MODELING EMERGENCE OF THE INTERBANK NETWORKS

Grzegorz Hałaj and Christoffer Kok

MACROPRUDENTIAL RESEARCH NETWORK

In 2014 all ECB publications feature a motif taken from the €20 banknote.

Grzegorz Hałaj (corresponding author)
European Central Bank; e-mail: grzegorz.halaj@ecb.europa.eu

Christoffer Kok
European Central Bank

Abstract

Interbank contagion has become a buzzword in the aftermath of the financial crisis that led to a series of shocks to the interbank market and to periods of pronounced market disruptions. However, little is known about how interbank networks are formed and about their sensitivity to changes in key bank parameters (for example, induced by common exogenous shocks or by regulatory initiatives). This paper aims to shed light on these issues by modelling endogenously the formation of interbank networks, which in turn allows for checking the sensitivity of interbank network structures and hence their underlying contagion risk to changes in market-driven parameters as well as to changes in regulatory measures such as large exposures limits. The sequential network formation mechanism presented in the paper is based on a portfolio optimisation model whereby banks allocate their interbank exposures while balancing the return and risk of counterparty default risk and the placements are accepted taking into account funding diversification benefits. The model offers some interesting insights into how key parameters may affect interbank network structures and can be a valuable tool for analysing the impact of various regulatory policy measures relating to banks' incentives to operate in the interbank market.

Non-technical summary

A key characteristic about the recent financial crisis was the potential for shocks hitting specific financial institutions to spread (quickly) across the entire system – the Lehman Brothers default in the Autumn 2008 being the most prominent event. These experiences have therefore fostered a wealth of studies on financial contagion, many of which apply network theory, to better understand the inherent riskiness of the financial system via the interconnectedness of financial institutions. A key finding in the literature is that contagion risks in for example the interbank market is very much determined by the structure of the network through which banks are interconnected. In other words, the scope for contagious losses following an idiosyncratic or system-wide shock depends on the number of connections and the centrality of affected institutions in the network. However, little is known about how financial networks are formed and about their sensitivity to changes in key bank parameters (for example, induced by common exogenous shocks or by regulatory initiatives).

Focusing on interbank market contagion, this study adds to this literature by modelling endogenously the formation of interbank networks, which in turn allows for checking the sensitivity of interbank network structures and hence their underlying contagion risk to changes in market-driven parameters as well as to changes in regulatory measures. Especially the latter dimension is of relevance from a macro-prudential policy perspective. An in-depth knowledge about how network structures are affected by specific macro-prudential policy measures is crucial for assessing the effectiveness and relevance of such policies. In this paper, we focus on a macro-prudential (and micro-prudential) policy instrument that is embedded in the new Basel standards and are directly related to interbank interconnectedness; namely, the large exposures limits. However, the modelling approach could also be applied to evaluate other types of prudential policy measures, for instance risk weights to other financial institutions.

The methodology proposed in this paper for modelling endogenous network formation is related both to so-called agent-based modelling as well as to game theoretical concepts. Specifically, the sequential network formation mechanism presented in the paper is based on a portfolio optimisation model whereby banks allocate their interbank exposures while balancing the return and risk related to levels and volatility of market interest rates and counterparty default risk and the placements are accepted taking into account funding diversification benefits. More precisely, the interbank network is an outcome of a sequential game played by banks trying to invest on the interbank market and to borrow interbank funding. We take a sample of 80 large EU banks and based on their balance sheet composition assume that they optimise their interbank assets taking into account risk and regulatory constraints as well as the demand for the interbank funding. This optimisation process results in a preferred interbank portfolio allocation for each bank in the system. For what concerns the funding side, banks define their most acceptable structure of funding sources with the objective of limiting refinancing (rollover) risk. Banks meet in a bargaining game in which the supply and demand for interbank lending is determined by allowing banks to marginally deviate from their optimal interbank allocations and the prices they offer on those. In order to account for the quite complex aspects of the interbank market formation we propose a sequential optimisation process, each step of which consisting of four distinctive rounds. The sequential optimisation process is repeated in an iterative manner until a satisfactory level of convergence (i.e. full allocation of interbank assets) is achieved.

The model offers some interesting insights into key parameters affecting interbank network

structures and can be a valuable tool for analysing the impact of various regulatory policy measures relating to banks' incentives to operate in the interbank market.

We find that the endogenous network model produces realistic, and complex, network configurations. Our approach allows for assessing the derived interbank structures against relevant network benchmarks and for verifying their sensitivity to key driving parameters, such as correlation (Q), loss given default (λ), investment risk aversion (κ), funding risk aversion (κ^F), interbank interest rate elasticity (α) and capital allocated to the interbank portfolios (e^I). In general, we find that interbank asset correlation appears to have a sizable impact on the structure of the endogenous network formation whereby high correlation translates into low diversification potential which tends to lead to a concentration of exposures in a few banks. In general, the interbank network structures derived by the model vary in a limited way to changes in key parameters and the variability that is observed tends to have intuitive explanations. For instance, increasing investment risk aversion leads on average to the decrease in the number of linkages. The structure of the endogenously derived interbank network (the number of linkages between banks) is also found to be sensitive to deposit interest rate elasticity and the level of allocated capital to the interbank portfolios.

In terms of policy experiments, we examine the impact of the network structures and the contagion risks related to internal risk limit systems and regulatory instruments aimed at limiting banks' risk to counterparty exposures, such as the Credit Valuation Adjustment charge on the capital allocated to the interbank asset portfolios and the regulatory large exposure limits already embedded in current regulatory frameworks. The EU directive on prudential supervision from June 2013 introduces special risk weights for exposures to financial institutions as a new macro-prudential tool and the concept of Credit Valuation Adjustment can be applied to test performance of the risk sensitive risk weights for the exposures to financial institutions. Apart from the evaluation of relevant regulatory instruments, the endogenous network model can also be employed in macro-prudential analysis in a broader sense. Namely, it can improve the assessment of the impact of the materialisation of systemic risks on the banking sector while taking into account the dynamic network formation resulting from changes in key real economic and financial variables. In particular, as an adverse shock to the economy would typically result in the deterioration of some counterparties' creditworthiness (e.g. as reflected in banks' CDS spreads) and hence affect the optimal allocation of interbank assets and liabilities. In other words, a given adverse scenario is likely to result in a rewiring of linkages in the interbank market and hence to also affect the interbank contagion risks that ultimately may result from such adverse shocks. Our findings suggest that in particular the setting of large exposure limits can have a pronounced impact of the level of contagion risks embedded in the interbank market, whereas the effects from the CVA appear more limited and ambiguous. All in all, while the reported results obviously hinges on the specific characteristics of the banks included in the network system and on the specific adverse scenarios considered, the overriding conclusion from these policy experiments is that macro-prudential policies can make a significant difference through their impact on the network formation and ultimately on the risk of interbank contagion to adverse shocks. From this perspective, the modelling approach presented in this paper can be employed for conducting impact assessments of selected macro-prudential policy instruments and in this way help inform the calibration of such tools.

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1 Introduction

The interbank market was one of the main victims of the financial crisis erupting in 2007. The crisis led to a general loss of trust among market participants and resulted in severe interbank market disruptions. Moreover, failures of some key market players triggered concerns about risks of interbank contagion whereby even small initial shocks could have potentially detrimental effects on the overall system. As a results of these concerns, and also reflecting a broader aim of making the financial sector more resilient, in recent years financial regulators have introduced various measures that aim at mitigating (and better reflecting) the risks inherent through the bilateral links between banks in the interbank network. These international reform initiatives range inter alia from limits on large counterparty exposures, higher capital requirements on counterparty exposures related to the OTC derivatives and requirements to settle standardised OTC derivatives contracts via central counterparty clearing (CCP) houses. While it seems plausible that these initiatives should help alleviate contagion risks in the interbank market, there is still only little research aiming to quantify and understand the effects of these reforms on network structures and the contagion risk that might emerge from these structures.

Against this background, this paper aims to help fill this gap in the literature by improving our understanding of risks stemming from bank interconnectedness and how specific regulatory measures can affect interbank network structures and hence contagion risk. When trying to assess how different policy measures are likely to impact on interbank network formation, it will be crucial to also take into account how banks could be expected to react to these measures. For this reason, the starting point of the analysis presented in this paper is to establish a setting whereby network structures emerge on the basis of banks' endogenous reactions to changes in the environment affecting their optimal asset and liability mix (and hence also their decision to lend in the interbank market).

For this purpose, the paper presents a model to derive interbank networks that are determined by certain characteristics of banks' balance sheets, the structure of which is assumed to be an outcome of banks' risk-adjusted return optimisation of their assets and liabilities. The model of bank balance sheet optimisation is combined with the random network generation technique presented in (Hałaj and Kok, 2013b). This allows us to study the endogenous network formation based on optimising bank behaviour.

The model can thus help to understand the foundations of topology of the interbank network. It furthermore provides a tool for analysing the sensitivity of the interbank structures to the heterogeneity of banks (in terms of size of balance sheet, capital position, general profitability of non-interbank assets, counterparty credit risk) and to changes of market and bank-specific risk parameters. Such parameter changes could for example be due to regulatory policy actions (for example, pertaining to capital buffers as well as the size and diversity of interbank exposures) aiming at mitigating systemic risk within the interbank system. The framework developed in this paper can therefore be used to conduct a normative analysis of macro and micro-prudential policies geared towards more resilient interbank market structures.

The paper is related to research on network formation which was only recently pursued in finance. Understanding the emergence process of the interbank networks can be critical to control and mitigate these risks. Endogenous networks (and their dynamics) are a difficult problem since the behaviour of the agents (banks in particular) is very complex. In other areas of social studies, the network formation was addressed by means of network game techniques

(Jackson and Wolinsky, 1996). In financial networks, researchers also applied recently game theoretical tools (Acemoglu et al., 2013; Cohen-Cole et al., 2011; Babus and Kondor, 2013; Bluhm et al., 2013; Gofman, 2013) or portfolio optimisation (Georg, 2011).[1] For instance, Acemoglu et al. (2013) shows that the equilibrium networks generated via a game on a space of interbank lending contracts posted by banks can be socially inefficient since financial agents "do not internalize the consequences of their actions on the rest of the network".[2] In Cohen-Cole et al. (2011) banks respond optimally to shocks to incentives to lend. Bluhm et al. (2013) approach to modelling endogenous interbank market is closely related to ours. However, the main distinctions from our approach are: risk neutrality of banks, riskiness of the interbank assets introduced to the model only via capital constraints and not including funding risk as having a potential impact on the interbank structure. Castiglionesi and Lavarro (2011) presented a model with endogenous network formation in a setting with micro-founded banking behaviour.[3] These advances notwithstanding, owing to the complexity of the equilibrium-based studies of network formation, agent-based modeling of financial networks is one promising avenue that can be followed (Markose, 2012; Grasselli, 2013).

This paper adds to this strand of the literature by taking a model of portfolio optimising banks to a firm-level data set of European banks, which in turn allows us to study within an endogenous network setting the impact of plausible internal limit systems based on credit valuation adjustments (CVA) accounting for counterparty credit risk (Deloitte and Partners, 2013) and various regulatory policy measures on interbank contagion risk. Apart from the asset-liability optimising behaviour that we impose on the agents (i.e. the banks), our network formation model also incorporates sequential game theoretical elements. If the portfolio optimisation of interbank investment and interbank funding does not lead to a full matching of interbank assets and liabilities, banks will engage in a bargaining game while taking into account deviations in their optimal levels of diversification of investment and funding risks (see e.g. Rochet and Tirole (1996)).[4] The sequence of portfolio optimisation and matching games is repeated until the full allocation of interbank assets at the aggregate level has been reached. The outlined mechanism is also related to studies on matching in the loan market (see e.g. (Fox, 2010; Chen and Song, 2013)). Furthermore, to further reduce mismatches between banks' funding needs and the available interbank credit emerging from the portfolio optimising choices, we introduce an interbank loan pricing mechanism that is related to models of money market price formation (see e.g. (Hamilton, 1998; Ewerhart et al., 2004; Eisenschmidt and Tapking, 2009)). Importantly, as argued by ad A. Kovner and Schoar (2011) such pricing mechanisms can be expected to be more sensitive to borrower characteristics (and risks) during periods of stress. The model presented here would be able to account for such effects.

The paper is structured as follows: Section 2 presents the model of network formation under optimising bank behaviour. In Section 3 some topology results from the network simulations are presented, while in Section 4 it is illustrated how the model can be applied for studying various macro-prudential policy measures. Section 5 concludes.

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[1]Some earlier contributions incorporating multi-agent network models, albeit with fixed network and static balance sheet assumptions, include (Iori et al., 2006; Nier et al., 2007).

[2]See also Gai and Kariv (2003) for an earlier contribution.

[3]Other studies in this direction include (Babus, 2011; Castiglionesi and Wagner, 2013).

[4]While not explicitly taken into account in this paper, this is related to the literature on interbank lending where due to asymmetric information banks are not able to perfectly monitor their peers. Such information asymmetries may be reinforced by adverse shocks as for example experienced during the recent financial crisis, see Heider et al. (2009).

