Credit and Liquidity in Interbank Rates: A Quadratic Approach

Simon Dubecq$^1$ Alain Monfort$^2$ Jean-Paul Renne$^3$
Guillaume Roussellet$^4$

$^1$European Central Bank

$^2$Crest, Banque de France, Maastricht University

$^3$Banque de France

$^4$Banque de France, Crest, Dauphine University

ECB Workshop on Non-Standard Monetary Policy Measures -
June 17, 2013

All the views presented here are those of the authors and should neither be associated with those of the Banque de France, nor of the ECB.
The interbank market risk is at the heart of the (on-going) financial crisis.

The IBOR-OIS spreads are some of the most scrutinized indicators of interbank-market risks.

During the crisis, conventional and unconventional actions taken by the central banks include:

- drop in the central bank interest rates,
- new facilities for liquidity providing to financial institutions (e.g. TAF in the US, VLTRO in the Euro-zone).

⇒ Have those unconventional actions been effective?
The Interbank Rates

- **EURIBOR rates**: unsecured interbank rates proxy. It contains:
  - *Credit risk*: default of the borrower before due date.
  - *Liquidity risk*: important liquidity need of the lender before due date $\implies$ Additional cost.

- **Overnight Indexed Swaps (OIS)**: riskless interbank rates proxy.
  - Fixed leg $\leftrightarrow$ Floating leg indexed on EONIA.
  - Netting and credit-enhancement mechanisms (margin calls).
  - Nearly no immobilisation of capital.
  $\implies$ Almost no credit and liquidity risk.
The Term Structure of Interbank Rates


3M EURIBOR and OIS rates

3M and 12M EURIBOR-OIS spread
Motivations

- Separate bank credit risk from liquidity risk in the IBOR-OIS spread.
  $\rightarrow$ *Observe the cause of fluctuations.*

- Extract the risk-premia linked to longer-term risk-bearing.
  $\rightarrow$ *Necessitate no-arbitrage term structure model.*

- Generate strictly positive spreads under both measures.
  $\rightarrow$ *Quadratic specification.*

Double decomposition to analyse monetary policy actions:

- Outright monetary transactions (late Aug. 2012).
Related literature

- **Quadratic term structure models**

- **Interbank rates modelling**
  Michaud & Upper (2008), Taylor & Williams (2009), Schwarz (2009), Filipovic & Trolle (2011), Christensen, Lopez & Rudebusch (2009), Angelini et al. (2011)

- **Decomposition of interest rates**
Contents

1 Introduction

2 The Model
   - QTSM Models: A General Framework
   - The EURIBOR-OIS Spread Modelling

3 Estimation
   - Identifying the Factors
   - Non-Linear Kalman Filtering for QTSM

4 Decomposition
   - Decomposition of the Spread
   - Decomposition of the Term Structure

5 Conclusion
Pricing the Interbank Risk-Free Rate

We denote:

\( r_t \) the short-term risk-free interest rate,

\( R^{OIS}_{t,h} \) the OIS rate at time \( t \) of maturity \( h \).

\[ \implies R^{OIS}_{t,1} = r_t. \]

Under the absence of arbitrage opportunities:

- existence of both a historical (\( \mathbb{P} \)) and a risk-neutral measure (\( \mathbb{Q} \)).

Pricing formula of secured rates under risk-neutral measure:

\[
R^{OIS}_{t,h} = -\frac{1}{h} \log \left( \mathbb{E}^Q_t \left[ \exp \left\{ -\sum_{k=0}^{h-1} r_{t+k} \right\} \right] \right)
\]
We denote:

- $d_t$ a dummy variable indicating either a default or an illiquidity event.
- $\lambda_t$ the intensity representing the underlying risks in the economy.

\[
\mathbb{P}(d_t = 1 | d_{t-1}, r_t, X_t) = 1 - \exp(-\lambda_t)
\]

Pricing formula of EURIBOR rates under risk-neutral measure:

\[
R_{t,h}^{EUR} = -\frac{1}{h} \log \left( \mathbb{E}_t^Q \left[ \exp \left\{ -\sum_{k=0}^{h-1} r_{t+k} + \lambda_{t+k+1} \right\} \right] \right)
\]
Standard Results in Term Structure Models

We denote:

\[ X_t \] a vector of factors in the economy.

If for all \( t \), \( r_t \) and \( \lambda_t \) are affine functions (resp. quadratic) of \( X_t \),
- the secured and unsecured rates are affine functions (resp. quadratic) of \( X_t \),
- these functions are available in closed-form for all maturities,
- the factor loadings are computable recursively.

