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By Miguel A. León-Ledesma[1],
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Abstract

Capital-labor substitution and total factor productivity (TFP) estimates are essential features of growth and income distribution models. In the context of a Monte Carlo exercise embodying balanced and near balanced growth, we demonstrate that the estimation of the substitution elasticity can be substantially biased if the form of technical progress is misspecified. For some parameter values, when factor shares are relatively constant, there could be an inherent bias towards Cobb-Douglas. The implied estimates of TFP growth also yield substantially different results depending on the specification of technical progress. A Constant Elasticity of Substitution production function is then estimated within a "normalized" system approach for the US economy over 1960:1–2004:4. Results show that the estimated substitution elasticity tends to be significantly lower using a factor augmenting specification (well below one). We are able to reject Hicks-, Harrod- and Solow-neutral specifications in favor of general factor augmentation with a non-negligible capital-augmenting component. Finally, we draw some important lessons for production and supply-side estimation.

JEL Classification: C15, C32, E23, O33, O51.
Keywords: Constant Elasticity of Substitution, Factor-Augmenting Technical Change, Technical Progress Neutrality, Factor Income share, Balanced Growth.

Non Technical Summary

Capital-labor substitution and overall productivity improvements (total factor productivity, TFP) estimates are essential features of growth and income distribution models. Both, however, in a production function framework require the modelling of technical change. Technical change captures the degree to which the output contribution of factor inputs (capital and labor) changes over time given fixed quantities of those factors – in effect, it captures quality improvements. The issue of the possible mis-specification of the form of technical change and its implications for the empirical estimates of the substitution elasticity and for TFP have been largely unexplored. We provide Monte Carlo (MC) evidence on the bias in the estimated substitution elasticity generated by mis-specifying the nature of technical change. To best isolate the effect of such biases, we use the so-called "\normalized" system approach. Although we find that the general factor augmenting specification correctly identifies technical progress, alternative neutrality specifications only work well when they correspond to the true data generation process. For parameter configurations that yield stable factor shares, the substitution elasticity is biased upwards (downwards), when its true value is below (above) unity. For plausible substitution values, this can often lead to biases in the estimated substitution elasticity towards unity. In the light of this, we then estimate a Constant Elasticity of Substitution production function is then estimated for the US economy for the 1960:1- 2004:4 period. Our results show that the estimated substitution elasticity tends to be significantly lower using a factor augmenting specification and is well below one. We are able to reject Hicks-, Harrod- and Solow-neutral specifications in favor of general factor augmentation with a non-negligible capital-augmenting component.

1 Introduction

Balanced growth defines a situation in which the capital-output ratio and factor income shares are constant (stationary). In terms of neoclassical growth theory, Uzawa (1961), it requires that technical progress should be Harrod Neutral or that production should be Cobb Douglas (i.e., a unitary elasticity of factor substitution). Although balanced growth is often considered a reasonable description of many economies, these two conditions underlying balanced growth are widely disputed.

For instance, there is now mounting evidence that aggregate production may be better characterized by a non-unitary substitution elasticity (e.g., Chirinko et al. (1999), Klump et al. (2007), León-Ledesma et al. (2010), Duffy and Papageorgiou (2000)). Further, Chirinko (2008)'s survey suggests that, across many different studies, evidence favors elasticities ranges of 0.4-0.6 for the US. Likewise, that all technical change must be labor augmenting is extremely restrictive. One perspective (Acemoglu (2003, 2007)) may be that while technical progress is asymptotically labor-augmenting, it may become capital-biased in transition reflecting incentives for factor-saving innovations. Such models of "biased technical change" attempt to reconcile historically-observed fluctuations in factor income shares with their apparent secular stability.

However, whatever its plausibility, dismissing the purely Harrod Neutral case risks the finding that *any* developmental pattern can be "fitted" by some suitable combination of technical progress and non-unitary substitution. Indeed, Diamond and McFadden (1965) (see also Diamond et al. (1978)) famously asserted that the elasticity and biased technical change could not be simultaneously identified. To counter this "impossibility theorem" researchers commonly make *a priori* assumptions about the direction of technical change (typically Hicks or Harrod neutral). Klump et al. (2007) and León-Ledesma et al. (2010) also argued that using the system approach (i.e., estimating the production function and capital and labor first order conditions jointly) with its implied cross-equation restriction vastly improved identification (an additional consideration, following the seminal work of La Grandville (1989) and Klump and de La Grandville (2000), was estimation in "normalized" form). Notwithstanding, a priori restrictions on the direction of technical change still imply a mis-specification error of some proportion. Understanding the implications of that mis-specification in a normalized context is the subject of this paper.

We provide Monte Carlo (MC) evidence on the bias in the estimated substitution elasticity generated by mis-specifying the nature of technical change. To best isolate the effect of such biases, we follow Klump et al. (2007) and León-Ledesma et al. (2010) and use the "normalized" system approach for estimation which was shown to dominate linear and nonlinear single equation approaches.[1] Although we find that the general factor augmenting specification correctly identifies technical progress, alternative neutrality specifications only work well when they correspond to the true data generation process (DGP). For parameter configurations that yield stable factor shares, the substitution elasticity is biased upwards (downwards), when its true value is below (above) unity. For plausible substitution values, this can often lead to biases in the estimated substitution elasticity towards unity.

In the light of this, we then estimate the supply system for the US economy for the 1960:1–2004:4 period under general factor-augmenting, Hicks-, Harrod-, and Solow-neutral specifications. Following the MC, we estimate a relatively simplified but most commonly used framework where growth in technical progress is constant (and without structural breaks) and where we abstract from time-varying factor utilization.[2]

Many of the lessons drawn from the MC find an echo in these empirical estimates. Although results yield very different values for the substitution elasticity, in all cases, our tests support the general factor-augmenting specification. Using the latter, the substitution elasticity for the US is around 0.5-0.6. We then derive estimates of Total Factor Productivity (TFP) growth and show and motivate relevant differences between specifications. Our preferred general factor-augmenting system captures a productivity acceleration during the second half of the 1990s consistent with that found in other studies (see Basu et al. (2003), Fernald and Ramnath (2004) and Jorgenson (2001)).

