Understanding the Gains from Wage Flexibility: The Exchange Rate Connection*

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Abstract

We study the gains from increased wage flexibility and their dependence on exchange rate policy, using a small open economy model with staggered price and wage setting. Two results stand out: (i) the impact of wage adjustments on employment is smaller the more the central bank seeks to stabilize the exchange rate, and (ii) an increase in wage flexibility often reduces welfare, and more likely so in economies under an exchange rate peg or an exchange rate-focused monetary policy. Our findings call into question the common view that wage flexibility is particularly desirable in a currency union.

Keywords: sticky wages, nominal rigidities, New Keynesian model, stabilization policies, exchange rate policy, currency unions, monetary policy rules.

JEL Classification No.: E32, E52, F41

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1 Introduction

The belief in the virtues of wage flexibility is widespread in policy circles. It manifests itself most clearly in the recurrent calls for wage moderation (or even outright wage cuts), issued by international policy institutions, and addressed to countries facing high unemployment. The Great Recession and the "crisis of the euro" have only reinforced those views.

The case for flexible wages rests on its perceived role as a factor of macroeconomic stability. Thus, a decrease in wages is expected to offset, at least partly, the negative effects on employment (and output) of an adverse aggregate shock. Conversely, the presence of rigid wages tends to amplify the employment and output effects of most aggregate shocks.\(^1\) Figure 1 illustrates that "classical" view, using a conventional labor market diagram.

The role of wages as a cushion is viewed as being particularly important in the context of economies that have joined a currency union or adopted any other form of hard peg, for in those cases the exchange rate is no longer available as an adjustment mechanism. In the face of a country-specific adverse shock that calls for a real exchange rate depreciation, a wage-based "internal devaluation" is warranted. The presence of wage rigidities, it is argued, will hinder that adjustment, and make it longer and more painful, by requiring, ceteris paribus, a higher rate of unemployment to bring about the needed adjustment in wages and prices. To the extent that wage flexibility acts as a substitute for exchange rate flexibility, it is viewed as particularly desirable in economies that have adopted a hard peg or joined a currency union.\(^2\)

The conventional wisdom described above ignores, however, the fact that in economies

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\(^2\) The analysis of the interaction between wage rigidities and the exchange rate regime has a long tradition in macroeconomics, tracing back to the seminal work of Friedman (1953) and Mundell (1961). Recent research on the consequences of wage rigidity in currency unions can be found Schmitt-Grohe and Uribe (2012) and Farhi et al. (2013).
with nominal rigidities the impact of wage adjustments on employment works to a large extent through its induced effect on the endogenous component of monetary policy, as the latter is loosened or tightened in response to lower or higher inflationary pressures. We refer to this as the "endogenous policy channel.". Thus, and as argued in Galí (2013) in the context of a closed economy model, whether an increase in wage flexibility raises welfare depends on the monetary policy rule in place and, in particular, on the strength of the central bank’s systematic response to inflation. If that response is weak, the benefits of increased wage flexibility in the form of more employment stability will be small and, in many cases, more than offset by the losses associated with greater volatility in price and wage inflation.

In the present paper we extend the analysis of the gains from wage flexibility to the case of an open economy. As we discuss below, openness brings about two additional factors with potentially counteracting implications. First, openness makes room for a "competitiveness channel", whereby a reduction in domestic wages leads to a terms of trade depreciation and, as a result, an increase in aggregate demand, output and employment. That mechanism should work to stabilize employment and, ceteris paribus, to strengthen the "endogenous policy channel". From the viewpoint of the "competitiveness channel", the degree of openness of the economy and the elasticity of net exports with respect to the real exchange rate would seem to be important determinants of the gains from greater wage flexibility.

On the other hand, monetary policy in the open economy may be driven, to a greater or lesser extent, by the desire to stabilize the exchange rate. In the absence of capital controls, maintaining a stable exchange rate requires that the interest rate does not deviate much from its relevant foreign counterpart. In that case, the "endogenous policy channel" will be dampened (or fully muted, in the case of a hard peg or a currency union), and so will be the effect of lower wages on aggregate demand and employment.

In order to understand the role played by the exchange rate in determining the gains
from wage flexibility, we develop a small open economy model with staggered price and wage setting, and study the impact of greater wage flexibility on macroeconomic stability and welfare, as a function of the exchange rate policy in place. Our model builds on the framework developed in Galí and Monacelli (2005), which we extend by allowing for nominal wage rigidities.3

Our analysis delivers two main findings. Firstly, we show that the impact of wage adjustments on employment is smaller the more the central bank seeks to stabilize the exchange rate. Hence, they are particularly ineffective in a currency union. Secondly, an increase in wage flexibility often reduces welfare, and more likely so in economies under an exchange rate-driven monetary policy. Our findings thus call into question the common view that wage flexibility is particularly desirable in a currency union.

The remainder of the paper is organized as follows. In Section 2 we describe our baseline model. In Section 3 we report our main findings regarding the role of exchange rate policy in determining the gains from increased wage flexibility. Section 4 analyzes the robustness of our findings to departures from our baseline calibration. Section 5 discusses the related literature. Section 6 summarizes the main lessons from the paper and concludes.

2 A New Keynesian Model of a Small Open Economy

In this section we describe the key ingredients of the model we use in our analysis of the gains from wage flexibility. Our model is one of a small open economy with staggered price and wage setting. It builds on the framework developed in Galí and Monacelli (2005), extending the latter by introducing sticky nominal wages (in addition to sticky prices), and a preference/demand shock (in addition to a technology shock).4 Since the

3The resulting framework is similar to the one used in Campolmi (2012) and Erceg et al. (2009).
4See, e.g. Campolmi (2012) and Erceg et al. (2009) for earlier examples of New Keynesian open economies with staggered nominal wage setting.
model is relatively standard, we restrict our exposition below to a description of the main assumptions, while relegating most derivations to an Appendix.

2.1 Households

We study a small open economy inhabited by a continuum of households, indexed by $i \in [0, 1]$, and with preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t(i), N_t(i); X_t)$$

where $N_t(i)$ denotes the amount of a differentiated labor service supplied by the household, $C_t(i)$ is a consumption index, and $X_t$ is an exogenous preference shifter, common to all domestic households. Period utility $U$ is assumed to take the form

$$U(C_t(i), N_t(i); X_t) = \left( \log C_t(i) - \frac{1}{1+\varphi} N_t(i)^{1+\varphi} \right) X_t$$

Under the assumption of complete financial markets, and given separable utility, consumption is equalized across domestic households. Thus, and in order to lighten the notation, we henceforth drop the index $i$ associated with household consumption.

The consumption index is defined by

$$C_t \equiv \left( (1-\nu)\frac{1}{\eta} C_{H,t}^{1-\frac{1}{\eta}} + \nu \frac{1}{\eta} C_{F,t}^{1-\frac{1}{\eta}} \right)^{\eta-1}$$

with $C_{H,t}$ being an index of domestic goods consumption given by the CES function

$$C_{H,t} \equiv \left( \int_0^1 C_{H,t}(j)^{\frac{1}{\eta}} dj \right)^{\frac{1}{\eta}}$$

where $j \in [0, 1]$ denotes the good variety.\footnote{As discussed below, domestic firms produce a continuum of differentiated goods, indexed by $j \in [0, 1]$.} $C_{F,t}$ is the quantity consumed of a composite foreign good. Parameter $\epsilon_p > 1$ denotes the elasticity

\footnote{For the limiting case of $\eta = 1$ the consumption index takes the form

$$C_t \equiv \Upsilon{(C_{H,t})^{1-v}(C_{F,t})^{v}}$$

where where $\Upsilon \equiv 1/(1-v)^{(1-v)\nu^v}$}
of substitution between varieties produced domestically. Parameter $\nu \in [0, 1]$ can be interpreted as a measure of openness.\footnote{Equivalently, and under the assumption that the domestic economy is infinitesimally small, $1 - \nu$ can be interpreted as a measure of home bias. See Galí and Monacelli (2005) for a discussion.}

The (log) preference shifter, $x_t \equiv \log X_t$, is assumed to follow an exogenous $AR(1)$ process:

$$x_t = \rho_x x_{t-1} + \varepsilon_t^x$$

The period budget constraint for the typical household is given by

$$\int_0^1 P_{H,t}(j)C_{H,t}(j)\,dj + P_{F,t}C_{F,t} + E_t\{Q_{t,t+1}D_{t+1}\} \leq D_t + W_t(i)N_t(i)$$

for $t = 0, 1, 2, \ldots$, where $P_{H,t}(j)$ is the price of domestic variety $j$. $P_{F,t}$ is the price of the imported good, expressed in domestic currency. $D_{t+1}$ is the nominal payoff in period $t + 1$ of the portfolio held at the end of period $t$ (which may include shares in domestic firms), $W_t(i)$ is the nominal wage for type $i$ labor. The previous variables are all expressed in units of domestic currency. $Q_{t,t+1} \equiv \beta(C_t/C_{t+1})(P_t/P_{t+1})$ is the relevant stochastic discount factor for one-period ahead nominal payoffs.

