

# Liquidity Trap and Excessive Leverage\*

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## Abstract

We investigate the role of macroprudential policies in mitigating liquidity traps driven by deleveraging, using a simple Keynesian model. When constrained agents engage in deleveraging, the interest rate needs to fall to induce unconstrained agents to pick up the decline in aggregate demand. However, if the fall in the interest rate is limited by the zero lower bound, aggregate demand is insufficient and the economy enters a liquidity trap. In such an environment, agents' *ex-ante* leverage and insurance decisions are associated with aggregate demand externalities. The competitive equilibrium allocation is constrained inefficient. Welfare can be improved by *ex-ante* macroprudential policies such as debt limits and mandatory insurance requirements. The size of the required intervention depends on the differences in marginal propensity to consume between borrowers and lenders during the deleveraging episode. Contractionary monetary policy is inferior to macroprudential policy in addressing excessive leverage, and it can even have the unintended consequence of increasing leverage.

**JEL Classification:** E32, E4

**Keywords:** Leverage, liquidity trap, zero lower bound, aggregate demand externality, efficiency, macroprudential policy, insurance.

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# 1 Introduction

Leverage has been proposed as a key contributing factor to the recent recession and the slow recovery in the US. Figure 1 illustrates the dramatic rise of leverage in the household sector before 2008 as well as the subsequent deleveraging episode. Using county-level data, Mian and Sufi (2012) have argued that household deleveraging is responsible for much of the job losses between 2007 and 2009. This view has recently been formalized in a number of theoretical models, e.g., Hall (2011), Eggertsson and Krugman (2012), and Guerrieri and Lorenzoni (2012). These models have emphasized that the interest rate needs to fall when constrained agents engage in deleveraging to induce unconstrained agents to make up for the lost aggregate demand. However, the nominal interest rate cannot fall below zero given that hoarding cash provides an alternative to holding bonds—a phenomenon also known as the *liquidity trap*. When inflation expectations are sticky, the lower bound on the nominal rate also prevents the real interest rate from declining, plunging the economy into a demand-driven recession. Figure 2 illustrates that the short term nominal and real interest rates in the US has indeed seemed constrained since December 2008.

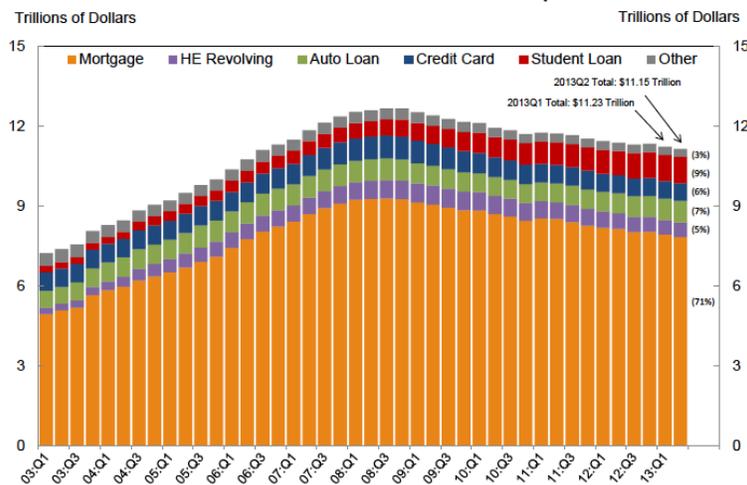


Figure 1: Evolution of household debt in the US over the last 10 years. Source: Quarterly Report on Household Debt and Credit (August 2013), Federal Reserve Bank of New York.

An important question concerns the optimal policy response to these episodes. The US Treasury and the Federal Reserve have responded to the recent recession by utilizing fiscal stimulus and unconventional monetary policies. These policies are (at least in part) supported by a growing theoretical literature that emphasizes the

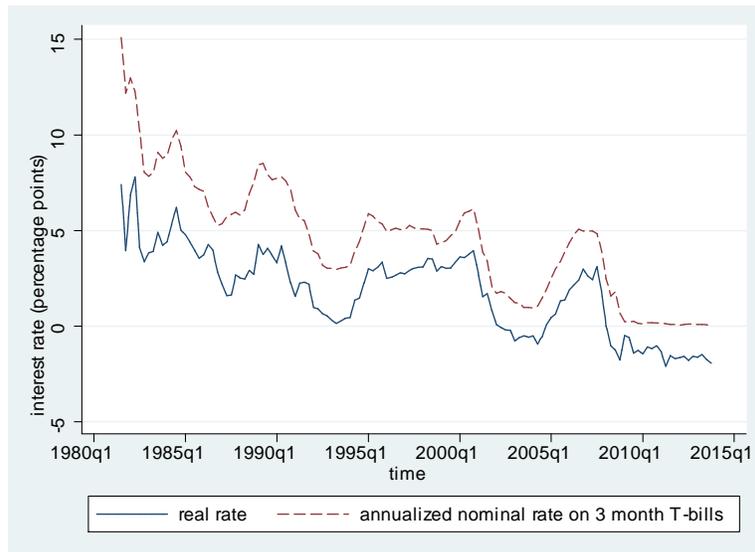


Figure 2: Nominal and real interest rates on 3 month US Treasury Bills between the third quarter of 1981 and the fourth quarter of 2013. The real interest rate is calculated as the annualized nominal rate minus the annualized current-quarter GDP inflation expectations obtained from the Philadelphia Fed’s Survey of Professional Forecasters.

benefits of stimulating aggregate demand during a liquidity trap. The theoretical contributions have understandably taken an ex-post perspective—characterizing the optimal policy once the economy is in the trap. Perhaps more surprisingly, both the practical and theoretical policy efforts have largely ignored the debt market, even though the problems are thought to have originated in the debt market.<sup>1</sup> In this paper, we analyze the scope for *ex-ante* macroprudential policies in debt markets—such as debt limits and insurance requirements.

To investigate optimal macroprudential policies, we present a tractable model, in which a tightening of borrowing constraints leads to deleveraging and may trigger a liquidity trap. The distinguishing feature of our model is that some agents, which we call borrowers, endogenously accumulate leverage—even though agents are aware that borrowing constraints will be tightened in the future. If borrowers have a sufficiently strong motive to borrow, e.g., due to impatience, then the economy enters a liquidity trap and features an anticipated demand driven recession.

<sup>1</sup>Several papers capture the liquidity trap in a representative household framework which leaves no room for debt market policies (see Eggertsson and Woodford (2003), Christiano et al. (2011), Werning (2012)). An exception is Eggertsson and Krugman (2011), which features debt but does not focus on debt market policies.

Our main result is that it is desirable to slow down the accumulation of leverage in these episodes. In the run-up to a liquidity trap, borrowers who behave individually rationally undertake *excessive leverage* from a social point of view. A simple macroprudential policy that restricts leverage (coupled with appropriate ex-ante transfers) could make all agents better off. This result obtains whenever ex-post deleveraging is severe enough to trigger a liquidity trap—assuming that the liquidity trap cannot be fully alleviated by ex-post policies.

The mechanism behind the constrained inefficiency is an *aggregate demand externality* that applies in environments in which output is influenced by aggregate demand. When this happens, agents' decisions that affect aggregate demand also affect aggregate output, and therefore other agents' income. Agents do not take into account these general equilibrium effects, which may lead to inefficiencies. In our economy, the liquidity trap ensures that output is influenced by demand and that it is below its (first-best) efficient level. Moreover, greater ex-ante leverage leads to a greater ex-post reduction in aggregate demand and a deeper recession. This is because deleveraging transfers liquid wealth from borrowers to lenders, but borrowers who delever have a much higher marginal propensity to consume (MPC) out of liquid wealth than lenders. Borrowers who choose their debt level (and lenders who finance them) do not take into account the negative demand externalities, leading to excessive leverage. In line with this intuition, we also show that the strength of the inefficiency—and therefore the size of the required intervention—depends on the MPC differences between borrowers and lenders.

Our model also provides a natural setting to contrast aggregate demand externalities with traditional *pecuniary externalities*. When borrowers' leveraging motive is relatively weak, the real interest rate during the deleveraging episode remains positive and the economy avoids the liquidity trap. In this region, ex-ante accumulation of leverage generates pecuniary externalities by lowering the ex-post real interest rate. These pecuniary externalities are harmful for lenders, who earn lower rates on their assets, but they are beneficial for borrowers, who pay lower interest rate on their debt. Indeed, the pecuniary externalities in our setting net out since markets are complete and, absent a liquidity trap, the equilibrium is constrained efficient. In contrast, when there is a liquidity trap, the pecuniary externalities are muted since the real interest rate is fixed at its lower bound, and greater leverage generates aggregate demand externalities. Unlike pecuniary externalities, aggregate demand externalities hurt all agents—since they operate by lowering incomes—which opens the door for inefficiencies.

In practice, the deleveraging episodes are highly uncertain from an ex-ante point of view. A natural question is whether agents share the risk associated with these episodes efficiently. Our second main result establishes that borrowers are also *under-insured* with respect to a deleveraging episode. A mandatory insurance requirement (coupled with ex-ante transfers) could make all households better off. Intuitively, borrowers' insurance purchases transfers liquid wealth in the deleveraging episode from lenders (or insurance providers) to borrowers who have a higher MPC. This increases aggregate demand and mitigates the recession. Agents do not take into account these positive aggregate demand externalities, which leads to too little insurance. Among other things, this result provides a rationale for indexing mortgage liabilities to house prices (along the lines proposed by Shiller and Weiss, 1999).

We also investigate whether preventive monetary policies could be used to address aggregate demand externalities generated by leverage. A common argument is that a contractionary policy that raises the interest rate in the run-up to the recent sub-prime crisis could have been beneficial in curbing leverage. Perhaps surprisingly, our model reveals that raising the interest rate during the leverage accumulation phase can have the unintended consequence of *increasing* leverage. A higher interest rate reduces borrowers' incentives to borrow keeping all else equal—which appears to be the conventional wisdom informed by partial equilibrium reasoning. However, the higher interest rate also creates a temporary recession which increases borrowers' incentives to borrow so as to smooth consumption. In addition, the higher interest rate also transfers wealth from borrowers to lenders, which further increases borrowers' incentives to borrow. In our model, the general equilibrium effects typically dominate (for example for constant elasticity preferences), and raising the interest rate has the perverse effect of raising leverage. This may contribute to explaining the continued increase in household leverage when the US Fed raised interest rates starting in June 2004, as illustrated in Figures 1 and 2.

There are versions of our model in which the conventional wisdom holds, and raising the interest rate lowers leverage (as in Curdia and Woodford, 2009). But even in these cases, the interest rate policy is inferior to macroprudential policies in dealing with excessive leverage. Intuitively, efficiency requires setting a wedge between borrowers' and lenders' *relative* incentives to hold bonds, whereas the interest rate policy creates a different intertemporal wedge that affects all agents' incentives equally. As a by-product, the interest rate policy also generates an unnecessary recession—which is not a feature of constrained efficient allocations. That said, a different preventive monetary policy, namely raising the inflation target, is supported

by our model as it would reduce the incidence of liquidity traps—and therefore, the relevance of aggregate demand externalities.

Our final analysis concerns endogenizing the debt limit faced by borrowers by assuming that debt is collateralized by financial assets, creating the potential for fire-sale effects. This introduces a new feedback loop into the economy, with two main implications. First, higher leverage lowers asset prices in the deleveraging phase, which in turn lowers borrowers' debt capacity and increases their distress. Hence, higher leverage generates fire-sale externalities that operate in the same direction as aggregate demand externalities. Second, an increase in borrowers' distress induces a more severe deleveraging episode and a deeper recession. Hence, fire-sale externalities exacerbate aggregate demand externalities. Conversely, lower aggregate output further lowers asset prices, exacerbating fire-sale externalities. These observations suggest that episodes of deleveraging that involve asset fire-sales are particularly severe.

The remainder of this paper is structured as follows. The next subsection discusses the related literature. Section 2 introduces the key aspects of our environment. Section 3 characterizes an equilibrium that features an anticipated demand-driven recession. The heart of the paper is Section 4, which illustrates aggregate demand externalities, contrasts them with traditional pecuniary externalities, and presents our main result about excessive leverage. This section also relates the strength of the inefficiency to empirically observable variables. Section 5 generalizes the model to incorporate uncertainty and presents our second main result about underinsurance. Section 6 discusses the role of preventive monetary policies in our environment. Section 7 presents the extension with endogenous debt limits and fire sale externalities, and Section 8 concludes. The appendix contains omitted proofs and derivations as well as some extensions of our baseline model.

## 1.1 Related literature

Our paper is related to a long economic literature studying the zero lower bound on nominal interest rates and liquidity traps, starting with Hicks (1937), and more recently emphasized by Krugman (1998) and Eggertsson and Woodford (2003, 2004). A growing recent literature has investigated the optimal fiscal and monetary policy response to liquidity traps (see e.g. Eggertsson, 2011; Christiano et al., 2011; Werning, 2012; Correia et al., 2013). Our contribution to this literature is that we focus on debt market policies, mainly from an ex-ante perspective.

Eggertsson and Krugman (2012) and Guerrieri and Lorenzoni (2012) describe how

financial market shocks that induce borrowers to delever lead to a decline in interest rates, which in turn can trigger a liquidity trap. Our framework is most closely related to Eggertsson and Krugman because we also model deleveraging between a set of impatient borrowers and patient lenders. They focus on the ex-post implications of deleveraging as well as the effects of monetary and fiscal policy during these episodes. Our contribution is to add an ex-ante stage and to investigate the role of macroprudential policies. Among other things, our paper calls for novel policy actions in debt markets that are significantly different from the more traditional policy responses to liquidity traps. Our paper also differs in terms of methodology: Instead of the New-Keynesian framework, we use a simple equilibrium concept with rationing in the goods market to describe liquidity traps, which enables us to obtain a sharp analytical characterization of the inefficiencies in debt markets.

The aggregate demand externality we focus on has first been discovered in the context of firms' price setting decisions, e.g., by Mankiw (1985), Akerlof and Yellen (1985) and Blanchard and Kiyotaki (1987). The broad idea is that, when output is not at its first-best level and influenced by aggregate demand, decentralized allocations that affect aggregate demand are socially inefficient. In Blanchard and Kiyotaki, output is not at its first-best level due to monopoly distortions, and firms' price setting affects aggregate demand due to complementarities in firms' demand. In our setting, output is not at its first-best level due to the liquidity trap. We also focus on agents' debt choices—as opposed to firms' price setting decisions—which affect aggregate demand due to differences in agents' marginal propensities to consume.

