



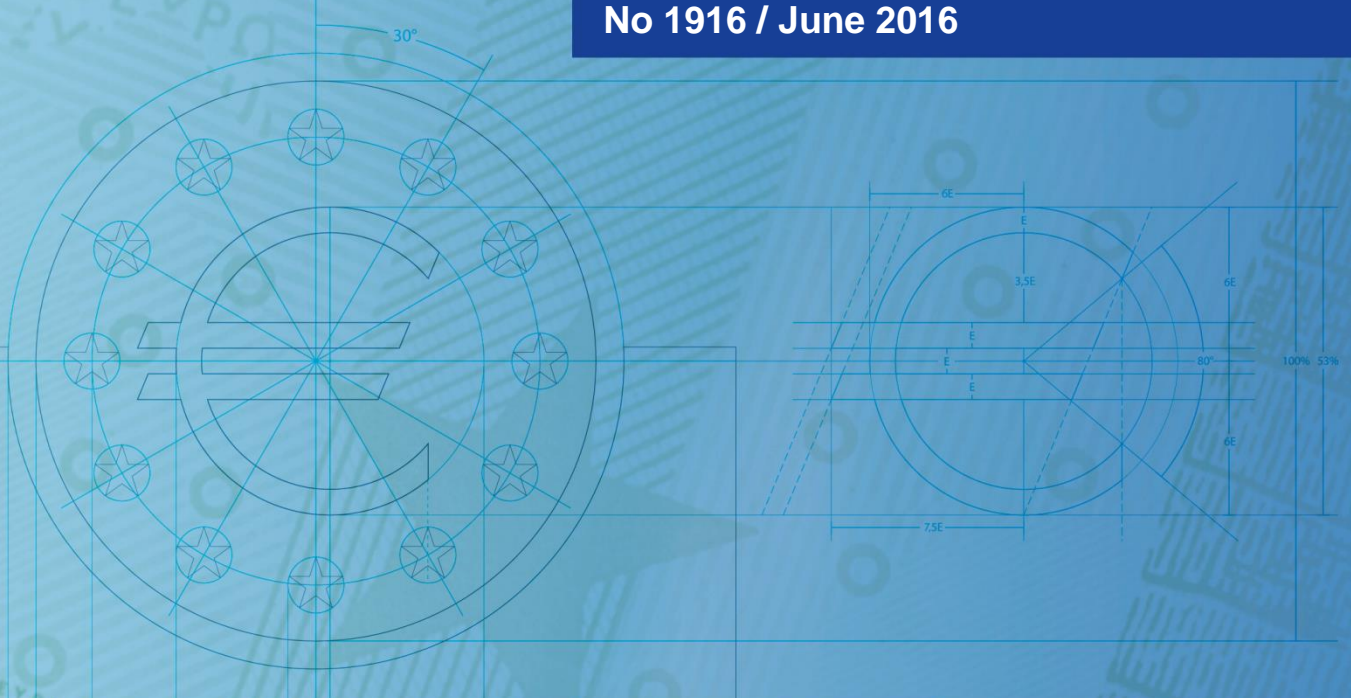
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Matthieu Darracq Pariès,  
Grzegorz Hałaj  
and Christoffer Kok

Bank capital structure and the  
credit channel of central bank  
asset purchases

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## Abstract

With the aim of reigniting inflation in the euro area, in early 2015 the ECB embarked on a large-scale asset purchase programme. We analyse the macroeconomic effects of the Asset Purchase Programme via the banking system, exploiting the cross-section of individual bank portfolio decisions. For this purpose, an augmented version of the DSGE model of Gertler and Karadi (2013), featuring a segmented banking sector, is estimated for the euro area and combined with a bank portfolio optimisation approach using granular bank level data. An important feature of our modelling approach is that it captures the heterogeneity of banks' responses to yield curve shocks, due to individual banks' balance sheet structure, different capital and liquidity constraints as well as different credit and market risk characteristics. The deep parameters of the DSGE model which control the transmission channel of central bank asset purchases are then adjusted to reproduce the easing of lending conditions consistent with the bank-level portfolio optimisation. Our macroeconomic simulations suggest that such unconventional policies have the potential to strongly support the growth momentum in the euro area and significantly lift inflation prospects. The paper also illustrates that the benefits of the measure crucially hinge on banks' ability and incentives to ease their lending conditions, which can vary significantly across jurisdictions and segments of the banking system.

**Keywords:** Portfolio optimisation, Banking, Quantitative Easing, DSGE

**JEL Classification:** C61, E52, G11

## Non-technical summary

In the aftermath of the global financial crisis, central banks have embarked on various forms of unconventional monetary policies to help reignite economic activity. One of the key instruments of this unconventional policy toolkit has been asset purchase (or quantitative easing) programmes, such as the Large-Scale Asset Purchase programmes of the US Federal Reserve, the Asset Purchase Facilities of the Bank of England and more recently the ECB's Asset Purchase Programme (henceforth APP).

There are several transmission channels through which central bank asset purchase programmes may affect the economy, such as direct effects on the asset price dynamics in the targeted market segments, changes in expectations due to the signalling effect of the programmes and more indirect effects they may have on the portfolio behaviour of banks and other financial institutions. On the latter, while some studies have examined the effect of purchase programmes on bank profitability, they have been inconclusive regarding potential “second-round” effects on credit supply and real economic activity which ultimately hinges upon bank portfolio allocation behaviour.

There are three main transmission channels through which a central bank asset purchase programme would affect bank balance sheets and ultimately the bank credit channel: (i) valuation effects on bank capital, (ii) income effects via a pass-through to funding costs and (iii) portfolio rebalancing effects as securities holdings become less attractive compared to other assets (e.g. loans). In terms of banks' credit supply responses to these three effects, we here focus on the price channel via effects on lending rates (as compared to quantity effects).

In a newly developed analytical approach, we combine a micro-level bank portfolio optimization model with a Dynamic Stochastic General Equilibrium model featuring a segmented banking sector and portfolio frictions. The combined framework allows for solving bank portfolio decisions after exogenous shocks and to consistently derive the macroeconomic implications of these adjustments.

On the macroeconomic modelling side, our strategy consists in introducing the minimal set of frictions into a standard medium-scale DSGE models so that (i) the model provides some micro-foundations for bank portfolio decisions between sovereign holdings and loan contracts, and (ii) the model has sufficient data consistency to provide a relevant quantification of APP macroeconomic multipliers. The model is estimated on euro area data.

The importance of the different channels and the overall magnitude of the effects will depend on the individual banks' portfolio characteristics and balance sheet constraints. A proper valuation of the APP would therefore need to account for such bank heterogeneity which may be pronounced not only across euro area countries but also within countries. In order to capture such distributional effects of central bank asset purchases, we employ a bank level portfolio optimization model in which banks maximise their risk-adjusted return on capital

given liquidity and solvency constraints.

Employing this framework, we conduct various counterfactual simulations on the impact of the APP for the euro area. Our findings suggest that such unconventional policies have the potential to strongly support the growth momentum in the euro area and significantly lift inflation prospects. The benefits of the policy measure rest on banks' ability and incentive to ease their lending conditions. The strength of the portfolio rebalancing channel through the banking system proves highly dependent on bank balance sheet conditions, and from this perspective, can have diverse impacts across euro area countries. Overall, however, the macro implications in terms of higher economic growth and inflation arising due to bank portfolio rebalancing effects are found to be positive for the euro area and for individual countries.

# 1 Introduction

In the aftermath of the global financial crisis to help reignite economic activity central banks have embarked on various forms of unconventional monetary policies. One of the key instruments of this unconventional policy toolkit has been asset purchase (or quantitative easing) programmes, such as the Large-Scale Asset Purchase programmes of the US Federal Reserve, the Asset Purchase Facilities of the Bank of England and more recently the ECB's Asset Purchase Programme (henceforth, APP).

There are several transmission channels through which asset purchase programmes may affect the economy, such as direct effects on the price of assets in the targeted market segment (see e.g. Meaning and Zhu (2011), Hamilton and Wu (2011), D'Amico and King (2013), D'Amico et al. (2012) and Hancock and Passmore (2011)), changes in expectations due to the signalling effect of the programmes (see inter alia Krishnamurthy and Vissing-Jorgensen (2011), Gagnon et al. (2011), Swanson (2011), Wright (2011), Gilchrist and Zakrajšek (2013) and Joyce et al. (2011b)) and the more indirect effects they may have on the portfolio behaviour of banks and other financial institutions. On the latter effect, while some studies have examined the effect of purchase programmes on bank profitability (see e.g. Lambert and Ueda (2014) and Montecino and Epstein (2014)) they have been inconclusive as to the "second-round" effect on credit supply and real economic activity which ultimately hinges upon how purchase programmes may affect banks' behaviour.

A number of mainly US and UK-based studies have attempted to quantify the macroeconomic implications of central bank asset purchase programmes using either VAR-type models or DSGE models, with overall rather wide-ranging outcomes in terms of the effects on output and prices but mostly suggesting that the asset purchase programmes have been effective in supporting economic growth. For instance, "US-based studies" include Chung et al. (2012), Fuhrer and Olivei (2011), Negro et al. (2011), Chen et al. (2012), Gertler and Karadi (2013), while "UK-based studies" include Joyce et al. (2011a), Goodhart and Ashworth (2012), Kapetanios et al. (2012), Bridges and Thomas (2012) and Pesaran and Smith (2012). For the euro area experiences, a few early studies include Lenza et al. (2010), Peersman (2011) and Altavilla et al. (2014). See also Martin and Milas (2012) for a survey. While many of these studies take a broad-based view of the impact of unconventional monetary policies including signalling, confidence and exchange rate effects, in this paper we take a somewhat narrower approach focusing on the bank credit channel via the portfolio rebalancing that central bank asset purchases may induce banks to undertake.

Our focus on the bank credit channel is motivated by the predominant role that banks play in the euro area financial system. We particularly emphasize the importance of taking into account the heterogeneous behaviour of banks in response to central bank asset purchases, especially when viewed against the background of the diverse and highly fragmented euro area banking sector. In other words, in a context where banks' business models and portfo-

lio composition vary and where their balance sheet constraints (e.g. solvency and liquidity requirements) notably differ, it seems reasonable to assume that asset purchase programmes will trigger different reactions across banks. Hence, also the ultimate macroeconomic effects of these unconventional monetary policies is likely to depend on the underlying diversity and heterogeneity characterising the banking sector at large.

Against this background, we argue in this paper that the financial propagation of the APP crucially depends on banks' incentives to rebalance their asset structure towards lending activity and the impact on their lending conditions, notably through lower lending margins. Following the sovereign yield compression that can be expected from the APP, banks can benefit from capital relief via positive valuation effects on their bond portfolios and lower funding costs. Besides, lower yields on new bond purchases would decrease the relative profitability of bond portfolios and therefore, encourage banks to expand lending and offer reduced lending margins. In order to quantify these effects for the euro area banking sector and ultimately for the economy at large, a portfolio optimisation model with heterogeneous banks is used to calibrate an APP counterfactual scenario in a medium-scale DSGE model with financial frictions. The paper is related to a small but emerging strand of the literature that analyses banks' portfolio choices in macro models, such as Adrian and Shin (2010), Gertler et al. (2012), Aoki and Sudo (2012), Aoki and Sudo (2013), He and Krishnamyrthu (2013), Adrian and Boyarchenko (2013a), Adrian and Boyarchenko (2013b), Benes et al. (2014a) and Benes et al. (2014b).

In order to capture banks' heterogeneous responses in a partial equilibrium setting, we use a multi-period model of a bank maximising its risk-adjusted return on capital given liquidity and solvency constraints (see Hałaj (2015)).<sup>1</sup> In line with the risk management literature (see for example Adam (2008)), banks are described as constrained portfolio managers maximising risk-adjusted returns (from loans and securities and taking into account funding costs) on capital subject to capital and liquidity constraints. The asset side of bank balance sheets consists of loans paying interest and subject to credit risk and securities characterized by the expected return and volatility parameters. On the liability side, two sources of funding are considered: customer and wholesale deposits paying fixed interest and subject to outflow (roll-over) risk and capital. The model reflects the regulatory risk constraint imposed on banks as well as the internal model-based risk limits: *(i)* regulatory constraint on the minimum capital ratio (RWA/Capital); *(ii)* Value-at-Risk: capital has to cover losses in 99% of the distribution of losses; *(iii)* Liquidity-at-Risk: liquidity buffer (securities after haircut) has to cover 99% of funding outflows. Banks' objective is to optimise the risk-adjusted return on capital, aggregated within the horizon of the optimisation, by choosing the lending volume and the purchase of securities, taking the risk-return profile of exposures as given.

For the assessment of the broader macroeconomic implications of the heterogeneous reac-

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<sup>1</sup>A related reference is Hałaj (2013)

tions of individual banks we employ a DSGE model including a segmented banking sector. Our modelling strategy consists in introducing the minimal set of frictions into existing DSGE models so that *i*) the model provides some micro-foundations for bank portfolio decisions between sovereign holdings and loan contracts, and *ii*) the model has sufficient data consistency to provide a relevant quantification of Asset Purchase Program macroeconomic multipliers. The basis for the general equilibrium model comes first from Smets and Wouters (2007) for the non-financial blocks and estimation strategy, and second from Gertler and Karadi (2013) for the intermediaries balance sheet constraints and approach to evaluate central bank asset purchases. We augment the model with segmented banks à la Gerali et al. (2010) and Darracq Pariès et al. (2011), notably introducing a loan contract à la Bernanke et al. (1999) with pre-determined lending rates. The DSGE model is estimated on euro area data following the approach of Smets and Wouters (2007). The main purpose of the empirical exercise is not to conduct an exhaustive review of the structural determinants of the euro area business cycle and evaluate the statistical performance of the model. Instead, by making use of the insights derived from our granular bank level optimisation approach we aim at narrowing down the plausible ranges for the deep parameters of the model, notably those to which the APP transmission would be most sensitive, and bring a satisfactory level of data consistency for the macroeconomic multipliers used in the quantitative exercises. In particular, we illustrate the sensitivity of the asset purchase propagation mechanism to three relevant dimensions of the parameter space: credit demand frictions, staggered lending rate setting, and frictions on portfolio decisions for households and bankers.

Overall, exploiting both the lending rate experiments derived from the cross-section of bank portfolio decisions as well as alternative estimations of the macroeconomic model, we conducted various counterfactual simulations on the impact of the APP for the euro area. The ranges of outcomes of our simulations suggest that such unconventional policies have the potential to strongly support the growth momentum in the euro area and significantly lift inflation prospects. The benefits of the APP rest on banks' ability and incentive to ease their lending conditions. The strength of the portfolio re-balancing channel through the banking system proves highly dependent on bank balance sheet conditions, and from this perspective, can have diverse impacts across jurisdictions and segments of the euro area banking system.

The rest of the paper is structured as follows: Section 2 describes the portfolio optimisation model. Section 3 presents the macroeconomic modelling framework and in Section 4 the estimation of the DSGE model is presented. Finally, in Section 5 the DSGE model simulations are presented while Section 6 concludes.



## 2 Bank portfolio rebalancing incentives

### 2.1 Banks' optimal responses to central bank asset purchases

There are three main transmission channels through which the APP would affect the economy via the bank credit channel: (i) valuation effects on bank capital, (ii) income effects via a pass-through to funding costs and (iii) portfolio rebalancing effects as securities holdings become less attractive compared to other assets (e.g. loans). In terms of banks' credit supply responses to these three effects, in this study we focus on the price channel via effects on lending rates (as compared to quantity effects). This is corroborated by recent observations both in terms of the sizeable changes to bank lending spreads since end-2014 (see Figure 1) and banks' responses to the ECB April 2015 bank lending survey suggesting that banks would mainly react to the APP by adjusting their terms and conditions rather than via quantities (see Figure 2).

While the valuation, income and portfolio allocation channels are likely to qualitatively affect all banks in the same way, the importance of the different channels and the overall magnitude of the effects will depend on the individual banks' portfolio characteristics and balance sheet constraints. A proper valuation of the APP would therefore need to account for such bank heterogeneity which may be pronounced not only across euro area countries but within countries. In order to capture such cross-distributional effects of central bank asset purchases we employ a bank level portfolio optimization model that is able to disentangle the effects of the three elements of the bank credit channel mentioned above. These distinct credit channel effects are subsequently used to inform key parameters in our macro model simulations (see Section 5).

The bank-level analysis needs to be conducted at a sufficiently granular bank-level data set to properly capture banks' heterogeneous portfolio optimisation responses to the APP while accounting for individual banks' idiosyncratic capital and liquidity constraints. For this reason, we apply the portfolio decision model to the consolidated balance sheet data reported by banks in the context of the 2014 Comprehensive Assessment: all the parameters of the model (volumes, interest rates and default probabilities) are inferred from the partly confidential 2014 Comprehensive Assessment stress test dataset. The Comprehensive Assessment stress test data covers 130 banks from all euro area countries, amounting to approximately 82% of total banking sector assets. In this paper, we focus primarily on the four largest euro area countries (i.e. Germany, France, Italy and Spain).

A few broad descriptive statistics concerning the sample of banks included in the analysis are worth highlighting. First, while the balance sheet structure is overall broadly similar across the banking sectors of the four largest euro area countries there are nevertheless some notable differences. For example, the split between private sector loans, other loans and securities portfolios differs across the banking sectors of the big four countries. Whereas private sector loans are the most dominant type of credit in all countries, they are relatively more important



in Spain, France and Italy. In Germany and France interbank lending is relatively important, while securities holdings are comparatively important in Germany and Italy. On the liabilities side, deposits is by far the most important funding source among Spanish and Italian banks, whereas German and French banks also rely strongly on market-based funding sources (such as covered bonds and short-term commercial papers).

Second, there is considerable heterogeneity across banks in terms of the relative importance of different balance sheet items (Figure 3). This suggests that ignoring the heterogeneous bank balance sheet information when assessing how banks would respond to asset purchase programmes could create misleading conclusions.

Third, as the APP targets government securities the relative importance of sovereign securities holdings across the banks is also worth highlighting (Figure 4). It is notable that the amount of sovereign securities held by German banks is overall substantially larger than those held by French, Italian and Spanish banks; although in terms of total assets Italian banks hold a relatively larger share of sovereign bonds on their balance sheets (c. 14 pct.). At the same time, most of the German banks' sovereign holdings are held to maturity whereas especially Italian but also Spanish and French banks hold a larger proportion of their sovereign securities in the available-for-sale and trading portfolios implying that a relatively larger share of their sovereign holdings will be marked to market.<sup>2</sup> Therefore, the price impact of an APP-induced sovereign yield shock will tend to more immediately affect these banks' profit and loss account and hence their capital. These banks' sovereign holdings will also be more easily sellable if banks were to rebalance their asset composition in response to the APP.

## 2.2 Modelling approach

We model banks' choices about their balance sheet structures using portfolio optimisation techniques that aim to reflect how banks' conduct their asset-liability management (ALM) in practice. The strategic actions taken by banks that change the composition of the balance sheet can be explained by the rational economic goals of maximal risk-adjusted return. The task of the balance sheet management is relatively complex. It is a multi-criteria problem with goals changing in time depending on the liquidity and solvency outlook. Banks, as all other firms try to maximise their profits but also have to build adequate buffers against possible fluctuations of their funding, especially given the high leverage of most banks' business model. Nonetheless, the portfolio choice problems studied in financial mathematics provide a rich, theoretically well-founded toolkit to describe the process of the risk-adjusted profit maximisation in which banks are involved on a regular basis in their risk management and ALM activities.

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<sup>2</sup>We account for the transitional arrangements in terms of removal of prudential filters relating to Basel III implementation in the EU. This implies that in our mark-to-market calculations for only 40 pct. of the available for sales portfolio the price revaluations are assumed to affect capital (corresponding to the phase-in arranged applied for 2015 in the 2014 comprehensive assessment).

In general, the setup of the balance sheet problem is based on a risk-adjusted maximisation of the return on capital. Technical details are available in the Appendix.<sup>3</sup> The risk of the balance sheet structure is related to uncertain funding sources (the risk of an outflow of deposits), the credit risk in the loan portfolio (outstanding and new volumes treated separately) and the volatile prices of the liquid securities. The bank operates in a 2-period time frame facing the risk of<sup>4</sup>:

- becoming illiquid when investing excessively into illiquid loans and exposing itself to a risk of having insufficient liquid funds to meet the potential outflow of funding at the end of each period;
- becoming insolvent if combined losses (loan losses and devaluation of securities) and interests due on funding sources erode the capital base.

The 2-period setup reflects a short term budget planning or ALM strategy (for instance in a one year horizon, see Adam (2008)) that takes into account potential consequences of the decisions taken in the first period for the second period (e.g. for the following year) and possible adjustment to the strategy after the first period following macro-financial developments affecting bank's capitalisation and profitability at the end of the budgeting horizon.

### 2.3 Heterogeneous bank responses to the asset purchase programme

The bank portfolio decision model can indirectly provide the partial equilibrium lending supply reaction of individual banks following the sovereign yield compression due to the APP. The approach taken to quantify the adjustment of bank lending policies to customers consists in finding the bank-specific lending rate spread decline that would stabilize banks market shares to the levels preceding the APP impact sovereign yields.

The pass-through of sovereign yield declines to lending rate spreads is computed in two steps. First the APP related yield compression affects the capital position of banks, their funding costs and the yield on new bond purchases, which condition the optimal structure of bank balance sheets. Three channels will be decomposed in the simulations broadly mirroring the three financial wedges embedded in the DSGE model:

- (i) *Direct (positive) impact on capital via revaluation of securities portfolio*: The shift of the yields translates into positive valuation effects on the securities portfolio. In accordance with observed market movements, it is assumed that those shocks are passed through 100% to the yields of the sovereign sub-portfolio and 75% to corporate bonds. The resulting revaluation is directly recognised in capital and hence implies a capital relief.<sup>5</sup>

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<sup>3</sup>See also Halaj (2015).

<sup>4</sup>The setup is flexible enough to be straightforwardly applied in a general multi-period model.

<sup>5</sup>A strong direct impact via the *capital channel* can be expected for banks with comparatively small capital

- (ii) *Funding shock* (reduction of the funding cost): This shift in the cost of funding exerts a positive impact on capital via the implied increase of net-interest income. In line with historical regularities, a 50% pass-through in retail deposits and 75% pass-through on the wholesale funding is assumed.
- (iii) *Impact on risk-return of exposures and portfolio decisions*: The new, lower yields computed in (i), change the risk-return parameters of the reinvestment portfolios. The shift of the bond yields is passed through to the expected coupon of securities over the optimisation horizon. This has a negative effect on the profitability of securities: lower coupons of new issuance and lower yield-to-maturity. Consequently, the Sharpe ratio of securities decreases and they become a less favourable investment option (as compared to loans for which the Sharpe ratio remains unchanged).

