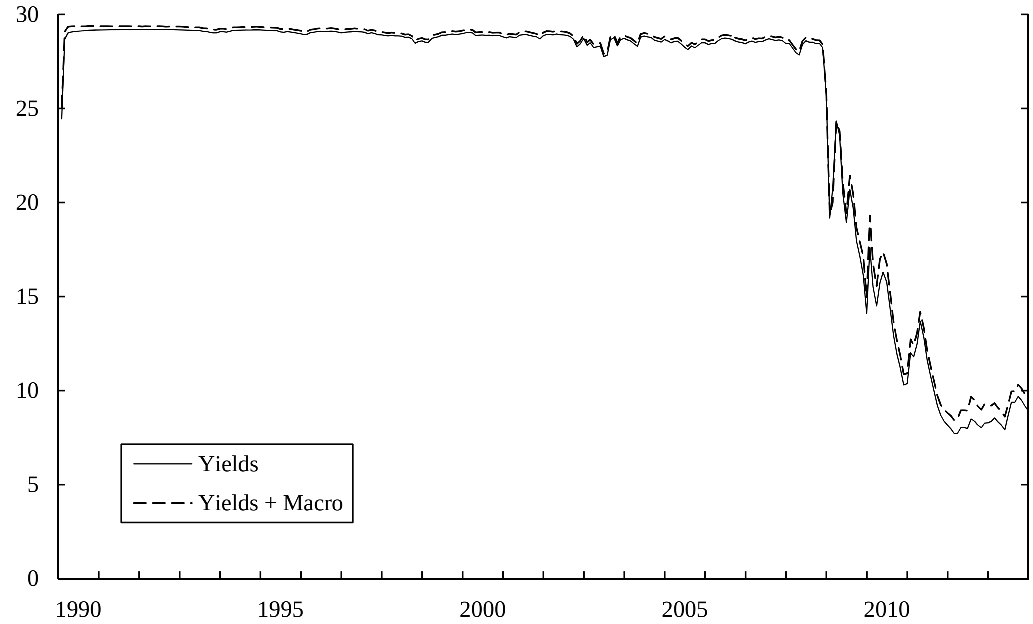


Figure 3: Precision gain of the filtered yield factors $X_{t|t}$ over a guess based on the unconditional distribution. The solid line is based on using only yield information. The dashed line is based on using both yield and macro information to filter the yield factors.

5 Interest Rate Expectations

5.1 The Taylor Rule Revisited

Even though macro variables do not enter explicitly into the shadow short rate specification (1), this does not imply lack of a relationship between r_t and the macro variables M_t. In particular, it follows from (2) and the properties of joint Gaussian random variables that

$$\mathbf{E}^{\mathbb{P}}[X_t|M_t] = \mu_X^{\mathbb{P}} + V_{XM}V_{MM}^{-1}(M_t - \mu_M^{\mathbb{P}}), \tag{3}$$

where $\mu_X^{\mathbb{P}}$ and $\mu_M^{\mathbb{P}}$ are the unconditional means of the yield factors and macro variables, respectively, and V_{XM} and V_{MM} are the upper-right and lower-right blocks of the unconditional covariance matrix of $Z_t = (X_t; M_t)$, $V = \text{Var}^{\mathbb{P}}(Z_t)$.[8] Together with (1), (3) implies that

$$\mathbf{E}^{\mathbb{P}}[r_t|M_t] = \alpha + \beta^{\top}(M_t - \mu_M^{\mathbb{P}}), \tag{4}$$

[8] As shown in Priebsch (2013), $\text{Var}^{\mathbb{P}}(Z_t) = \int_0^{\infty} e^{-K^{\mathbb{P}}s}\Sigma\Sigma^{\top}e^{-(K_1^{\mathbb{P}})^{\top}s}\,ds$.

where α and β are known up to the model parameters. Thus, the model implies a predictive relationship between r_t and M_t even though the macro variables do not enter into the shadow short rate equation (1). This is because assumption (1) is not a structural equation, not even a statistical relationship. It is a modeling device—representing the shadow short rate as a linear combination of three latent factors—that holds as an identity by definition. As such, it has no more empirical content than the tautological statement that $\mathbf{E}^{\mathbb{P}}[r_t|M_t, r_t] = r_t$; once we know the value of the shadow short rate, that value will no longer depend on other variables (including macro factors). On the other hand, (4) is a testable implication of the model that follows from (1) in combination with the other modeling assumptions in Section 2. It predicts a specific statistical association between r_t and the macro variables M_t. Hence, the estimated version of (4) can be compared to the reduced-form relationships between short rate and macro variables implied by other models. In particular, substituting the parameters estimated in Section 4 into (4), we obtain

$$\mathbf{E}^{\mathbb{P}}[r_t|M_t] = \underset{(0.0162)}{0.0407} - \underset{(0.5624)}{1.2038}(u_t + \underset{(0.0090)}{0.0003}) + \underset{(0.9679)}{1.9873}(\pi_t - \underset{(0.0042)}{0.0208}). \tag{5}$$

(with asymptotic standard errors computed by the delta method in parentheses). This equation bears striking resemblance to the Taylor (1993) rule, which can be written as[9]

$$r = 0.04 - 1.15u + 1.5(\pi - 0.02). \tag{6}$$

That is, based on the estimated interaction between yield factors and macro variables, the model *implies* a predictive (reduced-form) relationship between the shadow short rate and the macro variables that closely resembles the Taylor rule, without specifically *assuming* such a relationship. It is tempting to interpret (5) as an estimated monetary policy reaction function. However, we already know that the shadow short rate cannot literally be set according to a rule such as (6); this would imply that u_t and π_t are spanned by yields. Therefore, (5) is more appropriately interpreted as reflecting monetary policy *expectations* conditional on the realization of u_t and π_t. The model does not take a stand on the structural origins of these expectations.

Note that (5) applies to the *shadow* short rate rather than the observed short rate.

[9]Taylor's (1993) original rule and many of its subsequent variants are formulated in terms of inflation and the output gap. I convert between output gap and unemployment gap using an Okun's law coefficient of 2.3, as in Yellen (2012).

The model also implies a (necessarily nonlinear) version of (5) for the *observed* short rate, $\underline{r}_t$. Figure 4 overlays the model-implied expected *observed* short rate conditional on the macro factors, and the policy rate prescribed by the Taylor rule (4), separately for unemployment and inflation (in each case, keeping the other variable fixed at its unconditional expectation or, in the case of the Taylor rule, its neutral value). The two are remarkably similar away from the ZLB, but unlike the Taylor rule, the model does not imply a negative expected short rate when unemployment is particularly high or inflation is particularly low. Figure 5 plots the contours of the model-implied expected short rate, and the Taylor-rule-implied policy rate, as both unemployment and inflation vary. Again, the two match closely, both in terms of the level of the contours and their slope (the implicit tradeoff between unemployment and inflation), except for the bottom-right ZLB region. The model-implied expected short rate thus has the attractive feature of matching the Taylor rule closely during normal times, while remaining non-negative at the ZLB.

Furthermore, the model's implications extend immediately to expectations of future short rates as well as longer-term yields. Figure 6 plots the contours of the expected ten-year yield conditional on unemployment and inflation. As in Figure 5a, the expectation obeys the ZLB. However, unlike in the case of the the short rate, the implicit tradeoff between unemployment and inflation that keeps the conditional expectation of the ten-year yield fixed is not constant (the contours are not linear).

5.2 (Un)Conventional Policy Expectations

While conventional monetary policy chiefly operates through short-term interest rates, unconventional policy tools include guidance on the policy rate path going forward and large-scale asset purchases (Bernanke et al., 2004). These tools are aimed at lowering future expected short rates and term premiums, respectively, and by implication long-term yields.

