Endogenous Trade Participation with Incomplete Exchange Rate Pass-Through

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Bank of Canada

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The views expressed in this presentation, or in my remarks, are my own, and do not necessarily represent those of the Bank of Canada.
Exporter Characteristics and Dynamics

- **Exporter characteristics**
  - 12-18% more productive, 48% higher capital/worker, 20-45% more employment than non-exporters (Bernard & Jensen 1999)

- **Significant export costs**
  - 7-17% of a shipment’s value (Alessandria et al. 2010)

- **Time-varying extensive margin of trade**
  - Entry rate = 13.9%, exit rate = 12.6% (Bernard & Jensen 2004)

- **Incomplete exchange rate pass-through**
  - 23% in short run, 42% in long run (Campa & Goldberg 2005)
This paper studies implications of endogenous trade participation for

- international business cycles,
- dynamics of trade participation, and
- exchange rate pass-through

in an environment with nominal rigidities.
Main Findings

- With price rigidities, limited adjustments along intensive margin of trade lead to procyclical movements in the number of exporters, resulting in larger responses of export price and trade balance, in contrast to existing studies under flexible prices.

- Quantitative effect of extensive margin of trade on exchange rate pass-through is negligible, preserving incomplete pass-through in the presence of nominal rigidity.

- Entry and exit lead relatively more productive firms to dominate the export market.

- The model explains delayed response of trade balance and terms of trade to currency depreciation, with substantial adjustments along intensive and extensive margins.
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Related Literature

Exporter entry and exit

- Export hysteresis in partial equilibrium

- DSGE for business cycles
  Ghironi and Melitz (2005), Alessandria and Choi (2007)

Incomplete exchange rate pass-through

- Demand curvature and market structure
  Atkeson and Burstein (2008), Gust, Leduc and Vigfusson (2010)

- Local costs

- Price rigidities
  Devereux and Engel (2002), Bacchetta and van Wincoop (2003)
Model Overview: Two-country DSGE model

- **Representative household**

- **Competitive final-good producers**

\[ D_t = \left\{ \omega \left[ \int_0^1 y_t^H(i) \gamma^{-1} \gamma^\gamma \, di \right] \frac{\gamma - 1}{\rho - 1} \frac{\rho - 1}{\rho} + (1 - \omega) \left[ \int_{i \in \Theta_t} y_t^F(i) \gamma^{-1} \gamma^\gamma \, di \right] \frac{\gamma - 1}{\rho - 1} \frac{\rho - 1}{\rho} \right\} \frac{\rho}{\rho - 1} \]

- **Monopolistically competitive intermediate-good firms**
  - Each producing a differentiated product
  - Heterogeneous in prices, productivity, entry costs and continuation costs for exporting

  ⇒ These features drive individual firms’ state-dependent export decisions.
Model Overview

- All intermediate-good producers sell in their own country.

\[ y_t(i) = z_t(i)A_tK_t(i)\nu L_t(i)^{1-\nu} \]

- \( z_t(i) \) = current firm-specific productivity
- \( A_t \) = aggregate productivity

- To enter the export market, a firm pays entry cost, \( \eta \sim G^E(\eta) \).

- Upon entering the export market, an entrant sets a new price for its exports.

- To continue exporting, a firm pays a continuation cost, \( \xi \sim G(\xi) \).

- Price-adjustment hazard increasing in the age of a price
Potential entrant with productivity $z_c$ drawing entry cost $\eta$ solves:

$$V_t^E(z_c, \eta) = \max \left\{ \max_{P_{0,t}(z_c)} \left[ Q_t \frac{P_{0,t}(z_c)}{P^*_t} \tau y_{0,t}(z_c) - w_t L_{0,t}(z_c) - r_t K_{0,t}(z_c) - \eta w_t \right] \right\}$$

$$+ \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \sum_{\tilde{c} = 1}^{n_z} \pi_{c\tilde{c}} H_{1,t+1}(z_{\tilde{c}}, z_c, \xi') \right\}, \quad \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \sum_{\tilde{c} = 1}^{n_z} \pi_{c\tilde{c}} V_{t+1}(z_{\tilde{c}}, \eta') \right\}$$

