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Zachary Feinstein, Grzegorz Hałaj

Interbank asset-liability networks with fire sale management

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Abstract

Interconnectedness is an inherent feature of the modern financial system. While it contributes to efficiency of financial services, it also creates structural vulnerabilities: pernicious shock transmission and amplification impacting banks' capitalization. This has recently been seen during the Global Financial Crisis. Post-crisis reforms addressed many of the causes of this event, but contagion effects may not be fully eliminated. One reason for this may be related to financial institutions' incentives and strategic behaviours. We propose a model to study contagion effects in a banking system capturing network effects of direct exposures and indirect effects of market behaviour that may impact asset valuation. By doing so, we can embed a well-established fire-sale channel into our model. Unlike in related literature, we relax the assumption that there is an exogenous pecking order of how banks would sell their assets. Instead, banks act rationally in our model; they optimally construct a portfolio subject to budget constraints so as to raise cash to satisfy creditors (interbank and external). We assume that the guiding principle for banks is to maximize risk-adjusted returns generated by their balance sheets. We parameterize the theoretical model with publicly available data for a representative sample of European banks; this allows us to run simulations of bank valuations and asset prices under a set of stress scenarios.

JEL Codes: C62, C63, G11, G21

Keywords: systemic risk, interbank contagion, fire sales, optimal portfolio

Non-technical summary

We propose a model to study contagion effects in the banking system capturing network effects of direct exposures and indirect effects of market behaviour that may impact asset valuation. By doing so, we can embed a well-established fire sale channel into our model. We relax the typical assumption of an exogenous pecking order of how banks would sell their assets.

This modelling framework follows the structure of Feinstein (2019) to consider both default and price-mediated contagion. Banks in our model are assumed to act rationally. They optimally construct a portfolio subject to budget constraints so as to raise cash to satisfy creditors (interbank and external). We assume that the guiding principle for banks is to maximize risk-adjusted returns generated by their balance sheets. Selling assets exerts pressure on their prices, thus, directly impacting profit and losses of the banks transacting the assets and indirectly all banks holding these assets through mark-to-market revaluation. In coming to their optimal decisions, banks consider not only fluctuations in prices but the defaults of counterparties in a network of financial exposures, as well. Moreover, we explicitly model a market regulator with an objective to stabilize asset prices in the market by offsetting trades of the banks.

Our methodological contribution to the literature is about capturing strategic management actions of banks in a networked financial market and about embedding regulators' objectives to stabilize the market under stress given regulator's balance sheet size constraint and admissible quality of assets purchased by the regulator. Our empirical contribution is about bringing the model to the banking system of the European Europe. All of the data are publicly available, mostly in the transparency exercise data of the European Banking Authority, or they are some statistics on the interconnectedness borrowed from literature. This approach should help in replicability of the results.

In this set-up, we are able to demonstrate the existence of banks' strategies in equilibrium that combine interbank payments, asset holdings, and asset prices. Notably, even though we cannot theoretically exclude multiple equilibria, in the extensive set of simulations we conducted, our proposed numerical method converges to a consistent equilibrium.

We find that prices converge after the funding shocks considered in the simulations. Revaluation of assets differs across asset classes, notably equities are significantly more impacted by rebalancing of banks' portfolios. Market regulators can stabilize asset prices by offsetting transactions of commercial banks; however, a regulator's preferences regarding the quality and volume of assets it is willing to buy matters for the dynamics of the prices and the effectiveness of the interventions. Moreover, strategic interactions help to reduce the negative impacts on asset prices since they allow banks to internalize other banks' impact on prices. Finally, by running a model under the assumption that banks only consider the impacts of their own transactions, we show that abstracting from the strategic interaction in stress test application can lead to a significant overestimation of second round losses.

1 Introduction

Interconnectedness is an inherent feature of the modern financial system. These interconnections, in normal times, increase the efficiency of the banking system and financial services. This includes the ability for individuals and institutions to invest more appropriately to manage risk and hit target returns. In the best case, these interconnections in the financial system can even mitigate losses from idiosyncratic shocks because of increased diversification of investment opportunities. However, as witnessed in the Global Financial Crisis, the interconnections between financial institutions also create structural vulnerabilities that allow for contagion and amplification of systematic shocks. In this work we focus on two primary modes of financial contagion: interbank obligations and portfolio overlap. These modes of contagion are often referred to as *default contagion* and *price-mediated contagion* respectively.

Due to the Global Financial Crisis, reforms and new regulations have been implemented to address many of the causes of that specific crisis (FSB, 2016). These reforms include new regulations for "too big to fail" financial institutions and the encouragement of central clearing parties to reduce counterparty risks in many over-the-counter markets. Though these reforms are targeted at the precipitating causes of the Global Financial Crisis, financial contagion remains a threat to the health of the financial system. Financial contagion may be less related to direct interconnectedness via lending and borrowing between financial institutions and more to indirect links via similarities of their balance sheets and financial complexity. For instance, based on information collected by the Basel Committee on Banking Supervision to determine global systemically important banks (G-SIBs) we can see that even though interconnectedness dropped slightly in recent years, complexity did less so (see Figure 1). Moreover, international authorities such as the Financial Stability Board (see FSB (2020)) and the Bank of International Settlements emphasize that[1] "[...]despite progress made by macro prudential policy, we have been less good at making the global financial system more resilient as an interconnected system."

Default contagion and price-mediated contagion are financial concepts that are well-established in the mathematical finance literature. Default contagion occurs as a result of the direct exposures between financial institutions. We take the model of Eisenberg and Noe (2001) as the foundation for studying default contagion. Banks are connected through a network of obligations; the default of one bank causes losses in the balance sheet of its counterparties and thus can

[1] Remarks of Benoit Coeure "Learning the value of resilience and technology: the global financial system after Covid-19", `https://www.bis.org/speeches/sp200417.htm`.

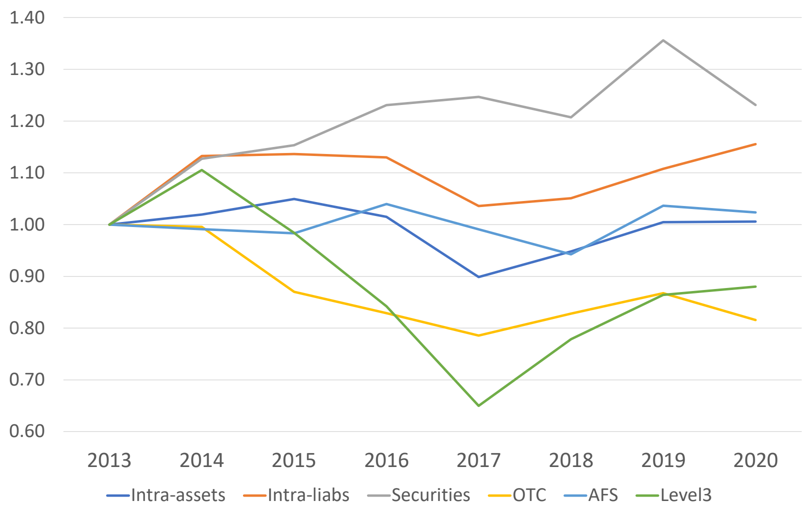


Figure 1: Indicators of interconnectedness and complexity used to determine G-SIB classification of banks. Dynamics normalized with 2013 at 1.0 (y-axis). Data from `https://www.bis.org/bcbs/gsib/hl_ind_since_2013.xlsx`.
Measure of interconnectedess:"Intra-assets" – intra-financial assets; "Intra-liabs" – intra-financial liabilities; ' "Securities" – securities outstanding;
"Complexity" – trading and available for sale securities indicator.
Measures of complexity: "OTC" – Notional amount of OTC derivatives; "AFS" – available for trading and trading securities portfolios; "Level3" – Level 3 assets as defined in Basel III in the context of Liquidity Coverage Ratio

trigger a cascade of further defaults. Though not utilized in this work, the framework of Eisenberg and Noe (2001) has been extended to include, e.g., bankruptcy costs (Rogers and Veraart, 2013), equity cross-holdings (Suzuki, 2002; Gouriéroux et al., 2012), and contingent claims such as credit default swaps (Schuldenzucker et al., 2019; Klages-Mundt and Minca, 2020; Banerjee and Feinstein, 2019). Price-mediated contagion occurs due to the indirect interactions between financial institutions via the price of commonly held assets; prices drop when a bank sells assets, and, following mark-to-market valuation, the balance sheets of all firms are impacted. This phenomenon is closely related to the fire sales when prices of assets in forced transactions deviate from their fundamental values. This was thoroughly described by Shleifer and Vishny (2011). These portfolio overlaps, which to a large extent are a consequence of risk regulation that promotes diversification of assets, can be a serious channel for contagion. We wish to highlight the work of Weber and Weske (2017), which summarizes multiple of these contagion models in a single work.

Many papers on price-mediated contagion and fire sales begin with the Eisenberg and Noe (2001) framework for interbank obligations. Within this framework, Cifuentes et al. (2005); Amini et al. (2016); Braverman and Minca (2018) and Calimani et al. (2019) study the existence and uniqueness of joint clearing payments and prices in systems with both interbank liabilities and a single marketable illiquid asset subject to fire sale dynamics. This notion is extended to include multiple illiquid assets in numerous works. The most common liquidation strategy undertaken is mechanistic and assumes proportional selling of all assets (Greenwood et al., 2015; Duarte and Eisenbach, 2018; Cont and Schaanning, 2019; Feinstein, 2020; Barucca et al., 2021; Aldasoro et al., 2022; Barnett et al., 2022; Ramadiah et al., 2022). However, recent literature has considered banks to be utility maximizers (Feinstein, 2017, 2019; Braouezec and Wagalath, 2019; Banerjee and Feinstein, 2020). Equilibrium interbank payments and prices of assets used to supplement liquidity after a shock were studied by Caballero and Simsek (2013) but with an assumption of an imperfectly observed structure of the market.