Based on the predictive relationships introduced in Section 5.1, we can use the model to analyze how realized short-term rates, expected *future* short-term rates, and long-term yields compared to their model-implied predicted values conditional on concurrent macro variables, both for the crisis period and before. Figure 7 plots the median forecasts (solid lines) and 70 percent prediction intervals (shaded areas) for the model-implied current short rate and one-year-ahead expected short rate,

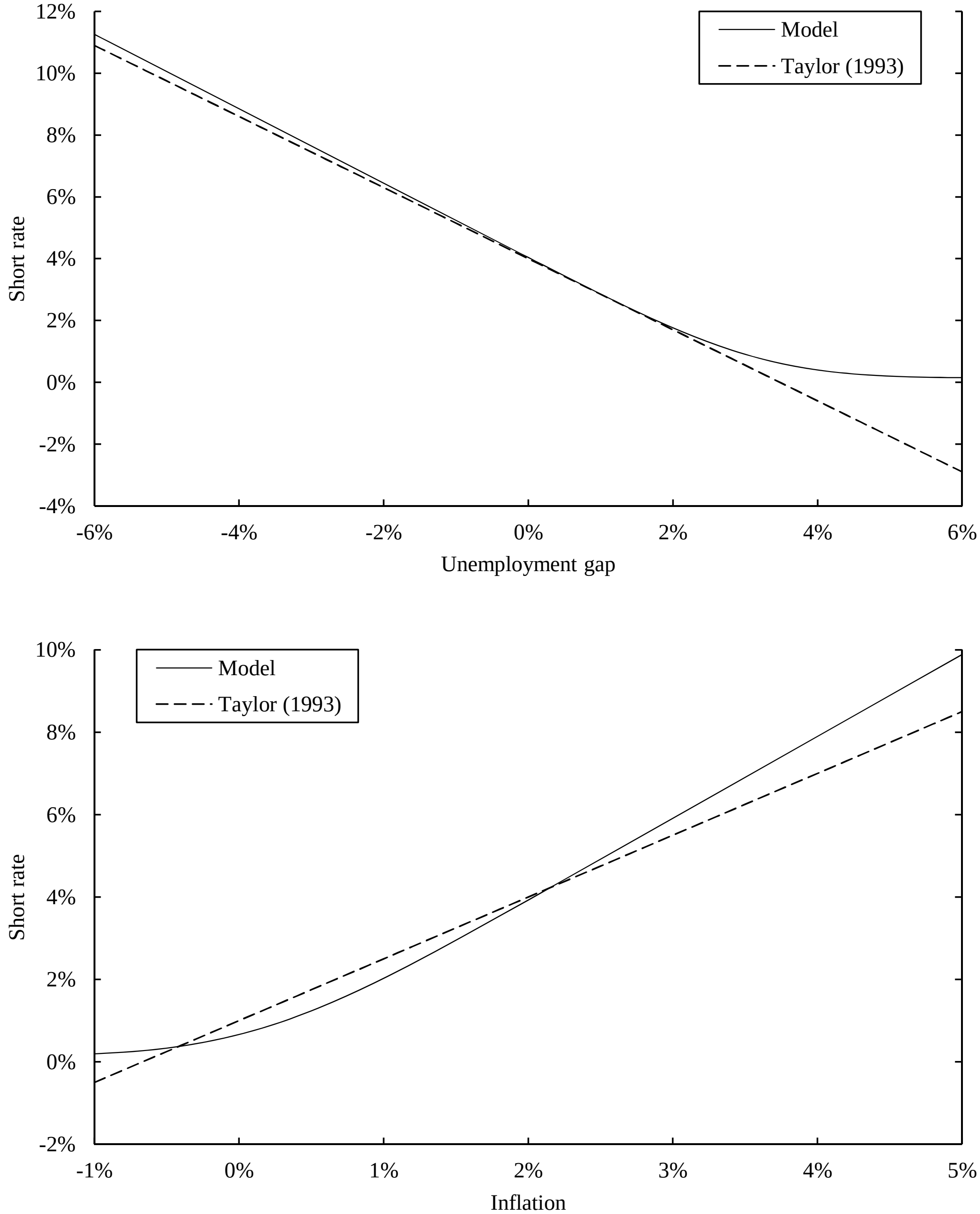
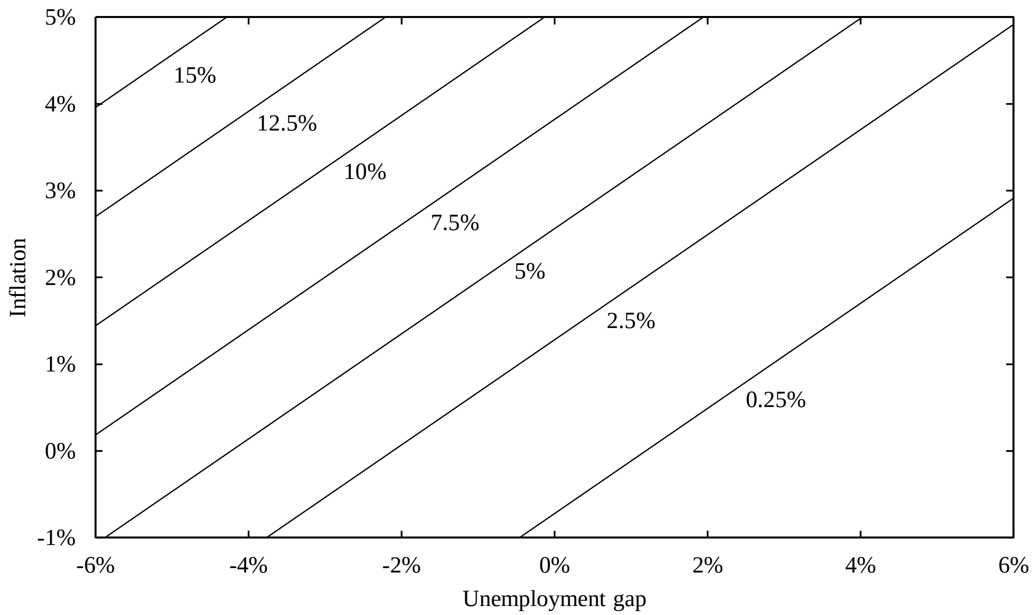
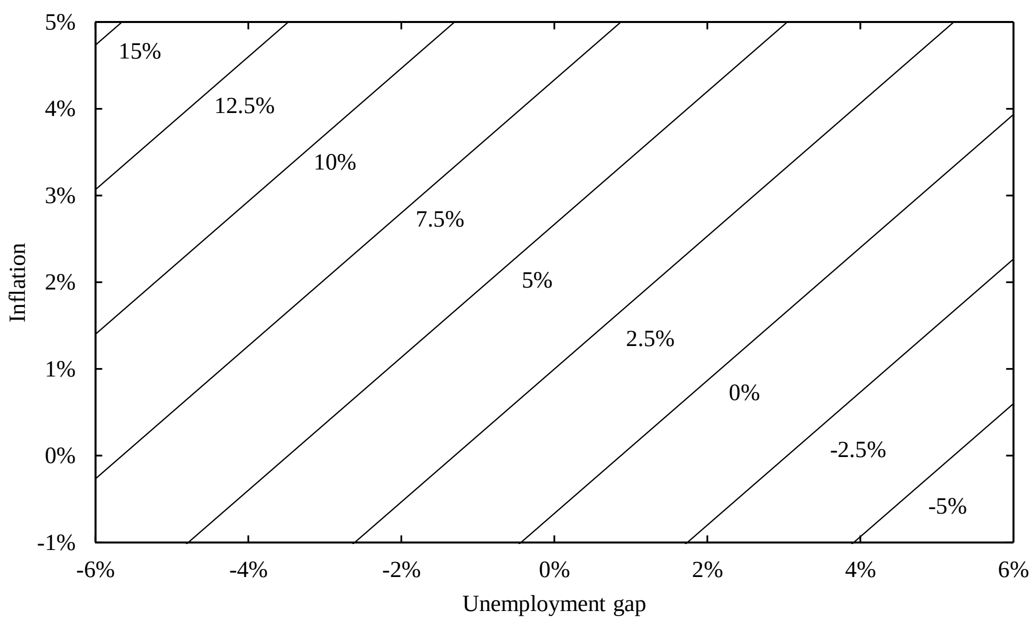


Figure 4: The solid lines show the expected model-implied short rate conditional on the unemployment gap (top panel) and inflation (bottom panel), $\mathbf{E}^{\mathbb{P}}[\underline{r}_t|M_t]$, in each case holding the other variable fixed at its unconditional expectation. The dashed lines represent the value of the policy rate prescribed by the Taylor (1993) rule.



(a) Model-implied expected short rate.



(b) Taylor-rule-implied policy rate.

Figure 5: Contours of the model-implied expected short rate conditional on the unemployment gap and inflation, $\mathbf{E}^{\mathbb{P}}[\underline{r}_t|M_t]$ (top panel), and contours of the policy rate prescribed by the Taylor (1993) rule (bottom panel).