Figure 1: The sequential four round procedure of the interbank formation
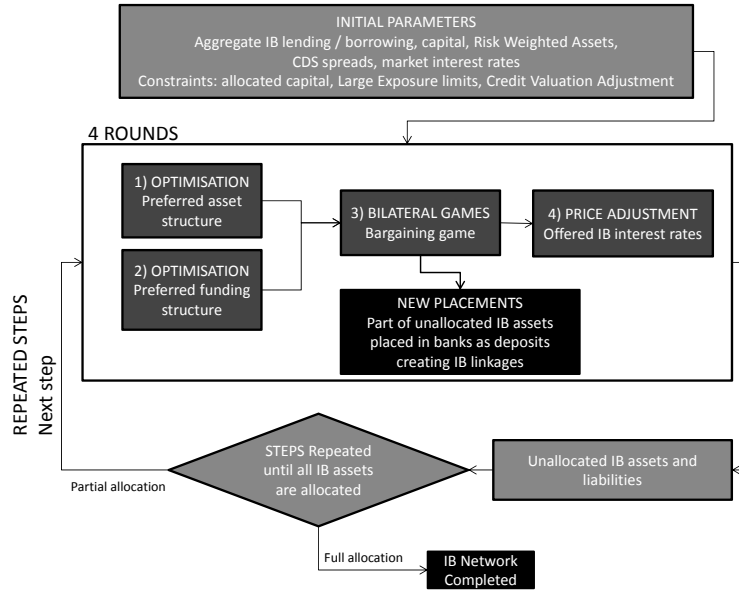
2 Model

2.1 Outline

The interbank network described in this paper is an outcome of a sequential game played by banks trying to invest on the interbank market and to borrow interbank funding. Banks optimise their interbank assets taking into account risk and regulatory constraints as well as the demand for the interbank funding and propose their preferred portfolio allocation. For what concerns the funding side banks define their most acceptable structure of funding sources with the objective to limit refinancing (rollover) risk. Banks meet in a bargaining game in which the supply and demand for interbank lending is determined. In order to account for the quite complex aspects of the interbank market formation we propose a sequential optimisation process, each step of which consisting of four distinctive rounds (see the block scheme in figure 2.1).

There are three main general assumptions of the model:

1. Banks know their aggregate interbank lending and borrowing as well as those of other banks in the system. It is a public information for all the banks in the sample.

2. Banks optimise the structure of their interbank assets, i.e. their allocation across counterparties.

3. Banks prefer diversified funding sources in terms of rollover risk (i.e. liquidity risk related to the replacement of the maturing interbank deposits).

This first, rather strong assumption has its motivation in the stable over time fraction of

interbank assets to total liabilities, confirmed empirically in a sample of 90 largest EU banks.[5] In theory, part of that assets and liabilities, in particular with the shortest maturities, can be volatile since it reacts to volatile banks' liquidity needs. However, the interbank investment portfolio and interbank funding portfolio may be much more stable since their volumes should result from a general asset-liability planning within the ALM process defining, inter alia a target for product mix of assets and funding sources and income parameters. The targets are managed by a system of limits on exposures or pricing offered by various business lines taking a holistic perspective on the balance sheet and taking into account market conditions, general goals set by executive board and strategies set by the ALCO. We do not model the outcomes of a preceding ALM process and treat the volumes of interbank assets and liabilities as given and deterministic. The uncertainty in the model stems from interest risk, counterparty credit risk and funding roll-over risk. The second assumption follows the standard portfolio choice theory. Optimisation is constraint by regulatory liquidity and capital rules and the relationship lending; banks are assumed to optimise their portfolio in a set of counterparties with whom they built up relationship lending. The third assumption refers to the set of banks' counterparties. Based on lending relationship, each bank has a subgroup of partners on the interbank market with whom it is likely to trade. It is reasonable to assume that banks try to minimise funding risk in their subgroups. Notably, there is some empirical evidence (Bräuning and Fecht, 2012) that the relationship lending may impact pricing of interbank loans and consequently also funding structure. All in all, the decision about the funding structure is a function of diversification needs and build-up of relationship lending.

In the *first round*, banks specify the preferred allocation of interbank assets by maximising the risk-adjusted return from the interbank portfolio. Banks are assumed to be risk averse which follows the approach taken in capital management, whereby accepted levels of exposure are commonly managed via RAROC[6] and RARORAC ALM indicators (Adam, 2008). In the traditional banking literature (Baltensperger, 1980; Boyd and Nicoló, 2005; Pelizzon and Schaefer, 2005) banks are assumed to take investment decisions under risk neutrality assumption.[7] Risk impacts banks' decisions via regulatory constraints. We follow an approach from a different strand of literature (Howard and Matheson, 1972; Danielsson et al., 2002; Cuoco and Liu, 2006) where decisions are risk sensitive. In fact, Baltensperger (1980) admits that joint modelling of loan or deposit volumes and diversification within these two portfolios can be approach rather by utility maximisation (implying risk sensitive decisions) than expected profit maximisation.

In this optimisation process, each bank first draws a sample of banks according to a pre-defined probability that a bank is related to another bank. The probability map was developed by Hałaj and Kok (2013b) using the geographical breakdown of banks' exposures disclosed during the EBA 2011 capital exercise. Second, they make offers of interbank placements at a current market rate trying to maximise the return adjusted by investment risk taking into account:

- expected interest income;

- risk related to interest rate volatility and potential default of counterparts, and correla-

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[5]A standard deviation of quarterly ratios of interbank assets or interbank liabilities to total assets amounts on average to 2.5%.

[6]Risk-Adjusted Return on Capital and Risk-Adjusted Return on Risk-Adjusted Capital

[7]See also Elyasiani et al. (1995); Balasubramanyan and VanHoose (2013)

tion among risks;

- internal risk limits for capital allocated to the interbank portfolio, based on the Credit Valuation Adjustment (CVA) concept[8] and regulatory constraints in form of the Large Exposure limits specifying the maximum size of an exposure in relation to the capital base;

- exogenous volume of total interbank lending.

Notably, the structure rather then the aggregate volume of lending is optimised. The aggregate interbank lending and borrowing of banks in the model is exogenous.

Obviously, the recipients of the interbank funding can have their own preferences regarding funding sources. Therefore, in the *second round* of the model, after the individual banks' optimisation of interbank assets, banks calculate their optimal funding structure, choosing among banks that offered funding in the first round. They decide about the preferred structure based on the funding risk of the resulting interbank funding portfolios. There is a clear motivation behind separating the optimisation process into interbank investment and funding choice. The model operates on two distinct types of portfolios: investment and funding. Asset and liability management process may give different priorities to management actions targeted at interbank assets and at interbank liabilities. For instance, banks may be more sensitive to the risk in their interbank funding portfolios since they may constitute a sizeable part of their funding sources. The setup of the model gives flexibility to parameterise risk aversion differently for optimisation of interbank assets and interbank liabilities but in the applications we use a common value for the two risk aversion parameters.

The offers of interbank placements may diverge from the funding needs of the other side of the interbank market. In the *third round* we therefore assume that pairs of banks negotiate the ultimate volume of the interbank deposit. We model these negotiations by means of a *bargaining game* in which banks may be more or less willing (or sensitive from an utility perspective) to deviate from their optimisation-based preferred asset-liability structures. Notably, also at this round banks take into account their risk and budget constraints.

Since interbank asset and interbank funding optimisation followed by the game may not result in full allocation of the predefined interbank assets and in satisfaction of all the interbank funding needs the prices on the interbank market may be adjusted. In the *fourth round* banks with an open funding gap are assumed to propose a new interest rate for the new interbank investors depending on the relative size of the gap to their total interbank funding needs. Implicitly, we do not model the role of the central bank which normally stands ready to provide liquidity.

The four consecutive rounds are repeated with a new drawing of banks to be included into sub-samples of banks with which each bank prefers to trade. Consequently, each bank enlarges the group of banks considered to be their counterparties on the interbank market and proposes a new preferred structure of the interbank assets and liabilities for the unallocated part in the previous step. In this way, the interbank assets and liabilities are incrementally allocated among banks.

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[8]This CVA element is not to be mistaken with the CVA capital charge on changes in the credit spread of counterparties on OTC derivatives transactions. However, the line of calculation is similar. Some banks use CVA internally to render exposure limits sensitive to the counterparty risk in a consistent, model-based way (Deloitte and Partners, 2013).

Modelling the network formation process in sequential terms, is obviously somewhat stylised as in reality banks are likely to conduct many of the steps described here in a simultaneous rather than sequential fashion. At the same time, the step-by-step approach is a convenient way of presenting the complex mechanisms that determine the formation of interbank linkages, which may realise in a very short time-span, even only several tick long.

The following subsections describe in details how the endogenous networks are derived. Some important notations used thereafter are introduced below.

Notation: $\bar{\mathbb{N}}$ stands for set $\{1, 2, \ldots, N\}$, '$*$' denotes entry-wise multiplication, i.e. $[x_1, \ldots, x_N] *$ $[y_1, \ldots, y_n] := [x_1 y_1, \ldots, x_N y_N]$, '$^\top$' is transposition operator and – for matrix X – $X_{\cdot j}$ denotes j^{th} column of X and $X_{i \cdot}$ denotes i^{th} row of X, $\#\mathcal{C}$ – number of elements in a set $\mathcal{C}$, $\mathbb{I}_A$ denotes indicator function of a set A.

2.2 Banks

First, a description of banks' balance sheet structures, interbank assets and liabilities in particular, is warranted. It is supposed that there are N banks in the system. Each institution i aims to invest a_i volume of interbank assets and collect l_i of interbank liabilities. These pre-defined volumes are dependent on various exogenous parameters. For instance, individual banks' aggregate interbank lending and borrowing can be an outcome of asset and liability modeling (ALM).[9] The interest rates paid by interbank deposits depend on:

- some reference market interest rates r^m (e.g. the 3-month offered interbank rate in country m),

- a credit risk spread (s_i) reflecting the credit risk of a given bank i,

- a liquidity premium q_i referring to the general market liquidity conditions and bank i's access to the interbank market[10],

- Loss Given Default (LGD) related to the exposure, denoted λ.

The LGD is assumed to be equal for all banks and exposures and amounting to 40%. We do not model maturity structure of the interbank assets and liabilities in the current setting, i.e. all interbank assets and liabilities have the same maturity.

The credit spread s_i is translated into a bank-specific interest rate paid by bank i to its interbank creditors – r_i. It is based on the notion of equivalence of the expected returns from interbank investment to a specific bank and from investing into the reference rate r^m,

$$r^m + q_i \equiv r_i p_i \lambda + (1 - p_i) r_i, \tag{1}$$

where p_i denotes marginal probability of default on the interbank placement extended to bank i and is calculated as

$$p_i := s_i / \lambda$$

Interest rate r_i can be interpreted as a rate that realises the expected return of r^m given the default risk captured by the spread s_i.[11] We use a very basic approximation of the default

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[9]Georg (2011) or Hałaj (2013) developed frameworks based on the portfolio theory to optimise the structure of investments and funding sources that could be followed.

[10]We assume for simplicity that $q \equiv 0$ while indicating how liquidity can be captured in the framework.

[11]Currency risk related to the cross-border lending between countries with different currencies is not addressed in the model.

probability p_i derived from the spread s_i but still we are able to gauge differences in default risk among bank and the definition of p_is is not key in developing the modelling framework for endogenous interbank networks.

Moreover, the cost – or a return from the interbank placement perspective – is risky. The riskiness is described by a vector $\sigma := [\sigma_1 \ldots \sigma_N]^\top$ of standard deviations of historical (computed) rates r_i and correlation matrix Q of these rates calculated from equation 1 taking into account time series of interbank rates and CDS spreads.[12] The riskiness stems from the volatility of market rates and variability of default probabilities. Likewise, correlation is related to:

- the common reference market rate for banks-debtors in one country or co-movement of reference rates between countries which the cost of interbank funding is indexed to;

- to the correlation of banks' default risk.[13]

Banks are also characterised by several other parameters not related to the interbank market but important in our framework from the risk absorption capacity perspective.

- capital e_i; and capital allocated to the interbank exposures e_i^I (e.g. economic capital budgeted for treasury management of the liquidity desk);

- risk weighted assets RWA_i – similarly, RWA_i^I risk-weighted assets calculated for the interbank exposures. This may depend on the composition of the portfolio, i.e. exposure to risk of different counterparts.

All the aforementioned balance sheet parameters are used in the following subsections to define banks' optimal investment and funding programs.

2.3 First round – optimisation of interbank assets

Each bank is assumed to construct its optimal composition of the interbank portfolio given market parameters, risk tolerance, diversification needs (also of a regulatory nature) and capital constraints (risk constraints including the Credit Valuation Adjustments (CVA) introduced within Basel III).