The importance of our subject matter is worth recalling. The shape of the production function (as captured by the substitution elasticity) plays a key role in models analyzing growth and convergence; income distribution; technical efficiency; labor-market outcomes, etc (e.g., see Klump and de La Grandville (2000), La Grandville (2009), Sato (2006), Rowthorn (1999), Chirinko (2008)). Moreover,

[1]Normalization essentially implies representing the production function in consistent indexed number form (see La Grandville (1989), Klump and de La Grandville (2000)). Without normalization the parameters of the production function have no economic interpretation since they are dependent on the normalization point and the elasticity of substitution. This feature significantly undermines estimation and comparative static exercises. Moreover normalization avoids the otherwise unusual situation whereby capital and labor output shares approach one half in the Leontief case.

[2]For attempts to model non-constant growth in technical change, see Klump et al. (2007). On the second point, namely time-varying factor utilization rates, this is essentially not problematic under the reasonable assumption of stationarity in such rates.

measurement of potential output is a key indicator for stabilization policy.[3] Likewise, changes in the direction of technical bias over time have contributed to our understanding of, e.g., labor-market inequality and the "skills premia" (Acemoglu (2002))[4]; factor income share movements (McAdam and Willman (2008)) and the welfare consequences of new technologies (Marquetti (2003)) etc. Finally, of course, since Solow (1957), the calculation of total factor productivity (TFP) growth has been a key application of production estimation.

The paper is organized as follows. In section 2 we present some relevant background on the more general Constant Elasticity of Substitution (CES) production function and in section 3 discuss the potential biases arising from mis-specification of technical change. In Section 4 we present the Monte Carlo setup and discuss the results. Section 5 present empirical results using US data. Section 6 concludes.

2 Theory Background

The CES production function allows the elasticity of capital and labor with respect to their relative price to be any constant between zero and infinity. This special type of production functions was formally introduced into economics by Arrow et al. (1961) and spawned a vast supporting literature (e.g., David and van de Klundert (1965), Kmenta (1967), Berndt (1976), Chirinko (2002), Klump et al. (2007)).

Furthermore, following the seminal work of La Grandville (1989) and Klump and de La Grandville (2000), the function is often expressed in "normalized" (or indexed) form since its parameters then have a direct economic interpretation.[5] Normalization also turns out to be important for estimation as emphasized by León-Ledesma et al. (2010). The normalized CES takes the form:

$$Y_t = F\left(\Gamma_t^K K_t,\, \Gamma_t^N N_t\right) = Y_0 \left[\pi_0 \left(\frac{\Gamma_t^K K_t}{\Gamma_0^K K_0}\right)^{\frac{\sigma-1}{\sigma}} + (1-\pi_0)\left(\frac{\Gamma_t^N N_t}{\Gamma_0^N N_0}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{1}$$

[3] Orphanides (2003) suggest mis-measurement of potential output in real time has been a historical constraint and tension for US monetary policy

[4] See also Greenwood et al. (1997) and Krussel et al. (2000).

[5] See also Klump and Preissler (2000), Klump and de La Grandville (2000), Klump and Saam (2008), and La Grandville (2009) for an analysis of the relevance of normalized production functions for growth theory. For any legitimate CES function, the value of the substitution elasticity depends on (i) a given level of capital deepening, (ii) a given marginal rate of substitution and (iii) a given level of per-capita production. Different CES functions are considered to be in the same "family" if they share common baseline values but differ only in their elasticity values and one point of tangency characterized by the given baseline values. Klump and de La Grandville (2000) then show how, given this normalization procedure, comparative statics on the elasticity substitution can be legitimately made.

where the point of time $t = 0$ represents the point of normalization, Y_t represents real output, K_t is the real capital stock and N_t is the labor input. The terms Γ_t^K and Γ_t^N capture capital and labor-augmenting technical progress. To circumvent problems related to the Diamond-McFadden impossibility theorem, researchers usually assume specific functional forms for technical progress, e.g., $\Gamma_t^K = \Gamma_0^K e^{\gamma_K t}$ and $\Gamma_t^N = \Gamma_0^N e^{\gamma_N t}$ where γ_i denotes growth in technical progress associated to factor i, t represents a time trend. This technical progress is alternatively Hicks neutral ($\gamma_K = \gamma_N > 0$), Harrod neutral ($\gamma_K = 0$,$\gamma_N > 0$) or, more seldom, Solow-Neutral ($\gamma_K > 0$,$\gamma_N = 0$). Hence a *general* factor-augmenting case ($\gamma_K > 0 \neq \gamma_N > 0$) is typically by-passed.

The capital income share at the point of normalization is $\pi_0 = \frac{r_0 K_0}{Y_0}$ (r denotes the real user cost of capital) and the elasticity of substitution between capital and labor inputs is given by the percentage change in factor proportions due to a change in the factor price ratio along an isoquant:

$$\sigma \in (0, \infty) = \frac{d \log (K/N)}{d \log (F_N/F_K)} \tag{2}$$

CES production function (1) nests Cobb-Douglas when $\sigma = 1$; the Leontief function (i.e., fixed factor proportions) when $\sigma = 0$; and a linear production function (i.e., perfect factor substitutes) when $\sigma \to \infty$.[6]

The higher is σ, the greater the similarity between capital and labor. Thus, when $\sigma < 1$, factors are gross complements in production and gross substitutes when $\sigma > 1$. Thus, it can be shown that with gross substitutes, substitutability between factors allows both the augmentation and bias of technological change to favor the same factor.[7] For gross complements, however, a capital-augmenting technological change, for instance, increases demand for labor (the complementary input) more than it does capital, and vice versa. By contrast, when $\sigma = 1$ an increase in technology does not produce a bias towards either factor (factor shares will always be constant since any change in factor proportions will be offset by a change in factor prices). Thus, the question of whether σ is above or below unity is arguably as important as its numerical value.