Maximum entry cost $\eta_t^E(z_c)$ this firm will pay to enter export market:

$$\beta E_t \frac{\lambda_{t+1}}{\lambda_t} \sum_{\tilde{c} = 1}^{n_z} \pi_{c\tilde{c}} V_{t+1}(z_{\tilde{c}}, \eta') = Q_t \frac{P_{0,t}(z_c)}{P^*_t} \tau y_{0,t}(z_c) - w_t L_{0,t}(z_c) - r_t K_{0,t}(z_c)$$

$$- \eta_t^E(z_c) w_t + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \sum_{\tilde{c} = 1}^{n_z} \pi_{c\tilde{c}} H_{1,t+1}(z_{\tilde{c}}, z_c, \xi')$$

where $H_{1,t}(z_c, z_s, \xi) = \alpha_1 V_{0,t}(z_c, \xi) + (1 - \alpha_1) V_{1,t}(z_c, z_s, \xi)$. 
Price-adjusting Incumbent

Price-adjusting incumbent exporter with current productivity $z_c$ drawing export cost $\xi$ solves:

$$V_{0,t}(z_c, \xi) = \max \left\{ \max_{P_{0,t}^X(z_c)} \left[ Q_t \frac{P_{0,t}^X(z_c)}{P_t^*} \tau y_{0,t}(z_c) - w_t L_{0,t}^X(z_c) - r_t K_{0,t}^X(z_c) - \xi w_t \right] + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \sum_{\tilde{c}=1}^{n_z} \pi_{c\tilde{c}} H_{1,t+1}(z_{\tilde{c}}, z_c, \xi') \right\}$$

Max. continuation cost $\xi^0_t(z_c)$ the firm will pay to continue exporting:

$$\beta E_t \frac{\lambda_{t+1}}{\lambda_t} \sum_{\tilde{c}=1}^{n_z} \pi_{c\tilde{c}} V_{t+1}^E(z_{\tilde{c}}, \eta') = Q_t \frac{P_{0,t}^X(z_c)}{P_t^X} \tau y_{0,t}(z_c) - w_t L_{0,t}^X(z_c) - r_t K_{0,t}^X(z_c) - \xi^0_t(z_c) w_t + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \sum_{\tilde{c}=1}^{n_z} \pi_{c\tilde{c}} H_{1,t+1}(z_{\tilde{c}}, z_c, \xi')$$
Non-price-adjusting Incumbent

Value of non-price-adjusting incumbent of type \((z_c, j, z_s)\) drawing continuation cost \(\xi:\)

\[
V_{j,t}(z_c, z_s, \xi) = \max \left[ \frac{Q_t}{P^X_t} \frac{P^X_{j,t}(z_s)}{P^X_t} \tau y_{j,t}(z_c, z_s) - w_t L^X_{j,t}(z_c, z_s) - r_t K^X_{j,t}(z_c, z_s) - \xi w_t 
+ \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \sum_{\bar{c}=1}^{nz} \pi_{c\bar{c}} H_{j+1,t+1}(z_{\bar{c}}, z_s, \xi'), \right.
\beta E_t \frac{\lambda_{t+1}}{\lambda_t} \sum_{\bar{c}=1}^{nz} \pi_{c\bar{c}} V_{t+1}^E(z_{\bar{c}}, \eta') \left. \right]
\]