Second, we simulate for each bank in our sample the impact of the three factors mentioned above on their portfolio decisions and numerically search for the changes in lending spreads which stabilise the loan market share to its pre-APP shock level.<sup>6</sup>

To illustrate the importance of bank balance sheet structure and return-risk characteristics of the loan and securities portfolios on the lending rate response to the APP-induced shocks to sovereign yields, Figure 5 graphically presents the composition of the balance sheets of two banks in the sample. Furthermore, Figure 6 shows the key parameter values of the two banks. The lending rate response of a negative 50 bps shock to sovereign bond yields differs between two banks with bank 1 reducing its lending rate by 50 bps while bank 2 only reduces its average lending rate by 25 bps. The different intensity of the lending rate impact is a result of the differences in the balance sheet structure and related parameter values. For instance, it is notable that the relative size of the securities portfolio is higher for bank 1, which implies that the bank can expect a higher direct *capital impact* due to revaluation effects and also has more bonds to sell (or not renew upon maturity) than bank 2. It also has a more wholesale based funding structure, which results in a larger *funding cost impact* compared to bank 2. It is also notable that the credit risk in the loan portfolio of bank 2 is substantially more elevated than buffers. As the lending rate response via the *capital channel* is primarily determined by the ratio between the revaluation effect on capital (which is broadly similar across banks in our sample) and the banks' excess capital buffer, a similar sized yield shock frees up relatively more capital to expand loan supply in banks that initially faced tighter capital constraints (in terms of low capital buffers).

<sup>6</sup>The interest rate paid by outstanding loan volumes is assumed to remain constant, while the interest related to the new loan production adjusts to stabilise the optimal lending activity to its baseline level. In the simulations, banks' default probabilities and loan loss distributions remain unchanged in the horizon of the optimisation, assuming that some risk exogeneity at the bank level is plausible. The risk parameters of the securities are quite uncertain and should proxy the market risk uncertainty, heterogeneous accounting rules within the securities portfolio and hedging activities which are not discernible in the aggregate reporting of banks. Therefore, the volatility of returns on the securities portfolio has been calibrated for each bank so that the optimal balance sheet structure in the baseline case (before the APP shock) matches the asset composition in the data sample.

for bank 1. At the same time, the Sharpe ratio on securities is larger for bank 2 compared to bank 1. These two elements are likely to produce a more muted lending rate response by bank 2. A factor which pulls in the opposite direction (i.e. a stronger average lending rate response of bank 2) is the larger share of maturing loans on the balance sheet of bank 2. However, this factor is dominated by the aforementioned factors allowing for a comparatively stronger response by bank 1.

Illustrating the importance of bank heterogeneity at the country aggregate level, Figure 7 shows the simulated pass-through of a uniform -50 bps shock to sovereign yields to lending rate spreads for the largest four euro area member states and decomposing the lending rate response of the three channels (namely, the revaluation impact on capital, the funding cost and the portfolio rebalancing (risk-return) effect). As expected, the consequences of the APP-related sovereign yield compression would give significant scope for banks to compress their lending rate margins, although with notable differences across banks in the four countries. Overall, the impact of a 50 bps negative shock to sovereign bond yields has the strongest aggregate impact on the lending rates of German and French banks which are reduced by around 35 bps and 29 bps, respectively. By comparison, the lending rate declines for Spain and Italy are below 10 bps and 15 bps respectively.

A first factor behind these responses is the stronger portfolio rebalancing effect for banks in France and to a lesser extent, in Germany.

Turning now to the role of the funding cost channel, the funding shock has a stronger effect on the lending rate response in Germany. This is partly due to the relatively high reliance on wholesale funding of banks in Germany, which implies a strong sensitivity of the loan portfolio decisions to changes in funding conditions. To illustrate this asymmetry, we computed the accounting change in banks' expenses relative to their capital position: Table 1 indeed shows a much higher influence of the shock on funding expenses in Germany (9.4%) than on the other countries (about 6.0% for France, Italy and Spain). The response of the banking system in France to the funding shock appears muted.

Finally, the valuation effect of bank capital position contributes to asymmetric lending rate responses across the largest euro area countries. In particular, Spanish banks stand out to be the least affected through this channel.<sup>7</sup>

Next, we examine the non-linear dependence of the lending rate response to the size of the sovereign bond yield shock. Figure 8 shows the aggregate banking sector responses across the four countries over a grid of sovereign bond yield shocks from 0 bps to 100 bps. The lending rate responses are broadly linear, although some 'cliff effects' are discernible in particular for Italian banks (and to a lesser extent, Spanish banks) with stronger declines in lending rates for large sovereign yield shocks. On average per 10 bps decline in the sovereign yield amount

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<sup>7</sup>Sensitivity analysis (available from the authors upon request) shows that the Spanish banks' optimal lending volumes are less sensitive to an improvement in capital.

to -6 bps for German banks, -3 bps for Spanish banks, -6 bps for French banks and -7 bps for Italian banks. For a 100 bps negative yield shock the banks reduce lending rates by around 51 bps in Germany, 26 bps in Spain, 64 bps in France and 51 bps in Italy.<sup>8</sup>

On this basis, we can then gauge what the impact on lending rates is due to the sovereign yield compression that occurred in the context of the ECB announcement (and subsequent implementation) of its expanded asset purchase programme on 22 January 2015. Arguably, the yield compression due to the APP started to occur already before the actual announcement as markets anticipated the upcoming purchase programme. It is obviously difficult to precisely determine when exactly markets started pricing the APP into euro area sovereign yields. We take a pragmatic approach and select mid-November 2014 as a cut-off date, which corresponds to the finalisation of the ECB Broad Macroeconomic Projections for Q4 2014 that eventually contributed to triggering the Governing Council's decision to initiate the APP in early-2015.<sup>9</sup> As input into the optimisation model we rely on estimates by Altavilla et al. (2015) who found that the APP resulted in a negative impact on euro area aggregate 10-year sovereign bond yields in the range of 30-50 basis points depending on the estimation approach. At the country level, they estimated a decline of 10-year sovereign bonds amounting to between 20 and 25 basis points in Germany, between 30 and 40 basis points in France, between 75 and 80 basis points in Italy and between 70-80 basis points in Spain.

Translating these "yield shocks" into lending rate responses via the bank optimisation model results in a median decline of lending rates amounting to 22 bps for the euro area as a whole. For the largest member states, the announcement effects proved already quite diverse and such a dispersion are compounded in the lending rate simulations through the asymmetric tightness of capital and liquidity constraints in particular. Indeed, the individual bank responses can be aggregated into country level lending rate declines of 15 bps in Germany, 22 bps in Spain, 21 bps in France and 32 bps in Italy.<sup>10</sup>

A final observation worth noting is that when looking across the sample of euro area banks about half of them do not find it optimal to rebalance their portfolio and hence the isolated impact of the APP has no impact on these banks' lending rate. Importantly, this does not

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<sup>8</sup>The optimisation model can also be used to simulate the needed sell-off of securities if instead banks would accommodate the sovereign bond yield shock by adjusting its securities portfolio (rather than by adjusting its lending rates). For example, for a 100 bps negative yield shock the securities sell-off accumulates to 17 pct. among German banks, 10 pct. for Spanish banks, 17 pct. for French banks and 8 pct. for Italian banks.

<sup>9</sup>Admittedly, euro area sovereign yields started to materially decline already earlier in 2014 which to some extent could also be attributed to the anticipation of the APP. Hence, an alternative cut-off date could be around July-August 2014. However, selecting an earlier cut-off date obviously risks contaminating the observed sovereign yield compression with factors other than the APP anticipation.

<sup>10</sup>In comparison to banks in Germany and France, the Sharpe ratio on Spanish and Italian banks' securities holdings is substantially higher than the Sharpe ratio on their loan book. This implies that a larger shock to securities returns of Spanish and Italian banks' holdings than in other countries is needed to induce those banks to reshuffle their portfolios from securities toward lending. This observation is consistent with the magnitude of the relative changes in Sharpe ratios presented in Table 1.

imply that the APP will not cause any changes in the lending rates of those banks. Only that those changes will not be caused by portfolio rebalancing considerations, but are likely to occur not least due to competitive pressures from those banks that find it optimal to reshuffle their portfolios towards higher lending. It is crucial to note that our approach consists of aggregating individual bank responses, while not accounting for potential strategic complementarities that may arise when banks start internalising the actions of other banks.

We now turn the general equilibrium analysis, starting with the specification of our DSGE model.

### 3 General equilibrium perspective

For the assessment of the broader macroeconomic implications of the heterogeneous reactions of individual banks we employ a DSGE model including a segmented banking sector. Our modelling strategy consists in introducing the minimal set of frictions into existing DSGE models so that (i) the model provides some micro-foundations for bank portfolio decisions between sovereign holdings and loan contracts, and (ii) the model has sufficient data consistency to provide a relevant quantification of APP macroeconomic multipliers.

The basis for the general equilibrium model comes first from Smets and Wouters (2007) for the non-financial blocks and estimation strategy, and second from Gertler and Karadi (2013) for the intermediaries balance sheet constraints and approach to evaluate central bank asset purchases. We augment the model with segmented banks à la Gerali et al. (2010) and Darracq Pariès et al. (2011), notably introducing a loan contract à la Bernanke et al. (1999) with pre-determined lending rates.

The main decision problems are reported below as well as the necessary notations related to the empirical exercise.<sup>11</sup> The model economy evolves along a balanced-growth path driven by a positive trend,  $\gamma$ , in the technological progress of the intermediate goods production and a positive steady state inflation rate,  $\pi^*$ . In the description of the model, stock and flow variables are expressed in real and effective terms (except if mentioned otherwise): they are deflated by the price level and the technology-related balanced growth path trend.

#### 3.1 Households behavior

The economy is populated by a continuum of heterogeneous infinitely-lived households. Each household is characterized by the quality of its labour services,  $h \in [0, 1]$ . At time  $t$ , the intertemporal utility function of a representative household  $h$  is

$$\mathcal{W}_t(h) = \mathbb{E}_t \sum_{j=0}^{\infty} (\beta\gamma^{1-\sigma_c})^j \varepsilon_{t+j}^b \mathcal{U}(C_{t+j}(h) - \eta C_{t+j-1}(h) / \gamma, N_{t+j}^S(h))$$

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<sup>11</sup>Details regarding the full set of equilibrium conditions can be obtained from the authors upon request.

Household  $h$  obtains utility from consumption of an aggregate index  $C_t(h)$ , relative to an internal habit depending on its past consumption  $\eta$ , while receiving disutility from the supply of their homogenous labour  $N_t^S(h)$ .  $\gamma$  is trend productivity growth and  $\beta$  is the preference rate. Utility also incorporates a consumption preference shock  $\varepsilon_t^b$ .

The instantaneous utility  $\mathcal{U}$  has the following functional form

$$\mathcal{U}(X_1, X_2) = \frac{X_1^{1-\sigma_c}}{1-\sigma_c} \exp\left(\tilde{L} \frac{(\sigma_c - 1)}{(1 + \sigma_l)} X_2^{1+\sigma_l}\right)$$

where  $\tilde{L}$  is a positive scale parameter and  $\sigma_c$  is the intertemporal elastasticity of substitution. Each household  $h$  maximizes its intertemporal utility under the following budget constraint:

$$\begin{aligned} & D_t(h) + Q_{B,t} \left[ B_{H,t}(h) + \frac{1}{2} \chi_H (B_{H,t}(h) - \bar{B}_H)^2 \right] + C_t(h) \\ = & \frac{R_{D,t-1}}{\pi_t} D_{t-1}(h) / \gamma + \frac{R_{G,t}}{\pi_t} Q_{B,t-1} B_{H,t-1} / \gamma \\ & + \frac{(1 - \tau_{w,t}) W_t^h N_t^S(h) + A_t(h) + T_t(h)}{P_t} + \Pi_t(h) \end{aligned}$$

where  $P_t$  is an aggregate price index,  $R_{D,t}$  is the one period ahead nominal gross deposit rate,  $D_t(h)$  are deposits,  $R_{G,t}$  is the nominal return on government securities,  $Q_{B,t}$  is the price of the government bond and  $B_{H,t}(h)$  is a government bond.  $W_t^h$  is the nominal wage,  $\pi_t$  is gross inflation,  $T_t(h)$  are government transfers (both expressed in effective terms) and  $\tau_{w,t}$  is a time-varying labor tax.  $\Pi_t(h)$  corresponds to the profits net of transfers from the various productive and financial segments owned by the households.  $\chi_H$  is the households' portfolio adjustment cost. A positive value of  $\chi_H$  prevents frictionless arbitrage of the returns on securities by the household sector. Finally  $A_t(h)$  is a nominal stream of income (both in effective terms) coming from state contingent securities and equating marginal utility of consumption across households  $h \in [0, 1]$ .

In equilibrium, households' choices in terms of consumption, hours and deposit holdings are identical.

More precisely, the first order conditions of the household problem with respect to consumption, labour, deposit and government bond holdings are

$$\Lambda_t = \mathcal{U}'_{1,t} - \beta \gamma^{-\sigma_c} \eta \mathbb{E}_t \mathcal{U}'_{1,t+1} \quad (1)$$

$$\Lambda_t \frac{W_t^h}{P_t} = \mathcal{U}'_{2,t} \quad (2)$$

$$\mathbb{E}_t \left[ \Xi_{t,t+1} \frac{R_{D,t}}{\pi_{t+1}} \right] = 1 \quad (3)$$

$$\mathbb{E}_t \left[ \Xi_{t,t+1} \frac{(R_{G,t+1} - R_{D,t})}{\pi_{t+1}} \right] = \chi_H (B_{H,t} - \bar{B}_H) \quad (4)$$



where  $\Lambda_t$  is the lagrange multiplier associated with the budget constraint and  $\Xi_{t,t+1} = \beta\gamma^{-\sigma_c} \frac{\Lambda_{t+1}}{\Lambda_t}$  is the period  $t$  stochastic discount factor of the households for nominal income streams at period  $t + 1$ .

### 3.2 Labor supply and wage setting

Intermediate goods producers make use of a labor input  $N_t^D$  produced by a segment of labor packers. Those labor packers operate in a competitive environment and aggregate a continuum of differentiated labor services  $N_t(i)$ ,  $i \in [0, 1]$  using a Kimball (1995) technology. The Kimball aggregator is defined by

$$\int_0^1 H\left(\frac{N_t(i)}{N_t^D}; \theta_w, \psi_w\right) di = 1$$

where we consider the following functional form:

$$H\left(\frac{N_t(i)}{N_t^D}\right) = \frac{\theta_w}{(\theta_w(1 + \psi_w) - 1)} \left[ (1 + \psi_w) \frac{N_t(i)}{N_t^D} - \psi_w \right]^{\frac{\theta_w(1 + \psi_w) - 1}{\theta_w(1 + \psi_w)}} - \left[ \frac{\theta_w}{(\theta_w(1 + \psi_w) - 1)} - 1 \right]$$

This function, where the parameter  $\psi_w$  determines the curvature of the demand curve, has the advantage that it reduces to the standard Kimball aggregator under the restriction  $\psi_w = 0$ .

The differentiated labor services are produced by a continuum of unions which transform the homogeneous household labor supply. Each union is a monopoly supplier of a differentiated labour service and sets its wage on a staggered basis, paying households the nominal wage rate  $W_t^h$ . Every period, any union faces a constant probability  $1 - \alpha_w$  of optimally adjusting its nominal wage, say  $W_t^*(i)$ , which will be the same for all suppliers of differentiated labor services. We denote thereafter  $w_t$  the aggregate real wage, expressed in effective terms, that intermediate producers pay for the labor input provided by the labor packers and  $w_t^*$  the effective real wage claimed by re-optimizing unions.

When they cannot re-optimize, wages are indexed on past inflation and steady state inflation according to the following indexation rule:

$$W_t(i) = \gamma [\pi_{t-1}]^{\xi_w} [\pi^*]^{1 - \xi_w} W_{t-1}(i)$$

with  $\pi_t = \frac{P_t}{P_{t-1}}$  the gross rate of inflation. Taking into account that they might not be able to choose their nominal wage optimally in a near future,  $W_t^*(i)$  is chosen to maximize their intertemporal profit under the labor demand from labor packers. Wages are subject to a time-varying tax rate  $\tau_{w,t}$  which is affected by an i.i.d shock defined by  $1 - \tau_{w,t} = (1 - \tau_w^*) \varepsilon_t^w$ . The recursive formulation of the aggregate wage setting is exposed in the appendix.

### 3.3 Entrepreneurs and loan officers

Every period, a fraction  $(1 - f)$  of household's members are workers while a fraction  $fe$  are entrepreneurs and the remaining mass  $f(1 - e)$  are bankers (see thereafter). Each entrepreneur faces a probability  $\zeta_e$  of staying entrepreneurs over next period and a probability  $(1 - \zeta_e)$  of becoming a worker again. To keep of share of entrepreneurs constant, we assume that similar number of workers randomly becomes entrepreneur. When entrepreneurs exit their accumulated earnings are transferred to the respective household. At the same time, newly entering entrepreneurs receive initial funds from their household. Overall, households transfer a real amount  $\Psi_{E,t}$  to the entrepreneurs for each period  $t$ . Finally, as it will become clear later, entrepreneurs decisions for leverage and lending rate are independent from their net worth and therefore identical. Accordingly, we will expose the decision problem for a representative entrepreneur.

At the end of the period  $t$  entrepreneurs buy the capital stock  $K_t$  from the capital producers at real price  $Q_t$  (expressed in terms of consumption goods). They transform the capital stock into an effective capital stock  $u_{t+1}K_t$  by choosing the utilisation rate  $u_{t+1}$ . The adjustment of the capacity utilization rate entails some adjustment costs per unit of capital stock  $\Gamma_u(u_{t+1})$ . The cost (or benefit)  $\Gamma_u$  is an increasing function of capacity utilization and is zero at steady state,  $\Gamma_u(u^*) = 0$ . The functional forms used for the adjustment costs on capacity utilization is given by  $\Gamma_u(X) = \frac{\overline{\kappa}}{\varphi} (\exp[\varphi(X - 1)] - 1)$ . The effective capital stock can then be rented out to intermediate goods producers at a nominal rental rate of  $r_{K,t+1}$ . Finally, by the end of period  $t + 1$ , entrepreneurs sell back the depreciated capital stock  $(1 - \delta)K_t$  to capital producer at price  $Q_{t+1}$ .

The gross nominal rate of return on capital across from period  $t$  to  $t + 1$  is therefore given by

$$R_{KK,t+1} \equiv \pi_{t+1} \frac{r_{K,t+1}u_{t+1} - \Gamma_u(u_{t+1}) + (1 - \delta)Q_{t+1}}{Q_t} \quad (5)$$

where  $\pi_{t+1}$  is the CPI inflation rate.

Each entrepreneur's return on capital is subject to a multiplicative idiosyncratic shock  $\omega_{e,t}$ . These shocks are independent and identically distributed across time and across entrepreneurs.  $\omega_{e,t}$  follows a lognormal CDF  $F_e(\omega_{e,t})$ , with mean 1 and variance  $\sigma_{e,t}$  which is assumed to be time-varying. By the law of large numbers, the average across entrepreneurs (denoted with the operator  $\tilde{E}$ ) for expected return on capital is given by  $\tilde{E}[\mathbb{E}_t(\omega_{e,t+1}R_{KK,t+1})] = \mathbb{E}_t(\int_0^\infty \omega_{e,t+1} dF_{e,t}(\omega) R_{KK,t+1}) = \mathbb{E}_t(R_{KK,t+1})$ .

Entrepreneur's choice over capacity utilization is independent from the idiosyncratic shock and implies that

$$r_{K,t} = \Gamma'_u(u_t). \quad (6)$$

Entrepreneurs finance their purchase of capital stock with their net worth  $NW_{E,t}$  and a

one-period loan  $L_{E,t}$  (expressed in real terms, deflated by the consumer price index) from the commercial lending branches:

$$Q_t K_t = NW_{E,t} + L_{E,t}. \quad (7)$$

In the tradition of costly-state-verification frameworks, lenders cannot observe the realisation of the idiosyncratic shock unless they pay a monitoring cost  $\mu_e$  per unit of assets that can be transferred to the bank in case of default. We constrain the set of lending contracts available to entrepreneurs, such that they can only use debt contracts in which the lending rate  $R_{LLE,t}$  is pre-determined at the previous time period. Default will occur when the entrepreneurial income that can be seized by the lender falls short of the agreed repayment of the loan. At period  $t + 1$ , once aggregate shocks are realised, this will happen for draws of the idiosyncratic shock below a certain threshold  $\bar{\omega}_{e,t}$ , given by

$$\bar{\omega}_{e,t+1} \chi_e R_{KK,t+1} \kappa_{e,t} = (R_{LLE,t} + 1) (\kappa_{e,t} - 1) \quad (8)$$

where  $R_{LLE,t}$  is the nominal lending rate determined at period  $t$  and  $\kappa_{e,t}$  is the corporate leverage defined as

$$\kappa_{e,t} = \frac{Q_t K_t}{NW_{E,t}} \quad (9)$$

$\chi_e$  represents the share of the entrepreneur's assets (gross of capital return) that banks can recover in case of default. When banks take over the entrepreneur's assets, they have to pay the monitoring costs.

The ex post return to the lender on the loan contract, denoted  $\tilde{R}_{LE,t}$ , can then be expressed as

$$\tilde{R}_{LE,t} = G(\bar{\omega}_{e,t}) \chi_e R_{KK,t} \frac{\kappa_{e,t-1}}{\kappa_{e,t-1} - 1} \quad (10)$$

where

$$G_e(\bar{\omega}) = (1 - F_e(\bar{\omega})) \bar{\omega} + (1 - \mu_e) \int_0^{\bar{\omega}} \omega dF_e(\omega).$$

The commercial lenders operate in perfect competition. They receive one-period loans from the retail lending branches who pay a gross nominal interest rate  $R_{LE,t}$ , to finance their extension of loan to entrepreneurs.

The loan officers have no other source of funds so that the volume of the loans they provide to the entrepreneurs equals the volume of funding they receive. Loan officers seek to maximise their discount intertemporal flow of income so that the first order condition of their decision problem gives

$$\mathbb{E}_t \left[ \Xi_{t,t+1} \frac{(\tilde{R}_{LE,t+1} - R_{LE,t})}{\pi_{t+1}} \right] = 0 \quad (11)$$

where  $\Xi_{t,t+1}$  is the period  $t$  stochastic discount factor of the households for nominal income streams at period  $t + 1$ .

