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Government guarantees and the two-way feedback between banking and sovereign debt crises

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Abstract

This paper studies the effects of government guarantees on the interconnection between banking and sovereign debt crises in a framework where both the banks and the government are fragile and the credibility and feasibility of the guarantees are determined endogenously. The analysis delivers some new results on the role of guarantees in the bank-sovereign nexus. First, guarantees emerge as a key channel linking banks' and sovereign stability, even in the absence of banks' holdings of sovereign bonds. Second, depending on the specific characteristics of the economy and the nature of banking crises, an increase in the size of guarantees may be beneficial for the bank-sovereign nexus, in that it enhances financial stability without undermining sovereign solvency.

Keywords: bank runs, sovereign default, strategic complementarity, government bond yield.

JEL classifications: G01, G18, H63

Non-Technical Summary

Public guarantees to financial institutions are considered to be an effective tool to prevent banking crises and mitigate their negative effects. However, they have also been blamed for the large costs they entail for the government providing them and, ultimately, for taxpayers. The recent Eurozone crisis has contributed to renew the interest about the desirability and design of public intervention in the banking sector, given that they proved to be an important channel feeding the negative spillovers between banks' and sovereign's stability.

As the Irish crisis in 2008 has shown, the provision of guarantees may entail an actual and potentially large disbursement for the government, to the point of threatening its solvency. The threat of a sovereign default is not just bad per se, but it also introduces new types of risks associated with guarantees designed to prevent banking crises. In particular, the deterioration of public finances reduces the credibility of the guarantees and, thus their effectiveness in preventing instability in the banking sector, with the consequence of magnifying the need and cost of public intervention. This sequence of negative effects, where banks and sovereigns stability affect each other has been labelled the bank-sovereign nexus and received increased attention by academics and policymakers in recent years.

Are public guarantees a triggering factor for the vicious circle between banks and sovereigns? What are the consequences of their introduction for banks' and sovereigns' stability? Is an increase in the level of guarantees always associated with an increase in sovereign instability and, in turn, to reduced credibility and effectiveness of the guarantees in preventing banking crises? Do guarantees interact with other factors, such as the nature of crises, the prospects of the economy and the health of the government, in determining the emergence of negative spillovers between banks' and sovereign's stability?

This paper tackles these questions and provides new insights about the role that guarantees play for the bank-sovereign nexus and their effects on both banks' and sovereign's stability. In the paper, I develop a theoretical framework where both banks and sovereign are fragile in that they are exposed to roll over risk and their ability to repay creditors depends on the realization of the state of the domestic economy. Both banking and sovereign debt crises either are associated with very low realization of the state of the economy or can be triggered by a coordination failure among creditors.

In this framework, guarantees to financial institutions play a key role for the emergence of banking and sovereign debt crises and their interaction, as they link depositors' and sovereign creditors' withdrawal decisions. This arises because the provision of the guarantees depends on the resources available to the government, which in turn are determined by sovereign creditors' rollover decision, and on the number of depositors running. The fewer investors roll over their investment in sovereign bonds, the less the resources available to the government to pay the guarantees and so the lower the repayment accrued by depositors. Symmetrically, the more depositors run, the larger the amount that the government needs to transfer to the banking sector, with the consequence that the government's budget is tighter, sovereign creditors' expected repayment is lower and the probability of a sovereign default is higher. It is precisely this strategic complementarity between depositors' and sovereign creditors' actions that sets the ground for the negative feedback loop between banks and their sovereign, where the increase in the probability of a sovereign default, induced by the provision of the guarantees, translates into reduced effectiveness of the guarantees in limiting instability in the banking sector.

The analysis in the paper provides a number of novel results about the effect of guarantees on the banks-sovereign nexus. First, guarantees play a key role for the emergence of the bank-sovereign nexus and they are an important channel for spillovers between banks and sovereign. Given this role, interesting interactions are at play between the guarantees and banks' holdings of sovereign bonds, which have been so far considered as the main transmission channel in the bank-sovereign nexus. This suggests that policies addressing the design of guarantee scheme and its funding are important in limiting the complementarity between banks and sovereign fragility and should be designed together with other policies tackling the spillovers between banking and sovereign debt crises.

Second, guarantees affect the probability of a banking crisis and a sovereign default in a non-trivial way. By improving banks' available resources, guarantees have a direct beneficial effect on financial stability. However, by tightening the government's budget constraint, they have a direct detrimental effect on sovereign solvency. Besides these direct effects, guarantees also indirectly affect each probability, because the likelihood of a banking crisis and that of a sovereign default affect one another. By reducing the probability of a banking crisis, guarantees reduce the instances in which the government transfers resources to the banking sector, thus improving sovereign stability. Symmetrically, by increasing the probability of a sovereign default, and so limiting the amount of resources available for public intervention, guarantees may become less credible, thus reducing banks' stability. Whether guarantees are beneficial or detrimental for financial and sovereign stability, it depends on which of these effects dominates.

An increase in the size of guarantees leads to a reduction in the probability of a banking crisis without undermining sovereign solvency when the direct effect on banks' stability is large. In this case, by significantly reducing the probability of a banking crisis, guarantees reduce the instances in which the government transfers resources to the banking sector, thus reducing the expected disbursement associated with the guarantees and, as a result, improving sovereign stability. The analysis in the paper suggests that such a result is more likely to emerge when the prospects for the economy are good, the government has a sound budget and banking instability is due to liquidity problems rather than solvency issues. In this respect, the analysis makes it possible to capture the cross-country differences in the effects of the increase in support offered by the governments to their banking sectors that emerged during the recent crisis and provides interesting insights into the variables that may play a role in the bank-sovereign nexus.

Overall, the analysis in the paper contributes to the discussion about the desirability of a new resolution framework and other policies aimed at limiting government support to banks, as well as the one about a pan-European deposit insurance scheme.

1 Introduction

Public guarantees to financial institutions are considered an effective tool for preventing banking crises and mitigating their negative effects. However, they have also been criticized for the large costs they entail for the government providing them and, ultimately, for taxpayers. The recent crisis has contributed to renewed interest in the desirability and design of public intervention in the banking sector, given the massive use of public funds and significant amendments to the existing guarantee schemes.

So far, the academic and policy debate (see Allen, Carletti, Goldstein and Leonello, 2015, for a survey) has focused on the distortions of banks' risk-taking associated with the provision of public guarantees: as banks do not fully bear the costs of their potential failure, public guarantees induce them to take excessive risks. As a consequence, the provision of guarantees, or even the mere announcement of their introduction, may lead to an increase in the likelihood (and possibly also in the severity) of a crisis, and thus in the need for and costs of public intervention.

While certainly important, the moral hazard problem induced by the guarantees is not the only form of risk - and source of costs - associated with their provision. As the recent events during the euro area crisis have shown, even in the absence of moral hazard considerations, the provision of guarantees may entail an actual, and potentially large, disbursement for the government to the point of threatening the solvency of the sovereign. The threat of a sovereign default is not just bad per se, but also introduces new types of risks associated with guarantees designed to prevent banking crises. In particular, the deterioration of public finances reduces the credibility of the guarantees and thus their effectiveness in preventing instability in the banking sector, with the consequence of magnifying the need for and cost of public intervention.

The Irish crisis in 2008 offers a clear illustration of this chain of events and the role that government guarantees played in triggering it. At the end of September 2008, the Irish Government announced the introduction of blanket guarantees as a response to the increased instability in the domestic banking sector, triggered by the failure of Lehman Brothers. This announcement led to an immediate increase in the sovereign credit default swap (CDS) spread and then to a co-movement between bank and sovereign CDS in the post-guarantee phase, as illustrated in Figure 1.

Insert Figure 1

This sequence of negative effects, where the banks' and the sovereign's stability affect each other, has been labelled the bank-sovereign nexus and has received increased attention from academics and policymakers in recent years. In the academic literature, guarantees represent only one of the channels linking the stability of banks and sovereigns and are not responsible for the entire feedback loop. These contributions mainly focus on banks' holdings of sovereign bonds and identify them as the main transmission channel of crises between banks and their sovereign: banks' bond holdings allow the distress of the sovereign to spread directly to banks and, furthermore, by affecting the government's decisions on bailouts and strategic defaults, they indirectly determine the losses borne by domestic banks in the case of sovereign default (see for example Acharya et al., 2014; Gennaioli et al., 2014 and 2015; and Faia, 2016). In this context, guarantees only play an ancillary role. Understanding the role of public guarantees in triggering and shaping the bank-sovereign nexus is, however, of paramount importance and has so far been overlooked both by the existing literature on guarantees and by that on the bank-sovereign nexus.

Are public guarantees a factor in triggering the vicious circle between banks and sovereigns? What are the consequences of their introduction for banks' and sovereigns' stability? Is an increase in the level of guarantees always associated with an increase in sovereign instability and, in turn, with reduced credibility and effectiveness of the guarantees in preventing banking crises? Do guarantees interact with other factors, such as the nature of crises, the prospects for the economy and the health of the government, in determining the emergence of negative spillovers between banks' and the sovereign's stability? These questions are also important in light of the recent crisis, during which economies implementing similar measures to restore financial stability differed significantly in terms of the impact that public intervention had on government solvency. As illustrated in Figure 2, the case of Germany differed significantly from that of Ireland, described above. In Germany neither the injection of funds into Hypo Real Estate Holding nor the approval by the Parliament of the Act on the Establishment of a Financial-Market Stabilization Fund in October 2008 had significant negative consequences for German public finances. While these interventions seemed effective in reducing the instability in the financial industry, German Bund yields remained low.

Insert Figure 2

To analyze the role that government guarantees play in the bank-sovereign nexus, I develop a global game model with a banking sector and a sovereign bond market, where both the banks and the government are fragile in that they are exposed to rollover risk. This framework makes it possible to endogenously determine the likelihood of a banking and a sovereign debt crisis, as well as the role that the provision and size of public guarantees play for banks' and the sovereign's stability and their interaction.

In the model, banks raise domestic consumers' after-tax endowments and invest them in a risky investment, the return on which depends on the prospects for the domestic economy. Domestic consumers are risk averse and derive their utility from both a private and a public good. The public good is provided by the government at date 2. At date 0, the government raises resources from two sources. It taxes domestic agents and issues bonds to foreign investors. All these resources are invested in a risky project, the return on which also depends on the prospects for the domestic economy. At the interim date, each depositor and investor receives an imperfect signal about the prospects for the domestic economy and, based on this signal, they decide whether to run (if a depositor) and whether or not to roll over (if an investor). In taking this decision, they compare the payoffs they would get from withdrawing at date 1 with those from rolling over their investment until date 2.

In this framework, guarantees to financial institutions play a key role in the emergence of banking and sovereign debt crises and their interaction, as they link depositors' and investors' withdrawal decisions and are the source of strategic complementarity between depositors' and sovereign creditors' actions. This arises because the provision of the guarantees depends on the resources available to the government, which in turn are determined by sovereign creditors' rollover decision, and on the number of depositors running. The fewer investors roll over their investment in sovereign bonds, the less the resources available to the government to pay the guarantees and so the lower the repayment accrued by depositors. Symmetrically, the more depositors run, the larger the amount that the government needs to transfer to the banking sector, with the consequence that the government's budget is tighter, sovereign creditors' expected repayment is lower and the probability of a sovereign default is higher. It is precisely this strategic complementarity that sets the ground for the negative feedback loop between banks and their sovereign, where the increase in the probability of a sovereign default, induced by the provision of the guarantees, translates into reduced effectiveness of the guarantees in limiting instability in the banking sector.

This result suggests a number of important observations about the contribution of the model and its policy implications. First, guarantees play a key role in the emergence of the bank-sovereign nexus. This suggests that policies addressing the design of guarantee schemes and their funding can play an important role in mitigating the negative spillovers between banks and sovereigns. The analysis in the paper thus contributes to the discussion about the desirability of a new resolution framework and other policies aimed at limiting government support to banks, as well as that on a pan-European deposit insurance scheme. Furthermore, extending the analysis to account for banks' holdings of sovereign bonds shows that there are relevant interactions between guarantees and bond holdings to be taken into account when designing policies to tackle the complementarity between banking and sovereign debt crises.

Second, guarantees affect the probability of a banking crisis and a sovereign default in a non-trivial way. The standard view in the literature is that guarantees are beneficial for banks' stability, as they improve banks' available resources and so lead to a reduction in the probability of a banking crisis, but are detrimental for sovereign solvency, as they entail a disbursement for the government and lead to an increase in the probability of a sovereign default. Besides these direct effects, guarantees also indirectly affect each probability, because the likelihood of a banking crisis and that of a sovereign default affect one another. By reducing the probability of a banking crisis, guarantees reduce the instances in which the government transfers resources to the banking sector, thus improving sovereign stability. Symmetrically, by increasing the probability of a sovereign default and so limiting the amount of resources available for public intervention, guarantees may become less credible, thus reducing banks' stability. These indirect effects would not emerge in more standard frameworks without complementarity between banks' and sovereign's stability.

Third, since the direct and indirect effects work in opposite directions, the overall effect of the guarantees on the bank-sovereign nexus can either be positive or negative. The former occurs when an increase in the size of the guarantee scheme leads to such a significant reduction in the probability of a banking crisis that, despite the increased disbursement for the government, the probability of a sovereign default also drops. This positive feedback loop between banks and their sovereign induced by an increase in the level of guarantees is not attainable in more standard models where the probabilities of crises are exogenous, independent of each other and not affected by changes in the level of guarantees.

Lastly, the specific characteristics of the economy and the nature of crises play an important role in determining the effect of guarantees on the bank-sovereign nexus. The analysis in the paper suggests that when the prospects for the economy are good, the government has a sound budget and banking instability is due to liquidity problems rather than solvency issues, an increase in the level of guarantees is more likely to have a beneficial effect on the bank-sovereign nexus. In this respect, the theoretical model developed in the paper makes it possible to capture the cross-country differences in the effects of the increase in support offered by the governments to their banking sectors that emerged during the recent crisis and provides interesting insights into the variables that may play a role in the bank-sovereign nexus.

The novelty of the paper is to analyze the effects of guarantees to financial institutions in a context where the government's budget is limited and both the banks and the government are fragile.

The paper belongs to the large literature on public intervention in banking (see e.g., Allen, Carletti and Leonello, 2011 for a survey) and challenges two crucial assumptions in this literature regarding the feasibility and credibility of guarantees. In the traditional literature on guarantees (see e.g., Diamond and Dybvig, 1983; Cooper and Ross, 2002), it is assumed that governments always have a way to raise resources to finance the intervention and that once guarantees are announced, banks' creditors are sure to receive the guaranteed amount. By contrast, in this paper neither the feasibility nor the credibility of the guarantees is taken for granted, rather they are determined endogenously in the model. This is in line with what we observed during the recent euro area crisis. When countries do not have the possibility to monetize the guarantees, they may not be able to raise the resources they need to pay them by raising taxes and/or issuing bonds and, as a result, the beneficiaries of the guarantees are no longer sure that they will receive the guaranteed amount.[1]

The paper also contributes to a new and growing literature on the bank-sovereign nexus. This literature features the seminal contributions by Bolton and Jeanne (2011) and Gennaioli, Martin and Rossi (2014), both developing a model where sovereign default negatively affects the stability of the banking sector. Among the papers analyzing the feedback loop between banks and sovereign, the closest to my paper are Cooper and Nikolov (2013), Acharya, Drechsler and Schnabl (2014) and Farhi and Tirole (2014).

There are significant differences between these three papers and mine. First, in my framework the guarantees are the key channel linking bank's and sovereign's stability, while in all the above mentioned papers, banks' holding of domestic sovereign bonds represent the channel through which the deterioration in the stability of the sovereign spills over to the banking sector. Second, in my model, both panic- and fundamental-based banking and sovereign debt crises can occur and their probabilities are endogenous and uniquely pinned down. This is an important difference as it allows to characterize the existence of a positive feedback loop between banking and sovereign debt crises induced by an increase in the guarantees.

The importance of endogenizing crises probability and thus, the use of global games, to analyze guarantees is also a central element in Allen, Carletti, Goldstein and Leonello (2017). In a model where the government budget is exogeous and it has always enough resources to pay the guarantees, they focus on the effect that guarantees have on the interaction between banks' liquidity creation and the likelihood of runs. I abstract from these aspects in their analysis, which makes it possible to focus on the issues related to the feasibility and credibility of guarantees that are, instead, missing from their paper. Specifically, unlike their

[1] Even in the case of countries like United States and United Kingdom, where guarantees can be monetized, there are always costs associated with this (inflation for example). Those costs represent a limit on the amount that the government can credibly guarantee and, in turn, on the feasibility of the scheme.

paper, I fully endogenize the government budget and allow the government to default. These are important differences since they determine the need to explicitly account for the presence of a sovereign bond market and to model sovereign default. Moreover, the presence of sovereign bond investors and guarantees introduces another source of strategic complementarity in the model on top of the intra-group strategic complementarity present in both models.

By deriving a two-sector model with two types of strategic complementarity, my model is in the spirit of Goldstein (2005), who analyses the interaction between the probabilities of a banking and a currency crisis based on the existence of strategic complementarity between depositors and investors. This paper is different in two respects. First, it deals with a more involved framework, since depositors' incentives to run do not monotonically increase with the number of depositors running and of investors not rolling over the investment in sovereign bonds. Second, it endogenizes agents' payoffs, namely banks' choice of the deposit contract and the yields on the sovereign bonds.

König, Anand and Heinemann (2014) also analyze the bank-sovereign nexus using global games and show that the credibility and effectiveness of the guarantees are intertwined with the sovereign funding risk. My paper differs from theirs in two important respects. First, I endogenize the creditors' payoffs and discuss how they are affected by the guarantees. Second, by focusing on the case where the prospects for the banks and those for the government are affected by the same variable, I deal with a richer set of possible outcomes in terms of interdependence between banking and sovereign debt crises.

Lastly, the paper speaks to the literature on commitment and bailouts (e.g. Farhi and Tirole, 2012; Keister, 2016). Those papers look at the ex post versus ex ante optimality of government intervention, its effect on banks' risk-taking and, hence, financial stability. The idea in these papers is that public intervention distorts banks' incentives to take risk and may not be desirable. This affects the credibility of the guarantees. In my paper, the government fully commits to providing the guarantees and yet the guarantees still may not be fully credible, as a consequence of the fact that their provision is not always feasible.

The paper proceeds as follows. Section 2 presents the model. Section 3 characterizes the equilibrium of the model and highlights the role of guarantees in the bank-sovereign nexus. Section 4 solves for the optimal deposit contract and the interest rate on sovereign bonds, thus highlighting the effect of guarantees on the cost of debt for both the banks and the sovereign. Section 5 extends the baseline model in two directions: it allows for banks' holdings of sovereign bonds and discusses the role of austerity measures in reducing the negative spillovers between banking and sovereign crises. Finally, Section 6 concludes. All proofs are in the

Appendix.

2 The model

The model is a standard banking model, in the spirit of Goldstein and Pauzner (2005), augmented by a government and a sovereign bond market.

2.1 The environment

Consider a three date economy ($t = 0, 1, 2$) with a banking sector and a sovereign bond market. In the banking sector, there is a continuum $[0, 1]$ of banks and a continuum $[0, 1]$ of risk-averse domestic consumers holding deposits in every bank. Consumers have to decide whether they want to withdraw their funds at date 1 or to wait until date 2.

In the sovereign bond market, a continuum $[0, 1]$ of risk-neutral foreign investors hold government bonds and they have to decide whether or not to roll over their investment in sovereign bonds at date 1. As I will explain in details below, all agents take their decisions after observing an imperfect signal about the growth rate of the domestic economy Y, which is a uniformly distributed random variable with $Y \sim U[0, 1]$. The variable Y depicts the state of the domestic economy and positively affects both the prospects for the bank and those for the government.

The banking sector

At date 0, each bank raises consumers' after-tax endowments $1 - t$ in exchange for a demandable deposit contract $(r_{B0}, \widetilde{r}_{B1})$ and invests those funds in a risky technology. For each unit invested at date 0, the investment returns one unit if liquidated at date 1 and a stochastic return $R(Y)$ at date 2, with $R'(Y) > 0$ and $E_Y [R(Y)] > 1$.

The banking sector is perfectly competitive, thus banks maximize depositors' expected utility and make zero profits. The deposit contract features a fixed repayment r_{B0} to depositors withdrawing at date 1 and a payment $\widetilde{r}_{B1}$ equal to a share of the return of bank's non-liquidated portfolio for those waiting until date 2. The repayment offered to early withdrawing depositors must lie in the range $\left[1 - t, \frac{1-t}{\lambda}\right)$ as (net) deposit rates are assumed to be always positive, whereby $r_{B0} \geq 1 - t$, and as a bank cannot promise a repayment that is not able to honour, whereby $r_{B0} < \frac{1-t}{\lambda}$.

Domestic consumers derive utility from two sources: they derive utility $u(c)$ from the consumption of the payments obtained from banks and $v(g)$ from the consumption of a public good provided by the government.

A consumer's preference is then given by

$$U(c,g) = u(c) + v(g),$$

with $u'(c) > 0, u''(c) < 0$, $v'(g) > 0, v''(g) < 0$, $u(0) = v(0) = 0$ and the relative risk aversion coefficient $\frac{-cu''(c)}{u'(c)} > 1$, for any $c > 1$.

Depositors are identical ex ante but can be of two types ex post: with probability λ, a preference shock hits and a depositor wants to consume the private good at date 1. With probability $1-\lambda$ no preference shock hits and a depositor is willing to wait and consume at date 2. For the Law of Large Numbers, λ and $1-\lambda$ represent the fraction of depositors who want to consume early (i.e., impatient depositors) and that of depositors that want to consume late (i.e., patient consumers), respectively. A consumer's type is private knowledge.

At date 1, banks satisfy consumers' withdrawals by liquidating their investment and by using the funds provided by the government as part of the guarantee scheme. If the sum of the liquidation proceeds and the guarantee is not enough to repay r_{B0} to the n withdrawing depositors, a bank is declared bankrupt and an orderly liquidation procedure starts. As a result, all assets of the bank are liquidated at date 1 and shared equally among the bank's creditors. Each of them is then entitled to receive an amount $1-t+\Gamma$, where $1-t$ is the liquidation value of the bank's assets and Γ represents the guarantee paid by the government as specified in details below.

The sovereign bond market

At date 0, the government raises resources from two sources: it taxes domestic consumers and issues bonds to foreign investors. Investors have an endowment of one unit at date 0 and nothing thereafter and no profitable alternative investment opportunities (they can only store their endowment if not investing in sovereign bonds). The resources raised by the government are invested in a productive technology in order to finance the provision of the public good g to domestic agents at date 2 and that of a guarantee scheme.

Such an investment technology returns one unit if liquidated at date 1 and a stochastic return at date 2 denoted with $\widetilde{G}(Y, I(i,t,r_{G0},\Gamma))$. This return depends positively on the growth rate of the domestic economy Y and on the number of units $I(i,t,r_{G0},\Gamma)$ that the government invests until date 2, where

$$I(i,t,r_{G0},\Gamma) = \max\{0, 1+t-ir_{G0}-\Gamma\}.$$

The amount of resources $I(i,t,r_{G0},\Gamma)$ invested by the government between date 1 and 2 is given by the difference between the initial investment $1+t$ and the units liquidated at date 1 to pay the interest rate r_{G0}

to the i investors withdrawing at date 1 minus the amount Γ transferred to the banking sector as specified in the guarantee scheme. Assumption 1 summarizes the properties of the function $\widetilde{G}(Y, I(i, t, r_{G0}, \Gamma))$.