Maximum cost \(\xi^j_t(z_c, z_s)\) the firm will pay to continue exporting:

\[
\beta E_t \frac{\lambda_{t+1}}{\lambda_t} \sum_{\bar{c}=1}^{nz} \pi_{c\bar{c}} V_{t+1}^E(z_{\bar{c}}, \eta') = \frac{Q_t}{P^X_t} \frac{P^X_{j,t}(z_s)}{P^X_t} \tau y_{j,t}(z_c, z_s) - w_t L^X_{j,t}(z_c, z_s) - r_t K^X_{j,t}(z_c, z_s) 
- \xi^j_t(z_c, z_s) w_t + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \sum_{\bar{c}=1}^{nz} \pi_{c\bar{c}} H_{j+1,t+1}(z_{\bar{c}}, z_s, \xi')
\]
Probabilities of Export Participation

- Probabilities of entering export market:
  \[ \zeta_t^E(z_c) = G^E(\eta_t^E(z_c)) \quad \text{for} \quad c = 1, \ldots, n_z \]

- Probabilities of remaining in the export market:
  a) if adjusting prices
  \[ \zeta_t^0(z_c) = G(\xi_t^0(z_c)) \quad \text{for} \quad c = 1, \ldots, n_z \]
  b) if not adjusting prices
  \[ \zeta_t^j(z_c, z_s) = G\left(\xi_t^j(z_c, z_s)\right) \quad \text{for} \quad c = 1, \ldots, n_z, \quad s = 1, \ldots, n_z \quad \text{and} \quad j = 1, \ldots, J - 1 \]
Representative household chooses $C_t, L_t, K_{t+1}, B_{t+1}(s^{t+1}),$ and $M_t$:

$$\max E_t \sum_{t=0}^{\infty} \beta^t \left[ \log C_t + \chi_1 \log \left( \frac{M_t}{P_t} \right) + \chi_2 (1 - L_t) \right]$$

subject to

$$P_t C_t + P_t I_t + \sum_{s^{t+1}} q(s^{t+1} | s^t) B(s^{t+1}) + M_t \leq P_t w_t L_t + P_t r_t K_t + B(s^t) + M_{t-1} + P_t d_t + T_M^t$$

$$K_{t+1} = (1 - \delta) K_t + I_t - \phi \left( \frac{I_t}{K_t} \right) K_t$$

where

$B(s^{t+1}) = \text{holdings of state-contingent, home-denominated bond}$

$q(s^{t+1} | s^t) = \text{price of } B(s^{t+1}) \text{ in units of home currency in state } s^t$

$T_M^t = \text{lump-sum government transfer} = M_s^t - M_{t-1}^s$

$M_s^t = \text{money supply}, \ M_t^s = \mu_t M_{t-1}^s, \ \log \mu_{t+1} = \rho_\mu \log \mu_t + \varepsilon_{t+1}^{\mu}$

$\phi(\cdot) = \text{convex capital adjustment cost function}, \ \phi(\delta) = 0, \ \phi'(\delta) = 0$
The model frequency is quarterly.

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
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<tbody>
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<td>Mass of exporters</td>
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<td>Bernard et al. (2003)</td>
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<td>Continuation rate</td>
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<td>frequency</td>
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<td>Nakamura &amp; Steinsson (2008)</td>
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## Business Cycle Moments

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| **Correlations with GDP** |       |       |
| Consumption              | 0.84  | 0.87  |
| Investment               | 0.94  | 0.94  |
| Labor                    | 0.86  | 0.79  |
| Net exports/GDP          | -0.31 | -0.35 |

| **Autocorrelations**     |       |       |
| GDP                      | 0.86  | 0.66  |
| Consumption              | 0.88  | 0.80  |
| Investment               | 0.88  | 0.50  |
| Labor                    | 0.90  | 0.40  |

| **International correlations** |       |       |
| GDP                      | 0.41  | 0.34  |
| Consumption              | 0.21  | 0.56  |
| Investment               | 0.18  | 0.17  |
| Labor                    | 0.27  | 0.46  |
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Role of Entry and Exit: Flexible prices

- **Exports**:
  - % deviation over time.
  - Blue line: entry & exit.
  - Red line: no entry & exit.

- **Imports**:
  - % deviation over time.
  - Blue line: entry & exit.
  - Red line: no entry & exit.