We assume that the law of one price holds at the level of each individual variety, implying

$$P_{F,t} = \mathcal{E}_t P^*_t$$

where $\mathcal{E}_t$ is the nominal exchange rate and $P^*_t$ is the foreign price level (in foreign currency). With little loss of generality, the latter is assumed to be constant and normalized to unity.

Each household is specialized in the provision of some differentiated labor service, for which firms generate an isoelastic demand (see below) and for which each household sets the corresponding nominal wage.\footnote{Alternatively, one can think of many households supplying each type of labor, with a union representing them setting the wage on their behalf.} Each period only a fraction $1 - \theta_w$ of households, drawn randomly from the population, reset their nominal wage in a way consistent with utility maximization, subject to the demand for their labor services (current and future).
The remaining fraction $\theta_w$ of households keep their nominal wage unchanged. Parameter $\theta_w \in [0, 1]$ can be thus seen as an index of nominal wage rigidities. Much of the analysis below explores the consequences of changes in that parameter.

As in Galí and Monacelli (2005), we assume domestic households have access to a complete set of state-contingent securities, traded domestically and internationally.

### 2.2 Firms

The home economy has a continuum of domestic firms, indexed by $j \in [0, 1]$. A typical firm produces a differentiated good using the technology

$$Y_t(j) = A_t N_t(j)^{1-\alpha}$$

where $Y_t(j)$ is output and $N_t(j) \equiv \left( \int_0^1 N_t(i,j)^{\frac{w-1}{w}} di \right)^{\frac{1}{w-1}}$ is a CES function of the quantities of different types of labor services hired. Parameter $\epsilon_w > 1$ denotes the elasticity of substitution between labor service varieties. $A_t$ is a stochastic technology parameter, common to all firms. Its logarithm, $a_t \equiv \log A_t$, follows an exogenous AR(1) process:

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a$$

Employment is subject to a proportional payroll tax $\tau_t$, common to all labor types, so that the effective cost of hiring one unit of type $i$ labor service is $W_t(i)(1 + \tau_t)$.

Each period, a subset of firms of measure $1 - \theta_p$, drawn randomly, reoptimize the price of their good, subject to a sequence of demand schedules for the latter. The remaining fraction $\theta_p$ keep their price unchanged. Parameter $\theta_p \in [0, 1]$ can thus be interpreted as an index of price rigidities. Prices are set in domestic currency and are the same for both the domestic and export markets. All firms meet the demand for their respective goods at the posted prices.

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<sup>9</sup>Note that a negative value for $\tau_t$ should be interpreted as an employment subsidy.
2.3 Demand for Exports

We assume that the demand for domestic good $j$ coming from the rest of the world is given by:

$$C^*_t(j) = \left( \frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\epsilon_p} C^*_t,$$

for $j \in [0,1]$, where $P_{H,t} \equiv \left( \int_0^1 P_{H,t}(j)^{1-\epsilon_p} dj \right)^{-\frac{1}{1-\epsilon_p}}$ is the domestic price index. Aggregate exports, $C^*_t$, are in turn given by

$$C^*_t = \nu \left( \frac{P_{H,t}}{P_{F,t}} \right)^{-\eta} C^*_t,$$

where, without loss of generality, the units in terms of which world consumption is expressed have been normalized so that in a symmetric steady state $P_H = P_F$, $C^*_H = \nu C^*$, and $C = C^*$.

For simplicity, and with little loss of generality, we assume that aggregate output and consumption in the world economy are constant, and equal to one.

2.4 Monetary Policy

The monetary authority in the home economy is assumed to follow an interest rate rule of the form:

$$i_t = \rho + \phi_{\pi} \pi_{H,t} + \frac{\phi_e}{1 - \phi_{e}} e_t,$$

where $i_t$ is the short-term policy rate, $\pi_{H,t} \equiv p_{H,t} - p_{H,t-1}$ denotes domestic inflation and $e_t \equiv \log E_t$ is the (log) nominal exchange rate. $\phi_{\pi} \geq 1$ and $\phi_e \in [0,1]$ are coefficients determining the strength of the central bank’s response to deviations of inflation and the (log) nominal exchange rate from their respective targets (normalized to zero). Note that in the limiting case of $\phi_e \to 1$ we have $e_t = 0$ for all $t$, which corresponds to an

\[\text{See Monacelli (2004) for an analysis of fixed exchange rates when alternative monetary policy regimes are specified according to rule (4).}\]
Throughout, and with little loss of generality, we assume that the interest rate in the rest of the world is constant, and normalized to zero ($i_t^* = 0$).

### 2.5 Equilibrium

In the Appendix we derive the (standard) optimality conditions for the problem facing households and firms. Combined with the market clearing conditions and after log-linearization around the zero inflation steady state, they can be used to determine the set of conditions characterizing the equilibrium of the small open economy. That equilibrium can be represented by means of the following system of difference equations (with lower case letters denoting the natural logarithms of the original variables and with constants ignored):

**Aggregate demand block:**

\[ y_t = (1 - \nu) c_t + \eta \nu (2 - \nu) s_t \]  \hspace{1cm} (5)

\[ c_t = x_t + (1 - \nu) s_t \]  \hspace{1cm} (6)

\[ c_t = E_t \{ c_{t+1} \} - (1 - \nu) (i_t - E_t \{ \pi_{H,t+1} \}) + (1 - \rho_x) x_t \]  \hspace{1cm} (7)

\[ i_t = \phi_x \pi_{H,t} + \frac{\phi_c}{1 - \phi_c} e_t \]  \hspace{1cm} (8)

\[ s_t \equiv e_t - \pi_{H,t} \]  \hspace{1cm} (9)

\[ n_t = \frac{1}{1 - \alpha} (y_t - a_t) \]  \hspace{1cm} (10)

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11 Alternatively, one may assume the rule:

\[ i_t = \phi_x \pi_{H,t} + \frac{\phi_c}{1 - \phi_c} e_t + \phi_{\Delta} \Delta e_t \]

a particular case of which, given by $\phi_{\Delta} = \phi_x \nu$, corresponds to

\[ i_t = \phi_x \pi_t + \phi_c e_t \]

where $\pi_t \equiv p_t - p_{t-1}$ is CPI inflation.
Aggregate supply block:

\[
\pi^p_{H,t} = \beta E_t\{\pi^p_{H,t+1}\} + \frac{\lambda_p\alpha}{1-\alpha} \bar{y}_t + \lambda_p \bar{\omega}_t + \lambda_p \nu \bar{\pi}_t + \lambda_p \tau_t \tag{11}
\]

\[
\pi^p_{H,t} \equiv p_{H,t} - p_{H,t-1} \tag{12}
\]

\[
\pi^w_t = \beta E_t\{\pi^w_{t+1}\} + \frac{\lambda_w\phi}{1-\alpha} \bar{y}_t + \lambda_w \bar{\omega}_t - \lambda_w \bar{\pi}_t \tag{13}
\]

\[
\pi^w_{w,t} \equiv w_t - w_{t-1} \tag{14}
\]

\[
\omega_t \equiv w_t - (p_{H,t} + \nu \bar{s}_t) \tag{15}
\]

where variables with a "\(\bar{\}\)" denote deviations from their natural (i.e. flexible price and wage) equilibrium counterparts (e.g., \(\bar{y}_t \equiv y_t - y^n_t\) denotes the output gap, with \(y^n_t\) being the natural level of output).

The aggregate demand block includes equation (5) determining output as a function of aggregate demand, which in turn is expressed as a function of consumption \(c_t\) and the terms of trade \(s_t\) (defined in (9)). Consumption evolves according to Euler equation (7), and thus responds to changes in the domestic real rate and the preference shifter.\(^{12}\) In addition, domestic consumption satisfies the risk sharing condition (6).\(^{13}\) Equation (8) is the interest rate rule introduced earlier. Equation (10) determines employment as a function of aggregate output, given technology.

The aggregate supply block consists of two equations, (11) and (13), describing the evolution of aggregate (domestic) price and wage inflation (defined, respectively, by (12) and (14)), as a function of the output, consumption and real wage gaps (as well as the

\(^{12}\)Notice that the (log-linearized) consumption Euler equation features a dependence of expected consumption growth on the real interest rate measured in units of domestic goods, \(i_t - E_t\{\pi_{H,t+1}\}\). In the Appendix we show how this can be derived from the original Euler equation featuring the CPI-based real interest rate, once the same condition is combined with a log-linear UIP condition and the definition of CPI inflation.