Assumption 1 *The function* $\widetilde{G}(Y, I(i, t, r_{G0}, \Gamma))$ *is*

i) increasing in Y *and* t *i.e.,* $\frac{\partial \widetilde{G}(Y,I(i,t,r_{G0},\Gamma))}{\partial Y} > 0$ *and* $\frac{\partial \widetilde{G}(Y,I(i,t,r_{G0},\Gamma))}{\partial t} > 0$;

ii) decreasing in i, r_{G0} *and* Γ *i.e.,* $\frac{\partial \widetilde{G}(Y,I(i,t,r_{G0},\Gamma))}{\partial i} < 0$, $\frac{\partial \widetilde{G}(Y,I(i,t,r_{G0},\Gamma))}{\partial r_{G0}} < 0$ *and* $\frac{\partial \widetilde{G}(Y,I(i,t,r_{G0},\Gamma))}{\partial \Gamma} < 0$; *and*

iii) $\frac{\partial \widetilde{G}(Y,I(i,t,r_{G0},\Gamma))}{\partial I(.)}$ *is independent of* Y.

The long term return on the government's investment $\widetilde{G}(Y, I(i, t, r_{G0}, \Gamma))$ positively depends on the growth rate of the domestic economy Y. Moreover, it also increases with the amount of resources that the government keeps invested until date 2 $I(i, t, r_{G0}, \Gamma)$. While the marginal productivity of a unit invested is independent of the state of the domestic economy Y, it depends on the number of investors that do not rollover the bonds i, the interest rate promised to investors withdrawing at date 1 r_{G0}, the guarantees transferred to the banking sector Γ, as well as the tax revenue accrued by the government t. The first three variables have a negative impact on $\widetilde{G}(Y, I(i, t, r_{G0}, \Gamma))$, while an increase in the tax revenue t leads to a higher return, via its positive effect on $I(i, t, r_{G0}, \Gamma)$.

Government bonds pay a (gross) interest rate $r_{G0} \geq 1$ to the investors withdrawing at date 1 and $r_{G1} \geq r_{G0}$ to those who roll over their investment until date 2.[2] While r_{G0} is always paid, the interest rate r_{G1} is only repaid if the government is solvent at date 2. The government defaults at date 2 if the return on the investment $\widetilde{G}(Y, I(i, t, r_{G0}, \Gamma))$ is not enough to pay r_{G1} to the $1-i$ investors that roll over the investment in sovereign bonds and to finance a minimum level of public expenditure $\overline{g}$. Formally, this is the case when

$$\widetilde{G}(Y, I(i, t, r_{G0}, \Gamma)) - (1 - i)r_{G1} - \overline{g} < 0.$$

The variable $\overline{g}$ represents the minimum amount of public expenditure required for the domestic economy to function. It can be interpreted as the expenditure for the health system, defence or education.[3] When the government is solvent and there is an excess of resources at date 2, such an amount- corresponding to $\widetilde{G}(Y, I(i, t, r_{G0}, \Gamma)) - (1 - i)r_{G1} - \overline{g} > 0$- is transferred to the domestic consumers in the form of a public good. When the government defaults, neither investors nor domestic consumers receive anything.[4]

[2] Investors always invest in government bonds at date 0. This is a consequence of the fact that they can always decide not to rollover their investment in sovereign bonds at date 1 and in this case they obtain $r_{G0} \geq 1$.

[3] All results hold also in the case $\overline{g} = 0$. However, having a positive $\overline{g}$ makes it possible to analyze the effect of austerity measures on the benefits and costs associated with the guarantees.

[4] The assumption that in the event of government default neither domestic consumers nor foreign investors receive anything

The guarantee scheme

At date 0, the government announces the introduction of a guarantee scheme Γ. It promises to transfer resources to the banking sector after the first $\gamma \in [\lambda, 1)$ depositors have withdrawn so as to guarantee that depositors will receive a fixed payoff as if only γ have withdrawn. To achieve this goal, the government needs to transfer the amount

$$\Gamma = \max\left\{0, \frac{(n-\gamma)\left(r_{B0}-(1-t)\right)}{1-\gamma}\right\}$$

to banks after the first γ depositors have withdrawn.

The parameter γ determines the size of the transfer of resources from the government to the banking sector. The lower γ, the larger the size of the transfer that the government promises to the banking sector- i.e., the size of the guarantees. In this respect, the parameter γ is an inverse measure of the size of the guarantee scheme Γ. The cases $\gamma = \lambda$ and $\gamma = 1$ correspond, respectively, to the full guarantee and the no guarantee scenario. In the former, the government commits to transfer resources to the banking sector at date 1 as soon as the first λ depositors withdraw, in an attempt to eliminate the occurrence of panic-driven runs. In the latter, instead, the government does not transfer any resources to the bank as it waits for all depositors to run and the bank to liquidate the entire investment at date 1. [5]

While the government **promises** to transfer $\frac{(n-\gamma)(r_{B0}-(1-t))}{1-\gamma}$ resources to a bank at date 1, the amount it **actually** transfers can be lower than $\frac{(n-\gamma)(r_{B0}-(1-t))}{1-\gamma}$. This is the novel feature of the theoretical framework and results from the government not having access to unlimited resources, but rather relying on creditors' rollover decisions to fund its expenses. Denote as $\Gamma(i, n, r_{B0}, r_{G0}, t)$ the actual amount transferred by the government. It is equal to

$$\Gamma(i, n, r_{B0}, r_{G0}, t, \gamma) = \begin{cases} \frac{(n-\gamma)(r_{B0}-(1-t))}{1-\gamma} & \text{if } \frac{(n-\gamma)(r_{B0}-(1-t))}{1-\gamma} \leq 1+t-ir_{G0} \\ 1+t-ir_{G0} & \text{if } \frac{(n-\gamma)(r_{B0}-(1-t))}{1-\gamma} \geq 1+t-ir_{G0} \end{cases}. \tag{1}$$

As stated in (1), $\Gamma(i, n, r_{B0}, r_{G0}, t)$ depends on the proportion n of depositors withdrawing at date 1, on the government's budget at date 1, as given by the resources $1+t-ir_{G0}$ remaining after paying the investors, and on the size of the intervention γ. It is easy to see that an increase in n and/or i makes it more difficult for the government to pay the promised guarantee.

captures the existence of large costs associated with sovereign default. From a purely technical perspective, it simplifies the analysis, but the results would also be valid in the case where both groups of agents could recover a fraction of government resources in the case of a default.

[5] In the analysis, I focus on the case where $1-t > \gamma r_{B0}$. This means that the government's transfer is made before the bank has already exhausted all its resources at date 1 and the liquidation procedures has not started yet. This is the relevant case to consider when the goal of the guarantees is to prevent runs. When $1-t < \gamma r_{B0}$, offering the guarantees would only affect depositors' repayment from the liquidation and, as a result, have the effect of increasing depositors' incentive to runs.

Information

The realization of the state of the economy Y at date 1 is not publicly observed. Instead, I assume that, before the government chooses the interest rate on sovereign bonds r_{G1} at date 1, all agents in the economy receive a public signal

$$x_P = Y + \varepsilon_P,$$

where the noise term $\varepsilon_P \sim U[-\pi, \pi]$. The public signal can be thought as the publication of a sovereign rating or a report on the state of the economy. Thus, the interest rate set by the government reflects the information contained in the public signal.

Depositors and investors base their decisions not only on the public signal, but also on additional information about the state of the economy that they can extract from different sources. This takes the form of a private signal x_{kj} that each agent j in group $k = \{B, G\}$ receives before taking their withdrawal decision, but after the interest rate r_{G1} has been chosen

$$x_{kj} = Y + \varepsilon_{kj},$$

where the noise term $\varepsilon_{kj} \sim U[-\varepsilon, \varepsilon]$ and it is i.i.d. across agents within each group.

The existence of both the private and the public signal allows the government's choice of the interest rate r_{G1} at date 1 to be endogenized, while still preserving the uniqueness of equilibrium in the depositors' and investors' problem.[6] Moreover, this formulation captures the idea of a constant flow of information reaching the market(s) that agents- in this case depositors and investors, use to take their decisions.

The timing of the model is as follows. At date 0, domestic consumers deposit their after tax endowment with a bank and foreign investors invest in sovereign bonds. The bank chooses the deposit contract $(r_{B0}, \widetilde{r}_{B1})$ and invests. Similarly, the government sets the interest rate r_{G0}, raises taxes t and invests. At date 1, after the public signal has arrived, the government chooses the interest rate r_{G1}. Depositors and investors receive the private signal about Y and take their withdrawal decisions. At date 2, the return on banks' and government's investment matures and they pay their remaining creditors, if solvent.

2.2 Discussion

Before moving to the characterization of the equilibrium of the model, it is worth to discuss some of the features and assumptions of the theoretical framework described above.

[6] The use of a public and private signal to avoid the signaling problem in a global game setting was proposed by Szkup (2013).

Orderly liquidation and depositors' payoff in the event of bank default

In the traditional banking literature, it is common to assume that in the case of a bank's failure, depositors are served according to a sequential service constraint. This means that the first depositors in line receive the promised repayment, while the others receive nothing. Despite its frequent use in banking models, the sequential service constraint does not really capture how a bank's bankruptcy procedure works in reality (see also the discussion in Matta and Perotti, 2015 about sequential service constraint versus orderly liquidation).

In the real world, in the event of default, a bank's assets are frozen, administrated and liquidated by the competent authority. All creditors with the same level of seniority are entitled to receive the same share of the liquidation proceeds. In the paper, this corresponds to the amount $1 - t + \Gamma(i, n, r_{B0}, r_{G0}, t, \gamma)$, with $\Gamma(i, n, r_{B0}, r_{G0}, t, \gamma)$ as given in (1).

From the perspective of the model, the assumption of orderly liquidation instead of the sequential service constraint in the event of a bank's default does not affect the main properties of the model. In particular, as I will discuss in details in Section 3, depositors' withdrawal decision is still characterized by the property of one-sided strategic complementarity as in Goldstein and Pauzner (2005).

The guarantee scheme

There are a few features of how guarantees are modelled which are worth discussing. First, they are designed in the model to limit a bank's premature liquidation of its assets. As a consequence, guarantees emerge as a tool to prevent panic-driven banking crises due to a bank being illiquid, rather than fundamental ones due to a bank becoming insolvent. In other words, efficient banking crises still occur in the model as guarantees do not prevent insolvent banks from facing a run and defaulting. In this respect, the proposed guarantee scheme echoes the one in Diamond and Dybvig (1983) and the analysis in the paper could be seen as the study of the benefits and costs of a guarantee à la Diamond and Dybvig in a context in which guarantees entail an actual disbursement and their provision is neither fully feasible nor credible.

Second, the way guarantees are modelled makes it possible to consider different sizes of intervention, ranging from no intervention (i.e., $\gamma = 1$) to full guarantees (i.e., $\gamma = \lambda$). This is important as it makes possible to clearly disentangle the direct and indirect effects associated with the guarantees.

Third, I assume that the guarantees are paid at date 1. The properties of the equilibrium and the results of the paper would not be qualitatively different if considering a guarantee scheme that still aimed at protecting depositors against liquidity risk (and not insolvency), but that, unlike the one in the paper, entailed a transfer from the government to the banking sector at date 2. This would be true as long as this transfer was increasing with the number of depositors running.

Banks' holdings of sovereign bonds

In the model, government bonds are held only by foreign investors. Domestic consumers do not hold them, either directly or via their bank. This implies that the guarantees represent the only channel linking the banks' and the sovereign's stability. From the model perspective this assumption perfectly serves the aim of the paper, which is to clearly identify the role that the guarantees play in the bank-sovereign nexus. This is because without banks' holdings of sovereign bonds the spillovers between banks and their sovereign are only due to the guarantees. From a more policy-oriented perspective, it suggests the importance of intervention focused specifically on guarantees to limit the extent of negative spillovers between banks and sovereigns. The recent regulatory reforms on bail-ins and the discussion in Europe about a pan-European deposit insurance scheme support this view.

The drawback of this approach is that, by disregarding banks' holdings of sovereign bonds, the model abstracts from a key transmission mechanism of crises (see, e.g., Arteta and Hale, 2008; Gennaioli, Martin, and Rossi, 2014, 2015; Ongena, Popov, and Van Horen, 2016; Altavilla, Pagano, and Simonelli, 2015; Acharya and Steffen, 2015), which may also have implications for the effectiveness and optimality of guarantees. To account for this, in Section 5.1, I extend the baseline model by allowing banks to hold sovereign bonds.

A *common fundamental for both the banks and the sovereign*

In the model, both the prospects for the banks and those for the sovereign depend on the same variable Y. This assumption does not qualitatively affect the results, which would still hold even in the case of completely independent fundamentals. The idea behind this is that Y is a stylized representation of the real sector, which, although for different reasons, affects both the banking sector and the government.[7] Importantly, having a common fundamental is not a confounding effect for the guarantees. In the model, it is still possible to clearly identify the role that the guarantees play in linking the banks' and the sovereign's stability in that, as I will show in detail below, there are crises that occur only because depositors expect a sovereign default to occur and vice versa.

3 Government guarantees, banking crisis and sovereign default

The provision of the guarantee scheme, coupled with the need for the government to issue bonds to raise funds, links depositors' and sovereign investors' withdrawal decisions and sets the ground for the feedback

[7] Bocola (2016) highlights the role that the real sector plays in the banks-sovereign nexus.

loop between banking and sovereign default crises.

To see this, in this section, I disentangle the effects that the provision and size of the guarantee scheme have on the likelihood of a bank run and a sovereign default, as well as on their interaction. I start by characterizing creditors' withdrawal decision and, thus the probability of a bank and sovereign default. Then, I analyze how changes in the size of the guarantees affect the likelihood of both crises, as well as, their interaction. In doing so, I take as given the deposit contract set by the bank and the interest rates on sovereign bonds, which I will then endogenize in Section 4.

In the model, banks' and sovereign creditors' payoffs depend on the state of the domestic economy Y and on the actions taken by all other agents in the economy. A key feature of the theoretical framework is the existence of two types of strategic complementarity: *within*- and *between*- groups strategic complementarity. The former refers to the fact that, within each group of agents, a creditor's incentive to withdraw at date 1 increases with the proportion of other agents in their own group withdrawing at the interim date. The latter, instead, captures the fact that the incentive to withdraw at date 1 also increases with the proportion of agents in the other group taking a similar action.

While the existence of *within*-groups strategic complementarity is independent of the provision of guarantees (see e.g., Diamond and Dybvig, 1983; Morris and Shin, 1998 and 2003; Goldstein and Pauzner, 2005), the *between*-groups one is a direct consequence of the guarantees and its strength depends on their size γ. This is because the proportion of depositors running n and investors not rolling over i affects the amount of resources that the government needs to subtract from its budget and transfer to the banking sector and the resources available to pay the guarantees, respectively.

In the sovereign bond market, an investor's incentive to withdraw at date 1 monotonically increases with both the proportion i of investors not rolling over their investment in sovereign bonds and that of depositors running at date 1 n. As both i and n increase, the government's budget at date 2, namely $\widetilde{G}(Y, I(i, t, r_{G0}, \Gamma)) - (1-i)r_{G1} - \overline{g}$, becomes tighter, thus reducing the probability of an investor receiving the promised repayment r_{G1}, while the repayment from withdrawing at date 1 is always equal to r_{G0}. To see this formally, recall that amount of resources invested by the government until date 2 depends on the guarantee $\Gamma(i, n, r_{B0}, r_{G0}, t)$ and is given by

$$I(i, n, r_{G0}, r_{B0}, t, \gamma) = \max\left\{0, 1 + t - ir_{G0} - \frac{(n-\gamma)\,(r_{B0} - (1-t))}{1-\gamma}\right\}.$$

Denote as $\alpha(Y, r_{G0}, r_{G1}, i, t, n, \gamma)$, an investor's expected payoff from rolling over the investment in sovereign bonds. It is equal to

$$\alpha(Y, i, n, r_{G0}, r_{B0}, t, \gamma) = r_{G1}[1 - \Pr(\widetilde{G}(Y, I(i, t, r_{G0}, n, \gamma)) - ir_{G1} - \overline{g} < 0)]. \tag{2}$$

The larger the proportion n of depositors withdrawing at date 1, the larger the guarantee paid by the government, as shown in (1), with the consequence that $I(i, n, r_{G0}, r_{G1}, t, \gamma)$ and, in turn, $\widetilde{G}(Y, I(i, n, r_{G0}, r_{G1}, t, \gamma))$ become smaller. Thus, $\frac{\partial \alpha(Y,i,n,r_{G0},r_{B0},t,\gamma)}{\partial n} < 0$ and the strategic complementarity between investors' and depositors' actions holds. Regarding the strategic complementarity within the group of foreign investors, it holds as long as $\frac{\partial \widetilde{G}(Y,I(i,t,r_{G0},\Gamma))}{\partial I(.)} r_{G0} > r_{G1}$, as in this case an increase in i makes the government budget tighter and, in turn, reduces $\alpha(Y, i, n, r_{G0}, r_{B0}, t, \gamma)$.[8] Moreover, it is easy to see that $\alpha(Y, i, n, r_{G0}, r_{B0}, t, \gamma)$ also increases with the size of guarantees, as $I(i, n, r_{G0}, r_{B0}, t, \gamma)$ and, in turn, $\widetilde{G}(Y, I(i, t, r_{G0}, n, \gamma))$, increases with it. This means that more generous guarantees (i.e., a lower γ) are associated with a lower expected payoff for sovereign creditors at date 2, and thus, ceteris paribus, with a higher incentive not to roll over the investment in sovereign bonds at date 1.

In the banking sector, a depositor's incentive to withdraw early and run on the bank also increases with the proportion n of depositors running and that of investors not rolling over i, although not monotonically. To see this formally, notice that the amount of resources available to a bank at date 1 to repay r_{B0} to the n depositors withdrawing early can be written as

$$1 - t + \Gamma(i, n, r_{B0}, r_{G0}, t, \gamma) = 1 - t + \min\left\{\frac{(n-\gamma)\,(r_{B0} - (1-t))}{1-\gamma}, 1 + t - ir_{G0}\right\}, \tag{3}$$

using the expression for $\Gamma(i, n, r_{B0}, r_{G0}, t, \gamma)$ in (1). Using (3), a depositor's payoff from withdrawing at date 2 is a function of n and i and is given by

$$\widetilde{r}_{B1} = \begin{cases} R(Y)\frac{1-t-nr_{B0}+\min\left\{\frac{(n-\gamma)(r_{B0}-(1-t))}{1-\gamma},1+t-ir_{G0}\right\}}{1-n} & \text{if } \lambda \leq n \leq \overline{n} \\ 1 - t + \min\left\{\frac{(n-\gamma)(r_{B0}\quad(1\quad t))}{1-\gamma}, 1 + t - ir_{G0}\right\} & \text{if } \overline{n} \leq n \leq 1 \end{cases}, \tag{4}$$

with $\overline{n} = \frac{1-t+\min\left\{\frac{(n-\gamma)(r_{B0}-(1-t))}{1-\gamma},1+t-ir_{G0}\right\}}{r_{B0}}$ identifying the number of depositors running above which the bank exhausts all available resources- its own and those provided by the government as guarantees− at date 1.

To see how a depositor's incentive to run varies with both i and n denote as $v(Y, n, i)$ a depositor's utility

[8] The government's budget at date 2 $\widetilde{G}(Y, I(i, t, r_{G0}, \Gamma)) - (1-i)r_{G1} - \overline{g}$ is decreasing in i as long as $\frac{\partial \widetilde{G}(Y,I(i,t,r_{G0},\Gamma))}{\partial i} + r_{G1} = -\frac{\partial \widetilde{G}(Y,I(i,t,r_{G0},\Gamma))}{\partial I(.)} r_{G0} + r_{G1} < 0$. I am assuming this now and I will subsequently show that this condition is always satisfied in equilibrium in Section 4, when solving for the optimal $\{r_{G0}, r_{G1}\}$.

differential between withdrawing at date 2 and running at date 1. The function $v(Y,n,i)$ is equal to

$$v\left(Y,n,i\right)=\begin{cases} u\left(R(Y)\frac{1-t-nr_{B0}+\min\left\{\frac{(n-\gamma)(r_{B0}-(1-t))}{1-\gamma},1+t-ir_{G0}\right\}}{1-n}\right)-u(r_{B0}) & \text{if } \lambda\leq n<\overline{n} \\ u\left(\widetilde{r}_{B1}\right)-u\left(\widetilde{r}_{B1}\right) & \text{if } \overline{n}\leq n\leq 1\end{cases}, \quad (5)$$

where, from (4), $\widetilde{r}_{B1}=1-t+\min\left\{\frac{(n-\gamma)(r_{B0}-(1-t))}{1-\gamma},1+t-ir_{G0}\right\}$ in the range $\overline{n}\leq n\leq 1$ because of the orderly liquidation assumption. It is easy to see that the function $v\left(Y,n,i\right)$ decreases with n and i in the range $\lambda\leq n\leq\overline{n}$, while it is a constant equal to zero in the range $\overline{n}\leq n\leq 1$. [9]

Creditors take their withdrawal decisions at date 1, after observing the private signal x_{kj} and choose the action that entails the highest payoff. As shown above, each creditor's payoff depends on the state of the economy Y and the actions taken by all other creditors. Since the private signal provides information about the realization of Y, as well as the actions of the other agents in the economy, each depositor and investor bases his decision on the signal they receives. When receiving a high signal, a creditor attributes a high posterior probability to the good realization of Y and, at the same time, infers that others have also received a high signal. This lowers their beliefs about the likelihood of a run and of a sovereign default and, in turn, increases the incentive to withdraw at date 2, if a depositor, and to roll over his investment in sovereign bonds, if an investor.

Because of the strategic complementarity between agents' actions, depositors' withdrawal decision and investors' rollover choice can be computed using the global games methodology. Following the literature on global games (see e.g., Morris and Shin, 2003), the uniqueness of the equilibrium in depositors' withdrawal and investors' rollover decisions is guaranteed by the existence of two extreme regions where depositors and investors have a dominant strategy.

I start with the depositors. Denote as $\underline{Y}_B$ the upperbound of the region in which withdrawing at date 1 is a dominant strategy for each depositor. This is the case when the payoff and so the utility that can obtained at date 2 are lower than that which can be obtained at date 1, even if no runs occur. Since $\underline{Y}_B$ is computed under the assumption that there is no run and only the early types withdraw (i.e., $n=\lambda$), no guarantee is paid and $\underline{Y}_B$ is simply the solution to

$$u\left(r_{B0}\right)=u\left(R\left(Y\right)\frac{1-t-\lambda r_{B0}}{1-\lambda}\right), \quad (6)$$

[9] Using the same terminology as in Goldstein and Pauzner (2005), the fact that $v\left(Y,n,i\right)$ does not decrease monotonically with n and i means that depositors' withdrawal decision only exhibits the property of one-sided strategic complementarity. By contrast with Goldstein and Pauzner (2005), where there is only strategic complementarity within group, in this framework, the concept of one-sided strategic complementarity applies to both the within- and between-groups strategic complementarity.

