

## THE NEW EURO AREA YIELD CURVES

*Yield curves describe the relationship between the residual maturity of financial instruments and their associated interest rates. This article describes the various ways of presenting this relationship using spot rates, forward rates or par yields and the model used by the ECB for its daily estimates, calculated from euro area central government bonds. It also provides detailed information on the data used and their selection and on data checking procedures. As a result of the interplay between the chosen dataset, the applied quality checks and the model used for the yield curve estimation, the term structure is smooth and fits the market data very well. The ECB yield curves provide a consistent reference term structure to market participants on a daily basis.*

### I INTRODUCTION

The level of market interest rates typically depends, among many other factors, on the residual maturity of the underlying financial instrument. The relationship between interest rates and the residual maturity is referred to as the term structure of interest rates and is depicted by the yield curve. Yield curves – via the calculation of implied forward rates – contain information on market participants' expectations vis-à-vis future short-term interest rates. Since January 1999, the ECB has regularly reported in the Monthly Bulletin on developments in the implied forward overnight interest rate curve for the euro area. Since 10 July 2007, the ECB estimates and releases yield curves calculated from euro area central government bonds on a daily basis.<sup>1</sup> These yield curves, particularly the curve based on AAA-rated bonds, can be considered as risk-free and as a benchmark, since

these bonds have a negligible expected default rate. The ECB yield curves provide a consistent reference term structure to market participants on a daily basis. This article explains how yield curves can be represented and provides background information on the data and model selection for the daily computation of the ECB's euro area yield curves. Box 1 provides information on the additional usefulness of the yield curve for monetary policy purposes, while Box 2 compares the new government bond yield curves with the curves derived on the basis of interest rate swaps, which have been used by the ECB since January 1999.

<sup>1</sup> The daily releases, including charts and tables, are available at <http://www.ecb.europa.eu/stats/money/yc/html/index.en.html>.

#### Box 1

#### USEFULNESS OF THE YIELD CURVE FOR MONETARY POLICY PURPOSES

Bond prices, like other financial asset prices, aggregate market participants' views about the future and are therefore inherently forward-looking in nature. In this regard, the yield curve offers a particularly rich and useful set of information for monetary policy purposes. This box focuses on information about the expected path of future short-term rates and the outlook for economic activity and inflation.<sup>1</sup> For this purpose, this section also elaborates on possible decompositions of the yield curve into real and purely nominal components.

<sup>1</sup> A much broader review of the information content of interest rates can be found in the article entitled "The information content of interest rates and their derivatives for monetary policy" in the May 2000 issue of the Monthly Bulletin.

### **The yield curve and expectations on future developments in short-term interest rates**

The yield curve depicts interest rates with different remaining maturities. The relative level of short and long-term interest rates at a certain date (the slope of the yield curve) should depend on market participants' expectations on future short-term interest rates.<sup>2</sup> According to what is known as the pure expectations hypothesis of the yield curve, arbitrage operations establish a close relationship between the slope of the yield curve and interest rate expectations. Instead of buying a long-term bond, an investor could also consider rolling over investments in short-term bonds over a period of the same length as the remaining maturity of the long-term bond. Disregarding risk considerations, the total return on the investment in the long-term bond should be equal to the expected cumulative return on the revolving investment in short-term bonds. This also implies that the average expected future short-term interest rate over the investment horizon should equal the long-term interest rate. For example, an upward sloping yield curve, featuring higher long-term interest rates than short-term rates, would then imply an expected increase in short-term rates.

However, the pure expectations hypothesis is strongly rejected by the empirical facts. Instead, the expected average short-term interest rate over a certain horizon tends to deviate from a corresponding long-term interest rate because long-term rates also contain unobservable risk or term premia. For that reason, the yield curve is upward sloping on average. Hence, in order to extract market expectations about future short-term interest rates from the yield curve, one needs an estimate of these term premia. However, this task is complicated in particular by the fact that term premia seem to fluctuate over time in a way that is not yet sufficiently understood. Nevertheless, the expectations hypothesis still appears to be a reasonable starting point for gauging interest rate expectations from the yield curve. Notably for shorter horizons, term premia tend to be relatively low and stable.

In summary, the yield curve provides two key pieces of information on market sentiments: an indication of market expectations on interest rates and global investors' perception of risk. Both are essential for monetary policy decisions and financial stability analysis.