We assume that entrepreneurs are myopic and the end-of-period  $t$  contracting problem for entrepreneurs consists in maximising the next period return on net worth for the lending rate and leverage:

$$\max_{\{R_{LLE,t}, \kappa_{e,t}\}} \mathbb{E}_t [(1 - \chi_e \Gamma_e(\bar{\omega}_{e,t+1})) R_{KK,t+1} \kappa_{e,t}]$$

subject to the participation constraint of the lender (11), the equation (8) for the default threshold  $\bar{\omega}_{e,t+1}$ , and where

$$\Gamma_e(\bar{\omega}) = (1 - F_e(\bar{\omega}))\bar{\omega} + \int_0^{\bar{\omega}} \omega dF_e(\omega).$$

After some manipulations, the first order conditions for the lending rate and the leverage lead to

$$\mathbb{E}_t [(1 - \chi_e \Gamma_e(\bar{\omega}_{e,t+1})) R_{KK,t+1} \kappa_{e,t}] = \frac{\mathbb{E}_t [\chi_e \Gamma'_e(\bar{\omega}_{e,t+1})]}{\mathbb{E}_t [\Xi_{t,t+1} G'_e(\bar{\omega}_{e,t+1})]} \mathbb{E}_t [\Xi_{t,t+1}] R_{LE,t} \quad (12)$$

where

$$\Gamma'_e(\bar{\omega}) = (1 - F_e(\bar{\omega})) \text{ and } G'_e(\bar{\omega}) = (1 - F_e(\bar{\omega})) - \mu_e \bar{\omega} dF_e(\bar{\omega}).$$

As anticipated at the beginning of the section, the solution to the problem shows that all entrepreneurs choose the same leverage and lending rate. Moreover, the features of the contracting problem imply that the *ex post* return to the lender  $\tilde{R}_{LE,t}$  will differ from the *ex ante* return  $R_{LE,t-1}$ . Log-linearising equation (12) and the participation constraint (11), one can show that innovations in the ex post return are notably driven by innovations in  $R_{KK,t}$ .

The loan contract introduced in this section is different from the one of Bernanke et al. (1999) in two respects: first, we impose that the contractual lending rate is predetermined and second, we assume limited seizability of entrepreneurs' assets in case of default. In Bernanke et al. (1999), it is the return to the lender that is predetermined while the contractual lending rate is state contingent. This implies that from period  $t$  to  $t + 1$ , the realisation of aggregate shocks has no impact on the lender's balance sheet. The assumption of predetermined contractual lending rates relaxes this property, as it allows for innovations on the lender's return. Besides, the restrictions imposed on the contracting problem imply that it is not optimal in the sense of Carlstrom et al. (2013) and Carlstrom et al. (2014).

Finally, aggregating across entrepreneurs, a fraction  $\zeta_e$  continues operating into the next period while the rest exits from the industry. The new entrepreneurs are endowed with starting net worth, proportional to the assets of the old entrepreneurs. Accordingly, the aggregate dynamics of entrepreneurs' net worth is given by

$$NW_{E,t} = \zeta_e (1 - \chi_e \Gamma_e(\bar{\omega}_{e,t})) \frac{R_{KK,t}}{\pi_{t-1}} \kappa_{e,t-1} NW_{E,t-1} / \gamma + \Psi_{E,t} \quad (13)$$

In the estimation, we also introduce a shock on the net worth of entrepreneurs which can be rationalised either as time-varying transfers to new entrepreneurs  $\Psi_{E,t}$ , or as a multiplicative shock on the survival probability of entrepreneurs,  $\varepsilon_t^{\zeta_e}$ .

### 3.4 The Banking sector

The banking sector is owned by the households and is segmented in various parts. First, bankers collect household deposits and provide funds to the retail lending branches. As in Gertler and Karadi (2011) and Gertler and Karadi (2013), bankers can divert funds and depositors enforce on them an incentive constraint which forces the bankers to hoard a sufficient level of net worth. This creates a financing cost wedge related to bank capital frictions. Second, retail lending branches receive funding from the bankers and allocate it to the loan officers. In the retail segment, a second wedge results from banks operating under monopolistic competition and facing nominal rigidity in their interest rate setting. In the third segment of the banking sector, loan officers extend loan contracts to entrepreneurs as explained previously which implies a third financing cost wedge related to credit risk compensation.

#### 3.4.1 Retail lending branches

A continuum of retail lending branches indexed by  $j$ , provide differentiated loans to loan officers. The total financing needs of loan officers follow a CES aggregation of differentiated loans  $L_{E,t} = \left[ \int_0^1 L_{E,t}(j)^{\frac{1}{\mu_E^R}} dj \right]^{\mu_E^R}$ . Differentiated loans are imperfect substitutes with elasticity of substitution  $\frac{\mu_E^R}{\mu_E^R - 1} > 1$ . The corresponding average return on loans is  $R_{LE} = \left[ \int_0^1 R_{LE}(j)^{\frac{1}{1-\mu_E^R}} dj \right]^{1-\mu_E^R}$ .

Retail lending branches are monopolistic competitors which levy funds from the *bankers* and set gross nominal interest rates on a staggered basis *à la* Calvo (1983), facing each period a constant probability  $1 - \xi_E^R$  of being able to re-optimize. If a retail lending branch cannot re-optimize its interest rate, the interest rate is left at its previous period level:

$$R_{LE,t}(j) = R_{LE,t-1}(j)$$

The retail lending branch  $j$  chooses  $\hat{R}_{LE,t}(j)$  to maximize its intertemporal profit

$$\mathbb{E}_t \left[ \sum_{k=0}^{\infty} (\beta \gamma^{-\sigma_c} \xi_E^R)^k \frac{\Lambda_{t+k}}{\Lambda_t} \left( \hat{R}_{LE,t}(j) L_{E,t+k}(j) - R_{BLE,t+k}(j) L_{E,t+k}(j) \right) \right]$$

where the demand from the loan officers is given by

$$L_{E,t+k}(j) = \left( \frac{\hat{R}_{LE,t}(j)}{R_{LE,t}} \right)^{-\frac{\mu_E^R}{\mu_E^R - 1}} \left( \frac{R_{LE,t}}{R_{LE,t+k}} \right)^{-\frac{\mu_E^R}{\mu_E^R - 1}} L_{LE,t+k}$$

and  $R_{BLE,t}$  is the gross funding rate on the loans from the *bankers*.

The first order condition can be rearranged into the following recursive formulation which determines the aggregate gross funding rate for the loan officers  $R_{LE}$  :

$$\mathcal{Z}_{E1,t}^R = \frac{R_{BLE,t}}{R_{LE,t}} \Lambda_t L_{E,t} + \xi_E^R \beta \gamma^{-\sigma_c} \mathbb{E}_t \left[ \left( \frac{R_{LE,t+1}}{R_{LE,t}} \right)^{\frac{\mu_E^R}{\mu_E^R - 1} + 1} \mathcal{Z}_{E1,t+1}^R \right] \quad (14)$$

$$\mathcal{Z}_{E2,t}^R = \varepsilon_{E,t}^R \Lambda_t L_{E,t} + \xi_E^R \beta \gamma^{-\sigma_c} \mathbb{E}_t \left[ \left( \frac{R_{LE,t+1}}{R_{LE,t}} \right)^{\frac{\mu_E^R}{\mu_E^R - 1}} \mathcal{Z}_{E2,t+1}^R \right] \quad (15)$$

$$1 = \xi_E^R \left( \frac{R_{LE,t}}{R_{LE,t-1}} \right)^{\frac{1}{\mu_E^R - 1}} + (1 - \xi_E^R) \left( \mu_E^R \frac{\mathcal{Z}_{E1,t}^R}{\mathcal{Z}_{E2,t}^R} \right)^{\frac{1}{1 - \mu_E^R}}. \quad (16)$$

Lending rate dispersion indexes are then given by

$$\Delta_{E,t}^R = (1 - \xi_E^R) \left( \mu_E^R \frac{\mathcal{Z}_{E1,t}^R}{\mathcal{Z}_{E2,t}^R} \right)^{-\frac{\mu_E^R}{\mu_E^R - 1}} + \xi_E^R \Delta_{E,t-1}^R \left( \frac{R_{LE,t}}{R_{LE,t-1}} \right)^{\frac{\mu_E^R}{\mu_E^R - 1}} \quad (17)$$

The staggered lending rate setting acts in the model as maturity transformation in banking activity and leads to imperfect pass-through of market interest rates on bank lending rates.

### 3.4.2 Bankers

As explained before, every period, a fraction  $f(1 - e)$  of the representative household's members are bankers. Like entrepreneurs, bankers face a probability  $\zeta_b$  of staying banker over next period and a probability  $(1 - \zeta_b)$  of becoming a worker again. When a banker exits, accumulated earnings are transferred to the respective household while newly entering bankers receive initial funds from their household. Overall, households transfer a real amount  $\Psi_{B,t}$  to new bankers for each period  $t$ . As shown later in this section, bankers' decisions are identical so we will expose the decision problem for a representative banker.

Bankers operate in competitive markets providing loans to retail lending branches,  $L_{BE,t}$ , and purchasing government securities,  $B_{B,t}$ , at price  $Q_{B,t}$ . To finance their lending activity, Bankers receive deposits,  $D_t$ , from households, with a gross interest rate,  $R_{D,t}$ , and accumulate net worth,  $NW_{B,t}$ . Their balance identity, in real terms, reads

$$L_{BE,t} + Q_{B,t} B_{B,t} = D_t + NW_{B,t}. \quad (18)$$

The accumulation of the bankers' net worth from period  $t$  to period  $t + 1$  results from the gross interest received from the loans to the retail lending bank, the gross return on government bond holdings,  $R_{G,t+1}$ , the lump-sum share of profits (and losses) coming from retail lending and loan officers activity,  $\Pi_{B,t+1}^R$ , per unit of each banker's net worth, minus the gross interest paid on deposits:

$$NW_{B,t+1} \pi_{t+1} = \frac{R_{N,t+1}^B}{\pi_{t+1}} NW_{B,t} / \gamma.$$

with

$$R_{N,t+1}^B \equiv (R_{BLE,t} - R_{D,t}) \kappa_{B,t}^l + (R_{G,t+1} - R_{D,t}) \kappa_{B,t}^g + R_{D,t} + \Pi_{B,t+1}^R \quad (19)$$

$$\kappa_{B,t}^l \equiv \frac{L_{BE,t}}{NW_{B,t}} \text{ and } \kappa_{B,t}^g \equiv \frac{Q_{B,t} B_{B,t}}{NW_{B,t}} \quad (20)$$

Iterating this equation backward implies

$$NW_{B,t+1} = \tilde{R}_{N,t+1-s,t+1}^B NW_{B,t+1-s} / \gamma^s \quad (21)$$

where  $\tilde{R}_{N,t+1-s,t+1}^B = \prod_{i=0}^s \left\{ \frac{R_{N,t+1-i}^B}{\pi_{t+1-i}} \right\}$  and  $\tilde{R}_{N,t+1-s,t+1-s}^B = 1$ . The bankers' objective is to maximise their terminal net worth when exiting the industry, which occurs with probability  $(1 - \zeta_b)$  each period. The value function for each banker is therefore given by

$$\mathcal{V}_{B,t} = (1 - \zeta_b) \sum_{k=0}^{\infty} (\zeta_b)^k \Xi_{t,t+k+1} NW_{B,t+k+1}$$

Using (21), the value function can be written recursively as follows

$$\mathcal{V}_{B,t} = (1 - \zeta_b) NW_{B,t} (\mathcal{X}_{B,t} - 1)$$

with

$$\mathcal{X}_{B,t} = 1 + \zeta_b \mathbb{E}_t \left[ \Xi_{t,t+1} \frac{R_{N,t+1}^B}{\pi_{t+1}} \mathcal{X}_{B,t+1} \right].$$

As in Gertler and Karadi (2013), bankers can divert a fraction of their assets and transfer them without costs to the households. In this case, the depositors force the default on the intermediary and will only recover the remaining fraction of the asset. The corresponding incentive compatibility constraint is

$$\begin{aligned} \mathcal{V}_{B,t} &\geq \lambda_b (L_{BE,t} + \delta_{b,t} Q_{B,t} B_{B,t}) \\ &\geq \lambda_b \left( \kappa_{B,t}^l + \delta_{b,t} \kappa_{B,t}^g \right) NW_{B,t}. \end{aligned} \quad (22)$$

The diversion rate for private loans is  $\lambda_b$  and  $\lambda_b \delta_{b,t}$  for government securities. We allow  $\delta_{b,t}$  to be time-varying. Under the parameter values considered thereafter, the constraints are assumed to always bind in the vicinity of the steady state.

Given their initial net worth, the end-of-period  $t$  contracting problem for bankers consists in maximising  $\mathcal{V}_{B,t}$  for the exposures to private sector loans  $\kappa_{B,t}^l$  and government securities  $\kappa_{B,t}^g$  subject to the incentive constraint (22) :

$$\mathcal{V}_{B,t} = \max_{\{\kappa_{B,t}^l, \kappa_{B,t}^g\}} \left\{ \zeta_b \tilde{\mathcal{X}}_{B,t} NW_{B,t} \right\} \quad (23)$$

where we denoted  $\tilde{\mathcal{X}}_{B,t} \equiv (\mathcal{X}_{B,t} - 1) \frac{(1 - \zeta_b)}{\zeta_b}$  and  $\tilde{\mathcal{X}}_{B,t}$  follows



$$\tilde{\mathcal{X}}_{B,t} = \mathbb{E}_t \left[ \Xi_{t,t+1} \frac{R_{N,t+1}^B}{\pi_{t+1}} \left( \zeta_b \tilde{\mathcal{X}}_{B,t+1} + (1 - \zeta_b) \right) \right]. \quad (24)$$

Note that the stream of transfers  $\Pi_{B,t+1+s}^R$  is considered exogenous by bankers in their decision problem which implies that  $\frac{\partial \Pi_{B,t+1+s}^R}{\partial \kappa_{B,t}^l} = 0$ . The first order conditions for this problem can then be formulated as

$$\begin{aligned} \mathbb{E}_t \left[ \Xi_{t,t+1} \frac{\partial R_{N,t+1}^B}{\partial \kappa_{B,t}^l} \left( \zeta_b \tilde{\mathcal{X}}_{B,t+1} + (1 - \zeta_b) \right) / \pi_{t+1} \right] &= \mu_t \lambda_b \\ \mathbb{E}_t \left[ \Xi_{t,t+1} \frac{\partial R_{N,t+1}^B}{\partial \kappa_{B,t}^g} \left( \zeta_b \tilde{\mathcal{X}}_{B,t+1} + (1 - \zeta_b) \right) / \pi_{t+1} \right] &= \mu_t \lambda_b \delta_{b,t} \end{aligned}$$

where  $\mu_t$  is the lagrange multiplier related to the incentive constraint.

The first order conditions force a proportionality relationship between the excess return on government bonds and the excess return on private lending:

$$\begin{aligned} &\mathbb{E}_t \left[ \Xi_{t,t+1} \left( \zeta_b \tilde{\mathcal{X}}_{B,t+1} + (1 - \zeta_b) \right) \frac{(R_{G,t+1} - R_{D,t})}{\pi_{t+1}} \right] \\ &= \delta_{b,t} \mathbb{E}_t \left[ \Xi_{t,t+1} \left( \zeta_b \tilde{\mathcal{X}}_{B,t+1} + (1 - \zeta_b) \right) \frac{(R_{BLE,t} - R_{D,t})}{\pi_{t+1}} \right] \end{aligned} \quad (25)$$

Aggregating across bankers, a fraction  $\zeta_b$  continues operating into the next period while the rest exits from the industry. The new bankers are endowed with starting net worth, proportional to the assets of the old bankers. Accordingly, the aggregate dynamics of bankers' net worth is given by

$$NW_{B,t} = \zeta_b \frac{R_{N,t}^B}{\pi_t} NW_{B,t-1} / \gamma + \Psi_{B,t}. \quad (26)$$

### 3.5 Capital producers

Using investment goods, a segment of perfectly competitive firms, owned by households, produce a stock of fixed capital. At the beginning of period  $t$ , those firms buy back the depreciated capital stocks  $(1 - \delta)K_{t-1}$  at real prices (in terms of consumption goods)  $Q_t$ . Then they augment the various stocks using distributed goods and facing adjustment costs. The augmented stocks are sold back to entrepreneurs at the end of the period at the same prices. The decision problem of capital stock producers is given by

$$\max_{\{K_t, I_t\}} \mathbb{E}_t \sum_{k=0}^{\infty} \Xi_{t,t+k} \left\{ Q_{t+k} (K_{t+k} - (1 - \delta)K_{t+k-1} / \gamma) - I_{t+k} \right\}$$

subject to the constraints

$$K_t = (1 - \delta)K_{t-1} / \gamma + \left[ 1 - S \left( \gamma \frac{I_t \varepsilon_t^I}{I_{t-1}} \right) \right] I_t$$

where  $S$  is a non-negative adjustment cost function formulated in terms of the gross rate of change in investment and  $\varepsilon_t^I$  is an efficiency shock to the technology of fixed capital accumulation. The functional form adopted is  $S(x) = \phi/2 (x - \gamma)^2$ .

The resulting first order conditions read

$$Q_t \left[ 1 - S \left( \frac{I_t \varepsilon_t^I}{I_{t-1}} \right) - \gamma \frac{I_t \varepsilon_t^I}{I_{t-1}} S' \left( \frac{I_t \varepsilon_t^I}{I_{t-1}} \right) \right] + \beta \gamma^{-\sigma_e} \mathbb{E}_t \left[ Q_{t+1} \frac{\Lambda_{t+1}}{\Lambda_t} \left( \gamma \frac{I_{t+1} \varepsilon_{t+1}^I}{I_t} \right)^2 S' \left( \frac{I_{t+1} \varepsilon_{t+1}^I}{I_t} \right) \right] = 1 \quad (27)$$

### 3.6 Final and intermediate goods producers

Final producers are perfectly competitive firms producing an aggregate final good  $Y_t$ , expressed in effective terms, that may be used for consumption and investment. This production is obtained using a continuum of differentiated intermediate goods  $Y_t(z)$ ,  $z \in [0, 1]$  (also expressed in effective terms) with the Kimball (1995) technology. Here again, the Kimball aggregator is defined by

$$\int_0^1 G \left( \frac{Y_t(z)}{Y_t}; \theta_p, \psi \right) dz = 1$$

with

$$G \left( \frac{Y_t(z)}{Y_t} \right) = \frac{\theta_p}{(\theta_p(1+\psi) - 1)} \left[ (1+\psi) \frac{Y_t(z)}{Y_t} - \psi \right]^{\frac{\theta_p(1+\psi)-1}{\theta_p(1+\psi)}} - \left[ \frac{\theta_p}{(\theta_p(1+\psi) - 1)} - 1 \right].$$

The representative final good producer maximizes profits  $P_t Y_t - \int_0^1 P_t(z) Y_t(z) dz$  subject to the production function, taking as given the final good price  $P_t$  and the prices of all intermediate goods.

In the intermediate goods sector, firms  $z \in [0, 1]$  are monopolistic competitors and produce differentiated products by using a common Cobb-Douglas technology:

$$Y_t(z) = \varepsilon_t^a (u_t K_{t-1}(z) / \gamma)^\alpha [N^D(z)]^{1-\alpha} - \Omega \quad (28)$$

where  $\varepsilon_t^a$  is an exogenous productivity shock,  $\Omega > 0$  is a fixed cost and  $\gamma$  is the trend technological growth rate. A firm  $z$  hires its capital,  $\tilde{K}_t(z) = u_t K_{t-1}(z)$ , and labor,  $N_t^D(z)$ , on a competitive market by minimizing its production cost. Due to our assumptions on the labor market and the rental rate of capital, the real marginal cost is identical across producers. We introduce a time varying tax on firm's revenue is affected by an i.i.d shock defined by  $1 - \tau_{p,t} = (1 - \tau_p^*) \varepsilon_t^p$ .

In each period, a firm  $z$  faces a constant (across time and firms) probability  $1 - \alpha_p$  of being able to re-optimize its nominal price, say  $P_t^*(z)$ . If a firm cannot re-optimize its price, the

nominal price evolves according to the rule  $P_t(z) = \pi_{t-1}^{\xi_p} [\pi^*]^{(1-\xi_p)} P_{t-1}(z)$ , i.e. the nominal price is indexed on past inflation and steady state inflation. In our model, all firms that can re-optimize their price at time  $t$  choose the same level, denoted  $p_t^*$  in real terms.

The first order condition associated with the maximization of the intertemporal profit can be expressed in a recursive form as shown in the appendix.

### 3.7 Government

Public expenditures  $G^*$ , expressed in real terms, are subject to random shocks  $\varepsilon_t^g$ . The government finances public spending with labour tax, product tax and lump-sum transfers so that the government debt  $Q_{B,t}B_G$ , expressed in real effective terms, accumulates according to

$$Q_{B,t}B_{G,t} = \frac{R_{G,t}}{\pi_t} Q_{B,t-1}B_{G,t-1} / \gamma + G^* \varepsilon_t^g - \tau_{w,t} w_t L_t - \tau_{p,t} Y_t - T_t. \quad (29)$$

In the empirical analysis, we neglect the dynamics of public debt and assume that lump-sum taxes  $T_t$  are adjusted to ensure that

$$B_{G,t} = \overline{B_G}, \quad \forall t > 0.$$

In order to introduce long-term sovereign debt, we assume that government securities are perpetuities which pay geometrically-decaying coupons ( $c_g$  the first period,  $(1 - \tau_g)c_g$  the second one,  $(1 - \tau_g)^2 c_g$  the third one, etc...). The nominal return on sovereign bond holding from period  $t$  to period  $t + 1$  is therefore

$$R_{G,t+1} = \frac{c_g + (1 - \tau_g)Q_{B,t+1}}{Q_{B,t}}.$$

The standard monetary policy instrument is the deposit interest rate  $R_{D,t}$ . The monetary authority follows an interest rate feedback rule which incorporates terms on lagged inflation, lagged output gap and its first difference as in Smets and Wouters (2007). The output gap is defined as the log-difference between actual and flexible-price output. The reaction function also incorporates a non-systematic component  $\varepsilon_t^r$ .