\(^{13}\)The intertemporal optimality conditions of the domestic and foreign consumers can be combined to yield, as a first order approximation, the interest parity condition

\[
i_t = E_t\{\Delta e_{t+1}\}
\]

We do not list that condition separately since it can be obtained by combining (6) and (7).
payroll tax in the case of price inflation). Finally, (15) defines the real (consumption) wage, as a function of the nominal wage, the domestic price and the terms of trade.\textsuperscript{14}

As derived in the Appendix, natural employment, which we denote by $n_t^n$ is given by

$$n_t^n = \Omega(\eta - 1)\nu(2 - \nu)a_t - \Omega\nu[1 + (\eta - 1)(2 - \nu)]x_t - n\tau_t$$

where $\Omega \equiv \frac{1}{1-\alpha+(\alpha+\varphi)[1+(\eta-1)\nu(2-\nu)]} > 0$. Note that under our assumptions on technology and preferences, and in the absence of variations in the employment subsidy, natural employment would be constant in a closed economy (i.e. under $\nu = 0$). For the open economy natural employment remains independent of technology in the special case of $\eta = 1$, but is still affected by the demand shock (through the risk sharing condition).

The previous expression can be combined with other equilibrium conditions to derive the natural values of the remaining variables. Thus, and ignoring constants,

$$y_t^n = a_t + (1 - \alpha)n_t^n$$

$$s_t^n = a_t - x_t - \tau_t - (\alpha + \varphi)n_t^n$$

$$c_t^n = x_t + (1 - \nu)s_t^n$$

$$\omega_t^n = a_t - \alpha n_t^n - \tau_t - \nu s_t$$

\textbf{2.6 Calibration}

Table 1 lists the baseline settings for the model parameters, which we use in many of the simulations below. Most of those settings are pretty standard. The curvature of labor disutility, $\varphi$, is set to 5, a value consistent with a Frisch labor elasticity of 0.2. The discount factor $\beta$ is set to 0.99. Parameter $\alpha$, measuring the degree of decreasing returns to labor, is set to 0.25. Parameters $\epsilon_p$ and $\epsilon_w$ are set, respectively to values 4.52 and 9. As discussed in Galí (2011), the former is consistent with a steady state unemployment rate of 5 percent, while the latter implies a steady state price markup of 12.5 percent.

\textsuperscript{14}Note that $p_H,t + \nu s_t = p_t$ corresponds to the (log) CPI.
The baseline setting for the Calvo price and wage stickiness parameters, $\theta_p = \theta_w = 0.75$, imply an average duration of individual prices and wages of one year, in a way consistent with much of the micro evidence.\textsuperscript{15} Much of the analysis below, however, consists of determining the consequences of variations in $\theta_w$, an its interaction with the exchange rate coefficient $\phi_e$. The inflation coefficient in the interest rate rule is set to 1.5, the value proposed by Taylor (1993). We set the baseline elasticity of substitution between domestic and foreign goods, denoted by $\eta$, to unity (a convenient case, as discussed below), and the openness parameter, $\nu$, to 0.4 (implying a steady state import share of 0.4). In the robustness section we explore alternative settings for both parameters, as well as for $\theta_p$. Finally, we choose 0.9 as a baseline value for persistence parameters $\rho_x$ and $\rho_u$.

3 The Impact of Labor Costs on Employment: The Role of Exchange Rate Policy

The extent to which wage flexibility may play a stabilizing role depends on the influence that wages (or other labor costs) may have on employment itself. In this section we "dissect the mechanism" through which that influence manifests itself in our model economy.

As argued in Galí (2013), the mechanism through which adjustments in wages end up affecting employment in the New Knesian model is very different from that in a classical economy. In the latter, a change in the real wage directly affects the quantity of labor demanded by firms, which is determined by the equality between the marginal product of labor and the wage. By way of contrast, in a Keynesian environment the amount of labor hired is determined by the quantity of output firms want to produce (given technology), which in turn is determined by aggregate demand. Thus, a change in wages ends up affecting employment through its (sequential) impact on marginal cost, inflation and --through the policy rule-- nominal and real interest rates and, hence, consumption (or

\textsuperscript{15}See, e.g., Taylor (1999), Nakamura and Steinsson (2007) and Baratieri, Basu and Gottschalk (2013).
other interest rate-sensitive components) and aggregate demand. Thus, as demonstrated in Galí (2013), the strength of the central bank’s response to variations in inflation is a key factor in determining the response of employment to a change in wages (or other labor costs). This is what we refer to as the "endogenous policy channel". In the open economy, the extent to which domestic monetary policy is constrained by the desire (or need) to stabilize the exchange rate, will influence the central bank’s response to the changes in inflation brought about by a wage adjustment and, as a result, the ultimate impact on employment of such an adjustment.

In order to illustrate the previous point, we simulate the response of employment to an exogenous decline in the payroll tax, a component of the labor cost which our model treats as exogenous. In a classical economy, that policy intervention would have a direct effect on labor demand and would raise employment. This is not the case in a Keynesian environment like the one analyzed here, in which the response of employment will depend to a great extent on how the central bank reacts to the "disinflationsary" pressures triggered by the payroll tax cut. More specifically, we study how the response of employment to the payroll tax cut depends on the strength of the central bank’s response to the exchange rate, as measured by $\phi_e$.

We assume that the payroll tax follows an exogenous $AR(1)$ process with autoregressive coefficient of 0.9, and simulate the impact of a 1 percent drop. Figure 1.a displays the implied impulse response of employment to the payroll tax cut as a function of $\phi_e$. When the central bank’s concern for exchange rate stability is weak (i.e., for values of $\phi_e$ close to zero) employment increases substantially in response to that policy intervention. As we increase the value of $\phi_e$ the response of employment becomes more muted and is almost negligible when $\phi_e$ is close to unity.

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16To be clear: we do not think that exogenous variations in payroll taxes or employment subsidies are an important source of fluctuations in actual economies. But a change in the payroll tax provides a clean experiment to examine the impact of changes in labor costs on employment.

17Unless stated otherwise, the remaining parameters are set at their baseline values in all simulations.
What explains the inverse relation between the size of the employment response and the value of $\phi_e$? Equation (5) makes clear that the response of output (and, hence, of employment, given an unchanged technology) is in turn a function of the consumption and terms of trade responses. Next we discuss the determinants of those responses.

To understand the determinants of the consumption response, we solve equation (7) forward to yield:

$$c_t = x_t - (1 - \nu) \sum_{k=0}^{\infty} E_t\{r_{t+k}\}$$

where $r_t \equiv i_t - E_t\{\pi_{H,t+1}\}$ is the real interest rate, measured in terms of domestic goods. Thus, in the absence of a preference shock, we see that the response of consumption is inversely related to the sum of current and expected future real rates. It is easy to show that a similar result holds for the terms of trade: by combining (6) and (7), one can derive a "real" version of uncovered interest parity condition

$$s_t = -r_t + E_t\{s_{t+1}\}$$

which in turn can be solved forward to yield

$$s_t = -\sum_{k=0}^{\infty} E_t\{r_{t+k}\}$$

Thus, we see that the dependence of the employment effects of a payroll tax reduction on the exchange rate coefficient $\phi_e$ must involve the latter’s differential impact on the path of real interest rates. This is confirmed by Figure 1.b, which plots the response of the real interest rate to the same policy intervention, as a function of $\phi_e$ (note that the directions of the axes has been reversed for better viewing). Thus, for low values of $\phi_e$ the real rate goes down and remains persistently below its initial level in response to the payroll tax cut. By contrast, for higher values of $\phi_e$ the real rate rises in response to the same shock.\textsuperscript{18} The behavior of the real interest rate is, given the sluggishness of prices,

\textsuperscript{18}Note, however, that independently of the value of $\phi_e$ the cumulative sum $\sum_{k=0}^{\infty} E_t\{r_{t+k}\}$ is negative, since employment rises in all cases. Thus, the initial increase in the real rate observed for high values of $\phi_e$ is more than offset by a subsequent decline below the steady state level.
very much influenced by the response of the nominal interest rate, displayed in Figure 2.c. That response is always persistently negative (given the deflationary pressures), though its size is strongly dependent on $\phi_e$. The larger is the latter coefficient, the more muted is the reduction in the nominal rate in response to the payroll tax cut, for the central bank seeks to limit to a greater extent the resulting depreciation of the nominal exchange rate. For sufficiently large values of $\phi_e$, the reduction in the nominal rate is more than offset in the short run by the fall in expected inflation, thus leading to a higher real rate.