2.3.1 Prerequisites

Let L_{ij} denote an interbank placement of bank j in bank i. Bank risk aversion is measured by $\kappa \geq 0$. CVA is assumed to impact the economic capital and, consequently the potential for interbank lending.[14] For simplicity, we assume that an interbank exposure of volume L_{ij} requires $\gamma_i L_{ij}$ to be deducted from capital e_j^I, for γ_i being bank specific CVA factor, to account for the market based assessment of the credit risk related with bank i. A possible way to calculate CVA is presented in the appendix. The parameter γ can also be viewed

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[12]Other measures can be applied, e.g. VaR-based, reflecting the tail risks or some multiples of standard deviation (2-, 3-times standard deviation).

[13]Reason: banks operate on similar markets, have portfolios of clients whose credit quality depends on similar factors, their capital base is similarly eroded by the deteriorating market conditions, etc.

[14]BCBS (2011) stipulates rules to account for the counterparty risk in the regulatory capital. From that viewpoint and for consistency, e_i^I can also be treated as regulatory capital.

as a risk sensitive add-on to the risk weights applied to the interbank exposures. The later interpretation gives rise to the macro-prudential application of the CVA concept and it is illustrated in section 4.

Banks are assumed to trade *most likely* with banks with which they have an established customer relationship. This is proposed to be captured by banks' geographical proximity as well as the international profile of the bank. It is assumed that banks are more likely to trade with each other if they operate on the same market. The probability map (P) of interbank linkages, introduced by Hałaj and Kok (2013b) and calculated based on the banks' geographical breakdown of exposures, is used to sample banks with which a given bank intends to trade. The maturity is standard and common across the market and the rate is determined by the reference rate and the credit quality of the borrower (see identity 1).

2.3.2 Procedure

Since the formation of the interbank network is modelled in a sequential way, we set the initial values of banks' assets and liabilities to be matched on the interbank market at the steps $k = 1,\ 2,\ 3...$ and of a structure of the interbank network, i.e. for $k = 0$

$$\bar{l}^0 = l,$$

$$\bar{a}^0 = a,$$

$$\bar{L}^0 = 0^{N \times N}$$

Vectors $\bar{a}^k$, $\bar{l}^k$ denote banks' aggregate interbank lending and borrowing which is still not allocated among banks before step k. A matrix $\bar{L}^k$ denotes the structure of linkages on the interbank market created up to the step k of the algorithm. Additionally, for notational convenience we denote $\bar{B}_j^0 = \emptyset$ the initial empty set of banks in which a given bank j intends to invest.

At step k, bank j draws a sample of banks $B_j^k \subset \bar{\mathbb{N}}/\{j\}$. More specifically, each counterparty i of the bank j is accepted with probability P_{ij}.[15] Banks from the set B_j^k are assumed to enlarge the set of investment opportunities of bank j, i.e. $\bar{B}_j^k = \bar{B}_j^{k-1} \cup B_j^k$. At step k, the bank considers (optimally) extending interbank placements to banks $\bar{B}_j^k$.

Bank j is assumed to maximise the following risk-adjusted return form the interbank investment:

$$J(L_{1j}^k, \ldots, L_{Nj}^k) = \sum_{i|i \neq j} r_i^k L_{ij}^k - \kappa_j (\sigma * L_{\cdot j}^k)^\top Q (\sigma * L_{\cdot j}^k), \qquad (2)$$

where $r^1 \equiv r$ and rates r^k in steps $k \geq 2$ of the endogenous network algorithm can vary according to adjustments related to the funding needs of banks that have problems with finding enough interbank funding sources (see subsection 2.6). The vector of risk measures σ was defined in section 2.2. The interest rates r^k paid by the interbank deposits are the transaction rates defined by equation 1 and the risk – both related to market interest rate risk and default risk – is captured by the covariance $(\sigma * L_{\cdot j}^k)^\top Q (\sigma * L_{\cdot j}^k)$.

Given the drawn sample $\bar{B}_j^k$, the set of admissible strategies is

$$\mathcal{A}_j^k: = \{y \in \mathbb{R}_+^N | \text{for all } n \notin \bar{B}_j^k \ y_n = 0\}$$

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[15]It can be thought of as a trader of bank j calling they counterparties randomly but potentially with higher chance of selecting banks more closely related in trading with j.

subject to further constraints related to risk and regulations. $\mathcal{A}_j^k$ can be interpreted as set of bank j's actions allowing for investing only in the drawn subsample. Obviously, starting from a different seed the sampling may cover any configurations of banks which are allowed by the probability map P.

The maximum value of the functional (2) always exists; however, it may not be unique. This may happen if there are banks with the same characteristics of return and risk. Theoretically is is highly unlikely, however in practice, we use peers' parameters for banks with unavailable individual data on interbank interest rates and credit default spreads. Having two identical banks with respect to return and risk parameters means that other market participants can be indifferent to which of them to lend. In our setting, only the size of banks and their customer relationship P matter. Therefore, we calculate the theoretically optimal breakdown of the interbank placements taking into account a random representative for a group of identical banks and then average out the results.

In the baseline setting of the endogenous networks we do not restrict the size of exposures a bank is allowed to hold against another bank. However, in practice banks are constrained by so-called "large exposure limits" (LE).[16] To account for such regulations, we impose one additional condition:

each exposure should not exceed $\chi > 0$ fraction of the total regulatory capital.

In the current EU Capital Requirements Directive χ is assumed to be equal to 0.25. Moreover, there is the additional requirement that the sum of all exposures that (individually) exceed 10 per cent of the capital should not surpass 800 per cent of capital. The second requirement would introduce a nonlinearity in the set of constraints in our model and we decide not to include it. However, the large exposure limit imposed on the individual interbank placement proves to be a more stringent constraint and its severity can be tuned and tested by shifting it sufficiently below 25%. All in all, our baseline setup of the model excludes the large exposure limit constraints which are introduced for sensitivity analysis of the network structures.[17]

The maximisation of the functional (2) is subject to some feasibility and capital constraints.

1. budget constraint – $\sum_{j|j \neq i} L_{ij}^k = \bar{a}_j^k$ and $L_{jj}^k = 0$, where – just to remind – $\bar{a}_i^0$ is exogenously determined;

2. counterpart's size constraint – $L_{ij}^k \leq \bar{l}_i^k$;

3. capital constraint – $\sum_{i|i \neq j} \omega_i(\bar{L}_{ij}^k + L_{ij}^k) \leq e_j^I - \gamma \cdot (\bar{L}_{\cdot j}^k + L_{\cdot j}^k)$ or equivalently $\sum_{i|i \neq j}(\omega_i + \gamma_i)(\bar{L}_{ij}^k + L_{ij}^k) \leq e_j^I$;

4. (optionally) large exposure limit constraint – $(\bar{L}_{ij}^k + L_{ij}^k) \leq \chi e_j$.

Given the risk constraints on the one hand and the riskiness of the interbank lending on the other hand, it may not be possible for a bank i to place exactly $\bar{a}_i^k$ interbank deposits in total in step k. Therefore, the budget constraint may not be plausible – as a consequence the bank i should consider lending less.[18] We apply the following compromising iterative procedure.

---

[16]See Article 111 of Directive 2006/48/EC that introduces the limits.

[17]More discussions of the LE impact on the structure of the interbank system can be found in Hałaj and Kok (2013a).

[18]In an extreme case, also the LE constraints may prove to be too severe. The system is not solvable if there exists a pair (k, j) such that $\chi \sum_{j|j \neq k} e_j < l_k$, which means that bank k is not able to find the predefined volume l_k of the interbank funding.

The bank is assumed to solve a problem with budget constraint $\bar{a}_i^k$ replaced with $\bar{a}_i^k - \Delta\bar{a}_i^k$, for some (small enough and positive) $\Delta\bar{a}_i^k$. If the resulting optimisation has still too stringent constraints, then the bank continues with $\bar{a}_i^k - 2\Delta\bar{a}_i^k$, $\bar{a}_i^k - 3\Delta\bar{a}_i^k$,... until $\bar{a}_i^k - n_i\Delta\bar{a}_i^k$, with $n_i \in \bar{\mathbb{N}}$ such that $\bar{a}_i^k - n_i\Delta\bar{a}_i^k > 0$ and

$$\sum_{i|i\neq j} L_{ij}^k = \bar{a}_i^k - n_i\Delta\bar{a}_i^k$$

is a feasible constraint.[19] The procedure can be interpreted as banks' gradual adjustments the total interbank assets until the risk requirements are satisfied.

To simplify notation, the outcome of all banks' optimisation of the interbank assets is a matrix $L^{I,k}$:

$$
\begin{matrix}
(= \bar{a}^k - n\Delta\bar{a}^k) \\
\begin{bmatrix} L_{11}^{I,k} & \dots & L_{1N}^{I,k} \\ \vdots & \ddots & \vdots \\ L_{N1}^{I,k} & \dots & L_{NN}^{I,k} \end{bmatrix} \quad (\neq '?')
\end{matrix}
\tag{3}
$$

The sum of elements in a given column j of matrix $L^{I,k}$ equals $\bar{a}_j - k_j\Delta\bar{a}_j$ but the sum of elements in row i may also exceed $\bar{l}_i^k$. This may happen if a bank is a particularly attractive borrower on the market given its level of counterparty credit risk. This can also be interpreted as tension between demand for interbank funding resulting from the overall ALM process and the supply contingent on the banks' optimal interbank investment plans.

2.4 Second round – accepting placements according to funding needs

The funding side of the market is assumed to accept placements according to their funding structure preferences, while applying the funding diversification risk criteria.

In order to quantify the funding risk, let us suppose that X_j is a random variable taking values 0 and 1: 0 with probability p_j inferred from the credit default spreads s_j (see 2.2) and 1 with probability $1 - p_j$. Obviously, p_j is also a random variable. For a uniformly distributed u_j on the interval $[0,1]$, independent of p_j and u_i for $i \neq j$, X_j has the following concise representation:

$$X_j = \mathbb{I}_{\{u_j > p_j\}}$$

The variable X_j represents a rollover risk of a bank accepting funding from bank j due to default probability of j. Let $\bar{D}_X^2$ denote the covariance matrix of $[X_1, \dots, X_N]$ with the underlying correlation of X_is being matrix Q^X. The covariance has a representation in a closed form formula, the derivation of which is presented in appendix.

Each bank i aims at minimising the funding risk. It is assumed that a default of a creditor results in an inability to roll over funding which means materialisation of the funding risk. The risk is measured by the variance of the funding portfolio. For a vector of deposits $[L_{i1}^k, \dots, L_{iN}^k]$ it is quantified by $F \colon \mathbb{R}_+^N \to \mathbb{R}$ defined:

$$F(L_{i1}^k, \dots, L_{iN}^k) = \kappa^F [L_{i1}^k \ \dots \ L_{iN}^k] \bar{D}_X^2 [L_{i1}^k \ \dots \ L_{iN}^k]^\top, \tag{4}$$

---

[19]In order to guarantee that the iterative procedure gives a solution for some n_i, the fraction Δ should be defined as $\bar{a}_i^k/K$, for some large enough $K \in \mathbb{N}$. Then, in the worst case, $n_i = K$ is a feasible constraint preventing bank i from any interbank lending due to the already high risk accumulated in its balance sheet that has to be covered by the capital base.

where κ^F is funding risk aversion parameter. We assume different risk aversions for the two different optimisation processes reflecting two distinct types of risks (investment and funding). However, in the applications we use the same value for the two parameters.

Banks need to choose the composition of their interbank funding portfolios taking as a constraint the set $\bar{B}_j^k$ of banks that, first considered an option to extend a placement to them and, second the total capacity of their counterparties at step k. Formally, the admissible set $\mathcal{A}_i^F$ of a bank i is defined as:

$$\mathcal{A}_i^F := \{y \in \mathbb{R}_+^N | j \in \bar{B}_j^k \Rightarrow y_j \leq \bar{a}_j^k \text{ and } j \notin \bar{B}_j^k \Rightarrow y_j = 0\}$$

In other words, the non-zero components of vectors belonging to $\mathcal{A}_i^F$ can only be those js that satisfy: $\bar{B}_j^k \ni i$, i.e. bank j has drawn bank i to be a candidate of a counterparty for its interbank investment portfolio.

Consequently, minimisation of the funding risk for bank i means solving the following program:

minimise $F(y)$ on $\mathcal{A}_i^F$
subject to

- budget constraint:
$$\sum_j y_j = \bar{l}_i^k,$$

- limit on cost of funding: $(\bar{L}_{i\cdot}^k + L_{i\cdot}^k)r \leq r_i^l$. Banks are willing to pay on their interbank funding rates on average r_i^l. This internal limit is related to the expected profitability of assets.[20] It is assumed that if the average cost of funding exceeds the limit, the bank's return on interbank liabilities is negative.

The minimising vector is denoted $L_i^{F,k}$. The optimisation of the funding portfolio is performed by all the banks in the system simultaneously.

The budget constraint may be too stringent simply because of an insufficient supply of the interbank funding following the first round of the optimisation process. Analogously to the interbank asset optimisation, bank i tries to solve the funding problem with a *slightly* relaxed budget constraint, i.e. replacing $\bar{l}_i^k$ with $\bar{l}_i^k - \Delta\bar{l}_i^k$, $\bar{l}_i^k - 2\Delta\bar{l}_i^k$,... until for some $n_i^{F,k} \in \bar{\mathbb{N}}$, depending on the step k, $\bar{l}_i^k - n_i^{F,k}\Delta\bar{l}_i^k$ is a feasible constraint.