A number of recent papers, e.g., Farhi and Werning (2012ab, 2013) and Schmitt-Grohe and Uribe (2012abc), also analyze aggregate demand externalities in contexts similar to ours. Schmitt-Grohe and Uribe analyze economies with fixed exchange rates that exhibit downward rigidity in nominal wages. They identify negative aggregate demand externalities associated with actions that increase wages during good times, because these actions lead to greater unemployment during bad times. In Farhi and Werning (2012ab), output responds to aggregate demand because prices are sticky and countries are in a currency union (and thus, under the same monetary policy). They emphasize the inefficiencies in cross-country insurance arrangements. In our model, output is demand-determined because of a liquidity trap, and we emphasize the inefficiencies in household leverage in a closed economy setting. In parallel and independent work, Farhi and Werning (2013) develop a general theory of aggregate demand externalities in the presence of nominal rigidities and constraints on monetary policy, with applications including liquidity traps and currency unions. Our

framework falls into this broad class of aggregate demand externalities, but we focus in depth on the externalities created by deleveraging in a liquidity trap.

Our results on excessive borrowing and risk-taking also resemble the recent literature on pecuniary externalities, including Caballero and Krishnamurthy (2003), Lorenzoni (2008), Bianchi and Mendoza (2010), Jeanne and Korinek (2010ab) and Korinek (2011). In those papers, agents do not internalize the impact of individual decisions on asset prices. A planner can improve welfare by moving asset prices in a way that relaxes financial constraints. The aggregate demand externality of this paper works through a completely different channel. In fact, the externality applies not when prices are volatile, but in the opposite case when a certain price—namely the real interest rate—is fixed. We discuss the differences with pecuniary externalities further in Section 4, and illustrate the interaction of our mechanism with fire-sale externalities in Section 7.

## 2 Environment and equilibrium

The economy is set in infinite discrete time  $t \in \{0, 1, \dots\}$ , with a single consumption good. There are two types of households, borrowers and lenders, denoted by  $h \in \{b, l\}$ . There is an equal measure of each type of households, normalized to 1/2. Households are symmetric except that borrowers have a weakly lower discount factor than lenders,  $\beta^b \leq \beta^l$ .

Our central focus is to analyze how the debt or asset holdings of the two types of households interact with aggregate output. Let  $d_t^h$  denote the outstanding debt—or assets, if negative—of household  $h$  at date  $t$ . Households start with initial debt or asset levels denoted by  $d_0^h$ . At each date  $t$ , they face the one-period interest rate  $r_{t+1}$  and they choose their outstanding debt or asset levels for the next period,  $d_{t+1}^h$ .

Our first key ingredient is that, from date 1 onwards, households are subject to a borrowing constraint constraint, that is,  $d_{t+1}^h \leq \phi$  for each  $t \geq 1$ . Here,  $\phi > 0$  denotes an exogenous debt limit as in Aiyagari (1994), or more recently, Eggertsson and Krugman (2012). The constraint can be thought of as capturing a financial shock in reduced form, e.g. a drop in loan-to-value ratios or collateral values, that would force households to reduce their leverage. In contrast, we assume that households can choose  $d_1^h$  at date 0 without any constraints. The role of these ingredients is to generate household leveraging at date 0 followed by deleveraging at date 1 along the lines of Figure 1. Moreover, to study the efficiency of agents' ex-ante decisions, we assume that the future financial shock is anticipated at date 0. In our baseline

model, we abstract away from uncertainty so that the shock is perfectly anticipated. In Section 5, we will introduce uncertainty about the financial shock.

Our second key ingredient is a lower bound on the real interest rate:

$$r_{t+1} \geq \underline{r}_{t+1} \text{ for each } t. \quad (1)$$

As suggested by Figure 2, real interest rates in the US appear to be bounded from below in recent years. Japan had a similar experience during two decades of deflation. We take the lower bound as exogenous here and investigate its implications for aggregate allocations and optimal macroprudential policies. For our baseline analysis, we normalize the lower bound to zero (see Section 6 for the effects of changing the lower bound).

In practice, the bound on the real interest rate emerges from a combination of the zero lower bound on the nominal interest rate and stickiness of inflation expectations.<sup>2</sup> The bound on the nominal interest rate is uncontroversial—as it emerges from a no-arbitrage condition between money and government bonds. There are different approaches to “microfounding” the stickiness of inflation expectations. We describe a particular microfoundation in Appendix A.1 in which monetary policy is set according to a standard Taylor rule designed to set inflation equal to a constant target. This leads to the bound in (1) with  $\underline{r}_{t+1}$  equal to the negative of the inflation target.<sup>3</sup> The New-Keynesian framework provides an alternative microfoundation by positing that nominal prices or wages are sticky—which naturally translates into stickiness of inflation, and thus, inflation expectations. Yet another microfoundation is provided by a model in which households are boundedly rational in a way that their inflation expectations are based on limited or past information, as documented in recent work by Malmendier and Nagel (2013). We remain agnostic about the source of the stickiness of inflation expectations by taking the lower bound in (1) as exogenous. Aside from being consistent with the recent US experience, this provides us with a tractable environment to obtain clean analytic insights into inefficiencies in debt markets.

The demand side of the model is described by households’ consumption-savings

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<sup>2</sup>These two ingredients, combined with the Fisher equation,  $1 + r_{t+1} = (1 + i_{t+1}) E_t \left[ \frac{P_t}{P_{t+1}} \right]$ , lead to a lower bound on the real interest rate.

<sup>3</sup>In fact, in our setting the Taylor rule with a zero inflation target is the optimal time-consistent policy if there is some cost of inflation: it is ex-post efficient although, ex-ante, it generates a bound on the real rate and a recession. So our explanation emphasizes the difficulty of monetary policymakers to commit to inflation.

decision.<sup>4</sup> For the baseline model, we assume households' state utility function over consumption  $\tilde{c}_t^h$  and labor  $n_t^h$  takes the particular form,  $u(\tilde{c}_t^h - v(n_t^h))$ . We define  $c_t = \tilde{c}_t^h - v(n_t^h)$  as *net consumption*. Households' optimization problem can then be written as:

$$\begin{aligned} \max_{\{c_t^h, d_{t+1}^h, n_t^h\}_t} \quad & \sum_{t=0}^{\infty} (\beta^h)^t u(c_t^h) \\ \text{s.t. } \quad & c_t^h = e_t^h - d_t^h + \frac{d_{t+1}^h}{1+r_{t+1}} \text{ for all } t, \\ \text{where } \quad & e_t^h = w_t n_t^h + \Pi_t - v(n_t^h) \text{ and } d_{t+1}^h \leq \phi_{t+1} \text{ for each } t \geq 1. \end{aligned} \quad (2)$$

Here  $w_t$  denotes wages,  $\Pi_t$  denotes profits from firms that are described below, and  $e_t^h$  denotes households' *net income*, that is, their income net of labor costs. The preferences,  $u(\tilde{c}_t^h - v(n_t^h))$ , provide tractability but are not necessary for our main results (see Appendix A.5). As noted in Greenwood, Hercowitz and Huffman (GHH, 1988), the specification implies that there is no wealth effect on labor supply. As a result, the efficient output level is constant.

The supply side is described by a linear technology that can convert one unit of labor to one unit of the consumption good. The efficient level of net income is then given by:

$$e^* \equiv \max_{n_t} n_t - v(n_t).$$

However, the equilibrium does not necessarily feature efficient production due to the constraint in (1). When this constraint binds, the interest rate is too high relative to its market clearing level. Since the interest rate is the price of current consumption good (in terms of the future consumption good), an elevated interest rate leads to a demand shortage for current goods and a rationing of supply.

To capture the possibility of rationing, we modify the supply side of the Walrasian equilibrium to accommodate the constraint in (1). In particular, we consider a competitive goods sector that solves the following optimization problem:

$$\Pi_t = \max_{n_t} n_t - w_t n_t \quad \text{s.t.} \quad \begin{cases} 0 \leq n_t, & \text{if } r_{t+1} > 0 \\ 0 \leq n_t \leq \frac{\tilde{c}_t^h + c_t^h}{2}, & \text{if } r_{t+1} = \underline{r}_{t+1} \end{cases}. \quad (3)$$

When the real interest rate is above the lower bound,  $r_{t+1} > \underline{r}_{t+1}$ , the sector optimizes

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<sup>4</sup>To keep the analysis simple, we ignore investment and focus on inefficiencies associated household leveraging. But our main results also have implications for inefficiencies associated with investment and firms' leverage that we discuss in the concluding section.

as usual. When the interest rate is at its lower bound,  $r_{t+1} = \underline{r}_{t+1}$ , the sector is subject to an additional constraint that supply cannot exceed the aggregate demand for goods,  $\frac{\tilde{c}_t^b + \tilde{c}_t^l}{2}$ . When this constraint binds, the sector is making positive profits, and firms are in principle willing to increase their output. However, their output is rationed due to a shortage of aggregate demand. We assume that the available demand is allocated symmetrically across firms in this case. The equilibrium output is then determined by aggregate demand at the bounded interest rate,  $\underline{r}_{t+1}$ . We also assume households have equal ownership of firms so that each household receives profits,  $\Pi_t = n_t - w_t n_t$ .

**Definition 1** (Equilibrium). *The equilibrium is a path of allocations,  $\{c_t^h, d_{t+1}^h, n_t^h, e_t^h\}_t$ , and real prices and profits,  $\{w_t, r_{t+1}, \Pi_t\}_t$ , such that the household allocations solve problem (2), the final good sector solves problem (3) and markets clear.*

**Remark** (Comparison with Keynesian Models with rationing). Our equilibrium notion is similar to the rationing equilibria analyzed by a strand of the macroeconomics literature, e.g., Clower (1965), Barro and Grossman (1971), Malinvaud (1977), and Benassy (1986). We focus on the special case in which there is rationing in the goods market—and only when the lower bound on the real interest rate binds—but no rationing in the labor market. We adopt this case since it features the minimally required departure from a Walrasian equilibrium to capture a liquidity trap. Adding rationing to the labor market could further exacerbate the outcomes but would not change our qualitative conclusions.

**Remark** (Comparison with New-Keynesian models). Our equilibrium notion generates a similar rationing outcome as standard New-Keynesian models. There, monopolistic firms with pre-set prices (above their marginal costs) would be willing to supply the level of output demanded at the exogenous interest rate,  $r_{t+1} = \underline{r}_{t+1}$ . The main difference is that the New-Keynesian models generate an additional prediction that prices should fall during a liquidity trap. This prediction did not hold in the data for the most recent US experience, which stimulated a recent literature (see, for instance, Ball and Mazumder, 2011; Coibion and Gorodnichenko, 2013; Hall, 2013). Our equilibrium notion enables us to abstract away from inflation—and the ongoing debate about missing disinflation—so as to focus on real allocations in debt markets. A number of recent papers take an approach similar to ours, e.g., Hall (2011), Kocherlakota (2012), and Caballero and Farhi (2013).

### 3 An anticipated demand-driven recession

This section characterizes the decentralized equilibrium and describes a recession that is anticipated by households. The next section analyzes the efficiency properties of this equilibrium. We simplify the notation as follows. First note that equilibrium labor supply is the same for both types of households,  $n_t^h = (v')^{-1}(w_t)$ , which implies that their net income,  $e_t^h$ , is also the same. Hence, we let  $e_t = n_t - v(n_t)$  denote this common value of net income. Second, market clearing for debt implies  $d_t^l = -d_t^b$ . Hence, we drop the superscript and denote the debt level of borrowers at a given date by  $d_t^b = d_t$ , and that of lenders by  $d_t^l = -d_t$ .

To characterize the equilibrium, we normalize the lower bound on the interest rate in (1) to zero in the current section, that is, we set  $\underline{r}_{t+1} = 0$  for each  $t \geq 1$ . We investigate the effects of changes in the lower bound in Section 6. Furthermore, we set  $\underline{r}_1 = -\infty$  in the initial period, which enables us to abstract away from the possibility of a liquidity trap at date 0.<sup>5</sup> In addition, we make the standard assumptions about preferences: that is  $u(\cdot)$  and  $v(\cdot)$  are both strictly increasing,  $u(\cdot)$  is strictly concave and  $v(\cdot)$  is strictly convex, and they satisfy the conditions  $\lim_{c \rightarrow 0} u'(c) = \infty$ ,  $v'(0) = 0$  and  $\lim_{n \rightarrow \infty} v'(n) = \infty$ . We also assume  $\frac{u'(2e^*)}{u'(e^* + \phi(1 - \beta^l))} < \beta^l$ , which allows for the constraint on the real rate to bind.

**Steady state** We will focus on equilibria in which borrowers' constraint binds at all dates, that is,  $d_{t+1} = \phi$  for each  $t \geq 1$  (and lenders' constraints do not bind). To characterize these equilibria, first consider dates  $t \geq 2$ . At these dates, the economy is in a steady-state. Since borrowers are constrained, the real interest rate is determined by the discount factor of lenders and is constant at  $r_{t+1} = 1/\beta^l - 1 > 0$ . At a positive interest rate, aggregate demand is not a constraining factor and firms are optimizing as usual so that equilibrium wages are given by  $w_t = 1$  [cf. problem (3)]. The optimization problem of households (2) then implies that their net income is at its efficient level and consumption is given by:

$$c_t^b = e^* - \phi(1 - \beta^l) \quad \text{and} \quad c_t^l = e^* + \phi(1 - \beta^l) \quad \text{for } t \geq 2 \quad (4)$$

**Deleveraging** Next consider date  $t = 1$ . Borrowers' consumption is given by  $c_1^b = e_1 - \left(d_1 - \frac{\phi}{1+r_2}\right)$ . In particular, the larger the outstanding debt level  $d_1$  is relative

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<sup>5</sup>Alternatively, we could impose assumptions on parameters such as initial debt to rule out the possibility of a liquidity trap at date 0.

to the debt limit, the more borrowers are forced to reduce their consumption. The resulting slack in aggregate demand needs to be absorbed by an increase in lenders' consumption:

$$c_1^l = e_1 + \left( d_1 - \frac{\phi}{1 + r_2} \right).$$

Since lenders are unconstrained, their Euler equation holds

$$\frac{u'(c_1^l)}{\beta^l u'(c_2^l)} = 1 + r_2,$$

where  $c_2^l$  is characterized in (4). Hence, the increase in lenders' consumption at date 1 is mediated through a decrease in the real interest rate,  $r_2$ . The key observation is that the lower bound on the real interest rate effectively sets an upper bound on lenders' consumption in equilibrium,  $c_1^l \leq \bar{c}_1^l$ , given by the solution to

$$u'(\bar{c}_1^l) = \beta^l u'(e^* + \phi(1 - \beta^l)). \quad (5)$$

The equilibrium at date 1 then depends on the relative size of two terms:

$$d_1 - \phi \lesseqgtr \bar{c}_1^l - e^*.$$

The left hand side is the amount of deleveraging borrowers are forced into given that the borrowing limit falls to  $\phi$  (and the real rate is at its lower bound). The right hand side is the maximum amount of demand the unconstrained agents can absorb when the real rate is at its lower bound. If the left side is smaller than the right side, then the equilibrium features  $r_2 \geq 0$  and  $e_1 = e^*$ . In this case, the effects of deleveraging on aggregate demand are offset by a reduction in the real interest rate and aggregate supply is at its efficient level  $e^*$ . The left side of Figure 3 (the range corresponding to  $d_1 \leq \bar{d}_1$ ) illustrates this outcome.