In the range $Y < \underline{Y}_B$, runs are fundamental-driven in that they only depend on the bad realization of Y. Symmetrically, when the state of the domestic economy Y is very good$-$i.e., $Y > \overline{Y}_B-$, irrespective of what other depositors and investors do, it is optimal for a patient depositor to wait and withdraw at date 2. I refer to the range $\left(\overline{Y}_B, 1\right]$ as the upper dominance region where no runs ever occur. For this to be the case, it must hold that the repayment they can accrue at date 2 is higher than r_{B0}. To determine the threshold $\overline{Y}_B$, I follow Goldstein and Pauzner (2005) and make two assumptions. I assume that when $Y > \overline{Y}_B$, the per unit liquidation value of the project at date 1 also improves increasing from 1 to $\frac{1}{\lambda}$ and that, in this range, $R(Y) > \frac{1}{\lambda}$. Since $r_{B0} < \frac{1-t}{\lambda}$, these two assumptions imply that the bank does not need to liquidate more than one unit for each withdrawing depositor and that a late depositor's repayment at date 2 is always higher than what they can obtain at date 1.

Consider now the foreign investors. The determination of the lower and upper dominance regions and their respective thresholds is analogous. The threshold $\underline{Y}_G$ is computed under the assumption that $i = 0$ and no runs occur (i.e., $n = \lambda$) and, thus no guarantees are paid. Thus, $\underline{Y}_G$ is the solution to

$$\widetilde{G}(Y, I(0, t, r_{G0}, \lambda, \gamma)) - r_{G1} - \overline{g} = 0,$$

with $I(0, t, r_{G0}, \lambda, \gamma) = 1 + t$. Similarly to the case of depositors, sovereign default crises occurring in the range $[0, \underline{Y}_G)$ are fundamental-driven, as they are only due to a bad realization of Y. Regarding the upper dominance region, the threshold $\overline{Y}_G$ is computed taking into account that the government fully liquidates the amount initially invested at date 1, as it would be the case if all depositors and sovereign creditors withdrew at date 1 (i.e., when $n = i = 1$), and it corresponds to the case $I(1, t, r_{G0}, 1, \gamma) = 0$. The inequality $\underline{Y}_G < \overline{Y}_G$ holds since $\widetilde{G}(Y, I(i, t, r_{G0}, n, \gamma))$ is increasing in Y and decreasing in both n and i.

Besides these extreme ranges of Y, depositors' and foreign investors' actions depend crucially on what all the other agents-both within their group and in the other group- do. In other words, for intermediate values of Y, banking and sovereign default crises occur and they are panic-driven in that they are the result of a coordination failure. In these intermediate ranges, depositors and investors withdraw at date 1 out of the self-fulfilling belief that other depositors and investors would do the same, thus leading to panic-driven banking crises and sovereign defaults.

The following proposition characterizes depositors' withdrawal decision and foreign investors' rollover choice.

Proposition 1 *For given r_{B0}, r_{G0} and r_{G1}, the model has a unique threshold equilibrium where all depositors* *run if they receive a signal below* x_B^g *and do not run above and all investors withdraw if they receive a signal below* x_G^g *and roll over otherwise. The equilibrium thresholds* $\{x_B^g, x_G^g\}$ *satisfy* $x_B^g = x_B(x_G^g)$ *and* $x_G^g = x_G(x_B^g)$*, with* $0 < x'_B(x_G) < 1$ *and* $0 < x'_G(x_B) < 1$*, and are the solution to the following system:*

$$\begin{aligned} &\int_{\lambda}^{\gamma} u\left(R(x_B^g + \varepsilon(1 - 2\tfrac{n-\lambda}{1-\lambda}))\tfrac{1-t-nr_{B0}}{1-n}\right) dn + \int_{\gamma}^{\widetilde{n}(x_B^g, x_G^g)} u\left(R(x_B^g + \varepsilon(1 - 2\tfrac{n-\lambda}{1-\lambda}))\tfrac{1-t-\gamma r_{B0}}{1-\gamma}\right) dn + \\ &+ \int_{\widetilde{n}(x_B^g, x_G^g)}^{\overline{n}(x_B^g, x_G^g)} u \quad R(x_B^g + \varepsilon(1 - 2\tfrac{n-\lambda}{1-\lambda}))\frac{2 - \frac{x_G^g - x_B^g}{2\varepsilon} r_{G0} + \frac{n-\lambda}{1-\lambda} r_{G0} - n r_{B0}}{1-n} \Bigg) - \int_{\lambda}^{\overline{n}(x_B^g, x_G^g)} u\left(r_{B0}\right) dn = 0 \end{aligned} \tag{7}$$

$$\widetilde{G}\left(x_G^g - \frac{r_{G0}}{r_{G1}} 2\varepsilon + \varepsilon, 1 + t - \frac{r_{G0}^2}{r_{G1}} - \frac{\left(\lambda + (1-\lambda)\left(\frac{x_B^g - x_G^g}{2\varepsilon} + \frac{r_{G0}}{r_{G1}}\right) - \gamma\right)(r_{B0} - (1-t))}{1-\gamma}\right) - (r_{G1} - r_{G0}) - \overline{g} = 0, \tag{8}$$

with $\widetilde{n}(x_B^g, x_G^g) = \frac{\left[(1-\gamma)(1-\lambda) + \gamma(r_{B0} - (1-t)) + \lambda(1-\gamma) r_{G0} - (1-\lambda)(1-\gamma)\frac{x_G^g - x_B^g}{2\varepsilon} r_{G0}\right]}{(1-\lambda)(r_{B0} - (1-t)) + r_{G0}(1-\gamma)}$ *and* $\overline{n}(x_B^g, x_G^g) = \frac{2(1-\lambda) - (1-\lambda)\frac{x_G^g - x_B^g}{2\varepsilon} r_{G0} + \lambda r_{G0}}{r_{B0(1-\lambda)} + r_{G0}}$.

The equilibrium thresholds $\{x_B^g, x_G^g\}$ are the solution of a system of two equations. Equation (7) represents a depositor's indifference condition between withdrawing at date 2 and date 1. Equation (8), instead, represents government's solvency constraint when it is at the margin between being solvent and defaulting and an investor is exactly indifferent between withdrawing and rolling over the investment in sovereign bonds at date 1.

It emerges from the two expressions in the proposition that depositors' and investors' actions depend on the growth rate of the domestic economy Y as well as on the actions taken by all other agents in the economy−both in their own and in the other group−. The greater the likelihood of a banking crisis, the larger the probability of a sovereign default and viceversa since the thresholds x_B^g and x_G^g are positively related. This implies that there are ranges of Y where a banking and sovereign debt crisis only occurs because each depositor and each investor believe that a sovereign default and a banking crisis, respectively, are going to occur.

The positive correlation between depositors' and investors' equilibrium thresholds is relevant in two respects. First, it is a direct implication of the *between-groups* strategic complementarity induced by the guarantees and captures the existence of the feedback loop between banking and sovereign debt crises, which I analyze in detail below. In an economy without guarantees, each depositor and investor would take their withdrawal decisions independently from each other (and only caring about the action of the other agents in their respective group) despite the common fundamental Y affecting both the prospects for the banks and those for the government. In this case, the occurrence of a banking crisis and a sovereign default would also be independent.

Second, the positive correlation between investors' and depositors' equilibrium thresholds highlights the fact that the feasibility of a guarantee scheme matters not only for its effectiveness in limiting the occurrence of banking crises, but also for the costs associated with its provision. For a given size of the guarantee scheme γ, a higher threshold for the investors leads to a higher threshold for depositors, that is to greater instability in the banking sector. This is the consequence of the fact that less resources are available to pay the guarantees and so their effectiveness in limiting panic runs is reduced. Symmetrically, a higher threshold for depositors leads to higher equilibrium thresholds for the investors and thus to more instability in the sovereign debt market. This is a consequence of the higher disbursement incurred by the government to provide the guarantees.

The following proposition formally discusses the two features of the equilibrium just described.

Proposition 2 *Denote as x_B^{ng} and x_G^{ng} the equilibrium thresholds in the economy without guarantees and as x_B^f and x_G^f those in the case where there no concerns about the feasibility of the guarantee scheme. Given the size of the guarantee scheme γ and the interest rates r_{B0}, r_{G0} and r_{G1}, it holds that*

i) $x_B^{ng} > x_B^g$ and $x_G^{ng} < x_G^g$;

ii) $x_B^g > x_B^f$ and $x_G^g > x_G^f$.

The proposition highlights two important results. First, it points out the direct effects that the provision of the guarantees has on the probability of a banking crisis and sovereign default, respectively: the introduction of a guarantee scheme reduces the former, while it increases the latter. The intuition behind this result is simple. On the one hand, the introduction of a guarantee scheme improves financial stability as banks have, ceteris paribus, more resources to meet early withdrawals, thus reducing depositors' incentives to run. On the other hand, the introduction of guarantees adds an extra disbursement for the government, which, ceteris paribus, makes its resource constraint more binding, thus reducing investors' incentive to roll over the investment in sovereign bonds. Second, the proposition shows that the uncertainty associated with the provision of guarantees limits their effectiveness in preventing banking crises and makes their costs higher. This result is obtained by comparing the equilibrium thresholds x_B^g and x_G^g, characterized in Proposition 1, with two "artificial" thresholds computed under the assumption that the government always has enough resources to pay the guarantees and so the depositors believe that they will always receive the guaranteed amount.

The intuition behind the second result in the proposition is less straightforward. Importantly, this has to do with the feasibility of the guarantees, rather than with the limited commitment that may characterize government's choice and thus, to the possibility that the government chooses to default and not to provide the guarantees. The reduced effectiveness of the guarantees in preventing runs and the higher sovereign instability they induce are a consequence of the fact the government is fragile and does not have the possibility to raise unlimited and costless resources to provide the guarantees. In this context, the provision of guarantees triggers a downward spiral, where the reduced effectiveness of the guarantees in preventing banking crises exacerbates the cost for the government and, in turn, leads to increased instability in the sovereign bond market and to even lower credibility and effectiveness of the guarantees. In other words, the difference $x_B^g - x_B^f$ identifies the range of Y where a banking crisis only occurs because of the risk of a sovereign default. Similarly, the difference $x_G^g - x_G^f$ identifies the range where a sovereign default only occurs because of the fear of a bank run.

This negative feedback loop between banks and sovereign generated by the provision of the guarantees represents a novel contribution of this paper to the literature on public intervention in the financial sector and to that on the bank-sovereign nexus. Regarding the former, it highlights the existence of costs associated with the provision of guarantees− namely the threat to sovereign solvency and the consequent reduced effectiveness of public intervention− that have been overlooked by the existing literature. Regarding the latter, it shows that a feedback loop between banking and sovereign debt crises arise even when banks do not hold any sovereign bonds, thus highlighting the key role that guarantees play for the complementarity between banking and sovereign debt crises.

3.1 Increase in the size of the guarantee γ and the positive feedback loop between banking crisis and sovereign default

So far, I have shown that the introduction of the government guarantees generates a trade-off. On the one hand, it reduces the probability of a banking crisis by limiting the coordination failure among depositors (i.e., $x_B^g < x_B^{ng}$). On the other hand, the introduction of the guarantees is associated with an increase in the instability in the sovereign bond market, in that it reduces investors' incentive to roll over (i.e., $x_G^g > x_G^{ng}$). I have also shown that concerns about the feasibility of the announced guarantee scheme foster even greater instability, as they lead to an increase in instability in both the banking sector and sovereign bond market (i.e., $x_B^g > x_B^f$ and $x_G^g > x_G^f$). When all these elements are taken into account, announcing the introduction of the guarantees at date 0 is optimal only when the benefits in terms of reduced costs of a banking crisis offset the costs associated with the increased sovereign instability. This is ultimately an empirical question.

A more interesting exercise is to analyze the effect of an increase in the support offered by the government to banks− namely an increase in the size of the guarantees− on both financial and sovereign stability. What makes such analysis interesting is that, unlike existing models, the probabilities of a banking crisis and a sovereign default are not fixed and interact with each other. This implies that the overall effect of an increase in the size of guarantees on the likelihood of a banking crisis and a sovereign default is not only determined by their respective direct effects, but also depends on the indirect ones.

For banks, the direct effect captures the reduction in the bank's liquidation needs resulting from the provision of larger guarantees. The direct effect on sovereign stability, instead, captures the increase in the disbursement that the government suffers in the case a run occurs and thus, a more binding resource constraint. The indirect effects for both the groups of agents come from the fact that, as stated in Proposition 1, their threshold is an increasing function of that of the other group $\left(\text{i.e., } \frac{dx_B^g(x_G)}{dx_G^g} > 0 \text{ and } \frac{dx_G^g(x_B)}{dx_B^g} > 0\right)$. Since the direct and indirect effects move in opposite directions, a number of novel results regarding the impact of guarantees on financial and sovereign stability and their interaction emerge. They are illustrated in the following proposition.

Proposition 3 *Denote as $\Delta_{\gamma B}$ and $\Delta_{\gamma G}$ the direct effect of an increase in the level of guarantees (i.e., a lower γ) on depositors' and investors' equilibrium thresholds, respectively. The following cases can be distinguished:*

i) If $|\Delta_{\gamma G}| < \frac{dx_G^g}{dx_B^g}\Delta_{\gamma B}$, an increase in the size of guarantees leads to lower equilibrium thresholds for both depositors and investors;

ii) If $\frac{dx_G^g}{dx_B^g}\Delta_{\gamma B} < |\Delta_{\gamma G}| < \frac{\Delta_{\gamma B}}{\frac{dx_B^g}{dx_G^g}}$, an increase in the size of guarantees leads to a lower equilibrium threshold for the depositors, but to a higher one for the investors;

iii) If $|\Delta_{\gamma G}| > \frac{\Delta_{\gamma B}}{\frac{dx_B^g}{dx_G^g}}$, an increase in the size of guarantees leads to higher equilibrium thresholds for both depositors and investors.

The above proposition is illustrated in Figure 3 and shows that whether an increase in the size of the guarantees has a beneficial or detrimental effect on both financial and sovereign stability depends crucially on the magnitude of the direct and indirect effects that it has on the probability of each crisis. When the direct effect of an increase in the size of guarantees on sovereign stability is small $\left(\text{i.e., for } |\Delta_{\gamma G}| < \frac{dx_G^g}{dx_B^g}\Delta_{\gamma B}\right)$, larger guarantees improve both financial and sovereign stability. The reason is that, in this range, by reducing depositors' incentives to run, larger guarantees also lower the government disbursement associated with the provision of the guarantees. Thus, in this case, an increase in the size of guarantees has a stabilizing effect in both the banking sector and the sovereign debt market, as it triggers a positive feedback loop between banking and sovereign debt crises. As $\Delta_{\gamma G}$ increases $\left(\text{i.e., for } \frac{dx_G^g}{dx_B^g}\Delta_{\gamma B} < |\Delta_{\gamma G}| < \frac{\Delta_{\gamma B}}{\frac{dx_B^g}{dx_G^g}}\right)$, the beneficial effect of guarantees in terms of a reduction in depositors' incentives to run is not enough to compensate for the larger disbursement that the government is facing when a banking crisis occurs. As a result, an increase in the size of guarantees reduces sovereign investors' rollover incentives, thus worsening sovereign stability. Still the detrimental effect on sovereign stability is not too large, so that the presence of larger guarantees reduces depositors' incentives to run. As $\Delta_{\gamma G}$ increases further $\left(\text{i.e., for } |\Delta_{\gamma G}| > \frac{\Delta_{\gamma B}}{\frac{dx_B^g}{dx_G^g}}\right)$, the negative effect of larger guarantees on investors' incentives to roll over and, in turn, on sovereign stability hurts the credibility and so the effectiveness of the guarantees and results into an increase in the probability of a banking crisis. This last case reflects closely what we observed in Ireland in 2008.

Insert figure 3

An interesting implication of Proposition 3 concerns the role that the specific characteristics of the economy− such us, the nature of banking crises, the size of the banking sector, as well as the soundness of the government's budget− play in the result in the above proposition, as they determine the magnitude of the direct and indirect effects illustrated above.

Since in this framework guarantees only affect the probability of panic-driven bank runs, if banking crises are mostly fundamental driven, the effectiveness of guarantees in reducing depositors' incentives to run is limited (i.e., $\Delta_{\gamma B}$ is small), while they still lead to a disbursement for the government (i.e., $\Delta_{\gamma G}$ is large). Thus, in such a scenario, it is more likely that an increase in the size of guarantees leads to an increase in the probability of both banking and sovereign debt crises. As clearly emerged in the Irish crisis in 2008, the size of the banking sector relative to that of the public sector also matters. A larger banking sector makes guarantees more costly and the sovereign more constrained, thus making it more likely that larger guarantees will trigger a negative feedback loop between banking and sovereign debt crises. More generally, the direct (detrimental) effect that guarantees have on the sovereign and, in turn, on the bank-sovereign nexus, tends to be less pronounced when the sovereign fiscal position is stronger. Along this line, in Section 5.2, I will discuss in detail the effect of austerity measures, such as reducing public expenditure and increasing taxes, on the feedback loop between a banking crisis and sovereign default.

4 Deposit contract and interest rates on sovereign bonds

In this section, I characterize the equilibrium values for r_{B0}, r_{G0} and r_{G1}.

The equilibrium thresholds $\{x_B^g, x_B^g\}$ characterized in Proposition 1 depend on the deposit contract offered by banks and interest rates set by the government. I have the following results.

Lemma 1 *The equilibrium thresholds* $\{x_B^g, x_B^g\}$ *increase with both* r_{B0} $\left(i.e., \frac{\partial x_B^g}{\partial r_{B0}} > 0 \text{ and } \frac{\partial x_G^g}{\partial r_{B0}} > 0\right)$ *and* r_{G0} $\left(i.e., \frac{\partial x_B^g}{\partial r_{G0}} > 0 \text{ and } \frac{\partial x_G^g}{\partial r_{G0}} > 0\right)$. *The effect of* r_{G1} *on investors' equilibrium threshold* x_G^g *is not monotone:* $\frac{\partial x_G^g}{\partial r_{G1}} < 0$ *for* $r_{G1} < \widehat{r}_{G1}^g$ *and* $\frac{\partial x_G^g}{\partial r_{G1}} > 0$ *otherwise.*

A change in the repayment offered by the bank to the depositors withdrawing early r_{B0} affects both depositors' and investors' equilibrium thresholds. Symmetrically, a change in the interest rate on government bonds promised at date 1 and 2 affects all equilibrium thresholds. Consider first r_{B0}. Its effect on investors' threshold is twofold. First, a higher r_{B0} increases the amount of resources that the government needs to transfer to the banking sector to pay the guarantees. Second, a higher r_{B0} increases depositors' incentives to withdraw at date 1 (i.e., x_B^g increases, ceteris paribus), thus causing x_G^g to increase too. The case for r_{G0} is analogous.

The effect of r_{G1} on the thresholds is more involved given the non-monotonicity of x_G^g to changes in r_{G1}. The intuition behind this result is simple and can be easily grasped by looking at (2).[10] For a fixed probability of default of the government, an increase in r_{G1} implies that sovereign creditors receive a higher payoff when they roll over their investment in sovereign bonds and the government is solvent, thus increasing their incentive to roll over between date 1 and 2. However, an increase in r_{G1} affects the probability of a sovereign default, in that, ceteris paribus, it makes the government's budget constraint more binding. As a result, an increase in r_{G1} increases investors' incentives not to roll over. Depending on which of the two effects dominates, an increase in r_{G1} leads to either a decrease or an increase in the probability of a sovereign default. The lemma shows that the first effect dominates the second one when r_{G1} is sufficiently low, while the opposite is true for large values of r_{G1}. In other words, the equilibrium threshold x_G^g is a convex function of the interest rate r_{G1} and $\widehat{r}_{G1}^g$ represents the interest rate that minimizes the probability of a sovereign

[10] Recall that investors receive the fixed repayment r_{G0} if they do not roll over the investment in sovereign bonds at date 1.

debt crisis.

Having shown how the equilibrium thresholds change with the terms of the deposit contract and the interest rates set by the government, I can now move to their equilibrium choice. To do so, I focus on the limit case, when both the public and private signals are very precise, that is when $\pi \to 0$, $\varepsilon \to 0$ and $\frac{\varepsilon}{\pi} \to 0$. In this case, all agents receive approximately the same signal and so take the same actions and $x_B^g \to Y_B^g$ and $x_G^g \to Y_G^g$. This means that in the interval $Y < Y_B^g$, all depositors run and the bank fails at date 1. Symmetrically, in the interval $Y < Y_G^g$, no investors roll over the investment in sovereign bonds and the government defaults.

I start from the government. It chooses the interest rates $\{r_{G0}, r_{G1}\}$ so to maximize domestic agents' (i.e., depositors) expected utility. This is given by

$$\max_{r_{G1}} \begin{cases} \displaystyle\int_{Y_G^g}^{1} v\left(\widetilde{G}(Y, I(0,t,r_{G0},r_{B0},\lambda,\gamma)) - r_{G1} - \overline{g}\right) f(Y)\, dY \\ \displaystyle + \int_{0}^{Y_B^g} u(2-r_{G0}) f(Y)\, dY + \int_{Y_B^g}^{1} \lambda u(r_{B0}) + (1-\lambda)\, u\left(R(Y)\frac{1-t-\lambda r_{B0}}{1-\lambda}\right) f(Y)\, dY & \text{if } Y_G^g \geq Y_B^g \\ \displaystyle\int_{Y_G^g}^{Y_B^g} v\left(\widetilde{G}(Y, I(0,t,r_{G0},r_{B0},1,\gamma)) - r_{G1} - \overline{g}\right) f(Y)\, dY + \\ \displaystyle + \int_{Y_B^g}^{1} v\left(\widetilde{G}(Y, I(0,t,r_{G0},r_{B0},\lambda,\gamma)) - r_{G1} - \overline{g}\right) f(Y)\, dY & \text{if } Y_G^g < Y_B^g \\ \displaystyle + \int_{0}^{Y_B^g} u(r_{B0}) f(Y)\, dY + \int_{Y_B^g}^{1} \left[\lambda u(r_{B0}) + (1-\lambda)\, u\left(R(Y)\frac{1-t-\lambda r_{B0}}{1-\lambda}\right)\right] f(Y)\, dY \end{cases}, \quad (9)$$

where $I(0,t,r_{G0},r_{B0},\lambda,\gamma) = 1+t$, $I(0,t,r_{G0},r_{B0},1,\gamma) = 1+t-[r_{B0}-(1-t)] = 2-r_{B0}$ and $f(Y)$ is the density function of Y.

Depositors' expected utility, as given in (9), is a piecewise function as it depends on whether $Y_G^g \gtrless Y_B^g$. The terms in the two expressions in (9) have a similar meaning in the two cases. The first term in the case $Y_G^g \geq Y_B^g$ and the first two in the case $Y_G^g < Y_B^g$, represent depositors' expected utility from the public good. This is given by the government's available resources at date 2 when it is solvent (i.e., when $Y > Y_G^g$). In this range, when $Y > Y_B^g$, no runs occur and the government does not pay any guarantees and invest $I(0,t,r_{G0},r_{B0},\lambda,\gamma) = 1+t$ resources between date 1 and 2, thus generating a return $\widetilde{G}(Y, I(0,t,r_{G0},r_{B0},\lambda,\gamma))$ at date 2. In this case, each depositor receives an amount of the public good equal to $\widetilde{G}(Y, I(0, t, r_{G0}, r_{B0}, \lambda, \gamma)) - r_{G1} - \overline{g}$. In the range, $Y \leq Y_B^g$, instead, a run occurs and the government needs to transfer $r_{B0} - (1-t)$ resources to the banking sector. As a consequence, only $I(0, t, r_{G0}, r_{B0}, 1, \gamma) = 2 - r_{B0}$ resources are invested between date 1 and 2, thus generating a return $\widetilde{G}(Y, I(0, t, r_{G0}, r_{B0}, 1, \gamma))$. In this case, then each depositor receives an amount of the public good equal to $\widetilde{G}(Y, I(0, t, r_{G0}, r_{B0}, 1, \gamma)) - r_{G1} - \overline{g}$.