General pricing formulae for QTSM

\[
R_{t,h}^{OIS} = a_h^{OIS} + b_h^{OIS} X_t + X_t c_h^{OIS} X_t
\]

\[
R_{t,h}^{EUR} = a_h^{EUR} + b_h^{EUR} X_t + X_t c_h^{EUR} X_t
\]
Implicitly, EURIBOR and OIS are considered as zero-coupons rates,

We assume the short-term rate is independent from the intensity:

Spread formula

\[
S(t, h) = R_{t,h}^{EUR} - R_{t,h}^{OIS} \\
= -\frac{1}{h} \log \left( \mathbb{E}_t^Q \left[ \exp \left\{ -\sum_{k=1}^{h} \lambda_{t+k} \right\} \right] \right)
\]

\[\Rightarrow\text{ No need to express } r_t \text{ for the spread modelling.}\]

Remark: \[\lambda_t \geq 0 \Rightarrow S(t, h) \geq 0.\]
What We Need

- Definition of factors with
  - $\mathbb{P}$-dynamics,
  - $\mathbb{Q}$-dynamics,
- Specification of intensity $\lambda_t = f(X_t)$,
- Identification constraints.
Credit and liquidity latent risk factors: \( X_t = (x_{c,t}, x_{l,t})' \).

\( x_{c,t} \) and \( x_{l,t} \) are not instantaneously correlated.

VAR(1) representation with independent idiosyncratic shocks.

\[
\begin{pmatrix}
    x_{c,t} \\
    x_{l,t}
\end{pmatrix}
= 
\begin{pmatrix}
    \mu_c \\
    \mu_l
\end{pmatrix} + 
\begin{pmatrix}
    \varphi_{1,1} & \varphi_{1,2} \\
    \varphi_{2,1} & \varphi_{2,2}
\end{pmatrix}
\begin{pmatrix}
    x_{c,t-1} \\
    x_{l,t-1}
\end{pmatrix} + 
\begin{pmatrix}
    \sigma_c & 0 \\
    0 & \sigma_l
\end{pmatrix}
\begin{pmatrix}
    \varepsilon_{c,t} \\
    \varepsilon_{l,t}
\end{pmatrix}
\]

where \((\varepsilon_{c,t}, \varepsilon_{l,t})' \sim \mathcal{IN}^\mathbb{P}(0, I_2)\).

For identification purposes, \( \sigma_c^2 + \sigma_l^2 = 1 \).

We also define \( x_t = x_{c,t} + x_{l,t} \).
Risk-Neutral Specification and Intensity Process

- Also VAR(1) dynamics under $\mathbb{Q}$-measure with constraints
  \[ \implies \text{AR}(1) \mathbb{Q}\text{-dynamics for } x_t. \]

  \[ x_t = \mu^* + \varphi^* x_{t-1} + \varepsilon^*_t \quad \text{where} \quad \varepsilon^*_t \sim \mathcal{IIN}^{\mathbb{Q}}(0, 1) \]

- Intensity is one-factor dependent:

  \[ \lambda_t = \lambda_0 + \lambda_1 x_t + \lambda_2 x_t^2 \quad \text{with:} \]

  \[ \lambda_0 \geq \frac{\lambda_1^2}{4\lambda_2} \implies \lambda_t \geq 0 \]

Reduced-form pricing formulas

\[ S(t, h) = \theta_{0, h} + \theta_{1, h} x_t + \theta_{2, h} x_t^2 \]

with $\theta_{i, h}$ functions of $(\lambda_0, \lambda_1, \lambda_2, \mu^*, \varphi^*)$ computable recursively.
Contents

1 Introduction

2 The Model
   • QTSM Models: A General Framework
   • The EURIBOR-OIS Spread Modelling

3 Estimation
   • Identifying the Factors
   • Non-Linear Kalman Filtering for QTSM

4 Decomposition
   • Decomposition of the Spread
   • Decomposition of the Term Structure

5 Conclusion
Identifying the Factors

Identification Strategy

- Proxy for credit risk $P_{c,t} \rightarrow$ first PC of 36 Euro-zone bank CDS
- Proxy for liquidity risk $P_{l,t} \rightarrow$ first PC of
  - 5Y KfW-Bund spread
  - Spread of 3M general collateral repo rate versus 3M German treasury bill
- Bank Lending Survey data (BLS): percentage of ‘−’ and ‘−−’ answers to the question over the past three months, how has your bank’s liquidity position affected the credit standards as applied to the approval of loans or credit lines to enterprises?

