Financial indicators and density forecasts for US output and inflation

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The presentation does not reflect the official view of Banca d'Italia.

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Introduction

Questions

- Are financial indicators useful in forecasting output and inflation?
- Does the answer depend on what kind of **events** the forecaster is interested in predicting? (central case/bad scenarios)
- Does the answer depend on what kind of **models** the forecaster relies on? (linear/nonlinear)
- Was the Great Recession predictable on the basis of real-time financial information?

Introduction

Answers/conjectures

1. Yes (with qualifications)
2. Yes: financial info might be particularly useful in predicting "tail outcomes" and recessions.
3. Yes: nonlinear models account for the fact that the role of financial markets in generating/propagating shocks may change over time.
4. No idea

The paper in a nutshell (1)

Data and models

We cast the analysis as a density prediction problem:

$$pdf^m\left(y_{t+k}\,|I_t\right) = m\left(y_t, f_t, X_t\right)$$

- Monthly US data, 1973-2012
- y_t: industrial production growth, CPI inflation.
- f_t: Financial Condition Index (FCI) published by St Louis Fed.
- m: linear VAR *versus* Threshold VAR (potentially capturing normal times/crises).

The paper in a nutshell (2)

Results

1. VAR gives better point forecasts.
2. TAR gives better density forecasts.
3. f_t improves both, but works best in density space: finance helps in predicting off-equilibrium paths.
4. TAR with finance-driven regimes could have anticipated (up to a point...) the Great Recession.

Broader implications:

- Non-linearities matter

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- Predictive distributions are useful to study the finance-macro nexus

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Broader implications:

- Non-linearities matter
- Predictive distributions are useful to study the finance-macro nexus
- Given (1, 2), objectives and risk preferences of the forecaster become crucial.

Literature (1)

1. Forecasting with financial indicators (Stock-Watson 2003, 2012; Gilchrist-Yankov-Zakrajšek 2009, 2012; Ng-Wright, 2013; ...). Emphasis on point forecasts and linear models.
2. Density forecasting in macro (eg. Clark, 2011). No specific analysis of the role of financial factors.
3. Early warnings and crisis prediction (Borio-Lowe, 2002; Barro-Ursua, 2009; Lo Duca-Peltonen, 2011). Low frequency data and arbitrary/restrictive definition of "crises".

This paper

Contributes to (2), proposes density forecasting as a generalisation of (1) and a link between (1) and (3)

Literature (2)

4 GE models with financial shocks (Gertler-Kiyotaki 2010; Jermann-Quadrini 2012; Kiyotaki-Moore 2012; Liu-Wang-Zha 2013; ...). GE models with occasionally binding credit constraints (Bianchi 2012; Bianchi-Mendoza 2011; Guerrieri-Iacoviello 2013).

5 Evidence of nonlinear, regime-dependent, transmission of macrofinancial shocks (McCallum 1991; Balke 2004; GI 2013). Emphasis on impulse-response analysis.

Bottomline: financial shocks matter, and may have different implications in good and bad (credit-constrained) times.

This paper

Studies/exploits the nonlinearity modelled in (4) and documented in (5) from a forecasting perspective (see toy P.E. model in the paper)

- Data
- Models
- Simulating and evaluating distributions
- Results
- Conclusions