- **Net Exports**:
  - % of output over time.
  - Blue line: entry & exit.
  - Red line: no entry & exit.

- **Export Price Index**:
  - % deviation over time.
  - Blue line: entry & exit.
  - Red line: no entry & exit.

- **Number of Exporters**:
  - % deviation over time.
  - Blue line: entry & exit.
  - Red line: no entry & exit.

- **Real Exchange Rate**:
  - % deviation over time.
  - Blue line: entry & exit.
  - Red line: no entry & exit.

**Notes**:
- IRFs
- Yuko Imura (Bank of Canada)
- Endogenous Trade Participation
- June 2013
Role of Entry and Exit: Nominal rigidities

- Exports
- Imports
- Net exports
- Export price index
- Number of exporters
- Real exchange rate

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\[ \Delta p_t^X* = \alpha + \sum_{n=0}^{8} \beta_n \Delta e_{t-n} + \sum_{n=0}^{8} \gamma_n \pi^*_t - n + \delta \Delta y_t + \epsilon_t \]

Full model generates short-run incomplete pass-through.
\[ \Delta p_t^{X*} = \alpha + \sum_{n=0}^{8} \beta_n \Delta e_{t-n} + \sum_{n=0}^{8} \gamma_n \pi_{t-n}^* + \delta \Delta y_t + \epsilon_t \]

With flexible prices, pass-through is complete immediately.
\[
\Delta p_t^{X^*} = \alpha + \sum_{n=0}^{8} \beta_n \Delta e_{t-n} + \sum_{n=0}^{8} \gamma_n \pi^*_{t-n} + \delta \Delta y_t + \epsilon_t
\]

The added flexibility of endogenous trade participation does not overturn incomplete pass-through arising from nominal rigidity.
Pass-Through: Composition of exporters

Model without entry and exit

Price age

Current productivity

% of firms

Time

Current productivity

% of firms

Time

Yuko Imura (Bank of Canada)  Endogenous Trade Participation  June 2013
Pass-Through: Composition of exporters

Model without entry and exit

Price age

Current productivity

Model with entry and exit
In the presence of price rigidity, endogenous trade participation influences international business cycles, in contrast to earlier findings under flexible prices.

This suggests that market structure and pricing conventions may be critical in analyzing the role of exporter entry and exit for aggregate dynamics and international transmission of shocks.

Endogenous trade participation and incomplete exchange rate pass-through have important implications for the effects of currency movements and the conduct of monetary policy at both micro and macro levels.
Exporter Dynamics

![Graph showing the number of exporters and Real GDP from 1996 to 2010. The number of exporters increases over time, reaching a peak around 2008, followed by a decline. Real GDP also shows an upward trend, peaking around 2006 and then declining slightly.]

- **Number of Exporters**: The number of exporters increases from 1996 to 2008, reaching a peak of around 140,000 exporters, followed by a decline to around 120,000 by 2010.
- **Real GDP**: Real GDP increases from 1996 to 2006, peaking at around 200 trillion dollars, followed by a decline to around 18 trillion dollars by 2010.

**Key Points**:
- **1996**: Number of exporters is approximately 8,000, Real GDP is around 8 trillion dollars.
- **2000**: Number of exporters increases to around 10,000, Real GDP is around 12 trillion dollars.
- **2004**: Number of exporters reaches a peak of around 14,000, Real GDP is around 14 trillion dollars.
- **2008**: Number of exporters slightly decreases to around 13,000, Real GDP peaks at around 16 trillion dollars, followed by a decline.
- **2010**: Number of exporters decreases to around 12,000, Real GDP is around 14 trillion dollars.