[6] Going back to Hicks (1932), the value of the substitution elasticity is often viewed as reflecting economic flexibility and thus deep institutional factors such as labor bargaining power, the taxation burden, degree of economic openness, the characteristics of national education system, etc. Accordingly, some view changes in the substitution elasticity as potential drivers of endogenous growth and potentially even more important than traditionally-studied growth factors such as savings and technical progress, La Grandville (2009), Yuhn (1991). See also Bairam (1991).

[7] In other words, if $\sigma < 1$ and $\gamma_i > \gamma_j$ this implies that $F_i > F_j$ plus that there is a relative rise in the income share of factor i . Hence we can say that technical change related to factor i "favors" factor i in the gross complements case.

3 Some Possible Pitfalls In Supply Estimation

The CES function and the first-order conditions form a highly nonlinear system. This points to the advantage of Monte Carlo methods in detecting and quantifying mis-specification issues. However, before that, we discuss the general issues at stake and analytically derive some potential estimation problems.

First, in sections (3.1) and (3.2), we consider the particular impact of mis-specification of technical progress on the estimation of the elasticity of substitution, and then on TFP estimates and its decompositions. Second, in section (3.3) we touch on the possibility of observational equivalence; the properties of the CES function in admitting gross substitutes / complements in production can imply a similar evolution of, for instance, factor income shares across distinct technical parameters.

These examples, note, are meant to be primarily motivational: they usefully highlight many of the issues that will become apparent in both the MC and data estimation sections.

3.1 Mis-Specified Technical Change: Parameter Inference

The capital-to-labor income share, given a competitive goods market and profit maximization, can be expressed as,

$$\Theta_t = \frac{r_t}{w_t}\frac{K_t}{N_t} = \frac{\pi_0}{1-\pi_0}\left(\frac{\Gamma_t^K K_t/K_0}{\Gamma_t^N N_t/N_0}\right)^{\frac{\sigma-1}{\sigma}} \tag{3}$$

Whilst Θ is observed, neither the substitution elasticity nor technical change are. For Θ to be constant requires the familiar balanced growth cases of $\sigma = 1$ or Harrod neutrality. But can $d\Theta \approx 0$ when we purposefully depart form these two restrictive assumptions? And what would be the likely estimation consequences?

Equation (3) shows that if we assume Hicks neutrality, stable factor shares would require $\widehat{\sigma} \rightarrow 1$ to offset the trend in capital deepening.[8] Antràs (2004) uses this argument to rationalize Berndt (1976)'s widely-cited finding of Cobb-Douglas for US manufacturing. The same is true of Solow neutrality. Another possibility, for factor-augmenting technical progress, is that stable factor shares hold if the growth of technical bias offsets that of capital deepening.

Likewise, independent from the size of σ, Θ would remain broadly constant outside the balanced growth path if r_t "absorbs" some of the trend in capital aug-

[8]Capital deepening, K/N, grows at the same rate as labor-augmenting technical progress plus population growth. Thus, $\lim_{t\rightarrow\infty} K_t/N_t \rightarrow \infty$.

mentation. This, though, violates our priors that the real interest rate (and thus the real user cost) is stable.[9] However, we can show that this trend absorption need only be modest. If the user cost only partially absorbs the capital-augmenting technical progress, there will be trends also in the factor income shares, but these may be weak when coupled with a moderate pace of capital augmentation.[10],[11] Hence, the *relative* stability of factor income shares is not a sufficient condition for the correctness of either Cobb Douglas or Harrod neutrality.

We have seen that the assumption of Hicks neutrality can bias σ towards unity when the true DGP is Harrod-neutral. Correspondingly, we can show that quite generally (although not universally) also the Harrod-neutral specification can result in σ estimates that are either upwards or downwards biased when the true DGP contains capital-augmenting technical progress.

The *lhs* of equation (4) below corresponds to the "true" DGP for the observed capital income share and the *rhs* to the mis-specified Harrod-neutral (h) version:

$$\pi_0 \left(\frac{\Gamma_t^K K_t}{K_0 Y_t} \right)^{\frac{\sigma-1}{\sigma}} = \pi_0 \left(\frac{K_t}{K_0 Y_t} \right)^{\frac{\widehat{\sigma}^h - 1}{\widehat{\sigma}^h}} \tag{4}$$

Taking logs and rearranging,

$$\frac{\sigma - 1}{\sigma} \log \Gamma_t^K = \frac{\widehat{\sigma}^h - \sigma}{\widehat{\sigma}^h \sigma} \log \left(\frac{K_t}{K_0 Y_t} \right) \tag{5}$$

In the data, $\frac{K_t}{K_0 Y_t} = \left(\Gamma_t^K\right)^{\sigma-1} \left(\frac{r_0}{r_t}\right)^{\sigma}$. Assume $r_t = r_0 \left(\Gamma_t^K\right)^{\alpha}$, $\alpha \in (0,1]$. This then implies that the real user cost *partly* absorbs the trend in capital-augmenting technology. It can be shown that with values of $\alpha > \frac{\sigma-1}{\sigma}$, the negative trend in the capital-output ratio corresponds to the positive trend of Γ_t^K. When this condition holds, then in the interval $\alpha \in (0,1]$, $\widehat{\sigma}^h > \sigma$ and with $\sigma > 1$, in turn, $\widehat{\sigma}^h < \sigma$. However, when $\alpha = 0$ and $\sigma > 1$, then the capital-output ratio has a positive trend

[9] However, rather than exhibiting global stability, real interest rates are commonly thought of as regime-wise stationary, e.g., Rapach and Wohar (2005). Also, depreciation rates (another component of the user cost) have trended upwards over this sample - see Evans (2000). This is compatible with the commonly-held view that the share of equipment in capital has increased while the share of structures has decreased and hence investment is characterized by shorter mean lives.