Our methodological contribution to the literature is about capturing strategic management actions of banks in a networked financial market. In this work, banks are assumed to be utility maximizers who may choose to fully rebalance their portfolios. To the best of our knowledge, such a specification of banks' objectives to maximize utility from the rebalancing of the entire portfolio to raise cash and satisfy interbank and external creditors is a novelty in the literature. In coming to their optimal decisions, banks consider fluctuations in prices, the defaults of counterparties in a network of financial exposures, or even the market transactions of other banks. This modelling framework follows the structure of Feinstein (2019) to consider both default and price-mediated contagion. In this setup, we are able to demonstrate the existence of banks' strategies in equilibrium that combine interbank payments, asset holdings, and asset prices. Notably, by straightforward modification of portfolio constraints, the model can be set up to study the price impact of either reinvestment decisions (i.e., rebalancing of assets) or optimal liquidation strategies (i.e., selling of assets). The generality of this framework additionally allows us to study the impact of a market regulator on financial stability.

To study systemic risk, it is important not to neglect financial agents' management actions. Banks are required to have sound risk management and risk appetite frameworks. This will determine their reaction function in response to stress, including securities holdings. Typically, a pro-rata approach to the liquidation of assets is assumed in the literature. The particular choice of liquidation strategies has material implications for the spreading of contagion shocks, see Sydow et al. (2021). Therefore, we build a framework that sheds light on the optimal selection of liquidated types of assets, i.e., consistent with risk management considering, jointly, immediate price impact of the sales and resulting revaluation of securities holding, expected risk and return realized on the post-liquidation portfolios, and risk appetite.

We apply our theoretical model to the European banking system data. The asset prices in equilibrium seem to be uniquely determined; even though, theoretically, we are not able to exclude multiple equilibria, empirically, in the simulations we conducted, we observe only unique price equilibria. All of the data used to calibrate the model are publicly available or they are some statistics on the interconnectedness borrowed from literature. This is done to aid in the reproducibility of the results and for the application of the model to financial systems of other jurisdictions. Notably, even though we source balance sheet information from the supervisory confidential data, the data points are at a high level of aggregation and can be found in banks' financial reports or repositories of data vendors. We run some counterfactual stress-test scenario analysis on this dataset. Rather than focusing solely on clearing equilibria, the modelling and simulations consider the procedure in which markets reach clearing through the tâtonnement process. We also show the flexibility of our framework by incorporating a market regulator whose objective is to stabilize liquidity in the distressed financial system.

We find that prices converge after the funding shocks considered in the simulations. Revaluation of assets differs across asset classes; notably, equities are significantly more impacted by rebalancing of banks' portfolios. A market regulator can stabilize asset prices by offsetting transactions of commercial banks, however, regulator's preferences regarding the quality and volume of assets it is willing to buy matters for the dynamics of the prices and the effectiveness of the interventions. Moreover, strategic interactions help to reduce negative impacts on the prices since they allow banks to internalize other banks' impacts on the asset prices. Finally, we show that abstracting form the strategic interactions in stress test applications can lead to a significant overestimation of second-round losses.

This paper is organized as follows. In Section 2, we present the theoretical model utilized in this work. In order to do that we present the stylized balance sheet and rules for interactions between banks. Existence of equilibria is proven and the tâtonnement process is proposed. In

Section 3, we present the utility function utilized in our case studies that encodes the risk-adjusted returns taking other banks' actions into account. We additionally provide a method for calibrating this utility function to data. In Section 4, we calibrate this theoretical model to a comprehensive set of European banks and undertake stress tests of this banking system in our framework; Section 5 concludes.

2 Market clearing and the tâtonnement process

Consider a system of N banks indexed by $i \in \{1, \ldots, N\}$.[2] Each bank is interconnected in two ways: through interbank obligations and through portfolio overlaps. More explicitly, the balance sheet of bank i is made up of assets:

- cash account ($a_i \geq 0$);
- M marketable external assets ($x_{im} \geq 0$ for asset $m \in \{1, \ldots, M\}$);
- a pool of illiquid, non-marketable assets ($\tilde{x}_i \geq 0$); and
- interbank assets ($\sum_{j=1}^{N} L_{ji}$ for nominal obligations $L_{ji} \geq 0$ from bank j)

and liabilities:

- interbank funding, i.e., the sum of deposits from all other banks ($\bar{p}_i := \sum_{j=1}^{N} L_{ij}$);
- external funding ($z_i \geq 0$); and
- capital (c_i).

A fraction $\delta_i \in [0, 1]$ of the external funding sources is subject to funding risk. This assumption reflects rollover risk (i.e., the risk that maturing funding is not renewed or it is impossible to replace it) or the risk of runs (i.e., if funding providers call the debt that they granted). This parameter δ will be used in simulations of funding shocks in case studies presented in Section 4. For notational simplicity, we will denote the total funding on bank i's balance sheet that has to be satisfied as $\bar{P}_i := \bar{p}_i + \delta_i z_i$. Following, e.g., Eisenberg and Noe (2001), we assume that no bank accumulates any positive equity until all debts are paid in full and, consequently, the

[2]Consideration of a central bank or market regulator is introduced in Section 4.2.2.

balance sheet identity holds for any bank i

$$c_i = a_i + \tilde{x}_i + \sum_{m=1}^{M} x_{im} + \sum_{j=1}^{N} L_{ji} - (1-\delta_i)z_i - \bar{P}_i.$$

Assumption 2.1. *As validated in the data, we will assume throughout this work that all banks i have positive external assets $a_i + \sum_{m=1}^{M} x_{im} > 0$ and liabilities $\bar{P}_i > 0$.*

The clearing model under consideration is constructed from two interrelated contagion mechanisms: interbank payments and price impacts.

- For the interbank payments, we follow limited liabilities in that no bank will pay more than its total available assets. Following the rule set from Eisenberg and Noe (2001), we additionally assume throughout this work that all obligations have the same seniority. We consider the relative exposures of bank i to bank j by $\pi_{ji} := L_{ji}/\bar{P}_j$. As such, the inflows to bank i from interbank payments are given by $\sum_{j=1}^{N} \pi_{ji} p_j$, where $p_j \in [0, \bar{P}_j]$ is the payments made by bank j, which is determined in an equilibrium (clearing) procedure detailed below in (1).

- For the price impacts on marketable external assets, we introduce the collection of inverse demand functions $f_m : \mathbb{R} \to [\underline{q}_m, \overline{q}_m] \subseteq \mathbb{R}_{++}$ for each asset type $m \in \{1, \ldots, M\}$. Each inverse demand function provides a price generated by the market based on the aggregate liquidations in that asset. As undertaken in Feinstein (2019), these inverse demand functions can accept positive inputs (the asset is, on net, being sold by the banks) or negative inputs (the asset is, on net, being bought by the banks); this is in contrast to earlier works on price-mediated contagion, e.g., (Greenwood et al., 2015; Amini et al., 2016; Feinstein, 2017), in which banks are constrained so as to only liquidate assets. Without loss of generality, consider $f_m(0) = 1$ for every asset type m so that x_{im} denotes both the physical units of asset m held by bank i and the (pre–fire sale) value of those assets. Additionally, these inverse demand functions are assumed to be continuous and non-increasing in net asset liquidations. As such, the cash extracted from the marketable external assets (if positive) by bank i is provided by $\sum_{m=1}^{M} f_m(\sum_{j=1}^{N} v_{jm}) v_{im}$, where v_{jm} is the volume of assets of type m that are liquidated by bank j. Note that the impact of transacted volumes on prices may induce banks to alter their trading strategies in the equilibrium (clearing) procedure.

Assumption 2.2. *For the remainder of this work, we will assume that each inverse demand function has the linear structure* $f_m(v) := 1 - b_m v$ *for* $b_m \in (0, \frac{1}{2\sum_{i=1}^N x_{im}})$*. With this construction, the lower bound on the price of asset* m *is thus given by*

$$\underline{q}_m := 1 - b_m \sum_{i=1}^N x_{im} \in (\frac{1}{2}, 1)$$

and the upper bound is given by

$$\bar{q}_m := \frac{1 + \sqrt{1 + 4b_m \sum_{i=1}^N \left(a_i + \sum_{\tilde{m} \neq m} x_{i\tilde{m}} \underline{q}_{\tilde{m}} + \sum_{j=1}^N L_{ji} - \bar{P}_i\right)^+}}{2} \geq 1.$$

The clearing payments are determined in an Eisenberg-Noe framework when the prices of all marketable assets $q \in [\underline{q}_1, \bar{q}_1] \times \cdots \times [\underline{q}_M, \bar{q}_M]$ are fixed. That is, define the payment clearing mapping $\Psi : [0, \bar{P}_1] \times \cdots \times [0, \bar{P}_n] \times [\underline{q}_1, \bar{q}_1] \times \cdots \times [\underline{q}_M, \bar{q}_M] \to [0, \bar{P}_1] \times \cdots \times [0, \bar{P}_n]$ for bank i as

$$\Psi_i(p, q) = \left(a_i + \sum_{m=1}^M q_m x_{im} + \sum_{j=1}^N \pi_{ji} p_j\right) \wedge \bar{P}_i. \tag{1}$$

The clearing payment vector p^* under current market prices q is found as the fixed point of Ψ. Denote $p^*(q) = \Psi(p^*(q), q)$ to be the Eisenberg-Noe clearing payment vector under market prices $q \in [\underline{q}_1, \bar{q}_1] \times \cdots \times [\underline{q}_m, \bar{q}_m]$; $p^*(q)$ is unique due to Assumption 2.1 and (Eisenberg and Noe, 2001, Theorem 2).