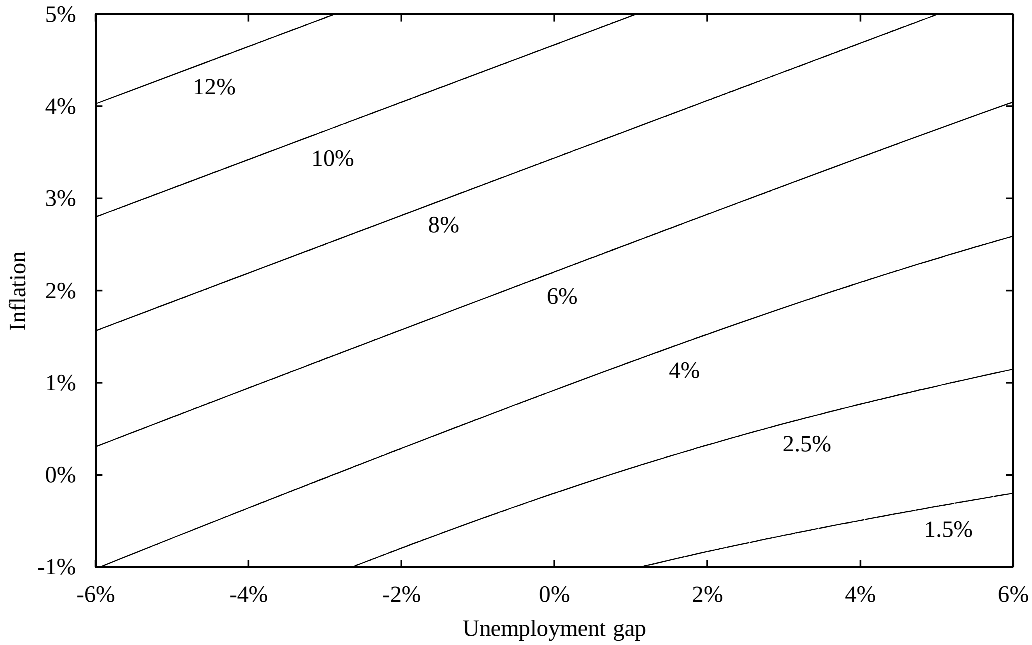


Figure 6: Contours of the model-implied expected ten-year yield conditional on the unemployment gap and inflation, $\mathbf{E}^{\mathbb{P}}[\underline{y}_t^{10}|M_t]$.

and Figure 8 similarly plots the model-implied prediction for the ten-year yield and ten-year yield term premium, conditional on the contemporaneously observed macro variables. Both figures also show the actual realizations of the predicted variables (dashed lines). One preliminary observation is that the model is able to predict the ten-year yield and corresponding term premium more precisely than short-term rates, as reflected in the narrower prediction intervals. This is due to the fact that long-term yields tend to be less volatile than short-term rates.

Figure 7 shows that, prior to the ZLB period, the actual short rate was relatively high (when compared to the model's prediction based on macro variables) in the mid to late 1990s (cf., Taylor, 1999), and relatively low in 2001–2004 (cf., Taylor, 2007). Meanwhile, as seen in Figure 8, the ten-year yield held steady in 2004 and 2005, despite a notable increase in its predicted value, apparently due to a decline in the ten-year yield term premium well below the level predicted by the model in light of the macro variables at the time. This has been called the "conundrum" period in the literature (for example, Rudebusch et al., 2006; Backus and Wright, 2007).

During the ZLB period, the current short rate was close to its median prediction given macro conditions, as shown in the top panel of Figure 7. The actual (model-implied) one-year-ahead expected short rate remained close to the ZLB, even as the model's median prediction rose to about one percent by late 2013, consistent with the FOMC's forward guidance to keep rates low. Nevertheless, the low one-year-ahead short rate expectations remained within the model's 70 percent prediction interval. Given macro conditions, only the ten-year yield was "unusually" low (relative to the 70 percent prediction interval) in 2011 and 2012, coinciding with the Fed's large-scale purchases of long-term Treasury securities. As the bottom panel in Figure 8 shows, this seems to be largely due to a lower-than-predicted term premium in those years.

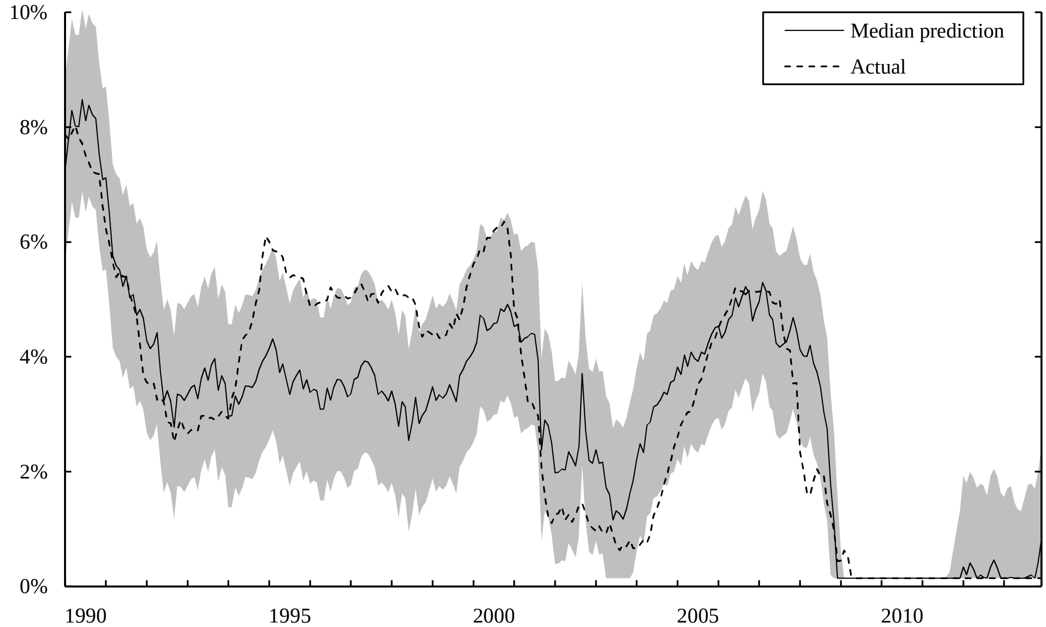
5.3 Dynamic Policy Expectations and the Neutral Policy Rate

Much like the original Taylor (1993) rule, the interest rate expectations considered so far are static in nature—they relate current rates to current macro conditions. Following the logic used to derive (4), we can, for any horizon $s > 0$, derive expectations of future interest rates conditional on *future* macro variables and the *current* state variables. For example, for the shadow short rate,

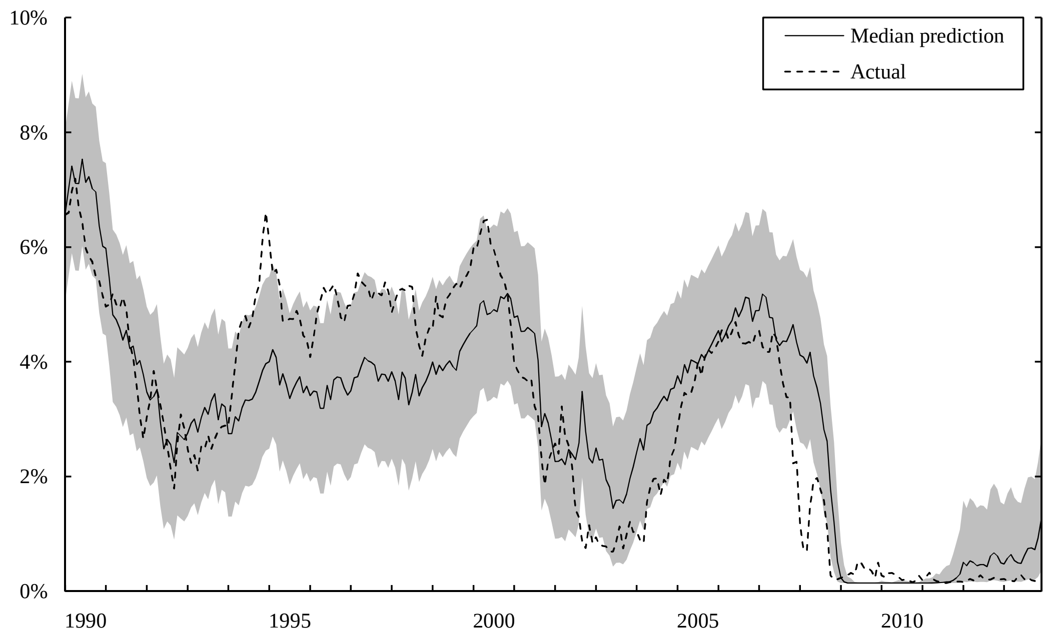
$$\mathbf{E}^{\mathbb{P}}[r_{t+s}|Z_t, M_{t+s}] = r^*_{t,s} + \beta_s^\top (M_{t+s} - \mu_M^{\mathbb{P}}), \tag{7}$$