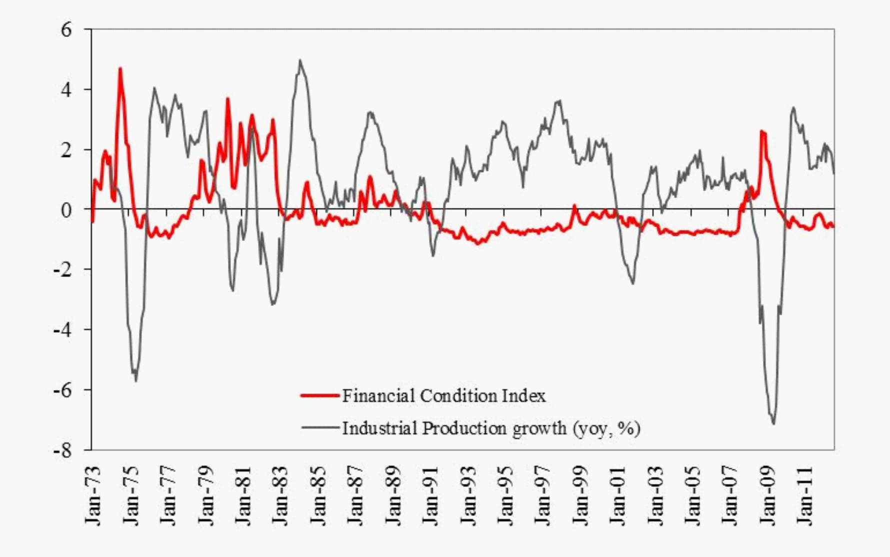
Data

US data, March 1973 – August 2012.

y_t : Industrial Production growth
π_t : CPI inflation
r_t : Fed Funds rate
f_t : Financial Conditions Index

FCI is a dynamic factor constructed from an unbalanced panel of 100 mixed-frequency indicators of financial activity (Brave & Butters 2012; Chicago Fed). Real time, very broad coverage (debt and equity markets, financial sector leverage, ...).

Financial Condition Index



- Data
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Forecasting models

Financial information and non-linearities on a 2x2 grid:

	No Finance	Finance
Linear	$VAR^{\S}$	VAR
Nonlinear	$(MSVAR)$	TAR

- $VAR^{\S}$ = linear VAR without f_t

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- TAR = two-state Threshold VAR with regime switches caused by f_t
- ($MSVAR$ = Markov-switching VAR, not shown for brevity)

VAR

$$Y_t = c + \sum_{j=1}^{P} B_j Y_{t-j} + \Omega^{1/2} e_t, \quad e_t \sim N(0, I) \tag{1}$$

We set $P = 13$ and study two specifications

- $VAR^{\S}$: $\quad Y_t = (y_t, \pi_t, r_t)$
- VAR: $\quad Y_t = (y_t, \pi_t, r_t, f_t)$

Natural conjugate prior (N, IW) as in e.g. Banbura-Giannone-Reichlin (JAE, 2010). All variables treated as independent AR(1) processes:

$$\begin{aligned} Y_t &= c + \Gamma Y_{t-1} + \Sigma e_t \\ \Gamma &= diag(\gamma_1, .., \gamma_N) \\ \Sigma &= diag(\sigma_1, ..., \sigma_N) \end{aligned}$$

TAR

$$
\begin{aligned}
Y_t &= c_{S_t} + \sum_{j=1}^{P} B_{S_t,j} Y_{t-j} + \Omega_{S_t}^{1/2} e_t, \quad e_t \sim N(0, I) && (2)\\
S_t &= \{0, 1\} && (3)\\
S_t &= 1 \Longleftrightarrow f_{t-d} \leq f^* && (4)
\end{aligned}
$$

where $Y_t = (y_t, \pi_t, r_t, f_t)$. Note f_t impacts (y_t, π_t, r_t) through $B_{S_t,j}$ *and* drives the transitions across regimes.

Symmetric natural conjugate prior for the two regimes, plus agnostic prior for (f^*, d):

$$
\begin{gathered}
f^* \sim N\left(\frac{\Sigma_t f_t}{T}, \bar{k}\right) \\
d \sim U\{1, ..., 13\}
\end{gathered}
$$

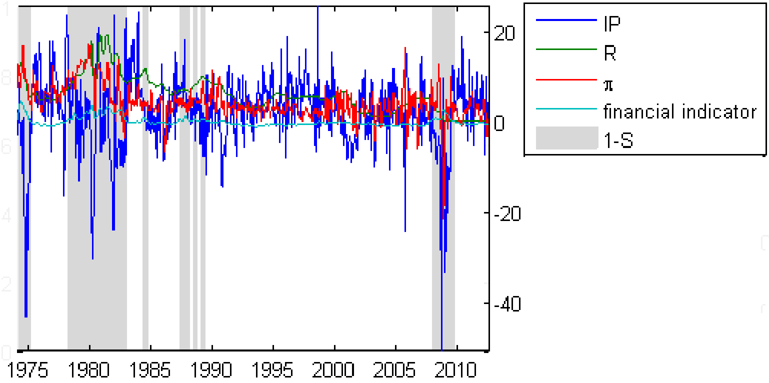
Estimation

Note: the priors are uninformative and a-theoretical. One could use theory to impose structure on the differences between regimes.