Otherwise, equivalently when the outstanding debt level is strictly above a threshold

$$d_1 > \bar{d}_1 = \phi + \bar{c}_1^l - e^*, \quad (6)$$

then the constraint on the real rate binds,  $r_2 = 0$ . The interest rate cannot fall sufficiently to induce lenders to consume the efficient level of output. In this case, households' net consumption is given by  $c_1^b = e_1 - d_1 + \phi$  and  $c_1^l = \bar{c}_1^l$ . Firms' demand for labor is determined by aggregate demand for consumption,  $n_1 = \frac{\bar{c}_1^b + \bar{c}_1^l}{2}$ . Hence, households' net income,  $e_1 = n_1 - v(n_1)$ , is also determined by aggregate demand for

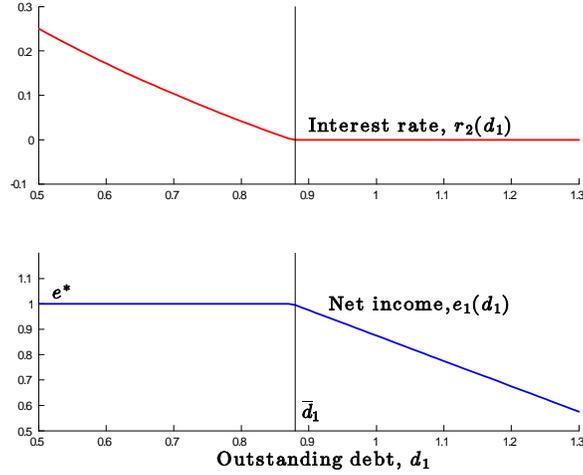


Figure 3: Interest rate and net income at date 1 as a function of outstanding debt  $d_1$ .

net consumption:

$$e_1 = \frac{c_1^b + c_1^l}{2} = \frac{e_1 - (d_1 - \phi) + \bar{c}_1^l}{2}. \quad (7)$$

After rearranging this expression, the equilibrium level of net income is given by:

$$e_1 = \bar{c}_1^l + \phi - d_1 < e^*. \quad (8)$$

In words, there is a demand shortage and rationing in the goods market, which in turn lowers wages and employment in the labor market, creating a demand driven recession. The right side of Figure 3 (the range corresponding to  $d_1 \geq \bar{d}_1$ ) illustrates this outcome.

Eq. (7) illustrates that there is a Keynesian cross and a Keynesian multiplier in our setting. The right hand side of Eq. (7) shows that an increase in borrowers' liquid wealth by one unit, e.g., through an increase in their net income, increases the aggregate demand by 1/2 units. This is because borrowers' population share is 1/2 and their marginal propensity to consume (MPC) out of liquid wealth is 1. The left hand side illustrates that net income is in turn determined by aggregate demand as in a typical Keynesian cross. This dependence also captures a Keynesian multiplier: An increase in borrowers' liquid wealth by one unit increases net income by 1/2 units, which in turn further increases borrowers' liquid wealth, which in turn increases net income by another 1/4 units, and so on.

Eq. (8) characterizes the equilibrium net income and illustrates that a greater level

of outstanding debt leads to a deeper recession. Intuitively, an increase in leverage transfers wealth at date 1 from borrowers to lenders. This in turn decreases aggregate demand and output since borrowers in our model have a much higher MPC of liquid wealth, namely 1, compared to lenders. The feature that borrowers' MPC is equal to 1 enables us to illustrate our inefficiency results sharply, but it is not necessary. Section 4.3 shows that net income is declining in outstanding debt,  $\frac{\partial e_1}{\partial d_1} < 0$ , as long as borrowers' MPC is greater than lenders' MPC. As we will see, this feature is all we need for aggregate demand externalities to be operational and to generate inefficiencies.

**Date 0 Allocations** We next turn to households' financial decisions at date 0. We conjecture an equilibrium in which the net income is at its efficient level,  $e_0 = e^*$ . Since households are unconstrained at date 0, the Euler equations of both of them hold

$$\frac{1}{1+r_1} = \frac{\beta^l u'(c_1^l)}{u'(c_0^l)} = \frac{\beta^b u'(c_1^b)}{u'(c_0^b)}. \quad (9)$$

The equilibrium debt level,  $d_1$ , and the interest rate,  $r_1$ , are determined by these equations. We next identify two conditions under which households choose a sufficiently high debt level that triggers a recession at date 1,  $d_1 > \bar{d}_1$ .

**Proposition 1.** *There is a deleveraging-induced recession at date 1 if the borrower is sufficiently impatient or sufficiently indebted at date 0. Specifically, for any debt level  $d_0$  there is a threshold level of impatience  $\bar{\beta}^b(d_0)$  such that the economy experiences a recession at date 1 if  $\beta^b < \bar{\beta}^b(d_0)$ . Conversely, for any level of impatience  $\beta^b$  there is a threshold debt level  $\bar{d}_0(\beta^b)$  such that the economy experiences a recession at date 1 if  $d_0 > \bar{d}_0(\beta^b)$ .*

We derive the relevant threshold levels in Appendix A.2. Under these conditions, the appendix establishes that the economy experiences a demand driven recession and liquidity trap at date 1.

## 4 Excessive leverage

This section analyzes the efficiency properties of equilibrium and presents our main result. We first illustrate the aggregate demand externalities in our setting. We then illustrate that the competitive equilibrium is constrained inefficient and that it can

be Pareto improved with simple macroprudential policies. The last part relates the strength of the inefficiency to the difference between borrowers' and lenders' MPCs.

## 4.1 Aggregate demand externalities

We consider a constrained planner at date 0 that can affect the amount of debt  $d_1$  that individuals carry into date 1 (through policies we will describe) but cannot interfere thereafter. We focus on constrained efficient allocations with  $d_1 \geq \phi$ , so that conditional on  $d_1$ , the economy behaves as we analyzed in the previous section for date 1 onwards.

Let  $V^h(d_1; D_1)$  denote the utility of a household of type  $h$  conditional on entering date 1 with an individual level of debt  $d_1$  and an aggregate level of debt  $D_1$ . The aggregate debt level  $D_1$  enters household utility because it determines the interest rate or net income at date 1. More specifically, we have:

$$\begin{aligned} V^b(d_1, D_1) &= u\left(e_1(D_1) - d_1 + \frac{\phi}{1 + r_2(D_1)}\right) + \sum_{t=2}^{\infty} (\beta^b)^t u(c_t^b) \\ V^l(d_1, D_1) &= u\left(e_1(D_1) + d_1 - \frac{\phi}{1 + r_2(D_1)}\right) + \sum_{t=2}^{\infty} (\beta^l)^t u(c_t^l) \end{aligned} \quad (10)$$

where  $r_2(D_1)$  and  $e_1(D_1)$  are characterized in the previous section and the continuation utilities from date 2 onwards do not depend on  $d_1$  or  $D_1$  [cf. Eq. (4)].

In equilibrium, we will find that  $D_1 = d_1$  since individual agents of type  $h$  are symmetric. But taking  $D_1$  explicitly into account is useful to illustrate the externalities. In particular, the private marginal value of debt for an individual household is given by  $\frac{\partial V^h}{\partial d_1} = u'(c_1^h)$ , whereas the social marginal value is  $\frac{\partial V^h}{\partial d_1} + \frac{\partial V^h}{\partial D_1}$ . Hence, the externalities from leverage in this setting are captured by  $\frac{\partial V^h}{\partial D_1}$ , which we characterize next.

**Lemma 1.** (i) If  $D_1 \in [\phi, \bar{d}_1)$ , then  $\frac{\partial V^h}{\partial D_1} = \begin{cases} -\eta u'(c_1^h) < 0, & \text{if } h = l \\ \eta u'(c_1^h) > 0, & \text{if } h = b \end{cases}$ , where  $\eta \in (0, 1)$ .

(ii) If  $D_1 > \bar{d}_1$ , then

$$\frac{\partial V^h}{\partial D_1} = \frac{\partial e_1}{\partial D_1} u'(c_1^h) = -u'(c_1^h) < 0, \text{ for each } h \in \{b, l\}. \quad (11)$$

The first part of the lemma illustrates the usual pecuniary externalities on the

interest rate in the case in which the debt level is sufficiently low so that output is not influenced by demand, that is  $e_1(D_1) = e^*$ . Higher aggregate debt  $D_1$  induces greater deleveraging at date 1. This reduces the interest rate to counter the reduction in demand. The reduction in the interest rate in turn generates a redistribution from lenders to borrowers (captured by  $\eta$ , which is characterized in the appendix). Consequently, deleveraging imposes positive pecuniary externalities on borrowers but negative pecuniary externalities on lenders. In fact, since markets between date 0 and 1 are complete, these two effects “net out” from an ex-ante point of view. In particular, the date 0 equilibrium is constrained efficient in this region (see Proposition 2 below).

The second part of the lemma illustrates the novel force in our model, *aggregate demand externalities*, and contrasts them with pecuniary externalities. In this case, the debt level is sufficiently large so that the economy is in a liquidity trap, which has two implications. First, the interest rate is fixed,  $r_2(D_1) = 0$ , so that the pecuniary externalities do not apply. Second, net income is decreasing in leverage,  $\frac{\partial e_1}{\partial D_1} < 0$ , through a reduction in aggregate demand (see Figure 3). Consequently, an increase in aggregate leverage reduces agents’ welfare, which we refer to as an aggregate demand externality.

A noteworthy feature about this externality is that it hurts all agents—because it operates through lowering incomes. This feature suggests that, unlike pecuniary externalities, aggregate demand externalities can lead to constrained inefficiencies, which we verify next.

## 4.2 Excessive leverage

In our setting, the equilibrium can be Pareto improved by reducing leverage. One way of doing this is ex-post, by writing down borrowers’ debt. To see this, suppose lenders forgive some of borrowers’ outstanding debt so that leverage is reduced from  $d_1$  to the threshold,  $\bar{d}_1$ , given by Eq. (6). By our earlier analysis, the recession is avoided and the net income increases to its efficient level,  $e^*$ . Borrowers’ net consumption and welfare naturally increases after this intervention. Less obviously, lenders’ net consumption remains the same at the upper bound,  $\bar{c}_1^l$ . The debt writedown has a direct negative effect on lenders’ welfare by reducing their assets, as captured by  $-\frac{\partial V^l}{\partial d_1} = -u'(c_1^l) < 0$ . However, the debt writedown also has an indirect positive effect on lenders’ welfare through aggregate demand externalities. Lemma 1 shows that the externalities are sufficiently strong to fully counter the direct effect,  $-\frac{\partial V^l}{\partial D_1} = u'(c_1^l) > 0$ , leading to ex-post Pareto improvement.

From the lenses of our model, debt-writedowns are always associated with aggregate demand externalities. However, these externalities are not always sufficiently strong to lead to a Pareto improvement.<sup>6</sup> Furthermore, ex-post debt writedowns are difficult to implement in practice for a variety of reasons, e.g., legal restrictions, concerns with moral hazard, or concerns with the financial health of intermediaries (assuming that some lenders are intermediaries). Therefore we do not analyze our results on ex-post inefficiency further.

An alternative, and arguably more practical, way to reduce leverage is to prevent it from accumulating in the first place. This creates a very general scope for Pareto improvements. To capture this possibility, suppose households' date 0 leverage choices are subject to an additional constraint,  $d_1^h \leq D_1$ , where  $D_1$  is an endogenous debt limit (which will also be the equilibrium debt limit, hence the abuse of notation). To trace the constrained efficient frontier, we also allow for a transfer of wealth,  $T_0$ , at date 0 from lenders to borrowers so that the outstanding debt becomes  $d_0 - T_0$ .

Our main result characterizes the allocations that can be implemented with these policies. To state the result, consider a hypothetical planner that chooses the date 0 allocations of households,  $(c_0^h, n_0^h)_h$ , as well as the debt level carried into the next date,  $D_1$ , and leaves the remaining allocations starting date 1 to the market. We say that an allocation  $((c_0^h, n_0^h)_h, D_1)$  is *constrained efficient* if it is optimal according to this planner, that is, if it solves

$$\max_{((c_0^h, n_0^h)_h, D_1)} \sum_h \gamma^h (u(c_0^h) + \beta^h V^h(D_1, D_1)) \quad \text{s.t.} \quad \sum_h c_0^h = \sum_h n_0^h - v(n_0^h), \quad (12)$$

where  $\gamma^h \neq 0$  captures the relative welfare weight assigned to type  $h$  agents. We next characterize the constrained efficient allocations, and show that these allocations can be implemented with the simple policies described above.

**Proposition 2** (Excessive Leverage). *An allocation  $((c_0^h, n_0^h)_h, D_1)$ , with  $D_1 \geq \phi$ , is constrained efficient if and only if output at date 0 is efficient, i.e.,  $e_0 = e^*$ ; and the consumption and debt allocations satisfy one of the following:*

- (i)  $D_1 < \bar{d}_1$  and the Euler equation (9) holds.

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<sup>6</sup>For instance, with separable preferences,  $u(c) - v(l)$ , analyzed in the appendix, debt writedowns do not generate ex-post Pareto improvement. This is also the case for the extension analyzed in Section 4.3 with heterogeneous borrowers.

(ii)  $D_1 = \bar{d}_1$  the following inequality holds:

$$\frac{\beta^l u'(c_1^l)}{u'(c_0^l)} \geq \frac{\beta^b u'(c_1^b)}{u'(c_0^b)}. \quad (13)$$

Moreover, every constrained efficient allocation of this type can be implemented as a competitive equilibrium with the debt limit,  $d_1^h \leq \bar{d}_1$  for each  $h$ , combined with an appropriate ex-ante transfer,  $T_0$ .