where, given s, $r^*_{t,s}$ depends only on the model parameters and Z_t, and β_s depends only on the model parameters. The expectation (7) can be interpreted as the sum of a time- and horizon-dependent "neutral" shadow short rate—the shadow rate that, as of time t, would be expected to prevail at time $t + s$ if macro variables returned to their unconditional expectations by that time—and a term that reflects the usual dependence on the realized macro variables. As before, we can derive an analogous (nonlinear) version of (7) for the *observed* future short rate, which, in turn, can be used to find the observed neutral short rate $\underline{r}^*_{t,s}$. For a horizon s of one year, Figure 9 plots the model-implied neutral short rate $\underline{r}^*_{t,1}$, translated to an ex ante real rate by subtracting model-implied expected inflation $\mathbf{E}^{\mathbb{P}}[\pi_{t+1}|Z_t]$, as well as a measure of the natural policy rate estimated by Laubach and Williams (2003) based on a framework incorporating only macroeconomic information. While the model-implied estimate is more volatile—as might be expected from a measure based on financial market information—it shows broad agreement with the Laubach/Williams neutral rate. In particular, at the end of the sample period, both rates are slightly negative.

Figure 10 shows how the coefficients β_s on the unemployment gap and inflation

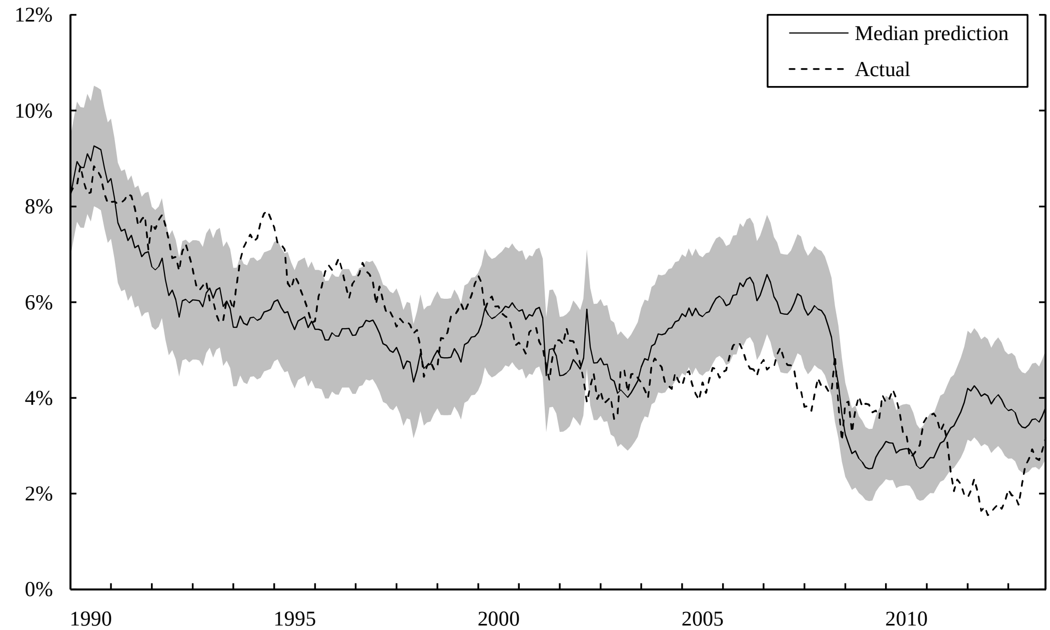


(a) Current short rate, $\underline{r}_t$

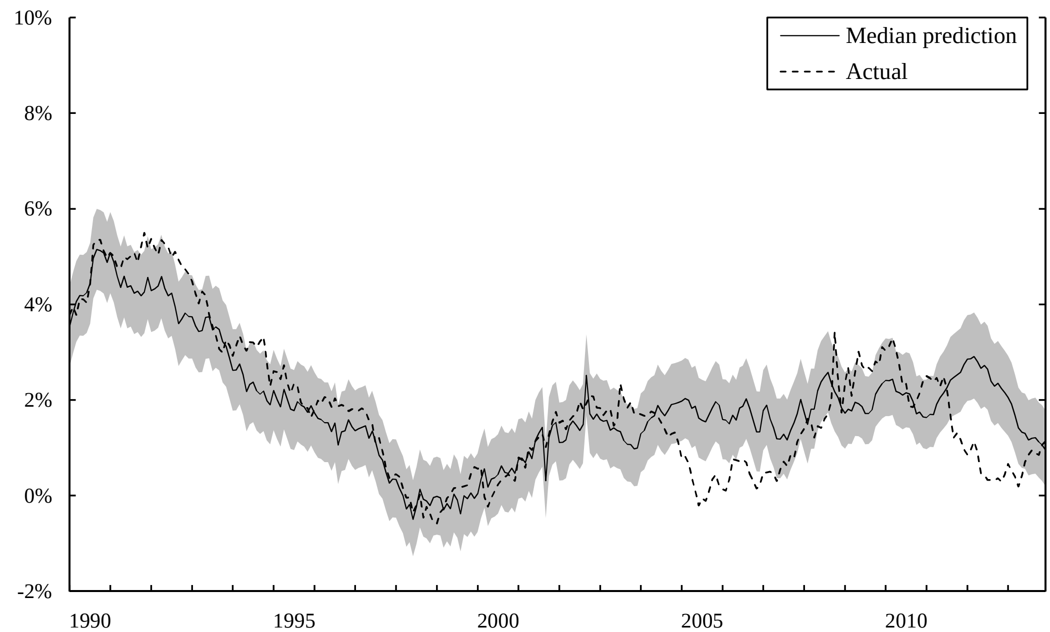


(b) One-year-ahead expected short rate, $\mathbf{E}_t[\underline{r}_{t+1}]$

Figure 7: Model predictions of current short rate and one-year-ahead expected short rate (median and 70 percent prediction interval), conditional on observed macro variables; observed counterparts.



(a) Ten-year yield, $\underline{y}_t^{t+10}$



(b) Ten-year yield term premium, $\underline{tp}_t^{t+10}$

Figure 8: Model predictions of the ten-year yield and the ten-year yield term premium (median and 70 percent prediction interval), conditional on observed macro variables; observed counterparts.

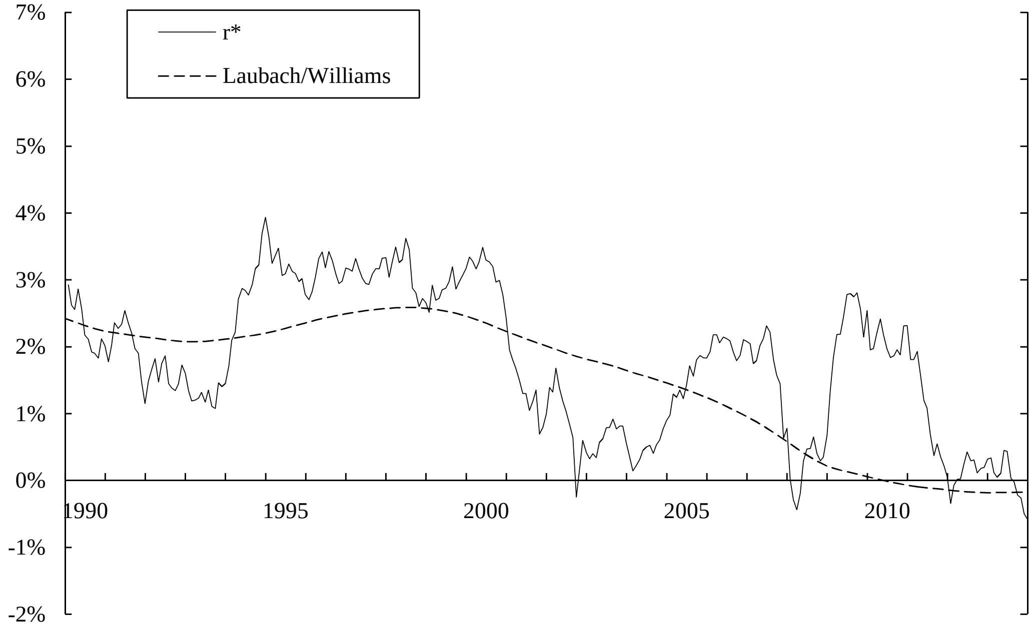


Figure 9: Model-implied "neutral" ex ante real short rate $\underline{r}^*_{t,1} - \mathbf{E}^{\mathbb{P}}[\pi_{t+1}|Z_t]$, and the natural real policy rate estimated by Laubach and Williams (2003).