The remaining terms in (9) represent depositors' expected utility from the private good. In the range $Y > Y_B^g$, no runs occur, only λ early depositors withdraw at date 1 and receive r_{B0}. The $1-\lambda$ late depositors receive a share of the return on the non liquidated units, as given by $R(Y)\frac{1-t-\lambda r_{B0}}{1-\lambda}$. In the range, $Y < Y_B^g$, a run occurs and all depositors receive the liquidation proceeds $2 - r_{G0}$ if $Y_G^g > Y_B^g$ and r_{B0} if $Y_G^g < Y_B^g$.

I start characterizing the choice of r_{G0}, as it is straightforward. At date 0, the government chooses the lowest possible interest rate, that is $r_{G0}^g = 1$. This is a consequence of the fact that investors are risk-neutral and both thresholds Y_B^g and Y_G^g are an increasing function of r_{G0}. The choice of r_{G1} is more involved and it is taken at date 1: The government chooses r_{G1}^g after receiving the public signal, but before depositors and investors take their withdrawal decisions. The following result holds.

Proposition 4 *At the limit when $\pi \to 0$, $\varepsilon \to 0$ and $\frac{\varepsilon}{\pi} \to 0$, the equilibrium interest rate on government bonds $r_{G1}^g < \widehat{r}_{G1}^g$ is given by*

$$r_{G1}^g = \begin{cases} r_{G0}^g & \text{if } Y > \overline{Y}_G \\ r_{G1}^* & \text{if } Y_G^g(\widehat{r}_{G1}^g) \leq Y \leq \overline{Y}_G \\ \emptyset & \text{if } Y < Y_G^g(\widehat{r}_{G1}^g) \end{cases},$$

where r_{G1}^* is the solution to $Y = Y_G^g(r_{G1})$.

In order to maximize depositors' expected utility, the government chooses the lowest possible interest rate conditional on being solvent. In other words, it chooses the lowest interest rate that induces investors to roll over the investment in sovereign bonds. In order to do this, the government needs to compensate investors for the risk they take, if any.

The proposition is illustrated in Figure 4, which shows how the interest rate r_{G1} changes with the growth rate of the domestic economy. When $Y > \overline{Y}_G$, there is no risk associated to the rollover of the government bonds and the equilibrium interest rate is $r_{G1}^g = r_{G0}^g = 1$. As the prospects for the economy deteriorate, as captured by a lower Y, the government needs to increase the interest rate offered to investors, until it reaches $\widehat{r}_{G1}^g$, above which there is no interest rate that will induce investors to roll over.

Insert Figure 4

An interesting implication of Proposition 4 is that panic-driven banking crises do not occur as long as the government is able to induce investors to roll over the bonds and sovereign default is avoided. In this case, in fact, the guarantees are always feasible and depositors' expect to receive the guaranteed amount. To see this clearly, consider the case in which the government offers full guarantees- namely the case with $\gamma = \lambda$. The following result holds.

Proposition 5 *When $\gamma = \lambda$, at the limit when $\pi \to 0$, $\varepsilon \to 0$ and $\frac{\varepsilon}{\pi} \to 0$, no bank runs occur in the range $Y_G^g(\widehat{r}_{G1}^g) \leq Y$ unless, $Y_G^g(\widehat{r}_{G1}^g) \leq \underline{Y}_B$.*

The above proposition is illustrated in Figure 5 and has a simple interpretation. As long as all investors roll over the investment in sovereign bonds (i.e., in the range $Y_G^g(\widehat{r}_{G1}^g) \leq Y$), depositors expect to always receive the guaranteed amount, which in the case of full guarantees (i.e., for $\gamma = \lambda$) means that their withdrawal decision is no longer driven by the fear that other depositors will run. This occurs because, when all investors roll over and sovereign default does not occur, the government has $1 + t$ to pay the guarantees that added to the $1 - t$ resources available at the bank sums up to $2 > r_{B0}^g$, as depositors' repayment cannot exceed the maximum amount of resources available in the economy. Hence, a depositor knows that the bank does not have to liquidate anything and that they are guaranteed a payoff at date 2 as if only λ early depositors were running. Thus, they only run when expecting the bank to default for fundamental reasons.

The proposition focuses on the extreme case where $\gamma = \lambda$ and full guarantees are provided. However, the result also holds for any value of $\gamma > \lambda$. In this more general case, the guarantees are not designed to fully eliminate panic runs but rather only to limit their occurrence. It follows that panic runs still occur in equilibrium even if sovereign default is avoided, but, similarly to the result illustrated in the proposition, only if $Y_G^g(\widehat{r}_{G1}^g) \leq Y_B^f$, as characterized in Proposition 2.[11] This is the case because as long as a sovereign default is avoided, depositors expect the government to have enough resources to pay the guarantees (i.e., as if there were no concerns about the feasibility of the scheme) and they only run in this case if, despite the guarantees, their expected payoff at the final date is lower than what they can obtain at date 1.

Insert Figure 5

This result suggests the importance of having a sound sovereign for the effectiveness and credibility of the guarantees and potentially for the feedback loop between banks and sovereigns. To this end, I will discuss the role and the effects of different austerity measures in Section 5.2.

[11] This condition is obtained in the proof of Proposition 5, which is derived for the general case $\gamma \geq \lambda$.

To complete the analysis, I analyze the choice of the deposit contract r_{B0}. Each bank chooses the interest rate at date 0 so as to maximize depositors' expected utility. In doing so, it anticipates the choice of r_{G1} by the government and thus, that runs will occur only in in the interval $Y < \max\left\{Y_G^g(\widehat{r}_{G1}^g), Y_B^f\right\}$. The bank's problem is as follows:

$$\max_{r_{B0}} \begin{cases} \int_0^{Y_G^g(\widehat{r}_{G1}^g)} u(1)\, dY + \int_{Y_G^g(\widehat{r}_{G1}^g)}^{Y_B^f} u(r_{B0})\, dY + \\ + \int_{Y_B^f}^{1} \lambda u(r_{B0}) + (1-\lambda)\, u\left(R(Y)\frac{1-t-\lambda r_{B0}}{1-\lambda}\right) dY + E[v(g)]dY. & \text{if } Y_B^f \geq Y_G^g(\widehat{r}_{G1}^g) \\ \int_0^{Y_G^g(\widehat{r}_{G1}^g)} u(1)\, dY + \\ + \int_{Y_G^g(\widehat{r}_{G1}^g)}^{1} \lambda u(r_{B0}) + (1-\lambda)\, u\left(R(Y)\frac{1-t-\lambda r_{B0}}{1-\lambda}\right) dY + E[v(g)]dY. & \text{if } Y_B^f < Y_G^g(\widehat{r}_{G1}^g) \end{cases} \tag{10}$$

The bank's objective function is a piecewise function as it depends on whether a run is associated with a sovereign default or is fundamental based. The first two terms in the case $Y_B^f \geq Y_G^g(\widehat{r}_{G1}^g)$ and the first one in the case $Y_B^f < Y_G^g(\widehat{r}_{G1}^g)$ represent depositors' expected utility in the event of a run. As shown above a bank run takes place only into two circumstances: when a sovereign default occurs; and ii) when no sovereign default occurs, but, despite the guarantees being fully feasible, depositors still have an incentive to runs when they expect other depositors to do the same. In both cases, depositors receive the liquidation proceeds. However, this amount is different in the two cases. If the run occurs when sovereign default does not-which is the case for any $Y < Y_B^f$ and $Y_B^f \geq Y_G^g(\widehat{r}_{G1}^g)$, the government transfers $r_{B0} - (1-t)$ resources and each depositor receives a repayment r_{B0}. If the bank run is associated with a sovereign default, instead, depositors receive $1 + t + 1 - t - r_{G0}^g = 2 - r_{G0}^g = 1$. The intermediate term, in both cases, represents depositors' utility when there is no run and only the λ impatient depositors withdraw at date 1. In this case, the impatient depositors receive the promised repayment r_{B0}, while the patient ones wait and receive $R(Y)\frac{1-t-\lambda r_{B0}}{1-\lambda}$ at date 2. The last term is the utility depositors accrue from the provision of the public good. As banks are atomistic, this amount does not depend on their individual choices and, thus, does not play a role in the choice of r_{B0} by an individual bank.

The problem is complicated and does not lend itself to a simple analytical solution, but it can still be shown that banks choose $r_{B0}^g > 1 - t$. When $r_{B0}^g = 1 - t$, the benefits of an increase in r_{B0} are maximal, while the losses approach zero as the bank always has enough resources (including those provided by the government) to repay early withdrawing depositors without liquidating more than one unit of the asset. As a consequence, when $r_{B0}^{g} = 1 - t$, runs, if they occur, are only fundamental-driven and entail no losses in terms of depositors' utility since $u\left(R(Y)\frac{1-t-\lambda r_{B0}}{1-\lambda}\right) - u\left(r_{B0}\right)\Big|_{Y=\underline{Y}_B} = 0$. [12]

5 Extensions

In this section, I extend the baseline model in two directions. First, I consider the possibility that banks hold sovereign bonds in their portfolio. Second, I analyze how the implementation of austerity measures, aimed at improving the state of public finances, affects the equilibrium.

5.1 Banks' holdings of sovereign bonds

In the baseline model, banks can only invest in a risky asset. In this section, I allow banks to invest a fraction b of their resources in long-term sovereign bonds at date 0. The remaining fraction $1 - b$ is invested in the risky asset as in the baseline model. The investment in sovereign bonds yields a per unit return $r_G^L > 1$ at date 2 if the government is solvent and 0 otherwise. In the spirit of Broner, Martin and Ventura (2010), I consider the presence of a secondary market where banks can sell bonds for a price $p_b = r_G^L \Pr(Y > Y_G | \, x_P)$ at date 1. This price is determined before the arrival of the private signal and reflects the fundamental value of the bonds (i.e., their expected return) conditional on the public signal x_P.

To keep the model tractable, I make some simplifying assumptions. First, I assume that, unlike investors, banks invest in long-term sovereign bonds, which can be sold at date 1 to raise liquidity.[13] Second, I assume that banks use their stock of bonds to satisfy early withdrawals, and only start liquidating the risky asset when this stock is depleted. Third, I restrict the analysis to the case where $\gamma r_{B0} > bp_b$, that is guarantees are only paid after the a bank has sold the entire stock of sovereign bonds in its portfolio. Finally, I set the taxes t and the level of public expenditure $\overline{g}$ to be equal to 0. The rest of the model stays the same.

Given the assumptions above, the amount Γ_b that the government commits to transfer to banks is now given by

$$\Gamma_b = \max\left\{0, \frac{(n-\gamma)\left(r_{B0} - (1-b) - p_b b\right)}{1-\gamma}\right\},$$

and the following proposition characterizes the equilibrium for depositors' and investors' rollover decisions.

[12] Proving that $r_{B0} > 1 - t$ is important as otherwise there would not be any strategic complementarity between depositors' actions, thus there would be no panic runs and, as a consequence, no need to introduce the guarantees.

[13] This assumption is important as it makes it possible to avoid a complicated coordination game between banks and foreign investors at date 1.

Proposition 6 *For given r_{B0}, r_{G0}, r_{G1}, γ and b, the model has still a unique threshold equilibrium where all depositors run if they receive a signal below x_B^b and do not run above and all investors withdraw if they receive a signal below x_G^b and roll over otherwise. The equilibrium thresholds $\left\{x_B^b, x_G^b\right\}$ are the solution to the following system:*

$$\int_{\lambda}^{\gamma} u\left(R(x_B^b+\varepsilon(1-2\tfrac{n-\lambda}{1-\lambda}))\frac{1-b-[nr_{B0}-bp_b]^+ + \left[b-\frac{nr_{B0}}{p_b}\right]^+ r_G^L}{1-n}\right)dn + \int_{\gamma}^{\widetilde{n}(x_B^b,x_G^b)} u\left(R(x_B^g+\varepsilon(1-2\tfrac{n-\lambda}{1-\lambda}))\frac{1-b-\gamma r_{B0}+bp_B}{1-\gamma}\right)dn +$$
$$+\int_{\widetilde{n}(x_B^b,x_G^b)}^{\overline{n}(x_B^b,x_G^b)} u \quad R(x_B^b+\varepsilon(1-2\tfrac{n-\lambda}{1-\lambda}))\frac{2+p_b b-\frac{x_G^b-x_B^b}{2\varepsilon}r_{G0}+\frac{n-\lambda}{1-\lambda}r_{G0}-nr_{B0}}{1-n}\Bigg) - \int_{\lambda}^{\overline{n}(x_B^b,x_G^b)} u\left(r_{B0}\right)dn +$$
$$-\int_{\lambda}^{\gamma}\pi^*\left[u \quad R(x_B^b+\varepsilon(1-2\tfrac{n-\lambda}{1-\lambda}))\frac{1-b-[nr_{B0}-bp_b]^+ + \left[b-\frac{nr_{B0}}{p_b}\right]^+ r_G^L}{1-n}\Bigg) - u\left(R(x_B^b+\varepsilon(1-2\tfrac{n-\lambda}{1-\lambda}))\frac{1-b-nr_{B0}+bp_b}{1-n}\right)\right]dn = 0 \tag{11}$$

$$\widetilde{G}\left(x_G^b - \frac{r_{G0}}{r_{G1}}2\varepsilon+\varepsilon, 1+b-\frac{r_{G0}^2}{r_{G1}} - \frac{\left(\lambda+(1-\lambda)\left(\frac{x_B^b-x_G^b}{2\varepsilon}+\frac{r_{G0}}{r_{G1}}\right)-\gamma\right)\left(r_{B0}-p_b b-(1-b)\right)}{1-\gamma}\right) - (r_{G1}-r_{G0}) - br_G^L = 0, \tag{12}$$

where $\pi^* = \left[\frac{x_G^b-x_B^b}{2\varepsilon}+\left(1-\frac{r_{G0}}{r_{G1}}\right)\right]$. The expressions for $\widetilde{n}(x_B^b, x_G^b)$ and $\overline{n}(x_B^b, x_G^b)$ are specified in the appendix.

The proposition shows that a unique threshold equilibrium also exists in the extended framework. As in the baseline model, the equilibrium is given by $\left\{x_B^b, x_G^b\right\}$, that is by the intersection of the curves (11) and (12) in the proposition. The interpretation of the two expressions, as well as their basic properties, are as in the baseline model. Most notably, depositors' and investors' actions still depend on the growth rate of the domestic economy Y, and, on the actions taken by all other agents in the economy. As a result, the thresholds x_B^b and x_G^b are positively related.

Banks' holdings of sovereign bonds crucially contribute to the interaction between x_B^b and x_G^b. In particular, unlike the baseline model, the risk of a sovereign default– as captured by x_G^b– does not only affect banks' stability (i.e., x_B^b) via its impact on the actual amount of guarantees provided, but also, more directly, through its effect on the repayment that patient depositors expect to receive at the final date. This is captured by the last term in (11). When a sovereign default occurs, which happens with probability $\pi^* = Pr(Y < Y_G^b) = \left[\frac{x_G^b-x_B^b}{2\varepsilon}+\left(1-\frac{r_{G0}}{r_{G1}}\right)\right]$, banks accrue a net loss on their investment in sovereign bonds equal to r_G^L. As a result, they have less resources to distribute to the waiting late depositors, who, in turn, run in greater numbers.

Despite the simplifying assumptions described above, the framework with banks' holdings of sovereign bonds is quite rich and does not allow a sharp characterization of the banks' and the government's choices.

However, it is still possible to highlight the channels through which both guarantees and banks' holdings of sovereign bonds link the banks' and the sovereign's stability and to elaborate on how banks' holdings of sovereign bonds interact with the guarantees. To this end, I proceed as follows. First, I disentangle the effects that banks' bondholdings have on the equilibrium thresholds and, thus, on the stability of both the banks and the government. In doing this, I show that the price of sovereign bonds, which can be thought as a measure of how good they are as store of liquidity, plays a crucial role in determining the sign of the various effects. Second, I show that the effects of guarantees and those of banks' holding of sovereign bonds on the equilibrium thresholds and, in turn, on the bank-sovereign nexus are interdependent and I elaborate on how such an interdependence hinges on the effect that guarantees have on the price of sovereign bonds.

Effects of sovereign bond holdings on the equilibrium threshold

As for the guarantees, banks' holdings of sovereign bonds b generate both direct and indirect effects on the equilibrium thresholds $\{x_B^b, x_G^b\}$. The direct effect of banks' holdings of sovereign bonds on investors' incentives not to rollover (i.e., on x_G^b) is twofold, as also illustrated in Figure 6. First, an increase in banks' holdings of sovereign bonds increases the resources available at date 2 by an amount $\frac{\partial \widetilde{G}(.)}{\partial I(.)} - r_G^L > 0$, thus easing the government's budget constraint and increasing investors' rollover incentives as a result.[14] Second, b also affects the disbursement associated with the provision of guarantees. A higher b leads to a higher or lower disbursement depending on the price of bonds in the secondary market p_b. In other words, depending on the extent to which holdings of sovereign bonds are a good store of liquidity for banks (i.e., whether $p_b \gtrless 1$), an increase in banks' holdings of sovereign bonds is either beneficial (if $p_b > 1$) or detrimental (if $p_b < 1$) for sovereign stability.

Insert Figure 6

The direct effect of banks' holdings of sovereign bonds on depositors' withdrawal decisions (i.e., on x_B^b) is threefold, as also illustrated in Figure 7. First, b affects depositors' date 2 repayment, as captured by the derivative of the first three terms in (11). In the range $[\widetilde{n}(x_B^b, x_G^b), \overline{n}(x_B^b, x_G^b)]$, such an effect is always positive, since in this case, a larger b translates into more resources available for the government and, in turn, into a larger transfer associated with the provision of the guarantees. In the other ranges (i.e., for $n \in [\lambda, \gamma]$ and $n \in [\gamma, \widetilde{n}(x_B^b, x_G^b)]$), the sign of this effect depends on whether $p_b \gtrless 1$, as it determines the amount

[14] Condition $\frac{\partial \widetilde{G}(.)}{\partial I(.)} - r_G^L > 0$ holds as the government chooses r_G^L in such a way that the marginal cost r_G^L is not higher than the marginal return $\frac{\partial \widetilde{G}(.)}{\partial I(.)}$.

of liquidity available to the bank to meet early withdrawals. When $p_b > 1$, a larger b is associated with a larger depositors' date 2 repayment, and, in turn, with a smaller x_B^b. In the opposite case, when $p_b < 1$, the effect is negative in that a larger b leads to a lower date 2 repayment and, in turn, to a higher x_B^b. Second, b affects $\overline{n}(x_B^b, x_G^b)$ and so it determines when the orderly liquidation procedure starts. A larger b implies a larger $\overline{n}(x_B^b, x_G^b)$, that is, as b increases, the orderly liquidation procedure is delayed. Since depositors benefit from a timely initiation of an orderly liquidation procedure, this leads to a positive effect of b on x_B^b. Third, because $r_G^L > 1$, the larger b is, the larger the losses are in terms of date 2 repayment when the sovereign defaults. This effect is captured by the derivative of the last term in (11) with respect to b and supports a positive effect of b on x_B^b. Given all these effects, the overall direct effect of b on x_B^b is generally ambiguous and tends to be negative when $p_b > 1$, that is when sovereign bonds provide additional liquidity to banks at date 1.

Insert Figure 7

Consider now the indirect effects. As in the baseline model, indirect effects are captured by the fact that the two equilibrium thresholds affect each other, that is $\frac{dx_B^b(x_G^b)}{dx_G^b} > 0$ and $\frac{dx_G^b(x_B^b)}{dx_B^b} > 0$. These effects are also present in the extended framework, as shown by Proposition 6, but there is an important caveat. While the indirect effect of banks' holdings of sovereign bonds on x_B^b is present irrespective of the presence of guarantees, the effect on x_G^b crucially depends on whether there is a guarantee scheme in place or not. When guarantees are not provided (i.e., $\gamma = 1$), a change in the probability of a run does not affect foreign investors' rollover decision, while a change in the investors' rollover decision still affects x_B^b, as captured by the last term in (11).[15]

Interaction between guarantees and banks' holdings of sovereign bonds

Banks' holdings of sovereign bonds significantly enrich the analysis of the effect of an increase in the size of the guarantees scheme. Besides the effects described in Section 3.1, guarantees also influence how a change in banks' bondholdings b affects the equilibrium thresholds $\{x_B^b, x_G^b\}$, that is the sign of the direct and indirect effects described above and their magnitude. Thus, non-trivial interactions between guarantees γ and bondholdings b may arise and the price p_b seems to play an important role in such interactions.

Consider, first, the case where a change in the size of the guarantees improves sovereign stability and leads

[15] The absence of an indirect effect of b on x_B^b when $\gamma = 1$ results from the simplifying assumptions illustrated above. In a more general framework, an increase in the probability of a run would, for example, force banks to withdraw their (short-term) investment in sovereign bonds, in order to satisfy the increased withdrawals, thus, tightening the government's budget constraint.

to $p_b > 1$. Then, an increase in banks' holdings of sovereign bonds b is likely to lead to a further increase in both financial and sovereign stability, which, in turn, reduces the need for and the cost of guarantees. In this respect, guarantees and banks' holdings of sovereign bonds are to some extent substitutes in terms of their effects on sovereign and financial stability.

Consider now the case, where a change in γ leads to an increase in sovereign instability, causing $p_b < 1$. In this case, given the set of effects described above, an increase in bond holdings is likely to reinforce the negative impact that guarantees have on both sovereign and financial fragility. Thus, also in this case, γ and b appear to be substitutes, with changes in bond holdings reinforcing the effects of guarantees.

The interactions between guarantees and banks' holdings of sovereign bonds suggest that different policies may be effective in ameliorating the negative spillovers between banking and sovereign debt crises and also highlight the importance of coordinating such policies.

5.2 Austerity measures

The analysis in the baseline model suggests that the effectiveness of the guarantees in preventing banking crises and its costs in terms of a higher instability in the sovereign bond market are influenced by the specific characteristics of the economy and also by the state of public finances.

The potential destabilizing effects of the two-way feedback between banking and sovereign debt crises are more pronounced the more limited government resources are and the more the government depends on the issuance of bonds to finance its expenditures. If the government had a sounder budget and more resources to finance the scheme, besides those raised from foreign investors by issuing bonds, the interdependence between banking and sovereign debt crises would be lower and so would the likelihood of each type of crisis. Thus, one possibility for the government to reduce the likelihood of banking and sovereign debt crises is to implement austerity measures so to improve the state of public finances. In the model, austerity measures can take two forms: a reduction in the public expenditure $\overline{g}$ and an increase in the tax t levied on domestic agents at date 0. Both measures improve government finances and thus, they should positively affect the credibility of the guarantee scheme and the solvency of the government. However, their effects on depositors' and investors' decisions can be very different, as illustrated in the following proposition.