### **The slope of the yield curve as an indicator for the outlook on economic activity**

The slope of the yield curve has often appeared to be a useful indicator to predict future economic activity. A steepening of the curve often anticipated an acceleration of economic activity while a flattening, and in particular an inversion of the curve, often indicated an imminent slowdown.

The explanation for this stylised fact is that a large positive spread between long and short-term interest rates may indicate that the market anticipates an increase in short-term interest rates because of a more positive outlook for economic growth. If such expectations are not systematically wrong, the term spread tends to predict economic activity relatively well. At the same time, the above-mentioned term premia may sometimes blur the yield curve's usefulness as a leading business cycle indicator. In fact, recent analysis by ECB staff has shown that the strong decline in the slope of the euro area yield curve in the last few years has been mainly driven by a decline in the term premium and not by increasingly lower interest rate expectations. As a

<sup>2</sup> Forward rates based on the yield curve have been used by the ECB as short-term interest rate assumptions since June 2006. See Box 10 entitled "Technical assumptions" in the June 2006 issue of the Monthly Bulletin.

result, it is likely that the euro area yield curve flattened during these years without anticipating a decline in domestic economic activity.<sup>3</sup>

### Decomposing the yield curve into real and inflation components

In addition to growth expectations, the longer end of the yield curve may also mirror market participants' views about trend developments in inflation (the Fisher hypothesis). In order to disentangle real interest rates from inflation expectations, one needs either a measure of proper ex ante real interest rates or a measure of market participants' inflation expectations. In the euro area, the yields on inflation-linked government bonds provide information about ex ante real interest rates. Although only relatively few government bonds are still linked to the euro area HICP, an estimation of euro area real yield curves is nevertheless possible.<sup>4</sup>

Subtracting the yield curve for inflation-linked bonds from a comparable yield curve for conventional nominal government bonds provides an estimate of the term structure of so-called break-even inflation rates. Break-even inflation rates can only be interpreted as "pure" inflation expectations if one abstracts from the existence of an inflation risk premium. However, this assumption appears to be rejected by the empirical facts. Decomposing break-even inflation rates into pure inflation expectations and the inflation risk premium requires a model-based analysis of the interaction between nominal and real yield curves on the one hand and inflation dynamics and corresponding inflation expectations on the other. Using real and nominal yield curves as an input, a model-based decomposition of the euro area yield curve has suggested that movements in the real components and not movements in the nominal (inflationary) components have been the main driver of the changes in the euro area yield curve since the start of Monetary Union in 1999.<sup>5</sup>

Despite the fact that time-varying term premia complicate the interpretation of movements in the yield curve, the yield curve remains a crucial source of information for monetary policy. This is especially the case in the light of recent advancements in modelling and understanding movements in term premia.

3 See Box 3 entitled "The recent flattening of the euro area yield curve: what role was played by risk premia?" in the December 2006 issue of the Monthly Bulletin.

4 See Box 3 entitled "Estimation of constant-maturity index-linked bond yields and break-even inflation rates for the euro area" in the July 2006 issue of the Monthly Bulletin.

5 See Box 3 entitled "Long-term real and inflation risk premia in the euro area bond market" in the April 2007 issue of the Monthly Bulletin.

## 2 METHODS TO REPRESENT THE YIELD CURVE

A yield curve is a graphical representation of the so-called term structure of interest rates, i.e. the relationship between the residual maturities of a homogeneous set of financial instruments and their computed interest rates. This computation is made on the basis of market prices for the underlying financial instruments, for instance government bonds that are traded on stock exchanges. In normal circumstances,

yield curves are upward sloping; that is, longer-term securities give a higher rate of return than shorter-term securities, since the longer the lender has to wait for the amortisation of his loan, the higher the expected risk (or term) premium (see also Box 1).

The relationship between the redemption value and the current market price determines the (annual) rate of return – also called yield to maturity – of an investment in fixed income

instruments such as bonds. Calculating this interest rate is straightforward in the case of zero coupon bonds, which provide only one payment. However, in practice these bonds are not always available, especially for residual maturities exceeding 12 months. Indeed, most debt securities in the euro area pay regular interest and are thus coupon-bearing. When calculating the yield to maturity of coupon-bearing bonds, all payment flows (coupons and redemptions) are discounted to current values at the same rate – i.e. the yield to maturity. Unless a constant discount rate applies to all maturities, i.e. the term structure is flat, the (zero coupon) interest rates – also referred to as spot rates – and yields to maturity of coupon bonds will differ.