in the dynamic shadow short rate expectation (7) vary as a function of horizon s. For every horizon, the coefficient on inflation is positive and the coefficient on unemployment is negative, as in the static expectation (5). However, both start at substantially smaller absolute values and converge to their static counterparts only gradually, especially in the case of inflation. This pattern can be interpreted as inertia in short-rate expectations, much as has been discussed for monetary policy more broadly (see Rudebusch, 2006): Conditional on an increase in inflation by one percentage point, the contemporaneous expected shadow short rate increases by only around 30 basis points. Conditional on inflation remaining at this elevated level, the expected future short rate increases gradually as a function of the horizon s; by one percentage point after about $2^1/_2$ years, and by 1.5 percentage points after about $5^1/_2$ years. The case of a change in the unemployment gap is analogous, although the shadow short rate would be expected to adjust more rapidly.

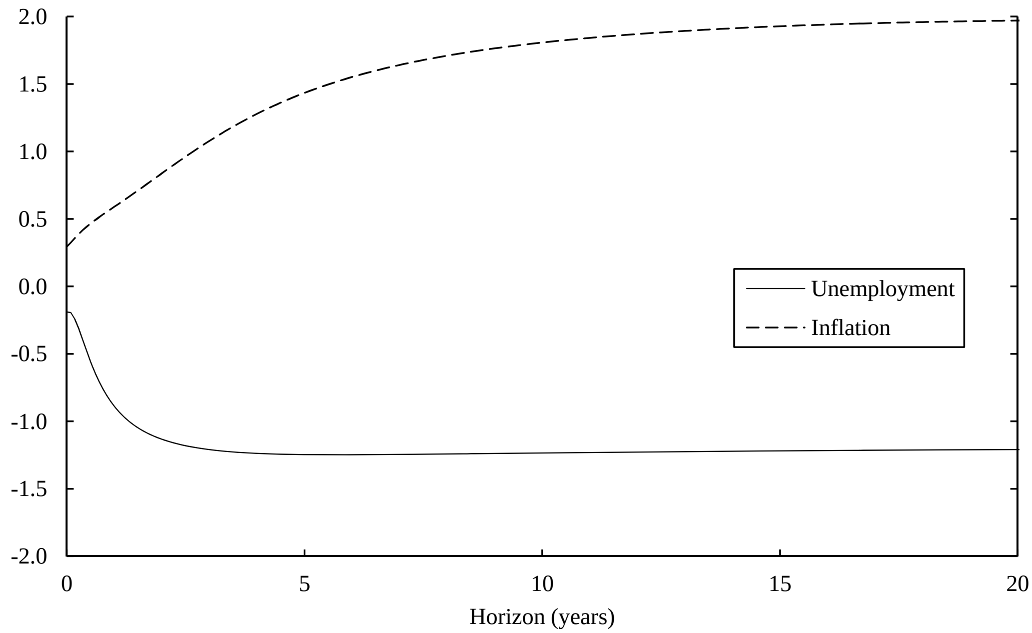


Figure 10: Coefficients β_s on the unemployment gap and inflation in the dynamic shadow-short rate expectation (7), as a function of horizon s.

5.4 The Tradeoff Between Inflation and Unemployment at Liftoff

Recall from Section 4 that unemployment and inflation are negatively correlated, both in sample and under the estimated stationary distribution. It turns out they are also *conditionally* negatively correlated at any given forecast horizon. On the other hand, changes in unemployment and inflation that leave the stance of monetary policy unchanged tend to be *positively* correlated: Higher unemployment generally reduces the optimal policy rate, but higher inflation generally raises it.

To the extent that the FOMC similarly trades off high inflation against high unemployment in the timing of its decision to first raise short-term interest rates—as indeed would be suggested by its forward guidance language—, this tradeoff should be reflected in a positive association between inflation and unemployment *at the time of liftoff*. That is, if inflation is running high, the FOMC may decide to raise rates sooner—at a higher rate of unemployment—than it otherwise would, and vice versa. Indeed, Figure 11 shows that the conditional model-implied correlation between the unemployment gap and inflation at liftoff, $\text{Corr}_t(u_\tau, \pi_\tau)$, where liftoff is the stopping

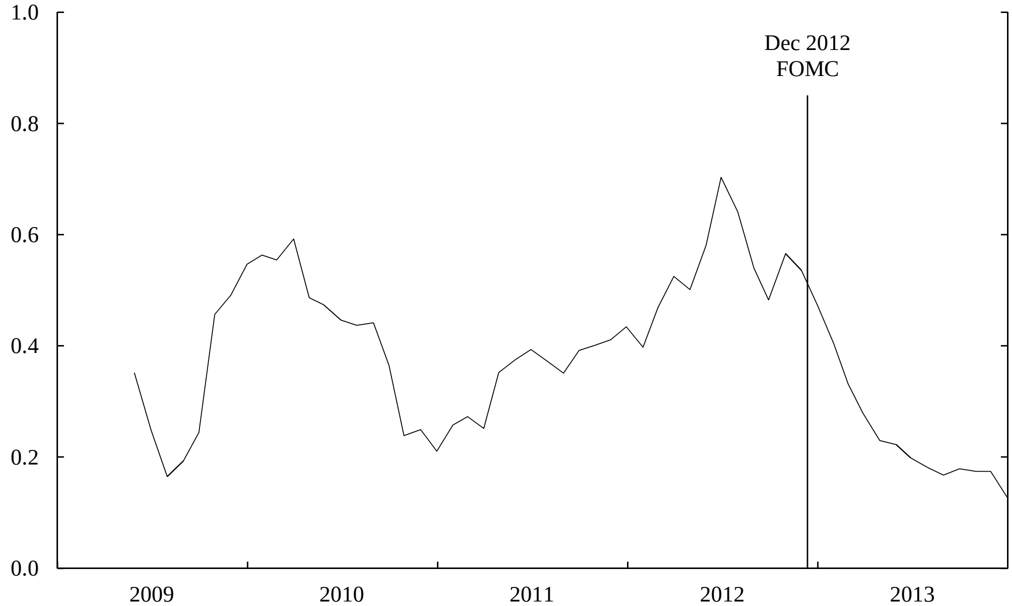


Figure 11: Model-implied correlation between the unemployment gap and inflation at liftoff, $\mathrm{Corr}_t(u_\tau, \pi_\tau)$.

time $\tau = \inf\{t > t_0 : r_t \geq r_{\min}\}$, is positive throughout the ZLB period, varying from just below 0.2 to above 0.7. Of note, the correlation declined considerably after the FOMC introduced an explicit unemployment rate threshold in December 2012.