Written in deviation from the steady state, the interest rule used in the estimation has the form:

$$\hat{R}_{D,t} = \rho \hat{R}_{D,t-1} + (1 - \rho) [r_\pi \hat{\pi}_{t-1} + r_y \hat{y}_{t-1}] + r_{\Delta y} \Delta \hat{y}_t + \log(\varepsilon_t^r) \quad (30)$$

where a hat over a variable denotes log-deviation of that variable from its deterministic steady-state level.

Finally, we assume as in Gertler and Karadi (2013) that the monetary authority can manage a bond portfolio  $B_{CB,t}$ . Central bank asset purchases operate in our model as a negative shock on the fixed supply of bonds.

### 3.8 Market clearing conditions

Market clearing condition on goods market is given by:

$$Y_t = C_t + I_t + G^* \varepsilon_t^g + \Psi(u_t) K_{t-1} / \gamma + \mu_e \int_0^{\bar{\omega}} \omega dF_e(\omega) K_{t-1} / \gamma \quad (31)$$

$$\Delta_{pk,t} Y_t = \varepsilon_t^a (u_t K_{t-1} / \gamma)^\alpha (N_t^D)^{1-\alpha} - \Omega \quad (32)$$

with  $\Delta_{pk,t}$  is a price dispersion index whose dynamics is presented in the appendix.

Equilibrium in the labor market implies that

$$\Delta_{wk,t} N_t^D = N_t^S \quad (33)$$

with  $N_t^D = \int_0^1 N_t^D(z) dz$  and  $N_t^S = \int_0^1 N_t^S(h) dh$ . The dynamics of the wage dispersion index  $\Delta_{wk,t}$  is also described in the appendix.

On the credit market, due to nominal rigidity in the setting of interest rate by retail banking branches, the following conditions holds

$$L_{BE,t} = \Delta_{E,t}^R L_{E,t} \quad (34)$$

where  $\Delta_{E,t}^R = \int_0^1 \left( \frac{R_{E,t}(j)}{R_{E,t}} \right)^{-\frac{\mu_E^R}{\mu_E^R - 1}} dj$  is the dispersion index among retail bank interest rates.

Moreover, in equilibrium the lump-sum transfer to bankers per unit of net worth from retail lending and loan officer profits and losses is given by

$$\Pi_{B,t+1}^R = \left( \tilde{R}_{LE,t+1} - R_{BLE,t} \right) \kappa_{B,t}^l. \quad (35)$$

We can now rewrite the recursive formulation of the bankers value function  $\mathcal{V}_{B,t}$  from equation (24) using bankers incentive constraint (22) and the first order condition (25). This gives a relationship between bank leverage and intermediation spreads:

$$\lambda_b \tilde{\kappa}_{B,t} / \zeta_b = \mathbb{E}_t \left[ \Xi_{t,t+1} \left( \frac{R_{BLE,t} - R_{D,t}}{\pi_{t+1}} \tilde{\kappa}_{B,t} + \frac{\tilde{R}_{LE,t+1} - R_{BLE,t}}{\pi_{t+1}} \kappa_{B,t}^l + R_{D,t} \right) (\lambda_b \tilde{\kappa}_{B,t+1} + (1 - \zeta_b)) \right] \quad (36)$$

where we denote  $\tilde{\kappa}_{B,t} \equiv \kappa_{B,t}^l + \delta_{b,t} \kappa_{B,t}^g$ .

Finally, on the government bond market, the fixed supply is distributed across holdings by households, bankers and the central bank:

$$B_{H,t} + B_{B,t} + B_{CB,t} = \overline{B}_G.$$

## 4 DSGE estimation

In this section, we present the DSGE model estimation, which follows Smets and Wouters (2007). The main purpose of the empirical exercise is not to conduct an exhaustive review of the structural determinants of the euro area business cycle and evaluate the statistical performance of the model. Instead, by making use of the insights derived from granular bank level optimisation approach outlined in Section 2 we aim at narrowing down the plausible ranges for the deep parameters of the model, notably those to which the APP transmission would be most sensitive, and bring a satisfactory level of data consistency for the macroeconomic multipliers used in the quantitative exercises of the last section.

The model is estimated on euro area data using Bayesian likelihood methods. We consider 10 key macroeconomic quarterly time series from 1995q1 to 2014q2: output, consumption, fixed investment, hours worked, real wages, the CPI inflation rate, the three-month short-term interest rate, bank loans, bank lending spreads and the (weighted) 10-year euro area sovereign spread. The data are *not* filtered before estimation with the exception of loans which are linearly detrended.

The exogenous shocks can be divided in four categories<sup>12</sup>:

1. Efficient shocks: AR(1) shocks on technology  $\epsilon_t^a$ , investment  $\epsilon_t^I$ , public expenditures  $\epsilon_t^g$  and consumption preferences  $\epsilon_t^b$ .
2. Inefficient shocks: ARMA(1,1) shocks on price markups  $\epsilon_t^p$ , and AR(1) on wage markups  $\epsilon_t^w$ .
3. Financial shocks: AR(1) shock on entrepreneurs idiosyncratic risk  $\epsilon_t^{\sigma^e}$ , on entrepreneurs net worth accumulation  $\epsilon_t^{\zeta^e}$ , as well as on banker's diversion rate related to sovereign bond holdings  $\epsilon_t^{\delta^b}$ .
4. Policy shocks: AR(1) shock on short term interest rates  $\epsilon_t^r$ .

We limit the number of shocks to be equal to the number of observed variables. As in Smets and Wouters (2007), we introduce a correlation between the government spending shock and the productivity shock,  $\rho_{a,g}$ .

### 4.1 Data

Data for GDP, consumption, investment, employment, wages and consumption-deflator are based on Fagan et al. (2001) and Eurostat. Employment numbers replace hours. Consequently,

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<sup>12</sup>All the AR(1) processes are written as:  $\log(\varepsilon_t^x) = \rho_x \log(\varepsilon_{t-1}^x) + \epsilon_t^x$  where  $\epsilon_t^x \sim \mathcal{N}(0, \sigma_{\varepsilon^x})$ . ARMA(1,1) are of the form  $\log(\varepsilon_t^x) = \rho_x \log(\varepsilon_{t-1}^x) - \eta_x \varepsilon_{t-1}^x + \epsilon_t^x$ . All shock processes  $\varepsilon_t^x$  are equal to one in the steady state.

as in Smets and Wouters (2005), hours are linked to the number of people employed  $e_t^*$  with the following dynamics:

$$e_t^* = \beta \mathbb{E}_t e_{t+1}^* + \frac{(1 - \beta \lambda_e)(1 - \lambda_e)}{\lambda_e} (l_t^* - e_t^*)$$

The three-month money market rate is the three-month Euribor taken from the ECB website and we use backdated series for the period prior to 1999 based on national data sources. Data on MFI loans are taken from the ECB website. Data prior to September 1997 have been backdated based on national sources. Meanwhile, data on retail bank loan and deposit rates are based on official ECB statistics from January 2003 onwards and on ECB internal estimates based on national sources in the period before. The lending rates refer to new business rates. For the period prior to January 2003 the euro area aggregate series have been weighted using corresponding loan volumes (outstanding amounts) by country.

For the estimation, the data are transformed in the following way. We take the quarterly growth rate of GDP, consumption, investment and loans, all expressed in real terms and divided by working age population. The employment variable is also divided by working age population. Real wages are measured with respect to the consumption deflator. Interest rates and spreads are measured quarterly. With the exception of loan growth and employment rate for which specific trend developments are not pinned down by the model, transformed data are not demeaned as the model features non-zero steady state values for such variables. A set of parameters are therefore estimated to ensure enough degrees of freedom to account for the mean values of the observed variables. Trend productivity growth  $\gamma$  captures the common mean of GDP, consumption, investment and real wage growth;  $\bar{L}$  is a level shift that we allow between the observed detrended employment rate and the model-consistent one;  $\bar{\pi}$  is the steady state inflation rate which controls for the CPI inflation rate mean; and we also estimate the preference rate  $r_\beta = 100(1/\beta - 1)$  which, combined with  $\bar{\pi}$  and  $\gamma$ , pins down the mean of the nominal interest rate. Regarding spreads, the bank lending spread mean is related to the monopolistic markup  $r_\mu = 100 \left( \frac{R_{LE} - R_{BLE}}{\pi} \right)$  while the sovereign spread mean depends on the bankers intermediation margin  $\frac{R_{BLE} - R_D}{\pi}$  and the diversion rate  $\delta_b$ .

We will consider two possible sets of credit data in our estimation and adjust the model calibration accordingly. Loans and lending rates can be either for the non-financial corporate sector or for the non-financial private sector (including households). Indeed, our model does not distinguish the financing of housing from non-housing productive capital stock, or business investment from residential investment. Therefore we will estimate the model under both configurations, which implies different calibration for the parameter sets of the entrepreneur decision problem.

## 4.2 Calibrated parameters and prior distributions

Like in Smets and Wouters (2007), some parameters are treated as fixed in the estimation. The depreciation rate of the capital stock  $\delta$  is set at 0.025 and the share of government spending in output at 18%. The steady state labor market markup is fixed at 1.5 and we chose curvature parameters of the Kimball aggregators of 10.

We fix in the steady state calibration the ratio of banks' holdings of government bonds to their loan book,  $\alpha_B = \frac{\kappa_B^g}{\kappa_B^l}$ , at 12%, in line with aggregate BSI statistics from the ECB. The total outstanding amount of sovereign debt in the steady state is assumed at 60% of annual GDP.

In order to calibrate and choose the prior distribution for the parameters in the financial block of the model, the steady state level of lending rate spreads  $\frac{R_{LLE}-R_D}{\pi}$  can be decomposed in three financial wedges.

- First the *credit risk compensation* corresponds to the spread between the lending rate applied by loan officers and the return on the overall loan portfolio for the retail bankers:  $r_{risk} = 100 \frac{R_{LLE}-R_{LE}}{\pi}$ .
- Second, the lending rate *competitive margin* is related to the retail banking monopolistic segment which applies a markup on financing rate provided by the bankers:  $r_{\mu} = 100 \frac{R_{LE}-R_{BLE}}{\pi}$ .
- Finally, the *bank capital channel* spread results from the decision problems of bankers and requires in equilibrium a higher return on private sector intermediation than on deposits,  $r_B = 100 \frac{R_{BLE}-R_D}{\pi}$ .

Starting with the entrepreneurs, we target default frequencies for firms of 0.7% and a credit risk compensation on lending rate of 50 bps (in annual terms) which broadly corresponds to one third of the sample mean of the lending spreads. The external finance premium  $100 \left( \frac{R_{KK}}{R_{LE}} - 1 \right)$  is set at 200 bps (in annual terms). We also aim at a matching a credit to GDP ratio consistent with the loan data under consideration. Four parameters are assigned to those targets: the monitoring costs  $\mu_e$ , the standard deviation of the idiosyncratic shock  $\sigma_e$ , the limited seizability parameter  $\chi_e$  and the entrepreneurs survival probability  $\zeta_e$ .

Then, the competitive margin  $r_{\mu}$  is a free parameter in the estimation and its prior distribution has a mean of 40 bps (in annual terms). We also estimate the Calvo lottery parameter related to retail lending rate setting,  $\xi_E^R$ , for which we choose a relatively uninformative prior distribution.

Let us now consider the banker's parameter space. In the steady state, equation (36) which links bank leverage to intermediation spreads is given by

$$\lambda_b \tilde{\kappa}_B / \zeta_b = \beta \gamma^{-\sigma_C} \left( \frac{r_B}{100} \tilde{\kappa}_B + \frac{r_{\mu}}{100} \kappa_B^l + R_D \right) (\lambda_b \tilde{\kappa}_B + (1 - \zeta_b)).$$



Assuming a fixed ratio of government bonds to loans in bank balance sheet,  $\alpha_B$ , then this relation can be re-written as

$$\beta\gamma^{-\sigma_C} \left( \frac{r_B}{100} + \frac{r_\mu}{100(1 + \delta_b\alpha_B)} \right) = \frac{\lambda_b\tilde{\kappa}_B - \zeta_b}{\tilde{\kappa}_B \left( \zeta_b + \lambda_b\tilde{\kappa}_B \frac{\zeta_b}{(1-\zeta_b)} \right)}. \quad (37)$$

For given values of  $\lambda_b$  and  $\zeta_b$ , intermediation spreads are a non-monotonic function of bank leverage,  $f_{\lambda_b, \zeta_b}(\tilde{\kappa}_B)$ . Moreover, steady state levels for the intermediation spreads and bank leverage can be consistent with multiple combinations of  $\lambda_b$  and  $\zeta_b$ . Therefore, in order to reduce the parameter space in the estimation and bring back monotonicity in this steady state relationship, we restrict the steady state allocations for values of  $\tilde{\kappa}_B^*(\lambda_b, \zeta_b)$  which maximize  $f_{\lambda_b, \zeta_b}(\tilde{\kappa}_B)$ . This is the case for

$$\lambda_b\tilde{\kappa}_B^* = \zeta_b + \sqrt{\zeta_b} \quad (38)$$

implying intermediation spreads of

$$\beta\gamma^{-\sigma_C} \left( \frac{r_B}{100} + \frac{r_\mu}{100(1 + \delta_b\alpha_B)} \right)^* = \frac{\zeta_b(1 - \zeta_b)}{\tilde{\kappa}_B(\zeta_b + \sqrt{\zeta_b})}. \quad (39)$$

Under such constraints, the intermediation spread  $\frac{r_B}{100} + \frac{r_\mu}{100(1+\delta_b\alpha_B)}$  is a decreasing function of bank leverage  $\tilde{\kappa}_B$  which depends only on  $\zeta_b$ . Moreover, bank leverage and the survival probability of bankers determine uniquely the diversion rate parameter  $\lambda_b$ . Then, in our calibration strategy, we set first  $\tilde{\kappa}_B$  at 8 (i.e. “weighted” capital ratio of 12.5%). Then we estimate  $\zeta_b$ , choosing a prior mean which implies a bank capital channel spread  $r_B$  of around 50 bps (in annual terms). This is consistent with a prior value for  $\lambda_b$  of around 0.3. Finally, the steady state value of initial transfers to new bankers,  $\Psi_B$ , is endogenously set so that the bank net worth accumulation holds (see equation (26)).

From equation (25), we see that the steady state level of sovereign spread is linked to  $r_B$  by

$$\frac{(R_G - R_D)}{\pi} = \frac{r_B}{100\delta_b}. \quad (40)$$

We estimate  $\delta_b$  using a prior distribution of mean 1. We set the geometric-decay of the perpetual coupons on sovereign bond  $\tau_g$  so that the duration of the securities is 10 years. The initial coupon level is adjusted to ensure that the steady state sovereign bond price  $Q_B$  equals 1.

Regarding households’ portfolio decisions, the adjustment cost parameter on the holding of sovereign securities,  $\chi_H$ , is left free in the estimation, choosing a prior distribution of mean 0.1. For the household first order condition on sovereign bond holdings to be consistent with the steady state sovereign spread and the share of bank holding of sovereign bonds, we let  $\bar{B}_H$  clear the steady state relationship associated with equation (4).

Regarding the other structural parameters, the prior distributions are similar to Smets and Wouters (2007) and are reported in Tables 2 and 3. The main differences relate to the choice of uniform priors for the standard deviations of the exogenous shocks.

### 4.3 Posterior parameter distributions

As mentioned previously, we estimated two versions of the model depending on the observed credit variables. In the first estimation, loan and lending rate data are for the non-financial corporate sector while the second estimation considers loans and rates for both non-financial corporations and households. We then examine the robustness of the key parameter estimates to alternative set of credit data and evaluate the sensitivity of our results to the calibration of credit demand frictions.

The posterior distributions, characterised by the mean and the 80% density intervals, are reported in Tables 2 and 3 for the estimation based both on non-financial corporation credit variables and on total economy credit variables. For deep parameters related to preferences, technology as well as real and nominal frictions, the posteriors are broadly similar between the two estimations. Some exceptions are noticeable though: in the estimation based on total economy credit variables, the investment adjustment cost  $\phi$  is higher than in the other one; the backward indexation in price and wage setting,  $\xi_p$  and  $\xi_w$ , is more pronounced; and the Taylor-rule coefficient on the level inflation  $r_\pi$  is somewhat lower.

Turning to the set of estimated parameters related to the financial block,  $\xi_E^R$ ,  $r_\mu$ ,  $\zeta_b$ ,  $\delta_b$ ,  $\chi_H$ , the differences are more striking. All these parameters are well-identified in both estimations: posterior distributions are sizeably narrower and shifted compared with the prior distributions (see plots in Figure 10).

First, and as expected, the degree of nominal rigidities in lending rate setting  $\xi_E^R$  is higher for the estimation based on total economy lending rate (incl. mortgages). Indeed, a common observation is that there are differences across retail bank products in terms of the speed and degree with which banks pass-through changes in policy rates facing their borrowers. These differences can be due to the maturity of the interest rate fixation in the loan contract or hinge on the degree of market power the bank has in particular segments. For instance, it can be assumed that large firms are in a better bargaining position vis-à-vis the bank than are its retail customers. Accordingly, it is often found that corporate loan rates adjust to policy rate changes in a speedier and sometimes more complete way than rates on loans to households. Darracq Pariès et al. (2014) summarise existing time-series evidence showing a more sluggish pass-through of monetary policy rate to mortgages than to corporate lending rates.

Second,  $r_\mu$ ,  $\zeta_b$  are lower in the estimation based on total economy credit variables. As those parameters pin-down the steady state financial wedges underlying the lending rate spread, it implies smaller *competitive margin* and wider *bank capital channel* spread.

Third, the posterior distribution of the adjustment cost on household portfolio decisions  $\chi_H$  is low in both estimations (with mean values below the one of the prior distribution, at less than 0.01) and relatively smaller in the case of total economy credit variables. Finally, the bankers diversion rate for sovereign bond holdings  $\delta_b$  features a mean posterior distribution around 1. The posterior mean is somewhat higher for the estimation based on non-financial

corporate credit variables. The calibration values of Gertler and Karadi (2013) for  $\chi_H$  and  $\delta_b$  are far from our posterior mean estimates, and are not even covered by the 80% shortest density interval.

## 5 The credit channel of central bank asset purchase programme

The central bank asset purchases are treated in the model as an exogenous process  $\epsilon_t^{QE}$  affecting the supply of government debt:

$$B_{G,t} = \overline{B_G} \epsilon_t^{QE}$$

where  $\log(\epsilon_t^{QE})$  follows a linear time-series process.

This formulation neglects the liability side of the central bank balance sheet and is similar to Gertler and Karadi (2013). The model economy is cashless and does not consider central bank reserves. Introducing relevant liquidity frictions in the banking system, preference for liquidity and imperfect substitution between government bonds and bank deposits in the household sector would help investigating other channels of the central bank asset purchases. We leave these aspects for further research.

In this section, we run first some sensitivity analysis on the macroeconomic propagation of one-off unexpected increase in  $\epsilon_t^{QE}$  allowing the monetary policy rate to endogenously react in line with the estimated Taylor rule. Thereafter, we aim at a more realistic formulation of the ECB's asset purchase programme and use the insights from the bank-level portfolio optimisation of section 2.3 to re-calibrate the strength of the credit channel in the DSGE model.

### 5.1 Sensitivity analysis on the transmission mechanism of central bank asset purchases in the DSGE model

The two estimated versions of the model form a good basis to investigate the sensitivity of the asset purchase transmission mechanism to selected dimensions of the parameter space.

For the purpose of this subsection, we characterise the asset purchase of the central bank as a one-period unexpected reduction in the outstanding amount of sovereign bonds by 10% of GDP. The central bank is then assumed to hold to maturity the securities purchased. Accordingly we assume that  $\log(\epsilon_t^{QE})$  follows an AR(1) process with an autoregressive coefficient of 0.96. The IRFs of the APP shock are presented in Figure 11 for both estimated versions of the model (see Model (1) and Model (2)).

On impact, bank sales of government bonds account for roughly two thirds of the central bank asset purchases, which is broadly similar for both model versions. Sovereign spread  $Spread_{RG,t+1}$  narrows by around 100 bps (in annual terms) in the model estimated on non-financial corporate (NFC) credit data, but by less than 40 bps (in annual terms) in the other specification. As we will show subsequently, the more muted price response of central bank

purchase in the latter case comes both from milder portfolio frictions for households (and to a lesser extent for bankers) and from stronger financial accelerator effects on the side of entrepreneurs, which everything else being equal imply smaller asset price changes for given quantity adjustments.

Further down in the intermediation chain, the pass-through of sovereign spreads to the required return on loans by retail lender,  $Spread_{R_{LE},t+1}$ , is around 0.8 for the model estimated on NFC credit variables and is lower, at around 0.5 in the other version, mainly due to higher nominal rigidities in the retail segment. In both specifications, the pass-through is higher for lending rates on entrepreneurs loan contracts  $Spread_{R_{LLE},t+1}$ , which is due to the fact that the expansionary effect of the asset purchases improves asset values, in other words entrepreneurs' net worth, and therefore reduces credit risk compensation demanded by banks on non-financial corporate loans. Given that entrepreneurs lending rates are predetermined, the unexpected decline in corporate default risk implies higher *ex post* return of the loan portfolio  $Spread_{\tilde{R}_{LE},t}$  than the *ex ante* required return  $Spread_{R_{LE},t+1}$ , thereby supporting on impact bankers' net worth. Overall, the easing in financial conditions spurs investment and output, generating inflationary pressures and countercyclical monetary policy adjustment. The output multiplier ends up being twice smaller in the model estimation on total economy credit variables, in line with the much more subdued sovereign spread reaction.

The differences in IRFs between the two estimated versions of the model, illustrate the sensitivity of the asset purchase propagation mechanism to three relevant dimensions of the parameter space: credit demand frictions, staggered lending rate setting, and frictions on portfolio decisions for households and bankers.

In order to first isolate the role of **credit demand frictions**, we run the same simulations using the model estimated with NFC credit variables but setting parameters of the entrepreneur and capital producer blocks as in the other estimated version. The corresponding IRFs are introduced in Figure 12 with red dotted lines. It turns out that this "credit demand" calibration (see Model(3) in Figure 11) explains almost fully the differences in IRFs for balance sheet variables and broadly half of the gaps for the other variables, with the exception of sovereign bond sales by households and bankers. By reinforcing the financial accelerator mechanism, the sovereign portfolio allocation implies higher sales of bankers bonds and more limited spread adjustment.