**Endogenous Trade Participation**

- Yuko Imura (Bank of Canada)
- June 2013
Final-good producers combine home- and foreign-produced intermediate goods to produce final goods $D_t$.

$$\max_{y^H_t(i), y^F_t(i)} P_tD_t - \int_0^1 P^D_t(i)y^H_t(i)di - \int_{i \in \Theta_t} P^X_t(i)y^F_t(i)di$$

subject to

$$D_t = \left\{ \omega \left[ \int_0^1 y^H_t(i)^{\frac{\gamma}{\gamma-1}} \frac{\rho}{\rho-1} \right]^{\frac{\gamma-1}{\gamma-1} \frac{\rho-1}{\rho}} + (1 - \omega) \left[ \int_{i \in \Theta_t} y^F_t(i)^{\frac{\gamma}{\gamma-1}} \frac{\rho}{\rho-1} \right]^{\frac{\gamma-1}{\gamma-1} \frac{\rho-1}{\rho}} \right\}^{\frac{\rho}{\rho-1}}$$

$\gamma = \text{elasticity of substitution b/w goods produced in the same country}$

$\rho = \text{elasticity of substitution b/w home and foreign goods}$

$\Theta_t = \text{time-varying set of foreign goods available in home country}$

$P_t = \text{consumer price index}$
Price-adjusting firm with current productivity $z_c$ chooses $P_{0,t}^D(z_c)$:

$$V_{0,t}^D(z_c) = \max_{P_{0,t}^D(z_c)} \left\{ \frac{P_{0,t}^D(z_c)}{P_t} y_{0,t}^D(z_c) - w_t L_{0,t}^D(z_c) - r_t K_{0,t}^D(z_c) 
+ \beta \mathbf{E}_t \frac{\lambda_{t+1}}{\lambda_t} \left[ \alpha_1 \sum_{\tilde{c}=1}^{n_z} \pi_{c\tilde{c}} V_{0,t+1}^D(z_{\tilde{c}}) + (1 - \alpha_1) \sum_{\tilde{c}=1}^{n_z} \pi_{c\tilde{c}} V_{1,t+1}^D(z_{\tilde{c}}, z_c) \right] \right\}$$

$\alpha_j = \text{probability of price adjustment for firms with } j\text{-period old price}$

$\pi_{c\tilde{c}} = \text{probability of moving from } z = z_c \text{ to } z' = z_{\tilde{c}}$
Intermediate-Good Firm: Domestic market

Value of non-price-adjusting firm of type \((z_c, j, z_s)\)

\[
V_{j,t}^D (z_c, z_s) = \frac{P_{j,t}^D (z_s)}{P_t^D} y_{j,t}^D (z_c, z_s) - w_t L_{j,t}^D (z_c, z_s) - r_t K_{j,t}^D (z_c, z_s) \\
+ \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left[ \alpha_{j+1} \sum_{\tilde{c}=1}^{n_z} \pi_{c\tilde{c}} V_{0,t+1}^D (z_{\tilde{c}}) + (1 - \alpha_{j+1}) \sum_{\tilde{c}=1}^{n_z} \pi_{c\tilde{c}} V_{j+1,t+1}^D (z_{\tilde{c}}, z_s) \right]
\]

\(z_c = \) current productivity

\(j = \) number of periods since the price was last set

\(z_s = \) productivity at the time of last price adjustment
Evolution of the distribution of domestic firms

\[ \theta_{j,t} (z_c, z_s) = \text{Number of firms starting date } t \text{ as type } (z_c, j, z_s) \]

Evolution for non-price adjusting firms:

\[ \theta_{j+1,t+1} (z_{\tilde{c}}, z_{\tilde{s}}) = (1 - \alpha_j) \sum_{c=1}^{n_z} \pi_{c \tilde{c}} \theta_{j,t} (z_c, z_{\tilde{s}}) \]

Total number of firms starting \( t + 1 \) as \( (z_{\tilde{c}}, 1, z_{\tilde{s}}) \):

\[ \theta_{1,t+1} (z_{\tilde{c}}, z_{\tilde{s}}) = \pi_{\tilde{c} \tilde{s}} \sum_{j=1}^{J} \sum_{s=1}^{n_z} \alpha_j \theta_{j,t} (z_{\tilde{s}}, z_s). \]
Evolution of the distribution of exporters

\[ \psi_{j,t}(z_c, z_s) = \text{Number of incumbents starting } t \text{ as type } (z_c, j, z_s) \]