Figure 12 plots confidence contours for the model-implied conditional joint pdf of the unemployment gap u_τ and inflation π_τ at the time of liftoff τ, as of March 2010 (corresponding to the first peak in Figure 11) and December 2013 (the last observation in the sample). Both distributions reflect the positive association seen in Figure 11, but also the weakening of this association later in the sample period. This appears to be primarily the result of a reduction in uncertainty about the unemployment gap at liftoff; the uncertainty about inflation at liftoff seems to have remained roughly unchanged. Of particular note is that the March 2010 distribution is bimodal (the dashed contours represent a confidence region of about 60 percent), reflecting two different likely scenarios for the future path of monetary policy. The first mode suggests an unemployment gap and inflation rate at liftoff close to the then-current values (the gray dot), whereas the second mode features a substantially lower unemployment gap as well as moderately lower inflation. The first mode is consistent with a rising inflation outlook and an FOMC compelled to raise rates at a still relatively large unemployment gap. The second mode is consistent with a more benign inflation trajectory, allowing the FOMC to keep short-term rates at zero until the unemployment gap has narrowed notably. The two modes also reflect potential uncertainty at the time about how much improvement in economic conditions the FOMC would have to see before it would consider raising rates. The March 2010 FOMC statement characterized labor market conditions as "stabilizing," and the FOMC concluded its current asset purchase program in the same month. While the statement continued to express the expectation of exceptionally low (although not necessarily zero) rates for an "extended period," dissent had recently begun to emerge within the FOMC.[10] The first mode can thus be interpreted to represent the scenario where rates are raised from the lower bound relatively swiftly (although perhaps kept at a subdued level for a prolonged period), while the second mode represents a scenario closer to the one that actually unfolded after March 2010. In particular, it it quite striking that the second mode corresponds to an unemployment rate quite close to (within about half a percentage point) the threshold later (in December 2012) announced by the FOMC. Figure 12a also shows two expected paths for unemployment and inflation. The black path represents the expectation $\mathbf{E}_t[(u_{t+s}, \pi_{t+s})]$, with each arrow marking one year. It indicates expected convergence back to the unconditional expectation (the unfilled black dot) over several decades. The gray path similarly traces out $\mathbf{E}_t[(u_{t+s}, \pi_{t+s})|\tau > s]$, the expected path conditional on short-term rates remaining at the lower bound. This path suggests slow convergence toward a state of secular stagnation—with low inflation and a persistent unemployment gap—, remarkably reminiscent—both qualitatively and quantitatively—of Japan's "lost decade" experience (see, for example, Krugman et al., 1998). Of note, the model is able to sketch this scenario based on a sample that does not include a similar event, and without any access to forward-looking information such as surveys. The difference between the black and gray paths is also an indication of the importance attributed by the model to the linkages between yield and macro variables, as the two paths differ only in the way they condition on the evolution of yields.

By December 2013, the FOMC had made efforts to clarify its objectives and preferences. The joint conditional distribution of the unemployment gap and inflation at liftoff as of December 2013, Figure 12b, thus has a single mode at an unemployment gap of approximately 0.7 percent and inflation at roughly 1.3 percent, indicating only

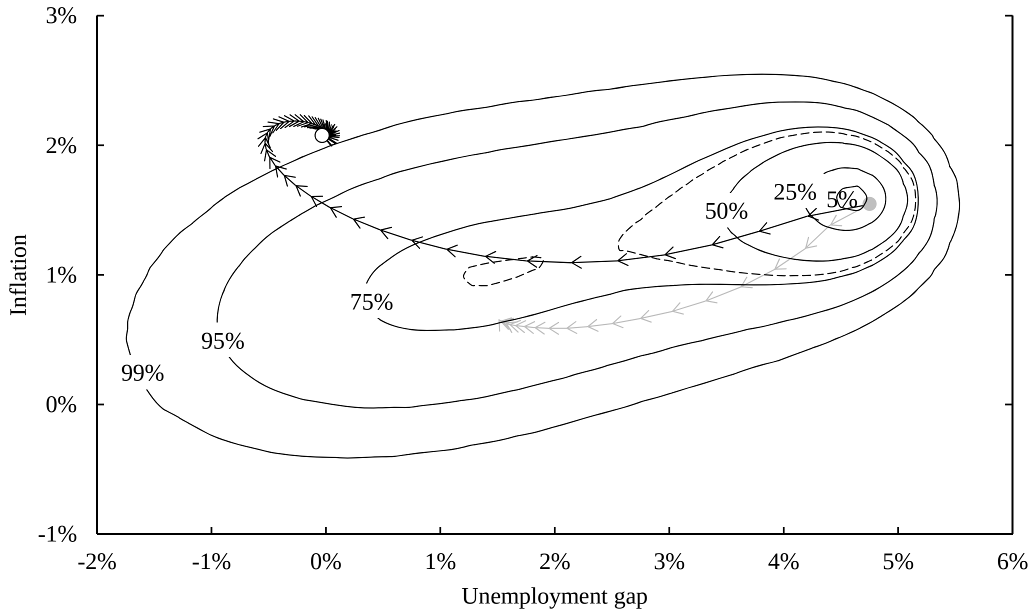
[10] Kansas Fed President Hoenig "believed that continuing to express the expectation of exceptionally low levels of the federal funds rate for an extended period was no longer warranted."

a slight rise in inflation but a more noticeable narrowing of the unemployment gap—by about 1/2 percentage point—, by the time of short-rate liftoff. The distribution bears some resemblance to what we might expect the distribution in March 2010, but conditioned on the second scenario (corresponding to the second mode in Figure 12a), to look like.

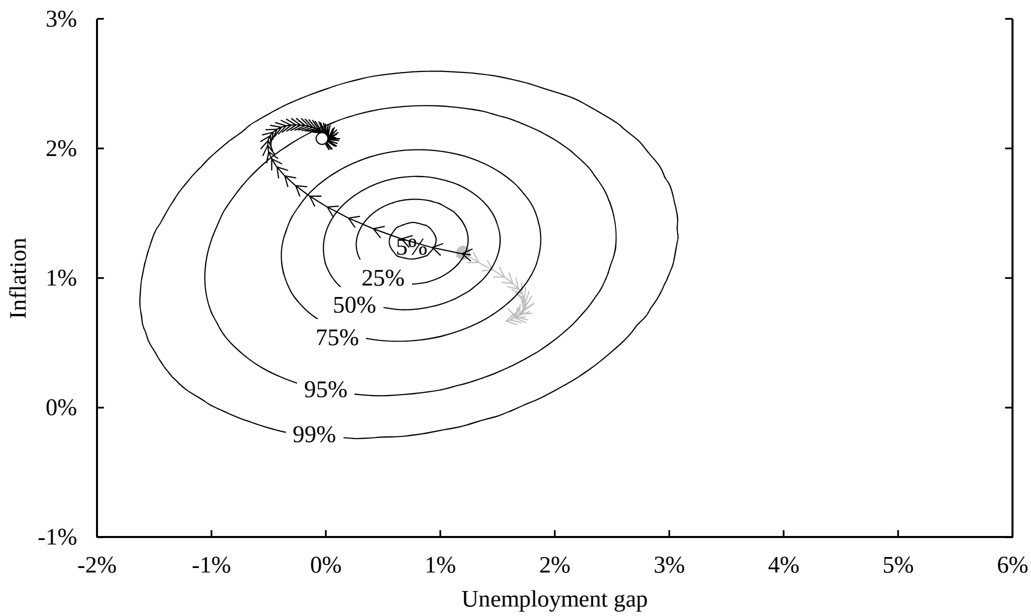
6 Interest Rate Uncertainty

Section 5 analyzed the relationships between interest rate *expectations* and macro variables. However, macro information is also a potential driver of interest rate *uncertainty.* While the underlying Gaussian structure of the model limits its ability to capture any time variation in interest rate uncertainty, it can nonetheless shed light on the extent to which this uncertainty can be accounted for by shocks to the macro variables. Consider the model-implied standard deviation of future rates, conditional on the current state variables. This uncertainty will likely reflect, at least in part, uncertainty about the evolution of the macroeconomy. The contribution of macro uncertainty to interest rate uncertainty can be quantified by computing the model-implied standard deviation of future rates, conditional on the current state variables *and future macro variables.* This amounts to asking how much interest rate uncertainty remains if we condition on a specific future path of the macroeconomy. Figure 13 plots the standard deviation of the future shadow short rate r_{t+s} (top panel), and the future shadow ten-year yield y^{10}_{t+s} (bottom panel), as a function of the forecast horizon s. The solid lines condition only on time t state variables, while the dashed lines condition, in addition, on the future macro variables M_{t+s}. The figures show that for a horizon of one to two years, most interest rate uncertainty is unrelated to the macro variables, while an increasingly large fraction of interest rate uncertainty at longer horizons can be attributed to macro uncertainty. For very long horizons, more than half of the shadow short rate uncertainty, and just below half of the shadow ten-year yield uncertainty, is eliminated when we condition on future macro variables.