The explanation for the main finding of this section is thus clear: in a New Keynesian open economy, a reduction in labor costs (exemplified above by a cut in payroll taxes) does not have a direct effect on employment; instead it ends up influencing it through its downward effect on price inflation and the consequent loosening of monetary policy (determined by $\phi_e$). When the exchange rate is not a concern for monetary policy, the fall in nominal and real interest rates is large, with the implied expansionary effect on consumption being complemented by the stimulus resulting from the implied nominal and real exchange rate depreciation. By contrast, when nominal exchange rate stability is given a significant weight as a monetary policy objective, the reduction in nominal (and real) interest rates triggered by the downward inflationary pressures is dampened by the desire to avoid a large nominal exchange rate depreciation, thus leading to a weaker aggregate demand stimulus and a smaller employment response. Note that for values of $\phi_e$ sufficiently close to unity, the response of the nominal rate is negligible (zero in the limiting case of an exchange rate peg, $\phi_e = 1$), implying a rise in the real interest rate in the short run.\(^{19}\) Note, however, that what matters for the response of both consumption and the terms of trade is not the immediate response of the real rate, but its expected

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\(^{19}\)There is an obvious (though far from complete) analogy between the limited (or even perverse) response of the interest rate to a disinflationary shock due to the exchange rate stability concerns emphasized here and that resulting from the zero lower bound (ZLB) constraint on the nominal interest rate becoming effective (see, e.g. Eggertsson and Woodford (2003)). The differences between the two include the non-linear nature of the ZLB constraint as well as its "inescapability" (though the latter may apply in practice to economies that belong to a currency union).
cumulative response. For the limiting case of an exchange rate peg, we know that the nominal interest rate does not change and the price level must eventually revert back to its original level, implying that the response of \( P_1 \) to the payroll tax cut (or any other shock) is equivalent to that of \( p_t \), which is negative.\(^{20}\) Thus, both consumption and the terms of trade must increase, and so must employment, even though the latter’s response is shown to be more muted than under flexible exchange rates.

To summarize the main finding of this section: we have shown how the effects on employment of exogenous changes in labor costs are strongly mediated by the response of monetary policy. The latter is, in turn, strongly shaped by preferences and/or commitments regarding the nominal exchange rate. When the exchange rate is fixed, as in a currency union, or zealously managed so that it does not deviate much from target, supply side interventions aimed at stimulating employment through a reduction in labor costs will be little effective, however well intended. The previous finding suggests that in those cases, an increase in wage flexibility, with its consequent greater sensitivity of labor costs to cyclical conditions, may not bring the employment stability benefits that may be expected from it. An analysis of those benefits is the focus of the next section.

4 Wage Flexibility, Exchange Rate Policy and Welfare

The previous section has focused on the role of a country’s exchange rate policy in determining the employment effects of an exogenous change in labor costs (in the form of a payroll tax cut). In actual economies, however, such exogenous shocks to wages or other labor cost components are likely to be rare events. Instead, labor costs are better viewed as endogenous, with wages adjusting to changes in economic conditions resulting from a

\(^{20}\)More generally, when \( \phi_e \) is positive the price level becomes stationary, implying that the initial fall in inflation is followed by an eventual increase in that variable, thus reinforcing the effect of the nominal rate response on the real rate, and accounting for the eventual negative response of that variable.
variety of demand and/or supply shocks. Needless to say, that adjustment may be faster or slower, full or partial, depending on the degree of wage flexibility.

As argued in the introduction, the degree of wage flexibility, i.e., their sensitivity to changes in economic conditions, is generally viewed as a key determinant of employment stability. Thus, and in the face of an adverse shock, a reduction in the average wage is likely to insulate, at least partly, the impact on employment. But the findings in the previous section suggest that, in an open economy, the exchange rate policy in place will be an important determinant of the extent to which endogenous wage adjustments may play a role in stabilizing employment fluctuations. In particular, that role is likely to be limited when exchange rate stability has an important weight in the monetary policy strategy. The previous observation, combined with the fact that—as is the case in our model economy—(i) fluctuations in wage and price inflation are costly in their own right and (ii) the size of such fluctuations is likely to increase with wage flexibility, raises the possibility that a reduction in wage rigidities may be counterproductive from a welfare viewpoint, its stabilizing benefits being too small to offset its harmful side effects.

In the present section we analyze formally the welfare gains from greater wage flexibility and their dependence on exchange rate policy. In particular, we seek to uncover the conditions under which, contrary to conventional wisdom, "improvements" in wage flexibility may be welfare-reducing.

In the next subsection we restrict our analysis to the baseline calibration. Most importantly, the assumption of a unit elasticity of substitution between domestic and foreign goods ($\eta = 1$), which is part of that baseline calibration, allows us to derive a simple second order approximation to the welfare losses experienced by the household. Departures from that baseline calibration are discussed later in the robustness section.
4.1 Wage Flexibility, Exchange Rate Policy and Welfare: The Baseline Case

In the special case of a unit elasticity of substitution between domestic and foreign goods \(\eta = 1\) and under the assumption of an optimal employment subsidy, the average welfare losses of domestic households are proportional, up to a second order approximation, to:\(^{21}\)

\[
L \sim (1 + \varphi) \text{var}(\delta_t) + \left(\frac{\epsilon_p}{\lambda_p(1 - \alpha)}\right) \text{var}(\pi_t^p) + \left(\frac{\epsilon_w}{\lambda_w}\right) \text{var}(\pi_t^w) \tag{16}
\]

i.e. a linear combination of the variances of the employment gap, price inflation and wage inflation.

Figure 3 displays the average welfare loss experienced by domestic households as a function of (i) the degree of wage stickiness, \(\theta_w\), and (ii) the exchange rate coefficient in the interest rate rule, \(\phi_e\). The remaining parameters are set at their baseline values.\(^{22}\) For this first batch of results, we condition on fluctuations being driven by demand shocks only. Several results are worth emphasizing. First, note that the relationship between the welfare loss and the degree of wage rigidity is non-monotonic, independently of \(\phi_e\). Starting from a value of \(\theta_w\) close to unity (strong wage rigidities), a reduction in that parameter (i.e., making wages "more flexible") always raises welfare losses. On the other hand, if wages are sufficiently flexible to begin with (i.e., \(\theta_w\) is sufficiently low), a further reduction in that parameter leads to a decline in welfare losses. Thus, an increase in wage flexibility may raise or lower welfare, depending on the initial degree of wage rigidities. Note also that the shape of the welfare loss function varies considerably with \(\phi_e\). Next we seek to understand the factors behind such patterns.

Figure 4 displays the three components of the welfare loss function, each being associated with one of the three terms in (16). The graph for the first component, associated

\(^{21}\)The derivation of the welfare loss function (to be written up) combines elements of the derivations of the corresponding function in Galí-Monacelli (2005) with those related to staggered-wage setting in Erceg et al. (2000). See Campolmi (2012). The \(\eta = 1\) case, combined with our assumption of log utility is a special case often referred to in the literature as the Cole-Obstfeld case.

\(^{22}\)We do not attempt to calibrate the variance of shocks, which we just normalize to unity. Thus, the reader should not attach any weight to the absolute value of the welfare losses reported.
with employment gap fluctuations, shows that an increase in wage flexibility always reduces the contribution of that component to the overall welfare losses. Yet, it is clear that the size of the reduction of that component of welfare losses is faster and more prominent when $\phi_e$ is zero or close to zero than when it is close to unity, a result consistent with the findings of the previous section. Turning to the second component, we observe that an increase in wage flexibility always raises the volatility of price inflation, and thus the contribution of the latter to welfare losses. The size of that effect seems largely independent of the exchange rate policy.

Note that the shape of the welfare surface resembles closely the one of the wage inflation component, so the contribution of the latter is particularly important to explain the results. The key feature is that, for any given value of $\phi_e$, the contribution of wage inflation volatility to welfare losses is non-monotonic. The explanation for that non-monotonicity is straightforward. On the one hand, the variance of wage inflation increases monotonically as wages become more flexible. This effect, which tends to raise welfare losses, is dominant when $\theta_w$ is relatively large. On the other hand, the weight associated with wage inflation volatility in the loss function, $\epsilon_w/\lambda_w$, goes down rapidly as wages become more flexible, accounting for the positive relation between welfare losses and $\theta_w$ when the latter parameter is below a certain level.$^{23}$

Given that the weight $\epsilon_w/\lambda_w$ is independent from $\phi_e$, the fact that the welfare losses are decreasing in $\theta_w$ for a larger range of values of the latter parameter when $\phi_e$ is large must be related to the implied behavior of wage inflation volatility. Thus, when $\phi_e$ is low, the increase in wage volatility resulting from an increase in wage flexibility is relatively small, compared to the case of a high value of $\phi_e$. The change in the volatility of the "drivers" of wage adjustment, employment and prices, resulting from greater wage flexibility is (relatively) more favorable to wage stability when monetary policy is focused on stabilizing price inflation as opposed to the nominal exchange rate.