The optimisation across all the banks gives an alternative interbank matrix $L^{F,k}$ taking into account funding needs and risks. The matrix $L^{F,k}$ is composed of vectors $L_i^{F,k}$ in the following way:

$$L^{F,k} := \begin{bmatrix} (L_1^{F,k})^\top \\ \vdots \\ (L_N^{F,k})^\top \end{bmatrix}$$

---

[20]The monitoring of such limiting values are critical for banks' income management processes. Typically, limits are implied by budgeting / Funding Transfer Pricing (FTP) systems (see Adam (2008) for definitions and applications). In order to deactivate this option for a bank i, r_i^l needs to be set to a very large number.

2.5 Third round – bargaining game

The interbank structure $L^{I,k}$ may be, as is usually the case, different from $L^{F,k}$. In those instances banks may need to somewhat deviate from their optimised interbank asset-liability structure and therefore enter into negotiations with other banks in a similar situation. In order to address the issue about banks' willingness to accept a counteroffer to the optimisation-based placement, we consider each pair of banks entering a type of a bargaining game with utilities (or disutilities) reflecting a possible acceptable deviation from the optimal allocation of portfolios. The game is performed simultaneously by all pairs of banks. The disutility – which is assumed to be of a linear type – is measured by a change of the optimised functional to a change in the exposure between the preferred volumes $L^{I,k}_{ij}$ and $L^{F,k}_{ij}$.

More specifically the proposed games give one possible solution to the following question: what may happen if at step k bank j offers a placement of $L^{I,k}_{ij}$ in bank i and bank i would optimally fund itself by a deposit $L^{F,k}_{ij}$ from bank j, which is substantially different in volume from the offered one? Perhaps the banks would not reject completely the offer since it may be costly to engage in finding a completely new counterparty. By doing that they may encounter risk of failing to timely allocate funds or replanish funding since the interbank market is not granular. Instead, we assume that these 2 banks would enter negotiations to find a compromising volume. We model this process in a bargaining game framework. Banks have their disutilities to deviate from the optimisation based volumes. The more sensitive their satisfaction is to the changes in the individually optimal volumes, the less willing they are to concede. We assume that each pair of banks play the bargaining game at each step of the sequential problem in isolation taking into account their risk constraints. This is a key assumption bringing the framework to a tractable one. The details of the game setup, rather technical, are postponed to appendix D.

The outcome of the game played by each pair of banks (i, j) in step k is the volume of interbank lending from j to i denoted $\tilde{L}^k_{ij}$. It implies that the interbank network matrix at the next step $k + 1$ is given as

$$\bar{L}^{k+1} := \bar{L}^k + \tilde{L}^{G,k} \tag{5}$$

Since in this way part of unallocated interbank assets before step k is now invested then the $k + 1$ total interbank assets and liabilities are updated in the following way:

$$\bar{l}^{k+1}_i \ := \ \bar{l}^k_i + \sum_j \tilde{L}^{G,k}_{ij}$$

$$\bar{a}^{k+1}_j \ := \ \bar{a}^k_j + \sum_i \tilde{L}^{G,k}_{ij}$$

2.6 Fourth round – price adjustments

Both the individual optimisation and the bargaining game at round k may not lead to the full allocation of the interbank assets and there may still be some banks striving for interbank funding. By construction of the bargaining game, there are no banks with excess funding sources. In order to increase the chance of supplementing the interbank funding in the next step, banks with interbank funding deficiency adjust their offered interest rate. The adjustment depends on the uncovered funding gap. Let us assume that the market is characterised by

a price elasticity parameter α which translated the funding position into the new offered price. If at the step $k+1$ the gap amounts to $g_i^{k+1} := l_i - \sum_j \bar{L}_{ij}^{k+1}$ then the offered rate $r_i^{k+1} = r_i^k \exp(\alpha g_i^{k+1}/l_i)$.[21]

2.7   Repeated steps

The initially drawn sample of banks $\bar{B}_{ij}^1$ may not guarantee a full allocation of interbank assets across the interbank market. There are various reasons for that: some samples may be too small, consisting of banks that are not large enough to accept deposits or not willing to accept all offered deposits given their preferred interbank funding structure. Therefore, at each step the samples are enlarged by randomly drawing additional banks (again with the probability P). Each step of the sequence composed of the optimisation of the interbank assets (see subsection 2.3) followed by the selection of the preferred interbank funding structure (see subsection 2.4), bargaining game (see subsection 2.5) and price adjustment of interbank deposits (see subsection 2.6) is repeated for the unallocated assets $\bar{a}^k$ and liabilities $\bar{l}^k$ until no more placements of significant volume are added to the network. The sequence of rounds 1-4 is terminated when the contribution of matrix $\tilde{L}^{G,k}$ is marginal comparing with the interbank network $\bar{L}^k$. This is verified by setting an accuracy threshold $\epsilon << 1$ and comparing it with

$$\max_i \frac{\sum_j \tilde{L}_{i,j}^{G,k}}{l_i} \quad \text{and} \quad \max_j \frac{\sum_i \tilde{L}_{i,j}^{G,k}}{a_j} \tag{6}$$

In most of the applications it takes about 10 steps to allocate more then 90% of the predefined interbank assets a. A thorough analysis of convergence is presented in subsection 3.2.

3   Results

3.1   Data

The model was applied to the EU banking system. The dataset regarding balance sheet structures of banks was the same as the one applied by Hałaj and Kok (2013b). Briefly, it contains:

- a sample of banks being a subset of EBA stress testing exercise disclosures of 2011 – $N = 80$;

- Bankscope van Dijk's data on individual banks' balance sheet aggregates of total assets (TA_i), interbank borrowing and lending, customer loans (L_i), securities holding (S_i) and capital position (e_i);

- Risk Weighted Assets of banks in the sample broken down (if available) by total customer loans, securities and interbank lending. These pieces of information are used to proxy the allocation of capital to the interbank exposures. Assuming the Basel II 20% Risk Weight (RW) for the interbank lending and calculating the average risk weights for customer

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[21]So far, we do not have a good calibration of α at hand and in the applications illustrated in section 4 we assume $\alpha = 0$. Nevertheless, sensitivity analysis of the model with respect to α is presented in 3.

loans and securities in the sample, denoted RW^L and RW^S, respectively. The allocated capital e^I is approximated in the following way:

$$e_i^I = \frac{20\% a_i}{20\% a_i + \mathrm{RW}^L L_i + \mathrm{RW}^S S_i} e_i$$

The averages of risk weight of customer loans and securities instead of the bank by bank weight were necessitated by gaps in the data set with respect to the portfolio breakdown of RWAs;

- The geographical breakdown of banks' aggregate exposures allow for parametrisation of the probability map P.

The straightforward caveat of the approximation of e^I is that the averaging of RW^L and RW^S across banks may lead to excessively stringent capital constraints for some of the banks. The compromising procedure of replacing of total interbank assets a_i with $a_i - k_i \Delta a_i$ accounts for that as well.[22]

Additionally, CDS spreads (s) – for individual banks if available, otherwise country-specific – and 3-month money market rates for EU countries (r^m) were used to approximate the bank-specific interbank rates and their riskiness measured by the standard deviation of rates. Some projected paths of the CDS spreads under the baseline economic scenario were applied to calculate the CVA of the interbank exposures.[23]

The estimation of the correlations Q and Q^X is followed by the testing of the statistical significance of all the entries. Insignificant ones (at the probability level of 5%) are replaced by zeros. Three years of data with monthly frequency are used for the estimation.

3.2 Convergence

The convergence of the proposed procedure has to be proved. In fact, stabilisation of the process is obvious – it is a non-decreasing, bounded-from-above process. Conditions under which the process converges to full allocation of interbank assets can easily be defined. A very simple, *negative* condition is the following:

Let $\mathrm{sgn}\colon \mathbb{R}^{N \times N} \to \{-1, 0, 1\}^{N \times N}$ be a sign function of elements of a given matrix. Let us assign $l^P \colon = \mathrm{sgn}(P) \cdot l$ and $a^P \colon = (a^\top \cdot \mathrm{sgn}(P))^\top$. Then, all the interbank assets and interbank liabilities cannot be fully matched if there exists bank i such that either $l_i^P < a_i$ or $a_i^P < l_i$.

This is a very simple criteria that is easy to verify. It provides information about the inherent inability of bank i to either place a pre-defined volume of money on the interbank or to find sufficient interbank funding sources because of its low connection to the system. The condition is satisfied in the analysed system.

The condition can be formulated also in expected terms by replacing $\mathrm{sgn}(P)$ with the original matrix P, with the outcome denoted l^E and a^E accordingly. It could roughly be interpreted as the average potential allocation across simulations. The comparison of the average allocation with the pre-defined total interbank assets a and liabilities l show that on

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[22]Some sensitivity analysis is provided in Appendix D.1, table 1.

[23]The projected series of bank individual CDS spreads were kindly provided to us by M. Gross and calculated according to a method developed in Gross and Kok (2013).

Figure 2: Convergence of the interbank structures
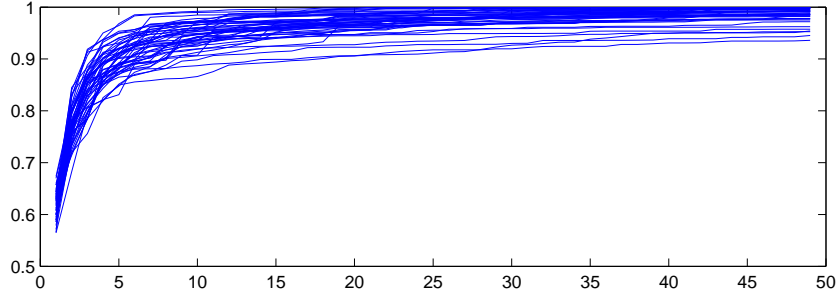
Note: Each curve represents ratios of the total allocated assets to the total initial interbank assets (50 simulations)

Source: own calculations

average 4% of interbank liabilities may not be matched with assets which is a relatively low number.[24]

Being analytically a very complicated problem, we study the convergence numerically. To present the convergence in a synthetic way, at each step k the ratio of the total allocated interbank assets $(\sum_j \bar{a}_j^k)$ to the total predefined interbank assets $(\sum_j a_j)$ was computed. The number of steps was set to a relatively large value of 50 and the interbank structure was simulated 50 times for various realisations of drawn subsamples of banks $(\bar{B}_j^k)_{1 \leq k \leq 50}$. The results are shown in figure 2. In just a few steps, the algorithm attains an allocation above 80% and the convergence pace decreases significantly. After 20 steps almost all paths lie above 95% of the allocation ratio. For practical implementation of our algorithm, apart from setting ϵ to 0.1%, in order to reduce the computation time, we choose that $k^{\max}$: $= 20$ steps as the additional stopping criteria for the algorithm.

3.3 Structure of endogenous interbank networks

It is far from obvious how the network resulting from the endogenous mechanism may look like. Some common statistical measures can help in understanding the structure at large. In general, the interbank networks are not complete. On average, bank-nodes have a degree of not more than 0.20 but the dispersion among nodes is substantial with some nodes having a degree of 0.30, while others only having a degree of 0.05 (see figure 3). The uniformness is more visibly violated for centrality measures that aim at gauging the importance of a node as a hub in the system. These measures are deemed particularly important for capturing the risk of contagion, by detecting the nodes that may be most prone to spreading contagion across the system. For instance, it is observed that betweenness centrality is several times higher for some particular nodes. Some studies focus on core / periphery properties which mean that there is a subset of nodes in the system that is fully connected whereas other nodes are only connected to that subset. There are various algorithms selecting the core and they may lead to a fuzzy classification – some nodes are 'almost' core or 'almost' periphery. In case of our endogenous networks we have not found any significant classification of nodes to the

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[24]The "4%" was calculated as $\sum_i (\min(1, a_i^E/l_i) - 1) * l_i) / \sum_i (l_i)$.