Proposition 7 *A reduction in the level of public expenditure $\overline{g}$ always leads to a lower probability of both banking and sovereign debt crises. An increase in the tax burden t can be counterproductive and lead to a* *higher probability of both crises when*

$$\left|\frac{\partial x_B^g}{\partial t}\right| > \left|\frac{\partial x_G^g}{\partial t}\right|.$$

The proposition shows that different austerity measures may have very different effects on the likelihood of banking and sovereign debt crises and their interaction. This result depends on whether a particular austerity measure only improves public finances or also has a direct effect on depositors' withdrawal decision.

In this framework, a reduction in the level of public expenditure $\overline{g}$ has only a direct effect on the government budget, but does not (directly) affect depositors' withdrawal decision. A lower $\overline{g}$ is associated with sounder public finances. As a consequence, the probability of a sovereign default is, ceteris paribus, lower and investors have a greater incentive to roll over the bonds. This indirectly benefits depositors and reduces their incentives to withdraw early, since the government has more resources to pay the guarantees at date 1 in the event of a run.

The effect of taxes is more complicated as they directly affect both public finances and the return on banks' investment in the opposite direction. On the one hand, an increase in the tax revenue improves the soundness of government budget. On the other hand, an increase in taxes reduces the initial investment of the bank and, thus, the expected payoff for the depositors waiting until date 2. When the latter effect dominates the former, the increase in taxes has a detrimental effect on sovereign stability and may also lead to an increase in the probability of a banking crisis, thus triggering overall a negative feedback loop between banking and sovereign debt crises.

6 Concluding remarks

In this paper, I analyze the effect of government guarantees on the probability of banking crises and sovereign default and on their interaction. To this end, I develop a model where both the banks and the sovereign are fragile in that they are exposed to roll-over risk. Panic- and fundamental-driven banking and sovereign debt crises emerge in this framework and their probability, as well as the deposit contract and the interest rate on sovereign bonds are determined endogenously. Thus, the paper offers a convenient framework for evaluating the role of government guarantees in triggering the feedback loop between banking and sovereign debt crises that we observed in the recent euro area crisis.

The analysis is extended to account for the interaction between guarantees and sovereign bond holdings by banks. This extension highlights that guarantees also affect the trade-off between the benefits of banks' holdings of sovereign bonds (e.g., their liquidity) and the costs of exacerbating the adverse effect of a sovereign default arising in a context where banks hold domestic sovereign bonds. Thus, it suggests that there are relevant interactions between guarantees and bondholdings to account for when designing policies to tackle the complementarity between banking and sovereign debt crises.

There are a number of interesting directions in which to further extend this paper. One interesting extension would be to consider the role of a supranational authority providing the guarantees instead of the national government. By extending this framework to a two-country model with spillovers across the two countries, it would be possible to identify the benefits and costs of a supranational authority providing the guarantees. This analysis could offer interesting insights into the introduction of a pan-European deposit insurance scheme. Finally, the paper abstracts from the analysis of the potential distortions in banks' behavior associated with the guarantees. Including those in the analysis would also represent an interesting extension as there the severity of such distortions should depend on sovereign funding risk.

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A Appendix

Proof of Proposition 1: Take the deposit contract r_{B0} and the interest rates r_{G0} and r_{G1} as given. The proof consists in a few steps. First, I prove that each group of agents behaves according to a threshold strategy when assuming that the other group also behaves according to a threshold strategy. Second, I characterize the two equilibrium thresholds. Finally, I show that they are unique.

When agents within each group behave according to a threshold strategy x_k^g with $k = B, G$, the proportion of agents withdrawing at date 1 is equal to the probability of receiving a signal below x_k^g. Thus, the proportion of investors withdrawing at date 1 $i(Y, x_G^g)$ is is given by

$$i(Y, x_G^g) = \begin{cases} 1 & \text{if } Y \leq x_G^g - \varepsilon \\ \frac{x_G^g - Y + \varepsilon}{2\varepsilon} & \text{if } x_G^g - \varepsilon \leq Y \leq x_G^g + \varepsilon \\ 0 & \text{if } Y \geq x_G^g + \varepsilon \end{cases}, \tag{13}$$

while the proportion of depositors withdrawing at date 1 $n(Y, x_B^g)$ is equal to

$$n(Y, x_B^g) = \begin{cases} 1 & \text{if } Y \leq x_B^g - \varepsilon \\ \lambda + (1-\lambda)\frac{x_B^g - Y + \varepsilon}{2\varepsilon} & \text{if } x_B^g - \varepsilon \leq Y \leq x_B^g + \varepsilon \\ \lambda & \text{if } Y \geq x_B^g + \varepsilon \end{cases}. \tag{14}$$

For both groups, when $Y < x_k^g - \varepsilon$, all agents in group k receive a signal below x_k^g and withdraw at date 1. The opposite is true when $Y > x_k^g + \varepsilon$. In this case, all agents receive a signal above x_k^g and wait until date 2. When Y is in the range $[x_k^g - \varepsilon, x_k^g + \varepsilon]$, there is a partial withdrawal as some agents receive a signal below x_k^g and decide to withdraw at date 1. In the case of depositors, the λ early depositors withdraw at date 1 irrespective of the signal received, that is even when $Y \geq x_B^g + \varepsilon$.

Having characterized the proportion of investors and depositors withdrawing at date 1 for any possible realization of Y given their respective threshold signals x_B^g and x_G^g, I can now move on to the conditions determining the equilibrium thresholds $\{x_B^g, x_G^g\}$. I start with the investors.

Denote as $\alpha(Y, r_{G0}, r_{G1}, i, t, n, \gamma)$ an investor' s expected payoff at date 2 from rolling over the investment in sovereign bonds. This is given by

$$r_{G1}\left[1 - \Pr\left(\widetilde{G}(Y, I(i, n\left(Y, x_B^g\right), r_{G0}, r_{B0}, t, \gamma)) - (1-i)r_{G1} - \overline{g} < 0\right)\right], \tag{15}$$

with $n(Y, x_B^g)$ as given in (14). It is easy to see that the expression in (15) increases with Y and monotonically decreases with the proportion i of investors withdrawing at date 1, when also taking into account the effect that Y has on the proportion of depositors running $n\left(Y, x_B^g\right)$. Since the payoff from not rolling over is fixed and equal to r_{G0}, investors' problem satisfies the conditions in Morris and Shin (1998) and Morris and Shin (2003). As a result, for a given x_B^g, there exists a unique x_G^g such that investors do not rollover if they observe a signal below x_G^g and do rollover otherwise.

To compute the exact expression for x_G^g, I proceed as follows. Investors' withdrawal decision is characterized by two equations. First, there exists a threshold value of the growth rate of the economy Y_G^g such that when $Y = Y_G^g$, the government is at the margin between defaulting and staying solvent. The threshold Y_G^g is the solution to

$$\widetilde{G}(Y_G^g, I(i\left(Y_G^g, x_G^g\right), n\left(Y_G^g, x_B^g\right), r_{G0}, r_{B0}, t, \gamma)) - (1 - i(Y_G^g, x_G^g))r_{G1} - \overline{g} = 0, \tag{16}$$

with $I(i\left(Y_G^g,x_G^g\right),n\left(Y_G^g,x_B^g\right),r_{G0},r_{B0},t,\gamma))=\max\left\{0,1+t-i\left(Y_G^g,x_G^g\right)r_{G0}-\frac{\left(n\left(Y_G^g,x_B^g\right)-\gamma\right)}{1-\gamma}\left(r_{B0}-(1-t)\right)\right\}$ and $i\left(Y_G^g,x_G^g\right)$ and $n(Y,x_B^g)$ are given by (13) and (14), respectively.

Second, an investor is indifferent between withdrawing at date 1 and rolling over the investment in sovereign bonds until date 2 when they receive the threshold signal x_G^g. Formally, this is the case when

$$r_{G1}\frac{x_G^g-Y_G^g+\varepsilon}{2\varepsilon}=r_{G0}, \tag{17}$$

where $\frac{x_G^g-Y_G^g+\varepsilon}{2\varepsilon}=1-\Pr(\widetilde{G}(Y_G^g,I(i\left(Y_G^g,x_G^g\right),n\left(Y_G^g,x_B^g\right),r_{G0},r_{B0},t,\gamma))-(1-i(Y_G^g,x_G^g))r_{G1}-\overline{g}<0)$ since, when the investor receives the signal x_G^g, the posterior distribution of the growth rate of the economy Y is given by $Y\sim U[x_G^g-\varepsilon,\ x_G^g+\varepsilon]$.

From (17) we have

$$Y_G^g=x_G^g-\frac{r_{G0}}{r_{G1}}2\varepsilon+\varepsilon. \tag{18}$$

Substituting it into (16) and given that the expression for $n\left(Y_G^g,x_B^g\right)$ simplifies to

$$n\left(Y_G^g,x_B^g\right)=\lambda+(1-\lambda)\left[\frac{x_B^g-x_G^g}{2\varepsilon}+\frac{r_{G0}}{r_{G1}}\right],$$

we have condition (8) as in the proposition.

Denoting as $f_G(x_B^g,x_G^g)=0$ the condition in (8) pinning down x_G^g as a function of x_B^g, it is easy to see that $f_G(x_B^g,x_G^g)$ decreases with x_B^g $\left(\text{i.e., }\frac{\partial f_G(x_B^g,x_G^g)}{\partial x_B^g}<0\right)$ and increases with x_G^g $\left(\text{i.e., }\frac{\partial f_G(x_B^g,x_G^g)}{\partial x_G^g}>0\right)$. Thus, using the implicit function theorem and denoting as $x_G\left(x_B^g\right)$ the solution to $f_G(x_B^g,x_G^g)=0$, it follows that $\frac{dx_G(x_B^g)}{dx_B^g}=-\frac{\frac{\partial f_G(x_B^g,x_G^g)}{\partial x_B^g}}{\frac{\partial f_G(x_B^g,x_G^g)}{\partial x_G^g}}>0$. Moreover, it holds that $\frac{dx_G^g(x_B^g)}{dx_B^g}<1$. This results from the fact that x_B^g only affects $f_G(x_B^g,x_G^g)$ via the change in $I\left(.\right)$, while x_G^g also directly affects $f_G\left(x_B^g,x_G^g\right)$ via the change in Y in the $\widetilde{G}\left(.\right)$.

Consider now depositors' withdrawal decision. This case is more involved since depositors' utility differential, as given by (5), does not monotonically decrease with n and i. In other words, a depositor's utility differential between withdrawing at date 2 and date 1 is maximal when $n=\overline{n}(i)$ rather than when $n=1$. Symmetrically, it is also the case that it is maximal when i solves $nr_{B0}=2-ir_{G0}$ rather than when $i=1$. This is the case because, for $n>\overline{n}(i)$, a depositor obtains the same repayment at date 2 and date 1 and $\overline{n}(i)$ is decreasing in i. So, using the terminology in Goldstein and Pauzner (2005), the depositors' problem exhibits only the property of one-sided global strategic complementarity in both depositors' and investors' actions. Despite this, building on their proof, it can be still shown that a unique threshold equilibrium exists for the depositors when they believe that investors behave according to the threshold strategy x_G^g.

Recall that the proportion of investors not rolling over and that of depositors running are given by (13) and (14), respectively. Then, for given x_B^g and x_G^g it follows that for any realization of Y, the proportion of depositors running and investors not rolling over are deterministic and I denote them simply as $i\left(Y\right)$ and $n(Y)$. The former (i.e., $i\left(Y\right)$) is a number between 0 and 1, the latter (i.e., $n\left(Y\right)$) is a number in the range $[\lambda,1]$.

Denote as $\Delta\left(x_{Bj},n\left(Y\right),i\left(Y\right)\right)$ a depositor j' s expected utility differential between withdrawing at date 2 and running at date 1, when they receive the signal x_{Bj}. It is equal to

$$\Delta\left(x_{Bj},n\left(Y\right),i\left(Y\right)\right)=\frac{1}{2\varepsilon}\int_{x_{Bj}-\varepsilon}^{x_{Bj}+\varepsilon}v(Y,n\left(Y\right),i\left(Y\right))dY. \tag{19}$$

The function $v(Y, n(Y), i(Y))$ comes from (5) and it is equal to

$$\frac{1}{1-\lambda}\left\{\int_{\lambda}^{\gamma}\left[u\left(R(Y)\frac{1-t-nr_{B0}}{1-n}\right)-u(r_{B0})\right]dn+\right. \tag{20}$$

$$\left.+\int_{\gamma}^{\widetilde{n}(i)}\left[u\left(R(Y)\frac{1-t-\gamma r_{B0}}{1-\gamma}\right)-u(r_{B0})\right]dn+\int_{\widetilde{n}(i)}^{\overline{n}(i)}\left[u\left(R(Y)\frac{2-ir_{G0}-nr_{B0}}{1-n}\right)-u(r_{B0})\right]dn\right\},$$

with $\widetilde{n}(i)$ being the solution to

$$1+t-ir_{G0}=\frac{(n-\gamma)(r_{B0}-(1-t))}{1-\gamma}, \tag{21}$$

and $\overline{n}(i)$ that to

$$1-t+1+t-ir_{G0}=2-ir_{G0}=nr_{B0}. \tag{22}$$

The lemma states a few properties of the function $\Delta(x_{Bj}, n(Y), i(Y))$.

Lemma 2 *The function* $\Delta(x_{Bj}, n(Y), i(Y))$

i) is continuos in x_{Bj}*;*

ii) for any $a>0$, $\Delta(x_{Bj}+a, (n+a)(Y), (i)(Y))$ *is non-decreasing in a;*

iii) in the range $\left[x_B^G-\varepsilon,\ x_B^G+\varepsilon\right]$, $\Delta(x_{Bj}+a, (n+a)(Y), (i)(Y))$ *is strictly increasing in a.*

Proof of Lemma 2: The function $\Delta(.)$ is continuous in x_{Bj} as a change in x_{Bj} only affects the limits of integration in (19). Point (ii) of the lemma implies that the function $\Delta(.)$ does not decrease as a consequence of a positive shifts in both the signal x_B^g and beliefs $n(Y)$. To prove this, I show that, accounting for the effect that Y has on the proportion of investors not rolling over $i(Y)$ and, in turn, on the guarantees $\Gamma(i, n, r_{B0}, r_{G0}, t, \gamma)$, the function $v(Y, n(Y), i(Y))$ is non-decreasing in Y. When a increases, depositors see the same distribution of n, while expecting Y to be larger. A larger Y has a twofold effect. Firstly, it positively affects $R(Y)$ and, in turn, a depositor's utility differential $v(Y, n(Y), i(Y))$. Secondly, for a given x_G^g, a higher signal and, in turn, better fundamentals, may be associated with a lower $i(Y)$. This has two effects on $v(Y, n(Y), i(Y))$. First, a lower $i(Y)$ implies that, in the range $[\widetilde{n}(i), \overline{n}(i)]$, a depositor's utility differential increases as $\frac{2-ir_{G0}-nr_{B0}}{1-n}$ increases. Second, a higher Y also positively affects the extremes the integrals $\widetilde{n}(i)$ and $\overline{n}(i)$, via the change in $i(Y)$. The extreme $\widetilde{n}(i)$ is more sensitive to changes in Y than $\overline{n}(i)$, as $\frac{d\widetilde{n}(i)}{dY}=\frac{d\widetilde{n}(i)}{di}\frac{di}{dY}=\frac{1}{2\varepsilon}\frac{r_{G0}(1-\gamma)}{r_{B0}-(1-t)}>\frac{d\overline{n}(i)}{dY}=\frac{d\overline{n}(i)}{di}\frac{di}{dY}=\frac{1}{2\varepsilon}\frac{r_{G0}}{r_{B0}}$ for $1-t>\gamma r_{B0}$. The condition $1-t>\gamma r_{B0}$ holds because guarantees are paid before the bank exhausts all its resources at date 1 and the orderly liquidation procedure starts. Thus, a positive shift in the signal enlarges the range $[\lambda, \widetilde{n}(i)]$, while it reduces the range $[\widetilde{n}(i), \overline{n}(i)]$. Since, $v(Y, n(Y), i(Y))$ is, ceteris paribus, larger in the range $[\lambda, \widetilde{n}(i)]$ than in the range $[\widetilde{n}(i), \overline{n}(i)]$, it follows that $v(Y, n(Y), i(Y))$ is non-decreasing in Y once I account for the effects of Y on $i(Y)$. Point (iii) of the lemma holds because in the range $[\lambda, \overline{n}(i)]$, $v(Y, n(Y, i(Y)))$ is strictly increasing in Y and, for $Y\in\left[x_B^G-\varepsilon,\ x_B^G+\varepsilon\right]$, there is a positive probability that n falls below $\overline{n}(i)$. Thus, $\Delta(x_{Bj}+a, (n+a)(Y), (i)(Y))$ is strictly increasing in a when $Y\in\left[x_B^G-\varepsilon,\ x_B^G+\varepsilon\right]$. QED.

Consider the expected utility differential $\Delta(.)$ of a depositor who has received exactly the threshold signal x_B^g. By Lemma 2, $\Delta(x_B^g, n(Y, x_B^g), i(Y, x_G^g))$ is continuous in x_B^g. Moreover, $\Delta(x_B^g, n(Y, x_B^g), i(Y, x_G^g))$ is negative in the lower dominance region, while it is positive in the upper dominance region. Thus, there exists a x_B^g at which $\Delta(x_B^g, n(Y, x_B^g), i(Y, x_G^g))=0$ holds. The uniqueness of x_B^g follows from the property (iii) that $\Delta(x_B^g, n(Y, x_B^g), i(Y, x_G^g))$ is strictly increasing in x_B^g when $Y\in[x_B^g-\varepsilon, x_B^g+\varepsilon]$.

To complete the characterization of depositors' withdrawal decision, I show that x_B^g is indeed a threshold equilibrium, that is no depositor has an incentive to deviate. Formally, this means that

$$\Delta\left(x_{Bj}, n\left(Y, x_B^g\right), i\left(Y, x_G^g\right)\right) < 0 \text{ for } x_{Bj} < x_B^g; \tag{23}$$

and

$$\Delta\left(x_{Bj}, n\left(Y, x_B^g\right), i\left(Y, x_G^g\right)\right) > 0 \text{ for } x_{Bj} > x_B^g. \tag{24}$$

To prove that (23) holds, I decompose the intervals over which the two integrals are computed into a common part $c = [x_{Bj} - \varepsilon,\ x_{Bj} + \varepsilon] \cap [x_B^g - \varepsilon,\ x_B^g + \varepsilon]$ and two disjoint parts $d_{Bj} = \frac{[x_{Bj}-\varepsilon,\ x_{Bj}+\varepsilon]}{c}$ and $d_B^g = \frac{\left[x_B^g-\varepsilon,\ x_B^g+\varepsilon\right]}{c}$. Then, it follows that

$$\Delta\left(x_{Bj}, n\left(Y, x_B^g\right), i\left(Y, x_G^g\right)\right) = \frac{1}{2\varepsilon}\int_{Y\in c} v\left(Y, n(Y, x_B^G), i\left(Y, x_G^G\right)\right) + \frac{1}{2\varepsilon}\int_{Y\in d_{Bj}} v\left(Y, n(Y, x_B^G), i\left(Y, x_G^G\right)\right),$$

and

$$\Delta\left(x_B^g, n(Y, x_B^G), i\left(Y, x_G^G\right)\right) = \frac{1}{2\varepsilon}\int_{Y\in c} v\left(Y, n(Y, x_B^G), i\left(Y, x_G^G\right)\right) + \frac{1}{2\varepsilon}\int_{Y\in d_B^g} v\left(Y, n(Y, x_B^G), i\left(Y, x_G^G\right)\right)).$$

For any $Y \in d_{Bj}$, $n = 1$ since $Y < x_B^g - \varepsilon$. Thus, $v\left(Y, n(Y, x_B^G), i\left(Y, x_G^G\right)\right) = 0$ and $\Delta\left(x_{Bj}, n\left(Y, x_B^g\right), i\left(Y, x_G^g\right)\right) = 0$ in that interval. In order to show that $\Delta\left(x_{Bj}, n\left(Y, x_B^g\right), i\left(Y, x_G^g\right)\right) < 0$, I need to show that

$$\frac{1}{2\varepsilon}\int_{Y\in c} v\left(Y, n(Y, x_B^G), i\left(Y, x_G^G\right)\right) < 0.$$

This is true because $\Delta\left(x_B^g, n\left(Y, x_B^g\right), i\left(Y, x_G^g\right)\right) = 0$ holds and the fundamental in the interval d_B^g are better than those in d_{Bj}, which implies that $\frac{1}{2\varepsilon}\int_{Y\in d_B^g} v\left(Y, n(Y, x_B^G), i\left(Y, x_G^G\right)\right)) > \frac{1}{2\varepsilon}\int_{Y\in d_{Bj}} v\left(Y, n(Y, x_B^G), i\left(Y, x_G^G\right)\right) = 0$.

Finally, the condition (7) in the proposition can be obtained by a simple change of variable. From $n(Y, x_B^g) = \lambda + (1-\lambda)\frac{x_B^g - Y + \varepsilon}{2\varepsilon}$ as specified in (14), I can rewrite $Y = x_B^g + \varepsilon\left(1 - 2\frac{n-\lambda}{1-\lambda}\right)$. Thus, the expression for $i\left(Y, x_G^g\right)$ in (13) becomes $i = \frac{x_G^g - x_B^g}{2\varepsilon} + \frac{n-\lambda}{1-\lambda}$ and the expression is as in the proposition. Substituting the expression $i = \frac{x_G^g - x_B^g}{2\varepsilon} + \frac{n-\lambda}{1-\lambda}$ into (21) and (22), I obtain the expressions for $\widetilde{n}\left(x_B, x_G\right)$ and $\bar{n}\left(x_B, x_G\right)$ as in the proposition.

Like in the case of investors, I denote as $f_B(x_B^g, x_G^g) = 0$ the condition (7) in the proposition and as $x_B\left(x_G^g\right)$ the solution to $f_B(x_B^g, x_G^g) = 0$. Given that $x_B(x_G^g)$ is the solution to $f_B(x_B^g, x_G^g) = 0$, the effect of x_G^g on $x_B(x_G^g)$ can be computed using the implicit function theorem as follows:

$$\frac{dx_B(x_G^g)}{dx_G^g} = -\frac{\frac{\partial f_B(x_B^g, x_G^g)}{\partial x_G^g}}{\frac{\partial f_B(x_B^g, x_G^g)}{\partial x_B^g}}.$$

Both conditions $\frac{\partial f_B(x_B^g, x_G^g)}{\partial x_B^g} > 0$ and $\frac{\partial f_B(x_B^g, x_G^g)}{\partial x_G^g} < 0$ follow from Lemma 2 and imply $\frac{dx_B(x_G^g)}{dx_G^g} > 0$. The condition $\frac{dx_B\left(x_G^g\right)}{dx_G^g} < 1$ holds because, while a change in x_G^g only affects $f_B(x_B^g, x_G^g)$ via a change in $i(Y, x_G^g)$, a change in the signal received by depositors, also affects the function via the direct effect on Y.