[10] Assuming capital augmenting-technical progress is 0.5% annually and even where that is fully absorbed by the real user cost, then the latter would rise from, for instance, 0.05 to 0.064 within 50 years.

[11] Jones (2003) also reports evidence showing capital shares for OECD countries frequently exhibit large variation and medium-run trends. These trends are certainly relevant for typical sample sizes available to researchers.

and $\widehat{\sigma}^h > \sigma > 1$.

Hence, mis-specified technical progress results in biased substitution elasticity estimates. For plausible values of σ (e.g., in the "Chirinko interval") this would quite often lead to a bias towards unity, i.e. upwards (downwards) biased when the true substitution elasticity is below (above) unity.

3.2 Mis-Specified Technical Change: TFP Estimates

Since Solow (1957) the calculation of TFP has been a key application of the production function literature. Predicated on Cobb Douglas, TFP calculations are invariably derived imposing Hicks Neutrality (the "Solow Residual" method). However, even if estimates of the *size* of TFP growth are robust to mis-specification, an accurate decomposition of TFP growth offers insights on the mechanisms underlining economic performance and may usefully inform various policy questions.

An exact (or *residual*) method to calculate the contribution of Log(TFP) to output is given by,

$$\log\left[\frac{F\left(\Gamma_t^K K_t,\, \Gamma_t^N N_t\right)}{F\left(\Gamma_0^K K_t,\, \Gamma_0^N N_t\right)}\right] = \frac{\sigma}{\sigma-1}\log\left[\frac{\pi_0\left(\frac{\Gamma_t^K K_t}{\Gamma_0^K K_0}\right)^{\frac{\sigma-1}{\sigma}} + (1-\pi_0)\left(\frac{\Gamma_t^N N_t}{\Gamma_0^N N_0}\right)^{\frac{\sigma-1}{\sigma}}}{\pi_0\left(\frac{K_t}{K_0}\right)^{\frac{\sigma-1}{\sigma}} + (1-\pi_0)\left(\frac{N_t}{N_0}\right)^{\frac{\sigma-1}{\sigma}}}\right] \tag{6}$$

For illustrative purposes, it is also useful to present a closed-form approximation for Log(TFP) separable from factor inputs. We follow Kmenta (1967) and Klump et al. (2007), by applying an expansion of the normalized log CES production function (1) around $\sigma = 1$:

$$\begin{aligned} y_t &= \pi k_t + a k_t^2 \\ &\quad \underbrace{+\,\pi\left[1+\frac{2a}{\pi}k_t\right]\gamma_K\cdot\widetilde{t} + (1-\pi)\left[1-\frac{2a}{(1-\pi)}k_t\right]\gamma_N\cdot\widetilde{t} + a\left[\gamma_K-\gamma_N\right]^2\cdot\widetilde{t}^2}_{\Phi=Log\ (TFP)} \end{aligned} \tag{7}$$

where $\widetilde{t} = t - t_0$, $y_t = \log[(Y_t/Y_0)\,/\,(N_t/N_0)]$, $k_t = \log[(K_t/K_0)\,/\,(N_t/N_0)]$, and where $a = \frac{(\sigma-1)\pi(1-\pi)}{2\sigma}$ and $\Gamma_t^i = \Gamma_0 e^{\gamma_i \widetilde{t}}$.[12]

Equation (7) shows that output-labor ratio can be decomposed into capital deepening and technical change, weighted by factor shares and the substitution elasticity

[12] Equation (7) is better understood as a relationship that may be exploited *after* estimation of a factor-augmenting production function, rather than a viable estimation form in itself. In terms of parameter identification it is clearly over-identified; León-Ledesma et al. (2010) provide some weak identification results based on a priori knowledge of the direction of technical bias.

(where $sign\,(a) = sign\,(\sigma - 1)$ and $\lim_{\sigma\in[0,\ \infty]} a \in \left[-\infty, \frac{1}{2}\pi\,(1-\pi)\right]$). In addition, (7) shows that, when $\sigma \neq 1$ and $\gamma_K \neq \gamma_N > 0$, additional (quadratic) curvature is introduced into the estimated production function.

The effect of capital deepening on $Log(TFP)$, given by $2a\widetilde{t}\,(\gamma_K - \gamma_N)$, switches sign depending on whether factors are gross substitutes or complements. However, although the transmission of individual technology changes to TFP is also a function of σ, generally its sign (and, in particular, the importance of gross substitutes or complements) is ambiguous.[13] The effect of σ on TFP through capital deepening can be given an economic interpretation, though. When $\sigma \neq 1$, capital deepening will be biased in favor of one factor of production (changing its income share). Hence, with factor augmenting technical change, an acceleration of capital deepening changes the estimated TFP growth simply because technical progress is biased in favor of one of the factors. If, for instance, $\sigma < 1$, capital deepening would increase the share of labor. If $(\gamma_K - \gamma_N) < 0$, then capital deepening would lead to an acceleration of the estimated TFP growth.

The expressions for Log(TFP) for the restricted neutrality cases are[14]:

$$\text{Harrod} : (1-\pi)\left[1 - \frac{2a}{(1-\pi)}k_t\right]\gamma_N \cdot \widetilde{t} + a\gamma_N^2 \cdot \widetilde{t}^2 \tag{8}$$

$$\text{Solow} : \pi\left[1 + \frac{2a}{\pi}k_t\right]\gamma_K \cdot \widetilde{t} + a\gamma_K^2 \cdot \widetilde{t}^2 \tag{9}$$

$$\text{Hicks} : \gamma \cdot \widetilde{t},\ \text{where } \gamma = \gamma_K = \gamma_N \tag{10}$$

The comparisons of (7) with variants (8)-(10) speak for themselves. For instance, in the Hicks case all improvements in TFP would be attributed to a single factor-neutral component, γ, excluding also any role for capital deepening.