Figure 3: Endogenous networks vs random graphs generated with parameters inherited from the endogenous ones
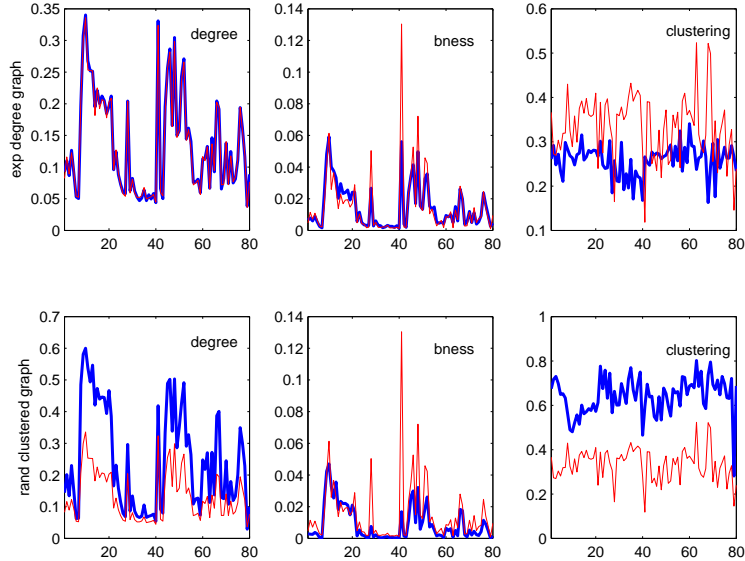
Note: x-axis: banks in the sample. y-axis: statistical measure of topological properties. Blue-wide lines: referring to endogenous networks (average in a random sample of 100 networks). Red-thin lines: referring to random graphs (top row: random degree graphs; bottom row: randomly clustered graph. *NetworkX* library in Python was used to generate and analyse the random graphs.)
Source: own calculations

core and periphery (using the Borgatti and Everett (1999) approach). This is probably due to the fact that we capture global, internationally active bank-hubs and domestic banks, usually strongly connected subsystems of the interbank networks. Overall, these findings suggests that the endogenous networks algorithm generates interbank structures that are not easy to be classified in a simple way by just a few topological parameters.

A usual approach to get a deeper understanding of the network structure is to compare it with some known, well-studied graphs that possess the same statistical (topological) properties. The best know example of a random graph is Erdös-Renyi model (E-R) constructing an edge between two given nodes with a given probability p, independent of all other pairs of nodes. Since we operate with a probability map assigning different probabilities to links between different banks, it is straightforward to imagine that the E-R approximation of endogenous networks should fail.[25] A more promising method in terms of accuracy of approximations is generated based on detailed information about degree and clustering of the endogenous networks. An *expected degree* graph (Chung and Lu, 2002) is the first example. In this model, links between nodes i and j are drawn with probability $\deg_i \deg_j / \sum_k \deg_k$, where $\deg_i$ is a degree of a node i. The second type of potentially useful graphs is a *random clustered* graph model (Newman, 2009). Given a degree sequence of all nodes and a triangle sequence of nodes[26] the random clustered algorithm chooses linkages uniformly from a joint distribution of

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[25]In order to save space in the paper, we do not report the results of the comparison which only confirm intuition.

[26]Triangle degree of a node is a number of triangles containing a given node. Triangle sequence is the sequence

possible set of triangles complying with the given degree sequence. In this way, the algorithm potentially has a better control not only of the degree distribution but also of clustering coefficients which are important indicators of contagion transmission channels.

The results of the comparison of the endogenous and random graphs are shown in Figure 3. Random graphs are constructed in such a way that for a given endogenous interbank network (EIN):

- the expected degree graph is generated using the degree sequence of nodes in EIN;

- the random clustered graph is generated with a sequence of pairs consisting of a degree and triangle degree of all nodes.

We analyse 100 realisations of endogenous networks and 100 corresponding random networks. The generated expected degree networks are almost identical to the endogenous networks with respect to the degree distribution. It is not surprising, given that degree of nodes is the only parameter of the expected degree graph algorithm. However, betweenness centrality, measuring direct and indirect connectivity of a given node with all other nodes in the system, proves to be less consistent. Some nodes of the endogenous networks are substantially more important in terms of centrality. The differences between endogenous and expected degree networks are even more striking for clustering measures gauging the concentration of linkages. The random clustered graphs do not perform better, even though their parameters have more degrees of freedom. The algorithm of random clustered networks preserves the ranking of the nodes in terms of degree measures but produces graphs with nodes possessing many more links than in the corresponding endogenous networks. The resulting clustering coefficients are in general higher as well. All in all, the complex topology of the endogenously modelled EU interbank network implies that random graphs may oversimplify their structure. This notwithstanding, random graphs offer a valuable benchmarking tool for understanding the relationship between various topological properties of networks.

4 Policy issues

On the basis of the network formation modelling approach presented in the previous sections, various pertinent policy questions can be addressed. For example, the approach can be employed to detect the impact of different policy measures on the endogenous emergence of network structures and the contagion risks related to those.

This is particularly useful for macro-prudential policy analysis purposes. As described in the following, the model can be used to evaluate the impact on interbank network structures of some regulatory instruments aimed at limiting banks' risk to counterparty exposures, such as the large exposure limits already embedded in current regulatory frameworks. The tool can therefore be used to help calibrate the optimal configuration of such macro-prudential and regulatory instruments.

Apart from the evaluation of relevant regulatory instruments, the endogenous network model can also be employed in macro-prudential analysis in a broader sense. Namely, for the assessment of the impact of the materialisation of systemic risks on the banking sector while taking into account the dynamic network formation resulting from changes in key real

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of triangle degrees of all the nodes in a graph.

economic and financial variables. In particular, as an adverse shock to the economy would typically result in the deterioration of some counterparties' creditworthiness (e.g. as reflected in banks' CDS spreads) and hence affect the optimal allocation of interbank assets and liabilities. In other words, a given adverse scenario is likely to result in a re-formation of linkages in the interbank market and hence to also affect the interbank contagion risks that ultimately may result from such adverse shocks.

The usage of the endogenous network model from these different policy perspectives is described in the following sub-sections.

4.1 Credit Valuation Adjustment

As described in Section 2, the CVA charge on economic capital allocated to interbank assets portfolio, usually applied by banks in their internal risk management systems, impacts the interbank asset structure decision-making process via the capital constraint whereby banks engaged in lending to riskier counterparties will generally face a comparatively higher capital charge to reflect the default risk of their interbank borrowers. In order to test the impact of CVA based capital surcharge (or equivalently the CVA based add-on to the risk weights) on the network structures, constraint 3 from subsection 2.3 including a sensitivity parameter c for CVA was modified as follows:

$$\sum_{i|i\neq j} (\omega_i + c\gamma_i)L_{ij} \leq e_j^I$$

By varying c from 0 to 1, the impact of CVA related to a particular configuration of exposures $L_{.1},... L_{.N}$ on the capital base increases gradually. Setting parameter c to 0 reflects no additional capital charges for counterparty risk related to the market perception of the credit risk of exposures against banks, whereas $c = 1$ portrays regime with the market-based credit risk valuation of the interbank exposures impacting the allocated capital to the interbank asset portfolio. Figure 4 presents the distribution of the size of linkages in networks constructed without and with the CVA add-on. The interbank networks structure does not change substantially except for some smaller (and weaker) banks that are forced to accept less diversified funding sources (in-degree measure drops for some of them). The reason for that is the shift of banks' interbank placements to more sound institutions as far as the market perception is concerned.

4.2 Large Exposure limits

With the objective of limiting counterparty concentration risk banking regulation (e.g. as stipulated in the Basel standards) imposes limits on the size of exposures banks are allowed to hold against other counterparties. As mentioned in subsection 2.3, the current EU standard for large exposure limits amounts to 25% of total regulatory capital.

In this subsection we test the sensitivity of the network structures to a variation of the 25%-threshold. the results are shown in Figures 5 and 6 for in-degree and out-degree measures, respectively. We observe that network structures (e.g. in terms of number of links of individual nodes) are relatively stable around and especially above the 25%-threshold. Raising the large exposure limits above the 25%-threshold on average would not seem to alter the network structure in any material way. By contrast, a more stringent approach to large exposure limits

Figure 4: Distribution of the volumes of exposures without and with CVA surcharge
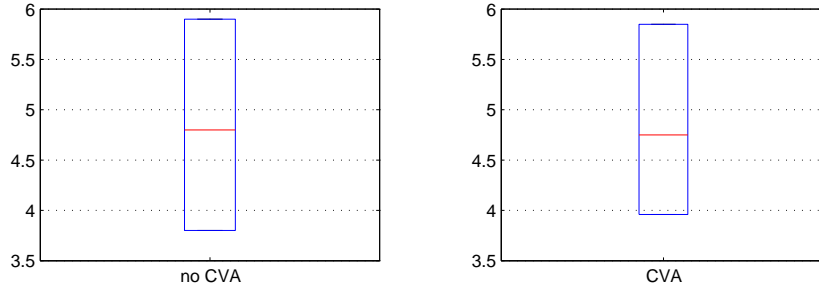
Note: y-axis: log size of exposures. Upper and bottom lines of the box reflect 75th and 25th percentile, the line inside the box – the median.
Source: own calculations

(i.e. moving the threshold towards 0%) could trigger substantial changes to the structure of banks' network connections. Intuitively, as limits on large exposures become more binding banks will have to reduce on the size of individual exposures and as result spread their interbank business across a wider range of counterparties. With some notable exceptions, this is indeed what we observe as reflected in increasing in- and out-degree measures when moving towards the 0%-threshold.

4.3 Effectiveness of policy instruments in adverse market conditions: Stress testing

The proposed approach to model the interbank networks opens many potential ways to study the effectiveness of various policy instruments in curbing contagion risk on the interbank market. We focus on the performance of the large exposure limits and in adverse market conditions.

The assessment is related to the dynamic balance sheet model of Hałaj (2013) that characterises banks' asset structures. By optimising the risk-adjusted profitability banks decide about the allocation of assets to the interbank portfolio. The optimisation involves many economic parameters describing banks' financial standing and their economic environment. This allows for passing through stress testing scenarios to project the evolution of the assets structures under adverse conditions and then – applying the framework for the endogenous interbank market formation – to analyse changes in the topology of the interbank market. The main drivers of the interbank structure would be (i) the projected bank individual total interbank assets, (ii) shifts in the profitability of the interbank investments (or from a different angle – availability of funding sources) resulting from the macro-financial scenario impact on interest rates and counterparty credit risk. Furthermore, by imposing large exposure limits under different economic and financial conditions the resilience of various structures to contagion propagation can be studied.

The methodology applied to analyse the impact on network structures under different macroeconomic conditions is as follows:

- We first compute each bank's total interbank investment and funding needs under a

Figure 5: Interbank network structures in various regimes of the large exposure limits (In-degree)
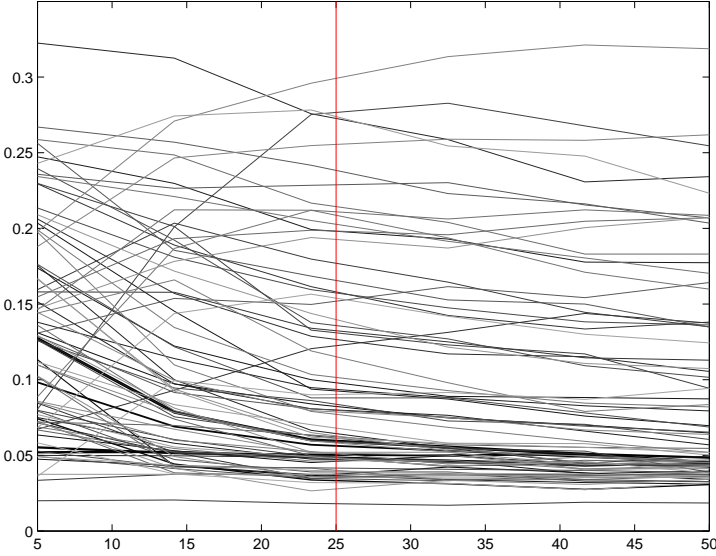
Note: x-axis: limit on exposures relative to the capital level (in %) y-axis: Out-degree of the networks. The darker the line, the larger the bank.
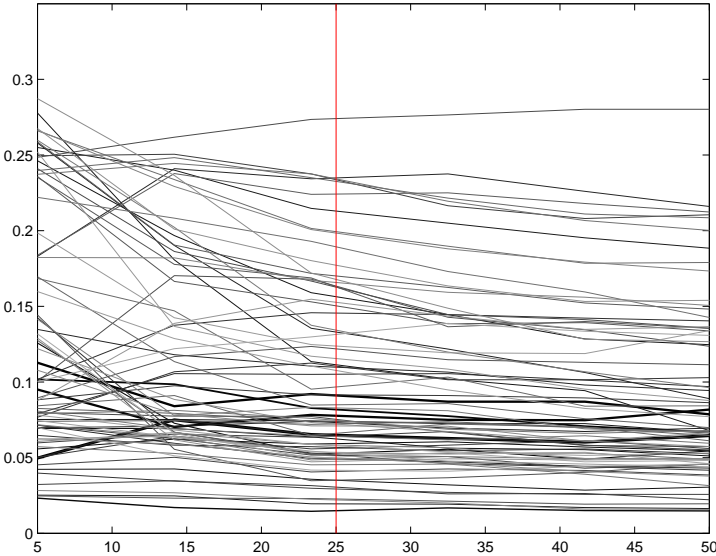Source: own calculations

Figure 6: Interbank network structures in various regimes of to the large exposure limits (Out-degree)

Note: x-axis: limit on exposures relative to the capital level (in %). y-axis: Out-degree of the networks. The darker the line, the larger the bank.
Source: own calculations

baseline macroeconomic scenario. The framework developed by Hałaj (2013) is used to translate the macroeconomic shock into the restructuring of the banks assets. The outcome of the model is the change of volume of broad balance sheet categories: customer loan portfolio, securities portfolio and interbank lending portfolio. The relative change of the volume of the interbank assets of bank j is used to scale the volume of the interbank funding of j.