To complete the proof of the proposition, I show that $\{x_B^g, x_G^g\}$ is unique. This follows from conditions $0 < \frac{dx_B\left(x_G^g\right)}{dx_G^g} < 1$ and $0 < \frac{dx_G\left(x_B^g\right)}{dx_B^g} < 1$, as they imply that the two curves, as given by $x_B\left(x_G^g\right)$ and $x_G\left(x_B^g\right)$, only cross once. The equilibrium $\{x_B^g, x_G^g\}$ corresponds exactly to the intersection of the two curves and the proposition follows. □

Proof of Proposition 2: In the economy without guarantees, depositors' and investors' withdrawal decisions are independent of each other and, thus, the two thresholds x_B^{ng} and x_G^{ng} can be computed separately. I start with the investors.

Assuming that all investors behave according to a threshold strategy x_G^{ng}, that is each investor rolls over the bonds if and only if they receive a signal above x_G^{ng} and they do not roll over otherwise, the proportion of investors withdrawing at date 1 is still given by (13), with the difference that we have now x_G^{ng} instead of x_G^g as the threshold signal.

The characterization of investors' rollover decisions follows exactly the same steps as in the proof of Proposition 1 and consists of two equations. First, there exists a threshold value of the growth rate of the economy Y_G^{ng} such that when $Y = Y_G^{ng}$, the government is at the margin between defaulting and staying solvent. The threshold Y_G^{ng} is the solution to

$$\widetilde{G}(Y_G^{ng}, I(i(Y_G^{ng}, x_G^{ng}), t, r_{G0})) - (1 - i(Y_G^{ng}, x_G^{ng}))r_{G1} - \overline{g} = 0, \tag{25}$$

with $I(i(Y_G^{ng}, x_G^{ng}), t, r_{G0}) = 1 + t - \frac{x_G^{ng} - Y_G^{ng} + \varepsilon}{2\varepsilon} r_{G0}$ and

Second, an investor is indifferent between withdrawing at date 1 and rolling over the investment in sovereign bonds until date 2 when he receives the threshold signal x_G^{ng}. Formally, this is the case when

$$r_{G1}\frac{x_G^{ng} - Y_G^{ng} + \varepsilon}{2\varepsilon} = r_{G0}, \tag{26}$$

where $\frac{x_G^{ng} - Y_G^{ng} + \varepsilon}{2\varepsilon} = 1 - \Pr(\widetilde{G}(Y, I(i, t, r_{G0})) - (1 - i)r_{G1} - \overline{g} < 0)$ when the depositor receives the signal x_G^{ng} since, given the signal, the posterior distribution of the growth rate of the economy Y is given by $Y \sim U[x_G^{ng} - \varepsilon,\, x_G^{ng} + \varepsilon]$.

From (26) we have

$$Y_G^{ng} = x_G^{ng} - \frac{r_{G0}}{r_{G1}} 2\varepsilon + \varepsilon.$$

Substituting it into (25) we obtain:

$$\widetilde{G}\left(x_G^{ng} - \frac{r_{G0}}{r_{G1}} 2\varepsilon + \varepsilon, 1 + t - \frac{r_{G0}^2}{r_{G1}}\right) - (r_{G1} - r_{G0}) - \overline{g} = 0, \tag{27}$$

where $1 + t - \frac{r_{G0}^2}{r_{G1}} = I\left(i(Y_G^{ng}, x_G^{ng}), r_{G0}, t\right)$.

Consider now depositors' withdrawal decision. As for the investors, the proof is analogous to the one in the proof of Proposition 1, albeit simpler given that depositors' and investors' withdrawal decisions are independent of each other.

The proportion of depositors running at date 1 is still given by (14), with the difference that we have now x_B^{ng} instead of x_B^g as the threshold signal. The equilibrium threshold x_B^{ng} corresponds to the solution of a depositor's indifference condition between withdrawing at date 1 or waiting until date 2. It is given by the solution to

$$\int_{\lambda}^{\frac{1-t}{r_{B0}}} \left[u\left(R(Y(n))\frac{1 - t - nr_{B0}}{1 - n}\right) - u\left(r_{B0}\right)\right] dn = 0, \tag{28}$$

where from (14), $Y(n) = x_B^{ng} + \varepsilon - 2\varepsilon \frac{(n-\lambda)}{1-\lambda}$ and $\frac{1-t}{r_{B0}}$ corresponds to the proportion of depositors running n for which the bank liquidates all its assets at date 1, that is $nr_{B0} = 1 - t$. Importantly, $\frac{1-t}{r_{B0}} < \overline{n}(x_B^g, x_G^g)$ as defined in Proposition 1. This is the case because $\overline{n}(x_B^g, x_G^g)$ solves $1 - t + (1 + t - ir_{G0}) = nr_{B0}$.

Having characterized the thresholds x_G^{ng} and x_B^{ng}, I need to compare them with those in the case where guarantees are in place, as given in Proposition 1. It is easy to see that (8) and (27) only differ in the expression for $I\left(.\right)$. In the economy with guarantees, the amount of resources invested by the government is smaller than that in the economy without guarantees as shown below when comparing $I\left(i(Y_G^{ng}, x_G^{ng}), r_{G0}, t\right)$ with $I\left(i(Y_G^g, x_G^g), n\left(Y_G^g, x_G^g\right) r_{G0}, r_{B0}, t, \gamma\right)$ evaluated at the same level of Y and x_G

$$1 + t - i(Y, x_G) r_{G0} > \max\left\{0, 1 + t - i\left(Y, x_G\right) r_{G0} - \frac{\left(n\left(Y, x_B\right) - \gamma\right)}{1-\gamma}\left(r_{B0} - (1-t)\right)\right\}.$$

Thus, since both (8) and (27) are increasing in Y, it follows that $x_G^{ng} < x_G^g$.

For the case of depositors, the proof is similar. Rearrange (7) as follows

$$\begin{aligned}
&\int_{\lambda}^{\gamma}\left[u\left(R(Y(n))\tfrac{1-t-nr_{B0}}{1-n}\right) - u\left(r_{B0}\right)\right]dn+ \\
&+\int_{\gamma}^{\widetilde{n}(x_B^g,x_G^g)}\left[u\left(R(Y(n))\tfrac{1-t-\gamma r_{B0}}{1-\gamma}\right) - u\left(r_{B0}\right)\right]dn+ \\
&+\int_{\widetilde{n}(x_B^g,x_G^g)}^{\overline{n}(x_B^g,x_G^g)}\left[u\quad R(Y\left(n\right))\frac{2-\frac{x_G^g-x_B^g}{2\varepsilon}r_{G0}+\frac{n-\lambda}{1-\lambda}r_{G0}-nr_{B0}}{1-n}\right)-u\left(r_{B0}\right)\Bigg]dn = 0
\end{aligned} \quad . \tag{29}$$

Comparing it with (28), it is easy to see that the expression in (29) is larger than the one in (28). The reason is that the repayment to late depositors withdrawing at date 2 is never lower than that in the economy without guarantees. Then, since both expressions are increasing in Y, it follows that $x_B^g < x_B^{ng}$, as stated in the proposition.

I now move on to prove the second result in the proposition. The two thresholds x_B^f and x_G^f corresponds to "artificial" thresholds computed under the assumption that guarantees are always feasible and so depositors are always sure to receive the guaranteed amount $R(Y)\frac{1-t-\gamma r_{B0}}{1-\gamma}$ when withdrawing at date 2. Under this assumption, the equilibrium threshold for the depositors x_B^f can easily be computed following the same steps as in the economy without guarantees.

Denote as x_B^f the threshold signal at which a depositor is exactly indifferent between withdrawing at date 1 and 2 when they expect to always receive the guaranteed amount. Then, the condition determining the threshold x_B^f is given by

$$\int_{\lambda}^{\gamma}\left[u\left(R(Y(n))\frac{1-t-nr_{B0}}{1-n}\right) - u\left(r_{B0}\right)\right]dn + \int_{\gamma}^{1}\left[u\left(R(Y(n))\frac{1-t-\gamma r_{B0}}{1-\gamma}\right) - u\left(r_{B0}\right)\right]dn = 0, \tag{30}$$

where $Y(n) = x_B^f + \varepsilon - 2\varepsilon\frac{(n-\lambda)}{1-\lambda}$. Comparing (29) with (30), it is easy to see that the former is smaller than the latter, thus, since both expressions are increasing in Y, it follows that $x_B^f < x_B^g$.

The threshold x_G^f is analogous to x_G^g. The condition $x_G^f < x_G^g$ follows from the fact that $x_B^f < x_B^g$ and x_G^g increases with depositors threshold signal as shown in Proposition 1. This completes the proof of the proposition.□

Proof of Proposition 3: Denote as $f_B(x_B^g, x_G^g, \gamma) = 0$ and $f_G(x_B^g, x_G^g, \gamma) = 0$ equations (7) and (8), respectively. To prove that x_B^g and x_G^g are increasing in γ, I use the implicit function theorem and obtain

$$\frac{dx_B^g}{d\gamma} = -\frac{\begin{vmatrix} \frac{\partial f_B(x_B^g,x_G^g,\gamma)}{\partial \gamma} & \frac{\partial f_B(x_B^g,x_G^g,\gamma)}{\partial x_G^g} \\ \frac{\partial f_G(x_B^g,x_G^g,\gamma)}{\partial \gamma} & \frac{\partial f_G(x_B^g,x_G^g,\gamma)}{\partial x_G^g} \end{vmatrix}}{\begin{vmatrix} \frac{\partial f_B(x_B^g,x_G^g,\gamma)}{\partial x_B^g} & \frac{\partial f_B(x_B^g,x_G^g,\gamma)}{\partial x_G^g} \\ \frac{\partial f_G(x_B^g,x_G^g,\gamma)}{\partial x_B^g} & \frac{\partial f_G(x_B^g,x_G^g,\gamma)}{\partial x_G^g} \end{vmatrix}}, \tag{31}$$

and

$$\frac{dx_G^g}{d\gamma} = -\frac{\begin{vmatrix} \frac{\partial f_B(x_B^g,x_G^g,\gamma)}{\partial x_B^g} & \frac{\partial f_B(x_B^g,x_G^g,\gamma)}{\partial \gamma} \\ \frac{\partial f_G(x_B^g,x_G^g,\gamma)}{\partial x_B^g} & \frac{\partial f_G(x_B^g,x_G^g,\gamma)}{\partial \gamma} \end{vmatrix}}{\begin{vmatrix} \frac{\partial f_B(x_B^g,x_G^g,\gamma)}{\partial x_B^g} & \frac{\partial f_B(x_B^g,x_G^g,\gamma)}{\partial x_G^g} \\ \frac{\partial f_G(x_B^g,x_G^g,\gamma)}{\partial x_B^g} & \frac{\partial f_G(x_B^g,x_G^g,\gamma)}{\partial x_G^g} \end{vmatrix}}. \tag{32}$$

The denominator in both (31) and (32) is the determinant of the *Jacobian* (J) and it is equal to

$$\begin{aligned} |J| &= \frac{\partial f_B(x_B^g, x_G^g, \gamma)}{\partial x_B^g}\frac{\partial f_G(x_B^g, x_G^g, \gamma)}{\partial x_G^g} - \frac{\partial f_B(x_B^g, x_G^g, \gamma)}{\partial x_G^g}\frac{\partial f_G(x_B^g, x_G^g, \gamma)}{\partial x_B^g} = \\ &= 1 - \frac{dx_B^g}{dx_G^g}\frac{dx_G^g}{dx_B^g} > 0 \end{aligned}$$

since $0 < \frac{dx_B^g}{dx_G^g} = -\frac{\frac{\partial f_B(x_B^g,x_G^g,\gamma)}{\partial x_G^g}}{\frac{\partial f_B(x_B^g,x_G^g,\gamma)}{\partial x_B^g}} < 1$ and $0 < \frac{dx_G^g}{dx_B^g} = -\frac{\frac{\partial f_G(x_B^g,x_G^g,\gamma)}{\partial x_B^g}}{\frac{\partial f_G(x_B^g,x_G^g,\gamma)}{\partial x_G^g}} < 1$ from Proposition 1. Thus, the signs of $\frac{dx_G^g}{d\gamma}$ and $\frac{dx_G^g}{d\gamma}$ are equal to the opposite signs of the numerators in expression (31) and (32), respectively.

In order for $\frac{dx_B^g}{d\gamma} > 0$ and $\frac{dx_G^g}{d\gamma} > 0$, it must hold that

$$\frac{\partial f_B(x_B^g, x_G^g, \gamma)}{\partial \gamma}\frac{\partial f_G(x_B^g, x_G^g, \gamma)}{\partial x_G^g} - \frac{\partial f_B(x_B^g, x_G^g, \gamma)}{\partial x_G^g}\frac{\partial f_G(x_B^g, x_G^g, \gamma)}{\partial \gamma} < 0,$$

and

$$\frac{\partial f_B(x_B^g, x_G^g, \gamma)}{\partial x_B^g}\frac{\partial f_G(x_B^g, x_G^g, \gamma)}{\partial \gamma} - \frac{\partial f_B(x_B^g, x_G^g, \gamma)}{\partial \gamma}\frac{\partial f_G(x_B^g, x_G^g, \gamma)}{\partial x_B^g} < 0,$$

respectively.

After dividing both expressions above by $\frac{\partial f_B(x_B^g,x_G^g,\gamma)}{\partial x_B^g}$ and $\frac{\partial f_G(x_B^g,x_G^g,\gamma)}{\partial x_G^g}$, they can be rewritten as follows

$$-\Delta_{\gamma B} - \frac{dx_B^g}{dx_G^g}\Delta_{\gamma G}, \tag{33}$$

and

$$-\Delta_{\gamma G} - \frac{dx_G^g}{dx_B^g}\Delta_{\gamma B}, \tag{34}$$

where $\Delta_{\gamma B} = -\frac{\frac{\partial f_B(x_B^g,x_G^g,\gamma)}{\partial \gamma}}{\frac{\partial f_B(x_B^g,x_G^g,\gamma)}{\partial x_B^g}}$, $\Delta_{\gamma G} = -\frac{\frac{\partial f_G(x_B^g,x_G^g,\gamma)}{\partial \gamma}}{\frac{\partial f_G(x_B^g,x_G^g,\gamma)}{\partial x_G^g}}$ and $\frac{dx_B^g}{dx_G^g} = -\frac{\frac{\partial f_G(x_B^g,x_G^g,\gamma)}{\partial x_G^g}}{\frac{\partial f_B(x_B^g,x_G^g,\gamma)}{\partial x_B^g}}$. The term $\Delta_{\gamma B}$ and $\Delta_{\gamma G}$ denote the direct effect of γ on x_B^g and x_G^g, respectively. From (7) and (8), it is easy to see that $\frac{\partial f_B(x_B^g,x_G^g,\gamma)}{\partial \gamma} < 0$ and $\frac{\partial f_G(x_B^g,x_G^g,\gamma)}{\partial \gamma} > 0$ so that $\Delta_{\gamma B} > 0$ and $\Delta_{\gamma G} < 0$. This is the case because a higher γ implies smaller guarantees and γ only affects x_B^g and x_G^g via the change in the repayment offered by banks to late depositors at date 2 and through the resources invested by the government until the final date $I(.) = \max\left\{0, 1+t-ir_{G0} - \frac{(n-\gamma)}{1-\gamma}\left[r_{B0}-(1-t)\right]\right\}$, respectively.

When $|\Delta_{\gamma G}| < \frac{dx_G^g}{dx_B^g}\Delta_{\gamma B}$, the expression in (34) is negative and, as a result, $\frac{dx_B^g}{d\gamma} > 0$. Since both $\frac{dx_G^g}{dx_B^g}$ and $\frac{dx_B^g}{dx_G^g}$ are smaller than 1, $|\Delta_{\gamma G}| < \frac{dx_G^g}{dx_B^g}\Delta_{\gamma B}$ implies that the expression in (33) is also negative. It follows that $\frac{dx_B^g}{d\gamma} > 0$.

When $|\Delta_{\gamma G}| > \frac{\Delta_{\gamma B}}{\frac{dx_G^g}{dx_B^g}}$, both (33) and (34) are positive. Thus, $\frac{dx_B^g}{d\gamma} < 0$ and $\frac{dx_G^g}{d\gamma} < 0$ holds. In the intermediate case, where $\frac{dx_G^g}{dx_B^g}\Delta_{\gamma B} < |\Delta_{\gamma G}| < \frac{\Delta_{\gamma B}}{\frac{dx_G^g}{dx_B^g}}$, (33) is negative, while (34) is positive. Then, $\frac{dx_G^g}{d\gamma} < 0$ and $\frac{dx_B^g}{d\gamma} > 0$ and the proposition follows. □

Proof of Lemma 1: To see how the equilibrium thresholds x_B^g and x_G^g change with the terms of the deposit contract $\{r_{B0}, \widetilde{r}_{B1}\}$ and the interest rate on sovereign bonds $\{r_{G0}, r_{G1}\}$, I use the implicit function theorem with the functions $f_B(x_B^g, x_G^g) = 0$ and $f_G(x_B^g, x_G^g) = 0$, as defined in (7) and (8), respectively.

Consider first the effect of a change in the interest rate r_{G1}. From (7), it is clear that r_{G1} does not affect x_B^g directly. A change in r_{G1} only determines a direct change in x_G^g via its effect on $f_G(x_B^g, x_G^g)$. Using the implicit function theorem and given that $\frac{\partial f_G(x_B^g, x_G^g)}{\partial x_G^g} > 0$, the sign of the effect of r_{G1} on x_G^g it is equal to the opposite sign of $\frac{\partial f_G(x_B^g, x_G^g)}{\partial r_{G1}}$, which is given by

$$\frac{\partial f_G(x_B^g, x_G^g)}{\partial r_{G1}} = \frac{\partial \widetilde{G}(Y, I(i,t,r_{B0},r_{G0},n,\gamma))}{\partial Y} 2\varepsilon \frac{r_{G0}}{r_{G1}^2} + \frac{\partial \widetilde{G}(Y, I(i,t,r_{B0},r_{G0},n,\gamma))}{\partial I(i,t,r_{B0},r_{G0},n,\gamma))}\left[\frac{r_{G0}^2}{r_{G1}^2} - \frac{r_{G0}}{r_{G1}^2}\frac{(r_{B0}-(1-t))}{1-\gamma}\right] - 1,$$

since $I(i,t,r_{B0},r_{G0},n,\gamma) = \max\left\{0, 1+t-\frac{r_{G0}^2}{r_{G1}} - \frac{\left(\frac{x_B^g - x_G^g}{2\varepsilon} + 1 - \frac{r_{G0}}{r_{G1}}\right)(r_{B0}-(1-t))}{1-\gamma}\right\}$.

The sign of the above expression cannot be easily determined since the first two terms have opposite signs. It follows that an increase in the interest rate r_{G1} has a non-monotone effect on the equilibrium threshold. Denote as $\widehat{r}_{G1}^g$ the solution to $\frac{\partial f_G(x_B^g, x_G^g)}{\partial r_{G1}} = 0$. Deriving $\frac{\partial f_G(x_B^g, x_G^g)}{\partial r_{G1}}$ with respect to r_{G1} again, I obtain:

$$\frac{\partial^2 f_G(x_B^g, x_G^g)}{\partial r_{G1}^2} = -\frac{\partial \widetilde{G}(x_G^g, I(i,t,r_{B0},r_{G0},n,\gamma))}{\partial Y} 4\varepsilon \frac{r_{G0}}{r_{G1}^3} - \frac{\partial \widetilde{G}(Y, I(i,t,r_{B0},r_{G0},n,\gamma))}{\partial I(i,t,r_{B0},r_{G0},n,\gamma))}\left[\frac{2r_{G0}^2}{r_{G1}^3} + \frac{2r_{G0}}{r_{G1}^3}\frac{(r_{B0}-(1-t))}{1-\gamma}\right] < 0.$$

It follows that for any $r_{G1} < \widehat{r}_{G1}^g$, $\frac{\partial f_G(x_B^g, x_G^g)}{\partial r_{G1}} > 0$ and so the equilibrium thresholds decreases with the interest rate r_{G1}. For $r_{G1} > \widehat{r}_{G1}^g$, instead, $\frac{\partial f_G(x_B^g, x_G^g)}{\partial r_{G1}} < 0$ implying that an increase in the interest rate leads to a higher equilibrium thresholds x_G^g. Since x_B^g is an increasing function of x_G^g, the effect of r_{G1} on the equilibrium threshold x_B^g is analogous.

Now consider the effect of r_{B0} and r_{G0}. These effects on the thresholds x_G^g and x_B^g are more involved since both r_{B0} and r_{G0} directly affect both $f_B(x_B^g, x_G^g)$ and $f_G(x_B^g, x_G^g)$.

Consider first r_{B0}. An increase in r_{B0} reduces depositors' payoff at date 2, while increasing that at date 1, as can easily be seen from (5). As a consequence, the function $f_B(x_B^g, x_G^g)$ decreases with r_{B0} and so x_B (x_G^g) increases with it.

Regarding the function $f_G(x_B^g, x_G^g)$, r_{B0} has also a direct effect on $f_G(x_B^g, x_G^g)$ as the number of units invested by the government until date 2 depends on r_{B0} as follows:

$$I(i,t,r_{B0},r_{G0},n,\gamma) = 1+t-\frac{r_{G0}^2}{r_{G1}} - \frac{\left(\frac{x_B^g - x_G^g}{2\varepsilon} + 1 - \frac{r_{G0}}{r_{G1}} - \gamma\right)(r_{B0}-(1-t))}{1-\gamma}. \quad (35)$$

Since $\frac{\partial I(i,t,r_{B0},r_{G0},n,\gamma)}{\partial r_{B0}} < 0$, the function $f_G(x_B^g, x_G^g)$ decreases with r_{B0} and so $x_G(x_B^g)$ increases. As a consequence, since, for given x_G^g and x_B^g, $x_B^g = x_B(x_G^g)$ and $x_G^g = x_G(x_B^g)$ increase respectively with r_{B0} and given that $\frac{\partial x_B(x_G^g)}{\partial x_G^g} > 0$ and $\frac{\partial x_G(x_B^g)}{\partial x_B^g} > 0$, it follows that the equilibrium thresholds $\{x_G^g, x_B^g\}$ increase with r_{B0}.

The effect of r_{G0} on $\{x_G^g, x_B^g\}$ is similar. An increases in r_{G0} reduces depositors' repayment at date 2, as emerges in (4), due to its negative effect on the actual guarantees paid by the government, thus leading to an increase of $x_B(x_G^g)$. From (35), an increase in r_{G0} reduces $I(i,t,r_{B0},r_{G0},n,\gamma)$, thus implying an increase also in $x_G(x_B^g)$. As in the previous case, since for given x_G^g and x_B^g, $x_B^g = x_B(x_G^g)$ and $x_G^g = x_G(x_B^g)$ increase respectively with r_{G0} and given that $\frac{\partial x_B(x_G^g)}{\partial x_G^g} > 0$ and $\frac{\partial x_G(x_B^g)}{\partial x_B^g} > 0$, it follows that the equilibrium thresholds $\{x_G^g, x_B^g\}$ increase with r_{G0}. This completes the proof of the Lemma. □

Proof of Proposition 4: The proof consists of two parts. First, I show that the equilibrium interest rate r_{G1}^g lies in the range $[1, \widehat{r}_{G1}^g]$ and then I show that investors are exactly indifferent between rolling over the bonds and not rolling over when offered r_{G1}^g. Consider the limit case when $\varepsilon \to 0$, $\pi \to 0$ and $\frac{\varepsilon}{\pi} \to 0$, and $x_B^g \to Y_B^g$ and $x_G^g \to Y_G^g$.