For values of K_t and N_t close to their normalization points, $k_t \approx 0$, one can also obtain two simpler approximation for Log(TFP):

$$\Phi^{Simple} = \pi\gamma_K \cdot \widetilde{t} + (1-\pi)\,\gamma_N \cdot \widetilde{t} + a\,[\gamma_K - \gamma_N]^2 \cdot \widetilde{t}^2 \tag{11}$$

$$\Phi^{LinearWeight} = \pi\gamma_K \cdot \widetilde{t} + (1-\pi)\,\gamma_N \cdot \widetilde{t} \tag{12}$$

The first abstracts from capital deepening. This may be considered informative

[13] Except in two cases, when $\gamma_K - \gamma_N > 0$:
$\frac{\partial\Phi}{\partial\gamma_N|\sigma<1} = (1-\pi)\,\widetilde{t}\left[\left\{1 - \frac{k_t\pi(\sigma-1)}{\sigma}\right\} - (\sigma-1)\,(\gamma_K - \gamma_N)\,\widetilde{t}\right] > 0$,
$\frac{\partial\Phi}{\partial\gamma_K|\sigma>1} = \widetilde{t}\left[\pi\left\{1 + \frac{k_t(1-\pi)(\sigma-1)}{\sigma}\right\} + (1-\pi)\,(\sigma-1)\,(\gamma_K - \gamma_N)\,\widetilde{t}\right] > 0$.

[14] Individual technical change cannot be identified in the Cobb-Douglas case.

regarding the contribution of capital deepening in TFP estimates based on (6) and (7) - especially so given the rapid capital deepening in the US towards the end of our sample. The second form, which is a simple linear weight of the two constant progress terms, discards all nonlinearities in TFP.

Although all cases coincide at the point of normalization, (11) by excluding capital deepening runs the risk that the nonlinearity in the TFP is not correctly captured. For instance, if the economy is characterized by Harrod neutrality, Φ^{Simple} implies the wrong sign for the quadratic effect term (being positive rather than negative).[15]

3.3 Identification Aspects: Iso-Shares

Assume $K_t = K_0 e^{\eta_K t}$, $N_t = N_0 e^{\eta_N t}$, $\Gamma_t^K = \Gamma_0^K e^{\gamma_K t}$ and $\Gamma_t^N = \Gamma_0^N e^{\gamma_N t}$. Assume further that although the histories of Θ, η_K and η_N are commonly observed, two separate studies arrive at the estimates: $\{\sigma_2, \gamma_{K,2}, \gamma_{N,2}\} \notin \{\sigma_1, \gamma_{K,1}, \gamma_{N,1}\}$. Given (3), we can derive the relationship between them as,

$$\sigma_2 = \frac{\phi}{1 - \sigma_1 (1 - \phi)} \tag{13}$$

with $\phi = \frac{\gamma_{K,2} - \gamma_{N,2} + \eta_K - \eta_N}{\gamma_{K,1} - \gamma_{N,1} + \eta_K - \eta_N}$, which we label the "bias ratio".[16] Expression (13) shows the combinations of σ's compatible with the same evolution of factor shares for given assumptions about the relative bias in technical progress. Hence, for a given ϕ we can derive a range of elasticities that generate the same factor income shares. For example, if $\phi = 2$ then, on a common dataset, $\sigma_1 = 0.25$, would imply $\sigma_2 = 1.33$, and $\sigma_1 = 1.25$ would imply $\sigma_2 = 0.95$.[17] We saw in section (2) how important the gross-substitutes/gross-complements distinction is, and here is a case where researchers on a common dataset would arrive at completely different conclusions.

In a system estimator with parameter restrictions, the estimated coefficients have to be compatible with the evolution of both output *and* factor payments, so the scope for this observational equivalence to affect estimation results is greatly reduced. However, if we restrict technical progress to take a particular form of

[15] In the Harrod neutral case $k_t = \gamma_N \cdot \widetilde{t}$. Substituting this into (8) results in the following form of the log(TFP): $\pi\gamma_K \cdot \widetilde{t} + (1-\pi)\gamma_N \cdot \widetilde{t} - a\gamma_N^2 \cdot \widetilde{t}^2$ and hence Φ^{Simple} implies the wrong sign for the quadratic term.

[16] Naturally, the trade off defined by (2), holds only exactly in a deterministic setting. However, we believe it to be indicative of trends in stochastic environments.

[17] More generally, $\sigma_1 \to \infty$, $\sigma_2 \to 0$ and naturally they cross at $\sigma_2 = \sigma_1 = 1 \forall \phi$ where we have a Cobb-Douglas technology with constant factor shares regardless of the direction or bias in absolute or relative technical progress.

augmentation such as Hicks- or Harrod-neutrality, then these identification issues become important. The estimate of σ will then bear the burden of fitting the data for output and factor payments, leading to estimation biases if the technical progress restriction is incorrect.

4 The Specification Bias: Monte Carlo Evidence

We now use a Monte Carlo simulation for a variety of parameter values of the supply side to quantitatively analyze the potential bias arising from mis-specification of technical progress discussed in the previous section. We simulate a consistent DGP for factor inputs, output, and factor payments, then estimate using the normalized system approach of Klump et al. (2007), and León-Ledesma et al. (2010) imposing particular forms of factor neutrality.[18]

The normalized system estimator of the parameters of the CES production function follows León-Ledesma et al. (2010). It consists of the joint estimation of (log-version of) the CES function (1) and the first order conditions for K and N. Normalization allows us to fix parameter π_0 to its observed value (capital income share in period 0) also simplifying the estimation problem. The 3-equation system of equations is then estimated jointly using a Generalized Nonlinear Least Squares (GNLLS) system estimator (which we also use for estimation with US data).