Turning to the implications of **sluggish lending rate pass-through**, we introduced another model calibration changing in the model estimated with NFC credit variables, the parameters for entrepreneurs and capital producers (like in Model (3)), but also the parameters of the retail lending segment. The simulations are reported in Figure 11 with crossed green lines. Compared with the simulations of the "credit demand" experiment, the higher degree of nominal rigidities in lending rate setting weakens the pass-through of the sovereign spread decline to intermediation spreads  $Spread_{R_{LE},t+1}$  and  $Spread_{R_{LLE},t}$ . Consequently, the positive

responses of investment and output are also dampened leading to less inflationary pressures and more limited monetary policy response, falling short though of explaining the remaining differences with the IRFs from the model estimated on total economy credit variables (see Model (2) in Figure 11). Regarding sovereign bond market variables and bankers' decisions, the lower pass-through due to higher nominal rigidities has negligible effect of the IRFs.

We focus now our sensitivity analysis on the **bank portfolio rebalancing frictions** which are governed by two parameters  $\delta_b$  and  $\chi_H$ . Four salient features of the macroeconomic transmission of the asset purchase programme are worth reviewing in this context. First, we need to consider the distribution of asset sales between the household and the financial sector: if households fully accommodate central bank purchases and behave as efficient marginal investors, the unconventional policy would become ineffective. Second, the magnitude of sovereign spread compression is a key indicator of the strength of the transmission channel: in our model, lower excess return on sovereign bonds requires a proportional decline in the excess return of banks' private sector intermediation. Third, the pass-through of sovereign yield changes to financing conditions faced by entrepreneurs for capital expenditures depends on the monopolistic retail banking segment and on the calibration of the corporate balance sheet channel. Fourth, we ultimately evaluate the unconventional policy on its output multipliers and notably on the strength of the credit channel.

Figures 12 to 16 document the sensitivity to those two parameters of the four main features of the central bank asset purchase IRFs, using the model estimated on NFC credit data. Each chart plots the iso curves for a given characteristic of the IRFs. Starting with the share of bond sales by bankers, it is generally increasing in the level of households' portfolio frictions  $\chi_H$  and decreasing in the level of bankers' portfolio frictions  $\delta_b$  (see 12). For  $\chi_H > 0.1$  this share could hardly fall below 80% for most values of  $\delta_b$  ranging from 0.001 to 2. For households to sell two thirds of the central bank purchases  $\chi_H$  has to be lower than 0.005 and  $\delta_b$  higher than 0.3.

Regarding the sovereign spread adjustment, the response is increasing in both parameters, varying from -0.1 to -2.6 (percentage deviation from the baseline quarterly spread) between low and high calibrations for both parameters (see Figure 13). However, the spread impact appears quite insensitive to  $\chi_H$  for low values of  $\delta_b$ : taking the Gertler and Karadi (2013) calibration of  $\delta_b = 0.5$ ,  $\chi_H$  would need to go below 0.005 to somehow decrease the impact on spreads. Conversely, when  $\chi_H$  is below 0.005, the response is relatively independent of  $\delta_b$  for values above 0.8.

Concerning the pass-through from sovereign yields to lending rate, it is an increasing function of  $\delta_b$  while  $\chi_H$  has almost no effect on it (see Figure 14). The pass-through declines to 0.5 for  $\delta_b$  around 2 and reaches 2 for  $\delta_b$  around 0.5. The pass-through iso curves are strongly affected by different degrees of nominal rigidities in lending settings: Figure 16 reports the same sensitivity analysis for the model estimated on total economy credit variables and shows

that the corresponding iso curves display stronger convexity (and overall more sluggishness) but still low dependence on  $\chi_H$ .

Finally, the output multipliers for the parameter ranges considered reach up to 0.7 in percentage deviation from baseline (see Figure 15). As off a certain level (around 0.1), the output response becomes insensitive to  $\chi_H$  while below this level, the dependence on  $\delta_b$  drops considerably: a multiplier of 0.4 for example, can be achieved either for  $\delta_b$  around 1.1 and any value of  $\chi_H$  above 0.1, or with  $\chi_H$  around 0.015 and any value of  $\delta_b$  above 1.3. Overall, this sensitivity analysis supports the choices made for the prior distributions on those parameters, as the plausible macroeconomic transmission of central bank asset purchase in our model would tend to require low values of  $\chi_H$ , notably for the sector distribution of bond sales, and levels of  $\delta_b$  higher than in Gertler and Karadi (2013) to be consistent with the pass-through regularities.

Overall, the sensitivity analysis conducted in this section shows that portfolio rebalancing frictions guide the bond market allocation and the lending rate pass-through of the APP. At the same time, credit demand frictions are found to have a meaningful impact on the strength of the credit channel and the output multipliers of the APP. These results need to be confronted with the micro-structure of bank portfolio decisions and their incentives to re-balance towards credit origination. This is the purpose of the next section.

## 5.2 Bank portfolio rebalancing and the macroeconomic impact of the ECB's APP

We propose a model-based evaluation of the January 2015 ECB's APP integrating both macro and micro perspectives. The model-based scenario will be conditioned on the observed financial market reactions and on the scope for portfolio rebalancing derived from the bank-level asset-liability management.

Moving towards more realistic counterfactual exercises, we consider the unexpected announcement of a central bank asset purchase programme, spread over one year and a half. In order to implement this programme in the DSGE model, we choose for  $\log(\epsilon_t^{QE})$  an AR(2) process as in Gertler and Karadi (2013). We scale the innovation and target the coefficients of the AR(2) process such that the purchases cumulate at 8.4 p.p. of GDP, 7 quarters after announcement, and gradually decline thereafter in line with the assumption of holding to maturity the purchased securities. This AR(2) process approximates well the main features of the ECB's APP of January 2015 which envisaged monthly purchases of EUR 60 billion from March 2015 until September 2016. Compared with the transmission of a one-off purchase of bonds of the same cumulated amount, as in the IRFs of the previous section, the effects on yields, lending spreads, output and inflation are broadly similar. In a companion paper, Darraçq Pariès and Kühl (2016) use the same DSGE model and study at length the implications of alternative formulation of the stochastic process for  $\log(\epsilon_t^{QE})$ . This is not the focus of the

present analysis.

The macroeconomic impact is derived from the DSGE model estimated on credit variables for the non-financial corporate sector. Departing from the posterior distributions, we constrain the portfolio frictions as to reproduce through the counterfactual *i*) the average response of sovereign spreads taken from event studies and *ii*) the pass-through of sovereign spreads to lending rates, as in section 2.3. In doing so, we increase  $\delta_b$  up to 2 while  $\chi_H$  is marginally changed. Monetary policy rate is assumed to remain constant for 2 years which mimics the presence of the effective lower bound on interest rate (ZLB). Darracq Pariès and Kühl (2016) also cover the sensitivity of APP propagation to an endogenous ZLB constraint so that we abstract here from this discussion.

The outcome of the scenario is presented in Figure 17 for the impact on output and in Figure 18 for the impact on inflation. Over the first 3 years of the simulation, output expands gradually, reaching a peak effect of almost 1% in the second year and mildly moderating thereafter. Inflationary pressures build up through several quarters with annual inflation rate at 0.6 p.p. above baseline two to three years after the start of the programme. As shown in section 2.3, the transmission of the asset purchase programme across euro area banking jurisdictions could be quite heterogeneous as banks differ in terms of capital and liquidity position, as well as in the relative riskiness of their credit exposures. Our DSGE model is designed as a closed economy and estimated on euro area wide data. Therefore, evaluating country-level implications of the APP with it would certainly stretch the boundaries of the model's validity. Nonetheless, the dispersion of bank-level responses to sovereign compression could provide some valuable sensitivity ranges for the strength of the portfolio rebalancing channel. We exploit this information in an illustrative manner by re-simulating the APP counterfactual and targeting a pass-through to lending rates as in the lowest and highest quartiles of the bank-level distribution. As before, the parameters  $\delta_b$  and  $\chi_H$  are set to match the yield and lending responses accordingly. These simulations provide some ranges in Figures 17 and 18. On the one hand, the more constrained credit channel, calibrated on the bottom quartile of the bank-level distribution of lending rate pass-through, would significantly dampen the macroeconomic multipliers of the APP. In this case, the stimulating effects on output and inflation would be around 30% smaller than in the benchmark case. Conversely, with a more supportive credit channel, calibrated on the top quartile of the pass-through distribution, the transmission of the APP to output and inflation would be around 10% stronger. The asymmetry between low and high credit channel simulations reflect the skewness in the bank-level distribution of portfolio rebalancing frictions: there is a moderate mass of banks which face severe constraints in pass-through the sovereign yield compression. This clearly illustrates the importance of accounting for banks' heterogeneous responses to central bank sovereign bond purchases. as far as their macroeconomic implications are concerned. Across jurisdictions, the impact of the asset purchases can also be quite heterogeneous due to



both cyclical factors as well as to the nature and structure of national banking sectors.

Overall, while the simulated macroeconomic impact based on our model could significantly vary depending on the strength of the credit channel, the results are mostly within a reasonable distance from each other. At the same time, our simulation results are found to be broadly consistent with the ranges observed across US and UK-based studies as depicted in Figure 19.<sup>13</sup> In general, for a given size of the programme, the output responses appear somewhat stronger in the US and especially UK studies compared to our simulation results. This may, however, partly owe to the fact that (as mentioned in the introduction) we only consider the macroeconomic propagation through the bank credit channel, whereas as broader effects on the economy due to for instance demand side and exchange rate effects are not considered.

## 6 Conclusion

In this paper, we have presented an estimated DSGE model for the euro area, which is well-suited for assessing the macroeconomic implications of central bank asset purchases in the spirit of Gertler and Karadi (2013). While building on the Gertler-Karadi model framework, we amend it somewhat to account for some of the key features of the euro area financial structures in particular by introducing a monopolistic banking sector.

The macro modelling framework that we present is particularly tailored to assessing the impact of asset purchase programmes via the bank credit channel. We argue in the paper that especially in the context of a diverse and fragmented euro area banking sector to properly assess the impact on output and inflation (also cross countries) it is crucial to account for banks' heterogeneous responses to the unconventional monetary policies. For this purpose, we complement the standard DSGE model simulation approaches with a bank level portfolio optimisation model that allows for gauging individual banks' responses to the sovereign yield compression resulting from central bank asset purchases.

Our findings suggest that such unconventional policies have the potential to strongly support the growth momentum in the euro area and significantly lift inflation prospects. The benefits of the policy measure rest on banks' ability and incentive to ease their lending conditions. The strength of the portfolio rebalancing channel through the banking system proves highly dependent on bank balance sheet conditions, and from this perspective, can have diverse impacts across euro area countries. Overall, however, the macro implications in terms of higher economic growth and inflation arising due to bank portfolio rebalancing effects are found to be positive for the euro area and for individual countries.

As noted above, our modelling approach especially focuses on the impact of asset purchase programmes via their direct effect on bank credit conditions arising due to portfolio rebalancing

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<sup>13</sup>Note that in order to make our model simulations broadly comparable to those of the US and UK asset programme studies, the model outcomes from the US and UK studies have been re-scaled to the size of the ECB APP; i.e. around USD 1 trillion.



incentives. Accordingly, there are a number of transmission channels that we do not explicitly cover in this paper, such as exchange rate effects, signalling and confidence effects that would affect aggregate demand independently of banks' responses to the unconventional monetary policies.

Whereas the focus of this paper was on the implications of unconventional monetary policies, our modelling setup can also be employed for banking sector stress test analysis. By combining a micro level bank optimization model with a DSGE model including a well-specified banking sector, the modelling framework can also be used to assess 'second-round' effects of adverse shocks hitting the banking sector, such as shocks to borrower PDs, asset returns and funding costs. The framework can thus be used as a complement to standard stress testing frameworks (see e.g. Henry and Kok (2013)). In contrast to traditional (partial equilibrium and reduced form) stress testing frameworks, in our setup the banks' dynamic responses to shocks would happen in a fully endogenous system where banks would internalise the effects of the adverse shocks in a instantaneous fashion.

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## A Portfolio optimisation model

In the following, the portfolio optimisation model and its calibration is described. The model setup is a variant of Hałaj (2015).

### A.1 Model description

The prerequisites for the optimisation model are information about the initial asset and funding structure, which define the budget and risk constraints. Moreover, information about return and risk parameters is needed for the optimisation. The model's balance sheet optimization algorithm takes as input these parameters as well as the preference structure and the goal function of banks to be optimised.<sup>14</sup> The outcomes of the optimisation can be measured in terms of the changes in balance sheet structure and the contributions to the P&L impact on capital. The framework allows for simulations of the distribution of capital projection of banks given the stochastic nature of the parameters. More importantly, in the context of the APP, it allows for sensitivity analysis of the optimal lending program given changes in the key parameters, in particular the expected return on loans. This sensitivity mechanism is applied to assess the potential decline of the lending spreads following the decline in sovereign yields.

The bank's funding volumes ( $F$ ) are assumed to be homogenous (i.e. consisting of only one type of funding or a constant mixture of funding sources) and to follow a simple autoregressive stochastic process. The riskiness is related to roll-over uncertainty, i.e. depositors may withdraw part of the funding sources. Funding risk is correlated, in particular with the value of securities portfolio. The change in funding may necessitate a "fire-sale" liquidation of part of the securities portfolio. Fire-sales are triggered by the drop in the stock of funding. The inflow of funding is favourable for banks. The loss due to the fire sales is proportional to the liquidated volume which involves a haircut to cover an outflow of funding.<sup>15</sup> Notably, banks may experience also an inter-temporal inflow of funding that can be reinvested in the available asset classes (loans and securities). Funding requires interest payments  $C_t := r^F F_t$ , where  $r^F$  is a given funding interest rate.

More formally, funding satisfies the following recursive equation<sup>16</sup>

$$F_{t+1} = F_t + \gamma F_t + \epsilon_{t+1}^F \quad (41)$$

where starting funding volume  $F_0$  is given and deterministic and  $\epsilon_t^F$  is a stochastic process describing the roll-over risk.<sup>17</sup> The fire-sales liquidation value is given by the following expression:  $(F_t - F_{t-1})^- / (1 - h)$  where  $h$  is a liquidity haircut and  $a^- := -\min(a, 0)$ .

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<sup>14</sup>Preferences are measured by the risk tolerance which is a parameter of the goal function defined as a risk-adjusted return. Notably, the optimization algorithm is flexible enough to account for other functional forms of banks' optimising functions.

<sup>15</sup>See Eser and Schwaab (2013) for an estimation for the euro area bond market.

<sup>16</sup>Fully rigorous mathematical formality is provided in Hałaj (2015).

<sup>17</sup>In the implementation of the model we assume that the risk factors ( $X$ , for instance  $X \equiv \epsilon_t^F$ ) are IID

The loan portfolio ( $L$ ) is homogenous and subject to default risk.<sup>18</sup> Loans pay a deterministic interest rate. The loan portfolio is perfectly illiquid, i.e. only the maturing part  $mL_t$ ,  $m \in [0, 1]$ , can be reinvested. The new business has its own default risk characteristics, correlated with the default risk of the outstanding business (as well as with risk factors of securities portfolio and funding). The outstanding volume of loans may default between  $t$  and  $t+1$ . The new business volumes are also subject to a default risk. In practice, they are functions of the probability of default (PD) distribution and Loss Given Default (LGD). This observation is important for the application of the model – loss rates are estimated by multiplying a random default probability with an average Loss Given Default.

In mathematical terms, let  $\rho$  and  $\rho^N$  be some stochastic processes on a properly defined probability space with filtration, taking values in  $(-\infty, 1]$ , describing the credit quality of the outstanding loan portfolio  $L$  and the new origination. In the applications we assume that risk factors of loans are log-normally distributed, i.e. for normally distributed  $v$  and  $v^N$ ,  $\rho = 1 - \exp(v)$  and  $\rho^N = 1 - \exp(v^N)$ . Then, the dynamics of the (balance sheet) volume of loans  $L$  satisfies

$$L_{t+1} = (1 - m)L_t\rho_{t+1} + q_t^L \rho_{t+1}^N \quad (42)$$

where  $q_t^L$  is a reinvestment strategy (subject to optimisation described later). Loans earn interest  $r$  that brings interest income  $I_t := rL_t$  at the end of each period  $[t-1, t]$ .

The interest income from loans is measured by the rate payment  $r$  multiplied by the end-of-period volume of the loans. Notably, the interest income of loans is affected by the defaulted volume of loans which is reflected by taking the volume of loans from the end of period  $[t, t+1]$  to compute interests earned in that period.

The part of the value of the balance sheet that is not invested in the loan portfolio is allocated into the securities portfolio ( $S$ ). The total reinvestment potential is equivalent to the sum of the maturing loans, the value of securities, the change in funding and the P&L impact of the fire-sales. Notably, the total reinvestment portfolio is impacted by the change in funding asymmetrically depending on the sign of the change. In case of the funding outflow, the bank “fire-sales” its securities to meet its obligations. The price of securities is risky and driven by a stochastic process  $\epsilon^S$ .

The law of motion for the securities is derived as

$$S_{t+1} = (mL_t + S_t + I_t - C_t + \Delta F_{K,t} - q_t^L)\epsilon_{t+1}^S \quad (43)$$

where  $\Delta F_{K,t} = F_t - F_{t-1} - \frac{h}{1-h}(F_t - F_{t-1})^-$  is the change of the volume of the reinvestment portfolio related to the volatile funding and  $\epsilon^S$  is a stochastic process representing a volatility of securities, such that of  $\epsilon_t^S$  are IID random variables.

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normally or log-normally distributed, with mean and standard deviation parameters denoted  $\mu_X$  and  $\sigma_X$  respectively.

<sup>18</sup>As a matter of fact we distinguish between customer lending and interbank lending and only customer loan portfolio is subject to optimisation, while lending to banks is assumed to remain constant.



Capital ( $K$ ) is a residual part of the balance sheet. At the end of each period, its level changes according to the accrued net interest income generated in period  $[t-1, t]$ , to valuation changes of the securities portfolio, to loan losses and to fire-sales of securities in case of funding outflows.

More specifically, as a consequence of the definitions of  $F$ ,  $L$  and  $S$  the capital evolves as follows:

$$K_0 = L_0 + S_0 - F_0$$

and for  $t > 0$

$$K_t = K_{t-1} + rL_t - r^F F_t + \Delta L_t + \Delta S_t - \frac{h}{1-h}(F_t - F_{t-1})^-$$

A bank is supposed to maximise the expected return on equity adjusted by the risk of that return and aggregated across periods. Return is measured by the aggregate net interest income, loan losses and valuation of securities realised within a given period divided by capital at the beginning of that period. Since the ratio is random, the risk of the return is simply gauged by the variance of that return-on-equity ratio.

There are two types of constraints imposed on the investment strategy: related to the liquidity (Liquidity-at-Risk) and solvency position (Value-at-Risk). Liquidity is understood as the balance sheet composition that allows for paying back due liabilities. We omit the cash flow balance of interest paid by loans and funding since we focus on liquidity shocks related to the fluctuations of deposits. For the liquidity purposes a shorter period is assumed – a holding period – in which the liquidity position cannot be adjusted. The investment strategy should then keep enough liquid resources to cover an outflow of deposits in  $1 - \alpha^F$  fraction of scenarios, at a given confidence level  $\alpha^F$ . The securities in the counterbalancing capacity can be liquidated with a haircut reflecting a discount that can be expected in case of the liquidation (potentially quite high in a “fire-sales” mode). Solvency is captured by the regulatory constraint, i.e. banks must keep their capital ratio (Capital/RWA) above a certain threshold and a more economic based principle to hold enough capital to absorb losses in a large majority of scenarios (i.e. in  $1 - \alpha$  fraction of scenarios of capital evolution).

The liquidity risk constraint (Liquidity-at-Risk) assumed to act in a given time horizon  $\Delta^l > 0$  can be formally put down in the following way:

$$\text{VaR}_\alpha (\mathbb{E} [(1-h)S_{t+\Delta^l} + (F_{t+\Delta^l} - F_t)|\mathcal{F}_t]) \geq 0 \quad (44)$$

where  $\text{VaR}_\alpha (\mathbb{E} [X|\mathcal{F}_t])$  is a conditional value-at-risk of a random variable  $X$  given information ( $\sigma$ -field)  $\mathcal{F}_t$  generated by random variables  $\epsilon_s^S$ ,  $\rho_s$ ,  $\rho_s^N$  and  $\epsilon_s^F$  for  $s \leq t$ .

In technical terms, the solvency constraints have two forms. For the risk weights  $\nu^L$  and  $\nu^S$ , and minimum capital ratio  $\kappa$  (e.g. equal to 10%):

$$\kappa(\nu^L((1-m)L_t + \bar{q}) + \nu^S(((m+r)L_t + S_t + \Delta F_{K,t} - \bar{q}))) \leq K_t \quad (45)$$

which translates into (assuming  $\nu^L > \nu^S$ ):

$$\bar{q} \leq \frac{K_t/\kappa - \nu^L(1-m)L_t - \nu^S((m+r)L_t + S_t + \Delta F_{K,t})}{\nu^L - \nu^S}$$

However, banks manage their investment portfolios taking worst case scenarios of the capital position in a given  $\Delta^K > 0$  period into account.<sup>19</sup> We consider  $\Delta^K$  period forward distribution of income and require that the capital of a bank covers the losses in  $(1-\alpha^K)*100\%$  of cases.

$$K_t + \text{VaR}_{\alpha^K}(\mathbb{E}[K_{t+\Delta^K} - K_t | \mathcal{F}_t]) > 0 \quad (46)$$

We assume that banks optimise the risk-adjusted return on capital, aggregated within the horizon of the optimisation. Given the 2-period implementation of the model, it only requires approximate (Monte-Carlo) methods to be implemented in the first period whereas at the second (and final) period the solution is explicit. Notably, it preserves all the important features of a  $T$ -period model; the inter-temporal effects resulting in a trade-off between investing more in profitable loans now and facing risk of illiquidity or generating less income but increasing survival probability. The optimisation problem is solved numerically by means of dynamic programming. It is a convenient way to derive the optimal portfolios in a backward manner.

More formally, the goal functional takes the following form:

$$J(l_0, s_0, f_0, \rho_0, q) = \mathbb{E} \sum_{t=1}^2 \delta^t (R_t - \beta \text{Var}(R_t)) \quad (47)$$

where  $\text{Var}(R_t) = \mathbb{E}[(R_t - \mathbb{E}R_t)^2 | \mathcal{F}_{t-1}]$  is a conditional variance process of the return on capital  $R_t := (K_t - K_{t-1})/K_{t-1}$ .