Evolution for non-price-adjusting incumbents:

\[ \psi_{j+1,t+1}(z_c, z_s) = (1 - \alpha_j) \sum_{c=1}^{n_z} \zeta_{j,t}^0(z_c, z_s) \cdot \pi_c \cdot \psi_{j,t}(z_c, z_s) \]

Total number of exporters of type \((z_c, 1, z_s)\):

\[ \psi_{1,t+1}(z_c, z_s) = \pi_s \cdot \zeta_{t}^0(z_s) \sum_{j=1}^{J} \sum_{s=1}^{n_z} \alpha_j \cdot \psi_{j,t}(z_s, z_s) + \pi_s \cdot N_t^E(z_s) \]

Number of entrants at time \(t\):

\[ N_t^E(z_c) = \zeta_{t}^E(z_c) \left[ \sum_{j=1}^{J} \sum_{s=1}^{n_z} \theta_{j,t}(z_c, z_s) - \sum_{j=1}^{J} \sum_{s=1}^{n_z} \psi_{j,t}(z_c, z_s) \right] \]
1. Price index for domestically-produced goods

\[ P^D_t = \left[ \sum_{j=1}^{J} \sum_{c=1}^{n_z} \sum_{s=1}^{n_z} \alpha_j \theta_{j,t}(z_c, z_s) P^D_{0,t}(z_c) \right]^{1-\gamma} \]

\[ + \sum_{j=1}^{J-1} \sum_{c=1}^{n_z} \sum_{s=1}^{n_z} (1 - \alpha_j) \theta_{j,t}(z_c, z_s) P^D_{j,t}(z_s) \right]^{1-\gamma} \]

2. Price index for imported goods

\[ P^X^*_t = \left[ \sum_{c=1}^{n_z} N^E_t(z_c) P^X^*_0(z_c) \right]^{1-\gamma} + \sum_{j=1}^{J} \sum_{c=1}^{n_z} \sum_{s=1}^{n_z} \alpha_j \cdot \zeta^0_t(z_c) \psi^*_j(z_c, z_s) P^X^*_0(z_c) \right]^{1-\gamma} \]

\[ + \sum_{j=1}^{J-1} \sum_{c=1}^{n_z} \sum_{s=1}^{n_z} (1 - \alpha_j) \cdot \zeta^*_t(z_c, z_s) \cdot \psi^*_j(z_c, z_s) \cdot P^X^*_j(z_s) \right]^{1-\gamma} \]

3. Price index for all the goods available in home country

\[ P_t = \left[ \omega_1^\rho \left( P^D_t \right)^{1-\rho} + \omega_2^\rho \left( P^X^*_t \right)^{1-\rho} \right]^{\frac{1}{1-\rho}} \]
### Parameter Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount factor</td>
<td>$\beta$</td>
</tr>
<tr>
<td>Weight on leisure in utility</td>
<td>$\chi_2$</td>
</tr>
<tr>
<td>s.s. labor $= 0.33$</td>
<td></td>
</tr>
<tr>
<td>Elasticity of substitution</td>
<td>$\gamma$</td>
</tr>
<tr>
<td>Ghironi &amp; Melitz (2005)</td>
<td></td>
</tr>
<tr>
<td>Armington elasticity</td>
<td>$\rho$</td>
</tr>
<tr>
<td>BKK(1995)</td>
<td></td>
</tr>
<tr>
<td>Capital share in production</td>
<td>$\nu$</td>
</tr>
<tr>
<td>Depreciation rate of capital</td>
<td>$\delta$</td>
</tr>
<tr>
<td>10% depreciation/year</td>
<td></td>
</tr>
<tr>
<td>Iceberg trade cost</td>
<td>$\tau$</td>
</tr>
<tr>
<td># of firm-specific productivity levels</td>
<td>$n_z$</td>
</tr>
</tbody>
</table>