- Bayesian approach
- VAR posterior is known analytically (Banbura et al, 2010).
- TAR and MSVAR posteriors can be simulated by Gibbs sampling (Chen & Lee, 1995; Amisano & Fagan, 2010)
- For each estimation we use 20,000 Gibbs sampling draws and discard the first 15,000

Estimation results (1)

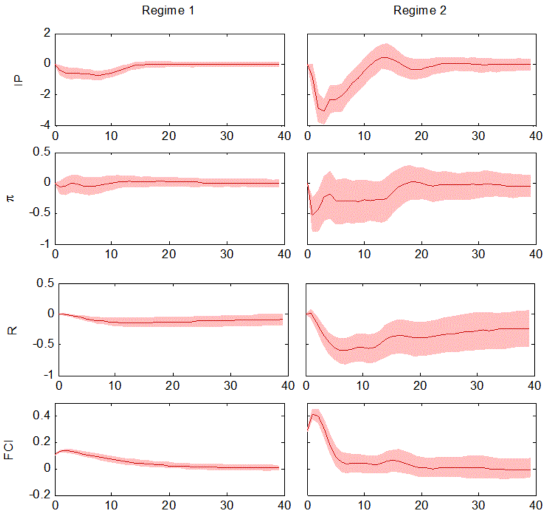
Financial regimes.



$(1-\hat{S}_t) = 1 \Leftrightarrow f_{t-d} > f^* \Leftrightarrow$ financial distress/binding credit constraints

Estimation results

A one standard deviation financial shock (recursive identification)



- Data
- Models
- **Simulating and evaluating distributions**
- Results
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Collect model's m parameters into Θ_t.The k-periods ahead PD is:

$$\begin{aligned} p_t^m &\equiv p^m\left(Y_{t+k} \,\middle|\, Y_t\right) \\ &= \int p\left(Y_{t+K} \,\middle|\, Y_t, \Theta_{t+k}\right) p\left(\Theta_{t+k} \,\middle|\, Y_t, \Theta_t\right) p\left(\Theta_t \,\middle|\, Y_t\right) d\Theta \end{aligned}$$

Simulating the PD:

1. draw $\Theta_t$from the posterior (3rd term)
2. simulate forward any time-varying parameters (2nd term)
3. use Θ_{t+k} to simulate paths for Y_{t+k} (1st term).

Generating the predictive densities

Models and data

- $m = VAR^{\S}, VAR, TAR$
- Recursive exercise: we start from 1973.03–1983.04 and reestimate all models adding one observation at a time.
- For each estimation sample $\{Y_{1,..,T}\}$ we simulate the models up to $K = 12$ months ahead.
- This gives us a set of 354 out-of-sample density forecasts $p^m(Y_{T+k} | Y_T)$ per model.

Evaluating the predictive densities

1. Calibration

Is any of the models "right"?
Probability integral transforms **(PIT)**, probability coverage ratios **(PCR)**

Intuition: the data should fall evenly across model-generated percentiles.

2. Accuracy

How to compare a pair of (potentially misspecified) models?
Log-scores **(LS)**, predictive Bayes factors **(BFs)**

Intuition: higher LS for models attaching higher likelihood to the events that actually occurred.

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Results
Overview

- For y_t, f_t improves both RMSE and LS. The LS gain is significant around the Great Recession.

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- RMSE and LS rank the models in a very different way:

$$\begin{aligned} RMSE &: VAR^{\S}, VAR \succ TAR \\ LS &: VAR^{\S}, VAR \prec TAR \end{aligned}$$

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- Most of these differences are predictable to some extent.