In much the same way as considered by Feinstein (2019), banks are free to rebalance their cash account and marketable assets but constrained so as to satisfy their obligations. We further impose a no-short-selling constraint. Rather than working directly on asset liquidations and purchases, consider the asset holdings $y_{im} \geq 0$ (for bank i and asset type m). The net amount of asset m liquidated by the banks is thus provided by $v_m = \sum_{j=1}^N [x_{jm} - y_{jm}]$. Furthermore, by buying and selling these marketable assets, each bank's cash account updates as well; given market prices q and clearing payments $p^*(q)$, this cash account for bank i would be given by

$$a_i^* := a_i + \sum_{m=1}^M q_m [x_{im} - y_{im}] + \sum_{j=1}^N \pi_{ji} p_j^*(q).$$

With the notion that banks do not accumulate equity until all debts are paid in full, it must follow that

$$a_i^* \geq \left(a_i + \sum_{m=1}^{M} q_m x_{im} + \sum_{j=1}^{N} \pi_{ji} p_j^*(q)\right) \wedge \bar{P}_i.$$

Therefore, in order to satisfy its obligations, bank i must invest so as to have enough cash to cover its payments, i.e.,

$$\sum_{m=1}^{M} q_m y_{im} \leq \left[a_i + \sum_{m=1}^{M} q_m x_{im} + \sum_{j=1}^{N} \pi_{ji} p_j^*(q) - \bar{P}_i\right]^+. \tag{2}$$

This constraint can be viewed as a budget constraint since it provides an upper bound on the investments made. The constraint is related to the capital value available to each bank given payments $p^*(q)$ and market prices q. Note that banks are allowed to hold excess cash at the end of the clearing procedure because of the inequality-based budget constraint.

Remark 2.3. The traditional fire sale literature, e.g., (Greenwood et al., 2015; Amini et al., 2016; Feinstein, 2017), constrains banks to only liquidate marketable assets, and to generate no excess cash reserves from doing so, during the clearing procedure. Such constraints can be included in the clearing procedure by including the additional rule that $y_{im} \leq x_{im}$ and forcing equality in the budget constraint (2). No other alterations would be required for that setting.

It remains to show how each bank will trade in order to determine their optimal portfolio given clearing payments $p^*(q)$ and market prices q. We assume that each bank is a portfolio optimizer following some strictly concave utility function. That is, bank i seeks to maximize the utility $u_i(y_i, y_{-i}^*)$. Consequently, bank i is a portfolio optimizer solving

$$y_i^\dagger(y^*, q) = \arg\max \left\{u_i(y_i, y_{-i}^*) \mid y_i \in \mathcal{A}_i(q)\right\}$$
$$\mathcal{A}_i(q) = \left\{y_i \in \mathbb{R}_+^M \;\middle|\; \sum_{m=1}^{M} q_m y_{im} \leq \left[a_i + \sum_{m=1}^{M} q_m x_{im} + \sum_{j=1}^{N} \pi_{ji} p_j^*(q) - \bar{P}_i\right]^+\right\}.$$

Notably, by the construction of $y^\dagger$, if a bank is defaulting (i.e., assets have less value than the total obligations), then it will hold no marketable assets because of the no-short-selling and budgetary constraints.

Assumption 2.4. *The utility function u_i is jointly continuous and $y_i \mapsto u_i(y_i, y_{-i}^*)$ is strictly concave for every bank i. As a consequence, $y_i^\dagger(y^*, q)$ is a singleton for any portfolio holdings y^**

and market prices q.

The full clearing procedure is joint in market prices q^* and portfolio holdings y^* described by

$$q^* = f\left(\sum_{i=1}^{N}[x_i - y_i^*]\right) \quad , \quad y^* = y^\dagger(y^*, q^*). \tag{3}$$

The clearing cash account for bank i would, as discussed above, be determined by

$$a_i^* = a_i + \sum_{m=1}^{M} q_m^*[x_{im} - y_{im}^*] + \sum_{j=1}^{N} \pi_{ji} p_j^*(q^*).$$

Excess cash reserves, after clearing, can then be computed simply as $(a_i^* - \bar{P}_i)^+$.

By construction of $y^\dagger$, any clearing portfolio holdings (and therefore also the cash account) is a Nash equilibrium. That is, fixing the strategy of all other banks y_{-i}^* and the prices q^*, bank i cannot obtain a higher utility than $u_i(y^*)$.

Proposition 2.5. *There exists a clearing solution q^*, y^* to* (3)*.*

Proof. First, by Lemma 5 of Eisenberg and Noe (2001), p^* is continuous as a function of the market prices q; and, by the Berge maximum theorem, $y^\dagger$ is jointly continuous in (y^*, q^*). Thus, existence trivially follows for $q^* = f\left(\sum_{i=1}^{N}[x_i - y_i^*]\right)$ and $y^* = y^\dagger(y^*, q^*)$ by the Brouwer fixed point theorem. □

Rather than concerning ourselves, explicitly, with considering an equilibrium solution, as in Feinstein (2019), we consider a specific tâtonnement process; the one we consider herein is on the space of portfolio holdings rather than prices as undertaken in Feinstein (2019). Namely, consider the following tâtonnement process:

$$dy_t = \left(y^\dagger(y_t, f\left(\sum_{i=1}^{N}[x_i - y_{i,t}]\right)) - y_t\right) \otimes \Delta_y dt, \qquad y_0 = x, \tag{4}$$

where Δ_y denotes the velocity of market orders and $\otimes$ denotes the Hadamard product. The resulting market prices over time can be computed explicitly by

$$q_t = f\left(\sum_{i=1}^{N}[x_i - y_{i,t}]\right)$$

with $q_0 = \vec{1}$ by construction.

Algorithm 2.6. The tâtonnement process can be simulated via an application of Euler's method as the following:

1. Initialize $t = 0$, $\Delta t > 0$, $\Delta_y \in (0,1)$, $q_t = \vec{1}$, and $y_t = x$;
2. While $t \leq T$ for some sufficiently large time T:
   (a) Initialize $k = 0$, $p^{(0)} = \bar{P}$, and $\mathcal{D}^{(0)} = \emptyset$;
   (b) Iterate $k = k + 1$;
   (c) Define $\mathcal{D}^{(k)} = \{i \mid a_i + \sum_m q_{t,m} x_{im} + \sum_j \pi_{ji} p_j^{(k-1)} < \bar{P}_j\}$;
   (d) If $\mathcal{D}^{(k)} = \mathcal{D}^{(k-1)}$ then go to step (2h) with $p^* = p^{(k-1)}$;
   (e) Set $\Lambda \in \{0,1\}^{N \times N}$ such that $\Lambda_{ii} = 1$ if $i \in \mathcal{D}^{(k)}$ and 0 otherwise;
   (f) Update $p^{(k)} = (I - \Lambda \Pi^\top)^{-1} \left[(I - \Lambda)\bar{P} + \Lambda \left(a + \sum_m q_{t,m} x_{\cdot m}\right)\right]$;
   (g) Return to step (2b);
   (h) For each bank i, define the feasible region $\mathcal{A}_i$ as

$$\mathcal{A}_i = \left\{ y_i \in \mathbb{R}^M_+ \;\middle|\; \sum_{m=1}^M q_{m,t} y_{im} \leq \left[a_i + \sum_{m=1}^M q_{m,t} x_{im} + \sum_{j=1}^N \pi_{ji} p_j^* - \bar{P}_i \right]^+ \right\};$$

   (i) For each bank i, define $y_i^\dagger$ as

$$y_i^\dagger = \arg\max\{u_i(y_i, y_{-i,t}) \mid y_i \in \mathcal{A}_i\};$$

   (j) Update $y_{t+\Delta t} = y_t + \Delta_y([y^\dagger - y_t])\Delta t$ and $q_{t+\Delta t} = f\left(\sum_{i=1}^N [x_i - y_{i,t+\Delta}]\right)$;
   (k) Increment $t = t + \Delta t$.

There are two possible conclusions from the tâtonnement process (4). If the tâtonnement process limits to a single point, then this is an equilibrium (clearing) vector of market holdings y^* with associated prices q^*; see Proposition 2.7. The associated payment vector can be found as $p^*(q^*)$. Notably, in numerical simulations, we have found that this tâtonnement process always converges; we refer the reader to the results in Section 4 for a demonstration of this convergence. As such, we use this tâtonnement procedure to compute the equilibrium solutions throughout this work. Though Proposition 2.5 provides existence criteria for an equilibrium solution, this tâtonnement process can be utilized even if that concavity criteria is not satisfied (though there may be challenges with computing $y^\dagger$ for the non-concave maximization problems).