- Second, we construct the interbank network applying the method proposed in section 2 under baseline scenario parameters and total interbank lending and borrowing in various regimes of Large Exposure limits.

- Finally, we impose an adverse macroeconomic shock to banks' capital position and subsequently run a contagion model of banks' defaults.[27] The clearing payments vector approach is used to measure the contagion effects (see Eisenberg and Noe (2001); Hałaj and Kok (2013b)).

Figure 7 illustrates the impact of an adverse scenario on the interbank network structures across different network indicators, as compared to the end-sample starting point. Deviations from the 45 degree line imply that the adverse shocks produce a different network configuration. Overall, it is difficult to gauge any systematic pattern in the responses of banks' interlinkages to the shocks. At the same time, it is notable that a considerable number of banks do deviate from the initial network characteristics, albeit the direction of the changes goes both ways. Some banks increase their degree of interconnectedness to the network, whereas other decrease it. In other words, as should intuitively be expected, a change in the macro-financial situation will endogenously lead to a change in network structures. We used statistical tests of differences in mean (t test), variance (F test) and whole distribution (Kolmogorov-Smirnoff test) of network measures (in- and out-degree, Katz indicator, DebtRank and betweenness centrality) to verify changes in the network structures. At significance level of 5% only a hypothesis about differences in variance of Katz ratio and betweenness cannot be rejected, i.e. a one-sided test indicates that in the adverse scenario some nodes become more interconnected with the whole system comparing to the baseline case. At the level of 10% significance in-degree measure shows more connections in the system under baseline scenario (the average number of linkages in the system is higher under the baseline). The outcomes of the statistical tests are intuitive. Under the adverse scenario, capital constraints and credit quality of some banks deteriorate resulting in less potential for them to accept interbank deposits (binding capital constraint) or to be offered a placement (binding credit quality). The optimisation mechanism proposed in the framework nicely captures the relationship between macro-financial conditions and network structures.

Apart from assessing whether adverse shocks impact the network formation process, our model can also be employed to evaluate whether contagion losses under an adverse scenario can be mitigated by adjusting certain regulatory (macro-prudential) instruments, such as large exposure limits or for example risk weights on exposures to other financial institutions.

For example, Figure 8 illustrates the impact of having different LE limit thresholds in the context of an adverse shock. Specifically, the y-axis illustrates the difference between networks formed under a 15% LE limit and under the standard 25% LE limit in terms of the capital loss following an adverse shock. A positive value implies that contagion losses rise when

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[27]To insure robustness of the results a couple of adverse scenarios was applied.

Figure 7: Impact of the adverse scenario on the interbank network structures
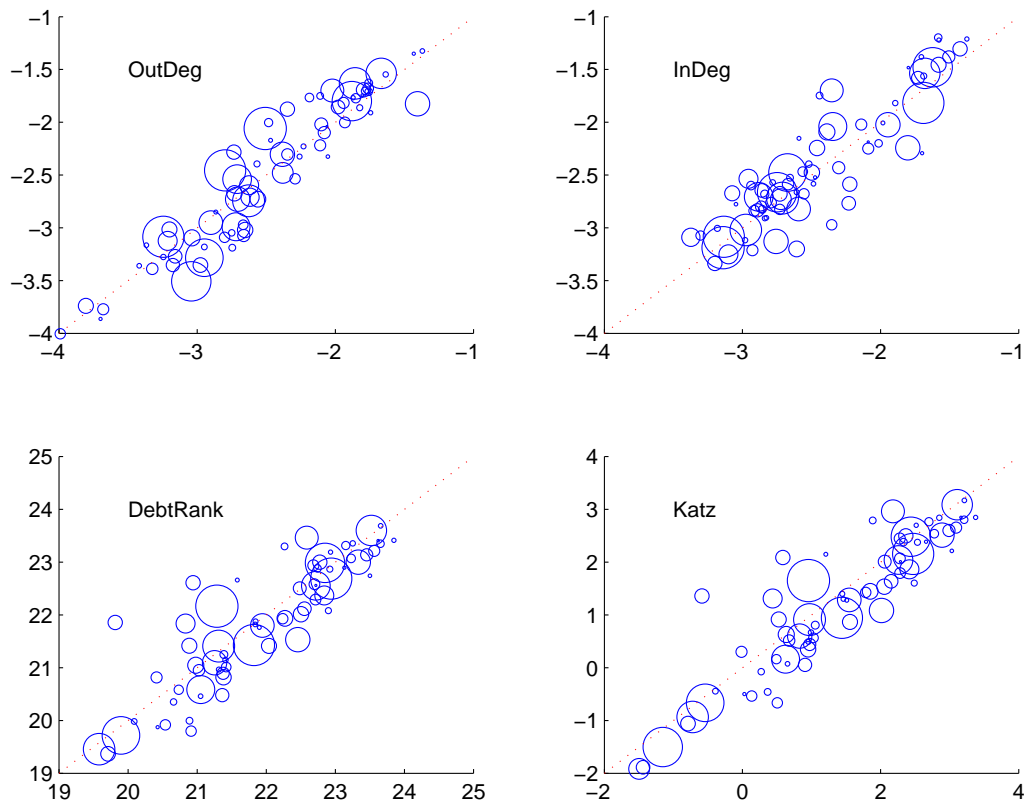
Note: Each circle represents a measures for a bank in a network constructed under end-sample estimates of the parameters (x-axis, log scale) and under an adverse scenario (y-axis, log scale). The size of a circle is proportional to bank's total assets.

Source: own calculations

Figure 8: Counterparty credit quality and the impact of LE limits on the losses incurred due to contagion
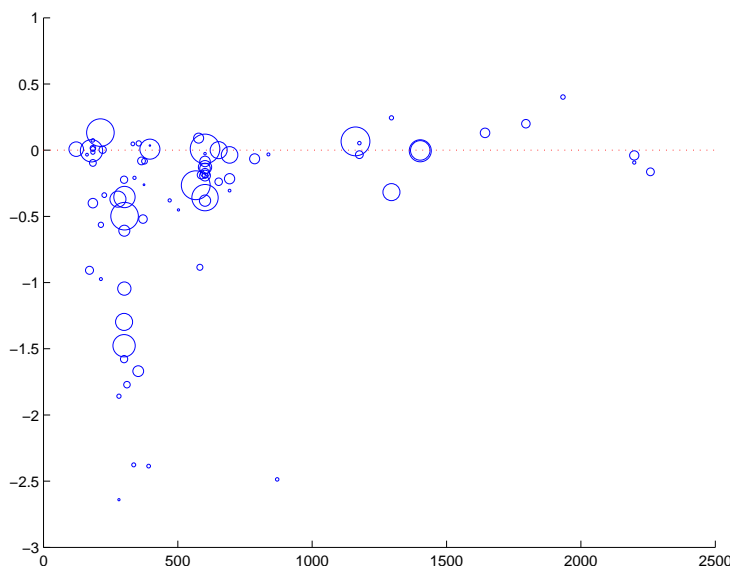
Note: x-axis: CDS spread (in bps). y-axis: difference of CAR after adverse stress testing shock between LE=15% and LE=25% regime (in pp, positive number means that by lowering LE limit contagion losses rise). No CVA adjustment (i.e. $\gamma \equiv 0$). The size of a circle is proportional to a bank's total assets. Source: own calculations

lowering the LE limit. On the x-axis, we plot the banks according to the size of their riskiness (measured in terms of their CDS spreads). It is observed that more stringent LE limits overall tend to lower contagion risk. Interestingly, this effect is especially pronounced for the group of banks perceived (by the markets) to be the soundest. In other words, the forced reduction of counterparty concentration risk that would be implied by a lowering of the LE limits would seem to particularly benefit the safest part of the banking system whereas the more vulnerable segments are found to be less affected by changes in the LE limits.

A similar analysis was carried out also for what concerns the CVA based add-on to the risk weights applied to the interbank exposures, see Figure 9. The Art. 458 of the CRDIV/CRR EU Directive[28] defines a macro-prudential instrument of specific risk weights attached to exposures to financial institutions. A rather broad formulation allows for a risk sensitive implementation of the instrument based on the CVA concept. In the context of our model, the parameter γ plays a role of that special add-on. As was the case for the large exposure limits, the impact of the CVA on contagion risk is mainly affecting the sounder banks. In the case of the CVA based add-on, the direction of changes is however not unambiguous as we observe that depending on the bank contagion risk can both increase and decrease when increasing the CVA charge.

[28]Directive 2013/36/EU of the European Parliament and of the Council of 26 June 2013 on access to the activity of credit institutions and the prudential supervision of credit institutions and investment firms, amending Directive 2002/87/EC and repealing Directives 2006/48/EC and 2006/49/EC

Figure 9: Counterparty credit quality and the impact of CVA capital charge on the losses incurred due to contagion
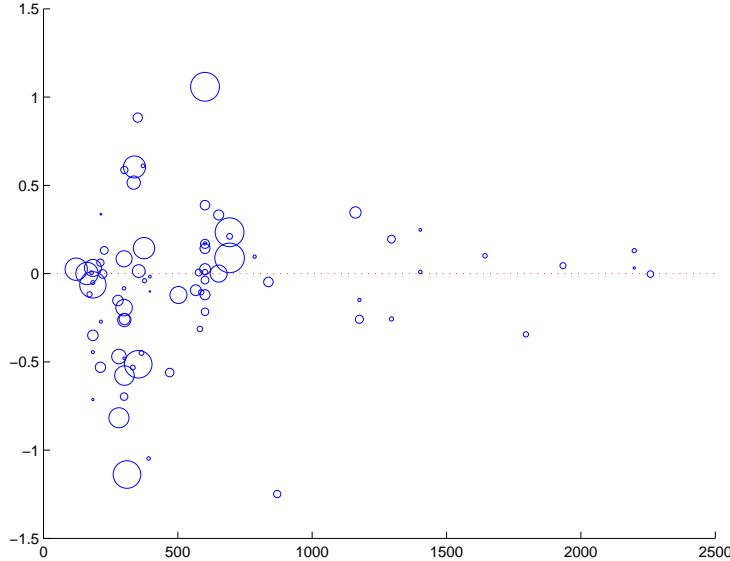
Note: x-axis: CDS spread (in bps). y-axis: difference of CAR after adverse stress testing shock between CVA and no CVA regime (in pp, positive number means that introduction of CVA charge increases contagion losses). LE= 25%. The size of a circle is proportional to a bank's total assets.
Source: own calculations

5   Conclusions

In this paper, we have tried to deviate from the standard, mechanistic cascading mechanism employed in traditional interbank contagion models. Instead, we have developed an agent-based model that is able to account for banks' dynamic, endogenous responses both in the formation process of the interbank network and in response to contagious shocks.

Apart from analysing network formation processes and the implications for interbank contagion risks in a setting where banks are dynamically optimising their actions, our model approach can also be used to assess the impact of different regulatory and macro-prudential policy instruments on the structure of interbank networks and their implied contagion risk. In this light, we presented a few policy experiments related to the effects of large exposure limits on counterparty credit risks. These macro-prudential policy instruments were analysed both in the context of network formation and in terms of their ability to mitigate interbank contagion risks under adverse circumstances.

All in all, while the reported results obviously hinges on the specific characteristics of the banks included in the network system and on the specific adverse scenarios considered, the overriding conclusion from these policy experiments is that macro-prudential policies can make a significant difference through their impact on the network formation and ultimately on the risk of interbank contagion to adverse shocks. From this perspective, the modelling approach presented in this paper can be employed for conducting impact assessments of selected macro-prudential policy instruments and in this way help inform the calibration of such tools.

Acknowledgements

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A   Credit Valuation Adjustment

The (OTC) interbank exposures carry risk related to the counterparts default. Banks will be obliged under Basel III agreement to quantify that risk by calculating the so-called Credit Valuation Adjustment (CVA) based on the market parameters reflecting market participants' perception of counterpart credit risk.[29] The advanced method of CVA calculation involves the observed credit default spreads of the counterparts to infer the market-based probability of default and consequently, given the assumed exposures and their maturities, the expected loss on the portfolio of the interbank assets. The CVA will become the additional capital charge for unhedged counterparty risk related to the OTC exposures (balance and off-balance). The CVA term is calculated in the following way :

$$\mathrm{CVA} = \lambda \sum_{n \leq N} \Big(\exp(-s_n t_n / \lambda) - \exp(-s_{n-1} t_{n-1} / \lambda) \Big) (EE_n d^n + EE_{n-1} d^{n-1})/2,$$

where λ is the market based assessment of the loss given default on the exposure against a given counterparty, t_n is the n-th point of time, cds_n is the projected CDS spread at time n, EE_n is the projected, netted exposure against the counterparty and d_n is a discount factor. Consequently, CVA accounts for the mark-to-market losses on the counterparty risk. If not available on the bank specific level, Basel iii allows for it to be replaced by an aggregate one based on rating, industry and region. Notably, the formula can be used by IRB banks and then the main difficulty of its implementation is finding appropriate profile for EE (in particular maturity t_N). The standardised approach banks are required to apply a simplified formula for which the profile of EE does not matter but still the maturity has to be imposed (at least the contractual of the analysed EAD).