From Lemma 1, it holds that $\frac{\partial Y_G^g}{\partial r_{G1}} < 0$ if $r_{G1} < \widehat{r}_{G1}^g$ and $\frac{\partial Y_G^g}{\partial r_{G1}} > 0$ if $r_{G1} > \widehat{r}_{G1}^g$. Choosing $r_{G1}^g > \widehat{r}_{G1}^g$ is never optimal since, on the one hand, it increases the probability of a sovereign default and, in turn, the probability of a banking crisis. On the other hand, it also implies a lower supply of public good when the government is solvent, which is not optimal since the government maximizes the utility of domestic consumers. Moreover, the interest rate cannot be smaller than 1 since $r_{G1}^g \geq r_{G0}^g \geq 1$. Thus,

$$1 \leq r_{G1}^g \leq \widehat{r}_{G1}^g.$$

Regarding the second part of the proof, it is useful to consider different ranges of Y separately. Consider first the case in which the government receives a signal that $Y \geq \overline{Y}_G$. At the limit, when $\pi \to 0$, $\varepsilon \to 0$ and $\frac{\varepsilon}{\pi} \to 0$, each investor is going to receive a signal that $Y \geq \overline{Y}_G$ and they always roll over since the government is solvent no matter what the other investors do. In this range, since the probability of a sovereign default is zero, investors do not have to be compensated for any risk when they decide to roll over the bonds and thus, it is enough for the government to offer $r_{G1}^g = r_{G0}^g = 1$ to induce investors to roll over.

Consider now the range $Y(\widehat{r}_{G1}^g) < Y < \overline{Y}_G$. I prove that, at the limit, when $\pi \to 0$, $\varepsilon \to 0$ and $\frac{\varepsilon}{\pi} \to 0$, the optimal interest rate $r_{G1}^g = r_{G1}^*$, where r_{G1}^* is the solution to

$$Y = Y_G^g(r_{G1}^g).$$

Assume by contradiction that the government chooses $r_{G1}^H > r_{G1}^*$ after observing the signal that the growth rate of the economy is $Y < \overline{Y}_G$. Being $\frac{\partial Y_G^g}{\partial r_{G1}} < 0$ for $r_{G1} < \widehat{r}_{G1}^g$, it follows that $Y_G^g(r_{G1}^H) < Y$, which implies that all investors will roll over the bonds. Setting an interest rate $r_{G1}^H > r_{G1}^*$ is not optimal since the government could reduce it and still induce investors to roll over, while providing a larger amount of the public good and thus higher utility to domestic consumers.

Assume again by contradiction that the government chooses $r_{G1}^L < r_{G1}^*$ after observing the signal that the growth rate of the economy is $Y < \overline{Y}_G$. As it is $\frac{\partial Y_G^g}{\partial r_{G1}} < 0$ for $r_{G1} < \widehat{r}_{G1}^g$, it follows that $Y_G^g(r_{G1}^L) > Y$, which implies that no investors will roll over the bonds. Setting an interest rate $r_{G1}^L < r_{G1}^g$ is not optimal since the government can do better by offering $r_{G1} = r^*_{G1}$, thus inducing investors to roll over and staying solvent.

Finally, consider the range $Y < Y(\widehat{r}^g_{G1})$. The proof that there is no interest rate that the investors are going to accept follows directly from Lemma 1 and from the fact that $\frac{\partial Y^g_G}{\partial r_{G1}} > 0$ if $r_{G1} > \widehat{r}^g_{G1}$. When the government observes $Y < Y(\widehat{r}^g_{G1})$, choosing $r^H_{G1} > \widehat{r}^g_{G1}$ implies that $Y(r^H_{G1}) > Y$ and no investors are willing to roll over and the government defaults. Choosing $r^L_{G1} < \widehat{r}^g_{G1}$ implies that $Y(r^L_{G1}) > Y(\widehat{r}^g_{G1}) > Y$ and again no investors are willing to roll over and the government defaults.

Notice that, in the limit case when $\varepsilon \to 0$, from Lemma 1, I can compute

$$\widehat{r}^g_{G1} = r_{G0}\sqrt{\left[1 - \frac{(r_{B0} - (1-t))}{(1-\gamma)\, r_{G0}}\right]}\sqrt{\frac{\partial \widetilde{G}(Y, I(i,t,r_{G0},\Gamma))}{\partial I(i,t,r_{G0},\Gamma)}}.$$

It follows that the condition $r^g_{G1} < r_{G0}\frac{\partial \widetilde{G}(Y,I(i,t,r_{G0},\Gamma))}{\partial I(i,t,r_{G0},\Gamma)}$ guaranteeing the existence of global strategic complementarity within investors' actions holds since $r^g_{G1} < \widehat{r}^g_{G1}$ and $r_{G0}\sqrt{\left[1 - \frac{(r_{B0}-(1-t))}{(1-\gamma)r_{G0}}\right]}\sqrt{\frac{\partial \widetilde{G}(Y,I(i,t,r_{G0},\Gamma))}{\partial I(i,t,r_{G0},\Gamma)}} = \widehat{r}^g_{G1} < r_{G0}\frac{\partial \widetilde{G}(Y,I(i,t,r_{G0},\Gamma))}{\partial I(i,t,r_{G0},\Gamma)}$. The proposition follows.□

Proof of Proposition 5: To prove the result in the proposition, I show that for any $Y > Y^g_G(\widehat{r}^g_{G1})$, the equilibrium threshold for the depositors Y^g_B is never larger than that of the investors Y^g_G unless $Y^g_B = \underline{Y}^g_B$. The proof is in the spirit of Goldstein (2005) and it is done for $\varepsilon \to 0$, $\pi \to 0$ and $\frac{\varepsilon}{\pi} \to 0$, that is both signals are very precise and all agents receive pretty much the same signal.

I start characterizing the equilibrium thresholds in the case where depositors and investors have extreme beliefs about the actions of agents in the other group. Denote as $Y^g_B\,(i=1)$ and $Y^g_B\,(i=0)$ depositors' equilibrium threshold in the case they expect that no investors roll over the investment in sovereign bonds and all investors roll over, respectively. Similarly, denote as $Y^g_G\,(n=1)$ and $Y^g_G\,(n=\lambda)$ investors' equilibrium thresholds in the case where they expect all late depositors to run and to wait until date 2, respectively. These thresholds under extreme beliefs can be computed following the same steps illustrated in Proposition 1 but fixing the proportion of investors and depositors running in the various cases.[16] Since in the case of the thresholds characterized in Proposition 1, $0 \le n \le 1$ and $0 \le i \le 1$ and depositors' and investors' actions are strategic complements, it follows that equilibrium thresholds Y^g_B and Y^g_G lie in the range $[Y^g_B(i=1), Y^g_B(i=0)]$ and $[Y^g_G(n=\lambda), Y^g_G(n=1)]$, respectively.

Having defined the upper and lower bound of the interval in which the equilibrium thresholds characterized in Proposition 1 lie, I now prove that $Y^g_B \le Y^g_G$.

Assume by contradiction that $Y^g_G < Y^g_B$. Since Y^g_B and Y^g_G lie in the range $[Y^g_B(i=0), Y^g_B(i=1)]$ and $[Y^g_G(n=\lambda), Y^g_G(n=1)]$, respectively, the inequality $Y^g_G < Y^g_B$ implies that $Y^g_B(i=1) > Y^g_G(n=\lambda)$.

Given $Y^g_B(i=1) > Y^g_G(n=\lambda)$, two cases can be distinguished:

1. Case I: $Y^g_G(n=1) > Y^g_B(i=0)$;
2. Case II: $Y^g_G(n=1) < Y^g_B(i=0)$.

Consider first Case I where $Y^g_G(n=1) > Y^g_B(i=0)$.

[16] Calculations from the author can be provided.

A depositor j receiving the signal $x_{Bj} = Y_B^g$ is indifferent between running and not running and believes that all investors roll over the bonds (i.e., $i = 0$) since $Y_G^g < Y_B^g$. This implies that depositors' equilibrium threshold Y_B^g would converge to $Y_B^g(i = 0)$. Symmetrically, an investor j receiving the signal $x_{Gj}^g = Y_G^g$ is exactly indifferent between rolling over and not rolling over their investment in sovereign bonds and believes that all depositors have withdrawn (i.e., $n = 1$) since $Y_G^g < Y_B^g$. This implies that investors' equilibrium threshold Y_G^g converges to $Y_G^g(n = 1)$. As $Y_G^g < Y_B^g$, it must then hold that $Y_G^g(n = 1) < Y_B^g(i = 0)$, which is a contradiction with the initial assumption $Y_G^g(n = 1) > Y_B^g(i = 0)$. The only equilibrium possible in this case is $Y_G^g = Y_B^g$.

Consider now Case II where $Y_G^g(n = 1) < Y_B^g(i = 0)$.

A depositor j receiving the signal $x_{Bj} = Y_B^g$ is indifferent between running and not running and believes that all investors roll over the bonds (i.e., $i = 0$) since $Y_G^g < Y_B^g$. This implies that depositors' equilibrium threshold Y_B^g would converge to $Y_B^g(i = 0)$. Symmetrically, an investor j receiving the signal $x_G^g = Y_G^g$ is exactly indifferent between rolling over and not rolling over his investment in sovereign bonds and believes that all depositors have withdrawn (i.e., $n = 1$) since $Y_G^g < Y_B^g$. This implies that investors' equilibrium threshold Y_G^g converges to $Y_G^g(n = 1)$. Thus, the equilibrium can feature $Y_G^g < Y_B^g$, but only if $Y_B^g = Y_B^g(i = 0)$. The threshold $Y_B^g(i = 0)$ is identical to the threshold Y_B^f characterized in Proposition 2, as given by the solution to equation (30), because when all investors roll over the investment in sovereign bonds there are enough resources in the economy to pay the guarantees. In the case $\gamma = \lambda$, the expression in (30) simplifies to

$$u\left(R\left(Y\right)\frac{1-t-\lambda r_{B0}}{1-\lambda}\right) - u\left(r_{B0}\right) = 0,$$

which is the same as condition (6) determining $\underline{Y}_B^g$ and the proposition follows. □

Proof of Proposition 6: The proof follows closely that of Proposition 1. When all investors and depositors behave accordingly to a threshold strategy− x_G^b and x_B^b, respectively− the proportion of investors withdrawing at date 1 $i(Y, x_G^b)$ and that of depositors running $n\left(Y, x_B^b\right)$ are still as given by (13) and (14).

Investors' rollover decision is computed as in the proof of Proposition 1. The only differences are in the government's solvency constraint because

$$I(.) = 1 + b - ir_{G0} - \frac{\left(n-\gamma\right)\left(r_{B0} - p_b b - (1-b)\right)}{1-\gamma},$$

and the amount $r_G^L b$ must be subtracted from the resources available at date 2. Thus, following the same steps as in the proof of Proposition 1, the condition in the proposition is obtained and it exhibits the same properties as the corresponding expression in the baseline model, that is $0 < \frac{dx_G^b\left(x_B^b\right)}{dx_B^b} < 1$.

Depositors' withdrawal decision is also computed as in the proof of Proposition 1. A depositor's utility differential between withdrawing at date 2 and running at date 1 is still given by (19), but the function

$v\left(Y, n\left(Y\right), i\left(Y\right)\right)$ is now given by

$$\frac{1}{1-\lambda}\left\{\int_{\lambda}^{\gamma}\left[u\left(R(Y)\frac{1-t-nr_{B0}}{1-n}\right)-u\left(r_{B0}\right)\right]dn+\right. \tag{36}$$

$$\left.+\int_{\gamma}^{\widetilde{n}(i)}\left[u\left(R(Y)\frac{1-t-\gamma r_{B0}}{1-\gamma}\right)-u\left(r_{B0}\right)\right]dn+\int_{\widetilde{n}(i)}^{\overline{n}(i)}\left[u\left(R(Y)\frac{2+p_b b-ir_{G0}-nr_{B0}}{1-n}\right)-u\left(r_{B0}\right)\right]dn\right\}+$$

$$-\pi^{*}\left[u\left(R(Y)\frac{1-b+\left[nr_{B0}-p_b b\right]^{+}+\left[b-\frac{nr_{B0}}{p_b}\right]^{+}r_G^L}{1-n}\right)-u\left(R(Y)\frac{1-b-nr_{B0}+p_b b}{1-n}\right)-u\left(r_{B0}\right)\right],$$

with $\widetilde{n}(i)$ being the solution to

$$1+b-ir_{G0}=\frac{\left(n-\gamma\right)\left(r_{B0}-p_b b-\left(1-b\right)\right)}{1-\gamma}, \tag{37}$$

and $\overline{n}(i)$ that to

$$1-b+p_b b+1+b-ir_{G0}=2+p_b b-ir_{G0}=nr_{B0}, \tag{38}$$

and π^{*}being the probability of a sovereign default, that is $Pr\left(Y<Y_G^b\right)=\frac{Y_G^b-x_B+\varepsilon}{2\varepsilon}$.

There are two important differences between (20) and (36). First, the amount that a depositor receives as part of the guarantee scheme is different, both when the government can pay the promised guarantees in full and when it cannot. Second, depositors' repayment at date 2 depends on whether the sovereign is solvent or not$-$ which in turn is determined by investors' rollover decisions$-$, as it determines whether the interest rate r_G^L on banks' sovereign bonds held until maturity is obtained or not. This is captured by the last term in (36).

Despite these differences, all properties of depositors' expected utility differential, illustrated in Lemma 2, still hold. In particular, once accounting for the effect of Y on i, the $v\left(Y, n\left(Y\right), i\left(Y\right)\right)$ function is still non-decreasing in Y, as the impact of i on the function is not affected by the presence of bonds and thus, it is the same as in the baseline model. As a result, following the same steps as in the proof of Proposition 1, it can be shown that depositors behave according to the threshold strategy x_B^b when investors behave according to the threshold strategy x_G^b.

The equilibrium threshold $x_B^b\left(x_G^b\right)$ is the solution to the condition in the proposition, which, as in the baseline model,can be obtained by a simple change of variable. From $n(Y, x_B^b)=\lambda+(1-\lambda)\frac{x_B^b-Y+\varepsilon}{2\varepsilon}$, I can rewrite $Y=x_B^b+\varepsilon\left(1-2\frac{n-\lambda}{1-\lambda}\right)$. Thus, the expression for $i\left(Y, x_G^b\right)$ in becomes $i=\frac{x_G^b-x_B^b}{2\varepsilon}+\frac{n-\lambda}{1\ \lambda}$ and the expression is as in the proposition. Substituting the expression $i=\frac{x_G^g-x_B^g}{2\varepsilon}+\frac{n-\lambda}{1-\lambda}$ into (37) and (38), I obtain $\widetilde{n}(x_B^b, x_G^b)=\frac{\left[(1+b)(1-\lambda)(1-\gamma)+\gamma(1-\lambda)(r_{B0}-p_b b-(1-b))+\lambda(1-\gamma)r_{G0}-(1-\lambda)(1-\gamma)\frac{x_G^g-x_B^g}{2\varepsilon}r_{G0}\right]}{(1-\lambda)(r_{B0}-p_b b-(1-b))+r_{G0}(1-\gamma)}$ and $\overline{n}(x_B^b, x_G^b)=\frac{(2+p_b b)(1-\lambda)-(1-\lambda)\frac{x_G^g-x_B^g}{2\varepsilon}r_{G0}+\lambda r_{G0}}{r_{B0(1-\lambda)}+r_{G0}}$.

As in the case of investors, $x_B^b\left(x_G^b\right)$ exhibits the same properties as in the baseline model, that is $0<\frac{dx_B^b\left(x_G^b\right)}{dx_G^b}<1$. The proposition follows. □

Proof of Proposition 7: Denote as $f_B(x_B^g, x_G^g, \overline{g}, t)$ and $f_G(x_B^g, x_G^g, \overline{g}, t)$, conditions (7) and (8), respectively. The effect of $\overline{g}$ and t on the equilibrium thresholds $\{x_B^g, x_G^g\}$ can be computed using the implicit function theorem. Consider first the effect of $\overline{g}$.

$$\frac{dx_B^g}{d\overline{g}} = -\frac{\begin{vmatrix} \frac{\partial f_B(x_B^g,x_G^g,\overline{g},t)}{\partial \overline{g}} & \frac{\partial f_B(x_B^g,x_G^g,\overline{g},t)}{\partial x_G^g} \\ \frac{\partial f_G(x_B^g,x_G^g,\overline{g},t)}{\partial \overline{g}} & \frac{\partial f_G(x_B^g,x_G^g,\overline{g},t)}{\partial x_G^g} \end{vmatrix}}{\begin{vmatrix} \frac{\partial f_B(x_B^g,x_G^g,\overline{g},t)}{\partial x_B^g} & \frac{\partial f_B(x_B^g,x_G^g,\overline{g},t)}{\partial x_G^g} \\ \frac{\partial f_G(x_B^g,x_G^g,\overline{g},t)}{\partial x_B^g} & \frac{\partial f_G(x_B^g,x_G^g,\overline{g},t)}{\partial x_G^g} \end{vmatrix}}, \tag{39}$$

and

$$\frac{dx_G^g}{d\overline{g}} = -\frac{\begin{vmatrix} \frac{\partial f_B(x_B^g,x_G^g,\overline{g},t)}{\partial x_B^g} & \frac{\partial f_B(x_B^g,x_G^g,\overline{g},t)}{\partial \overline{g}} \\ \frac{\partial f_G(x_B^g,x_G^g,\overline{g},t)}{\partial x_B^g} & \frac{\partial f_G(x_B^g,x_G^g,\overline{g},t)}{\partial \overline{g}} \end{vmatrix}}{\begin{vmatrix} \frac{\partial f_B(x_B^g,x_G^g,\overline{g},t)}{\partial x_B^g} & \frac{\partial f_B(x_B^g,x_G^g,\overline{g},t)}{\partial x_G^g} \\ \frac{\partial f_G(x_B^g,x_G^g,\overline{g},t)}{\partial x_B^g} & \frac{\partial f_G(x_B^g,x_G^g,\overline{g},t)}{\partial x_G^g} \end{vmatrix}}. \tag{40}$$

The denominators in (39) and (40) are positive, as established in the proof of Proposition 3.

Thus, the sign of $\frac{dx_B^g}{d\overline{g}}$ and $\frac{dx_G^g}{d\overline{g}}$ are given by the opposite sign of their respective numerators. I have the following

$$sign\frac{dx_B^g}{d\overline{g}} = -sign\left[\frac{\partial f_B(x_B^g,x_G^g,\overline{g},t)}{\partial \overline{g}}\frac{\partial f_G(x_B^g,x_G^g,\overline{g},t)}{\partial x_G^g} - \frac{\partial f_B(x_B^g,x_G^g,\overline{g},t)}{\partial x_G^g}\frac{\partial f_G(x_B^g,x_G^g,\overline{g},t)}{\partial \overline{g}}\right],$$

and

$$sign\frac{dx_G^g}{d\overline{g}} = -sign\left[\frac{\partial f_B(x_B^g,x_G^g,\overline{g},t)}{\partial x_B^g}\frac{\partial f_G(x_B^g,x_G^g,\overline{g},t)}{\partial \overline{g}} - \frac{\partial f_B(x_B^g,x_G^g,\overline{g},t)}{\partial \overline{g}}\frac{\partial f_G(x_B^g,x_G^g,\overline{g},t)}{\partial x_B^g}\right].$$

Since $\frac{\partial f_B(x_B^g,x_G^g,\overline{g},t)}{\partial \overline{g}} = 0$, the expressions above can be rewritten as follows:

$$sign\frac{dx_B^g}{d\overline{g}} = -sign\left[-\frac{\partial f_B(x_B^g,x_G^g,\overline{g},t)}{\partial x_G^g}\frac{\partial f_G(x_B^g,x_G^g,\overline{g},t)}{\partial \overline{g}}\right],$$

and

$$sign\frac{dx_G^g}{d\overline{g}} = -sign\left[\frac{\partial f_B(x_B^g,x_G^g,\overline{g},t)}{\partial x_B^g}\frac{\partial f_G(x_B^g,x_G^g,\overline{g},t)}{\partial \overline{g}}\right].$$

It follows that $\frac{dx_B^g}{d\overline{g}} > 0$ and $\frac{dx_G^g}{d\overline{g}} > 0$ since $\frac{\partial f_G(x_B^g,x_G^g,\overline{g},t)}{\partial \overline{g}} < 0$ and $\frac{\partial f_B(x_B^g,x_G^g,\overline{g},t)}{\partial x_B^g} > 0$ and $\frac{\partial f_B(x_B^g,x_G^g,\overline{g},t)}{\partial x_G^g} < 0$.

Consider now a change in the tax burden t. Again using the implicit function theorem, it is the case that

$$\frac{dx_B^g}{dt} = -\frac{\begin{vmatrix} \frac{\partial f_B(x_B^g,x_G^g,\overline{g},t)}{\partial t} & \frac{\partial f_B(x_B^g,x_G^g,\overline{g},t)}{\partial x_G^g} \\ \frac{\partial f_G(x_B^g,x_G^g,\overline{g},t)}{\partial t} & \frac{\partial f_G(x_B^g,x_G^g,\overline{g},t)}{\partial x_G^g} \end{vmatrix}}{\begin{vmatrix} \frac{\partial f_B(x_B^g,x_G^g,\overline{g},t)}{\partial x_B^g} & \frac{\partial f_B(x_B^g,x_G^g,\overline{g},t)}{\partial x_G^g} \\ \frac{\partial f_G(x_B^g,x_G^g,\overline{g},t)}{\partial x_B^g} & \frac{\partial f_G(x_B^g,x_G^g,\overline{g},t)}{\partial x_G^g} \end{vmatrix}},$$

and

$$\frac{dx_G^g}{dt} = -\frac{\begin{vmatrix} \frac{\partial f_B(x_B^g,x_G^g,\overline{g},t)}{\partial x_B^g} & \frac{\partial f_B(x_B^g,x_G^g,\overline{g},t)}{\partial t} \\ \frac{\partial f_G(x_B^g,x_G^g,\overline{g},t)}{\partial x_B^g} & \frac{\partial f_G(x_B^g,x_G^g,\overline{g},t)}{\partial t} \end{vmatrix}}{\begin{vmatrix} \frac{\partial f_B(x_B^g,x_G^g,\overline{g},t)}{\partial x_B^g} & \frac{\partial f_B(x_B^g,x_G^g,\overline{g},t)}{\partial x_G^g} \\ \frac{\partial f_G(x_B^g,x_G^g,\overline{g},t)}{\partial x_B^g} & \frac{\partial f_G(x_B^g,x_G^g,\overline{g},t)}{\partial x_G^g} \end{vmatrix}},$$

and

$$sign\frac{dx_B^g}{dt} = -sign\left[\frac{\partial f_B(x_B^g, x_G^g, \overline{g}, t)}{\partial t}\frac{\partial f_G(x_B^g, x_G^g, \overline{g}, t)}{\partial x_G^g} - \frac{\partial f_B(x_B^g, x_G^g, \overline{g}, t)}{\partial x_G^g}\frac{\partial f_G(x_B^g, x_G^g, \overline{g}, t)}{\partial t}\right],$$

and

$$sign\frac{dx_G^g}{dt} = -sign\left[\frac{\partial f_B(x_B^g, x_G^g, \overline{g}, t)}{\partial x_B^g}\frac{\partial f_G(x_B^g, x_G^g, \overline{g}, t)}{\partial t} - \frac{\partial f_B(x_B^g, x_G^g, \overline{g}, t)}{\partial t}\frac{\partial f_G(x_B^g, x_G^g, \overline{g}, t)}{\partial x_B^g}\right].$$

Consider first $\frac{dx_G^g}{dt}$: It is positive if

$$\frac{\partial f_B(x_B^g, x_G^g, \overline{g}, t)}{\partial x_B^g}\frac{\partial f_G(x_B^g, x_G^g, \overline{g}, t)}{\partial t} - \frac{\partial f_B(x_B^g, x_G^g, \overline{g}, t)}{\partial t}\frac{\partial f_G(x_B^g, x_G^g, \overline{g}, t)}{\partial x_B^g} < 0.$$

Divide the expression above by $\frac{\partial f_G(x_B^g, x_G^g, \overline{g}, t)}{\partial x_G^g}$ and $\frac{\partial f_B(x_B^g, x_G^g, \overline{g}, t)}{\partial x_B^g}$, which are both positive. The inequality above can be rewritten as

$$-\frac{\partial x_G^g}{\partial t} - \frac{dx_G^g}{dx_B^g}\frac{\partial x_B^g}{\partial t} < 0,$$

where $\frac{\partial x_B^g}{\partial t}$ and $\frac{\partial x_G^g}{\partial t}$ represents the direct effect of taxes on x_B^g and x_G^g, respectively. Since $\frac{\partial x_B^g}{\partial t} > 0, \frac{\partial x_G^g}{\partial t} < 0$ and $\frac{dx_B^g}{dx_G^g} < 1$, a necessary condition for $\frac{dx_G^g}{dt} > 0$ is $\left|\frac{\partial x_B^g}{\partial t}\right| > \left|\frac{\partial x_G^g}{\partial t}\right|$.