In order to derive the implicit average annual interest rate from the market price of a coupon-bearing bond, each future interest payment on this bond has to be discounted by the different current average interest rates related to the time at which the future payment occurs. This implies solving a set of equations with several unknowns. To facilitate the term structure estimation, it is useful to impose a functional form between interest rates and time to maturity. The term structure is then found via an iterative procedure. The ECB has chosen a functional form proposed by Nelson and Siegel and extended by Svensson.<sup>2</sup>

This model strikes a good balance between different criteria, including goodness of fit, smoothness and stability of the curve. Goodness of fit ensures that the derived yield curve indeed describes the observed data as well as possible. This can be measured by common error measures such as the mean absolute error (MAE). The Svensson model is generally also able to capture different movements in the underlying term structure (flexibility). This is particularly relevant for shorter maturities for which the yields react more to news or expectations of changes in policy interest rates. A crucial indicator for this criterion is the percentage of cases for which an estimated price for a bond lies within the bid-ask range for that bond. This is elaborated below.

The smoothness of a curve is important to ensure that expectations on future interest rates, as derived from the term structure, remain meaningful and do not become erratic. The yield curve should also be stable, even in the case of exceptionally mispriced instruments or missing data. Such rare events, which are made even less likely because of the data selection methods described below, should not have a noticeable impact on the curve as a whole. At the same time, changes in the pricing of bonds throughout the curve, for example as a result of news that changes expectations, should be appropriately reflected.

Chart 1 illustrates several points made above: while the population of bonds is rather large, some gaps remain, in particular for longer residual maturities where the number of issued debt securities is smaller. Comparing estimated yields for each maturity from the Svensson model with corresponding observations shows that the estimated values fit the observed yields rather well. This is attributable to the quality and the homogeneity of the dataset, which itself has almost the shape of a curve. It is worth mentioning that this fit is achieved despite the fact that the Svensson model is a parsimonious model, i.e. it is estimated with only a few parameters. This feature eases the communication of the yield curve, as it allows market participants to replicate the curve on the basis of a small set of parameters. The ECB releases these parameters every day, in addition to the set of interest rates that describes the term structure.

The estimated yield curve can be presented in various ways. The most basic is the presentation of spot (interest) rates explained above. From these spot rates, forward (interest) rates can be derived. These are interest rates for a future

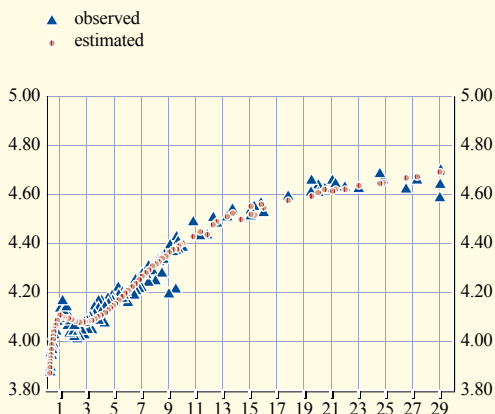
2 Svensson's functional form of the zero coupon rate  $z(TTM)$  is:

$$z(TTM) = \beta_0 + \beta_1 \left[ \frac{1 - e^{-\frac{-TTM}{\tau_1}}}{TTM/\tau_1} \right] + \beta_2 \left[ \frac{1 - e^{-\frac{-TTM}{\tau_1}}}{TTM/\tau_1} - e^{-\frac{-TTM}{\tau_1}} \right] + \beta_3 \left[ \frac{1 - e^{-\frac{-TTM}{\tau_2}}}{TTM/\tau_2} - e^{-\frac{-TTM}{\tau_2}} \right]$$

where TTM = term to maturity and  $\beta_1, \tau_1$  are the parameters to be estimated. See Svensson, 1994, "Estimating and Interpreting Forward Interest Rates: Sweden 1992-1994", NBER Working Paper No 4871.