$^{23}$Note that $\lim_{\theta_w \to \infty} \lambda_w = +\infty$
Figure 5 splits the \([\phi_e, \theta_w]\) parameter space in two regions, defined by the sign of the impact of wage rigidities on welfare. The boundary between the two regions is given by the value of \(\theta_w\) that maximizes welfare losses, as a function of \(\phi_e\). Note that, as \(\phi_e\) increases, i.e. as monetary policy becomes more focused on stabilizing the exchange rate, the range of \(\theta_w\) values for which a (marginal) increase in wage flexibility is undesirable from a welfare point of view becomes larger.\(^{24}\) In particular, in the limiting case of a hard peg or currency union, an increase in wage flexibility is welfare improving only for values of \(\theta_w\) below 0.15, i.e. much lower than implied by the empirical evidence.

Figures 6 through 8 report the corresponding findings when technology shocks are the only source of fluctuations. Note that, qualitatively, the Figures are very similar to the corresponding figures conditional on demand shocks, though the boundary which splits the two regions in the \([\phi_e, \theta_w]\) parameter space now appears to be more sensitive to the exchange rate coefficient for low values of the latter.

We conclude this subsection by pointing out two additional findings, both of which are captured in Figures 3 and 6. The first result has to do with the desirability or not of some concern for exchange rate stability in the design of monetary policy. We note that, independently of the value of \(\theta_w\), the welfare loss function is minimized for some positive (albeit small) value of \(\phi_e\). In other words, and conditional on the assumed interest rate rule (and given \(\phi_e = 1.5\)), there are gains from having the nominal rate respond somewhat to the nominal exchange rate, in order to stabilize the latter.\(^{25}\)

Secondly, note that for a broad range of values of \(\phi_e\) (its entire support, in the case of demand shocks), welfare losses for \(\theta_w = 1\) (fully rigid wages) are smaller than those associated with \(\theta_w = 0\) (fully flexible wages).\(^{26}\) In both cases the component of welfare

\(^{24}\)One can further show that, for the \(\eta = 1\) case considered here, the boundary between the two regions is invariant to the degree of openness.

\(^{25}\)That finding is not unrelated to the conclusions of Campolmi (2012), who shows that CPI inflation targeting is often more desirable than domestic inflation targeting in a model similar to ours. Note that CPI inflation is a weighted average of domestic inflation and the change in the (log) nominal exchange rate in our framework.

\(^{26}\)We thank our discussant, Stephanie Schmitt-Grohé, for pointing out this finding.
losses associated with wage inflation volatility is zero. Instead, we see that the gap in welfare between the two extreme environments associated with price inflation volatility (which favors fully sticky wages) more than offsets the corresponding gap associated with employment gap volatility (which favors fully flexible wages). That result thus hinges on the large weight associated with price inflation (relative to that of the employment gap) in the welfare loss function under our baseline calibration, and can be overturned when greater price flexibility is assumed, as discussed in the robustness section below. Moreover, the previous result is not invariant to the specific monetary policy rule assumed. In particular, conditional on the central bank following an optimal monetary policy, welfare losses are zero in the case of fully flexible wages, but strictly positive when wages display some stickiness (even if not full), at least as long as prices are sticky as well.\footnote{The optimal monetary policy in the case of flexible wages is known to involve strict domestic inflation targeting, i.e. $\pi_{H,t} = 0$, for all $t$. See Galí and Monacelli (2005).}

4.2 Wage Flexibility, Exchange Rate Policy and Welfare: Robustness

In the present subsection we analyze the sensitivity of our findings to a variety of departures from the baseline calibration studied above. In particular, we investigate the role of (a) the elasticity of substitution between domestic and foreign goods, (b) the degree of openness, and (c) the degree of price stickiness.

4.2.1 The Role of the Trade Elasticity

The analysis above was restricted to a specification of preferences featuring a \textit{unitary} value for the elasticity of substitution between domestic and imported goods (the \textit{trade elasticity}, for short), as well as a logarithmic utility of consumption. A recent literature has shown that in the more general case - either because utility is not logarithmic or the trade elasticity is different from 1 - it is no longer feasible to derive an accurate second order approximation of households’ welfare based only on a first order approximation of
the underlying equilibrium conditions.\textsuperscript{28}

In this subsection, we analyze the effects of variations in the degree of wage rigidity on welfare, and their interaction with the exchange rate policy, under alternative settings of the trade elasticity. For concreteness, we restrict our analysis to the case of demand-driven fluctuations.

Throughout we continue to assume log-consumption utility. We evaluate the conditional expected discounted utility (loss) of the representative agent by resorting to a second-order approximation of the equilibrium conditions. In particular, we measure expected conditional welfare as:

\begin{equation}
W_t = E_t \left\{ \sum_{j=0}^{\infty} \beta^j U(C_{t+j}, N_{t+j}; X_{t+j}) \right\} \tag{17}
\end{equation}

or, in recursive form:

\begin{equation}
W_t = U(C_t, N_t; X_t) + E_t \{ W_{t+1} \} \tag{18}
\end{equation}

To evaluate the welfare level associated to alternative combinations of policy parameters, we follow Schmitt-Grohe and Uribe (2004) and compute a numerical, second order approximation of $W_t$. In turn, this requires computing a second order accurate approximation to the full set of equilibrium conditions.\textsuperscript{29}

Figure 9, analogous to Figure 3 above, displays the effect on welfare losses of varying the degree of wage stickiness under alternative values of the exchange rate feedback parameter $\phi_e$. The figure has two panels, corresponding to two different values of the elasticity of substitution: $\eta = 0.5$ ("low elasticity") and $\eta = 2$ ("high elasticity"). Note that when the low trade elasticity is assumed, the shape of the welfare loss function is qualitatively equal to that shown in Figure 3, and our earlier main finding survives: given an initial value of $\theta_w$ is sufficiently low, making wages more flexible reduces welfare. In the case of a high trade elasticity (panel b), that finding also obtains but needs to be

\textsuperscript{28}See, e.g. de Paoli (2009).
qualified: it holds only for sufficiently large values of the exchange rate coefficient $\phi_e$. In both cases, however, it remains true that, contrary to conventional wisdom, the the range of $\theta_w$ values for which more wage flexibility is welfare reducing increases with the size of the exchange rate coefficient in the interest rate rule. In particular, it is largest in the case of an exchange rate peg or a currency union.

### 4.2.2 The Role of Trade Openness

Figures 10 shows the effect of the exchange rate coefficient $\phi_e$ on the threshold value for $\theta_w$ under three alternative values for the openness parameter $\nu$ (0.1, 0.3, and 0.5). We condition on two different calibrations of the trade elasticity, $\eta = 2$ (panel (a)) and $\eta = 1/2$ (panel (b)). Once again we report results for the case of demand shocks only.

As already mentioned above, for the case of a unitary trade elasticity (not shown), the degree of openness does not have any effect on the threshold value of $\theta_w$ and hence on the boundary between the two welfare impact regions. But this is not the case for non-unitary trade elasticities, as Figure 10 makes clear. First, we note that in both cases the threshold value for $\theta_w$ is decreasing in $\phi_e$, independently of the degree of openness. Thus, a key finding from the previous sections is shown to be robust to different degrees of openness, even in the case of non-unitary trade elasticities.

Beyond that basic result, we note that the sign of the effect of openness on the welfare regions turns out to depend on whether that elasticity is larger or smaller than unity. Thus, when $\eta = 2$, greater openness reduces the size of the region for which welfare losses are decreasing in $\theta_w$, for any given value of $\phi_e$ (see Figure 10.a). The opposite effect obtains when $\eta = 1/2$.

[The intuition for that result goes as follows. In our model the presence of nominal wage rigidity introduces an independent motive for stabilizing the exchange rate: fluctuations in the latter affect CPI inflation and, in turn, the wage markup and wage inflation.]

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30Recall that under complete exchange rate pass-through (and normalizing foreign inflation to zero), CPI inflation can be written, in logs, as a weighted average of domestic inflation and the rate of nominal
any given value of $\phi_e$, the larger the degree of openness, the larger the welfare benefit of stabilizing the nominal exchange. Hence the critical threshold of wage stickiness (at which welfare losses start decreasing when wage stickiness is reduced) rises with openness. To confirm this point, notice that the degree of openness becomes irrelevant when $\phi_e \to 1$. Under a fixed exchange rate, in fact, the nominal depreciation rate is zero at all times, and a marginal increase in openness does not bring any additional benefit in terms of higher CPI inflation and wage markup stability.\footnote{In the limit case of fully flexible wages, in fact, the threshold value for wage stickiness would always be at its maximum, and equal to 1.}

The above argument, however, is based on the assumption that the trade elasticity $\eta$ exceeds 1, which corresponds to the case of domestic and foreign goods being sufficiently substitutes. Figure 10b shows that when that elasticity has a value below 1, corresponding to the case of domestic and foreign goods being complements, higher openness lowers the threshold value for $\theta_w$.