Proxies equations

Proxies are assumed quadratic functions of the corresponding factor with measurement errors.

\[
\begin{align*}
    P_{c,t} & = \pi_{c,0} + \pi_{c,1}x_{c,t} + \pi_{c,2}x_{c,t}^2 + \sigma_{\nu_c}\nu_{c,t} \\
    P_{l,t} & = \pi_{l,0} + \pi_{l,1}x_{l,t} + \pi_{l,2}x_{l,t}^2 + \sigma_{\nu_l}\nu_{l,t}
\end{align*}
\]
Proxies

Credit risk proxy

Liquidity risk proxy
The state-space representation

Transition and measurement equations:

Transition Two-factor $P$-dynamics.

Measurement Spread pricing formulae and proxies specification.

$\implies$ Maximum likelihood estimation with the Quadratic Kalman Filter (Monfort, Renne, & Roussellet (2013)).

Estimation constraints:

- Intensity and proxies functions are monotonously increasing in most cases in both factors.
Non-Linear Kalman Filtering for QTSM

The state-space representation

Transition and measurement equations:

**Transition** Two-factor \( \mathbb{P} \)-dynamics.

**Measurement** Spread pricing formulae and proxies specification.

\[ \lambda_t(x_t) \implies \text{Maximum likelihood estimation with the Quadratic Kalman Filter (Monfort, Renne, & Roussellet (2013)).} \]

Estimation constraints:

- Intensity and proxies functions are monotonously increasing *in most cases* in both factors.
The state-space representation

Transition and measurement equations:

**Transition** Two-factor $\mathbb{P}$-dynamics.

**Measurement** Spread pricing formulae and proxies specification.

$\Rightarrow$ Maximum likelihood estimation with the Quadratic Kalman Filter (Monfort, Renne, & Roussellet (2013)).

Estimation constraints:

- Intensity and proxies functions are monotonously increasing *in most cases* in both factors.
## Results

**Table:** Risk-neutral and measurement parameter estimates

<table>
<thead>
<tr>
<th>Equation</th>
<th>Estimate</th>
<th>Estimate</th>
<th>Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_t$</td>
<td>$\mu^*$</td>
<td>0.2627***</td>
<td>$\varphi^*$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0387)</td>
<td></td>
</tr>
<tr>
<td>$P_{c,t}$</td>
<td>$\pi_{c,0}$</td>
<td>-8.9650***</td>
<td>$\pi_{c,1}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.4296)</td>
<td></td>
</tr>
<tr>
<td>$P_{l,t}$</td>
<td>$\pi_{l,0}$</td>
<td>-1.3098**</td>
<td>$\pi_{l,1}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.7577)</td>
<td></td>
</tr>
<tr>
<td>$\lambda_t$</td>
<td>$\lambda_0$</td>
<td>0.1015</td>
<td>$\lambda_1$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0666)</td>
<td></td>
</tr>
<tr>
<td>noise</td>
<td>$\sigma_{\nu_c}^2$</td>
<td>0.0081</td>
<td>$\sigma_{\nu_l}^2$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.4206)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\sigma_{\eta}^2$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Table: Risk-neutral and measurement parameter estimates

<table>
<thead>
<tr>
<th>Equation</th>
<th>Estimate</th>
<th>Estimate</th>
<th>Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_t$</td>
<td>$\mu^*$</td>
<td>$\varphi^*$</td>
<td>$\mu^*$</td>
</tr>
<tr>
<td>$P_{c,t}$</td>
<td>$\pi_{c,0}$</td>
<td>$-8,9650^{**}$</td>
<td>$-0,000006$</td>
</tr>
<tr>
<td>$P_{l,t}$</td>
<td>$\pi_{l,0}$</td>
<td>$-1,3098^{**}$</td>
<td>$0,1382^{**}$</td>
</tr>
<tr>
<td>$\lambda_t$</td>
<td>$\lambda_0$</td>
<td>$0,1015$</td>
<td>$0,0003$</td>
</tr>
<tr>
<td>noise</td>
<td>$\sigma^2_{\nu_c}$</td>
<td>$0,0081$</td>
<td>$0,1000$</td>
</tr>
</tbody>
</table>