Consider now $\frac{dx_B^g}{dt}$. It is positive if

$$\frac{\partial f_B(x_B^g, x_G^g, \overline{g}, t)}{\partial t}\frac{\partial f_G(x_B^g, x_G^g, \overline{g}, t)}{\partial x_G^g} - \frac{\partial f_B(x_B^g, x_G^g, \overline{g}, t)}{\partial x_G^g}\frac{\partial f_G(x_B^g, x_G^g, \overline{g}, t)}{\partial t} < 0.$$

Divide the expression above by $\frac{\partial f_G(x_B^g, x_G^g, \overline{g}, t)}{\partial x_G^g}$ and $\frac{\partial f_B(x_B^g, x_G^g, \overline{g}, t)}{\partial x_B^g}$, which are both positive. The inequality above can be rewritten as

$$-\frac{\partial x_B^g}{\partial t} - \frac{dx_B^g}{dx_G^g}\frac{\partial x_G^g}{\partial t} < 0,$$

Since $\frac{dx_B^g}{dx_G^g} < 1$, it follows that $\frac{dx_B^g}{dt} > 0$ when $\left|\frac{\partial x_B^g}{\partial t}\right| > \left|\frac{\partial x_G^g}{\partial t}\right|$. The proposition follows. □

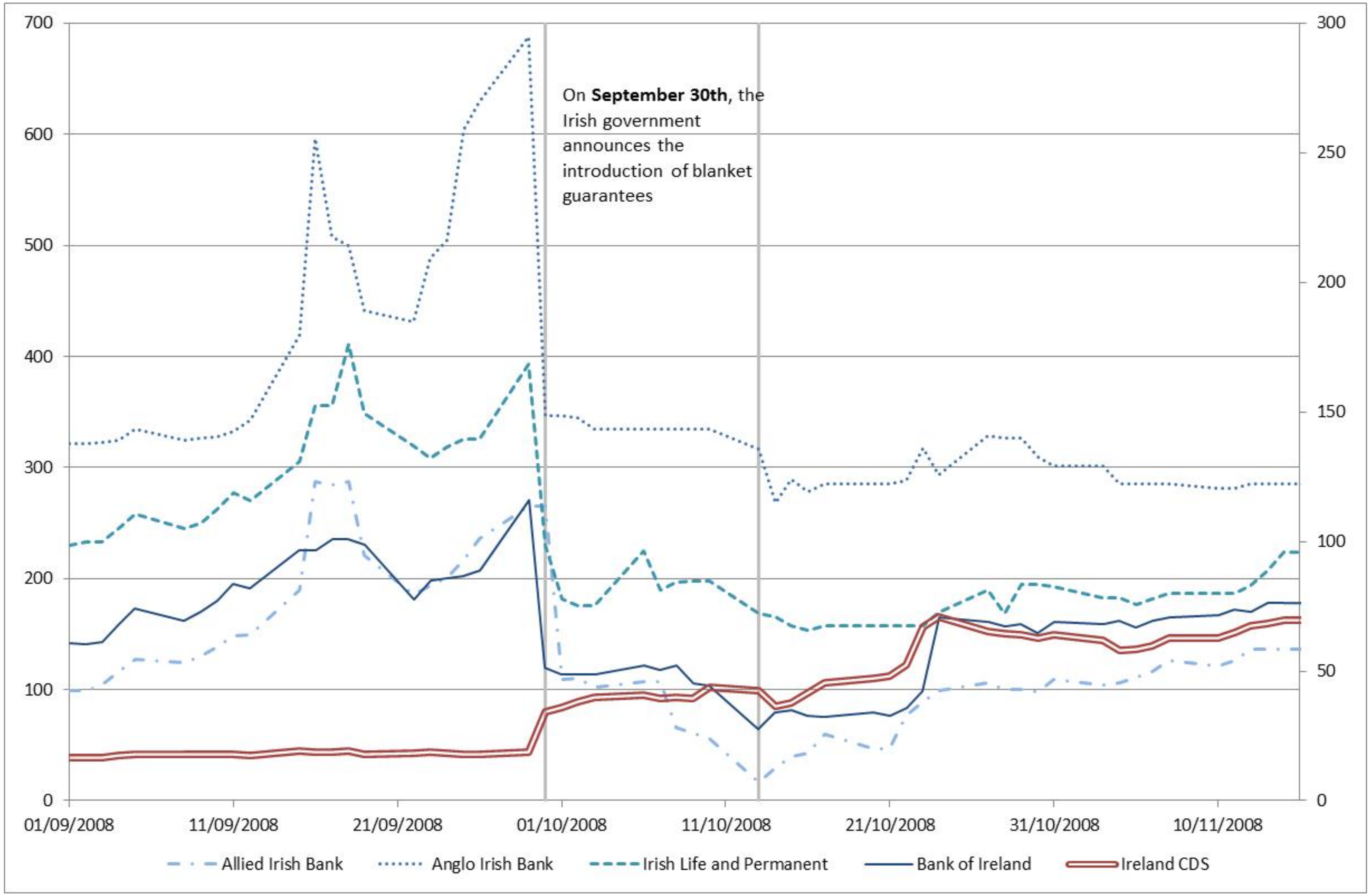

Figure 1: The Irish crisis.

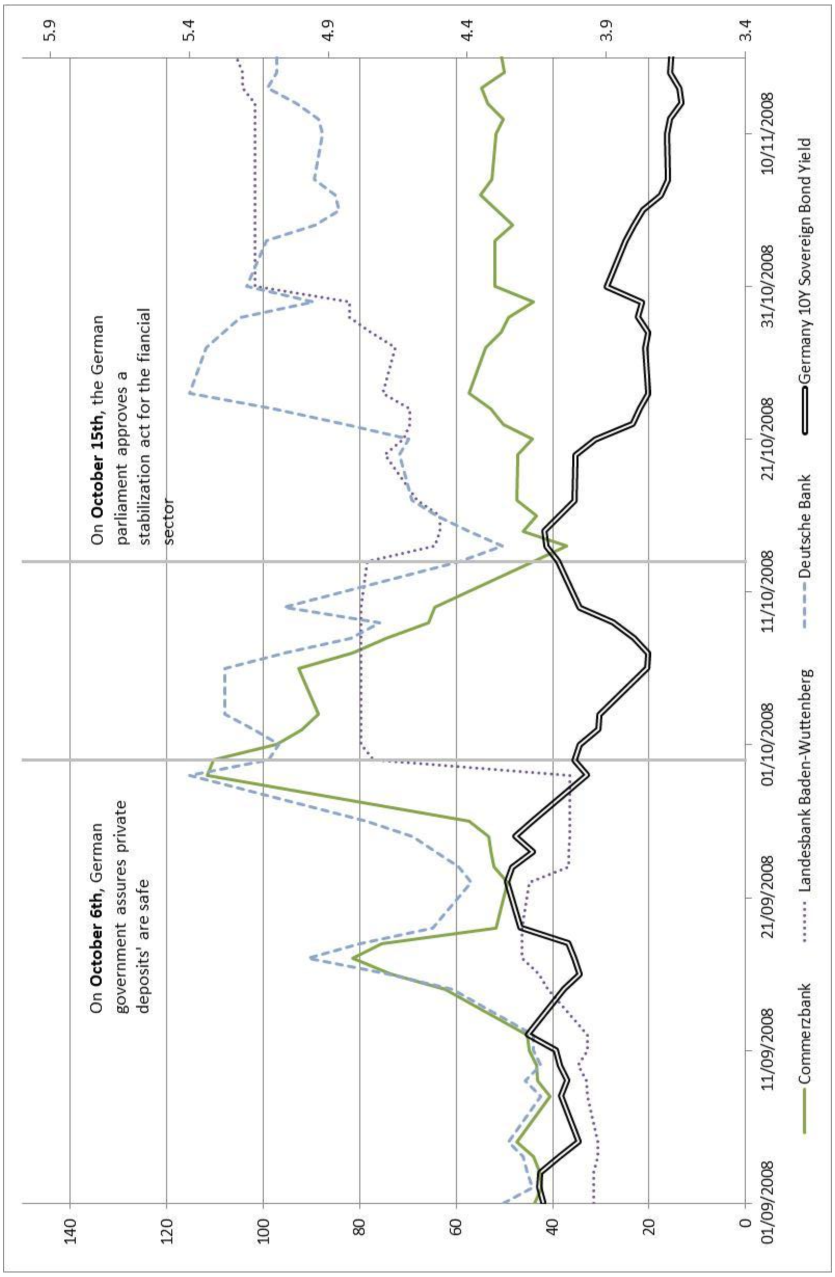

Figure 2: German banks CDS and sovereign bond yield.

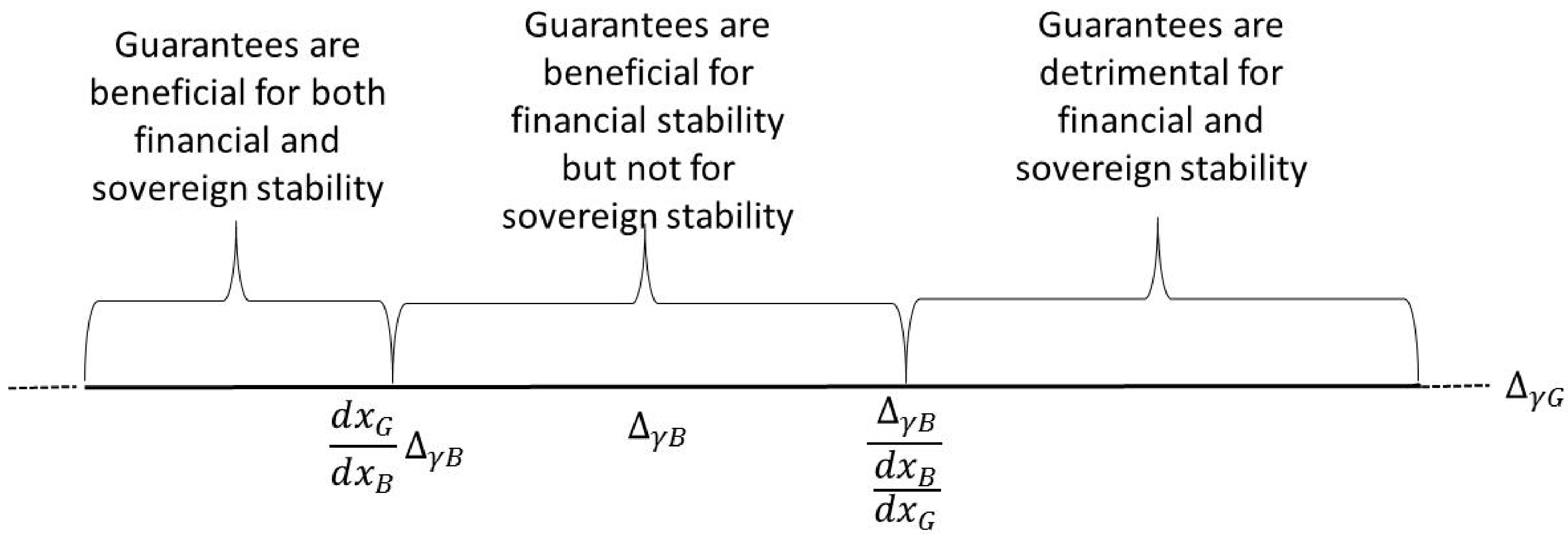

Figure 3: Guarantees and Crises. The figure shows how the overall effect of an increase of the size of the guarantees on the probability of a banking crisis and a sovereign default depends on the magnitude of the direct effect of the guarantees on the investors' threshold $\Delta_{\gamma G}$ and on how this compares with that on depositors' threshold $\Delta_{\gamma B}$. When $\Delta_{\gamma G} < \frac{dx_G}{dx_B}\Delta_{\gamma B}$, increasing the guarantees has a beneficial effect both on banks' and sovereign's stability. When $\frac{dx_G}{dx_B}\Delta_{\gamma G} < \Delta_{\gamma G} < \frac{\Delta_{\gamma B}}{\frac{dx_B}{dx_G}}$, increasing the size of the guarantees is only beneficial for financial stability. When $\frac{\Delta_{\gamma B}}{\frac{dx_B}{dx_G}} < \Delta_{\gamma G}$, it is detrimental for both financial and sovereign stability.

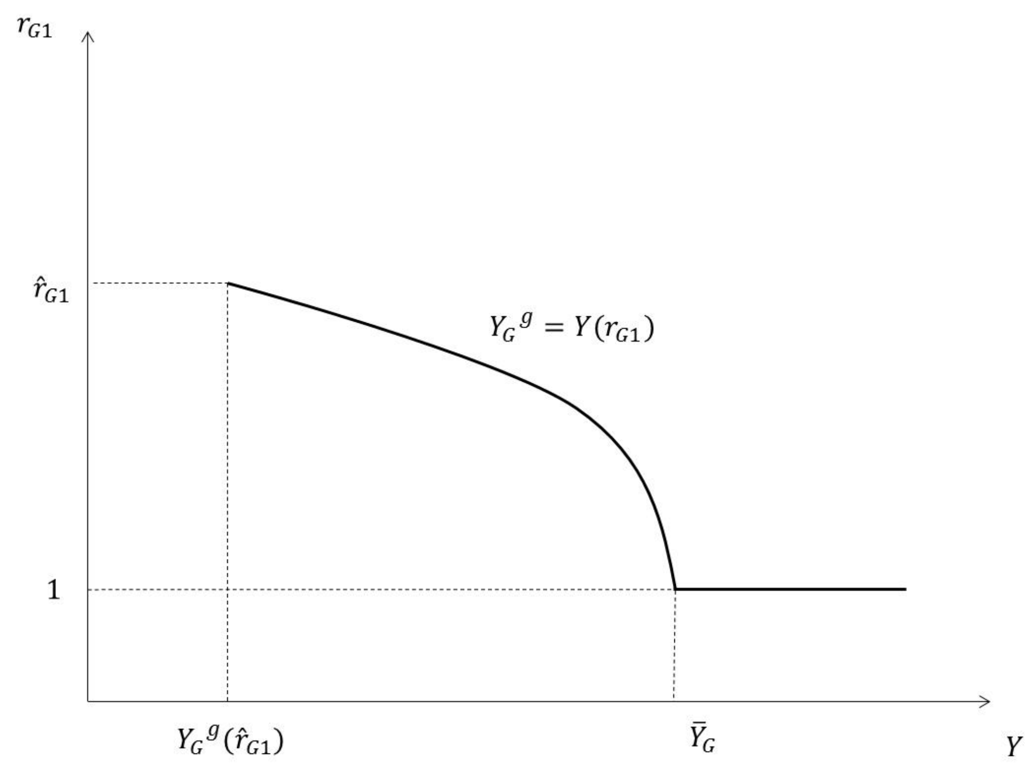

Figure 4: Sovereign bonds yield as a function of the growth rate of the economy Y. When the growth rate of the economy is very high ($Y > \bar{Y}_G$), the probability of a sovereign default is zero. In this range, the investors do not take any risk in rolling over the investment in sovereign bonds. As a consequence, the government offers them $r_{G1} = r_{G0} = 1$ and all investors roll over the investment in sovereign bonds. As the state of the economy worsens, the government has to offer a higher interest rate $r_{G1} > 1$ to induce investors to roll over. When the state of the economy Y falls below $Y_G^g(\hat{r}_{G1})$, there is no interest rate that will induce investors to roll over and the government defaults.

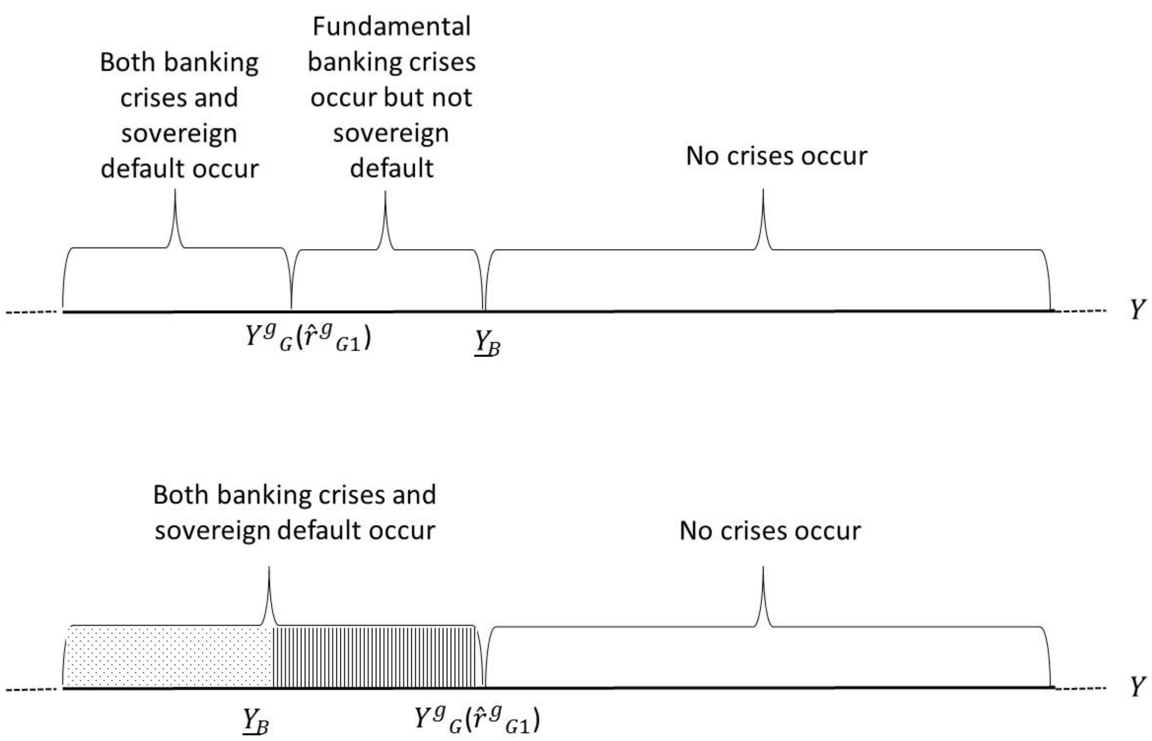

Figure 5: Crises as a function of the state of the economy Y. The figure illustrates the ranges of value of Y where a banking crisis and a sovereign default occur in the presence of full guarantees (i.e., when $\gamma = \lambda$) and the nature of banking crises. When $Y > Y^g{}_G(\hat{r}^g{}_{G1})$, the sovereign is always solvent. As a consequence, since guarantees are fully feasible and credible in this range, no panic-driven banking crises occur. When $Y < Y^g{}_G(\hat{r}^g{}_{G1})$, both a banking crisis and a sovereign default occur. Banking crises can be both panic-driven for $\underline{Y}_B < Y < Y^g{}_G(\hat{r}^g{}_{G1})$ and fundamental-driven for $Y < \underline{Y}_B$. In the upper panel, banking crises also occur when the government is solvent, in the range $Y^g{}_G(\hat{r}^g{}_{G1}) < Y < \underline{Y}_B$. In this case, however, they are only fundamental-driven and, thus, arise irrespective of the presence and credibility of guarantees.

	Government available resources	Government disbursement for the guarantees
Poorly liquid sovereign bonds ($p_b < 1$)	(-)	(+)
Highly liquid sovereign bonds ($p_b > 1$)	(-)	(-)

Figure 6: Direct effects of banks' holding of sovereign bonds b on investors' equilibrium threshold x_G^b. The figure disentangles the effects of an increase in banks' holdings of sovereign bonds on investors' equilibrium thresholds and, thus, on the probability of a sovereign default. The effect is twofold. First, an increase in banks' holdings of sovereign bonds improves government's available resources and, as a result, leads to a lower threshold irrespective of the price of sovereign bonds. Second, it affects the government's disbursement associated with the provision of guarantees. Such an effective is positive, whereby an increase in banks' bondholdings leads to a higher threshold, when $p_b < 1$ and negative, whereby it leads to a lower threshold when $p_b > 1$.

	Depositors' date 2 repayment	Start of the orderly liquidation procedure	Costs of a sovereign default for banks
Poorly liquid sovereign bonds ($p_b < 1$)	(-/+)	(+)	(+)
Highly liquid sovereign bonds ($p_b > 1$)	(-)	(+)	(+)

Figure 7: Direct effects of banks' holding of sovereign bonds b on depositors' equilibrium threshold x_B^b. The figure disentangles the effects of an increase in banks' holdings of sovereign bonds on depositors' equilibrium thresholds and, thus, on the probability of a banking crisis. The effect is threefold. First, an increase in banks' holdings of sovereign bonds affects depositors' repayment at date 2. This effect is positive, whereby an increase in banks' bondholdings leads to a higher threshold, when $p_b > 1$. When $p_b < 1$, instead, the effect on depositors' repayment can either be positive or negative depending on how low the price of sovereign bonds is. When the price is very low, the effect is positive and leads to a higher threshold; otherwise it is negative. Second, an increase in banks' holdings of sovereign bonds determines a delay in the start of the orderly liquidation procedure. This effect is always positive, irrespective of the price of sovereign bonds, thus leading to a higher equilibrium threshold. Third, it affects the costs of a sovereign default for banks. Such an effect is positive, whereby an increase in bondholdings lead to a higher threshold, irrespective of the price of sovereign bonds.

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