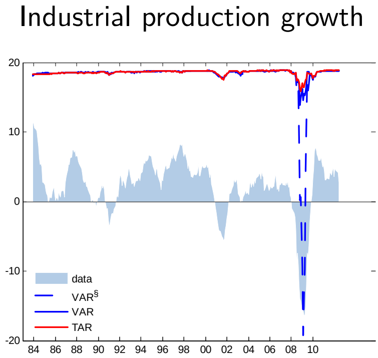
Results

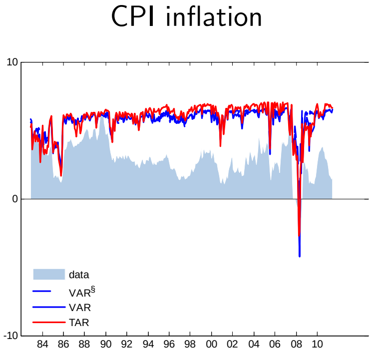
RMSE/LS for output and inflation

		RMSE				LS			
		1M	3M	6M	12M	1M	3M	6M	12M
VAR§	y	5.604	6.465	6.804	7.019	-3.674	-3.338	-3.418	-3.948
	r	0.167*	0.357	0.598	0.985	-0.675	-1.380	-1.754	-2.118
	π	2.078	2.607*	2.812*	3.077*	-2.584	-2.658	-2.266	-2.137
	f	–	–	–	–	–	–	–	–
VAR	y	5.446*	6.166*	6.558*	6.912*	-3.553	-3.156	-3.032	-2.964
	r	0.177	0.365	0.602	0.989	-0.645	-1.357	-1.723	-2.101
	π	2.067*	2.620	2.839	3.115	-2.583	-2.550	-2.339	-2.171
	f	0.102*	0.197	0.289	0.386	0.135	-0.649	-0.957	-1.130
TAR	y	5.491	6.187	6.594	6.934	-3.491*	-3.152*	-3.005*	-2.885*
	r	0.167	0.338*	0.555*	0.943*	0.022*	-0.778*	-1.364*	-1.999*
	π	2.115	2.667	2.864	3.116	-2.503*	-2.415*	-2.195*	-2.080*
	f	0.104	0.190*	0.271*	0.367*	0.496*	-0.122*	-0.431*	-0.717*

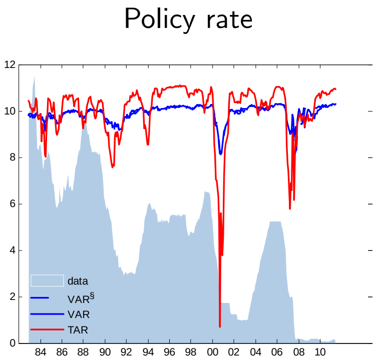
* denotes best model for each criterion/variable/horizon

Log-Scores (1)

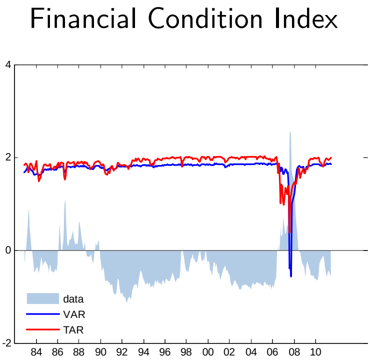

Industrial production growth
20
10
0
-10
-20
data
VAR§
VAR
TAR
84
86
88
90
92
94
96
98
00
02
04
06
08
10


CPI inflation
10
0
-10
data
VAR§
VAR
TAR
84
86
88
90
92
94
96
98
00
02
04
06
08
10

Log-Scores (2)



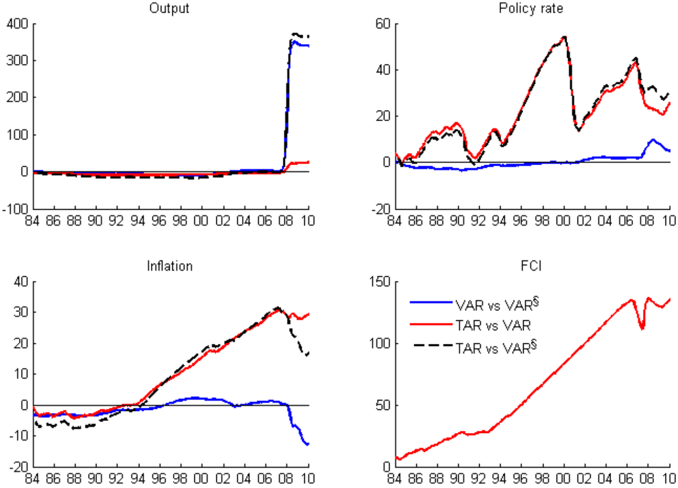
Policy rate
12
10
8
6
4
2
0
84 86 88 90 92 94 96 98 00 02 04 06 08 10
data
VAR§
VAR
TAR