The first part illustrates that equilibrium allocations in which  $d_1 < \bar{d}_1$  are constrained efficient. This part verifies that pecuniary externalities alone do not generate inefficiencies in our setting.

The second part, which is our main result, concerns equilibria in which  $d_1 \geq \bar{d}_1$  and aggregate demand externalities are active (on the margin). Constrained efficient allocations in this region are characterized by the debt level,  $D_1 = \bar{d}_1$ , and the inequality in (13), which we refer to as the *distorted Euler inequality*. As this inequality illustrates, at the optimal allocation borrowers would like to borrow—so as to increase their consumption at date 0 and reduce their consumption at date 1—but they are prevented from doing so by the planner. Indeed, the efficient allocations can be implemented by a simple debt limit applied to all agents (combined with an appropriate ex-ante transfer). A corollary is that the competitive equilibrium characterized in Proposition 1, which features  $d_1 > \bar{d}_1$  and the Euler equation (9), is constrained inefficient.

To understand the intuition for the inefficiency, observe that lowering debt when the economy is in a liquidity trap generates first-order welfare benefits because of positive aggregate demand externalities, as illustrated in Lemma 1. By contrast, distorting agents' consumption levels away from their privately optimal Euler equations in (9) generates locally second order losses. Given an appropriate date 0 transfer, everybody can be made better off. Intuitively, borrowers that choose their leverage (or equivalently, lenders that finance them) do not take into account the adverse general equilibrium effects on demand and output at date 1. A debt limit internalizes these externalities and leads to an ex-ante Pareto improvement. This policy naturally tilts the date 1 consumption from lenders to borrowers (and date 0 consumption from borrowers to lenders), as captured by the distorted Euler inequality (13). In our baseline setting, the externalities are so strong that the planner avoids the recession fully, that is, it is never optimal to choose  $D_1 > \bar{d}_1$ .

The ex-ante inefficiency result in Proposition 2 applies quite generally—except for the part that the recession is avoided fully (in general, the planner finds it optimal

to mitigate but not completely avoid the recession). For example, appendix A.5 establishes an analogous result for the case with separable preferences,  $u(c) - v(n)$ . We next present a different generalization of the result, which is also useful to gauge the magnitude of the inefficiency.

### 4.3 MPC differences and the magnitude of the inefficiency

Our analysis so far had the feature that borrowers' marginal propensity (MPC) to consume out of liquid wealth is equal to 1. This feature is useful to illustrate our welfare results sharply, but it is rather extreme. We next analyze a version of our model in which borrowers' MPC can be flexibly parameterized. This version is useful, among other things, to relate the strength of aggregate demand externalities to empirically observable variables.

The main difference is that there are now two groups of borrowers. These groups are identical except for their MPCs at date 1. A fraction  $\alpha \in [0, 1]$  of borrowers, denoted by type  $b_{high}$ , have high MPC at date 1 as before, while the remaining fraction, denoted by  $b_{low}$ , have lower MPC. Hence, borrowers as a group have an average MPC that is lower than 1 and that depends on the parameter  $\alpha$ . In practice, there are many sources of heterogeneity among borrowers that could generate MPC differences along these lines (e.g., heterogeneity in income shocks). In our analysis, we find it convenient to focus on heterogeneity in borrowers' constraints. Formally, suppose all borrowers are identical at date 0 but they face different constraints starting date 1. Type  $b_{high}$  borrowers are identical to the borrowers we have analyzed so far. In particular, they face the exogenous debt limit,  $\phi$ , described earlier. In contrast, type  $b_{low}$  borrowers are unconstrained at all dates.

To illustrate the basic effect of these changes at date 1, suppose all agents have log utility, that is,  $u(c) = \log c$ . Suppose also that type  $b_{low}$  borrowers have the same discount factor as lenders starting date 1, that is,  $\beta^{b_{low}} = \beta^l \equiv \beta$  (As before, all borrowers at date 0 have the discount factor  $\beta^b \leq \beta^l$ ). Let  $MPC_1^h$  denote the increase in type  $h$  households' consumption at date 1 in response to a transfer of one unit of liquid wealth at date 1, keeping their wages and interest rates at all dates constant. In view of log utility, lenders and type  $b_{high}$  borrowers consume a small and constant fraction of the additional income they receive—consistent with the permanent income hypothesis. More specifically

$$MPC_1^l = MPC_1^{b_{low}} = 1 - \beta. \tag{14}$$

In contrast, since type  $b_{high}$  borrowers are at their constraint, they consume all of the additional income,  $MPC_1^{b_{high}} = 1$ . Hence, the marginal propensity to consume of borrowers as a group is given by:

$$MPC_1^b \equiv \alpha + (1 - \alpha)(1 - \beta). \quad (15)$$

In particular, the parameter  $\alpha$  enables us to calibrate the MPC differences between borrowers and lenders.

To characterize the general equilibrium, we make a couple more simplifying assumptions—that can be dropped at the expense of additional notation. First, suppose borrowers do not know their types,  $\{b_{high}, b_{low}\}$ , at date 0 and receive this information at date 1. Second, borrowers also cannot trade assets whose payoffs are contingent on their idiosyncratic type realizations. These assumptions ensure each borrower enters date 1 with the same amount of outstanding debt, denoted by  $d_1$  as before. As before, there is a threshold debt level  $\bar{d}_1$ , such that the equilibrium features a liquidity trap only if  $d_1 > \bar{d}_1$ . The analysis in the appendix further shows that

$$\frac{\partial e_1}{\partial d_1} = -\frac{\alpha}{2 - \alpha} = -\frac{MPC_1^b - MPC_1^l}{2 - (MPC_1^b + MPC_1^l)}. \quad (16)$$

As before, an increase in outstanding leverage at date 1 leads to a deeper recession. Moreover, the strength of the effect depends on MPC differences between borrowers and lenders. Intuitively, greater leverage influences aggregate demand by transferring wealth at date 1 from borrowers to lenders. This transfer affects aggregate demand, and thus output, more when there is a greater difference between borrowers' and lenders' MPCs.

It is also instructive to consider the planner's constrained optimality condition—the analogue of Eq. (13) in this case—given by:

$$\frac{\beta^l u'(c_1^l)}{(1 - MPC_1^l) u'(c_0^l)} = \frac{\beta^b E_0 [u'(c_1^b)]}{(1 - MPC_1^b) u'(c_0^b)} \text{ for each } D_1 > \bar{d}_1, \quad (17)$$

where the expectation operator  $E_0[\cdot]$  is taken over borrowers' types at date 1. Observe that the planner weighs type  $h$  agents' consumption at date 1 by a factor  $\frac{1}{1 - MPC_1^h}$ . Given that borrowers have a higher MPC, the planner distorts agents' Euler equations towards providing more consumption to borrowers at date 1. Moreover, the planner's optimal intervention—measured as a wedge between borrowers' and lenders' Euler equations—also depends on the MPC differences between borrowers and lenders.

The empirical literature shows that borrowers' MPC was indeed significantly greater than lenders' MPC in the recent deleveraging episode. For example, Baker (2013) finds that a one standard deviation increase in a household's debt-to-asset ratio raises its MPC by about 20% (about 7 percentage points from a baseline of 37 percentage points—for a sample with median debt to asset ratio of approximately 0.4). See also the survey by Jappelli and Pistaferri (2010) and recent papers by Mian et al. (2013) and Parker et al. (2013). Our analysis suggest that the results from this literature can be used to guide optimal macroprudential policy.

## 5 Uncertainty and underinsurance

We next analyze the efficiency of households' insurance arrangements against deleveraging episodes. This requires extending our analysis to incorporate uncertainty. To this end, consider the baseline setting with a single type of borrower, but suppose the economy is in one of two states  $s \in \{H, L\}$  from date 1 onwards. The states differ in their debt limits. State  $L$  captures a deleveraging state with a debt limit as before,  $\phi_{t+1,L} \equiv \phi$  for each  $t \geq 1$ . State  $H$  in contrast captures an unconstrained state similar to date 0 of the earlier analysis, that is,  $\phi_{t+1,H} = \infty$  for each  $t \geq 1$ . We let  $\pi_s^h$  denote the belief of type  $h$  households for state  $s$ . We assume  $\pi_L^h > 0 \forall h$  so that the deleveraging episode is anticipated by all households.

We simplify the analysis by assuming that starting date 1, both types of households have the same discount factor  $\beta^b = \beta^l = \beta$ .<sup>7</sup> At date 0, however, borrowers are weakly more impatient than lenders,  $\beta_0^b \leq \beta_0^l$ . In addition, we also assume borrowers are (weakly) more optimistic than lenders about the likelihood of the unconstrained state,  $\pi_H^b \geq \pi_H^l$ . Neither of these assumptions is necessary, but since impatience/myopia and excessive optimism were viewed as important contributing factors to many deleveraging crises, they enable us to obtain additional interesting results.

At date 0, households are allowed to trade in a complete market of one-period ahead Arrow securities. Let  $q_{1,s}$  denote the price of an Arrow security that pays 1 unit of consumption good in state  $s \in \{H, L\}$  of date 1. Let  $d_{1,s}^h$  denote the security issuance of household  $h$  contingent on state  $s \in \{H, L\}$ . Household  $h$  raises  $\sum_s q_{1,s}^h d_{1,s}^h$  units of consumption good at date 0. Observe that the real interest rate at date 0 satisfies  $1 + r_1 = 1 / \sum_s q_{1,s}$ . Given this notation, the optimization problem of house-

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<sup>7</sup>This ensures that the equilibrium is non-degenerate in the high state  $H$ . Alternatively, we could impose a finite debt limit  $\phi_{t+1,H} < \infty$ .

holds and the definition of equilibrium generalize to uncertainty in a straightforward way.

The equilibrium in state  $L$  of date 1 conditional on debt level  $d_{1,s}$  is the same as described as before. In particular, the interest rate is zero and there is a demand driven recession as long as the outstanding debt level is sufficiently large,  $d_{1,L} > \bar{d}_1$ . The equilibrium in state  $H$  jumps immediately to a steady-state with interest rate  $1 + r_{t+1} = 1/\beta > 0$  and consumption  $c_{t,H}^b = e^* - (1 - \beta) d_{1,H}^b \forall t \geq 1$ .

The main difference concerns households' date 0 choices. In this case, households' allocations satisfy not only the analogue of the Euler equation (9) but also a *full-insurance equation* across the two states:

$$\frac{q_{1,H}}{q_{1,L}} = \frac{\pi_H^l u'(c_{1,H}^l)}{\pi_L^l u'(c_{1,L}^l)} = \frac{\pi_H^b u'(c_{1,H}^b)}{\pi_L^b u'(c_{1,L}^b)}. \quad (18)$$

We next describe under which conditions households choose a sufficiently high debt level for state  $L$  to trigger a recession,  $d_{1,L} > \bar{d}_1$ :

**Proposition 3.** *There is a deleveraging-induced recession in state  $L$  of date 1 if the borrower is either (i) sufficiently impatient or (ii) sufficiently indebted or (iii) sufficiently optimistic at date 0. Specifically, for any two of the parameters  $(\beta_0^b, d_0, \pi_L^b)$ , we can determine a threshold for the third parameter such that  $d_{1,L} > \bar{d}_1$  if the threshold is crossed, i.e. if  $\beta_0^b < \bar{\beta}_0^b(d_0, \pi_L^b)$  or  $d_0 > \bar{d}_0(\beta_0^b, \pi_L^b)$  or  $\pi_L^b < \bar{\pi}_L^b(\beta_0^b, d_0)$ .*

The thresholds are characterized in more detail in Appendix A.2. The first two cases are analogous to the cases in Proposition 1: if borrowers have a strong reason to take on leverage, they also place some of their debt in state  $L$ , even though this triggers a recession. The last case identifies a new factor that could exacerbate this outcome. If borrowers assign a sufficiently low probability to state  $L$ , relative to lenders, then they naturally have more debt outstanding in state  $L$  as opposed to state  $H$ . In each scenario,  $d_{1,L} > \bar{d}_1$  and there is a recession in state  $L$  of date 1.

To analyze constrained efficiency of this equilibrium, consider a planner who chooses households' allocations at date 0 and the outstanding leverage at date 1, but leaves the remaining allocations to the market. As before, we will see that the allocations chosen by this planner can be implemented with simple debt market policies.

The constrained planning problem can be written as:

$$\max_{((c_0^h, n_0^h)_h, (D_{1,s})_s)} \sum_h \gamma^h \left( u(c_0^h) + \beta_0^h \sum_s^h V_s^h(D_{1,s}, D_{1,s}) \right) \text{ s.t. } \sum_h c_0^h = \sum_h n_0^h - v(n_0^h). \quad (19)$$

Our next result characterizes the solution to this problem.

**Proposition 4** (Underinsurance). *An allocation  $((c_0^h, n_0^h)_h, (D_{1,s})_s)$ , with  $D_{1,L} \geq \phi$ , is constrained efficient if and only if output at date 0 is efficient, i.e.,  $e_0 = e^*$ ; households' substitution between date 0 and state  $H$  of date 1 is efficient,  $\frac{\beta_0^l \pi_H^l u'(c_{1,H}^l)}{u'(c_0^l)} = \frac{\beta_0^b \pi_H^b u'(c_{1,H}^b)}{u'(c_0^b)}$ ; and the remaining consumption and leverage allocations satisfy one of the following:*

- (i)  $D_{1,L} < \bar{d}_1$  and the full insurance equation (18) holds.
- (ii)  $D_{1,L} = \bar{d}_1$  and the distorted insurance inequality holds:

$$\frac{\pi_H^l u'(c_{1,H}^l)}{\pi_L^l u'(c_{1,L}^l)} \geq \frac{\pi_H^b u'(c_{1,H}^b)}{\pi_L^b u'(c_{1,L}^b)} \quad (20)$$

Moreover, every constrained efficient allocation of this type can be implemented as a competitive equilibrium with the mandatory insurance requirement,  $d_{1,L}^h \leq \bar{d}_1$  for each  $h$ , combined with an appropriate ex-ante transfer,  $T_0$ .

The second part illustrates our main result with uncertainty: Constrained efficient allocations satisfy the distorted insurance inequality in (20). Moreover, these allocations can be implemented with an endogenous limit on an agent's outstanding debt in state  $L$ ,  $d_1^b \leq D_{1,L}$ . Since this policy is equivalent to an insurance requirement that restricts agents' losses in the deleveraging state, we refer to it as a mandatory insurance requirement. In particular, the competitive equilibrium characterized in Proposition 3, which features  $d_{1,L} > \bar{d}_1$ , is constrained inefficient and can be Pareto improved with a simple insurance requirement.