**Chart 1 Observed yields versus estimated yields using the Svensson model**

(percentages per annum)

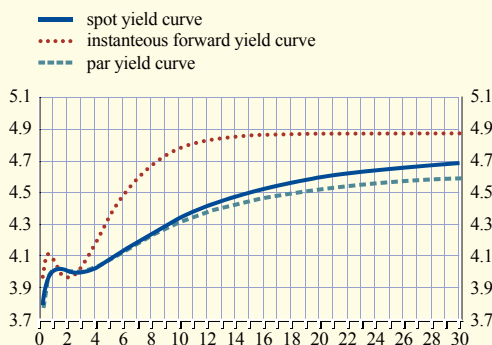


Sources: ECB calculations and EuroMTS; ratings provided by Fitch Ratings.

Note: Data for 14 December 2007. The horizontal axis refers to years to maturity. Due to the close fit of the estimated values to the observed values, the 160 observed points are not clearly visible.

**Chart 2 Estimated spot, instantaneous forward and par yield curves**

(percentages per annum)



Sources: ECB calculations and EuroMTS; ratings provided by Fitch Ratings.

Note: Data for 14 December 2007. The horizontal axis refers to years to maturity.

period and are implied in today's spot rates. For example, the forward interest rate for a one-year loan to be granted in two years' time is calculated from the difference between the three-year and the two-year spot rates.<sup>3</sup> When presenting forward rates derived using the Svensson model, the ECB displays the instantaneous forward curve, which reflects the interest rates applicable to the very near future (for example an overnight rate), for a bond issued at the chosen time. Since spot rates are calculated for maturities up to 30 years, the instantaneous forward rates are also available for a horizon of 30 years.

A third way to present the term structure of interest rates is to show the par yields. The spot rates are then transformed into the yields of coupon-bearing bonds that trade at par, i.e. their price is equivalent to their face values (redemption prices) and their yields are therefore equal to their coupon rates. This presentation is often favoured by market participants as they

typically trade coupon-bearing bonds. The values of the par yield curve are thus the closest to the yields of individual benchmark bonds that are quoted in the financial press. Chart 2 shows the different representations of the yield curve on a given day.

3 If the spot rates for two and three years are 4.0% and 4.5% respectively, the implied one-year forward rate, using a simple discrete formula, is 5.5% in two years. In this instance, it would make no difference to an investor if he were to place money at the three-year spot rate or invest initially at the two-year spot rate, followed by a deposit at the agreed one-year forward rate applicable in two years. In this simple example, the forward rate is given by  $\frac{R_2 T_2 - R_1 T_1}{T_2 - T_1}$ , where  $R_1$  is the spot rate at time  $T_1$ .

Here  $R_1 = 0.04$ ,  $R_2 = 0.045$ ,  $T_1 = 2$  years and  $T_2 = 3$  years. See, for example, J. Hull, *Introduction to Futures and Options Markets*, 1997, Prentice Hall.

Box 2

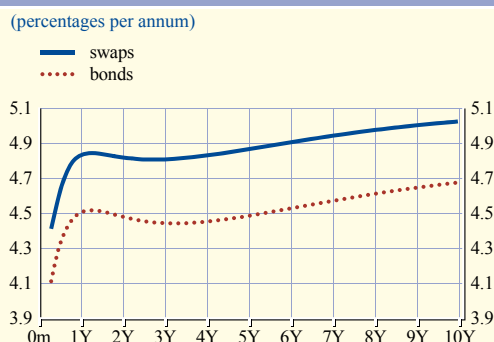
**COMPARING SWAP AND GOVERNMENT BOND YIELD CURVES**

Since January 1999 the ECB has regularly reported in the Monthly Bulletin on developments in the implied forward overnight interest rate curve for the euro area. This curve is estimated on the basis of interest rates observed on the fixed side of interest rate swap contracts for maturities of one to ten years and on money market rates for maturities of less than a year. Hence, such a forward curve should mirror market participants' expectations about the future path of short-term money market interest rates disregarding maturity and instrument-specific unobservable risk premia. The newly available government bond yield curve, as presented in this article, offers an alternative to compute an implied forward overnight interest rate curve for the euro area. Both curves provide complementary information and the choice between the two therefore depends on the analytical purpose at hand. Moreover, the swap curve is frequently used by the finance industry as the basis for their pricing of derivatives products, whereas for example long-term investors tend to prefer the government bond yield curve for their strategic asset allocation.