Yuko Imura (Bank of Canada)
Calibrated parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Home bias</td>
<td>$\omega$ 0.762</td>
</tr>
<tr>
<td>Entry costs $\sim U(0, \eta_U)$</td>
<td>$\eta_U$ 2.8</td>
</tr>
<tr>
<td>Continuation costs $\sim U(0, \xi_U)$</td>
<td>$\xi_U$ 0.17</td>
</tr>
<tr>
<td>Firm-specific productivity:</td>
<td>$\rho_z$ 0.81</td>
</tr>
<tr>
<td>$\log z' = \rho_z \log z + \epsilon'$, $\epsilon \sim N(0, \sigma_\epsilon)$</td>
<td>$\sigma_\epsilon$ 0.085</td>
</tr>
<tr>
<td>Price adjustment probability</td>
<td>$\alpha_j$ [0.05, 0.09, 0.25, 0.49, 0.70, 1.00]</td>
</tr>
</tbody>
</table>
Shock processes

- **Productivity** (Backus, Kehoe and Kydland, 1995)

\[
\begin{bmatrix}
A_t \\
A_t^*
\end{bmatrix} =
\begin{bmatrix}
0.906 & 0.088 \\
0.088 & 0.906
\end{bmatrix}
\begin{bmatrix}
A_{t-1} \\
A_{t-1}^*
\end{bmatrix} +
\begin{bmatrix}
\varepsilon_t^A \\
\varepsilon_{t*}^A
\end{bmatrix}
\]

where \( var(\varepsilon_t^A) = var(\varepsilon_{t*}^A) = (0.007)^2, \) \( corr(\varepsilon_t^A, \varepsilon_{t*}^A) = 0.258 \)

- **Money growth** (Chari, Kehoe and McGrattan, 2002)

\[
\begin{bmatrix}
\mu_t \\
\mu_t^*
\end{bmatrix} =
\begin{bmatrix}
0.68 & 0 \\
0 & 0.68
\end{bmatrix}
\begin{bmatrix}
\mu_{t-1} \\
\mu_{t-1}^*
\end{bmatrix} +
\begin{bmatrix}
\varepsilon_t^\mu \\
\varepsilon_{t*}^\mu
\end{bmatrix}
\]

where \( \bar{\mu} = 1.04^{1/4} \)

- I choose \( var(\varepsilon_t^\mu) = var(\varepsilon_{t*}^\mu) \) so that \( \sigma_Y = 1.42 \) in baseline model.
### Steady state

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Sticky</th>
<th>Flexible</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>EE</td>
<td>No EE</td>
</tr>
<tr>
<td>Entry and exit</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mass of exporters</td>
<td>0.21</td>
<td>0.21</td>
<td>0.21</td>
</tr>
<tr>
<td>Continuation rate</td>
<td>0.97/qtr</td>
<td>0.87</td>
<td>1</td>
</tr>
<tr>
<td>Entry rate</td>
<td>0.04/qtr</td>
<td>0.04</td>
<td>0</td>
</tr>
<tr>
<td>Exporter productivity</td>
<td>1.12-1.18</td>
<td>1.13</td>
<td>1</td>
</tr>
<tr>
<td>relative to nonexporters</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Imports/GDP ratio</td>
<td>0.12</td>
<td>0.12</td>
<td>0.11</td>
</tr>
<tr>
<td>Mean price duration</td>
<td>1.43-4.33qtrs</td>
<td>2.66</td>
<td>2.66</td>
</tr>
</tbody>
</table>
Impulse responses: Flexible-price model

- output
- consumption
- labor
- investment
- capital rental rate
- export price index
- number of exporters
- number of new exporters
- number of exiting exporters
- Home net exports
- real exchange rate
- productivity shock

Home country vs. Foreign country

Yuko Imura (Bank of Canada)  Endogenous Trade Participation  June 2013