We assume that the netted exposures of a given bank are constant within a given horizon t_N. It means that EE_n is constant, $EE_n = EE^*$ and the vector of proportionality parameters γ used in the definition of the capital constraints have entries defined as:

$$\gamma_i: \ = \frac{CVA_i}{EE^*} = \lambda \sum_{n \leq N} \Big(\exp(-s_{i,n} t_n / \lambda) - \exp(-s_{i,n-1} t_{n-1} / \lambda) \Big) (d^n + d^{n-1})/2$$

B   Modified Jaccard index – similarity of networks

In order to measure similarities between networks structures one can apply the so-called Jaccard index. It counts the number of linkages that appear in the two compared networks and relates it to the total number of linkages in both graphs. It therefore neglects the weight of the links, i.e. the importance of the overlapping linkages relative to the once uniquely present in one of the networks. We propose a slight modification to the index accounting for that

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[29]The most recent proposal can be found under the following address: http://www.bis.org/publ/bcbs227.htm

deficiency. For the 2 given networks $\mathcal{N}_1$ and $\mathcal{N}_2$ described by weighted adjacency matrices M^1 and M^2, span on the same set of N nodes, let us define 3 sets:

$$
\begin{aligned}
M12: &= \{(i,j) \in \bar{\mathbb{N}} \times \bar{\mathbb{N}} | (i,j) \in \mathcal{N}_1 \wedge (i,j) \in \mathcal{N}_2\} \\
M10: &= \{(i,j) \in \bar{\mathbb{N}} \times \bar{\mathbb{N}} | (i,j) \in \mathcal{N}_1 \wedge (i,j) \notin \mathcal{N}_2\} \\
M02: &= \{(i,j) \in \bar{\mathbb{N}} \times \bar{\mathbb{N}} | (i,j) \notin \mathcal{N}_1 \wedge (i,j) \in \mathcal{N}_2\}
\end{aligned}
$$

They describe the number of links that overlap between the graphs (M12) and those that are present in on graph but not in the other (M10 and M02). The modified Jaccard index ($J^m(\mathcal{N}_1, \mathcal{N}_2)$) is defined as:

$$
J^m(\mathcal{N}_1, \mathcal{N}_2) = \frac{\#M12 \sum_{(i,j) \in M12}(M_{i,j}^1 + M_{i,j}^2)}{\sum_{Z \in \{M12, M10, M02\}} \left(\#Z \sum_{(i,j) \in Z}(M_{i,j}^1 + M_{i,j}^2) \right)} \tag{7}
$$

The number of linkages within a given group (M12, M10 or M02) are weighted by the sum of exposures related to that links reflecting the importance of a given group of links.

C   Covariance of the funding risk X

Default probabilities p_j are modelled as random variables with variance estimated based on a time series of CDS spreads. Let $(\Omega, \mathcal{F}, \mathbf{P})$ be a probability space containing correlated random variables p_j taking values from [0,1] and independent (pairwise and with all p_js) random variables u_j, uniformly distributed on [0,1]. Let us define the σ-field generated by p_js, i.e.

$$
\mathcal{F}^p = \sigma \left(p_j^{-1}(C) | j \in \mathbb{N} \wedge C \in \mathcal{B}([0,1]) \right),
$$

where $\mathcal{B}([0,1])$ is a σ-field of Borel set on [0,1]. We use throughout the derivation the following fact:

For two given random variables Y_1 and Y_2, a σ-field $\mathcal{G}$, such that Y_1 is $\mathcal{G}$-measurable and Y_2 is independent of $\mathcal{G}$ and an integrable function f,

$$
\mathbf{E}[f(Y_1, Y_2)|\mathcal{G}] = \mathbf{E}[f(y, Y_2)]|_{y=Y_1}
$$

The proof of the fact is standard: first for the elementary functions, then for their linear combinations and finally in the closure of the resulting subspace in $L^1(\mathcal{F})$.

Additionally,

$$
\bar{X}_j: = \mathbf{E}[X_j|\mathcal{F}^p] = 1 - p_j
$$

since $\mathcal{F}^p$ is independent of u_j.

Case $i \neq j$

$$
\begin{aligned}
cov(X_i, X_j) &= \mathbf{E}[(X_i - \mathbf{E}X_i)(X_j - \mathbf{E}X_j)] = \mathbf{E}\left[\mathbf{E}\left[X_i X_j - \mathbf{E}\bar{X}_i \mathbf{E}\bar{X}_j|\mathcal{F}^p\right]\right] \\
&= \mathbf{E}\left[\mathbf{E}\left[\mathbb{I}_{\{u_i > \bar{p}_i\}} \mathbb{I}_{\{u_j > \bar{p}_j\}}\right] \bigg|_{(\bar{p}_i, \bar{p}_j)=(p_i, p_j)}\right] - (1 - \mathbf{E}p_i)(1 - \mathbf{E}p_j)
\end{aligned}
$$

$$\tag{8}$$

the last equality once again following independence of u_i, u_j and $\mathcal{F}^p$. Then, following independence of u_i and u_j

$$= \quad \mathbf{E}(1 - p_i)(1 - p_j) - (1 - \mathbf{E}p_i)(1 - \mathbf{E}p_j) = \mathbf{E}p_i p_j - \mathbf{E}p_i \mathbf{E}p_j = cov(p_i, p_j) \qquad (9)$$

<u>Case $i = j$</u>

$$cov(X_i, X_i) \quad = \quad \mathbf{E}\left[\left(\mathbb{I}_{\{u_i > p_i\}} \right)^2 \right] - (1 - \mathbf{E}p_i)^2$$

Since $\mathbb{I}_{\{u_i > p_i\}}$ takes only values 0 and 1, the square can simply be omitted in the first term under the expectation operator yielding

$$= \quad (1 - \mathbf{E}p_i) - (1 - \mathbf{E}p_i)^2 = \mathbf{E}p_i - (\mathbf{E}p_i)^2 \qquad (10)$$

D Setup and solution to the bargaining game

The aim of the bargaining game defined for all pairs of banks is to find an agreements between the offers of interbank placements and interbank funding needs. The key parameters to define the game are the sensitivities of a bank's optimised functional to a move from the optimum to the second player's optimum.[30] The asset side bank's satisfaction is measured by risk-adjusted return at the optimal allocation. The funding side bank's utility is gauged by the variance of the funding portfolio. Let us focus on a pair (i, j). Bank j is a lender, i is a borrower. For the bank j we define a function of its satisfaction $U_{ij}^{a,k}$ given a placement x to bank i *ceteris paribus*, i.e other volumes retained at the optimum level. Formally,

$$U_{ij}^{a,k}(x) = J(L_{1j}^{I,k}, \ldots, L_{i-1j}^{I,k}, x, L_{i+1j}^{I,k}, \ldots, L_{Nj}^{I,k})$$

Analogously, the utility $U_{ij}^{l,k}$ of the bank borrower i accepting placement x from bank j is measured at the optimal funding volumes from all other banks than j:

$$U_{ij}^{l,k}(x) = -F(L_{i1}^{F,k}, \ldots, L_{ij-1}^{F,k}, x, L_{ij+1}^{F,k}, \ldots, L_{iN}^{F,k})$$

Obviously, $U_{ij}^{a,k*} = U_{ij}^{a,k}(L_{ij}^{I,k})$ and $U_{ij}^{l,k*} = U_{ij}^{l,k}(L_{ij}^{F,k})$ are the measures of satisfaction at the optimum.

If $L_{ij}^{I,k} \neq L_{ij}^{F,k}$ then we define sensitivity measures of satisfaction of player j moving from the individually optimal allocation $L_{ij}^{I,k}$ to $L_{ij}^{F,k}$ and player i changing the funding volume obtained from player j from $L_{ij}^{F,k}$ to $L_{ij}^{I,k}$. We measure the sensitivity by means of the following ratios:

$$s_{ij}^{a,k} \quad = \quad \max\left(\frac{U_{ij}^{a,k}(L_{ij}^{I,k}) - U_{ij}^{a,k}(L_{ij}^{F,k})}{|L_{ij}^{F,k} - L_{ij}^{I,k}|}, 0 \right) \qquad (11)$$

$$s_{ij}^{l,k} \quad = \quad \max\left(\frac{U_{ij}^{l,k}(L_{ij}^{F,k}) - U_{ij}^{l,k}(L_{ij}^{I,k})}{|L_{ij}^{I,k} - L_{ij}^{F,k}|}, 0 \right) \qquad (12)$$

---

[30]We prefer this definition of sensitivity to an alternative one based on a unit change or derivative to mitigate problems of missing second order terms. Notably, the optimised functionals in round 1 and 2 are quadratic.

In this way, we implicitly assume that banks' dissatisfaction from abandoning the optimisation-based investment and funding portfolios is growing linearly along the allocation between $L_{ij}^{I,k}$ and $L_{ij}^{F,k}$. The max operation accounts for the fact that actions $L_{ij}^{I,k}$ and $L_{ij}^{F,k}$ are not globally optimal but rather on a constraint set of strategies.

In the interbank bargaining game at round k banks maximise the utility functional G_{ij}^k defined:

Case 1: $L_{ij}^{I,k} > L_{ij}^{F,k} \quad \Rightarrow$

$$G_{ij}^k(x) = \left[U_{ij}^{l,k*} - s_{ij}^{l,k} \cdot (x - L_{ij}^{F,k})\right] \left[U_{ij}^{a,k*} - s_{ij}^{a,k} \cdot (L_{ij}^{I,k} - x)\right] \tag{13}$$

maximised on $[L_{ij}^{F,k}, L_{ij}^{I,k}]$

Case 2: $L_{ij}^{I,k} < L_{ij}^{F,k} \quad \Rightarrow$

$$G_{ij}^k(x) = \left[U_{ij}^{a,k*} - s_{ij}^{a,k} \cdot (x - L_{ij}^{I,k})\right] \left[U_{ij}^{l,k*} - s_{ij}^{l,k} \cdot (L_{ij}^{F,k} - x)\right] \tag{14}$$

maximised on $[L_{ij}^{I,k}, L_{ij}^{F,k}]$

After basic calculations, the solution can simply be written as:

Case 1: the bargaining game equilibrium allocation $L^{G,k}$ satisfies:

$$L_{ij}^{G,k} = \max(\min(x_{ij}^{k*}, L_{ij}^{I,k}), L_{ij}^{F,k})$$

where

$$x_{ij}^{k*} = \frac{1}{2}\left[L_{ij}^{F,k} + L_{ij}^{I,k} + \frac{U_{ij}^{l,k*}}{s_{ij}^{l,k}} - \frac{U_{ij}^{a,k*}}{s_{ij}^{a,k}}\right]$$

Case 2: the bargaining game equilibrium allocation $L^{G,k}$ satisfies:

$$L_{ij}^{G,k} = \max(\min(x_{ij}^{k*}, L_{ij}^{F,k}), L_{ij}^{I,k})$$

where

$$x_{ij}^{k*} = \frac{1}{2}\left[L_{ij}^{F,k} + L_{ij}^{I,k} + \frac{U_{ij}^{a,k*}}{s_{ij}^{a,k}} - \frac{U_{ij}^{l,k*}}{s_{ij}^{l,k}}\right]$$

On aggregate, the outcome of the bargaining game may violate the $\bar{a}^k$ and $\bar{l}^k$ constraint, since $\sum_i L_{ij'}^{G,k} > \bar{a}_{j'}$ for some j' and $\sum_j L_{i'j}^{G,k} > \bar{l}_{i'}$ for some i'. Therefore, rows and columns of $L^{G,k}$ are transformed proportionately:
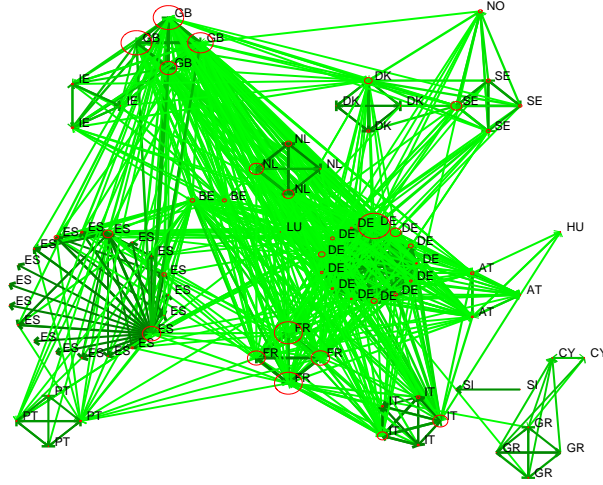
$$L_{ij}^{G,k} \to \hat{L}_{ij}^{G,k}: = L_{ij}^{G,k} \cdot \min(1, \frac{\bar{l}_i^k}{\sum_j L_{ij}^{G,k}}),$$

$$L_{ij}^{G,k} \to \breve{L}_{ij}^{G,k}: = L_{ij}^{G,k} \cdot \min(1, \frac{\bar{a}_j^k}{\sum_i L_{ij}^{G,k}})$$

The ultimate matrix of exposures $\tilde{L}^{G,k}$ realised in the step k is defined as the element-wise minimum of $\hat{L}^{G,k}$ and $\breve{L}^{G,k}$:

$$\tilde{L}_{ij}^{G,k} = \min(\hat{L}_{ij}^{G,k}, \breve{L}_{ij}^{G,k})$$

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Figure 10: Incompletness of the interbank network structure with the estimated correlation of risks

Note: The darker the line, the higher the probability that the link exists. The circles around the nodes indicate bank sizes (proportional to log of total assets).
Source: own calculations

D.1 Sensitivity analysis

To make sure that policy conclusions we draw are robust, we assess performance of the model verifying the interbank structures it produces and their sensitivity to key driving parameters, such as correlation (Q), loss given default (λ), investment risk aversion (κ), funding risk aversion (κ^F), interbank interest rate elasticity (α) and capital allocated to the interbank portfolios (e^I).