The intuition is as follows. In the case of low trade elasticity, any given degree of stabilization of the real exchange rate produces now two competing effects. On the one hand, it improves welfare, via a more stable wage markup. But at the same time it produces on average a more appreciated real exchange rate, which, in the case $\eta < 1$, reduces welfare.\footnote{As argued in De Paoli (2009) for a small open economy with flexible wages, a reduction in the volatility of the real exchange rate generates an average appreciation of the latter. This is welfare improving (reducing) if $\eta > 1$ ($\eta < 1$).} Since the former effect is second order, while the latter is first order, the net effect of higher openness is to make (nominal and real) exchange stabilization less desirable at the margin, in contrast with the previous case of trade elasticity $\eta > 1$.

### 4.2.3 The Role of Price Stickiness

The analysis of the previous sections has been conducted under the assumption of an unchanged degree of price stickiness ($\theta_p = 0.75$), corresponding to prices having an eaverage depreciation: $\pi_t = (1-\nu)\pi_{H,t} + \nu \Delta e_t$. Hence, the larger the degree of openness $\nu$, the larger the "weight" of the nominal depreciation rate in CPI inflation.
duration of four quarters. Figure 11 illustrates the effect on the welfare impact regions of varying the degree of price stickiness. Notice that, if $\theta_p$ is sufficiently high, the threshold value is generally decreasing in the exchange rate feedback coefficient $\phi_e$. As we approach full price rigidity, however, that threshold value becomes independent of $\phi_e$. In other words, when prices are completely rigid, it is irrelevant whether monetary policy is constrained or not, because domestic inflation will not react to the underlying disturbances and, as a result, neither will the nominal interest rate, given the assumed monetary policy rule (4).

Similarly, if prices are sufficiently flexible, the threshold value for wage stickiness is also largely independent of $\phi_e$. In that case, in fact, it is irrelevant whether or not monetary policy is unconstrained, because its ability to influence the real interest rate, via movements of the nominal interest rate, is impaired. Thus, in this vein, fully rigid and fully flexible prices are symmetric cases. Note, however, that the relative size of the welfare impact regions is very different in the two cases. In particular, the range of $\theta_w$ values for which welfare declines with greater wage flexibility tends to be larger when prices are stickier, for any given value of $\phi_e$.

Figure 12 plots the corresponding picture in the case of technology shocks. A qualitatively similar pattern emerges, with the boundary between the welfare impact regions being independent of $\phi_e$ either at extreme values of price stickiness or of price flexibility. Outside this parameter regions, and as illustrated in our previous section, the threshold value for wage stickiness is decreasing in the the exchange rate coefficient $\phi_e$, although it becomes extremely sensitive to $\phi_e$ at low values of the latter.

5 Related Literature

Friedman (1953) is classic reference on the interaction between nominal rigidities and the exchange rate regime. His "case for flexible exchange rates" rests on the usefulness of exchange rate adjustments as a substitute for nominal price and wage adjustments, when
the latter are difficult to bring about, in order to support a desirable or warranted change in the relative price of domestic and foreign goods. The presence of sufficiently flexible wages and prices as one of the criteria for the success of a currency union can be viewed as a corollary of Friedman’s argument (see, e.g. European Commission (1990), Mongelli (2002)). More recent theoretical work focusing on the costs of downward nominal wage rigidity under an exchange rate peg can be found in Schmitt-Grohé and Uribe (2012), among others.

A number of contributions have analyzed the consequences and desirability of increased price and wage flexibility in the closed economy. Thus, DeLong and Summers (1986) use a model with staggered Taylor contracts to show that an increase in wage flexibility (indexed by the responsiveness of wages to cyclical conditions) may be destabilizing due to a Mundell effect (i.e. the contractionary impact of falling prices, working through the expected real rate). Using a New Keynesian model, Battarai, Eggertsson, Schoenlen (2012) study the conditions under which an increase in price flexibility may have destabilizing effects on output and employment. This will be the case when demands shock prevail if interest rates do not respond strongly to inflation. By contrast, when supply shocks are dominant, greater price flexibility is destabilizing only if interest rates respond strongly to inflation. Galí (2013) addresses the same question with a focus on wage flexibility and its impact on welfare. He shows that an increase in wage flexibility may be welfare reducing if the interest rate is not too responsive to inflation. The three papers rely on a closed economy framework, and hence have nothing to say regarding the role of the exchange rate.

The constraints on monetary policy imposed by a currency union are similar to those implied by a binding zero lower bound on the nominal interest rate. In that context, Eggertsson, Ferrero and Raffo (2013) raise a warning on the possible contractionary effects of structural reforms (modelled as favorable supply shocks), due to the increase in

\[33\text{Similar, but not identical, as made clear by Erceg and Lindé (2012).}\]
real interest rates resulting from deflationary pressures combined with an unresponsive nominal rate.

6 Concluding Remarks

Calling for greater wage flexibility as a way of insulating employment from shocks has become part of the conventional policy advice kit. Under that conventional view, for countries under a hard peg or belonging to a currency union, wage flexibility is seen as being even more valuable, given the impossibility of using the exchange rate as a buffer.

The present paper calls into question that conventional wisdom. Using a standard New Keynesian open economy model, we have analyzed the impact of changes in the degree of wage rigidity on the economy’s equilibrium properties. Two findings stand out. Firstly, the effectiveness of labor cost adjustments on employment is inversely related to the degree to which the central bank seeks to stabilize the exchange rate. That effectiveness is minimal in a currency union.

Secondly, an increase in wage flexibility often reduces welfare, and more likely so in economies under an exchange rate peg or an exchange rate-focused monetary policy.

Our findings thus call into question the common view that wage flexibility is particularly desirable in a currency union.
APPENDIX

A.1. Households

The optimal allocation of any given expenditure on domestic goods yields the demand functions:

\[ C_{H,t}(j) = \left( \frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\epsilon_p} C_{H,t} \]  

(19)

for all \( j \in [0, 1] \), where \( P_{H,t} \equiv \left( \int_0^1 P_{H,t}(j)^{1-\epsilon_p} dj \right)^{\frac{1}{1-\epsilon_p}} \) is the domestic price index. Combining the optimality conditions in (19), with the definitions of \( P_{H,t} \) and \( C_{H,t} \) we obtain

\[ \int_0^1 P_{H,t}(j)C_{H,t}(j) dj = P_{H,t}C_{H,t}. \]

The optimal allocation of expenditures between domestic and imported goods in turn requires

\[ C_{H,t} = (1 - \nu)(P_{H,t}/P_t)^{-\eta}C_t \quad ; \quad C_{F,t} = \nu(P_{F,t}/P_t)^{-\eta}C_t \]

(20)

where \( P_t \equiv ((1 - \nu)P_{H,t}^{1-\eta} + \nu P_{F,t}^{1-\eta})^{\frac{1}{1-\eta}} \) is the consumer price index (CPI, for short).\(^{34}\) Accordingly, total consumption expenditures by each household are given by \( P_{H,t}C_{H,t} + P_{F,t}C_{F,t} = P_tC_t \). Note that in a symmetric steady state with \( P_H/P = 1 \) parameter \( \nu \) corresponds to the share of domestic consumption allocated to imported goods. It is also in this sense that \( \nu \) can be interpreted as an index of openness.

The household’s intertemporal optimality condition takes the form

\[ 1 = \beta (1 + i_t) E_t \left\{ \left( \frac{C_t}{C_{t+1}} \right) \left( \frac{X_{t+1}}{X_t} \right) \left( \frac{P_t}{P_{t+1}} \right) \right\} \]

where \( i_t \) is the interest rate on a one-period nominally riskless bond denominated in domestic currency.

We assume a complete set of state-contingent securities traded internationally. That

\(^{34}\)When \( \eta = 1 \) we have \( P_t \equiv (P_{H,t})^{1-\alpha}(P_{F,t})^{\alpha} \)
assumption implies a risk sharing condition of the form:

\[ C_t = \vartheta X_t C_t^* Q_t \]

where \( Q_t \equiv \frac{\xi_t P^t_t}{P_t} \) is the real exchange rate and \( C_t^* \) is world consumption. Without loss of generality we set \( \vartheta = 1 \). Letting \( S_t \equiv \frac{\xi_t P^t_t}{P_{H,t}} \) denote the terms of trade, note that \( S_t = Q_t (P_t / P_{H,t}) \) implies the monotonic relation \( Q_t = S_t \left( (1 - \nu) + \nu S_t^{1-\eta} \right)^{-\frac{1}{\nu-1}} \).

Each household is specialized in the provision of some differentiated labor service, for which firms generate an isoelastic demand (see below) and for which each household sets the corresponding nominal wage. Each period only a fraction \( 1 - \theta_w \) of households, drawn randomly from the population, reset their nominal wage in a way consistent with utility maximization, subject to the demand for their labor services (current and future). The remaining fraction \( \theta_w \) of households keep their nominal wage unchanged. Parameter \( \theta_w \in [0, 1] \) can be thus seen as an index of nominal wage rigidities.