## A.2 Model calibration and sensitivity analysis

The sensitivity of the optimal loan portfolio choice to various key parameters of the model provides information about the most important factors of banks' lending behaviour: the returns and risks of the loan and securities portfolios, the capital level and the cost of funding.<sup>20</sup>

Focusing first on the impact on the optimal size of the loan portfolio of changes to the yield on the securities portfolio, we find that, *ceteris paribus*, banks in France and to some extent Germany are relatively more sensitive to changes in securities' returns than banks in Italy and Spain. In other words, the same decline in the yield on securities has a stronger positive impact on the optimal loan portfolio for French banks compared to Italian and Spanish banks. The main reason is that the starting value of the return-risk (Sharpe) ratio of Spanish and Italian banks compared to the Sharpe ratio on their loan books is relatively higher than the

<sup>19</sup>E.g. under the ICAAP process.

<sup>20</sup>detailed results and illustrations are available from the authors.

same relationship characterising the average French (and German) bank. This implies that it takes a relatively stronger negative shock to returns on securities held by Spanish and Italian banks to induce them to rebalance their portfolio from securities to loans.

The sensitivity of the optimal lending volume to changes in lending rates differs somewhat across countries. Higher lending rates make it more attractive to lend. The average sensitivity of German, Italian and French banks is similar, whereas in the case of Spain it is much smaller. This observation is consistent with the fact that the average credit quality of loan portfolios in the sample of Spanish banks is lower than in portfolios of other banks in the sample; therefore, a higher increase of lending rates is needed to outweigh the expected credit losses.

For what concerns banks' optimal lending dynamics as a function of the volatility of prices of securities. A clear positive relationship between volatility of securities and loan supply is observed. In other words, customer lending tends to become attractive for the banks when the uncertainty about the valuation of the securities portfolio increases. The sensitivity proves to be quite homogenous across countries.

We also explore the sensitivity of optimal lending dynamics to changes in the riskiness of the loan portfolio as measured by PDs on loans. We find a clear negative relationship (as expected) meaning that when the riskiness of the loan book increases it will become less attractive for banks to lend.

Turning to the optimal lending as a function of the level of the capital two notable observations can be made from this sensitivity analysis: first, a reduction of the level of capital may restrain some banks to extend lending at all (i.e. the capital condition becomes too stringent) which is reflected by a kinky shape of the lines in the figure; second, the sign of the relationship may differ from bank to bank and along the simulated changes of capital since such changes impact banks' financial position in many directions (i.e. by changing the capital ratio and VaR constraints, affecting the sensitivity of ROE and its variance to changes in the asset composition).

Finally, regarding the impact of funding cost on the optimal lending portfolio choice the model points to a monotonic decreasing relationship between funding costs and the asset composition towards loans. However, the sensitivity appears quite different across countries, with the French banking jurisdiction displaying relatively more muted responses. By contrast, for Germany, the average bank loan portfolio decision turns out to be most sensitive to funding costs. This is partly due to the fact that on average, German banks are shown to be more reliant on wholesale funding.

## B Recursive formulation of price and wage settings

### B.1 Wage setting

In the following, given that the steady state model features a balanced growth path, all variables are appropriately deflated to be stationary in the stochastic equilibrium.

The first order condition of the union's program for the re-optimized wage  $w_t^*$  can be written recursively as follows:

$$w_t^* = \frac{\theta_w(1+\psi_w)}{(\theta_w(1+\psi_w)-1)} \frac{\mathcal{H}_{1,t}}{\mathcal{H}_{2,t}} + \frac{\psi_w}{(\theta_w-1)} (w_{t-1}^*)^{1+\theta_w(1+\psi_w)} \frac{\mathcal{H}_{3,t}}{\mathcal{H}_{2,t}} \quad (48)$$

with

$$\begin{aligned} \mathcal{H}_{1,t} = & \varepsilon_t^B \tilde{L} (N_t^S)^{1+\sigma_l} w_t^{\theta_w(1+\psi_w)} (C_t - \eta C_{t-1}/\gamma)^{(1-\sigma_c)} \exp\left(\tilde{L} \frac{(\sigma_c-1)}{(1+\sigma_l)} (N_t^S)^{1+\sigma_l}\right) \Delta_{w\lambda,t}^{\theta_w(1+\psi_w)/(1-\theta_w(1+\psi_w))} \\ & + \beta \gamma^{(1-\sigma_c)} \alpha_w \mathbb{E}_t \left[ \left( \frac{\pi_{t+1}}{\pi_t^{\xi_w} [\pi^*]^{(1-\xi_w)}} \right)^{\theta_w(1+\psi_w)} \mathcal{H}_{1,t+1} \right] \end{aligned} \quad (49)$$

$$\begin{aligned} \mathcal{H}_{2,t} = & (1 - \tau_{w,t}) \lambda_t N_t^S w_t^{\theta_w(1+\psi_w)} \Delta_{w\lambda,t}^{\theta_w(1+\psi_w)/(1-\theta_w(1+\psi_w))} \\ & + \beta \gamma^{(1-\sigma_c)} \alpha_w \mathbb{E}_t \left[ \left( \frac{\pi_{t+1}}{\pi_t^{\xi_w} [\pi^*]^{(1-\xi_w)}} \right)^{\theta_w(1+\psi_w)-1} \mathcal{H}_{2,t+1} \right] \end{aligned} \quad (50)$$

$$\mathcal{H}_{3,t} = (1 - \tau_{w,t}) \lambda_t N_t^S + \beta \gamma^{(1-\sigma_c)} \alpha_w \mathbb{E}_t \left[ \left( \frac{\pi_t^{\xi_w} [\pi^*]^{(1-\xi_w)}}{\pi_{t+1}} \right) \mathcal{H}_{3,t+1} \right] \quad (51)$$

The aggregate wage dynamics could also be expressed as

$$\begin{aligned} (w_t)^{1-\theta_w(1+\psi_w)} \Delta_{w\lambda,t} &= (1 - \alpha_w) (w_t^*)^{1-\theta_w(1+\psi_w)} \\ &+ \alpha_w \left( \frac{\pi_t}{\pi_{t-1}^{\xi_w} [\pi^*]^{1-\xi_w}} \right)^{\theta_w(1+\psi_w)-1} (w_{t-1})^{1-\theta_w(1+\psi_w)} \Delta_{w\lambda,t-1} \end{aligned} \quad (52)$$

The previous equations include a dispersion index  $\Delta_{w\lambda,t}$  which is related to the re-optimizing wage and the aggregate wage through the following conditions

$$1 = \frac{1}{1+\psi_w} \Delta_{w\lambda,t}^{1/(1-\theta_w(1+\psi_w))} + \frac{\psi_w}{1+\psi_w} \Delta_{wl,t} \quad (53)$$

$$\Delta_{wl,t} = (1 - \alpha_w) \left( \frac{w_t^*}{w_t} \right) + \alpha_w \left( \frac{w_t}{w_{t-1}} \frac{\pi_t}{\pi_{t-1}^{\xi_w} [\pi^*]^{1-\xi_w}} \right)^{-1} \Delta_{wl,t-1} \quad (54)$$

The market clearing condition linking total labor demand of intermediate firms and total labor supply of households includes a wage dispersion index given by

$$\Delta_{wk,t} = \frac{1}{1+\psi_w} \Delta_{w,t} \cdot \Delta_{w\lambda,t}^{\theta_w(1+\psi_w)/(1-\theta_w(1+\psi_w))} + \frac{\psi_w}{1+\psi_w} \quad (55)$$

with

$$\Delta_{w,t} = (1 - \alpha_w) \left( \frac{w_t^*}{w_t} \right)^{-\theta_w(1+\psi_w)} + \alpha_w \left( \frac{w_t}{w_{t-1}} \frac{\pi_t}{\pi_{t-1}^{\xi_w} [\pi^*]^{1-\xi_w}} \right)^{\theta_w(1+\psi_w)} \Delta_{w,t-1} \quad (56)$$

## B.2 Price setting

The first order condition of the intermediate firms profit maximization leads to

$$p_t^* = \frac{\theta_p(1+\psi)}{(\theta_p(1+\psi) - 1)} \frac{\mathcal{Z}_{1,t}}{\mathcal{Z}_{2,t}} + \frac{\psi}{(\theta_p - 1)} (p_t^*)^{1+\theta_p(1+\psi)} \frac{\mathcal{Z}_{3,t}}{\mathcal{Z}_{2,t}} \quad (57)$$

with

$$\begin{aligned} \mathcal{Z}_{1,t} &= \lambda_t m c_t Y_t \Delta_{p\lambda,t}^{\theta_p(1+\psi)/(1-\theta_p(1+\psi))} \\ &\quad + \beta \gamma^{(1-\sigma_c)} \alpha_p \mathbb{E}_t \left[ \left( \frac{\pi_{t+1}}{\pi_t^{\xi_p} [\pi^*]^{(1-\xi_p)}} \right)^{\theta_p(1+\psi)} \mathcal{Z}_{1,t+1} \right] \end{aligned} \quad (58)$$

$$\begin{aligned} \mathcal{Z}_{2,t} &= (1 - \tau_{p,t}) \lambda_t Y_t \Delta_{p\lambda,t}^{\theta_p(1+\psi)/(1-\theta_p(1+\psi))} \\ &\quad + \beta \gamma^{(1-\sigma_c)} \alpha_p \mathbb{E}_t \left[ \left( \frac{\pi_{t+1}}{\pi_t^{\xi_p} [\pi^*]^{(1-\xi_p)}} \right)^{\theta_p(1+\psi)-1} \mathcal{Z}_{2,t+1} \right] \end{aligned} \quad (59)$$

$$\mathcal{Z}_{3,t} = (1 - \tau_{p,t}) \lambda_t Y_t + \beta \gamma^{(1-\sigma_c)} \alpha_p \mathbb{E}_t \left[ \left( \frac{\pi_t^{\xi_p} [\pi^*]^{(1-\xi_p)}}{\pi_{t+1}} \right) \mathcal{Z}_{3,t+1} \right] \quad (60)$$

Aggregate price dynamics can then be written as

$$\Delta_{p\lambda,t} = (1 - \alpha_p) (p_t^*)^{1-\theta_p(1+\psi)} + \alpha_p \left( \frac{\pi_t}{\pi_{t-1}^{\xi_p} [\pi^*]^{1-\xi_p}} \right)^{\theta_p(1+\psi)-1} \Delta_{p\lambda,t-1} \quad (61)$$

Here again, compared with the Dixit-Stiglitz aggregator case, the previous equations include a dispersion index  $\Delta_{p\lambda,t}$  which is given by

$$1 = \frac{1}{1+\psi} \Delta_{p\lambda,t}^{1/(1-\theta_p(1+\psi))} + \frac{\psi}{1+\psi} \Delta_{pl,t} \quad (62)$$

$$\Delta_{pl,t} = (1 - \alpha_p) (p_t^*) + \alpha_p \left( \frac{\pi_t}{\pi_{t-1}^{\xi_p} [\pi^*]^{1-\xi_p}} \right)^{-1} \Delta_{pl,t-1} \quad (63)$$

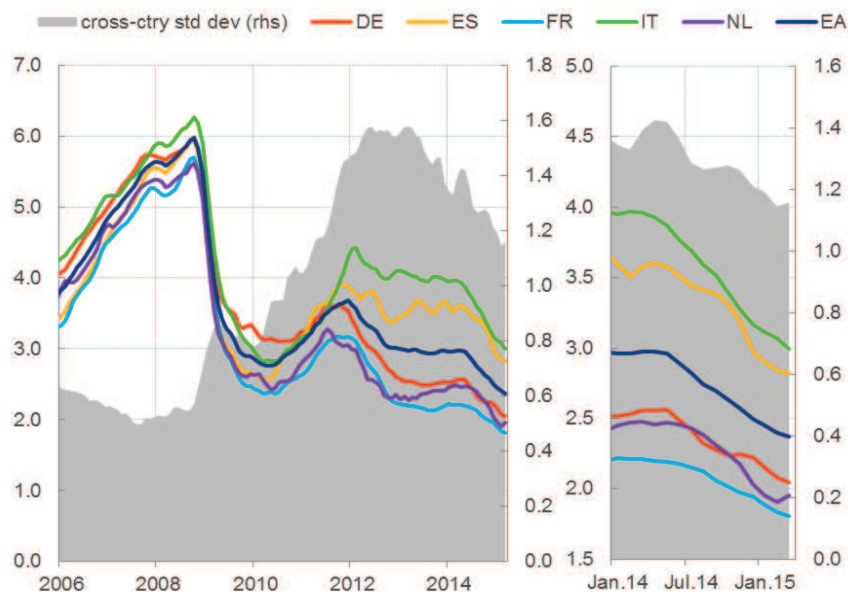
The market clearing conditions in the goods market also involves a price dispersion index given by

$$\Delta_{pk,t} = \frac{1}{1+\psi} \Delta_{p,t} \cdot \Delta_{p\lambda,t}^{\theta_p(1+\psi)/(1-\theta_p(1+\psi))} + \frac{\psi}{1+\psi} \quad (64)$$

with

$$\Delta_{p,t} = (1 - \alpha_p) (p_t^*)^{-\theta_p(1+\psi)} + \alpha_p \left( \frac{\pi_t}{\pi_{t-1}^{\xi_p} [\pi^*]^{1-\xi_p}} \right)^{\theta_p(1+\psi)} \Delta_{p,t-1} \quad (65)$$

Figure 1: Composite indicator of the nominal cost of bank borrowing for non-financial corporations in the euro area (percentage per annum; 3-month moving averages)



Note: The composite indicator of the cost of borrowing is calculated by aggregating short- and long-term rates using a 24-month moving average of new business volumes. The cross-country standard deviation is calculated over a fixed sample of 12 euro area countries.

Source: ECB.

Table 1: Impact of the 50 bp shock on the main portfolio characteristics of banks' balance sheets

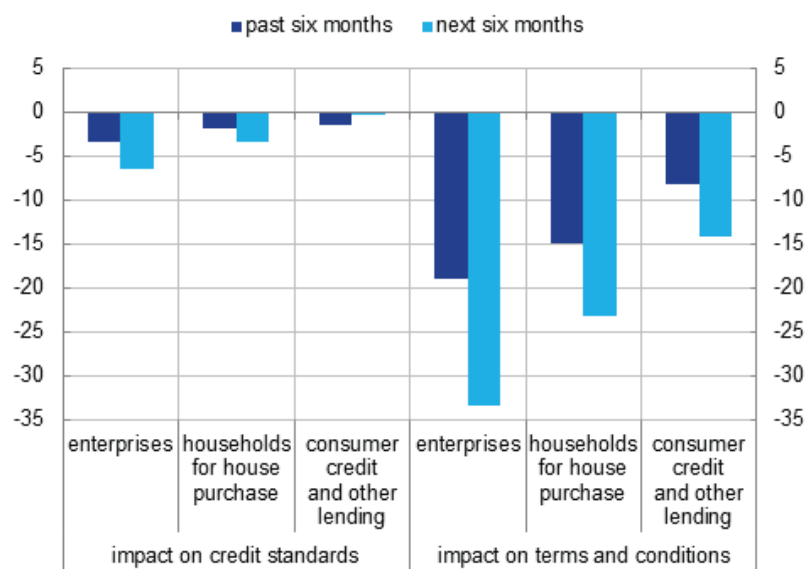
Trigger:	Impact measure:	DE	FR	IT	ES
valuation shock	valuation relative to capital (%)	3.3	1.6	2.6	2.2
yield shift	relative Sharpe ratio*	1.27	1.29	1.19	1.19
funding cost shock	change in expenses relative to capital (%)	-9.4	-5.4	-6.0	-6.1

Note: capital weighted averages

\*) Sharpe ratio of loans divided by Sharpe ratio of securities presented as a post- and pre-shock ratio, i.e. 1.27 for Germany means that – due to the 50 bp shock – the relative Sharpe ratio of loans vs securities increase by 27%.

Source: Source: own calculations

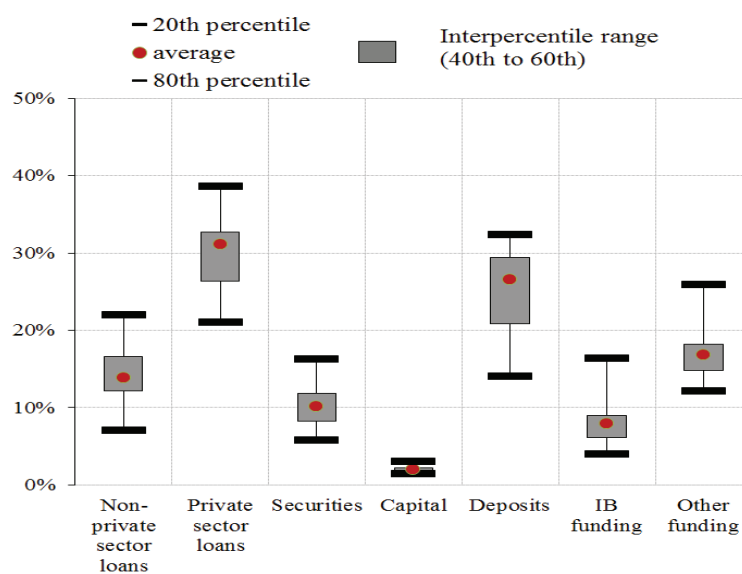
Figure 2: Impact of the expanded APP on bank lending conditions (net percentage of respondents)



Note: The net percentages are defined as the difference between the sum of the percentages for “tightened considerably” and “tightened somewhat” and the sum of the percentages for “eased somewhat” and “eased considerably”. The results shown are calculated as a percentage of the number of banks which did not reply “not applicable”.

Source: ECB, April 2015 Euro area bank lending survey.

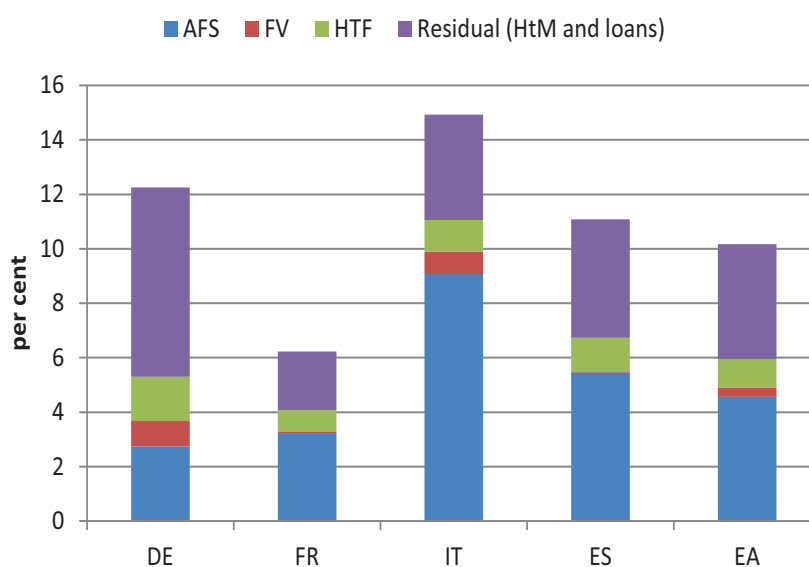
Figure 3: Balance sheet structure of banks by major items - cross-bank dispersion (in percentage to total assets)



Source: ECB Comprehensive Assessment Database



Figure 4: Sovereign securities holdings of Comprehensive Assessment banks by portfolio (in percentage to total assets)



Note: Net direct positions measure gross exposures (long) net of cash short positions of sovereign debt to other counterparties only where there is a maturity matching. AFS: available for sale; HFT: financial assets held for trading; FV: holdings designated at fair value through P&L banking book. Residual includes securities held to maturity and loans.