This result identifies a distinct type of inefficiency in our setting. Borrowers in a competitive equilibrium not only take on excessive leverage, but they also buy too little insurance with respect to severe deleveraging episodes. Intuitively, they do not take into account the positive aggregate demand externalities their insurance purchases would bring about.

Since housing price declines typically coincide with declines in the borrowing ability of households, this result provides a rationale for indexing mortgage liabilities

to housing prices. Home equity insurance along these lines has long been proposed as being privately beneficial for homeowners (see, for instance, Shiller and Weiss, 1999). Our model emphasizes the uninternalized social benefits and creates a rationale for making this type of insurance mandatory—especially with respect to severe and economy-wide downturns in house prices.

In practice, homeowners do not seem to be particularly interested in home equity insurance (see Shiller, 2003). One reason for this is borrowers' optimism (see Case, Shiller, and Thompson, 2012, for evidence of homebuyers' optimism in the run-up to the recent crisis). Our analysis—for the case of Proposition 3 in which borrowers and lenders disagree—illustrates that optimism and aggregate demand externalities are *complementary* sources of underinsurance. In particular, optimism generates a first source of underinsurance relative to a common belief benchmark, which is privately optimal for the agent. However, optimism also contributes to leverage and makes the aggregate demand externalities more likely to emerge. These externalities in turn generate a second source of underinsurance that is socially inefficient.

The inefficiency result in Proposition 4 generalizes to an economy in which financial markets are incomplete so that households only have access to noncontingent debt. This amounts to imposing the constraint  $d_1 \equiv d_{1,L} = d_{1,H}$  for households' problem in competitive equilibrium as well as for the constrained planning problem. The main difference in this case is that, since the interest rate  $r_2$  in state  $H$  is variable, the planner that sets  $D_1$  considers not only the aggregate demand externalities in state  $L$  but also the pecuniary externalities in state  $H$ . In fact, since agents' marginal utilities across states  $H$  and  $L$  are not equated, these pecuniary externalities by themselves could generate inefficiencies. However, in our setting with two continuation states, aggregate demand externalities are sufficiently powerful that the equilibrium always features too much leverage.

## 6 Preventive monetary policies

The analysis so far has focused on macroprudential policies in debt markets. A natural question is whether preventive monetary policies could also be desirable to mitigate the inefficiencies in this environment. In this section, we analyze respectively the effect of changing the inflation target and adopting contractionary monetary policy.

## 6.1 Changing the inflation target

Blanchard, Dell’Ariccia and Mauro (BDM, 2010), among others, emphasized that a higher inflation target could be useful to avoid or mitigate the liquidity trap. Appendix A.1 illustrates that a Taylor rule with a higher inflation target lowers the bound (1) on the real rate. More specifically, the bound is now given by  $\underline{r}_{t+1} = -\frac{\zeta}{1+\zeta}$  for each  $t \geq 1$ , where  $\zeta > 0$  corresponds to a positive inflation target. Consequently, a greater level of leverage is necessary to plunge the economy into a demand driven recession, consistent with BDM (2010). Our analysis adds further that this policy might also improve social welfare because the aggregate demand externalities emerge only when the real rate is constrained. These welfare benefits should be weighed against the various costs of a higher steady-state inflation.

## 6.2 Contractionary monetary policy

It has also been discussed that interest rate policy could be used as a preventive measure against financial crises. In fact, a number of economists have argued that the US Federal Reserve should have raised interest rates in the mid-2000s in order to lean against the housing bubble or to reduce leverage (see Woodford (2012) and Rajan (2010) for detailed discussions). We next investigate the effect of contractionary policy at date 0 on household leverage.

To this end, consider the baseline setting with a single type of borrower and no uncertainty. Suppose the conditions in Proposition 1 apply so that there is a liquidity trap at date 1. We capture contractionary monetary policy by considering a policymaker that sets the interest rate floor  $\underline{r}_1$  at date 0 to a level that is higher than the “natural” interest rate characterized by the Euler equations (9). Then, the equilibrium interest rate is given by this level,  $r_1 = \underline{r}_1$ , and the equilibrium features a policy-induced recession at date 0, that is, agents’ net income falls to  $e_0 < e^*$ . Moreover, agents’ Euler equations are now given by

$$\frac{1}{1 + \underline{r}_1} = \frac{\beta^l u'(e_1 + (d_1 - \phi))}{u'(e_0 + d_0 - \frac{d_1}{1 + \underline{r}_1})} = \frac{\beta^b u'(e_1 - (d_1 - \phi))}{u'(e_0 - (d_0 - \frac{d_1}{1 + \underline{r}_1}))}, \quad (21)$$

where  $e_1 = \bar{c}_1^l - (d_1 - \phi) < e^*$  as in (8). This describes two equations in two unknowns,  $e_0(\underline{r}_1)$ ,  $d_1(\underline{r}_1)$ , which can be solved as a function of the policy rate  $\underline{r}_1$ . Our next result characterizes the comparative statics with respect to  $\underline{r}_1$ .

**Proposition 5** (Contractionary Monetary Policy). *Consider the equilibrium described above with a liquidity trap at date 1 and the constrained interest rate  $r_1 = \underline{r}_1$  at date 0. Suppose  $-u''(x)/u(x)$  is a weakly decreasing function of  $x$ . Suppose also that  $d_0$  is sufficiently large so that  $d_0 - \frac{d_1(\underline{r}_1)}{1+\underline{r}_1} > 0$ . Then,  $e'_0(\underline{r}_1) < 0$  and  $d'_1(\underline{r}_1) > 0$ , that is: increasing the interest rate  $\underline{r}_1$  decreases the current net income and increases the outstanding debt level  $d_1$ .*

The proposition considers cases in which the utility function lies in the decreasing absolute risk aversion family—which encompasses the commonly used constant elasticity case—and lenders’ initial assets are sufficiently large so that their consumption exceeds borrowers’ consumption (see (21)). As expected, raising the interest rate in the run-up to a deleveraging episode creates a recession. Perhaps surprisingly, under quite natural assumptions, raising the interest rate in our setting also *increases* the equilibrium leverage. This in turn leads to a more severe recession at date 1.

To understand this result, suppose  $u(c) = \log c$  and  $\phi = 0$ . In this case, borrowers’ and lenders’ optimal debt choices have closed form solutions, conditional on the income levels  $e_0$  and  $e_1$ , given by

$$\begin{aligned} d_1^b &= \frac{1}{1 + \beta^b} (e_1 - \beta^b (1 + \underline{r}_1) (e_0 - d_0)) \\ d_1^l &= \frac{1}{1 + \beta^l} (e_1 - \beta^l (1 + \underline{r}_1) (e_0 + d_0)). \end{aligned} \tag{22}$$

In particular, keeping  $e_0$  and  $e_1$  constant, a higher  $\underline{r}_1$  reduces both  $d_1^b$  and  $d_1^l$ . Intuitively, the substitution effect induces borrowers to borrow less but also lenders to save more. This creates an excess demand in the asset market (that is,  $d_1^b + d_1^l$  falls below 0)—or equivalently, a shortage of demand in the goods market. To equilibrate markets, output falls and agents’ net income  $e_0$  declines. As this happens, both  $d_1^b$  and  $d_1^l$  increases: that is, borrowers borrow more and lenders save less so as to smooth their consumption. In our model, these effects are roughly balanced across borrowers and lenders since all agents share the same elasticity of intertemporal substitution. In fact, if  $d_0$  were equal to 0, the reduction in  $e_0$  would be (with log utility) just enough to counter the initial effect and the equilibrium debt level  $d_1 = d_1^b = -d_1^l$  would remain unchanged (see (22)). When  $d_0$  is sufficiently large, higher  $\underline{r}_1$  creates an additional wealth transfer from borrowers to lenders. This increases borrowers’ debt  $d_1^b$  further—while increasing lenders’ assets—generating a higher equilibrium debt level  $d_1 = d_1^b$ . The proof in the appendix uses more subtle arguments to establish the result

more generally.

Hence, the conventional wisdom—that raising the interest rate decreases leverage—fails in view of two general equilibrium effects on borrowers’ income and wealth. First, the higher interest rate creates a temporary recession, which reduces borrowers’ current income and induces them to take on greater debt. Second, the higher interest rate also transfers wealth from borrowers to lenders, which further increases borrowers’ debt. The combination of these two effects can—and in our model typically does—dominate the partial equilibrium effect of the higher interest rate, leading to greater debt in equilibrium.

We could construct variants of our model in which raising the interest rate decreases the outstanding leverage,  $d_1$ . For instance, if borrowers’ intertemporal substitution is more elastic than lenders’, as in Curdia and Woodford (2009), then the equilibrium debt level might decrease due to stronger a substitution effect for borrowers. However, even in these cases, the interest rate policy would not be the optimal instrument to deal with the excessive leverage problem. To see this, recall that the constrained efficient allocations in Proposition 2 do not feature a recession at date 0, and that they satisfy the distorted Euler inequality (13). In contrast, raising the interest rate creates an inefficient recession and continues to satisfy the Euler equations in (21).

One way to interpret these differences is that the interest rate policy sets a single wedge that affects the incentives for intertemporal substitution, whereas the constrained efficient allocations require setting wedges that differentially affect borrowers’ and lenders’ incentives to hold assets. Given that the interest rate policy targets “the wrong wedge,” it could at best be viewed as a crude solution for dealing with excessive leverage. In contrast, macroprudential policies, e.g., debt limits or insurance requirements, optimally internalize aggregate demand externalities created by leverage.

It is important to emphasize that contractionary monetary policy could well be desirable for reasons outside the scope of our model. For instance, raising the interest rate might be useful to mitigate inefficient investment booms and fire-sale externalities as in Lorenzoni (2008) or Stein (2012). A higher interest rate might also be useful to lean against asset price bubbles, e.g., by discouraging the “search for yield” phenomenon discussed in Rajan (2010). Our point is that contractionary monetary policy is not the ideal instrument to reduce household leverage, and in fact, might have the unintended consequence of raising leverage.

## 7 Aggregate demand and fire-sale externalities

In this section we endogenize the debt limit faced by borrowers by assuming that debt is collateralized by a financial asset, creating the potential for fire-sale effects. This introduces a new feedback loop into the economy: first, a decline in asset prices reduces the borrowing capacity of agents and forces them to delever, giving rise to financial amplification; secondly, in a liquidity trap, deleveraging leads to a demand-induced decline in output that triggers Keynesian multiplier effects. The two feedback effects mutually reinforce each other. As a result, a recession involving deleveraging and fire-sale effects of collateral assets may be particularly severe.<sup>8</sup>

We modify our earlier setup by assuming that borrowers hold one unit  $a_t = 1$  of a tree from which they obtain a dividend  $y_t$  every date. For simplicity, we assume that the tree only pays dividends if it is owned by borrowers so the tree cannot be sold to lenders. The tree trades among borrowers at a market price of  $p_t$ . We follow Jeanne and Korinek (2010b) in assuming that borrowers are subject to a moral hazard problem and have the option to abscond with their loans after the market for loans has closed. In order to alleviate the moral hazard problem, they pledge their trees as collateral to lenders. When a borrower absconds with her loan, lenders can detect this and can seize up to a fraction  $\phi_{t+1} < 1$  of the collateral and sell it to other borrowers. The borrowing constraint is therefore endogenous and given by:

$$d_{t+1}/(1 + r_{t+1}) \leq \phi_{t+1} a_{t+1} p_t.$$

Similar to earlier, we assume  $\phi_1 = 1$  and  $\phi_{t+1} = \phi < 1$  for each  $t \geq 1$ . Deleveraging may now be driven by two separate forces: a decline in the pledgeability parameter,  $\phi_t$ , and a decline in the price of the collateral asset. We will see shortly that declines in  $\phi_t$  are generally amplified by asset price declines.

In the following, we make two simplifying assumptions. First, starting date  $t = 2$ , we assume that the output from the tree is a constant  $y$  and there are no further shocks. Second, we let the discount factors of the two agents  $\beta^b = \beta^l = \beta$ . Together, these two assumptions imply that the economy will be in a steady state starting date 2 in which debt is constant at  $d_t = d_2$  and the asset price and consumption satisfy

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<sup>8</sup>The interaction between asset fire sales and aggregate demand has also been studied in Carlstrom and Fuerst (1997), Bernanke et al. (1999) and Iacoviello (2005). In these papers, monetary policy can mitigate the feedback effects resulting from tightening borrowing constraints. We consider the possibility of a lower bound on interest rates that prevents this, and we add a normative dimension focused on debt market policies.

$p_t = \frac{\beta}{1-\beta}y$ ,  $c_t^b = y + e^* - (1 - \beta) d_2$ ,  $c_t^l = e^* + (1 - \beta) d_2$  for  $t \geq 2$  respectively.

We next consider the equilibrium at date 1 at which the asset's dividend is given by some  $y_1 \leq y$ . As before, if the debt level is sufficiently large, that is,  $d_1 > \bar{d}_1$  for some threshold  $\bar{d}_1$ , then the economy is in a liquidity trap. In particular, borrowers are constrained,  $d_2 = \phi p_1$ , the interest rate is at zero,  $r_2 = 0$ , and output is below its efficient level,  $e_1 < e^*$ . Moreover, the equilibrium is determined lenders' Euler equation at the zero interest rate:

$$u'(e_1 + d_1 - \phi p_1) = \beta u'(e^* + (1 - \beta) \phi p_1). \quad (23)$$

The difference is that the asset price also enters this equation since higher prices increase the endogenous debt limit, which influences aggregate demand and output. The asset price is in turn characterized by:

$$p_1 = MRS(e_1, p_1) \cdot p_2 = \frac{u'(c_2^b)}{(1 - \phi) u'(c_1^b) + \phi \beta u'(c_2^b)} \cdot \frac{\beta y}{1 - \beta}, \quad (24)$$

where  $\begin{cases} c_2^b = e^* + y - (1 - \beta) \phi p_1 \\ c_1^b = e_1 + y_1 - d_1 + \phi p_1 \end{cases}$ .