Financial Condition Index
4
2
0
-2
84 86 88 90 92 94 96 98 00 02 04 06 08 10
data
VAR
TAR

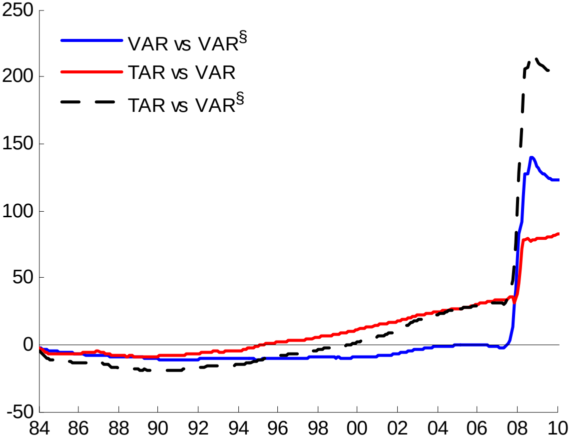
Log-Bayes Factors (1)

Marginal distributions



Log-Bayes Factors (2)

Joint distribution of IP and CPI



Giacomini-White decision criteria

Is the discrepancy between models itself predictable?

Following Giacomini-White (E 2006), we study the persistence of the *difference in performance* between pairs of models:

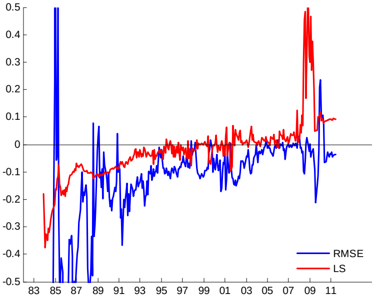
$$\begin{aligned} \Delta Loss_{t+\tau} &= \alpha + \delta \Delta Loss_t + \varepsilon_t \\ \text{where } \Delta Loss &\equiv Loss^{VAR} - Loss^{TAR} \\ \text{and} \quad Loss &\equiv RMSE, \ -LS \end{aligned}$$

Model selection criterion:

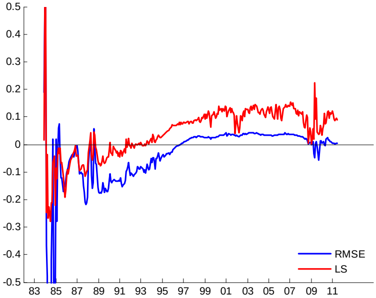
$$Use\ \ TAR \iff E_t \Delta Loss_{t+\tau} > 0 \iff (\hat{\alpha} + \hat{\delta} \Delta Loss_t) > 0$$

Blue (red) line = $E_t \Delta Loss_{t+12}$ for $Loss = RMSE$ ($-LS$).
Positives implies that *TAR* dominates *VAR*.

Industrial production

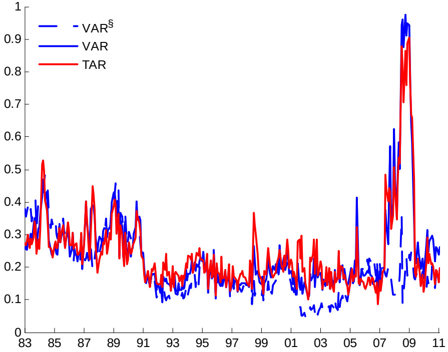


CPI inflation



Predictive densities and early warnings

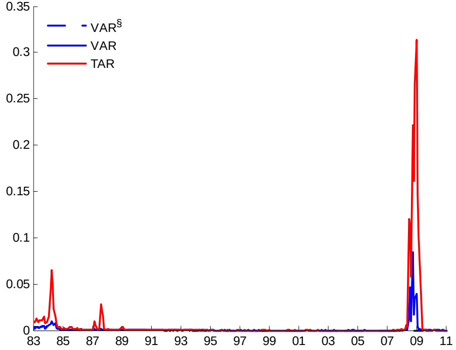
Ex-ante recession probability: $prob_t \left(\Sigma_{h=1}^{12} y_{t+h} < 0\right)$