Source: ECB Comprehensive Assessment Database

Figure 5: Heterogeneity of banks' balance sheets and lending rate response

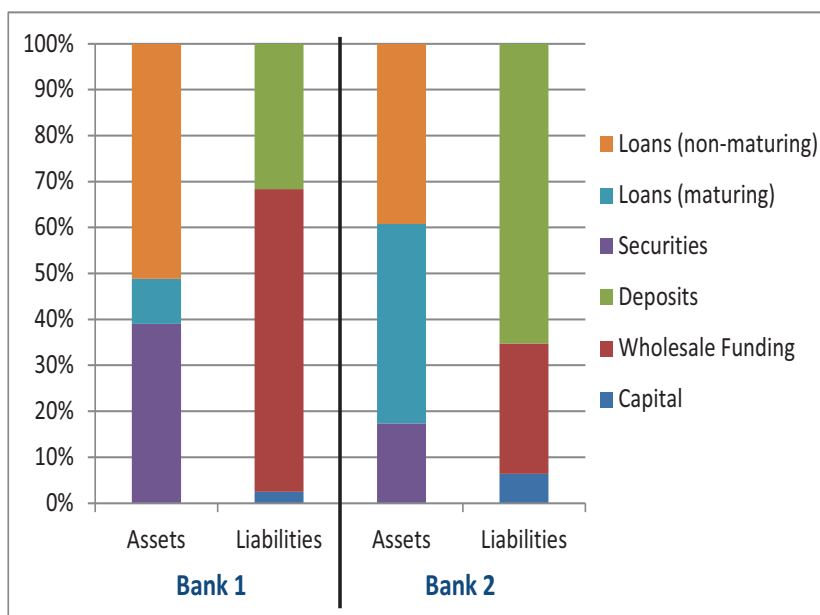
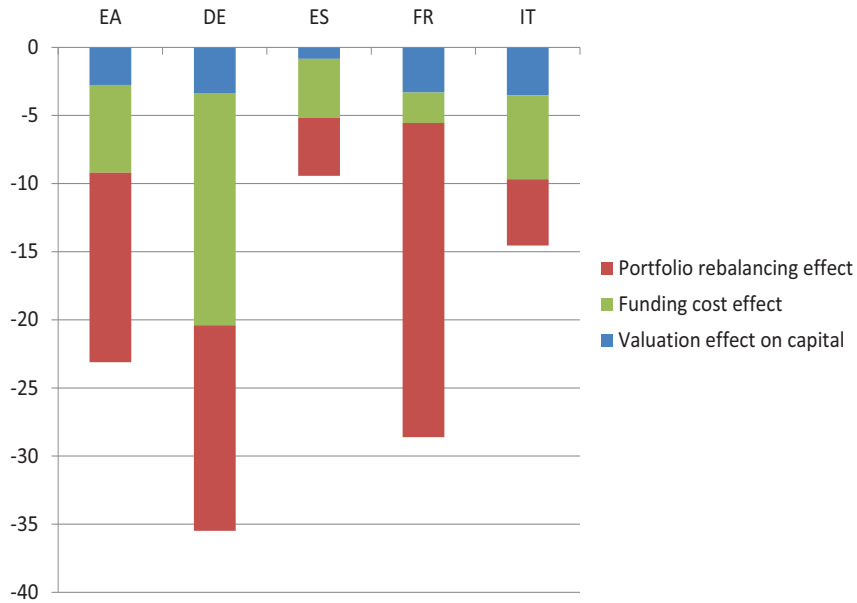


Figure 6: Balance sheet parameter values and lending rate response

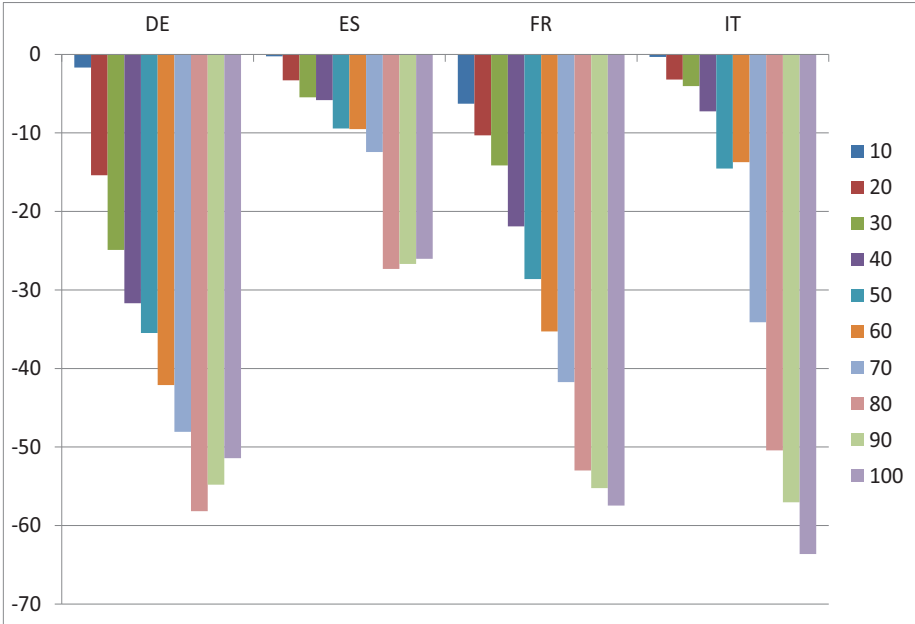
	Bank 1	Bank 2
<b>Loan rate response (in bps)</b>	<b>-50</b>	<b>-25</b>
<b>Parameter values:</b>		
CAR (in %)	10.6%	10.8%
PD loan book (in %)	0.2%	1.2%
loan rate (in %)	3.4%	6.1%
Return on securities (in %)	1.4%	3.1%
Sharpe ratio loans	1.94	0.91
Sharpe ratio securities	1.01	1.55

Figure 7: Decomposition of the factors behind the lending rate response to a uniform 50 bps decline in sovereign yields



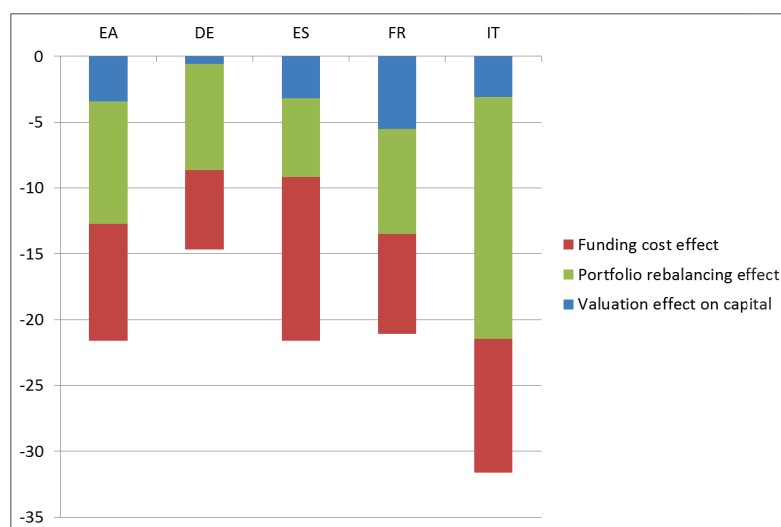
Source: own calculations

Figure 8: Lending rate responses to 10 bps incremental declines in sovereign yields; in basis points



Source: own calculations

Figure 9: Lending rate response to observed decline in sovereign yields; in basis points



Source: own calculations

Table 2: Parameter Estimates 1

Param		<i>A priori</i> beliefs			NFC credit variables <i>A posteriori</i> beliefs			Total credit variables <i>A posteriori</i> beliefs		
		Dist.	Mean	Std.	Mean	$\mathcal{I}_1$	$\mathcal{I}_2$	Mean	$\mathcal{I}_1$	$\mathcal{I}_2$
		$\sigma_c$	Intertemp. elasticity of subst.	gamma	1.50	0.38	1.57	1.20	1.99	1.57
$\eta$	Habit formation	normal	0.70	0.10	0.79	0.72	0.87	0.78	0.71	0.84
$\sigma_l$	Labor disutility	gamma	2.00	0.75	1.08	0.36	1.76	1.06	0.45	1.67
$\phi$	Investment adj. cost	normal	4.00	1.50	3.50	2.29	4.59	5.09	3.62	6.56
$\varphi$	Cap. utilization adj. cost	beta	0.50	0.15	0.63	0.48	0.79	0.60	0.43	0.78
$\alpha_p$	Calvo lottery, price setting	beta	0.50	0.10	0.54	0.43	0.64	0.52	0.42	0.62
$\xi_p$	Indexation, price setting	beta	0.50	0.15	0.32	0.14	0.49	0.40	0.22	0.58
$\alpha_w$	Calvo lottery, wage setting	beta	0.50	0.10	0.57	0.45	0.68	0.52	0.42	0.63
$\xi_w$	Indexation, wage setting	beta	0.50	0.15	0.24	0.10	0.37	0.30	0.13	0.47
$\xi_E^R$	Calvo lottery, lending rate	beta	0.50	0.20	0.07	0.02	0.11	0.46	0.39	0.53
$r_\mu$	Lending rate margin	gamma	0.15	0.05	0.10	0.05	0.15	0.04	0.01	0.07
$\delta_b$	Diversion rate for sov. Bonds	gamma	1.00	0.50	1.10	0.83	1.36	0.98	0.72	1.22
$\chi_H$	Portfolio adj. cost	gamma	0.10	0.05	0.014	0.007	0.021	0.005	0.003	0.007
$\zeta_b$	Prob. survival of bankers	beta	0.95	0.02	0.95	0.94	0.96	0.94	0.93	0.95
$\alpha$	Capital share	normal	0.30	0.05	0.28	0.24	0.32	0.26	0.22	0.30
$\mu_p$	Price markup	normal	1.25	0.12	1.39	1.24	1.54	1.40	1.26	1.56
$r_\beta$	Time-preference rate	gamma	0.25	0.10	0.11	0.04	0.17	0.10	0.04	0.16
$\gamma$	Trend productivity	normal	0.40	0.10	0.21	0.16	0.25	0.19	0.14	0.24
$\bar{L}$	Employment shift	normal	0.00	5.00	1.25	-1.56	4.00	1.07	-1.89	3.87
$\bar{\pi}$	SS inflation rate	gamma	0.50	0.25	0.62	0.44	0.80	0.61	0.41	0.80
$\rho$	Interest rate smoothing	beta	0.75	0.15	0.90	0.87	0.92	0.88	0.85	0.90
$r_\pi$	Taylor rule coef. on inflation	normal	1.50	0.25	1.77	1.50	2.05	1.53	1.30	1.76
$r_{\Delta\pi}$	Taylor rule coef. on d(inflation)	gamma	0.30	0.10	0.06	0.03	0.09	0.05	0.03	0.08
$r_{\Delta Y}$	Taylor rule coef. on d(output)	normal	0.12	0.05	0.07	0.05	0.10	0.07	0.05	0.10

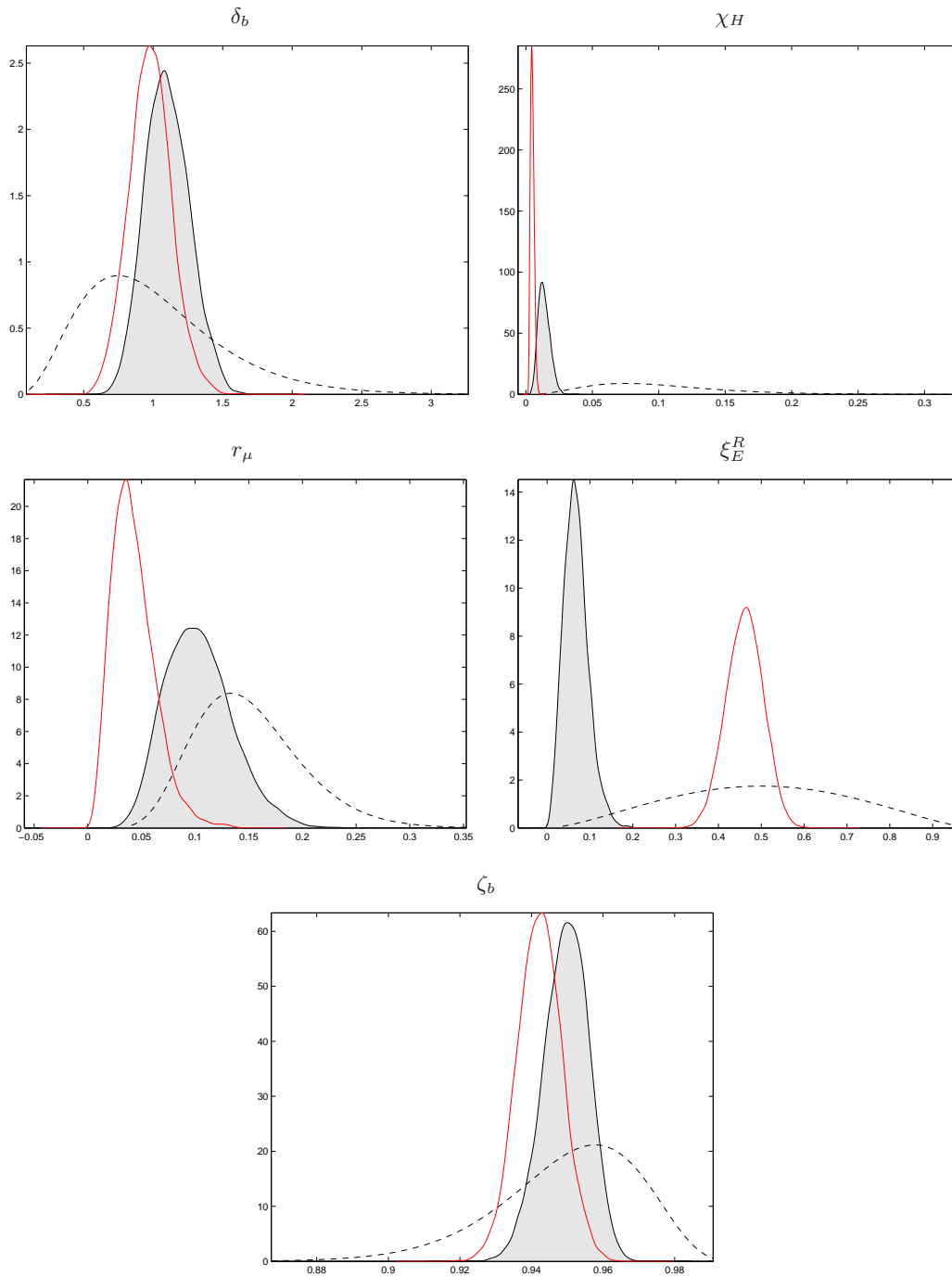
$[\mathcal{I}_1, \mathcal{I}_2]$  is the shortest interval covering eighty percent of the posterior distribution.

Table 3: Parameter Estimates 2

Param		<i>A priori</i> beliefs			NFC credit variables			Total credit variables		
		<i>A priori</i> beliefs			<i>A posteriori</i> beliefs			<i>A posteriori</i> beliefs		
		Dist.	Mean	Std.	Mean	$\mathcal{I}_1$	$\mathcal{I}_2$	Mean	$\mathcal{I}_1$	$\mathcal{I}_2$
$\lambda_e$	Employment adj. cost	beta	0.50	0.28	0.86	0.84	0.89	0.86	0.83	0.89
$\rho_{a,g}$	Corr(Tech.,Gov. Spend.)	uniform	4.50	3.18	0.63	-0.01	1.23	0.59	-0.02	1.27
$\rho_a$	AR(1) Technology	beta	0.50	0.25	0.88	0.81	0.96	0.91	0.86	0.97
$\rho_b$	AR(1) Preference	beta	0.50	0.25	0.20	0.01	0.38	0.16	0.01	0.31
$\rho_g$	AR(1) Gov. spending	beta	0.50	0.25	0.99	0.98	1.00	0.99	0.98	1.00
$\rho_I$	AR(1) Inv. Technology	beta	0.50	0.20	0.76	0.67	0.85	0.71	0.61	0.82
$\rho_p$	AR(1) Price markup	beta	0.50	0.20	0.99	0.97	1.00	0.98	0.96	1.00
$\eta_p$	MA(1) Price markup	beta	0.50	0.20	0.74	0.58	0.90	0.69	0.49	0.88
$\rho_w$	AR(1) Wage markup	beta	0.50	0.20	0.92	0.88	0.96	0.94	0.92	0.98
$\rho_{\sigma_e}$	AR(1) entrepr. risk	beta	0.50	0.20	0.00	0.00	0.01	0.01	0.00	0.01
$\rho_{\zeta_e}$	AR(1) entrepr. net worth	beta	0.50	0.20	0.46	0.29	0.64	0.54	0.37	0.72
$\rho_{\delta_b}$	AR(1) Bankers diversion rate	beta	0.50	0.25	0.94	0.89	0.98	0.85	0.78	0.93
<i>Std</i>										
$\epsilon_t^a$	Technology	uniform	5.00	2.89	0.76	0.49	1.01	0.73	0.51	0.96
$\epsilon_t^b$	Preference	uniform	5.00	2.89	2.26	1.48	2.99	2.04	1.45	2.64
$\epsilon_t^g$	Gov. spending	uniform	5.00	2.89	1.79	1.53	2.03	1.82	1.57	2.07
$\epsilon_t^I$	Inv. Technology	uniform	10.00	5.77	3.39	2.50	4.29	3.92	2.64	5.13
$\epsilon_t^p$	Price markup	uniform	0.25	0.14	0.19	0.15	0.24	0.18	0.14	0.22
$\epsilon_t^w$	Price markup	uniform	0.25	0.14	0.09	0.06	0.11	0.09	0.06	0.13
$\epsilon_t^r$	Wage markup	uniform	0.25	0.14	0.10	0.08	0.11	0.10	0.08	0.12
$\epsilon_t^{\sigma_e}$	Entrepreneurs risk	uniform	5.00	2.89	1.03	0.84	1.21	0.83	0.67	1.01
$\epsilon_t^{\zeta_e}$	Entrepreneurs net worth	uniform	0.50	0.29	0.15	0.12	0.17	0.43	0.35	0.50
$\epsilon_t^{\delta_b}$	Bankers diversion rate	uniform	5.00	2.89	3.07	1.60	4.49	3.44	2.18	4.60
$P_\lambda(\mathcal{Y})$					-102.5			-89.6		

$[\mathcal{I}_1, \mathcal{I}_2]$  is the shortest interval covering eighty percent of the posterior distribution.

Figure 10: PRIOR and POSTERIOR densities



Prior density (black dotted line), Posterior density from the estimation with NFC credit variables (shaded grey area), Posterior density from the estimation with total economy credit variables (red line)



Figure 11: Impulse Response Functions associated to a shock on  $\epsilon_t^{QE}$ . Model (1): estimation with NFC credit variables (plain lines and shaded grey areas); model (2): estimated with total economy credit variables (blue dotted lines with circle); model (3): model (1) with parameter for the entrepreneur and capital producers from model (2) (red dotted line); model (4): model (3) with parameter for the retail lending segment from model (2) (green line with crosses).

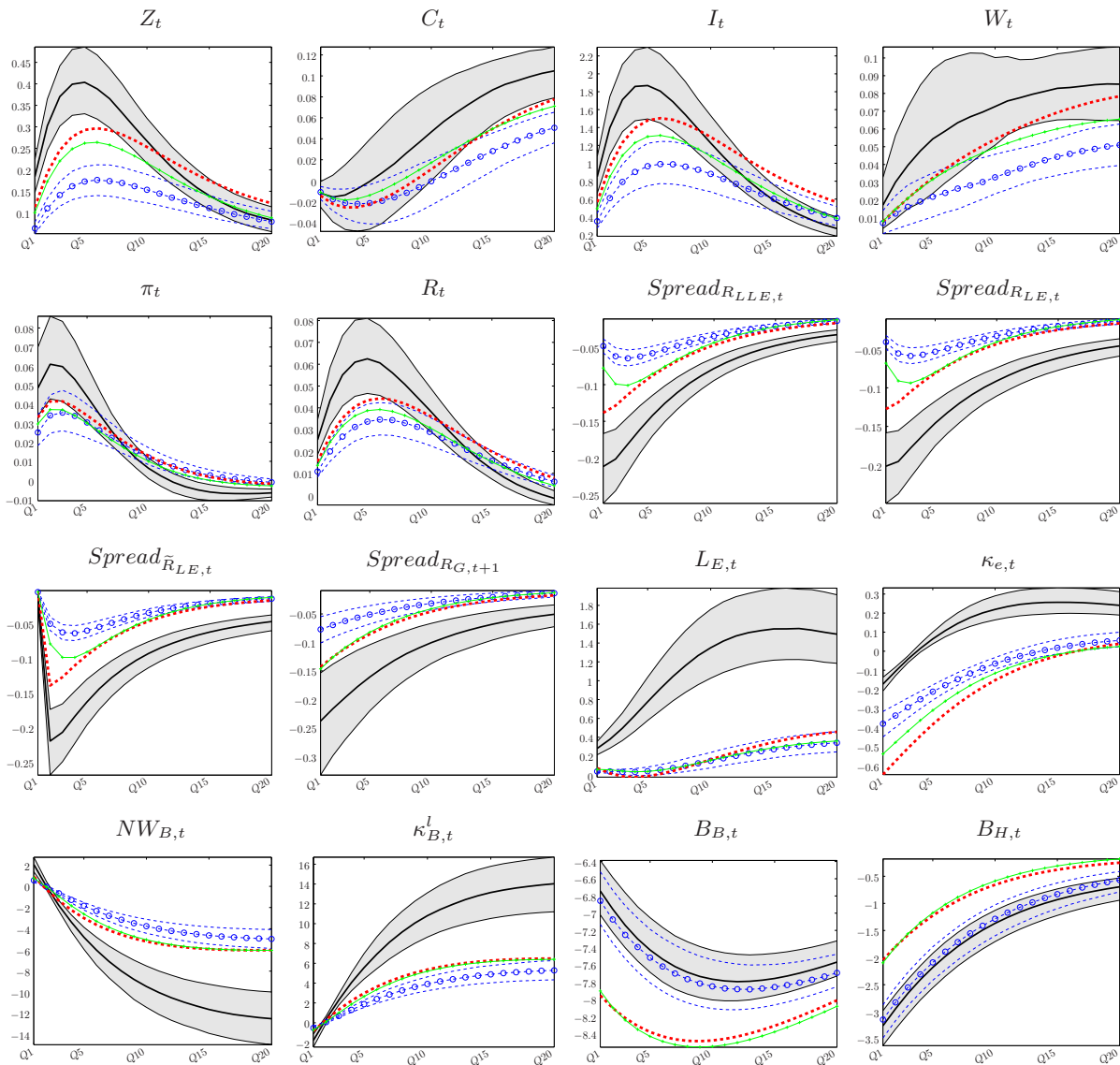
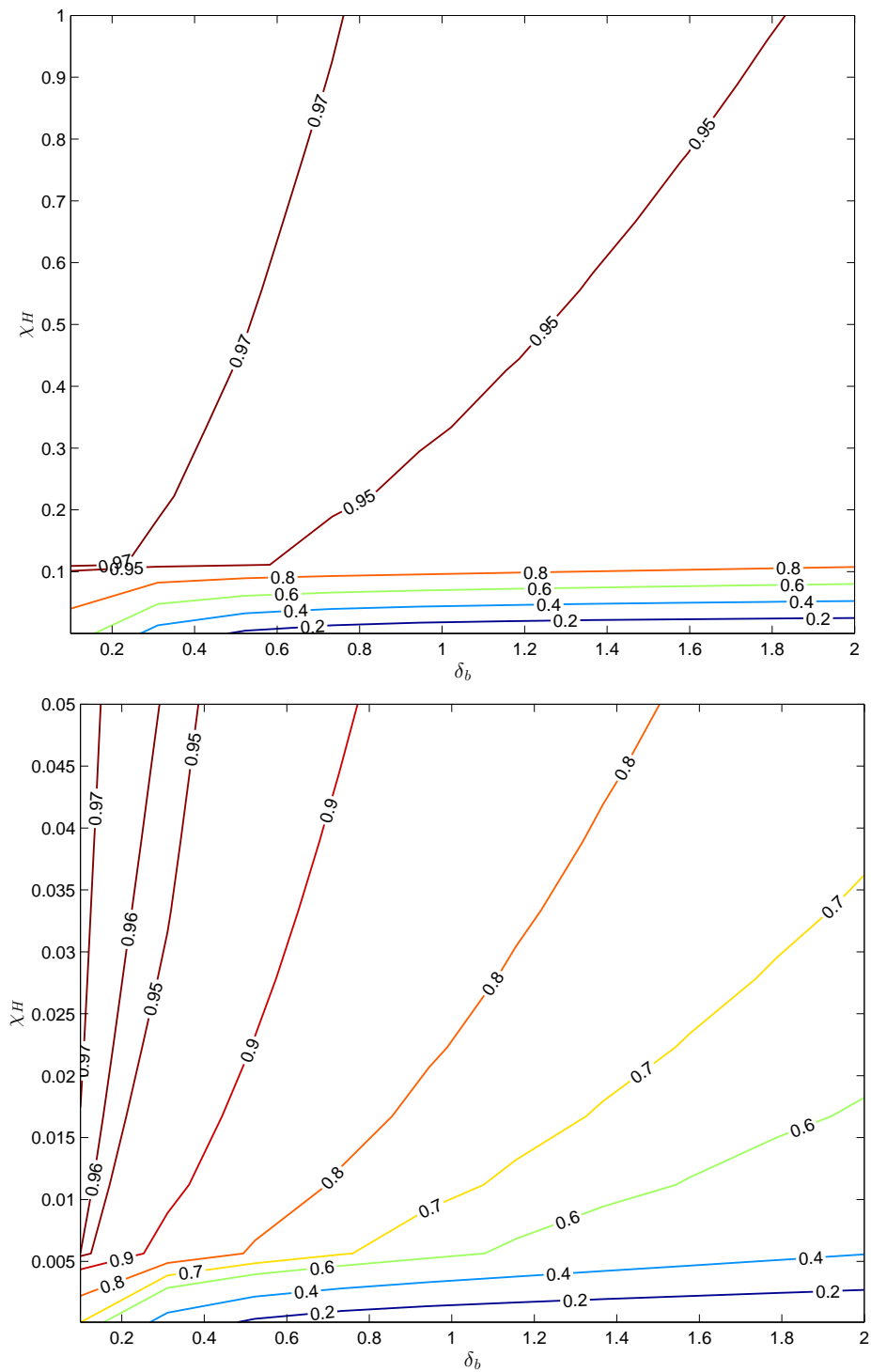
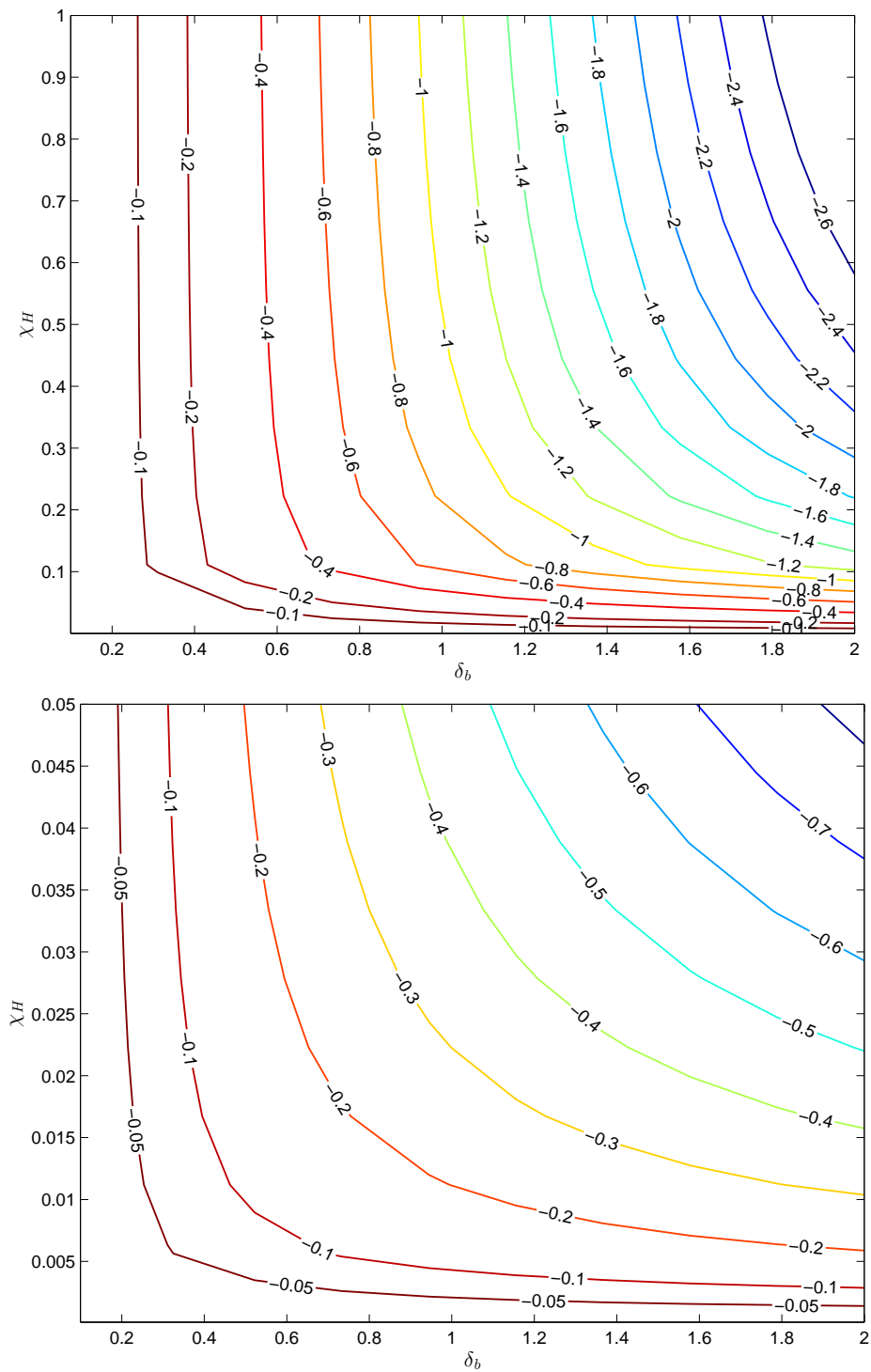