This captures that today's asset price is tomorrow's price  $p_2 = \frac{\beta y}{1-\beta}$  discounted by the *MRS* applicable to asset purchases, which in turn reflects that a fraction  $\phi$  of the asset can be purchased with borrowed funds. Since the extent of deleveraging at date 1 is endogenous to  $p_1$ , the *MRS* is itself a function of the asset price  $p_1$ . For the implicit asset price equation (24) to have a unique and well-defined solution, it is necessary that the slope of the left-hand side is higher than the slope of the right-hand side, i.e.  $p_2 \cdot \partial MRS / \partial p_1 < 1$ . (The condition is characterized in terms of fundamental parameters in the appendix.) We also observe that  $\partial MRS / \partial e_1 > 0$  as higher income today makes borrowers more willing to buy assets. Therefore the equilibrium asset price defined by the equation is increasing in current income,  $dp_1 / de_1 > 0$ . Furthermore, the asset price is increasing in the exogenous collateral limit,  $\phi$ , which can be understood from a collateral value channel: A higher  $\phi$  implies the asset is more useful to relax the borrowing constraint, which raises its price.

The equilibrium is characterized by two equations, (23) and (24), in two unknowns  $(e_1, p_1)$ . The first equation describes an increasing relation,  $e_1^{AD}(p_1)$ , that represents the aggregate demand effects of asset prices. Intuitively, a higher price raises the endogenous debt level, which in turn raises aggregate demand and output. The second equation describes the consumer's asset pricing relationship  $e_1^{AP}(p_1)$ , i.e. it captures

the level of income required to support a given asset price. It is also increasing under our earlier assumption on the *MRS*. Intuitively, supporting a higher asset price requires higher consumption and therefore a higher net income,  $e_1$ . Any intersection of these two curves, that also satisfies  $\partial e_1^{AP} / \partial p_1 > \partial e_1^{AD} / \partial p_1$ , is a stable equilibrium.

To analyze welfare, consider the externalities from leverage,  $\frac{\partial V^h}{\partial D_1}$ , which can now be written as:

$$\begin{aligned} \frac{\partial V^l}{\partial D_1} &= u'(c_1^l) \frac{de_1}{dD_1}, \\ \frac{\partial V^b}{\partial D_1} &= u'(c_1^b) \frac{de_1}{dD_1} + \phi \frac{dp_1}{dD_1} [u'(c_1^b) - \beta u'(c_2^b)], \end{aligned}$$

where  $\frac{de_1}{dD_1}$  and  $\frac{dp_1}{dD_1}$  are jointly obtained from expressions (23) and (24) and are both negative under the assumptions made earlier. Note that the expression for both types of households features aggregate demand externalities. The expression for borrowers features in addition fire sale externalities. Intuitively, a higher debt level lowers borrowers' consumption, which in turn lowers the asset price. The low price in turn tightens borrowing constraints and further reduces borrower' welfare. Recall also that a low price further reduces aggregate demand and output, which in turn generates even lower prices, and so on.

It follows that endogenizing the financial constraint as a function of asset prices reinforces the problems of excessive leverage and underinsurance through two channels. First, it introduces fire-sale externalities that operate on borrowers' welfare in the same direction as aggregate demand externalities. Second, it exacerbates aggregate demand externalities by tightening borrowing constraints further. The latter effect also illustrates an interesting mechanism through which asset price declines hurt all agents in the economy via aggregate demand effects, even if they do not hold financial assets. In this model, lenders do not hold the asset, but they are nonetheless hurt by the price decline because it leads to more deleveraging and magnifies the recession.

## 8 Conclusion

When borrowers are forced to delever, the interest rate might fail to decline sufficiently to clear the goods market, plunging the economy into a liquidity trap. This paper analyzed the role of preventive policies in the run-up to such episodes. We established that the competitive equilibrium allocations feature excessive leverage and underinsurance. A planner can improve welfare and implement constrained effi-

cient allocations by using macroprudential policies such as debt limits or mandatory insurance requirements. The size of the required intervention depends on the differences in marginal propensity to consume between borrowers and lenders during the deleveraging episode.

We also showed that contractionary monetary policy that raises the interest rate cannot implement the constrained efficient allocations in this setting. Moreover, due to general equilibrium effects, this policy can have the unintended consequence of increasing household leverage and exacerbating aggregate demand externalities. That said, a contractionary monetary policy could well be desirable for reasons outside our model. We leave a more complete analysis of preventive monetary policies for future work.

Although we focus on consumption and household leverage, our mechanism also has implications for investment and firms' leverage. Similar to households, firms feature a great deal of heterogeneity in their propensities to invest out of liquidity. Moreover, although there is no consensus, firms that are more financially constrained seem to have greater propensity to invest (see, for instance, Rauh, 2006) especially during a financial crisis (see Campello, Graham, Harvey, 2010). Hence, transferring ex-post wealth from borrowing firms to "lending" firms (those with large holdings of cash) is likely to decrease investment and aggregate demand. Our main results then suggests that firms will also borrow too much, and purchase too little insurance, in the run-up to financial crises. Just like with households, these inefficiencies can be corrected with macroprudential policies such as debt limits and capital/insurance requirements.

A growing literature on financial crises has emphasized various other factors that encourage excessive leverage, including fire-sale externalities, optimism, and moral hazard. Our analysis suggests these distortions are complementary to the aggregate demand externalities that we emphasize. For instance, asset fire sales reduce aggregate demand by tightening borrowing constraints, which in turn exacerbates aggregate demand externalities. Similarly, optimistic beliefs imply agents take on excessive leverage and do not want to insure, which makes it more likely that the economy enters the high-leverage conditions under which aggregate demand externalities matter. An interesting future direction is to investigate further the interaction between various sources of excessive leverage.

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# A Appendix: Extensions and omitted proofs

## A.1 Microfounding the lower bound on the real interest rate

In the main text, we took the lower bound (1) on the real interest rate as given. We next provide a microfoundation for this bound based on two assumptions related to nominal variables. Consider the cashless limit economy described in Woodford (2003). Let  $P_t$  denote the nominal price of the consumption good at date  $t$  and  $i_{t+1}$  denote the nominal interest rate.

**Assumption (A1).** There is a zero lower bound on the nominal interest rate:

$$i_{t+1} \geq 0 \text{ for each } t \geq 0. \quad (\text{A.1})$$

This assumption captures a no-arbitrage condition between money and government bonds.

**Assumption (A2).** The nominal interest rate,  $i_{t+1}$ , is set according to a standard Taylor rule adjusted for the zero lower bound, given by:

$$\begin{aligned} \log(1 + i_{t+1}) &= \max(0, \log(1 + r_{t+1}^n) + \psi(\log(P_t/P_{t-1}) - \log(1 + \zeta))), \quad (\text{A.2}) \\ \text{where } 1 + r_{t+1}^n &= \min_{h \in \{b, l\}} \frac{u'(c_t^h)}{\beta^h u'(c_{t+1}^h)} \text{ and } \psi > 1. \end{aligned}$$

This version of the Taylor rule is designed to set the gross inflation equal to the gross target,  $1 + \zeta$ , whenever possible. In fact, we will see below that the Taylor rule in our setting implies

$$P_{t+1}/P_t = 1 + \zeta \text{ for each } t \geq 1. \quad (\text{A.3})$$

Combining Eqs. (A.1) and (A.3) with the Fisher equation,  $1 + r_{t+1} = (1 + i_{t+1}) E_t \left[ \frac{P_t}{P_{t+1}} \right]$ , leads to the bound:

$$r_{t+1} \geq \underline{r}_{t+1} = -\frac{\zeta}{1 + \zeta} \text{ for each } t \geq 1.$$

The baseline analysis considers the case in which the target,  $\zeta$ , is normalized to 0. Section 6 discusses the case  $\zeta \geq 0$ .

Eq. (A.3) also implies that the Taylor rule with  $\zeta = 0$  is the optimal time-consistent policy if there is some cost to inflation, i.e. it is *ex-post efficient*. Hence, absent commitment, the optimal monetary policy naturally leads to the lower bound in (1). On the other hand, if the monetary policy could commit at date 0 to creating inflation at date 1, then the lower bound could be circumvented—as emphasized by Krugman (1998) and the subsequent literature.

It remains to show the claim that the Taylor rule in (A.2) implies Eq. (A.3). Recall that starting date 2 the real interest rate is constant and given by  $r_{t+1} = 1/\beta^l - 1 > 0$ . Assumption (A2) then implies that the inflation is at its target level, that is,  $P_t/P_{t-1} = 1 + \zeta$  for each  $t \geq 2$  as long as the nominal interest rate,  $i_{\tilde{t}+1}$ , is positive and finite for each  $\tilde{t} \geq 2$ . To see this, suppose  $P_t/P_{t-1} > 1 + \zeta$  for some  $t \geq 2$ . Then, the Taylor rule along with the Fisher equation,  $1 + r_{t+1} = (1 + i_{t+1}) \frac{P_t}{P_{t+1}}$ , implies  $\frac{P_{t+1}}{P_t} = \left( \frac{P_t}{P_{t-1}} / (1 + \zeta) \right)^\psi$ . Given the

Taylor coefficient  $\psi > 1$ , repeating this argument implies  $\lim_{\bar{t} \rightarrow \infty} \frac{P_{\bar{t}+1}}{P_{\bar{t}}} = \infty$ , which yields a contradiction. A similar contradiction is obtained if  $P_t/P_{t-1} < 1 + \zeta$  for some  $t \geq 2$ .<sup>9</sup>

## A.2 Omitted proofs for the baseline model

This section presents the proofs of the results for the baseline model and its variants analyzed in Sections 3, 4, 5, and 6.

**Proof of Proposition 1.** The result claims there is a recession at date 1 under appropriate conditions. To prove this result, suppose the contrary, that is,  $e_0 = e_1 = e^*$ . Let  $\bar{r}_1(d_0)$  denote the interest rate at which lenders would hold assets  $\bar{d}_1$  in equilibrium, defined by:

$$\frac{1}{1 + \bar{r}_1} = \frac{\beta^l u'(e^* + \bar{d}_1 - \phi)}{u'(e^* + d_0 - \bar{d}_1 / (1 + \bar{r}_1))}.$$

Note that for  $\bar{r}_1(d_0)$  is a decreasing function of  $d_0$ . This also implies that  $d_0 - \bar{d}_1 / (1 + \bar{r}_1(d_0))$  is increasing in  $d_0$ .

The equilibrium features  $d_1 > \bar{d}_1$  if the marginal rates of substitution of the two agents at the debt level  $\bar{d}_1$  satisfy:

$$\begin{aligned} & \left. \frac{\beta^l u'(c_1^l)}{u'(c_0^l)} \right|_{d_1 = \bar{d}_1} > \left. \frac{\beta^b u'(c_1^b)}{u'(c_0^b)} \right|_{d_1 = \bar{d}_1} \\ \text{or } & \frac{\beta^l u'(e^* + \bar{d}_1 - \phi)}{u'(e^* + d_0 - \bar{d}_1 / (1 + \bar{r}_1(d_0)))} > \frac{\beta^b u'(e^* - \bar{d}_1 + \phi)}{u'(e^* - d_0 + \bar{d}_1 / (1 + \bar{r}_1(d_0)))}. \end{aligned} \quad (\text{A.4})$$

Observe that the right hand side of this inequality is decreasing in  $\beta^b$ . Hence, for a given debt level  $d_0$ , there is a threshold level of impatience  $\bar{\beta}^b(d_0)$  such that the inequality holds for each  $\beta^b \geq \bar{\beta}^b(d_0)$ .

Similarly, since  $d_0 - \bar{d}_1 / (1 + \bar{r}_1(d_0))$  is increasing in  $d_0$ , the left hand side of (A.4) is increasing in  $d_0$ , while the right hand side is decreasing in  $d_0$ . Hence, for a given level  $\beta^b$ , there is a threshold level  $\bar{d}_0(\beta^b)$  such that the inequality holds for each  $d_0 > \bar{d}_0(\beta^b)$ . It follows that  $d_1 > \bar{d}_1$ , and thus, there is a recession at date 1, if the borrowers is sufficiently impatient or sufficiently indebted at date 0.  $\square$

**Proof of Lemma 1.** First consider the case  $d_1 > \bar{d}_1$ . Eq. (8) implies  $\frac{de_1}{dd_1} = -1$ . Eq. (10) then implies  $\frac{\partial V^h}{\partial D_1} = -u'(u_1^h) < 0$ .

Next consider the case  $d_1 < \bar{d}_1$ . In this case, differentiating lenders' Euler equation (9), we have:

$$\frac{dr_2}{dd_1} = \frac{u''(c_1^l)}{\beta^l u'(c_2^l) - u''(c_1^l) \phi / (1 + r)^2} < 0.$$

<sup>9</sup>We abstract away from the equilibria with self-fulfilling deflationary traps and inflationary panics (see Cochrane, 2011).

This in turn raises consumption of borrowers and lowers consumption of lenders by:

$$\frac{dc_1^b}{dd_1} = -\frac{\phi}{(1+r)^2} \cdot \frac{dr_2}{dd_1} \equiv \eta \text{ and } \frac{dc_1^l}{dd_1} = -\eta.$$

It can also be checked that  $\eta \in (0, 1)$ , completing the proof.  $\square$

**Proof of Proposition 2.** Let  $\nabla_{sub}V^h(d_1, D_1)$  denote the set of subgradients of function  $V^h(\cdot)$  with respect to its second variable (aggregate debt level). If  $D_1 \neq \bar{d}_1$ , the function  $V^h$  is differentiable in its second variable. In this case, there is a unique subgradient characterized by Lemma 1. If  $D_1 = \bar{d}_1$ , then the function  $V^h$  has a kink at  $\bar{d}_1$  due to the kink of the function  $e_1(D_1)$  (see Eqs. (10) and (8)). In this case, there are multiple subgradients characterized by:

$$\nabla_{sub}V^h(d_1, \bar{d}_1) = \begin{cases} [-u'(c_1^h), -\eta u'(c_1^h)] & \text{if } h = l \\ [-u'(c_1^h), \eta u'(c_1^h)] & \text{if } h = b \end{cases}. \quad (\text{A.5})$$

In particular, for each  $h$ , the the subgradients lie in the interval between the right and the left derivatives of the function  $V^h$  characterized in Lemma 1.

Next consider the optimality conditions for problem (12), which can be written as:

$$\frac{\beta^l [u'(c_1^l) + \delta^l]}{u'(c_0^l)} = \frac{\beta^b [u'(c_1^b) - \delta^b]}{u'(c_0^b)}, \quad (\text{A.6})$$

where  $\delta^h \in \nabla_{sub}V^h(D_1, D_1)$  denotes the subgradient evaluated at individual and aggregate debt levels,  $D_1$ . Note that we consider generalized first order conditions that apply also at points at which the objective function might have a kink. Conversely, it can also be seen that any allocation that satisfies these conditions, along with the intratemporal condition  $v'(n_0^h) = 1$  for each  $h$ , corresponds to a solution to problem (12) given Pareto weights that satisfy  $\frac{\gamma^b}{\gamma^l} = \frac{u'(c_0^b)}{u'(c_0^l)}$ . Hence, it suffices to characterize the allocations that satisfy condition (A.6).