One particular interbank structure estimated in the developed endogenous network model is presented in Figure 10. It is incomplete with the largest banks being the most interconnected nodes, which is in line with findings elsewhere in the literature.[31]

The correlation structure should in theory influence the concentration and completeness of the interbank network. Notably, from the perspective of the optimal asset structures, high correlation translates into low diversification potential implying that the assets are optimally allocated in just a few banks. However, it is not a priori clear how the network structures may respond to the increasing correlation of the interests paid by the interbank deposits. Figures 11 and 12 synthetically show sensitivities of some basic (and commonly used) topological measures of networks to the decreasing correlation. The systemic importance of some nodes gradually increases as the correlation falls from the estimated one to the state where there is no dependence between interbank interest rates (and default risk that influences the cost of funding). This is slightly more persistent for the larger banks whereas some of the small banks

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[31]For a few representative country-specific studies using real-time overnight transactions data or large exposure data as well as entropy approaches, see e.g. Furfine (2003), Upper and Worms (2004), Boss et al. (2004), van Lelyveld and Liedorp (2006), Soramaki et al. (2007) and Degryse and Nguyen (2007).

Figure 11: In-degree of banks in the simulated system vs the strength of correlation
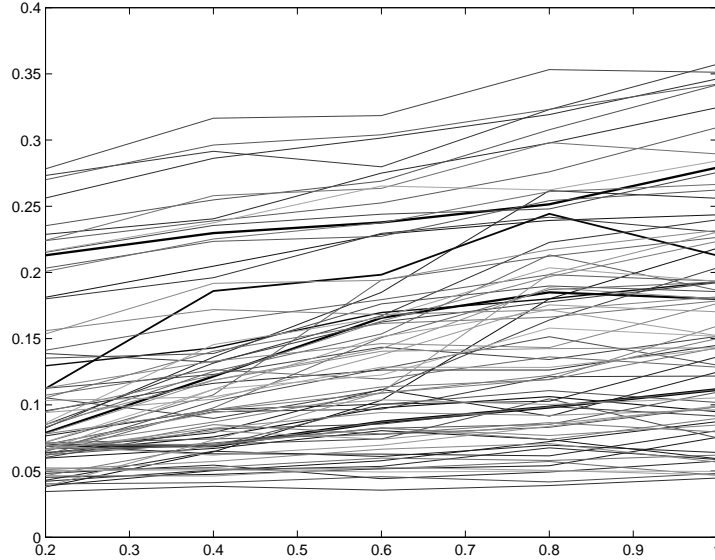
Note: x-axis: factor reducing the correlation of risk between banks, e.g. 0.4 means that the applied correlation matrix $Q_{0.4}$ is such that for all $i \neq j$ $Q_{0.4}(i,j) = (1-0.4)Q(i,j)$. The darker the line, the larger the bank. Investment risk aversion $\kappa = 1$. Funding sources diversification parameter is set to 0.5. No CVA adjustment (i.e. $\gamma \equiv 0$).
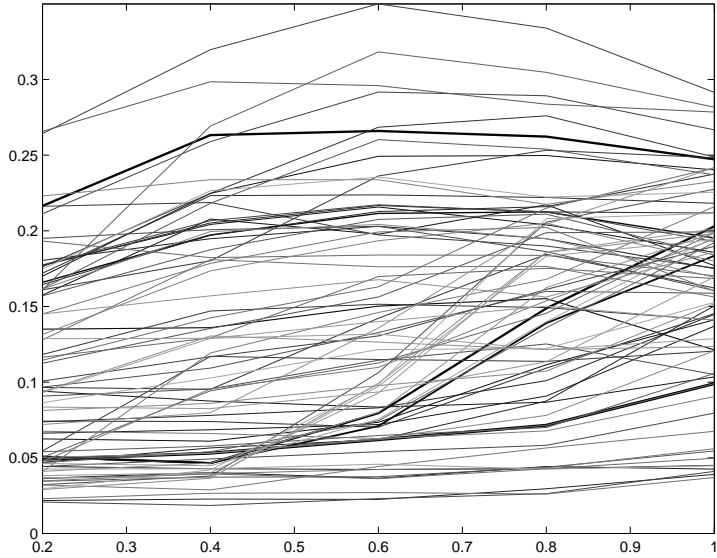Source: own calculations

react strongly as the correlation becomes close to 0 by spreading their interbank assets more widely across the system.

Given that the two measures of interconnectedness indicate the increase of linkages as correlation vanishes, it is interesting to study the importance of the sizes of linkages and the general network structure differences. In order to address the issues we plotted the distribution of the interbank exposures for different strengths of the correlation (see figure 13). The estimated dependence of the return and risk parameters of the interbank deposits, which is relatively high due to the correlation of the reference interest rates, leads to network with many banks connected by some very large exposures comparing to the median. As the correlation decreases, the network become more and more homogenous in terms of interbank deposit volumes. Notably, in general the size of exposures decreases.

The interbank networks may thus differ a lot for different degrees of correlation. However, it can be observed that the largest linkages, contributing to the core structure of the interbank graph, are generally stable and that the differences are related mainly to the smaller deposit amounts. This is illustrated in the lower pane of figure 14. We compared the original Jaccard index based on the adjacency matrix with a modified one accounting for the sizes of the exposures. It is observed that the curve produced by the modified index lies above the one obtained from the original one indicating that the linkages among the largest banks are more stable (i.e. less sensitive to correlation). This notwithstanding, in general correlation appears to have a sizable impact on the structure of the endogenous network formation.

There are several key parameters of the model that were set in an arbitrary way. We

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Figure 12: Out-degree of banks in the simulated system versus the strength of correlation of interbank returns


Note: The darker the line, the larger the bank. Investment risk aversion $\kappa = 1$. Funding sources diversification parameter is set to 0.5. No CVA adjustment (i.e. $\gamma \equiv 0$)
Source: own calculations

Figure 13: Distribution of linkages in the network vs correlation of the combined interest rate and counteparty risk
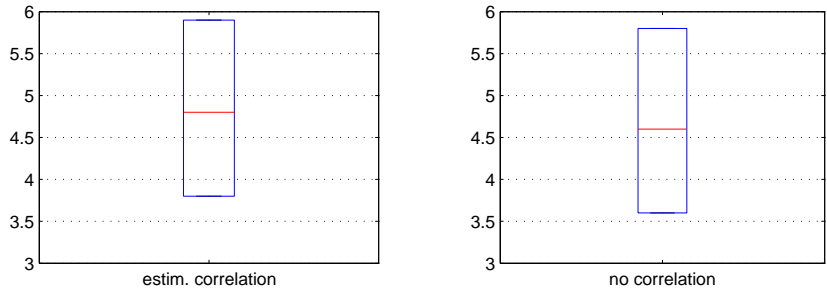
Note: y-axis: log size of exposures. Upper and bottom lines of the box reflect 75th and 25th percentile, the line inside the box – the median.
Source: own calculations

Figure 14: Network similarity measures versus the strength of the correlation.
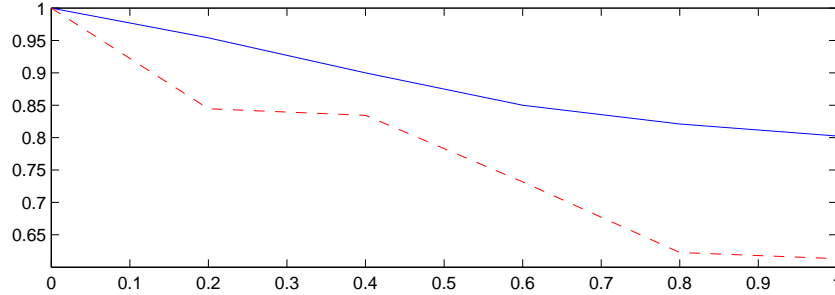
Note: x-axis: factor reducing the correlation of risk between banks, e.g. 0.4 means that the applied correlation matrix $Q_{0.4}$ is such that for all $i \neq j$ $Q_{0.4}(i,j) = (1 - 0.4)Q(i,j)$. dashed line – Jaccard index, solid line – modified Jaccard index (taking into account the size of the exposures (see Appendix B)).
Source: own calculations

Table 1: Sensitivity of networks structures to parameters

Parameter (baseline)	Tested value	Network measure – avg. weighted relative change vs baseline (%)				
		Out-deg	In-deg	Bness	Katz	DebtRank
LGD (0.4)						
	0.3	-1.6	0.2	1.3	1.5	2.3
		(0.1)	(0.4)	(0.0)	(0.3)	(0.1)
	0.5	1.8	-0.8	-0.5	-1.4	-0.1
		(0.2)	(0.3)	(0.1)	(0.1)	(0.0)
Inv. risk aversion (0.5)						
	1.5	-3.0	-1.0	0.2	0.5	0.9
		(0.1)	(0.2)	(0.5)	(0.1)	(0.3)
	2.5	-3.7	-1.0	-4.2	0.7	1.2
		(0.1)	(0.1)	(0.7)	(0.2)	(0.2)
Fund. risk aversion (0.5)						
	1.5	1.2	0.1	3.4	5.2	3.8
		(0.5)	(0.0)	(0.5)	(0.8)	(0.6)
	2.5	2.3	0.1	5.6	5.6	3.7
		(0.7)	(0.0)	(0.6)	(0.7)	(0.1)
Price elast. (0.0)						
	0.5	4.1	5.2	10.3	-0.6	-1.8
		(1.7)	(0.6)	(2.4)	(0.3)	(0.1)
	1.0	4.4	-2.3	12.8	3.2	-1.7
		(0.5)	(0.4)	(1.9)	(0.3)	(2.1)
Alloc. capital (eI)						
	$0.9eI$	2.6	0.6	10.0	-4.2	-3.7
		(0.2)	(0.3)	(0.6)	(0.7)	(0.8)
	$1.1eI$	5.9	3.4	3.7	-1.6	-2.7
		(0.3)	(0.3)	(0.6)	(0.6)	(0.5)

Note: relative change with respect to baseline value of the tested parameter; standard deviations in brackets
Source: own calculations

Figure 15: Distribution of Bargaining Game adjustment factors
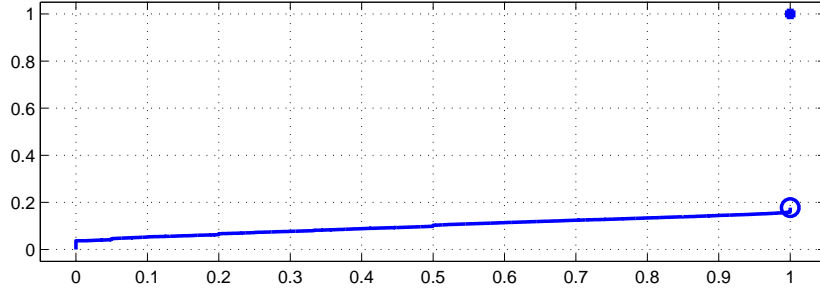
Note: adjustment factor: $\min(1, \frac{\bar{l}_i^k}{\sum_j L_{ij}^{G,k}})$ for any index j; distribution based on a simulation of 50 networks of 80 banks with 20 steps of optimisation of assets and liabilities followed by bargaining game $\Rightarrow$ 80000 calculated ratios.
Source: own calculations

address the issue of how sensitive the results are to the chosen specification and present the results in Table 1. The structure of the endogenously derived interbank network is most sensitive to deposit interest rate elasticity and the level of allocated capital to the interbank portfolios. It is most evident for the number of linkages between banks (in- and out-degree measures) and betweenness centrality. In general, however, the structures vary in a limited way and the variability that we do observe tends to have intuitive explanations. For instance, increasing investment risk aversion leads on average to the decrease in the number of linkages as measured by in- and out-degree. Consequently, some nodes in the network become more systemic, which can be measured by Katz and DebtRank indicators detecting nodes-hubs that directly and indirectly connect banks in the network.

The proportionate transformation $L^G \rightarrow \tilde{L}^G$ is designed to ensure that the budget constraint of total interbank assets a and liabilities l is satisfied after the bargaining game but may distort the outcomes of the game. We verified in how many instances the proportional adjustment is activated. As shown in figure 15 in more than 80% of cases no adjustment is required and the remaining part of the distribution is spread uniformly in $[0, 1]$ interval of adjustments.