While the realization of future macro variables is, of course, not known, conditioning on different economic scenarios allows us to generate different plausible interest rate forecast paths. Figure 14 plots the mean and median forecast as of December 2013 (the last date in the estimation sample) for the observed short rate $\underline{r}_{t+s}$, as well as 70 percent prediction intervals. As the top-left panel shows, even when condition-

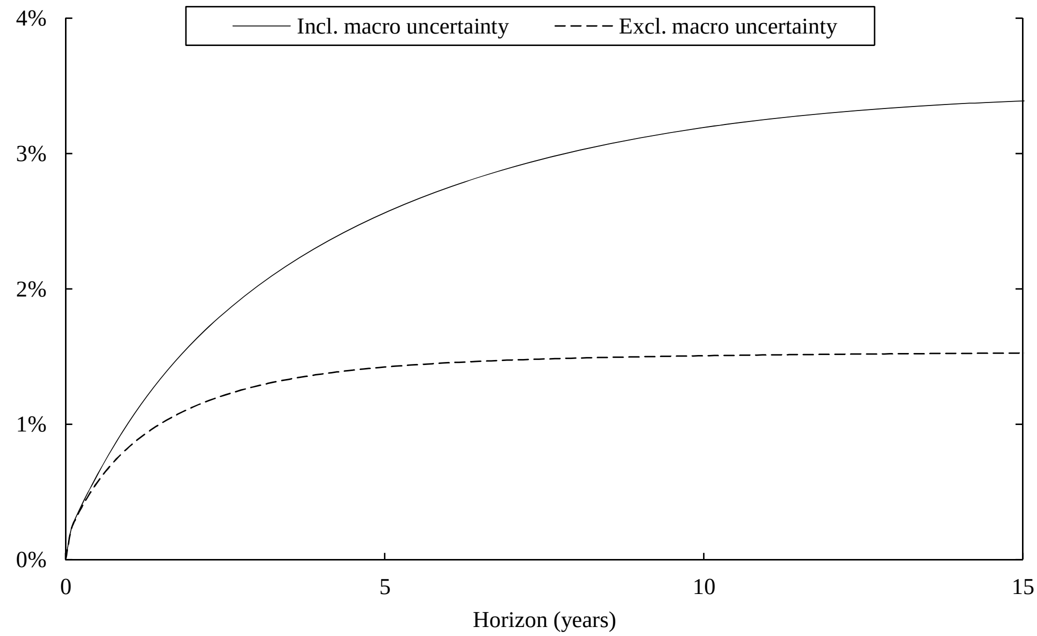


(a) March 2010

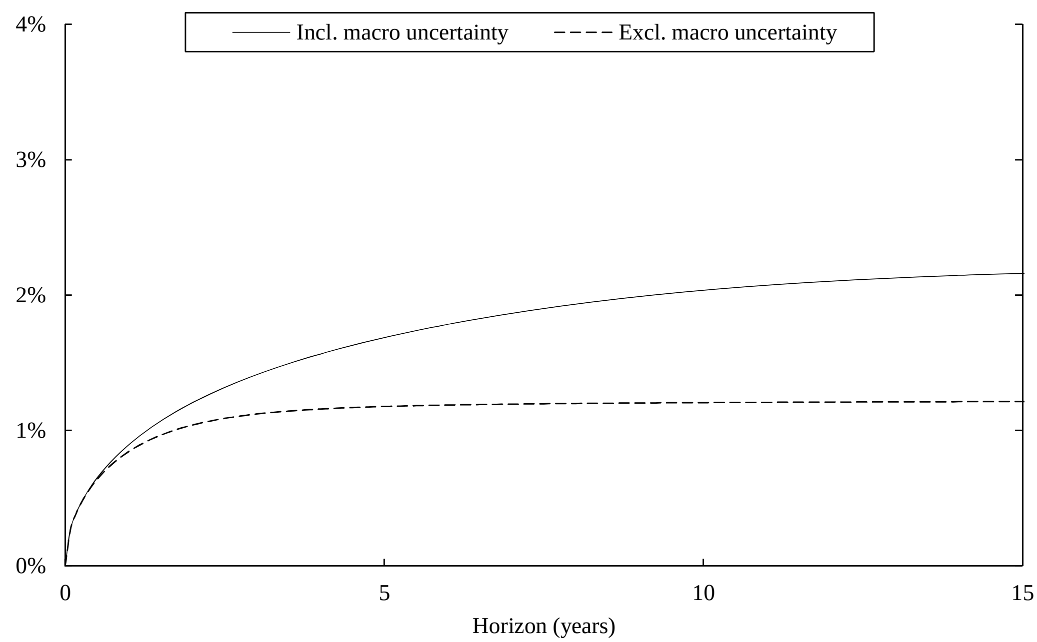


(b) December 2013

Figure 12: Confidence contours of the conditional time t joint pdf of the unemployment gap u_τ and the inflation π_τ at liftoff date τ. The gray dots represent the initial states (u_t, π_t). The unfilled black dots are the unconditional expectation under the stationary distribution, $\mathbf{E}[(u, \pi)]$. The black paths trace the expectation $\mathbf{E}_t[(u_{t+s}, \pi_{t+s})]$, with each arrow marking one year. The gray paths similarly trace the expectation $\mathbf{E}_t[(u_{t+s}, \pi_{t+s})|\tau > s]$.



(a) $\text{se}^{\mathbb{P}}(r_{t+s}|Z_t)$ (solid line) and $\text{se}^{\mathbb{P}}(r_{t+s}|Z_t, M_{t+s})$ (dashed line).



(b) $\text{se}^{\mathbb{P}}(y^{10}_{t+s}|Z_t)$ (solid line) and $\text{se}^{\mathbb{P}}(y^{10}_{t+s}|Z_t, M_{t+s})$ (dashed line).

Figure 13: Model-implied conditional standard deviation of the shadow short rate r_{t+s} (top panel) and the shadow ten-year yield y^{10}_{t+s} (bottom panel) as a function of the forecast horizon s. The solid lines plot the standard deviation when conditioning only on the information available at the initial time t. The dashed lines represent the standard deviation when also conditioning on the realization of the future time $t+s$ macro variables.

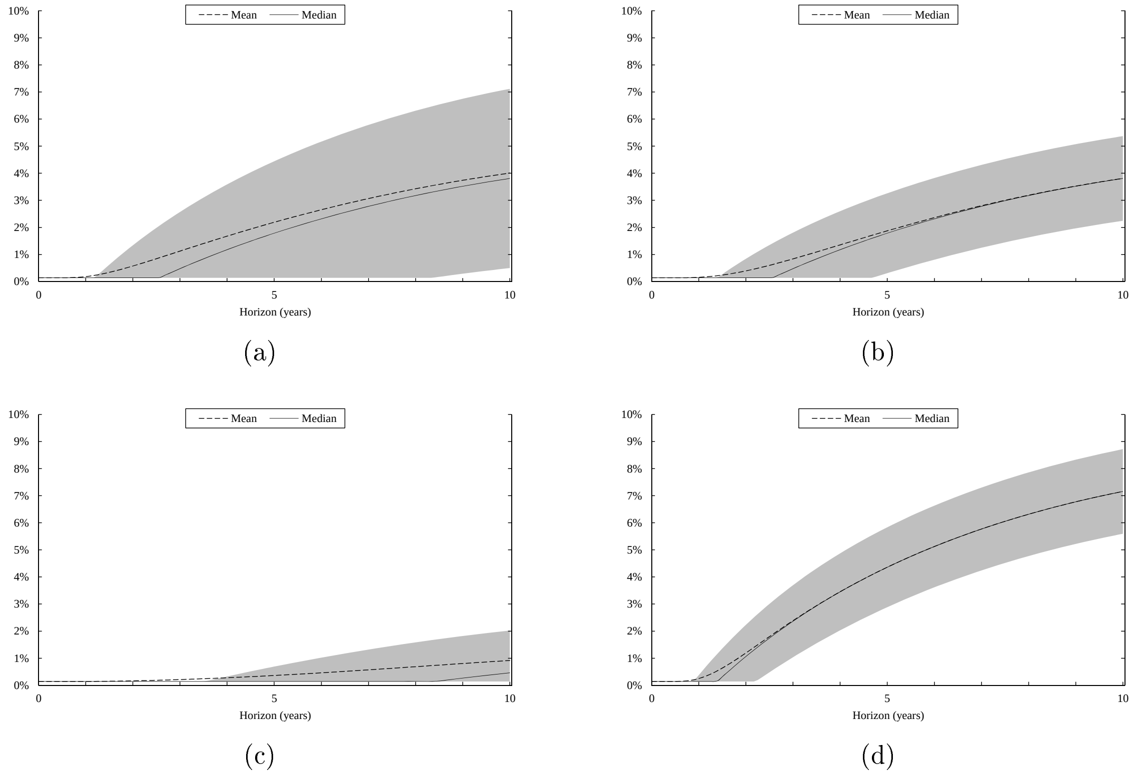
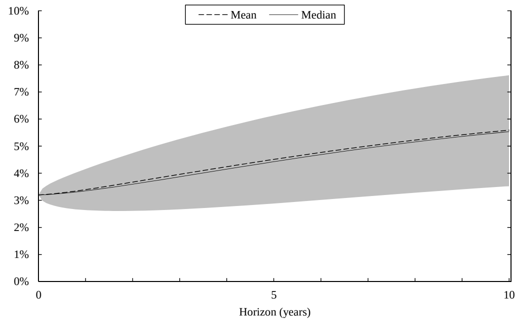