Proposition 2.7. *If the tâtonnement process* (4) *converges to a point* y^*, *then* $q^* := f(\sum_{i=1}^N [x_i - y_i^*])$ *and* y^* *define a clearing solution of* (3).

Proof. y^* is a point of convergence of (4) if and only if $y^\dagger(y^*, q^*) = y^*$ for $q^* = f(\sum_{i=1}^N [x_i - y_i^*])$. Recall the definition of a clearing solution from (3) and the proof is complete. □

3 Risk-adjusted return utility

In this section we propose a class of financially meaningful utility functions for consideration in Section 4. We wish to note that the tâtonnement process proposed above in (4) can be utilized with other utility functions (including heterogeneous utilities for different institutions). Broadly, this utility function corresponds to banks being risk-adjusted return optimizers.

More specifically, consider the setting in which bank i seeks to maximize risk-adjusted returns where returns are measured with respect to the inverse demand function f. That is, the return of a portfolio y_i is the aggregate of the returns $\mu^\top y_i$, losses on the liquidated volume $x - y$, and losses due to the revaluation of the holdings with a rebound factor $\beta_i \in [0,1]^M$. Furthermore, bank i will adjust its returns through the covariance structure C with risk aversion $\gamma_i \geq 0$. Mathematically, this is all combined into the single utility function

$$u_i(y) = y_i^\top \left(\mu - (I - \text{diag}(\beta_i))[\vec{1} - f(\sum_{j=1}^N [x_j - y_j])] \right) - (x_i - y_i)^\top \left(\vec{1} - f(\sum_{j=1}^N [x_j - y_j]) \right) - \frac{\gamma_i}{2} y_i^\top C y_i.$$

Thus, bank i's objective is to design asset liquidations (or purchases) to maximize the risk-adjusted return on the outstanding (post–fire sale) assets. To summarize, this objective comprises:

1. *Expected returns*: $\mu^\top y_i$;
2. *Liquidation costs* on volume $x_i - y_i$: $-(x_i - y_i)^\top \left(\vec{1} - f(\sum_{j=1}^N [x_j - y_j])\right)$;
3. *Revaluation losses* of holdings y_i with mark-to-market elasticity β_i: $-y_i^\top (I - \text{diag}(\beta_i)) [\vec{1} - f(\sum_{j=1}^N [x_j - y_j])]$, where $\text{diag}(\beta_i)$ is a diagonal matrix with entries of β_i on the diagonal;
4. *Risk-adjustment*: $-\frac{\gamma_i}{2} y_i^\top C y_i$.

Recall from Assumption 2.2 that (for simplicity) the inverse demand function $f_m(v) = 1 - b_m v$ has a linear structure with $b_m \in (0, \frac{1}{2\sum_{i=1}^N x_{im}})$. Thus, the utility function can be simplified to the quadratic structure:

$$u_i(y) = -\left[y_i^\top \left(\operatorname{diag}(\beta_i)\operatorname{diag}(b) + \frac{\gamma_i}{2}C\right) y_i - \left(\mu + \operatorname{diag}(b)\left[x_i + \operatorname{diag}(\beta_i)\left(\sum_{j=1}^N x_j - \sum_{j\neq i} y_j\right)\right]\right)^\top y_i\right]$$

with difference only up to a constant with respect to y_i. Thus, this objective is strictly concave in y_i so long as $\operatorname{diag}(\beta_i)\operatorname{diag}(b) + \frac{\gamma_i}{2}C$ is positive definite; this is guaranteed so long as $\beta_i, b \in \mathbb{R}^M_{++}$.

We wish to conclude this section with consideration of the calibration of the bank-specific parameters, i.e., the rebound factor $\beta_i \in [0,1]^M$ and the risk-aversion $\gamma_i \geq 0$. This calibration is done so that each bank i, when unconstrained, would (approximately) seek to invest in portfolio x_i. Though we present this calibration for the *quantities* invested, x_i is also the vector of (pre–fire sale) values invested in each asset by bank i. This is due to the normalization of the inverse demand function so that $f_m(0) = 1$ for each asset m. Therefore, the calibration presented can, equally, be viewed as a procedure on the *value* invested in each asset rather than the quantity.

In order to construct a tractable calibration, consider the first-order conditions for maximizing the utility u_i for bank i given that all other banks hold their initial portfolios x_{-i}, i.e., the optimal investment y_i^* satisfies

$$\nabla_{y_i} u_i(y_i^*, x_{-i}) = -2\left(\operatorname{diag}(b)\operatorname{diag}(\beta_i) + \frac{\gamma_i}{2}C\right) y_i^* + (\mu + \operatorname{diag}(b)[I + \operatorname{diag}(\beta_i)]x_i) = \vec{0}.$$

As we are seeking β_i, γ_i so that $y_i^* \approx x_i$, we consider the linear mapping

$$g_i(\beta_i, \gamma_i) := \operatorname{diag}(x_i)\operatorname{diag}(b)\beta_i + [Cx_i]\gamma_i - \mu - \operatorname{diag}(b)x_i$$

which is derived from $\nabla_{y_i} u_i(x_i, x_{-i})$ evaluated at β_i, γ_i. We can, therefore, calibrate the system by finding the bank-specific rebound factor β_i and risk-aversion γ_i as the minimizers of the quadratic program

$$\min\left\{\|g_i(\beta_i, \gamma_i)\|_2^2 \mid \beta_i \in [0,1]^M,\ \gamma_i \geq 0\right\}. \tag{5}$$

This calibration procedure is undertaken for all banks in the system to produce the heterogeneous utility functions u_i.

4 Case studies

In this section we consider an in-depth case study and stress test of the European banking system. First, in Section 4.1, we present the dataset and model calibration. This data is then used in Section 4.2 to complete numerical stress tests of the European banking system.

4.1 Banking system data

The EU-wide Transparency exercise conducted by the European Banking Authority complements banks' own Pillar 3 disclosures, as laid down in the EU Capital Requirements Directive. EBA discloses detailed bank-by-bank data, in a comparable and accessible format, for 120 banks across 25 EEA / EU countries data, for quarterly reference dates and we pick the most recent, 30 June 2021 snapshot. Below, we list data points we extracted and the rationale for using them in our model.

1. (number and size of banks) Complexity of the system is to a large degree determined by the number of banks and distribution of their sizes. This implies how the shocks may trigger contagion, how decisions of relatively large banks impact the market and how smaller banks may be affected. We consider a system of 120 bank with heterogeneous sizes measured by total assets (see Table 1).

2. (diversification of banks' balance sheets) Granularity of banks portfolios determines the extent of portfolio overlaps. The overlaps are the key driver of the price-mediated contagion. It means they would create an asset portfolio revaluation chains following liquidation of assets. The higher the number of uncorrelated assets, the higher the diversification of risks. However, the impact of diversification for systemic risk is non-trivial: assuming no strategic decisions about which asset to liquidate under stress, literature shows diversification benefits for small shocks and larger spill-overs for more extreme ones (see, e.g., Wagner (2011); Roncoroni et al. (2021)). However, the role of optimized liquidations is not clear. To shed light on the optimized liquidation of overlapping portfolios, we consider balance sheets with three broad securities classes: (i) government bonds, (ii) financial and non-financial corporate debt and (iii) equities. These breakdown is available in the EBA

core/periphery		mean	q25	median	q75
c	ta [billion €]	897.7	516.8	651.0	1321.1
	cash	0.3	0.1	0.2	0.4
	gov	6.1	2.8	5.2	8.7
	nfc	2.9	1.4	2.1	4.0
	equities	1.0	0.5	0.7	1.2
	iba	8.0	3.9	5.5	8.5
	loans	59.8	55.0	60.8	68.6
	wf	22.9	19.7	23.0	25.6
	ibl	5.6	3.1	4.2	6.3
p	ta [billion €]	82.2	31.6	56.5	96.8
	cash	0.5	0.0	0.2	0.6
	gov	11.2	4.0	8.1	16.4
	nfc	5.2	2.1	3.9	7.0
	equities	0.6	0.1	0.4	0.7
	iba	8.4	3.0	5.2	10.3
	loans	58.8	51.8	61.5	66.9
	wf	20.9	12.6	19.9	27.4
	ibl	4.6	0.8	2.3	5.2

Table 1: Statistics of data used to parameterize the model. Derived from transparency templates of the EBA data collection as of 2021-06-30.
'c' – core banks, with total assets > 300bn €, 'p' – periphery banks, with total assets ≤ 300bn €. Categories map to FINREP/COREP as follows: ta=total assets; cash=Cash, cash balances at central banks and other demand deposits; gov=Debt securities, including at amortised cost and fair value, general governments; nfc=Debt securities, including at amortised cost and fair value, credit institutions, other financial and non-financial corporations; equities=Equity exposure; loans=Loans and advances (including at amortised cost and fair value); wf=wholesale funding, incl. other financial institutions and non-financial corporations; ibl=Interbank funding. All variables other than total assets are reported as a percentage of total assets.

transparency templates. Within these three classes, we consider a number of more granular securities, i.e. 20 securities per class, chosen such that a predetermined measure of portfolio similarities is best matched. We sample the composition of assets to match one of the most commonly used similarity measures – the cosine portfolio similarity – computed for the European banking system. For a pair of banks, i and j, a cosine similarity of their portfolios $[x_{i1}, x_{i2}, \ldots, x_{iM}]$ and $[x_{j1}, x_{j2}, \ldots, x_{jM}]$ is

$$cos(x_{i\cdot}, x_{j\cdot}) = \frac{x_{i\cdot}^{\top} x_{j\cdot}}{\|x_{i\cdot}\| \|x_{j\cdot}\|}$$

From Sydow et al. (2021), we borrow a distribution of cosine measures across largest banks in the euro area. Using this data, we compute average similarity among largest banks (labelled *core*) and among smaller banks (labelled *periphery*) and we report the numbers in Table 2. The portfolio sampling procedure is describe in Appendix B.

	q25	mean	q75
core	0.122763	0.29560	0.433167
periphery	0.000000	0.08393	0.086088

Table 2: The table shows statistics of cosine similarity measures across banks in the sample. Specifically, banks are allocated into two groups: core banks, with total assets $>$ 300 billion eur; periphery banks, with total assets $\leq$ 300 billion eur. Statistics, i.e. 'q25'= 25th percentile, mean, median and 'q75'= 75th percentile, are for the distribution of cosine measures computed for pairs of banks separately within core group and within periphery group.