4.1 The Monte Carlo experiment

We generate data in a consistent way corresponding to a particular evolution of factor inputs, technical progress and output. This Monte Carlo data is estimated under both correctly specified and mis-specified systems. Hence, we draw M simulated stochastic processes for labor (N_t), capital (K_t), labor- (Γ_t^N) and capital-(Γ_t^K) augmenting technology. Using these, we then derive "potential" or "equilibrium" output (Y_t^*), observed output (Y_t) and real factor payments (w_t and r_t), for a range of parameter values and shock variances. The simulated system is consistent with the normalized approach, so that we ensure our parameters are *deep*, i.e. can be given an economic interpretation and are not the result of a combination of other parameters.

[18]León-Ledesma et al. (2010) also considered a MC exercise. Their objective, however, was to examine the power of different estimator *types*. They also abstracted from questions of whether the simulated data was plausible in terms of balanced or near balanced growth trajectories.

Given our emphasis on realistic settings, where the economy does not deviate in an evident way from the case of stable factor income shares, we first need to devise a way to set parameter values such that we exclude unrealistic income trends. We can do this by looking again at the expression for the capital-to-labor income share under competitive profit maximization,

$$\Theta_t = \frac{r_t K_t}{w_t N_t} = \frac{\pi}{1-\pi}\left(\frac{\Gamma_t^K K_t/K_0}{\Gamma_t^N N_t/N_0}\right)^{\frac{\sigma-1}{\sigma}}$$

Thus, if $\sigma \neq 1$, capital- and labor-augmenting technical change can lead to ever increasing or decreasing factor shares for given factor proportions. Hence, for given rates of technical progress, to obtain approximately constant shares, we set the rate of growth of K in such a way that we avoid any counter-factual trends in shares. One simple mechanism to achieve this, following our earlier discussion, is to allow r to absorb some fraction, α, of the trend in capital augmentation (assuming $\Gamma_0^K = \Gamma_0^N = 1$):

$$r_t = r_0 e^{\alpha\left(\gamma_K \cdot \tilde{t}\right)} \tag{14}$$

with capital then solved from its first order condition:

$$K_t = Y_t \left(\frac{\pi_0}{r_t}\right)^{\sigma}\left[\frac{Y_0}{K_0} e^{\left(\gamma_K \cdot \tilde{t}\right)}\right]^{\sigma-1} \tag{15}$$

If $\alpha = 0$ and/or $\gamma_K = 0$, the Harrod-neutral case, the real user cost and capital-output ratio are constant: $r = r_0$; $K/Y = \left(\frac{\pi}{r_0}\right)^{\sigma}\left(\frac{Y_0}{K_0}\right)^{\sigma-1}$. Whereas if $\alpha \neq 0$ and $\gamma_K > 0$, $r \to \infty$ and $K/Y \to 0$. Hence, once we decide α, for given technology parameters, we obtain r from (14). Given an exogenous law of motion for N, the CES function and (15) solve for K and Y. Using the value of K from this recursive system, we obtain the *average* rate of growth of K that we then use to build our stochastic DGP. This is the value compatible with factor shares and real interest rates that do not display counter-factual trends. Given that parameter α controls the rate of change of r, a sufficiently small value can be set to mimic empirically-relevant paths for r and hence K/Y and Θ. In our experiments, we set $\alpha = 0.5$.

The functional construct of (14) is not without an empirical counterpart. As we know, the real user cost comprises the nominal interest rate (i.e., the risk-free government bond rate or firms' market rates), inflation, capital depreciation, taxes, capital gains etc. All these are time-varying.[19] Thus, if there is technical change

[19] In Figure 5 below we plot our measure of the user cost series for the US. This is relatively

which is not solely Harrod neutral alongside approximately constant factor shares, factor payments must be compensating.[20]

Hence, once the rate of growth of K has been derived, we can then describe the full DGP for the MC simulations. Capital and labor evolve as stationary stochastic processes around a deterministic trend:

$$K_t = K_0 e^{\left(\kappa \widetilde{t} + \varepsilon_t^K\right)} \;,\; N_t = N_0 e^{\left(\eta \widetilde{t} + \varepsilon_t^N\right)} \tag{16}$$

where κ and η represent their respective mean growth rates. The initial value for N values was set to $N_0 = 1$, and $K_0 = \pi_0 / r_0$, with the real user cost at $r_0 = 0.05$.[21],[22]

The technical progress functions, as described before, are also assumed to be exponential with a deterministic and stochastic component (around a suitable point of normalization):

$$\Gamma_t^K = \Gamma_0^K e^{\left(\gamma_K \widetilde{t} + \varepsilon_t^{\Gamma^K}\right)}, \quad \Gamma_t^N = \Gamma_0^N e^{\left(\gamma_N \widetilde{t} + \varepsilon_t^{\Gamma^N}\right)} \tag{17}$$

where Γ_0^K and Γ_0^N are initial values for technology which we also set to unity.

We then obtain equilibrium output from the normalized CES function:

$$Y_t^* = Y_0^* \left[\pi_0 \left(\frac{K_t}{K_0} e^{\left(\gamma_K \widetilde{t} + \varepsilon_t^{\Gamma^K}\right)} \right)^{\frac{\sigma-1}{\sigma}} + (1-\pi_0) \left(\frac{N_t}{N_0} e^{\left(\gamma_N \widetilde{t} + \varepsilon_t^{\Gamma^N}\right)} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \tag{18}$$

with $Y_0^* = 1$. This "equilibrium" output is then used to derive the real factor payments from the FOCs, to which we add a multiplicative shock.

$$r_t = \frac{\partial Y_t^*}{\partial K_t} = \pi_0 \left(\frac{Y_0^*}{K_0} e^{\left(\gamma_K \widetilde{t} + \varepsilon_t^{\Gamma^K}\right)} \right)^{\frac{\sigma-1}{\sigma}} \left(\frac{Y_t^*}{K_t} \right)^{\frac{1}{\sigma}} e^{\varepsilon_t^r} \tag{19}$$

$$w_t = \frac{\partial Y_t^*}{\partial N_t} = (1-\pi_0) \left(\frac{Y_0^*}{N_0} e^{\left(\gamma_N \widetilde{t} + \varepsilon_t^{\Gamma^N}\right)} \right)^{\frac{\sigma-1}{\sigma}} \left(\frac{Y_t^*}{N_t} \right)^{\frac{1}{\sigma}} e^{\varepsilon_t^w} \tag{20}$$

Equations (19) and (20) imply that real factor returns equal their marginal product

simple and based on the real government bond yield and depreciation.