First consider the case  $D_1 < \bar{d}_1$ . Using Lemma 1, condition (A.6) becomes identical to the Euler equation (9), proving the first part. Next consider the case  $D_1 > \bar{d}_1$ . Using Lemma 1, condition (A.6) is violated since the left hand side is zero while the right hand side is strictly positive. Hence, there is no constrained efficient allocation with  $D_1 > \bar{d}_1$ . Finally consider the case  $D_1 = \bar{d}_1$ . Using (A.5), we have:

$$\frac{\beta^l [u'(c_1^l) + \delta^l]}{u'(c_0^l)} \in \left[ 0, \frac{(1-\eta)\beta^l u'(c_1^l)}{u'(c_0^l)} \right]$$

and:

$$\frac{\beta^b [u'(c_1^b) - \delta^b]}{u'(c_0^b)} \in \left[ \frac{(1-\eta)\beta^b u'(c_1^b)}{u'(c_0^b)}, \frac{2\beta^b u'(c_1^b)}{u'(c_0^b)} \right].$$

Combining these expressions with condition (A.6), we obtain  $\frac{\beta^l u'(c_1^l)}{u'(c_0^l)} \geq \frac{\beta^b u'(c_1^b)}{u'(c_0^b)}$ . Conversely,

for any allocation that satisfies this inequality, there exists subgradients  $\delta^l$  and  $\delta^b$  such that condition (A.6) holds.

It follows that the optimal allocations with  $D_1 \geq \bar{d}_1$  are characterized by  $D_1 = \bar{d}_1$  and the distorted Euler inequality (13). Note, from our analysis in Section 3, that these allocations feature  $e_1 = e^*$  and:

$$c_1^l = \bar{c}_1^l = e^* + (\bar{d}_1 - \phi) \text{ and } c_1^b = e^* - (\bar{d}_1 - \phi). \quad (\text{A.7})$$

Next consider  $(c_0^l, c_0^b, c_1^l, c_1^b)$  that satisfies the distorted Euler inequality (13) and Eq. (A.7), along with the resource constraints at date 0. We next claim that this allocation can be implemented with the endogenous debt limit  $d_1^h \leq \bar{d}_1$ , and an appropriate transfer  $T_0$ . The transfer can be equivalently thought of as setting borrowers' initial debt level at an alternative level  $\tilde{d}_0 = d_0 - T_0$ . With this debt level, the date 0 allocations are given by:

$$c_0^l = e^* + \tilde{d}_0 - \bar{d}_1 / (1 + r_1) \text{ and } c_0^b = e^* - \tilde{d}_0 + \bar{d}_1 / (1 + r_1).$$

By time discounting, we also have  $c_0^l \geq c_1^l > e^*$ . The debt limit does not bind for lenders, which implies the interest rate is given by:

$$\frac{1}{1 + r_1} = \frac{\beta^l u'(c_1^l)}{u'(c_0^l)} = \frac{\beta^l u'(e^* + (\bar{d}_1 - \phi))}{u'(e^* + \tilde{d}_0 - \bar{d}_1 / (1 + r_1))}.$$

As in the proof of Proposition 1, the expression,  $\tilde{d}_0 - \bar{d}_1 / (1 + r_1)$ , is increasing in  $\tilde{d}_0$ . Hence, there exists a level of  $\tilde{d}_0$  such that  $\tilde{d}_0 - \bar{d}_1 / (1 + r_1)$  is equal to  $c_0^l - e^* > 0$ . It follows that, this choice of  $\tilde{d}_0$  (and the corresponding  $T_0$ ), along with the debt limit,  $d_1^h \leq \bar{d}_1$ , implements the desired allocation.  $\square$

**Proof of Proposition 3.** Under either condition (i), (ii), or (iii), we claim that there exists an equilibrium in which  $d_{1,L} \geq \bar{d}_1$  and a recession is triggered in state  $L$  of date 1. The optimality conditions can be written as:

$$\begin{aligned} \frac{1}{q_{1,L}} &= \frac{u'(e^* + d_0 - q_{1,H}d_{1,H} - q_{1,L}d_{1,L})}{\pi_L^l \beta^l u'(\bar{c}_1^l)} = \frac{u'(e^* - (d_0 - q_{1,H}d_{1,H} - q_{1,L}d_{1,L}))}{\pi_L^b \beta^b u'(\bar{c}_1^l + 2(\phi - d_{1,L}))} \quad (\text{A.8}) \\ \frac{q_{1,L}}{q_{1,H}} &= \frac{\pi_L^l}{\pi_H^l} \frac{u'(\bar{c}_1^l)}{u'(e^* + (1 - \beta^l)d_{1,H})} = \frac{\pi_L^b}{\pi_H^b} \frac{u'(\bar{c}_1^l + 2(\phi - d_{1,L}))}{u'(e^* - (1 - \beta^l)d_{1,H})}. \end{aligned}$$

These expressions represent 4 equations in 4 unknowns,  $d_{1,H}, d_{1,L}, q_{1,L}, q_{1,H}$ . Given the regularity conditions, there is a unique solution. For the conjectured allocation to be an equilibrium, we also need the solution to satisfy  $d_{1,L} \geq \bar{d}_1$ . First consider conditions (i) or (ii), i.e., suppose  $\pi_L^b = \pi_L^l$ . In this case, a similar analysis as in the proof of Proposition 3 establishes that  $d_{1,L} \geq \bar{d}_1$  is satisfied when  $\beta_0^b \leq \bar{\beta}_0^b(d_0, \pi_L^b)$  or when  $d_0 \geq \bar{d}_0(\beta_0^b, \pi_L^b)$  (for appropriate threshold functions  $\bar{\beta}_0^b(\cdot)$  and  $\bar{d}_0(\cdot)$ ). Next consider condition (iii). It can be checked that  $d_{1,L}$  is decreasing in  $\pi_L^b$ , and that  $\lim_{\pi_L^b \rightarrow 0} d_{1,L} > \bar{d}_1$  (since  $\lim_{\pi_L^b \rightarrow 0} c_{1,L}^b = 0$ ). Thus, there exists a threshold function  $\bar{\pi}_L^b(\beta_0^b, d_0)$  such that  $d_{1,L} > \bar{d}_1$  whenever  $\pi_L^b <$

$\bar{\pi}_L^b(\beta_0^b, d_0)$ , completing the proof.  $\square$

**Proof of Proposition 4.** The proof proceeds similar to the proof of Proposition 2. Optimality conditions for problem (19) imply  $\frac{\beta_0^l \pi_H^l u'(c_{1,H}^l)}{u'(c_0^l)} = \frac{\beta_0^b \pi_H^b u'(c_{1,H}^b)}{u'(c_0^b)}$  and

$$\frac{\pi_L^l \left[ u'(c_{1,L}^l) + \delta^l \right]}{\pi_H^l u'(c_{1,H}^l)} = \frac{\pi_L^l \left[ u'(c_{1,L}^b) - \delta^b \right]}{\pi_H^l u'(c_{1,H}^b)}, \quad (\text{A.9})$$

Here,  $\delta^l$  and  $\delta^b$  denote respectively the subgradients of  $V^h(d_1, D_1)$  with respect to the second variable, evaluated at  $d_1 = D_1$ . Conversely, it can be seen that any allocation that satisfies these equations corresponds to a solution to the planner's problem with appropriate Pareto weights. Hence, it suffices to characterize the allocations that satisfy condition (A.6).

For the case  $D_{1,L} < \bar{d}_1$ , applying Lemma 1 in the numerator of Eq. (A.9) makes the condition equivalent to (18). For the case  $D_{1,L} > \bar{d}_1$ , the numerator on the left hand side of Eq. (A.9) is zero whereas the right-hand side remains positive, implying that  $D_{1,L} > \bar{d}_1$  is never optimal. Finally, for the case  $D_{1,L} = \bar{d}_1$ , it can be seen that condition (A.9) implies the insurance inequality (20). Conversely, given the inequality in (20), there exists subgradient such that the condition (A.9) holds. This completes the characterization of the solution to problem (19). As in the proof Proposition 2, it can also be seen that the allocation can be implemented with a mandatory insurance requirement,  $d_{1,L}^h \leq \bar{d}_1$  combined with an appropriate initial transfer  $T_0$ , completing the proof.  $\square$

We next establish the following lemma, which will be useful to prove Proposition 5.

**Lemma 2.** *Consider a strictly increasing and strictly concave function  $u(\cdot)$  such that  $-u''(x)/u'(x)$  is a weakly decreasing function of  $x$ . Then,*

$$\frac{d}{dx} \left( \frac{u'(x+y)}{u'(x-y)} \right) \geq 0 \text{ and } \frac{d}{dy} \left( \frac{u'(x+y)}{u'(x-y)} \right) < 0,$$

for each  $x, y \in \mathbb{R}_+$ .

**Proof of Lemma 2.** Note that

$$\frac{d}{dx} \left( \frac{u'(x+y)}{u'(x-y)} \right) = \frac{u'(x+y)}{u'(x-y)} \left( \frac{u''(x+y)}{u'(x+y)} - \frac{u''(x-y)}{u'(x-y)} \right) \geq 0,$$

where the inequality follows since  $-u''(x)/u'(x)$  is weakly decreasing in  $x$ . We also have:

$$\frac{d}{dy} \left( \frac{u'(x+y)}{u'(x-y)} \right) = \frac{u''(x+y)u'(x-y) + u'(x+y)u''(x-y)}{u'(x-y)^2} < 0,$$

where the inequality follows since  $u(\cdot)$  is strictly concave.  $\square$

**Proof of Proposition 5.** Let  $d_1(r_1)$  and  $e_0(r_1)$  denote the solution to Eqs. (21). It is also useful to define  $y(r_1) = d_0 - \frac{d_1(r_1)}{1+r_1}$ , which corresponds to lenders' consumption at date 0 in excess of their net income. Note that, by assumption, we have  $y(r_1) > 0$ .

We first show that  $e'_0(r_1) < 0$ . Suppose, to reach a contradiction,  $e'_0(r_1) \geq 0$ . Note that lenders' Euler equation implies [cf. Eq. (21)]:

$$\frac{1}{1+r_1} u'(e_0(r_1) + y(r_1)) = \beta^l u'(\bar{c}_1^l). \quad (\text{A.10})$$

Since  $e'_0(r_1) \geq 0$ , this expression implies  $y'(r_1) < 0$ . Using the definition of  $y(\cdot)$ , this further implies  $d'_1(r_1) > 0$ . Next consider borrowers' Euler equation [cf. Eq. (21)]:

$$\frac{1}{1+r_1} = \frac{\beta^b u'(\bar{c}_1^b - 2(d_1(r_1) - \phi))}{u'(e_0(r_1) - y(r_1))}.$$

The left hand side is strictly decreasing in  $r_1$ . However, since  $e'_0(r_1) \geq 0$ ,  $y'(r_1) < 0$  and  $d'_1(r_1) > 0$ , the right hand side is strictly increasing in  $r_1$ . This yields a contradiction and proves  $e'_0(r_1) < 0$ .

We next establish that  $d'_1(r_1) > 0$ . Suppose, to reach a contradiction,  $d'_1(r_1) \leq 0$ . Using the definition of  $y(\cdot)$ , this further implies  $y'(r_1) > 0$ . Combining lenders' and borrowers' Euler equations, we also have [cf. Eq. (21)]:

$$\frac{\beta^l u'(\bar{c}_1^l)}{\beta^b u'(\bar{c}_1^b - 2(d_1(r_1) - \phi))} = \frac{u'(e_0(r_1) + y(r_1))}{u'(e_0(r_1) - y(r_1))}.$$

The left hand side is weakly increasing in  $r_1$  since  $d'_1(r_1) \leq 0$ . However, since  $e'_0(r_1) < 0$  and  $y'(r_1) > 0$ , Lemma 2 implies the right hand side is strictly decreasing in  $r_1$ . This yields a contradiction and shows  $d'_1(r_1) \leq 0$ , completing the proof.  $\square$

### A.3 Extension with heterogeneous borrowers

This section completes the characterization of the model with two types of borrowers  $\{b_{high}, b_{low}\}$  described in Section 4.3. We first describe the equilibrium and then analyze its efficiency properties.

As described in the main text, at date 0 all borrowers choose the same debt level,  $d_1$ . Consider the equilibrium starting date 1. As before, there is a threshold debt level  $\bar{d}_1$ , characterized in Eq. (A.15) below, above which the equilibrium features a liquidity trap with  $r_2 = 0$  and  $e_1 \leq e^*$ . Suppose  $d_1 \geq \bar{d}_1$ , then type  $b_{high}$  borrowers are forced into deleveraging. Thus, their outstanding debt for the next date is  $d_2 = \phi$  and their consumption is given by:

$$c_1^{b_{high}} = e_1 - d_1 + \phi \text{ and } c_2^{b_{high}} = e^* - \phi(1 - \beta).$$

In contrast, type  $b_{low}$  borrowers choose their consumption and outstanding debt according

to the Euler equation:

$$u' \left( c_1^{b_{low}} \right) = \beta u' \left( c_2^{b_{low}} \right), \quad (\text{A.11})$$

$$\text{where } c_1^{b_{low}} = e_1 - d_1 + d_2^{b_{low}} \text{ and } c_2^{b_{low}} = e^* - d_2^{b_{low}} (1 - \beta).$$

As a group, borrowers' total outstanding debt at date 2 is then given by:

$$d_2 = \alpha \phi + (1 - \alpha) d_2^{b_{low}}. \quad (\text{A.12})$$

As before, lenders choose their consumption and outstanding debt according to the Euler equation:

$$u' \left( c_1^l \right) = \beta u' \left( c_2^l \right), \quad (\text{A.13})$$

$$\text{where } c_1^l = e_1 + d_1 - d_2 \text{ and } c_2^l = e^* + d_2 (1 - \beta).$$

Here, the second line uses the debt market clearing condition. The equilibrium triple,  $(e_1, d_2^{b_{low}}, d_2)$ , is found by solving the three equations (A.11), (A.12), (A.13). Using log utility, there is a closed form solution given by:

$$\begin{aligned} e_1 &= \frac{e^*}{\beta} - \frac{\alpha}{2 - \alpha} \left( d_1 - \frac{\phi}{\beta} \right) \\ d_2^{low} &= \beta \left( d_1 + \frac{\alpha}{2 - \alpha} \left( d_1 - \frac{\phi}{\beta} \right) \right) \\ \text{and } d_2 &= \frac{\alpha}{2 - \alpha} \phi + \frac{1 - \alpha}{2 - \alpha} 2\beta d_1 \end{aligned} \quad (\text{A.14})$$

The first equation implies Eqs. (16) in the main text. The same equation also implies there is a liquidity trap with  $e_1 < e^*$  and  $r_2 = 0$  as long as:

$$d_1 > \bar{d}_1 = e^* \left( \frac{1}{\beta} - 1 \right) \frac{2 - \alpha}{\alpha} + \frac{\phi}{\beta}. \quad (\text{A.15})$$

Next consider the equilibrium at date 0. As before, we conjecture an equilibrium in which the net income is at its efficient level,  $e_0 = e^*$ . Since households are unconstrained, both of their Euler equations hold:

$$\frac{1}{1 + r_1} = \frac{\beta^l u' \left( c_1^l \right)}{u' \left( c_0^l \right)} = \frac{\beta^b E_0 \left[ u' \left( c_1^b \right) \right]}{u' \left( c_0^b \right)}.$$

Here, the expectation is taken over borrowers' types at date 1, that is,

$$E_0 \left[ u' \left( c_1^b \right) \right] = \alpha u' \left( c_1^{b_{high}} \right) + (1 - \alpha) u' \left( c_1^{b_{low}} \right).$$

In view of our characterization starting date 1, the Euler equations represent two equations in two unknowns,  $(r_1, d_1)$ , which has a solution. As before, there is a deleveraging-induced

recession at date 1, that is,  $d_1 > \bar{d}_1$ , as long as borrower is sufficiently impatient or sufficiently indebted at date 0.