3. (loan portfolios) The focus of the model is on the marketable portfolios that banks can rebalance in a relatively short period of time. However, a large chunk of the balance sheets is typically locked in non-liquid investments like loans to households or corporations. They would not be part of the fire-sale mechanism but nevertheless consume capital and would effectively reduce capacity to absorb losses from revaluation of liquidated portfolios in fire sales.

4. (interbank and direct exposure to shocks from other banks in the system) Typically, direct exposures are relatively small, reduced by large exposure limits introduced after the GFC.[3] However, they still constitute a material part of the balance sheets relative to banks' capital. We know total interbank lending and borrowing of banks and we disaggregate this information to construct an interbank network using a simulated network approach of Hałaj and Kok (2013). The algorithm uses a so-called probability map defining likelihood of a connection between each pair of banks. In a nutshell, for a given probability map the simulated network procedure randomly samples interbank structures using a version of the accept-reject algorithm. To this end, linkages are drawn from a uniform distribution and accepted with a predefined probability p_{ij} of an exposure being extended between two given banks, i and j. By a specific assignment of probabilities to the links, the algorithm can yield a desired configuration of the network. The map is constructed based on the data published in the context of 2021 EBA transparency exercise disclosing for each bank in the sample a geographical distribution of this bank's exposures to credit institutions across

[3] See, e.g., `https://www.eba.europa.eu/regulation-and-policy/large-exposures`

countries.[4] For each country, we aggregate the exposures across core and periphery banks. A probability of an exposure of a bank i in country Cx, depending on whether it is core or periphery, to a bank j in country Cy is computed as ratio of core or periphery banks' exposures to credit institutions in country Cy to total exposures of core or periphery banks to all credit institutions.

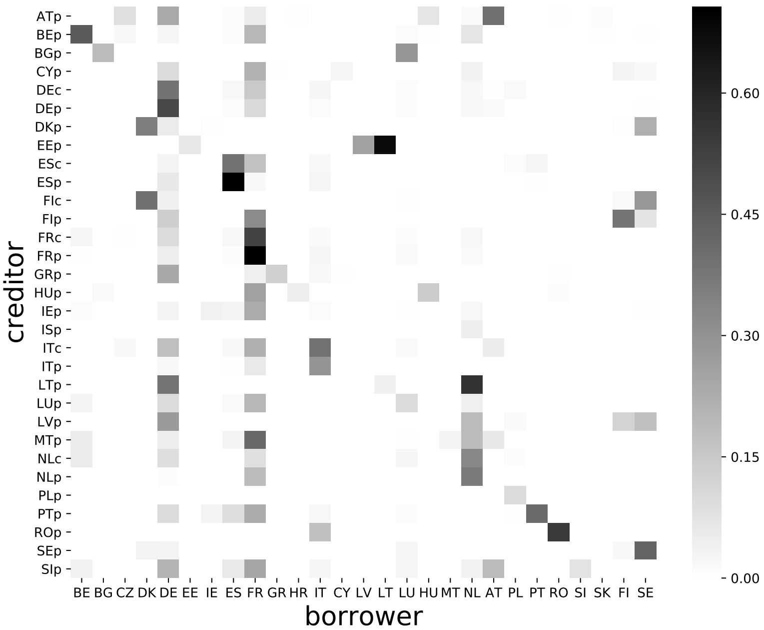


Figure 2: Heatmap represents a probability map derived form 2021 EBA transparency exercise. Each cell at the intersection XXc (or XXp) on the y-axis list shows a probability that the core (or periphery) bank-creditor in country XX is exposed to a bank-borrower in country YY. (using Seaborn in Python)

5. (return, risk and price-impact parameters) To complete the modelling of our system, we consider a few additional assumptions. First, we assume, as in Greenwood et al. (2015), linear inverse demand functions describe the liquidation exposures and provide the asset prices. We borrow sensitivities from Fukker et al. (2022). They estimate price impacts at different quantiles of market distress and we arbitrarily take a 25th percentile reflecting a moderate distress. In these conditions, selling 1bn euro of non-financial bonds results in a 2.5% drop in prices of those assets. Government bonds are 2.4 times less sensitive,

[4]See https://www.eba.europa.eu/risk-analysis-and-data/eu-wide-transparency-exercise/2021

implying slightly more than 1% (i.e., 1.04%) decline in prices of those bonds. Equities are by far more sensitive but, in general the sensitivity depends on the market capitalization of equities; the higher the capitalization the less impact of transacted volumes. Banks would tend to hold equities of larger companies and, following Fukker et al. (2022), we assume a 5% price impact of 1bn equities sold.

Second, we set return on assets and the volatility of returns on assets that feed into the risk matrix C. Return of government bonds is set to 5y government bond yields, equal to 1.6% as of June 2021. For NFC bonds we assume a credit spread which is weighted average of spread for investment-grade (IG) and high-yield (HY) bonds, weighting by the volume of the two categories of bonds held by banks (see Cappiello et al. (2021)), and amounts to 150bps, added on top of the government bond yields. Equity returns are proxied by a dividend yield of 2%. Volatility of government bonds is set to an index published in the ECB SDW database.[5] Its value as of June 2021 was 0.075. Absent a reliable index for non-financial corporate bond volatility we assume that the Sharpe ratio of government and corporate bonds is in parity. This implies 0.16 volatility of NFCs. The volatility of equities is parametrised using EURO STOXX 50 Volatility; it amounted to 25%. We assume that the correlations between assets are constant and equal to 20%.

Finally, the risk aversion parameter γ and the fractions β of asset revaluation recognized as losses are calibrated using equation (5). We are looking for bank- and period-specific values of γ and β such that the theoretically optimal bank asset structure is the closest to the observed one. The detailed outcomes of calibration are shown in Appendix A.

4.2 Numerical stress tests and simulations

We illustrate our model from Section 2 by conducting a set of simulations to determine how banks would (theoretically) react to funding shocks. Specifically, we consider two classes of funding shocks:

1. *Common shocks*: All banks in a given country experience a funding shock of the same magnitude, i.e., the same percentage outflow in a given funding category. This shock mimics a general market distress.

[5] Its reference identifier is CISS.D.U2.Z0Z.4F.EC.SS_BM.CON.

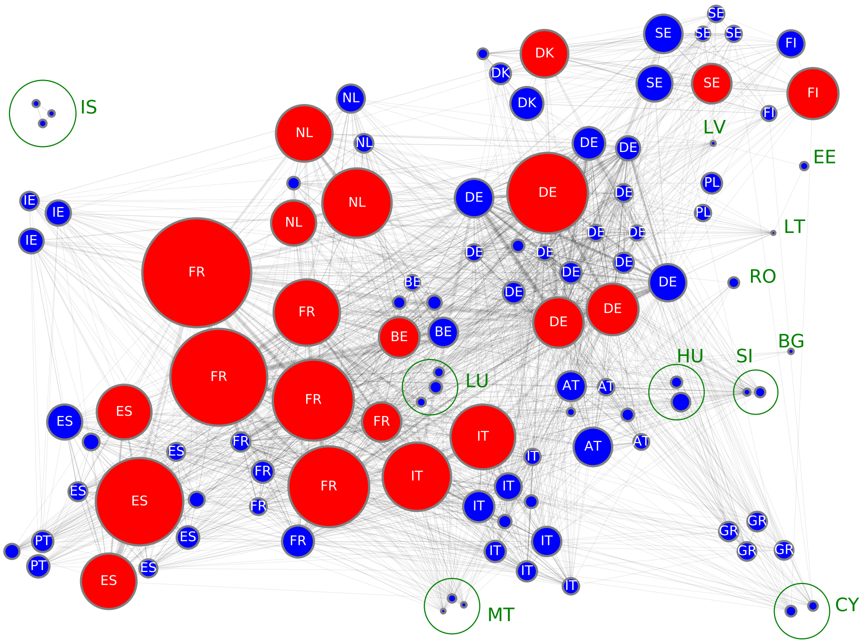


Figure 3: Network of interbank exposures in which nodes indicate banks, the width of edges are proportional to a logarithm of the exposures between the connected banks, i.e., exposures to risk related to interbank lending, debt instruments or derivatives.

2. *Idiosyncratic shocks*: Only one bank experiences a funding shock and other banks only react to the management actions of the initially stressed bank. This shock is larger in magnitude and mimics a situation of loss of trust to one bank that may be related to its deteriorated financial conditions.