[20]Note, we could also have allowed the real wage rate to absorb technology trends, but the motivation for this seems less well founded compared to that of the real user cost.

[21]For estimation, initial values for r_0 and K_0 do not affect the results if the system is appropriately normalized.

[22]For all the experiments we also simulated K_t and N_t such that they displayed stochastic, rather than deterministic, trends as in León-Ledesma et al. (2010). We report here the case of deterministic trends because it makes the discussion above about factor shares more transparent. However, the conclusions of the analysis did not change. Results are available on request.

times a multiplicative shock that temporarily deviate factor payments from equilibrium. All shocks, $\Lambda = [K, N, \Gamma^K, \Gamma^N, r, w]$, are assumed normally distributed iid: $\varepsilon_t^\Lambda \overset{iid}{\sim} N(0, \sigma_{\varepsilon^\Lambda})$.

Because we need to ensure that our artificial data is consistent with national accounts identities, we then obtain the "observed" output series using the identity:

$$Y_t \equiv r_t K_t + w_t N_t \tag{21}$$

We use the "observed" output series for estimation purposes. This ensures that, regardless of the shocks, factor shares sum to unity, which has to be the case in this artificial setting with absent mark-ups.

Hence, the experiment consists of, first, simulating the time series for factor inputs, technical progress, and equilibrium output. Second, from these we obtain factor payments and observed output. Finally, we estimate the normalized system, (18)-(20), imposing Hicks-, Harrod- and Solow neutrality in technical progress. We repeat these steps M times and analyze the possible biases arising from misspecification by looking at the difference between the true and estimated σ.

Table 1 lists the parameters used to generate the simulated series. We fixed the distribution parameter to 0.4.[23] The substitution elasticity is set to a neighborhood around Cobb-Douglas (0.9) and ± 0.4 (thus accommodating gross substitute and complements). Labor supply growth is set to 1.5% per year and capital stock growth to the values implied from our earlier discussion, so that κ changes for each experiment. We use a variety of values for technical progress, assuming a plausible summation of 2% per year; $\gamma_N = 2\%$ and $\gamma_K = 0\%$ (Harrod-neutral case); $\gamma_N = 0\%$, $\gamma_K = 2\%$ (Solow neutral); and $\gamma_N = \gamma_K = \gamma = 1\%$ (Hicks-neutral). Finally, we have two cases where technical progress is of the general factor augmenting form.

The standard errors of the shocks are chosen so that they also generate series with realistic behavior. We chose a value of 0.1 for the capital and labor stochastic shocks.[24] For the technical-progress parameters, following León-Ledesma et al. (2010), we used a value of 0.01 when the technical progress parameter is set to zero, so that the stochastic component of technical progress does not dominate. When technical progress exceeds zero we used a value of 0.05 so when technical progress is

[23] In practice, setting different values for π_0 did not affect the results.

[24] This is approximately the standard error of labor and capital equipment around a trend with US data from 1950 to 2005. The results, however, remained invariant when we used values of 0.2 and 0.05.

present it is also subject to larger shocks.[25] Finally, for shocks to factor payments, we used the standard deviation of the de-trended real wages and the standard deviation of demeaned user cost of capital for the US economy over 1950-2000.[26] These take values of 0.05 and 0.1 respectively, reflecting the larger volatility of the real user cost.

We used a sample size of 50 (years).[27] Finally, since nonlinear system estimators used require initial guesses for the parameters, which we set these to their true value following León-Ledesma et al. (2010).[28] Although this is relaxed in our estimation on US data (section 5.5).

4.2 Monte Carlo results

4.2.1 Median Estimates

Tables 2 to **4** report the Monte Carlo results when the data are generated according to the $\{\gamma_K\,,\,\gamma_N\,\}$ and $\{\sigma\}$ combinations given in Table 1 but then estimated for the respective cases of Hicks-, Harrod- and Solow neutrality. In the tables, we report the median parameter estimates across the 5,000 draws for the substitution elasticity (and its percentiles) and γ_i.

Where the imposed technical change corresponds to the true DGP (labeled "benchmark" in the tables), the parameters are very precisely estimated, reflecting the power of the normalized system, León-Ledesma et al. (2010). However, in non-benchmark gross complements cases (i.e., the first two columns in each table), systematic bias is almost always found, i.e.,:

$$\sigma^m - \sigma\,\{0.5,\ 0.9\} > 0$$

The gross-substitute, non-benchmarks cases are less clear cut. Whilst, in all but two cases (both relating to Harrod neutrality, Table 2) a gross substitutes production

[25] For robustness purposes, we also replicated the results assuming no shock when technical progress is zero and also equal shocks for both components. The results were not affected by these changes.

[26] From the Bureau of Economic Analysis.

[27] Using values of 100 and 30 led to very similar results, although, as expected, the range of estimated values for the parameters increased as we decrease the sample size.

[28] This facilitates comparisons across specifications and estimator types since we eliminate the effect of arbitrary starting values on our results.

function is correctly identified, almost in all cases there is a downward bias:

$$\sigma^m - \sigma\,\{1.3\} < 0, \text{ with } \sigma^m \approx 1$$

4.2.2 Distributions

The distribution of the substitution elasticities across the 5,000 draws shed further light on these results (**Figures 1, 2** and **3**). Regarding the $\sigma = 0.5$ case, we see that the general factor augmenting specification is always tightly distributed around the true value of the substitution elasticity. The Solow neutral specification, though, yields a bimodal distribution for the two cases in which technical progress is net labor-augmenting. To a smaller degree, the Harrod-neutral specification also shows bimodality in two cases. The distributions also tend to be more skewed when the specified model differs from the true DGP. To illustrate, under a Solow neutral DGP, the Hicks neutral estimation has a median substitution elasticity at $\sigma^m = 0.77$ as well as considerable positive skewness.