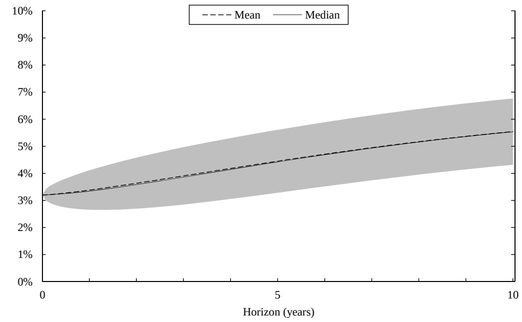
Figure 14: Mean (dashed lines) and median (solid lines) forecasts of the short rate, $\underline{r}_{t+s}$, and 70 percent prediction intervals (the shaded areas), as of December 2013. The top-left panel conditions only on yield and macro information at the time of the forecast. The top-right panel conditions on future macro variables realizing as expected at the time of the forecast. The bottom panels condition on future high unemployment and low inflation, and future low unemployment and high inflation.

ing on current yield and macro variables, there is substantial uncertainty around the short-rate forecast. The top-right panel illustrates that this uncertainty is reduced substantially if we condition on future macro variables being at their time t expected values. This leaves the median forecast for the future short rate unchanged, while slightly lowering the mean forecast through a Jensen's inequality effect. The bottom-left and bottom-right panels in the figure condition on economic scenarios associated with low and high future expected rates, respectively. In particular, the bottom-left panel is conditioned on the future unemployment gap being one standard deviation above its time t expected value, and future inflation being one standard deviation below. The bottom-right panel represents the reverse scenario.

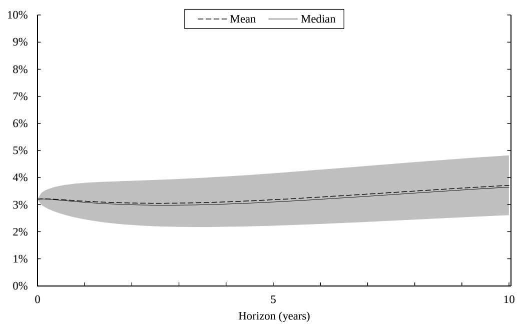
Figure 15 goes through the same exercise for the ten-year yield. Figure 16 similarly



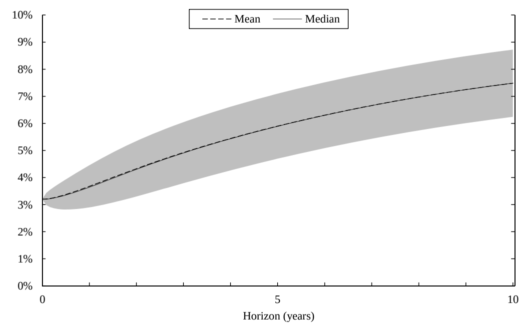
(a)



(b)



(c)



(d)

Figure 15: Mean (dashed lines) and median (solid lines) forecasts of the ten-year yield, $\underline{y}_{t+s}^{10}$, and 70 percent prediction intervals (the shaded areas), as of December 2013. The top-left panel conditions only on yield and macro information at the time of the forecast. The top-right panel conditions on future macro variables realizing as expected at the time of the forecast. The bottom panels condition on future high unemployment and low inflation, and future low unemployment and high inflation.

considers conditional and unconditional forecasts of the entire yield curve, based on the stationary distribution of the state variables.

7 Conclusion

This paper estimates a shadow-rate term structure model with unspanned macro variables and uses the model to analyze recent monetary policy developments as reflected in the term structure of U.S. Treasury yields. Based on the model-implied relationships between macro variables and observed short rates, short rate expectations, and long-term yields, only the ten-year yield appears to have been unusually low during parts of the ZLB period. The model further implies a positive correlation between in-

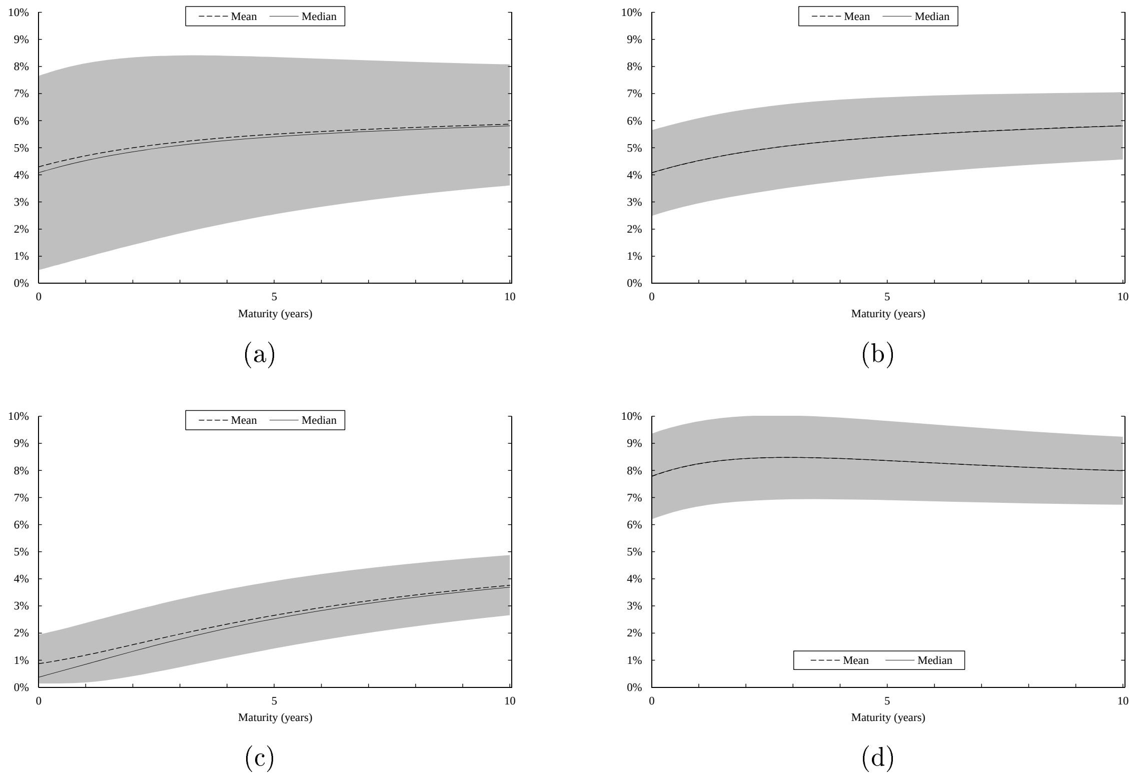


Figure 16: Mean (dashed lines) and median (solid lines) forecasts of the yield curve, $\underline{y}_t^{t+\tau}$, and 70 percent prediction intervals (the shaded areas), as a function of maturity τ. The top-left panel is unconditional. The top-right panel is conditional on macro variables being at their unconditional means. The bottom panels condition on high unemployment and low inflation, and low unemployment and high inflation.

flation and unemployment at the (stochastic) future time of short-rate liftoff from the ZLB. Conditional on remaining in the ZLB state, the model predicts convergence of the macro variables to a state of secular stagnation, with a persistent unemployment gap and below-target inflation.

Consistent with the findings in Laubach and Williams (2003), the model suggests that the neutral real short rate varies substantially over time. As of December 2013, it is slightly negative and at the low end of its historical range. The model also implies a substantial degree of inertia in short rate expectations, especially with respect to inflation. Finally, the model suggests that a substantial portion of medium- to long-term interest rate uncertainty is due to macro uncertainty. Large parts of the "black-box" prediction uncertainty around unconditional interest rate forecasts can be accounted for by conditioning on specific macroeconomic scenarios.

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