VAR/TAR virtually identical: all that matters is the presence of FCI

Predictive densities and early warnings

Ex-ante "great recession" probability: $prob_t\left(\Sigma_{h=1}^{12} y_{t+h} < -20\%\right)$



... But TAR anticipates a more severe downturn.

Robustness

- Data: "excess bond premium" (Gilchrist and Zakrajšek, 2012) instead of Financial Condition Index.

  $\longrightarrow$ Similar qualitative results.

- Models: rolling VAR, Markov-switching VAR with transition probabilities that depend on FCI.

  $\longrightarrow$ Both dominated by TAR. TAR appears to capture the "right" kind of time variation in parameters.

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Conclusions

1. **Method.** Predictive distributions are a better tool than point forecasts to study the predictive power of financial indicators.
2. **Financial indicators** improve both, but the improvement is more significant/stable for the densities.
3. **Models:** VAR is better (worse) than TAR for point (density) forecasting. With imperfect models, the risk preferences of the forecaster become crucial.
4. **Great Recession:** essentially unpredictable – but less so for a TAR with finance-driven regimes.

Next steps

- Work out distributional implications of credit constraints in a (more) general equilibrium model.
- Think formally about risk preferences and model selection.
- Refine priors on good/bad regimes
- More robustness (sample, prior hyperparameters, ...)

Thanks!

Reserve slides

Deaton (1991) revisited

Endowment economy with random income and consumption/saving decision subject to borrowing constraint:

$$\max_{(c_t, a_t)_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \Big(\mathcal{U}(c_t) + \mathcal{P}(a_t + \theta_t y) \Big) \tag{5}$$

$$c_t + \frac{a_t}{1+r} = a_{t-1} + y_t \tag{6}$$

$$y_t = e^{z_t}, \;\; z_t \sim N(0, \sigma_z) \tag{7}$$

$$\theta_t = \theta(1-\rho_\theta) + \rho_\theta \theta_{t-1} + \epsilon_t, \quad \varepsilon_t \sim N(0, \sigma_\varepsilon) \tag{8}$$

Deaton (1991) revisited

- **Penalty function**: $\mathcal{P}(a_t + \theta_t y) = \phi \log(a_t + \theta_t y)$.
  Borrowing $(a_t < 0)$ causes disutility, with $\mathcal{P} \to -\infty$ as $a_t \to -\theta_t y$.
  A trick to approximate an occasionally binding constraint:

$$\mathcal{P}(a_t + \theta_t y) \simeq a_t \geq -\theta_t y$$

- **Financial shock** ε_t: shifts the borrowing limit for a given income level. A proxy for collateral value or strength of lender's balance sheet.

Obviously a toy model, with exogenous income and interest rate, but useful to think about (linear/nonlinear) and (central/density) forecasting issue.

Calibration

β	r	θ	σ_z	σ_ε	ρ_θ	ϕ
0.90	0.03	1	0.1	0.01	0.5	0.05

Made up. Low β guarantees that agents borrow in equilibrium:
$-\theta y < a < 0$

Policy functions at II order

	$\hat{a}_{t-1}$	$\hat{\theta}_{t-1}$	z_t	ε_t	$\hat{a}_{t-1}\hat{\theta}_{t-1}$	$\hat{a}_{t-1}z_t$	$\hat{a}_{t-1}\varepsilon_t$	$\hat{\theta}_{t-1}z_t$	$\hat{\theta}_{t-1}\varepsilon_t$
$\hat{c}_t$	0.264	0.058	0.263	0.116	-0.068	-0.127	-0.135	-0.067	-0.085
$\hat{a}_t$	0.758	-0.060	0.754	-0.119	0.069	0.130	0.139	0.070	0.088

Selected terms. All in deviations from steady-state values.