We next analyze the efficiency of this equilibrium. As in the main text, let  $V^h(d_1; D_1)$  denote the expected utility of a household of type  $h$  at date 1 but before the realization of borrowers' type at date 1. With this notation, the aggregate demand externalities (for the region  $D_1 > \bar{d}_1$ ) are given by:

$$\begin{aligned} \frac{\partial V^l}{\partial D_1} &= \frac{\partial e_1}{\partial D_1} u'(c_1^l) = -\frac{\alpha}{2-\alpha} u'(c_1^l), \\ \text{and } \frac{\partial V^b}{\partial D_1} &= \frac{\partial e_1}{\partial D_1} E_0 [u'(c_1^b)] = -\frac{\alpha}{2-\alpha} E_0 [u'(c_1^b)]. \end{aligned} \tag{A.16}$$

The constrained planning problem is then still given by (12). Using Eq. (A.16), the first order condition in this case is given by:

$$\frac{\beta^l u'(c_1^l)}{u'(c_0^l)} = \frac{\beta^b E_0 [u'(c_1^b)]}{(1-\alpha) u'(c_0^b)} \text{ for each } D_1 > \bar{d}_1.$$

Plugging in the value of  $\alpha$ , this leads to Eq. (17). This equation can also be used to obtain an analogue of the main result, Proposition 2 for this case.

## A.4 Extension with fire sales

This section completes the characterization of the model with fire sales described in Section 7. To characterize the condition  $p_2 \cdot \partial MRS / \partial p_1 < 1$  in more detail, note that

$$\frac{\partial MRS}{\partial p_1} = \frac{-(1-\beta) \phi u''(c_2^b) [(1-\phi) u'(c_1^b) - 2\phi \beta u'(c_2^b)] - (1-\phi) \phi u''(c_1^b) u'(c_2^b)}{[(1-\phi) u'(c_1^b) + \phi \beta u'(c_2^b)]^2}$$

If we approximate  $\beta \approx 1$ , all but the last term disappear from the numerator. Furthermore, we approximate  $u'(c_1^b) \approx \beta u'(c_2^b)$  which holds exactly in the neighborhood of where the constraint becomes binding. This simplifies the expression in the denominator. Taken together,

$$\frac{\partial MRS}{\partial p_1} \approx -\frac{\phi(1-\phi) u''(c_1^b)}{u'(c_1^b)} = \frac{\phi(1-\phi)}{\sigma c_1^b} < \frac{1}{p_2} \quad \text{or} \quad \phi(1-\phi) < \sigma \frac{c_1^b}{p_2}$$

where  $\sigma$  is the intertemporal elasticity of substitution. In short, the solution to equation (24) is unique and well-defined if the leverage parameter is sufficiently small compared to the consumption/asset price ratio. If the condition was violated, an infinitesimal increase at date 1 consumption would lead to a discrete upward jump in the asset price and relax the constraint by more than necessary to finance the marginal increase in consumption. This would violate the assumption that the equilibrium exhibits a binding borrowing constraint. Observe also that this is a common type of condition in models of financial amplification to guarantee uniqueness (see e.g. Lorenzoni, 2008; Jeanne and Korinek, 2010b).

## A.5 Extension with separable preferences (Online appendix)

This section describes and analyzes a version of the baseline model in which households have separable preferences. We first characterize the equilibrium. We then characterize the constrained efficient allocations and establish the inefficiency of equilibrium—generalizing the main result, Proposition 2, to this case.

Consider the same model as in the main text with two differences. First, households' preferences are given by  $u(c) - v(n)$ , where  $u(\cdot)$  and  $v(\cdot)$  satisfy the standard assumptions. Second, suppose type  $h$  households hold all of the shares of firms in which they work (and none of the shares of other firms) so their equilibrium income is always given by  $n_1^h$ . The latter assumption is made only for simplicity and can be relaxed at the expense of additional notation.

As before, consider first dates  $t \geq 2$ , at which the level of consumer debt is constant at the maximum permissible level  $d_t = \phi$  and borrowers pay lenders a constant amount of interest  $\left(1 - \frac{1}{1+r_{t+1}}\right) \phi = (1 - \beta^l) \phi$  at every date. Households labor supply is given by:

$$u'(n^{l*} + (1 - \beta^l) \phi) = v'(n^{l*}), \text{ and } u'(n^{b*} + (1 - \beta^l) \phi) = v'(n^{b*}),$$

with  $n^{l*} < n^{b*}$  (since  $c^{l*} > c^{b*}$ ).

Now consider date 1. As in the main text, there is a threshold,  $\bar{d}_1$ , such that  $r_2 \leq 0$  only if  $d_1 \geq \bar{d}_1$ . To characterize this threshold, consider lenders' Euler equation at zero interest rate,  $v'(\bar{n}_1^l) = \beta^l v'(n_1^{l*})$ , which pins down their labor supply,  $\bar{n}_1^l$ . Their intratemporal optimality condition,  $u'(\bar{n}_1^l + \bar{d}_1 - \phi) = \beta^l v'(n_1^{l*})$ , then pins down  $\bar{d}_1$ .

The equilibrium when  $d_1 < \bar{d}_1$  is standard and characterized by the variables,  $(r_2 > 0, n_1^l, n_1^b)$ , that satisfy:

$$\begin{aligned} u'\left(n_1^b - d_1 + \frac{\phi}{1+r_2}\right) &= v'(n_1^b) \\ \text{and } u'\left(n_1^l + d_1 - \frac{\phi}{1+r_2}\right) &= v'(n_1^l) = \beta^l (1+r_2) v'(n_1^{l*}). \end{aligned}$$

For comparison with below, it is useful to note that  $\frac{\partial r_2}{\partial d_1} < 0$ ,  $\frac{\partial n_1^l}{\partial d_1} \in (-1, 0)$  and  $\frac{\partial n_1^b}{\partial d_1} \in (0, 1)$ . In particular, an increase in debt leads to a reduction in the interest rate, a decrease in lenders' labor supply and an increase in borrowers' labor supply (through a wealth effect). Note also that agents' labor supply responses are less than one-for-one (as usual).

The equilibrium when  $d_1 > \bar{d}_1$  features the liquidity trap. In this case,  $r_2 = 0$  and thus lenders' Euler equation can be written as:

$$u'(n_1^l + d_1 - \phi) = \beta^l u'(n^{l*} + (1 - \beta^l) \phi).$$

This implies

$$n_1^l(d_1) = \bar{n}_1^l + \bar{d}_1 - d_1,$$

which is the analogue of Eq. (8) in the main text. In particular, lenders' employment and

output decreases one-to-one with leverage as in the main text:  $\frac{\partial n_1^l}{\partial d_1} = -1$ . The equilibrium wage is given by:

$$w_1(d_1) = \frac{v'(n_1^l)}{u'(n_1^l + d_1 - \phi)} = \frac{v'(n_1^l)}{\beta^l v'(n^{l*})} = \frac{v'(n_1^l)}{v'(\bar{n}_1^l)} < 1.$$

The last inequality implies the labor wedge is strictly positive

$$\tau_1 = 1 - w_1(d_1) > 0.$$

That is, when  $d_1 > \bar{d}_1$ , the economy experiences a demand driven recession. Note also that the elasticity of the wage with respect to leverage is given by  $\frac{\partial w_1 / \partial d_1}{w_1 / d_1} = \frac{-1}{\eta^l} \frac{d_1}{n_1^l}$ , where  $\eta^l = \frac{v''(n_1^l) n_1^l}{v'(n_1^l)}$  is the Frish elasticity of lenders' labor supply. This expression illustrates that the wage response is influenced by two factors: The size of debt relative to output and the Frish elasticity.

Borrowers' labor supply (and consumption) is then determined by their intratemporal condition,  $v'(n_1^b) = w_1(d_1) u'(n_1^b - (d_1 - \phi))$ . Implicitly differentiating, and using the expression for the wage response, we obtain:

$$\frac{\partial n_1^b}{\partial d_1} \frac{(v''(n_1^b) - u''(c_1^b)) c_1^b}{w_1 u'(c_1^b)} = -\frac{1}{\eta^l} \frac{c_1^b}{n_1^l} + \frac{1}{\theta^b},$$

where  $\theta^b = 1 / \left( \frac{-u''(c_1^b) c_1^b}{u'(c_1^b)} \right)$  is defined as lenders' intertemporal elasticity of consumption.<sup>10</sup>

This expression also implies  $\frac{\partial n_1^b(d_1)}{\partial d_1} < 1$ , which when combined with  $\frac{\partial n_1^l}{\partial d_1} = -1$  implies:

$$\frac{\partial n_1^l}{\partial d_1} + \frac{\partial n_1^b}{\partial d_1} < 1. \tag{A.17}$$

In particular, the total employment is decreasing in leverage regardless of the parameters.

Date 0 equilibrium is then characterized by Euler equations (9). Under conditions similar to those in Proposition 1, the equilibrium features  $d_1 > \bar{d}_1$  and an anticipated recession.

We next analyze the efficiency properties of this equilibrium. First let  $V^h(d_1, D_1)$  denote the utility of a household of type  $h$  conditional on entering date 1 with an individual level

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<sup>10</sup>Hence, the sign of borrowers' labor supply response to leverage is in general ambiguous. On the one hand, greater leverage lowers borrowers' consumption and increases their marginal utility, which in turn induces them to work more: this is captured by the second term. On the other hand, greater leverage lowers aggregate demand and wages, which in turn induces borrowers to work less: this is captured by the first term. The net effect is likely to be negative if the labor elasticity,  $\eta^l$ , is low relative to the intertemporal elasticity,  $\theta^b$  (given  $\frac{c_1^b}{n_1^l}$  is likely to be close to 1).

of debt  $d_1$  and an aggregate level of debt  $D_1$ . We have:

$$V^b(d_1, D_1) = u\left(n_1(D_1) - d_1 + \frac{\phi}{1 + r_2(D_1)}\right) - v(n_1(D_1)) + \sum_{t=2}^{\infty} (\beta^b)^t u(c_t^b)$$

for borrowers and a similar expression for lenders. For  $D_1 > \bar{d}_1$ , taking the first order conditions with respect to  $D_1$ , we obtain:

$$\frac{\partial V^h}{\partial D_1} = u'(c_1^h) \frac{\partial n_1^h}{\partial D_1} \tau_1 \text{ for } D_1 > \bar{d}_1, \quad (\text{A.18})$$

which characterizes the aggregate demand externalities in this case. Note that the strength of the externalities for type  $h$  households depends on their employment response to leverage.

Next let  $W^h(D_1) = V^h(d_1, D_1)$  and consider the ex-ante constrained planning problem described in Section 4.2. The first order conditions imply Eq. (A.6) also in this case. Using the expression for externalities in (A.18), the planner's optimality condition can be written as:

$$\frac{\beta^l u'(c_1^l) \left(1 + \frac{\partial n_1^l}{\partial D_1} \tau_1\right)}{u'(c_0^l)} = \frac{\beta^b u'(c_1^b) \left(1 - \frac{\partial n_1^b}{\partial D_1} \tau_1\right)}{u'(c_0^b)} \text{ for } D_1 > \bar{d}_1. \quad (\text{A.19})$$

Recall from (A.17) that aggregate employment always declines with leverage. Combining this inequality with Eq. (A.19) shows the distorted Euler inequality (13) also holds in this case. The following result combines these observations to characterize the constrained efficient allocations in this setting.

**Proposition 6** (Excessive Leverage with Separable Preferences). *An allocation  $((c_0^h, n_0^h)_h, D_1)$ , with  $D_1 \geq \phi$ , is constrained efficient if and only if labor supply and output at date 0 is efficient, i.e.,  $u'(c_0^h) = v'(n_0^h)$  for each  $h$ ; and the consumption and debt allocations satisfy one of the following:*

(i)  $D_1 < \bar{d}_1$  and the Euler equation (9) holds.

(ii)  $D_1 \geq \bar{d}_1$ , and the planner's optimality condition in (A.19), and thus also the distorted Euler inequality in (13), holds. Moreover, every constrained efficient allocation of this type can be implemented as a competitive equilibrium with a debt limit,  $d_1^h \leq D_1$  for each  $h$ , combined with an appropriate ex-ante transfer,  $T_0$ .

This result is the analogue of our main result, Proposition 2, for separable preferences. The only difference is that debt levels strictly greater than  $\bar{d}_1$  can also correspond to constrained efficient allocations, as long as the allocation satisfies the planner's optimality condition in (A.19). In these cases, the planner mitigates but does not completely alleviate the recession. Intuitively, this is because the aggregate demand externalities with separable preferences depend on the size of the labor wedge,  $\tau_1$  [cf. Eq. (A.18)]. At  $D_1 = \bar{d}_1$ , the labor wedge is small,  $\tau_1 = 0$ , which implies that the planner might prefer to allow for a mild recession so as to improve ex-ante risk sharing. In contrast, the aggregate demand externalities with the GHH preferences analyzed in the main text are large regardless of the labor wedge [cf. Eq. (11)], which induces the planner to fully avoid the recession.