The wage newly set in period \( t \), denoted by \( W_t \), must satisfy the optimality condition:

\[
\sum_{k=0}^{\infty} (\beta \theta_w)^k E_t \left\{ \beta_t \frac{\xi_{t+k} U_{c,t+k}}{C_t} \left( \frac{W_t}{P_{t+k}} - \mathcal{M}^\mu \mathcal{M}RS_{t+k|t} \right) \right\} = 0 \tag{22}
\]

\[ 35 \text{To see this note that}
\]

\[
\left( \frac{\xi_t V_{t,t+1}}{P_t} \right) \left( \frac{X_t}{C_t} \right) = \xi_{t,t+1} \beta \left( \frac{X_{t+1}}{C_{t+1}} \right) \left( \frac{\xi_{t+1}}{P_{t+1}} \right) \tag{21}
\]

\[
\left( \frac{V_{t,t+1}}{P_t} \right) \left( \frac{1}{C_t} \right) = \xi_{t,t+1} \beta \left( \frac{1}{C_{t+1}} \right) \left( \frac{1}{P_{t+1}} \right)
\]

where \( V_{t,t+1} \) is the period \( t \) price (in foreign currency) of a one-period (Arrow) security that yields one unit of foreign currency if a specific state of nature is realized in period \( t + 1 \), and nothing otherwise, and where \( \xi_{t,t+1} \) is the probability of that state of nature being realized in \( t + 1 \) (conditional on the state of nature at \( t \)).

\[ 36 \text{Note that the equilibrium price of a riskless bond denominated in foreign currency is given, via arbitrage, by} \ (1+i_t)^{-1} = E_t \{ V_{t,t+1} \}. \text{ The previous pricing equation can be combined with the corresponding domestic bond pricing equation,} (1+i_t)^{-1} = E_t \{ V_{t,t+1} \xi_{t+1} \xi_t \} ^{-1} \text{ to obtain, after log-linearization, a familiar version of the uncovered interest parity condition:}
\]

\[ i_t = i_t^* + E_t \{ \Delta e_{t+1} \} \]

\[ 37 \text{When} \ \eta = 1 \text{ then} \ Q_t = \mathcal{S}_t^{1-\alpha} \]
where $\mathcal{M}^w \equiv \frac{\mu^w}{\epsilon_w - 1}$ is the frictionless wage markup and $MRS_{t+k|t} \equiv C_{t+k}N^\varphi_{t+k|t}$, with $N^\varphi_{t+k|t}$ denoting $t+k$ employment for a household who last set its wage in period $t$.

Log-linearization of the previous condition yields

$$\overline{w}_t = \mu^w + (1 - \beta \theta_w) \sum_{k=0}^{\infty} (\beta \theta_w)^k E_t \{ mrs_{t+k|t} + p_{t+k} \}$$

where $\mu^w \equiv \log \mathcal{M}^w$.

Define the economy’s *average* marginal rate of substitution as $MRS_t \equiv C_t N^\varphi_t$, where $N_t$ is aggregate employment. Thus, up to a first order approximation,

$$mrs_{t+k|t} = mrs_{t+k} + \varphi (n_{t+k|t} - n_{t+k})$$

(23)

$$= mrs_{t+k} - \epsilon_w \varphi (w_t^i - w_{t+k})$$

Furthermore, log-linearizing the expression for the aggregate wage index around a zero inflation steady state we obtain

$$w_t = \theta_w w_{t-1} + (1 - \theta_w) \overline{w}_t$$

(24)

We can finally combine equations (22) through (24) and derive the baseline wage inflation equation

$$\pi^w_t = \beta E_t \{ \pi^w_{t+1} \} - \lambda_w (\mu^w_t - \mu^w)$$

(25)

where $\pi^w_t \equiv w_t - w_{t-1}$ is wage inflation, $\mu^w_t \equiv w_t - p_t - mrs_t$ denotes the (log) *average* wage markup, and $\lambda_w \equiv \frac{(1 - \theta_w)(1 - \beta \theta_w)}{\theta_w (1 + \epsilon_w \varphi)} > 0$.

A.2 Firms

Cost minimization by firms implies a set of demand schedules for labor services of each type:

$$N_t(i, j) = \left( \frac{W(i)}{W_t} \right)^{-\epsilon_w} N_t(j)$$
for all $i \in [0, 1]$ and $j \in [0, 1]$. Note that aggregate demand for labor of type $i$ is thus given by:

$$N_t(i) = \int_0^1 N_t(i, j) dj = \left( \frac{W(i)}{W_t} \right)^{-\epsilon_w} N_t$$

where $N_t = \int_0^1 N_t(j) dj$ is aggregate employment.

Each period, a subset of firms of measure $1 - \theta_p$, drawn randomly, reoptimize the price of their good, subject to a sequence of demand schedules for the latter. The remaining fraction $\theta_p$ keep their price unchanged. Parameter $\theta_p \in [0, 1]$ can thus be interpreted as an index of price rigidities. All firms meet the demand for their respective goods at the posted prices.

As a result, and to a first-order approximation, the (log) domestic price level evolves over time according to the difference equation

$$p_{H,t} = \theta_p p_{H,t-1} + (1 - \theta_p) \bar{p}_{H,t}$$

where $\bar{p}_{H,t} \equiv \log \bar{P}_{H,t}$ is the (log) price newly set by firms adjusting the price in period $t$. When choosing that price $\bar{P}_{H,t}$, each firm seeks to maximize its value, subject to the sequence of demand constraints $Y_{t+k|t} = (\bar{P}_{H,t}/P_{H,t+k})^{-\epsilon_p} (C_{H,t+k} + C_{H,t})$, for $k = 0, 1, 2, ...$ consistent with the households’ optimality condition (??), where $Y_{t+k|t}$ denotes output at time $t + k$ of a firm that last reset its price in period $t$.

The resulting optimality condition is given by

$$\sum_{k=0}^{\infty} \theta_p^k E_t \left\{ Q_{t,t+k} Y_{t+k|t} \left( \bar{P}_{H,t} - M_p \Psi_{t+k|t} \right) \right\} = 0$$

where $Q_{t,t+k} \equiv \beta^k (C_t/C_{t+k})(X_{t+k}/X_t)(P_t/P_{t+k})$ is the relevant stochastic discount factor for nominal payoffs in period $t + k$, $\Psi_{t+k|t} \equiv \frac{W_{t+k}}{(1-\alpha)\epsilon_{t+k}N_{t+k|t}}$ is the marginal cost in period $t + k$ of a firm producing quantity $Y_{t+k|t}$, and $M_p \equiv \frac{\epsilon_p}{\epsilon_{p-1}}$ is the price markup under flexible prices.

Log-linearization of the previous optimality condition around the zero inflation steady state...
state yields

\[ \bar{p}_{H,t} = \mu^p + (1 - \beta \theta_p) \sum_{k=0}^{\infty} (\beta \theta_p)^k E_t \{ \psi_{t+k\mid t} \} \]  

(27)

where \( \mu^p \equiv \log M^p \) and \( \psi_{t+k\mid t} \equiv \log \Psi_{t+k\mid t} \). In words, firms adjusting their price in any given period choose the latter to equal the desired markup over a weighted average of current and future nominal marginal costs.

Define the average nominal marginal cost as \( \Psi_t = \frac{W_t}{(1-\alpha)A_t N_t} \). Taking logs and using the (first order) approximate aggregate relation \( y_t = a_t + (1 - \alpha) n_t \), it follows that

\[ \psi_{t+k\mid t} = \psi_{t+k} + \alpha(n_{t+k\mid t} - n_{t+k}) \]

\[ = \psi_{t+k} - \frac{\alpha \varepsilon_p}{(1 - \alpha)}(\bar{p}_{H,t} - p_{H,t+k}) \]

Combining the previous result with (26) and (27), one can derive the price inflation equation

\[ \pi^p_t = \beta E_t \{ \pi^p_{t+1} \} - \lambda_p (\mu^p_t - \mu^p) \]  

(28)

where \( \pi^p_t \equiv p_t - p_{t-1} \) denotes price inflation, \( \mu^p_t \equiv p_t - \psi_t \) is the average price markup and \( \lambda_p \equiv \frac{(1-\theta_p)(1-\beta \theta_p)}{\alpha} \frac{1-\alpha}{1-\alpha + \alpha \varepsilon_p} \). Thus, price inflation is driven by current and expected deviations of average price markups from desired markups. Note the symmetry between the price inflation equation (28) and its wage counterpart in (??).