We assume that in both classes of shocks it is unsecured wholesale and corporate funding that is in distress. These sources of funding are, in general, sensitive to the market conditions of banks' financial standing. Specifically, for the systematic shock scenario, we analyse a stylized run-off shock of 10% to all banks domiciled in one country, thus describing either a system-wide funding shock or the severity of how a given bank's funding conditions deteriorate. We apply those shocks to a snapshot of data as of June 2021. In the idiosyncratic case, we hit one bank with 25% funding outflow shock.[6] To reduce the impact of a selection bias, we run the model

[6]Additionally, we also study extremely severe scenarios of shocks above 66% run-off rates to explore the default contagion mechanism in the model but we do not report detailed results. Only for shocks of this very high magnitude, some of the banks in the sample would not have sufficient assets to liquidate to meet their obliged payments. Notably, this extremely severe event is highly unlikely to materialize, thus this type of simulation is only to illustrate the default contagion channel in the model.

for each country with more than 10 banks in the sample and present arithmetic mean of the results.

We solve the model by running Algorithm 2.6 with $\Delta_y = 0.1$ liquidation adjustment with 100 iterations of the tâtonnement process with step-size $\Delta t = 1$; as observed, this setup is sufficient to numerically reach an equilibrium. We report the evolution of prices that are a harmonized metric of distress. Below, we report some observations regarding our model and this dataset regarding, e.g., convergence, dependence of price impact on the shock size, and the dynamics of vulnerabilities.

4.2.1 Convergence

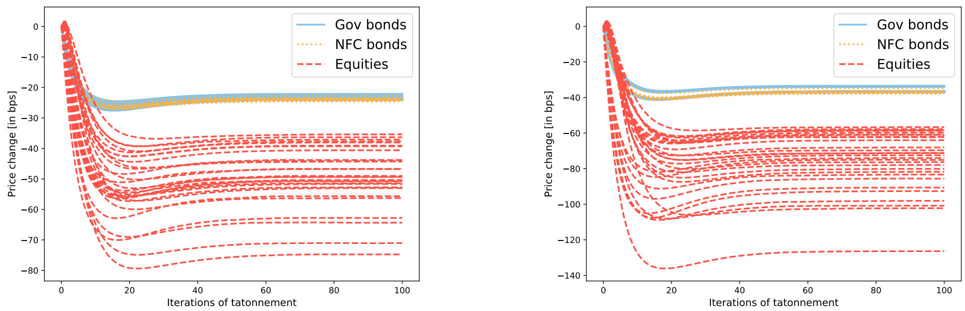
Asymptotic behaviour of Algorithm 2.6 indicates the observed post-stress response of the banks in the market. In Figure 4a, we show the behaviour of prices in one specific example of the system-wide shock (10%) and idiosyncratic shock, and one snapshot of data as of end of June 2021.

First, the process converges after about 60 steps of the (discretized) tâtonnement process as the curves representing prices and cash holdings flatten out. Interestingly, we obtain an equilibrium solution, even though, theoretically, this is not guaranteed for the tâtonnement process (due to the possibility of multiple equilibria prices after the shock). It may be a feature of the data or an empirical confirmation that in a real world applications of the model, prices converge without any ambiguity.

Second, revaluation of assets differs across asset classes. Assets with prices most sensitive to transacted volumes experience the deepest decline in equilibrium. Notably, this is not only a result of the assumed price impact functions but also an implication of the actual volumes traded. It happens that, in equilibrium, banks trade large enough quantities of more sensitive assets so that the average price drops more for equities and non-financial corporate bonds.

Third, price convergence is not monotonic, i.e., after the initial trough asset prices bounce back and stabilize, on average, after 50 iterations of the tâtonnement process.

Implicitly, the specific dynamics of the market after an idiosyncratic funding shock can also be interpreted as trading between two sides of the market. Banks not hit by the initial funding outflow would buy assets from the banks fire-selling to supplement liquidity. However, in our setup we cannot distinguish pairs of transacting banks. Models of market matching can address this issue (see, e.g., Cui and Radde (2019)), but we are focused not on the trading patterns but rather on the impact of trading on market prices.



(a) System-wide funding shock

(b) Idiosyncratic funding shock

Figure 4: Changes in prices of assets (y-axis, bps) in the steps (x-axis) of the tâtonnement process searching for the equilibrium given a shock corresponding to a 10% run-off rate of the unsecured wholesale funding affecting all banks in one country (left panel) or 25% affecting a single bank (right panel). Time is measured by the steps of the tâtonnement process.

4.2.2 Market regulator

One important caveat to the conclusions about price impact is related to the absence of a market regulator in the simulations. Market regulators like central banks have their mandate to watch market liquidity and restore it if it is impaired.[7] This power was demonstrated, for instance, right after the unexpected outbreak of the COVID-19 pandemic, which impacted market and funding liquidity in March and April, 2020; in that period, the market turmoil prompted central banks around the world to inject liquidity into the financial system and to roll out special liquidity facilities (Cavallino and Fiore, 2020). However, there is usually a reaction time that implies initial sell-off of assets and market liquidity dislocation, especially in the case of such an unprecedented shock as COVID-19 (Haas and Neely, 2020). Our model can provide insights into this crisis, helping to understand the impact of the policy support measures and to calibrate them, i.e. to determine their size and scope.

Still, our framework is general enough to allow us to embed a market regulator. The only requirement is that the regulator's actions can be formulated in terms of an optimization program. We illustrate its role in the system assuming that the regulator would try to strategically

[7]See for instance the framework of the Bank of Canada (`https://www.bankofcanada.ca/markets/market-operations-liquidity-provision/`) or ECB (`https://www.ecb.europa.eu/mopo/liq/html/index.en.html`).

minimize the price impact of banks' transactions following a shock. This is consistent with, for instance, central banks' role to maintain price stability and favourable liquidity conditions on the market.

Specifically, we add node $N+1$ to the model representing the regulator. We formulate the regulator's preferences as a penalty due to deviation of prices from the initial price equal to 1. We assume that the regulator could buy assets but only of a certain credit quality, denoted by an eligibility subset $Q \subseteq \{1, 2, \ldots, M\}$. Without loss of generality, we can assume that the regulator does not hold any assets at the start of the funding shock (i.e., $x_{N+1} = \vec{0}$), which means that its strategy y represents net purchasing.[8] Mathematically, the utility of the regulator is written down as

$$u_{N+1}(y) = -\left\| f\Big(\sum_{i=1}^{N+1}[x_i - y_i]\Big) - \vec{1}\right\|^2. \tag{6}$$

Moreover, the regulator has a budget constraint limiting its self-imposed size of the balance sheet (dollar amount B). Combining the budget constraint and eligibility of assets transacted by the regulator, we can write down the admissible set of actions as

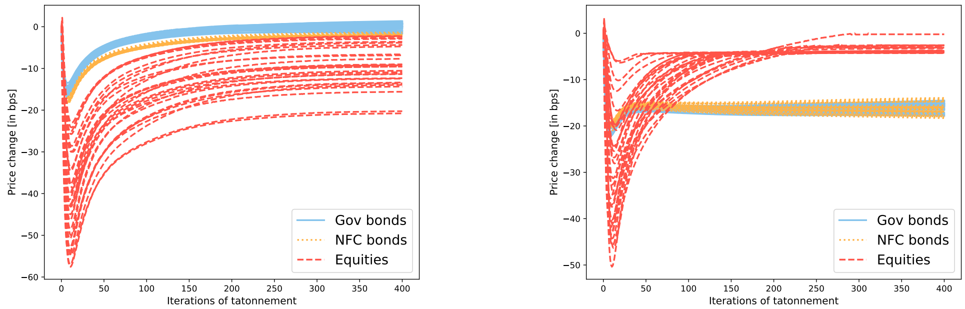
$$\mathcal{A}_{N+1}(q) = \left\{ y_{N+1} \in \mathbb{R}_+^M \;\middle|\; \sum_{m=1}^{M} q_m y_{(N+1),m} \leq B, \quad y_{(N+1),m} = 0 \quad \text{if} \quad m \notin Q \right\}. \tag{7}$$

The regulator maximizes its utility function subject to other banks' holdings of assets, i.e., a function $u_{N+1}(y_{N+1}, (y_1^*, \ldots, y_N^*))$, over $\mathcal{A}_{N+1}(q)$. Notably, since the regulator is represented as another node in the modelled system, the commercial banks $1, \ldots, N$ consider the actions of the regulator in their strategic decisions about selling or buying of assets.

For simplicity, the regulator is not present on the interbank market of direct lending and borrowing; therefore, it is excluded from the interbank network and, thus, the clearing payments algorithm. This assumption would not materially reduce the generality of the model since the regulator would in any case not be expected to default.

In such an augmented setup, we run two types of simulations. First, we illustrate how the set of eligible assets influence the impact of the regulator on the asset prices in equilibrium. Second, we analyse the impact of the budget constraint on the asset prices.

[8]The model allows for an easy extension to include also selling into the set of admissible action of the regulator. However, since we focus on modelling a funding stress situation it is reasonable to assume that the regulator would be buying securities liquidated by the commercial banks.



(a) Highest quality assets purchased by the regulators

(b) Constrained capacity of the regulator

Figure 5: Prices of assets (y-axis) in the equilibrium with a market regulator for 10% funding shock to unsecured wholesale and corporate funding of all banks in one country. Each line represent a different asset class.