- A negative financial shock $\varepsilon_t < 0$ depresses c and increases a, i.e. it leads to a cut in debt relative to equilibrium

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- Its impact is stronger when debt is already high ($\hat{a}_{t-1} < 0$) and/or borrowing conditions are tight ($\hat{\theta}_{t-1} < 0$)

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	$\hat{a}_{t-1}$	$\hat{\theta}_{t-1}$	z_t	ε_t	$\hat{a}_{t-1}\hat{\theta}_{t-1}$	$\hat{a}_{t-1}z_t$	$\hat{a}_{t-1}\varepsilon_t$	$\hat{\theta}_{t-1}z_t$	$\hat{\theta}_{t-1}\varepsilon_t$
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- Any prediction from a linear model ignores $a\theta$, az, $a\varepsilon$, θz, $\theta\varepsilon$

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- CFs $(E_t c_{t+1})$ from a nonlinear model leave out $a\varepsilon$, θz, $\theta\varepsilon$

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- CFs ($E_t c_{t+1}$) from a nonlinear model leave out $a\varepsilon$, θz, $\theta\varepsilon$
- PDs ($p_t(c_{t+1})$) from a nonlinear model capture all terms.

  For instance, the model should predict an increase in the *volatility* of c_t when θ_{t-1} or a_{t-1} are low (tight markets/high debt).

MS-VAR

$$
\begin{aligned}
Y_t &= c_{S_t} + \sum_{j=1}^{P} B_{j,S_t} Y_{t-j} + \Omega_{S_t}^{1/2} e_t, \quad e_t \sim N(0, I) && (9) \\
S_t &= \{0, 1\} && (10) \\
S_t &= 1 \Longleftrightarrow x_t^* \geq 0 && (11) \\
x_t^* &= \lambda_0 + \gamma_1 f_{t-1} + \lambda_1 S_{t-1} + \nu_t, \nu_t \sim N(0, 1) && (12)
\end{aligned}
$$

where $Y_t = (y_t, \pi_t, r_t)$ and x_t^* is an unobserved state.

Symmetric n.c. prior for the two regimes and agnostic prior for (λ_i, γ):

$$
\left[\begin{array}{ccc} \lambda_0 & \lambda_1 & \gamma_1 \end{array} \right]' \sim N\left(\left[\begin{array}{ccc} -2 & 4 & 0 \end{array} \right]', \bar{k} I \right)
$$

MS-VAR vs TAR

The MS-VAR incorporates a more flexible/possibly weaker role for finance:

- f_t does *not* have a direct impact on (y_t, π_t) through $B_{S_t,j}$
- f_t *may/may not* influence the transitions between regimes:

$\gamma_1 < 0 \Rightarrow$ high f_t increases the prob of entering/being stuck in S_0
$\gamma_1 = 0 \Rightarrow$ fixed, exogenous transition probabilities

Different story:
here financial distress does not cause recessions, but can bring about a state with e.g. lower average output growth and/or different transmission channels for non-financial (monetary, AS, AD) shocks.

MS-VAR

$$Y_t = c_{S_t} + \sum_{j=1}^{P} B_{j,S_t} Y_{t-j} + \Omega_{S,t}^{1/2} e_t \tag{13}$$

$$\begin{bmatrix} \Pr(0\,|0) & \Pr(0\,|1) \\ \Pr(1\,|0) & \Pr(1\,|1) \end{bmatrix} = \begin{bmatrix} P\,(f_{t-1}) & 1 - Q\,(f_{t-1}) \\ 1 - P\,(f_{t-1}) & Q\,(f_{t-1}) \end{bmatrix} \tag{14}$$