A.3. Equilibrium

Goods market clearing in the home economy thus requires

\[ Y_t(j) = C_t(j) + C^*_H(j) \]

\[ = \left( \frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\nu} \left( 1 - \nu \right) \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} C_t + \nu \left( \frac{P_{H,t}}{P_{F,t}} \right)^{-\eta} Y^*_t \]  

(29)

for all \( j \in [0, 1] \) and all \( t \), and where we have imposed goods market clearing at the world level as well, i.e. \( C^*_t = Y^*_t \).
Plugging (29) into the definition of aggregate domestic output

\[ Y_t \equiv \left( \int_0^1 Y_t(j)^{1 - \frac{1}{\nu}} dj \right)^{\frac{\nu}{\nu - 1}} \]

we obtain:

\[ Y_t = (1 - \nu) \left( (1 - \nu) + \nu S_t^{1 - \gamma} \right)^{\frac{\gamma}{\gamma - 1}} C_t + \nu S_t^{\gamma} Y_t^* \]

The corresponding log-linear approximation around the symmetric steady state is given by

\[ y_t = (1 - \nu) c_t + \nu y_t^* + \eta \nu (2 - \nu) s_t \tag{30} \]

The log-linearized Euler equation takes the form:

\[
\begin{align*}
c_t &= E_t \{ c_{t+1} \} - (i_t - E_t \{ \pi_{t+1} \}) + (1 - \rho_x) x_t \\
&= E_t \{ c_{t+1} \} - (1 - \nu)(i_t - E_t \{ \pi_{H,t+1} \}) - \nu(i_t^* - E_t \{ \pi_{t+1}^* \}) + (1 - \rho_x) x_t
\end{align*}
\]

where the second equality makes use of the fact that \( \pi_t = (1 - \nu) \pi_{H,t} + \nu(\Delta e_t + \pi_t^*) \) together with the log-linearized interest parity condition \( i_t = i_t^* + E_t \{ \Delta e_{t+1} \} \).

Noting that, up to a first order approximation, \( q_t = (1 - \nu) s_t \), we can write the risk sharing condition as:

\[ c_t = x_t + (1 - \nu) s_t + y_t^* \tag{31} \]

Employment is given by:

\[ (1 - \alpha) n_t = y_t - a_t \tag{32} \]

Next we derive expressions for the average price and wage markups as a function of the output and real wage gaps. Letting \( \omega_t \equiv w_t - p_t \) denote the (log) consumption wage, we can write the average wage markup as

\[ Y_t = (1 - \nu) S_t^\gamma C_t + \nu S_t^\gamma C_t^* \]

---

38 Note that in the special case of \( \eta = 1 \) we have

\[ Y_t = (1 - \nu) S_t^\gamma C_t + \nu S_t^\gamma C_t^* \]

33
\[ \mu^w_t \equiv \omega_t - mrs_t = \omega_t - (c_t + \varphi n_t) \] (33)

Thus, in deviations from the natural equilibrium

\[ \mu^w_t - \mu^w = \tilde{\omega}_t - \tilde{c}_t - \frac{\varphi}{1 - \alpha} \tilde{y}_t \]

We can thus rewrite the wage inflation equation as:

\[ \pi^w_t = \beta E_t \{ \pi^w_{t+1} \} + \frac{\lambda w\varphi}{1 - \alpha} \tilde{y}_t + \lambda w \tilde{c}_t - \lambda w \tilde{\omega}_t \] (34)

Similarly, the average price markup is given by

\[ \mu^p_t \equiv p_{H,t} - (\tau_t + w_t - m p n_t) = a_t - \alpha n_t - (\tau_t + \omega_t + \nu s_t) + \log(1 - \alpha) \] (35)

Hence,

\[ \mu^p_t - \mu^p = -\frac{\alpha}{1 - \alpha} \tilde{y}_t - \tilde{\omega}_t + \nu \tilde{s}_t - \tau_t \]

The domestic inflation equation can thus be written as

\[ \pi^p_{H,t} = \beta E_t \{ \pi^p_{H,t+1} \} + \frac{\lambda p \alpha}{1 - \alpha} \tilde{y}_t + \lambda p \tilde{\omega}_t + \lambda p \nu \tilde{s}_t + \lambda p \tau_t \] (36)

Finally, we need to take into account the following identities:

\[ \pi^p_{H,t} \equiv p_{H,t} - p_{H,t-1} \] (37)

\[ \pi^w_{w,t} \equiv w_t - w_{t-1} \] (38)

\[ \tilde{\omega}_t = w_t - (p_{H,t} + \nu \tilde{s}_t) - (\omega^n_t + \nu s^n_t) \] (39)
\[ s_t \equiv e_t - p_{H,t} \]  

\section*{A.4 Natural Equilibrium}

Setting price and wage markups to their frictionless levels in (33) and (35):

\[(\alpha + \varphi)n^n_t = a_t - c^n_t - \nu s^n_t - \mu + \log(1 - \alpha)\]

where \( \mu \equiv \mu^p + \mu^w \), and where variables with an \( n \) superscript denote the natural equilibrium values of the original variable. Combining the previous expression with the risk sharing condition

\[ c^n_t = x_t + (1 - \nu)s^n_t \]

we obtain

\[(\alpha + \varphi)n^n_t = a_t - x_t - s^n_t - \tau_t - \mu + \log(1 - \alpha) \] \hspace{1cm} (41)

Goods market clearing implies

\[ a_t + (1 - \alpha)n^n_t = (1 - \nu)c^n_t + \eta\nu(2 - \nu)s^n_t \]

which combined with the risk sharing condition implies:

\[ a_t + (1 - \alpha)n^n_t = (1 - \nu)x_t + (1 + (\eta - 1)\nu(2 - \nu))s^n_t \] \hspace{1cm} (42)

Finally, combining (41) and (42) to substitute \( s_t \) out yields an expression for natural employment, which we denote by \( n^n_t \):

\[ n^n_t = \Omega(\eta - 1)\nu(2 - \nu)a_t - \Omega\nu[1 + (\eta - 1)(2 - \nu)]x_t - \Omega[1 + (\eta - 1)(2 - \nu)]\tau_t + n \]

where \( n \equiv \Omega[1 + (\eta - 1)\nu(2 - \nu)](\log(1 - \alpha) - \mu) \) and \( \Omega \equiv \frac{1}{1 - \alpha + (\alpha + \varphi)[1 + (\eta - 1)\nu(2 - \nu) \mu]} > 0 \).

Note that in the special case of \( \eta = 1 \), natural employment is independent of technology, but is still affected by the demand shock.

The previous expression can be combined with other equilibrium conditions to derive the natural values of the remaining variables. Thus,

\[ y^n_t = a_t + (1 - \alpha)n^n_t \]
A.5. Equilibrium Dynamics in the Presence of Nominal Rigidities

Aggregate demand block:

\[ s^n_t = a_t - x_t - \tau_t - (\alpha + \varphi)n^n_t - \mu + \log(1 - \alpha) \]

\[ c^n_t = x_t + (1 - \nu)s^n_t \]

\[ \omega^n_t = a_t - \alpha n^n_t - \tau_t - \nu s_t - \mu + \log(1 - \alpha) \]

\[ y_t = (1 - \nu)c_t + \eta \nu (2 - \nu)s_t \]

\[ c_t = x_t + (1 - \nu)s_t \]

\[ c_t = E_t \{ c_{t+1} \} - (1 - \nu)(i_t - E_t \{ \pi_{H,t+1} \}) + (1 - \rho_p) x_t \]

\[ i_t = \phi_p \pi_{H,t} + \frac{\phi_e}{1 - \phi_e} e_t \]

\[ s_t = e_t - p_{H,t} \]

\[ n_t = \frac{1}{1 - \alpha} (y_t - a_t) \]

Aggregate supply block:

\[ \pi^p_{H,t} = \beta E_t \{ \pi^p_{H,t+1} \} - \lambda_p (\mu^p_t - \mu^p) \]

\[ \pi^p_{H,t} = p_{H,t} - p_{H,t-1} \]

\[ \mu^p_t = a_t - \alpha n_t - (\omega_t + \nu s_t) + \tau_t \]

\[ \pi^w_t = \beta E_t \{ \pi^w_{t+1} \} - \lambda_w (\mu^w_t - \mu^w) \]

\[ \pi^w_{w,t} = w_t - w_{t-1} \]

\[ \mu^w_t = \omega_t - (c_t + \varphi n_t) \]

\[ \omega_t \equiv w_t - (p_{H,t} + \nu s_t) \]
References


Cole and Obstfeld (1993):


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<th>Description</th>
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(a) Employment component

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(c) Wage inflation component
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Welfare Impact Regions: Demand Shocks

\[
\frac{\partial \text{Welfare}}{\partial \theta_w} > 0
\]

\[
\frac{\partial \text{Welfare}}{\partial \theta_w} < 0
\]
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Wage Flexibility, Exchange Rate Policy and Welfare: Technology Shocks
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(a) Employment component

(b) Price inflation component

(c) Wage inflation component
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\[ \frac{\partial \text{Welfare}}{\partial \theta_w} > 0 \]

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