To conduct the first type of simulations, we assume B is large enough, so that the regulator allows for unlimited expansion of its balance sheet. Then, we link a composition of Q with the haircuts of the assets, i.e., we parameterize $Q(h)$ as follows:

$$Q(h) = \{m \in \{1, ..., M\} \mid \text{liquidity}_m < h\}\,, \tag{8}$$

where liquidity_m is a liquidity parameter of asset. Government bonds would be considered high-liquid, whereas corporate bonds and equities less liquid assets. We assume a uniform funding shock across banks in one country equal to a 10% run-off rate of the unsecured funding sources. The results are illustrated in Figure 5.

We can clearly see that the negative asset price impact of the shock is mitigated by a regulator who has unlimited capacity and willingness to buy assets of the highest quality, i.e., government bonds, on the market. As depicted in Figure 5a, when the regulator is unconstrained in capacity, it is able to completely stabilize the price of the government bonds that converge to the initial price 1. Since in the tâtonnement process the financial agents – both banks and the regulator – transact only a fraction of what the theoretically optimal buying or selling is prescribed, the funding shock pushes the prices down before they recover. As the highest-quality assets are no longer subject to the fire sales, the banks are healthier and have less need to liquidate other, lower quality, assets. This has outsized beneficial effects for the financial system as the fire sale in other assets requires less rebalancing in total. Specifically, with the regulator, the corporate bonds drop approximately 5 bps and equities drop between 5 and 25 bps; this is in comparison to the baseline scenario (depicted in Figure 4a), i.e. without the intervention of the regulator, in which corporate bonds drop approximately 25 bps and equities drop between 35 and 80 bps.

In the second type of the simulations, we assume that the regulator can purchase assets in all categories held by the banks but we vary the budget constraint. We express the constraint as a fraction of the total volume of the liquid asset in the balance sheets of the commercial banks. The fraction is a parameter to study the regulator's propensity to increase the size of its balance sheet. The results when the regulator is constrained to grow its balance sheet by at most 10% of the total liquid and less liquid assets held by the commercial banks are shown in right subplot in the Figure 5. In that figure, we can see that the regulator is constrained in its ability to rescue the financial system; however, with only a slight larger budget, the regulator is able to stabilize the prices at their initial level of 1. In case the budget constraint is binding, we find that the price of all asset classes are adversely impacted, though to varying degrees. Interestingly, as evidenced in Figure 5b, the regulator appears to use its constrained capacity to stabilize the least liquid asset prices the most; we conjecture that this occurs because of the higher price impacts for equities leads to the smallest net impact (as measured by the Euclidean norm), i.e., these assets provide more bang for the buck. These simulations suggests that the capacity of the regulator to take assets onto its balance sheet should be a key to mitigate the consequences of the funding shock hitting the banking system.

In addition to the direct intervention of the market regulator, we can also investigate a more indirect approach to mitigate financial contagion. The regulator's role in stabilizing the market can also be seen from the accounting rules' perspective. Some studies indicated – although equivocally – that fair-value accounting rules may have potential to fuel the crisis (see Argimon et al. (2015)) since losses need to be recognized immediately after the market prices drop implying revaluation of assets. This, in turn may induce banks to sell even more assets, further depressing the prices and creating negative feedback between market prices and balance sheet management. As the regulator decreases the fraction of assets that needs to be recognised at the market prices (increases β), the drop in prices after a funding shock diminishes.[9] This suggests that fair-value accounting rules can be an instruments to stabilize market prices and reduce the depth of financial crisis.

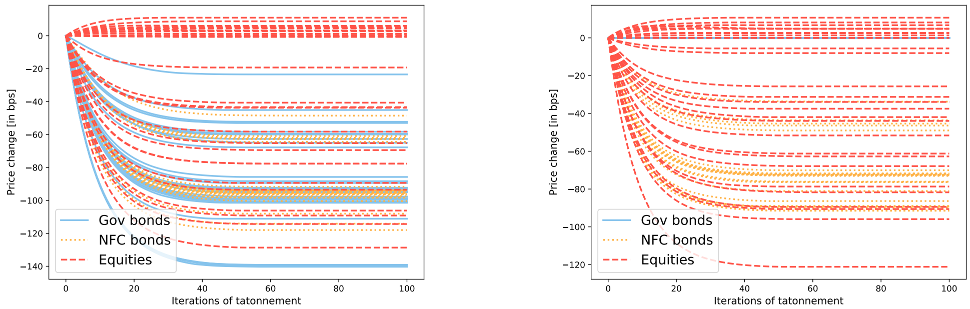
[9] For space considerations, simulations of the system response under varied β and the supporting graphs are not displayed within this text.

4.2.3 Strategic interactions

In convergence to the optimal structure of balance sheets, banks consider other banks' portfolio management actions in their optimization program. Therefore, we are looking at banks' best responses to other banks' moves; the equilibrium we consider combines a clearing payments vector on the interbank market and optimal transactions of assets held in liquidity buffers. For each bank, the optimization of its peers is prescribed in the utility functions through the price impact, which assumes that each bank optimizes its assets subject to funding shock and observed movements in asset prices. To do so, each bank has a perfect insight into other banks' portfolio structures so that it can consider the aggregate transacted volumes $\sum_{j=1}^{N}[x_j - y_j]$. Alternatively, we can expect that exact management actions of other banks are opaque and banks only see the changes in prices. In such a case, banks would only consider their own impact on the market, i.e., bank i analyzes the impact of its transacted volumes $x_i - y_i$ to optimize asset holdings y_i. We look at the implications of the prices in equilibrium of such a myopic optimization. We show the corresponding dynamic of prices in the tâtonnement process in Figure 6.

Remark 4.1. Though we formulate the utility functions *without* strategic interactions so that the clearing solutions would, mathematically, provide Nash equilibria, we do *not* consider such a setting as providing Nash equilibrium. Specifically, the modified utility function for bank i is given by $u_i(\cdot, x_{-i})$ where u_i is defined in Section 3 rather than $u_i(\cdot, y_{-i})$ for strategic interactions; this distinction is introduced due to the partial information available to the banks without strategic interactions. As the banks would ultimately wish to optimize the setting *with* strategic interactions, they may be able to modify their strategy y_i in order to improve the *true* utility (when fixing all other banks' actions y_{-i}).

Three observations can be made on the reaction and overreaction of bank strategies. First, banks that are optimizing balance sheets in isolation will depress asset prices much more than if they were strategically interacting with other banks. We conjecture this is because the "isolated" banks do not internalize other banks' potential actions causing additional pressures on the prices of assets; with interactions the banks may sell less, which corresponds with higher marginal cash raised from any liquidation. Interestingly, prices of some government bonds fall by up to 140 bps comparing with about 80 bp drop when consequences of other banks' actions are internalized by all the market players (compare Fig. 5 and Fig. 4). Bond and equity prices, which are much more sensitive to transacted volumes, exhibit a comparable fall under the funding shock when



(a) No strategic interactions

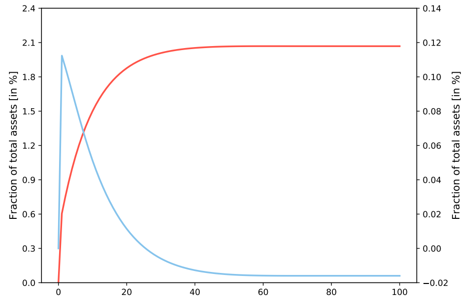
(b) No strategic interactions but with regulators' interventions

Figure 6: Tâtonnement process (x-axis) for 10% funding shock to unsecured wholesale and corporate funding of all banks in a given country. Each line represents an asset class.

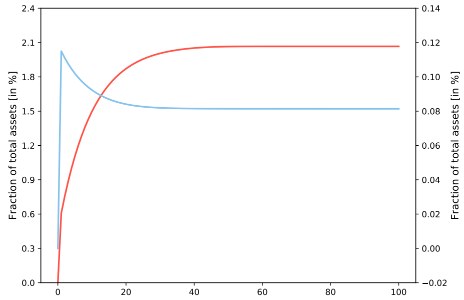
banks decide in isolation about how to raise liquidity.

Second, banks that are not subject to the initial funding shock are less likely to "overreact" in a flight to safety, i.e., sell more assets than strictly necessary, when taking other banks' actions into account. This can be measured by considering the cash held (a_i^*), which (1) can be computed from the surplus from the budget constraint if the bank is solvent and (2) is equal to the assets if the bank is insolvent; we compute and display this cash accumulation (as a fraction of total assets) in Figure 7.

We conjecture that these observations will generally hold since all banks have the same utility function (up to parameter differences) and, thus, all seek to liquidate similar assets. Therefore, when considering the utility function with full interactions, we find that each institution, by endogeonizing the impacts from other banks, internalizes the trade-offs in what it is selling. In contrast, without interactions, each institution does what is locally optimal for it, which can make the system worse off. Conversely, when considering strategic interactions, banks not hit initially by the funding shock are more prone to overreact in the first steps of the tâtonnement process and accumulate cash since banks consider that the value of assets – and, in turn, the capacity of their balance sheets to raise cash – can be impaired by the collective actions of the whole system of banks. Figure 7 shows that cash that initially increases by a similar amount stabilizes at a higher level for banks optimizing their balance sheets in isolation. All in all, there appears to be a trade-off between price impacts and the volatility of prices, as seen by comparing and contrasting the utility with strategic interactions to the utility without interactions.