Figure 12: Bank sales of sovereign bonds after QE



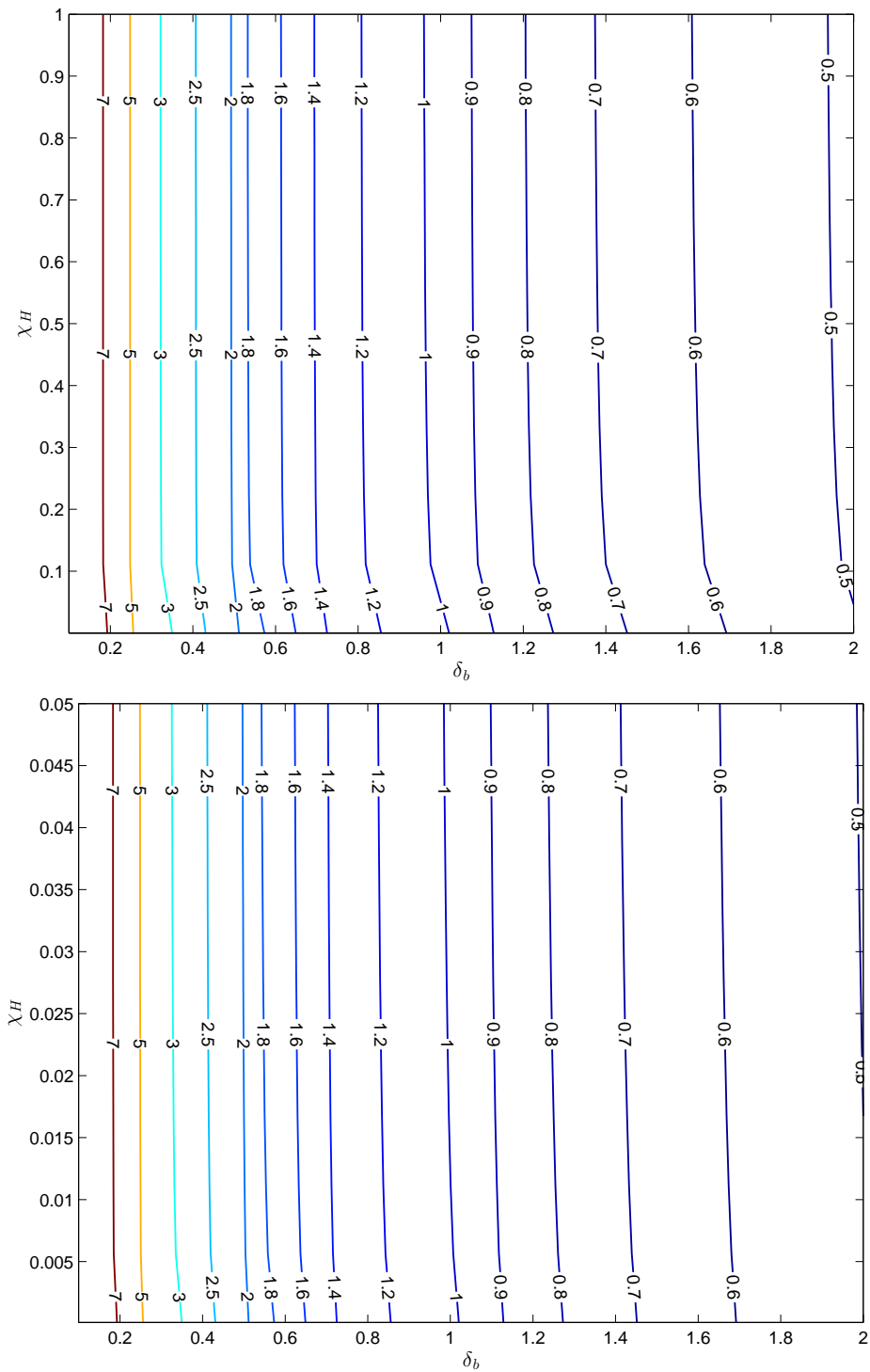
Sensitivity analysis on the average response of  $\frac{B_{B,t}}{B_{G,t}}$  over the first 6 quarters after a QE shock, for different values of  $\delta_b$  and  $\chi_H$

Figure 13: Sovereign yield multipliers of QE



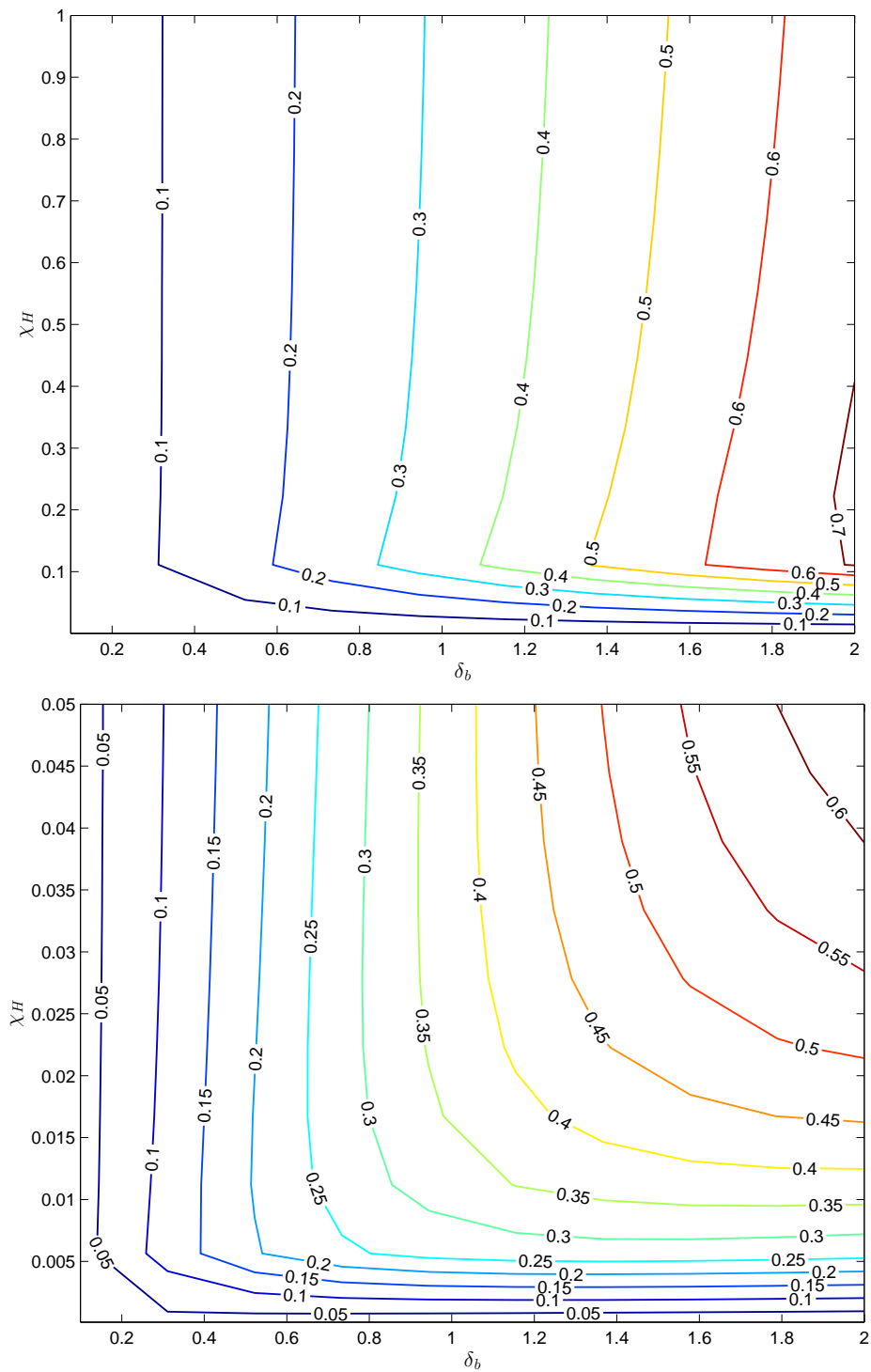
Sensitivity analysis on the initial response of sovereign yield to a QE shock for different values of  $\delta_b$  and  $\chi_H$

Figure 14: Pass-through from sovereign yield to lending rate after QE



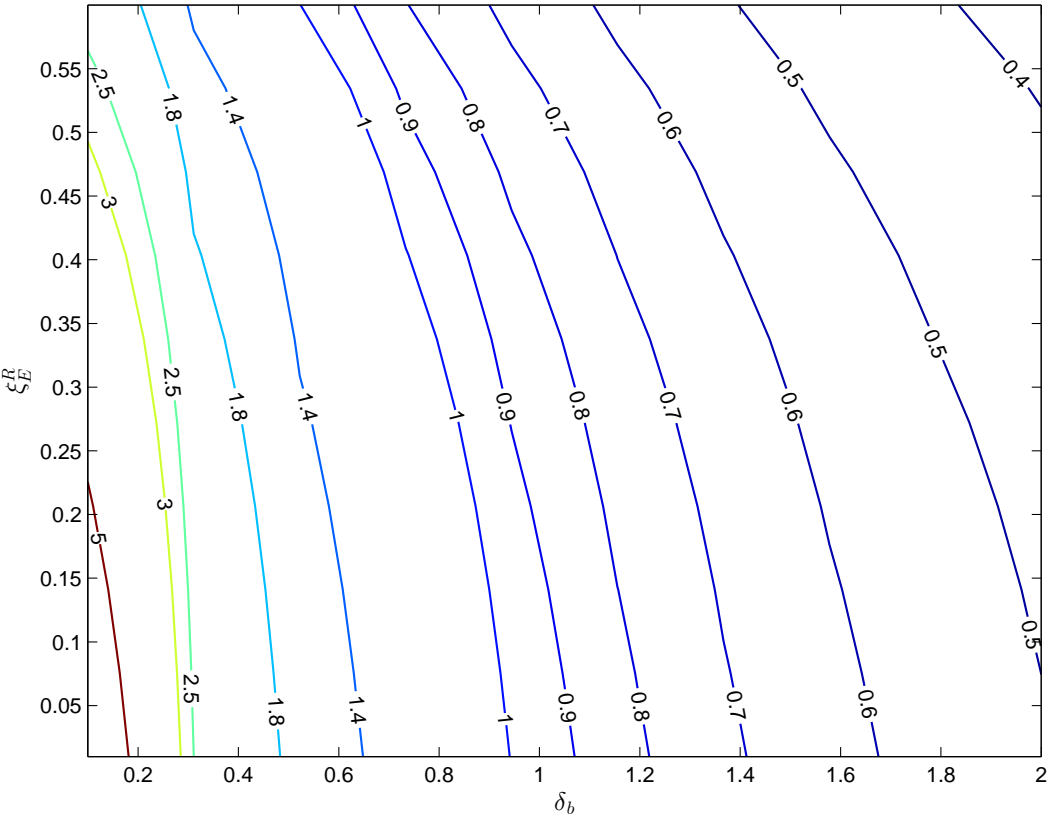
Sensitivity analysis on the initial response of  $\frac{R_{LLE,t}}{R_{G,t}}$  to a QE shock for different values of  $\delta_b$  and  $\chi_H$

Figure 15: Output multipliers of QE



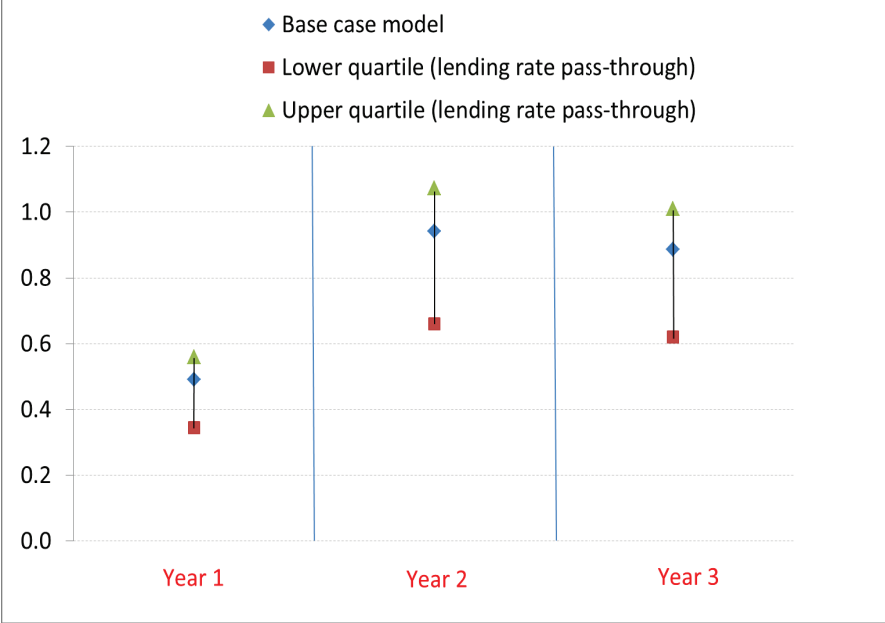
Sensitivity analysis on the response of output after 8 quarters to a QE shock for different values of  $\delta_b$  and  $\chi_H$

Figure 16: Estimation with total economy credit variables: Pass-through from sovereign yield to lending rate after QE



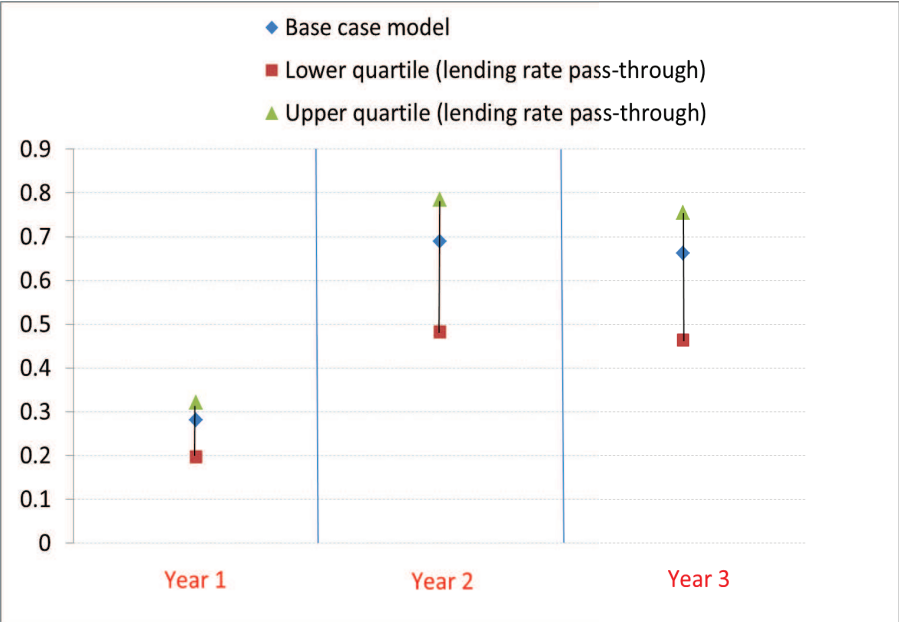
Sensitivity analysis on the initial response of  $\frac{R_{LLE,t}}{R_{G,t}}$  to a QE shock for different values of  $\delta_b$  and  $\xi_E^R$

Figure 17: GDP impact of APP for different degrees of bank portfolio rebalancing; in per cent deviation from baseline



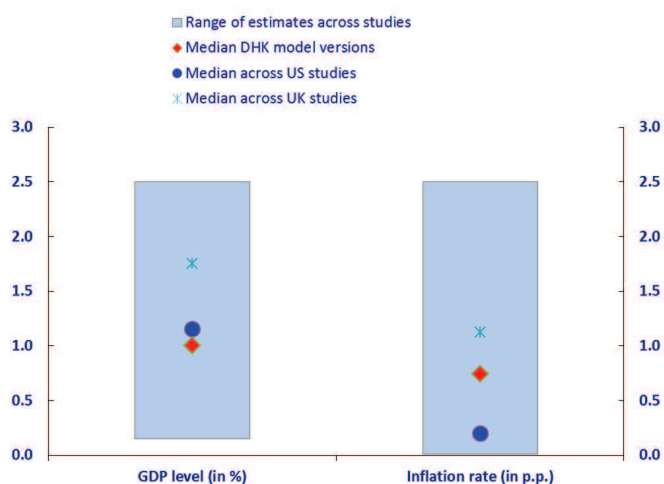
Source: own calculations

Figure 18: Inflation impact of APP for different degrees of bank portfolio rebalancing; in percentage point deviation from baseline



Source: own calculations

Figure 19: Peak impact on GDP and inflation of central bank asset purchases over 3-year horizon in comparison to US and UK-based studies (in per cent deviation from baseline and percentage point deviation from baseline)



Source: own calculations; "US studies" include Chung et al. (2011), Fuhrer and Olivei (2011), Del Negro et al. (2011), Chen et al. (2012), Gertler-Karadi (2013); "UK studies" include Joyce et al. (2011), Kapetanios et al. (2012), Bridges and Thomas (2012), Pesaran and Smith (2012).



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### Matthieu Darracq Pariès

European Central Bank; email: [matthieu.darracq\\_paries@ecb.int](mailto:matthieu.darracq_paries@ecb.int)

### Grzegorz Hałaj

European Central Bank; email: [grzegorz.halaj@ecb.int](mailto:grzegorz.halaj@ecb.int)

### Christoffer Kok

European Central Bank; email: [christoffer.kok@ecb.int](mailto:christoffer.kok@ecb.int)

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Postal address 60640 Frankfurt am Main, Germany

Telephone +49 69 1344 0

Website [www.ecb.europa.eu](http://www.ecb.europa.eu)

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