---

**Notes:**
- $\mu^*$, $\mu^*$, $\varphi^*$, $\lambda_0$, $\lambda_0$, $\sigma^2_{\nu_c}$
- $\pi_{c,0}$, $\pi_{l,0}$, $\pi_{c,1}$, $\pi_{l,1}$, $\lambda_1$, $\sigma^2_{\nu_l}$
- $\sigma^2_{\nu_c}$, $\sigma^2_{\nu_l}$
- $\lambda_2$, $\sigma^2_{\eta}$
Contents

1 Introduction

2 The Model
   - QTSM Models: A General Framework
   - The EURIBOR-OIS Spread Modelling

3 Estimation
   - Identifying the Factors
   - Non-Linear Kalman Filtering for QTSM

4 Decomposition
   - Decomposition of the Spread
   - Decomposition of the Term Structure

5 Conclusion
Decomposition of the spread

The credit spread $S(t, h)$ is given by:

$$S(t, h) = \theta_{0, h} + \theta_{1, h} x_t + \theta_{2, h} x_t^2$$

which can be further decomposed into:

- **Credit Spread**:
  $$\theta_{1, h} x_c, t + \theta_{2, h} x_c^2, t$$

- **Liquidity Spread**:
  $$\theta_{1, h} x_l, t + \theta_{2, h} x_l^2, t$$

- **Interaction**:
  $$2 \theta_{2, h} x_c, t x_l, t + \theta_{0, h}$$

- **Credit Risk Part**
  - Presence and comovement of credit risk in the economy.

- **Liquidity Risk Part**
  - Effect not attributable to any of the previous effects.

- **Interaction Part**: Presence and comovement of both risks in the economy.

\[ \Rightarrow \text{Decomposition in credit/liquidity and expected hypothesis component/term premia.} \]
Decomposition Results: 6M Spread

Credit and liquidity decomposition: Panel(a)

Credit and liquidity decomposition: Panel(b)

Term premia decomposition
**Decomposition of the Spread**

**Time series decomposition**

**Liquidity component:**
- High level on average and high-frequency fluctuations,
- represents most of the spread during Lehman crisis
- disappears at the end of the sample.

**Credit component:**
- Globally increasing and low-frequency fluctuations,
- represents more than 20 bps at the end of the sample.

**Interaction term:**
- Represents between 0 and 40 bps for the 6-month spread,
- fades out at the end of the sample.

**Term premia:**
- Possess similar features as the observed spread,
- fluctuates between 0 and 60 bps for the 6-month spread.
Decomposition of the Term Structure

11 Nov 2011

26 Oct 2012

bps

1m\textsuperscript{th}  3m\textsuperscript{th}  6m\textsuperscript{th}  9m\textsuperscript{th}  12m\textsuperscript{th}  15m\textsuperscript{th}
Efficiency of Unconventional Monetary Policies

**SMP**  No clear drop or increase in any spread component.
  \[\implies\]  No effect.

**VLTRO**  Significant drop after the announcement due mostly to liquidity and to a lesser extent to the interaction term. The two allotments do not change this trend.
  \[\implies\]  Nearly a 50 bps drop in 16 weeks.

**OMT**  Disappearing of both liquidity and interaction terms 2 months after Mario Draghi’s London Speech.
  \[\implies\]  Contributed to erase liquidity risk in the Euro Area.
<table>
<thead>
<tr>
<th>Contents</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1</strong> Introduction</td>
</tr>
<tr>
<td><strong>2</strong> The Model</td>
</tr>
<tr>
<td>- QTSM Models: A General Framework</td>
</tr>
<tr>
<td>- The EURIBOR-OIS Spread Modelling</td>
</tr>
<tr>
<td><strong>3</strong> Estimation</td>
</tr>
<tr>
<td>- Identifying the Factors</td>
</tr>
<tr>
<td>- Non-Linear Kalman Filtering for QTSM</td>
</tr>
<tr>
<td><strong>4</strong> Decomposition</td>
</tr>
<tr>
<td>- Decomposition of the Spread</td>
</tr>
<tr>
<td>- Decomposition of the Term Structure</td>
</tr>
<tr>
<td><strong>5</strong> Conclusion</td>
</tr>
</tbody>
</table>
In this paper,

- We use a quadratic no-arbitrage term structure model of EURIBOR-OIS spreads.
- We perform a decomposition of interbank spreads in credit and liquidity components.
- We extract the term premia from the observed spread.
- We show that the SMP program had no significant influence on interbank risk whereas the OMT contributed to erase the liquidity risk for all maturities.