What are the main differences between a government bond curve and a swap curve? The government bond yield curve can be considered as a risk-free yield curve if the default risk of government bonds is assumed to be negligible. This assumption is especially reasonable for a curve based on AAA-rated bonds. In contrast, the swap curve is constructed from instruments which are more vulnerable to default of the counterparties involved. In addition to credit risk considerations, government bonds differ from swap contracts because they can be used as collateral. Both aspects contribute to the spread between swap rates and the government bond yields for corresponding maturities.

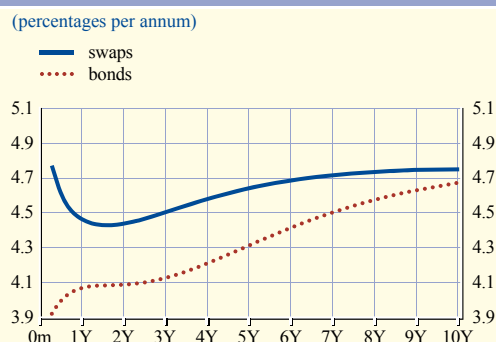
A comparison of movements in the government bond yield curve and the swap rate curve can provide valuable information, especially in periods of financial market stress. For example, forward curves derived from both interest rate swaps and government bonds contained roughly the same information at the end of June 2007 (see Chart A). At this time, both curves signalled market expectations of rising short-term interest rates over a horizon of about one year.

**Chart A Implied forward euro area overnight interest rates on 29 June 2007**



Source: ECB. Underlying data provided by Reuters and EuroMTS; ratings provided by Fitch Ratings.

**Chart B Implied forward euro area overnight interest rates on 31 August 2007**



Source: ECB. Underlying data provided by Reuters and EuroMTS; ratings provided by Fitch Ratings.

Additionally, the spread between both curves was rather constant across maturities. However, this picture changed dramatically during the money market tensions in August 2007 (see Chart B).

In fact, at the end of August, interest rates on the short end of the government forward curve had dropped substantially compared with end-June. This was probably attributable to a combination of a drop in the expected future short-term risk-free rate and flight-to-safety portfolio shifts of market participants. At the same time short-term forward rates derived from the swap curve increased significantly. This increase mainly reflected market participants' perception of a tight liquidity situation on the money market.

### 3 IMPLEMENTATION OF THE EURO AREA YIELD CURVE

#### 3.1 DATA SELECTION

On each trading day, the ECB receives and checks a set of files with prices and yields of the euro area government bonds that were traded/quoted during the previous trading day.<sup>4</sup> The initial basket of bonds undergoes some predefined quality checks and corrupted prices/yields, if any, are eliminated (for example, there should be no zero or negative prices/yields in the population). Afterwards, several selection criteria are applied to ensure that the yield curve is estimated on the basis of a sufficiently homogeneous population of bonds. Only euro area central government bonds issued in euro are considered. Special featured bonds and/or variable rate coupon bonds are removed.

Typically, yield curves are used to obtain risk-free benchmark rates. As a rule, the issuer with the lowest credit risk is the central government. In the euro area many central governments have AAA issuer rating. Currently, the ECB releases two different sets of euro area yield curves: (1) a reference curve based on AAA-rated government bonds; and (2) an additional curve based on central government bonds from all euro area countries regardless of their credit rating.

#### LIQUIDITY

Liquidity considerations play a major role in the selection process of the bonds. The more liquid a market is, the better the information content of prices in that market segment. Typical liquidity

measures are: total turnover<sup>5</sup>, average trade size and bid-ask spreads. Moreover, small bonds are often rather illiquid. In the production of the ECB's yield curves only bonds with a minimum trading volume of €1 million per day are used. In addition, only debt securities with a maximum bid-ask spread of 3 basis points are retained.

#### SYNCHRONICITY

Market expectations can be gauged from the yield curve if the underlying yields reflect a homogeneous information set. This requires that yield or price data are taken at about the same time. The dataset used for the estimation of the euro area yield curves comprises the close-of-market prices, i.e. the prices of the last executed transactions at 5 p.m. Central European Time (CET) for each bond. If there have been no transactions, quotes posted at that time are used.

#### MATURITY SPECTRUM

The euro area central government bond yield curves are estimated using debt securities with residual maturities above three months and below 30 years. Bonds with maturities below three months are less traded and thus typically have more volatile prices/yields than other bonds. On the other side of the spectrum, only a few bonds have a remaining maturity exceeding 30 years and often their price mainly reflects the exceptional demand of institutional investors that need assets with a long duration. In view of these possible distortions and their lack of

<sup>4</sup> Underlying data are provided by EuroMTS; ratings are provided by Fitch Ratings.