(a) Change in cash holdings with strategic interactions



(b) Change in cash holdings without strategic interactions

Figure 7: As a % of total assets, funding outflow exhibited by banks in one country. Red line – average for banks initially hit by the funding shock (LHS y-axis scale); Blue line – average for banks not hit initially by the funding shock (RHS y-axis scale).

Third, the regulator is less effective in stabilising prices of assets when banks do not internalize other banks' decisions. Comparing with the strategic decision making (Figure 4a), the regulator buying only the government bonds leaves the equity and non-financial bond markets vulnerable to the sell-off (the depth of price decline for equities and non-financial bonds is comparable in Figures 5a and 6b).

4.2.4 Fire sales

The fire-sale framework presented in Section 2 proposes a system in which banks are allowed to both buy and sell the assets. This is in contrast to much of the prominent literature on fire sales (see, e.g., Greenwood et al. (2015); Amini et al. (2016)) in which the banks are restricted to liquidating assets only (even if they have a surplus). Herein, we look at the impacts of allowing banks to act optimally, according to the utility function defined in Section 3, rather than being forced to follow a *proportional* liquidation strategy. Proportional liquidation means that a bank with a liquidity shortfall sells off its portfolio proportionally to the size of its holdings.

As depicted in Figure 8, the price impacts from our framework are nearly an order of magnitude smaller than the proportional liquidation setting prevalent in the literature (e.g., Greenwood et al. (2015)). This indicates that, by conducting stress tests without appropriately accounting for bank behavior, the health of the financial system can be grossly misspecified. The proportional liquidation setting presents much larger price impacts both due to the nonstrategic nature of the fire sale (i.e., because assets are liquidated without consideration of how this will impact

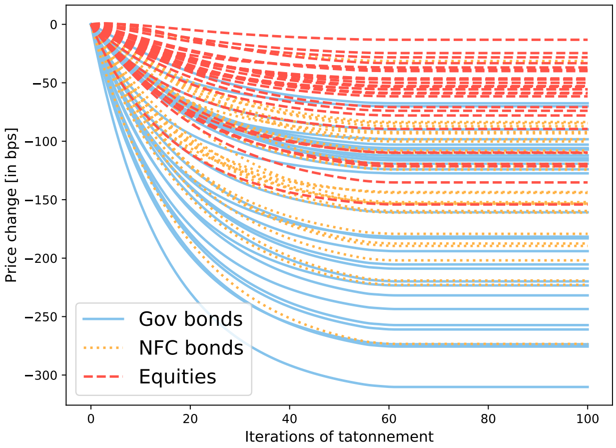


Figure 8: Prices in equilibrium when banks proportionally sell assets (like in Amini et al. (2016)) to meet a funding shock (x-axis, steps in tâtonnement process).

the prices) and because otherwise healthy institutions are unable to provide a backstop to the financial system by purchasing assets at a discount, which can then multiply through feedback effects inherent in price-mediated contagion. Taken together, this proportional liquidation strategy will, generally, overestimate the total liquidations, and thus underestimate the health of the financial system. The model presented in this work, in contrast, accounts for these strategic choices to more accurately simulate the price impacts under a stress scenario. When compared to real stress scenarios, the price impacts observed are much more in line with the single digit percentage price fluctuations than the double digit impacts observed by following the proportional liquidation strategy. Therefore, the magnitude of the second-round effects in stress tests abstracting from strategic balance sheet management under stress should be interpreted with caution.

5 Conclusions

A decade after the Global Financial Crisis, systemic risk is still not fully understood. Systemic risk is attributed to complex structures of financial system and management actions of financial agents that may have unpredictable consequences. We shed light on some aspects of systemic risk related to network effects in the interconnected interbank market and asset management under stress.

To this end, we build a model of the interbank market where banks are linked by direct exposures and may respond to financial shocks by rebalancing their assets to raise cash and maximize utility from investment opportunities. We bring the model to a comprehensive dataset on the European banking system. This allows us to measure financial vulnerabilities related to funding shocks and effectiveness of regulatory interventions to mitigate liquidity risk.

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A Calibration

To assess the accuracy of the calibration, we compare the observed composition of banks' assets with the asset structures obtained in the portfolio optimization using the calibrated values of risk aversion γ and mark-to-market elasticity β. On the whole, banks' total assets are recomputed from the calibrated values of risk aversion, and mark-to-market elasticities are very close to the observed values. On average, total assets in calibration are 96% of the observed total assets (average across banks and periods), with only a 1.6% standard deviation. However, there is more divergence between calibrated and observed asset structures. We measure the accuracy of the calibrated asset structures by the normalized difference between the observed and calibrated assets. The measures for each bank and for all points in time in the data sample are presented in Figure 2. The accuracy is generally between 5% and 20%, with one bank around 30%.

B Similarity

We group banks into two subgroups: (i) highly connected, (ii) periphery. The connectivity through portfolio similarity within group (i) is defines as c_C and between banks in group (ii) as c_P. For consistency, similarity of portfolios between core and periphery banks is rationalized in the following way. Since cosine similarity is invariant to proportional scaling up or down of the portfolios we can randomly generate structures based on versors, i.e. vectors in the space of $\{0,1\}^M$. Assuming that we sample entries of portfolios independently from a Bernoulli distribution with probability p_C of drawing 1 and $1-p_C$ of drawing 0 for highly connected banks, and p_P of drawing 1 and $1-p_P$ of drawing 0 for periphery banks, we get that for $i \in C$ and $j \in P$

$$\mathbf{E}cos(x_i, x_j) = \frac{Mp_C p_P}{\sqrt{Mp_C}\sqrt{Mp_P}} = \sqrt{p_C p_P} = \sqrt{c_C c_P},$$

where the last equality follows from the first when considering similarity for two core banks and for two periphery banks. Consequently, we obtain a similarity matrix S

$$S_{ij} = \begin{cases} c_C & i \in C,\ j \in C \\ c_P & i \in P,\ j \in P \\ \sqrt{c_C c_P} & \text{otherwise} \end{cases}$$

To randomly sample portfolios with a similarity property S, we rely on a fact related to a

	Item	Description	FINREP/ COREP codes
0	ta	Total assets	F01.01_380_010
1	cash	Cash, cash balances at central banks and other demand deposits	F01.01_010_010
2	gov	Debt securities, including at amortised cost and fair value, general governments	F18.00.a_030_010; F18.00.a_183_010; F18.00.a_213_010
3	nfc	Debt securities, including at amortised cost and fair value, credit institutions, other financial and non-financial corporations	F18.00.a_040_010; F18.00.a_184_010; F18.00.a_214_010; F18.00.a_050_010; F18.00.a_185_010; F18.00.a_215_010; F18.00.a_060_010; F18.00.a_186_010; F18.00.a_216_010
4	equity	Equity exposure	C07.00.a_010_010_016; C09.02_140_010_999—0
5	iba	Exposure to institutions (interbank lending)	C07.00.a_010_010_007; C08.01.a_010_020_005; C08.01.a_010_020_006
6	loans	Loans and advances(including at amortised cost and fair value	F18.00.a_070_010; F18.00.a_191_010; F18.00.a_221_010
7	wf	Wholesale funding, incl. other financial institutions and non-financial corporations	F08.01_210_010; F08.01_210_020; F08.01_210_030; F08.01_210_034; F08.01_210_035; F08.01_260_010; F08.01_260_020; F08.01_260_030; F08.01_260_034; F08.01_260_035
8	ibl	Interbank funding	F08.01_160_010; F08.01_160_020; F08.01_160_030; F08.01_160_034; F08.01_160_035

Table 3: Data points mapped to FINREP/ COREP codes.

Cholesky decomposition of matrices. The number of assets (M) in portfolios has to be equal or larger than the number of banks (N). First, for a matrix X representing portfolios of banks, similarity matrix $cos(X)$ satisfies

$$cos(X) = D_X^\top X^\top X D_X,$$

where D_X is a diagonal matrix with inverse of norms of columns in X. Therefore, we can see that similarity is invariant to a multiplication by any diagonal matrix. Moreover, suppose that $S = LL^\top$, with L being a Cholesky decomposition matrix. Then, for any $N \times M$ matrix O with rows pair-wise orthogonal, the following matrix $P_L(O) = OL$ defines portfolios with similarity S:

$$cos(P_L(O)) = cos(L^\top O^\top OL) = cos(L^\top DL) = cos(\sqrt{D}L^\top L\sqrt{D}) = S$$

$D := O^\top O$ is a diagonal matrix, since O is orthogonal and $\sqrt{D}$ is a diagonal with square root of diagonal entries of D.

Since O has negative entries, we need to adopt an approximate procedure to generate portfolios with similarity S applying the following steps:

1. Sample O from a multivariate normal distribution.
2. Assign $X_+ := \max\{0, OL\}$, i.e. replace negative entries with 0.
3. Scale up or down each column of X_+, so that the sum of entries equals the size of corresponding bank's securities portfolio.
4. Repeat 1-3 several times and choose a realization in step 3 that minimizes the difference in cosine similarities with respect to the given similarity S.

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Zachary Feinstein
Stevens Institute of Technology, Hoboken, United States; email: zfeinste@stevens.edu

Grzegorz Hałaj
European Central Bank, Frankfurt am Main, Germany; email: Grzegorz.Halaj@ecb.europa.eu

Postal address 60640 Frankfurt am Main, Germany
Telephone +49 69 1344 0
Website www.ecb.europa.eu

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