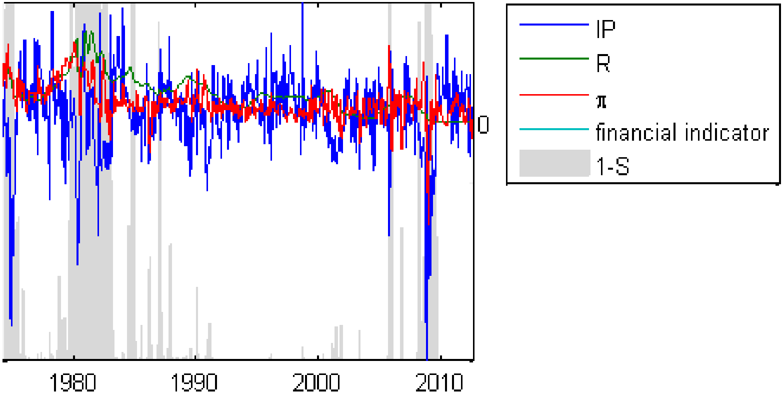
where $e_t \sim N(0, I)$, $Y_t = (y_t, \pi_t, r_t)$, and (P, Q) are Probit models:

$$P(f_{t-1}) = 1 - \Phi\,(\lambda_0 + \gamma_1 f_{t-1}) \tag{15}$$

$$Q(f_{t-1}) = \Phi\,(\lambda_0 + \lambda_1 + \gamma_1 f_{t-1}) \tag{16}$$

Estimation results, FCI specification

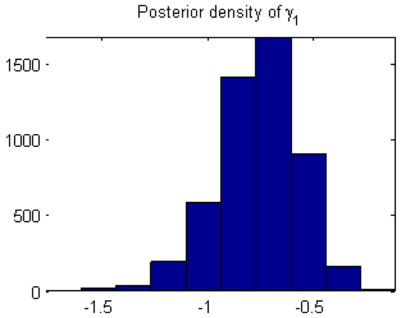
MSVAR regimes



Grey area = median estimate of $\Pr(\hat{S}_t = 0)$ based on full-sample information. Continuous values in $[0, 1]$

Estimation results, FCI specification

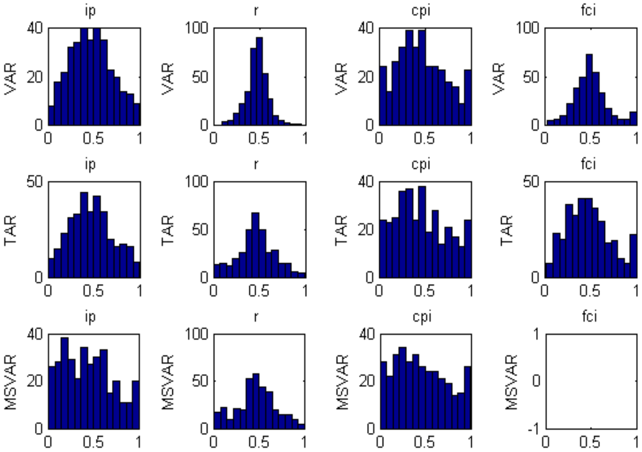
MSVAR posterior



- $\gamma_1 < 0$: financial instability increases the likelihood of entering the bad state
- The BS indicator delivers $\gamma_1 \simeq 0$, and EBP a counterintuitive $\gamma_1 > 0$.

PITs

Specifications based on the Financial Condition Index



Amisano-Giacomini weighted LS test

	Left tail				Both tails			
	y	r	π	f	y	r	π	f
Weighted log-scores:								
$VAR^{\S}$	-1.881	-0.513	-1.846	–	-0.924	-0.220	-0.914	–
VAR	-1.761	-0.491	-1.848	0.249	-0.816	-0.211	-0.927	-0.075
TAR	-1.698*	0.032	-1.779	0.479*	-0.753*	0.029	-0.866*	0.149*
MSVAR	-2.006	0.066*	-1.732*	–	-1.129	0.055*	-0.887	–
P-values:								
$VAR^{\S}$,VAR	0.050	0.000	0.230	–	0.139	0.021	0.181	–
TAR, VAR	0.370	0.000	0.674	0.000	0.401	0.000	0.801	0.000
MSVAR, VAR	0.517	0.000	0.425	–	0.025	0.000	0.334	–
MSVAR, TAR	0.101	0.228	0.098	–	0.535	0.026	0.333	–

(*) denotes the best model for each variable and weighting function