<sup>5</sup> Measured as total volume of daily trades.

liquidity, these very long-term bonds are not retained either.

### ADJUSTMENT FOR TAX AND MARKET CONVENTIONS

In the interests of comparability, yields before tax are used. The population of bonds used for the estimation of euro area yield curves comprises bonds traded in different markets with different trading calendars and market conventions. Some adjustments are therefore made to guarantee comparability, for example regarding the settlement and day-count conventions.

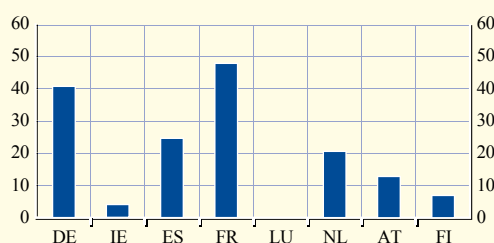
### OUTLIER DETECTION AND REMOVAL

Despite the above-mentioned selection criteria, the yields of a few bonds may still deviate significantly from the rest. To prevent noise in the yield curve estimation, these outliers are removed from the sample. Outliers are traced separately for a number of residual maturity brackets. Bonds with yields that deviate more than two standard deviations from the average are considered as outliers and are removed from the sample. Within each of these brackets, the average yield and standard deviation are calculated. This procedure is iterated in order to reduce the sensitivity of the analysis to potentially large outliers eliminated in the first round that could have distorted the average yield level and the standard deviation.

### 3.2 BOND POPULATION AND ESTIMATION RESULTS

Applying the above-mentioned selection criteria and checks results in a very stable population of

**Chart 3 Average number of AAA-rated central government bonds per country**



Sources: ECB and EuroMTS; ratings provided by Fitch Ratings. Note: Data refer to the period from 29 December 2006 to 14 December 2007.

bonds that is used for the actual estimation of the yield curves. It contains some 160 debt securities for the AAA curve and some 260 bonds for the curve with issues by all euro area central governments. The day-to-day fluctuations are negligible.

The sample is also very stable in terms of the country contributions. Chart 3 shows the average number of bonds per country for the AAA curve over the past 12 months.

The quality of the estimation becomes apparent when examining the difference between the observed yields and their theoretical counterparts derived from the computed yield curve (see also Chart 1).

The table below presents some statistics on the daily goodness of fit of the reference euro area yield curve. On average nearly 80% of the actual yields can be replicated with a margin of 3 basis points.

**Table Goodness-of-fit measures for the euro area AAA yield curve**

(basis points unless otherwise indicated)

	Hit rate: percentage of bonds for which the theoretical yield deviates from the observed yield by 3 basis points or less	Mean absolute error	Weighted mean absolute error	Root mean squared error
Average	78.4	2.08	1.27	2.93
Standard deviation	11.8	0.62	0.37	0.78

Source: ECB. Underlying data provided by EuroMTS; ratings provided by Fitch Ratings. Note: Results based on daily observations for the period from 29 December 2006 to 14 December 2007.

The average daily MAE for the period for which data are currently available was 2.08, which means that, on average, the absolute difference between the real observed yield of a bond and that estimated by the model is around 2 basis points. If weighted by the duration of each bond, the average absolute error for the period amounts to a mere 1.27 basis points. These results suggest a very good fit. Even during the recent financial market turmoil, when central government bond prices also became more volatile, the fit remained high, albeit not at the level witnessed in calmer periods (hit rate of up to 90%). In turn, this has led to an increase in the standard deviation of the above-mentioned quality indicators. Moreover, the large number of bonds included in the final population limits the impact of a slight mispricing of an individual bond.

As a result of the interplay between the chosen dataset, the applied quality checks and the model used for the yield curve estimation, the term structure is smooth and fits the market data very well.

#### 4 CONCLUSION

For monetary policy purposes and for analysing financial market developments, central banks and other market participants need reliable representations of the term structure of interest rates. The new euro area government bond yield curves released on a daily basis are a useful complement to the already existing information. In particular, the term structures based on AAA-rated instruments are considered free of credit risk and therefore provide a floor for the borrowing costs of the economy. As a result of a careful data selection procedure and various quality checks, the new euro area government bond yield curve is smooth and stable, and fits the market data and its purpose very well.