The $\sigma = 0.9$ case is interesting given its proximity to Cobb-Douglas, and thus the heightened relevance of the issues raised in Section 3. Note that the densities are now largely symmetric with little skew and limited dispersion, ($\sigma^m \mid \sigma = 0.9 \in [0.89, 1.03]$)[29] and most (12/15) detect gross complements at the median. Consistent with the $\sigma = 0.5$ case above, almost all median estimates exhibit upward biases. In this case, that bias is ostensibly to unity. As earlier discussed, a unitary substitution elasticity is a strong attractor: pulling estimates to the log-linear form captures the broadly balanced growth characteristics of the simulated data minimizing the cost of the imprecise technical change component. Recalling approximation (7), $\widehat{\sigma} \to 1$, neutralizes the effect of quadratic curvature in capital deepening and technical bias, and minimizes the weight given to the individual technical progress components. Furthermore, bi- or multi-modality is more severe than in the $\sigma = 0.5$ (or indeed $\sigma = 1.3$) case, even so for the cases where both forms of technical change are permitted; thus, even the factor-augmenting specification shows a (second) peak around unity in all cases.

For $\sigma = 1.3$ the distributions are, by contrast, much flatter, except for the Solow neutral specification. In the case where $\gamma_N = 0$ and $\gamma_K = 0.02$, the Hicks-neutral specification is very flat, although the scale of the graph makes it difficult to show the frequency variation. This explains the high values for the median σ reported in Table 2 for that case. This value, though, is hardly representative. The factor

[29]For the 0.5 and 1.3, the substitution elasticity ranges are respectively, 0.52-1.07 and 0.85-1.53.

augmenting specification, despite capturing very well the true values of σ, also tend to display a small local maximum around a value of one.

Our MC exercises were necessarily stylized. In particular, we analyzed an environment of balanced (or near balanced) growth. This has several advantages. First, it corresponds to situation common to many developed countries (over reasonably-sized samples). Second, it places our exercises within a familiar context, making the interpretation and motivation of results accordingly more transparent. However, third, it in fact makes for a particularly challenging exercise since estimates framed around something like a balanced growth path may degenerate to unitary elasticities and overlook or strongly bias the nature of technical change. Our next step is to analyze how these potential biases affect estimates of the supply-side parameters and estimates of TFP growth for the US economy.

5 CES Estimation of the US Economy

5.1 Data

We use quarterly seasonally-adjusted time series for the US from 1960:1 to 2004:4. Our principal source is the *NIPA* Tables (National Income and Product Accounts) for production and income.[30] The output series is calculated as Private Non-Residential output: thus, total output minus Indirect Tax Revenues, public-sector and residential output. After these adjustments, the output concept used is compatible with that of our capital stock series. Employment is defined as the sum of self-employed persons and the private sector full-time equivalent employees. To create quarterly private non-residential capital stock compatible with both the annual index of constant replacement cost capital stock (Herman (2000)) and the accumulated NIPA net investment, we first estimated the base value for the capital stock as a ratio:

$$KB = \frac{\sum_{t=0}^{T} \text{Net Investment}}{KI_T \; - \; KI_0} \tag{22}$$

where KI_T and KI_0 refer to the values of the capital stock index at the end and beginning of the sample, respectively. The quarterly constant price non-residential private capital stock was then calculated by accumulating the base level KB from the

[30] These series can be found at `http://www.bea.doc.gov/bea/dn/nipaweb/index.asp`. Moreover, all data, transformation and replication files (for both the MC and US estimation exercises) are available from the authors upon request.

midpoint of the sample by using the quarterly NIPA series of non-residential private net investment. This procedure ensures that the constructed quarterly capital stock has the same trend as the annual capital stock index. The time-varying depreciation rate was calculated as the ratio of NIPA consumption of non-residential capital to the capital stock lagged one period. The nominal user cost is defined as the product of the investment deflator and the real user cost, the latter being the sum of real interest rate (defined in terms of investment deflator inflation) and the depreciation rate. The underlying interest rate is the U.S. 5-year government securities rate.

Figures 4 and **5** present some variables of interest. The capital-output ratio appears to show a declining trend during this period (although it is not very pronounced and is more relevant during the first half of the sample). Both labor productivity and capital intensity show clear upward trends that are close to each other, although the former ratio has grown slightly faster, particularly from the mid-1990s. The share of labor in income shows sizeable variations: it declines up to 1965 to its minimum, then increases to remain at a higher level during the 1970s. The share then declines from the mid-1980s onwards, but not monotonically.[31].

Figure 5 shows the evolution of real wages together with labor productivity (in index form) and the real user cost of capital. The user cost shows considerable volatility from the mid-1970s to the mid-1980s mostly due to the volatility of quarterly inflation. We can also observe that since the mid-1960s, real wages grow slightly above labor productivity (which is informal evidence that the substitution elasticity lies below one) .

Standard ADF tests on capital and labor shares rejected the null of non-stationarity, although only marginally so. For the capital-output ratio, an ADF test rejects the null of non-stationarity with a significant deterministic trend. The magnitude of the trend, however, is very small (about 0.2% *per year*), indicating that the deviation from balanced growth is not dramatic. The existence of this trend, together with broadly stable factor shares, suggests that technical progress *cannot only be labor-augmenting.*

5.2 Specification

Given the practical existence of a markup over factor costs in the data, the estimated model includes an extra parameter $\mu \geq 0$ which captures an estimated average mark-

[31] In our data, capital and labor shares are defined in terms of factor incomes and thus do not sum to unity owing to the existence of an aggregate mark-up (this is later introduced as a parameter in the normalized system)

up. Further, as the real user cost of capital occasionally takes negative values, the FOC for capital enters in levels.

Also, with real data, to diminish the size of stochastic component in the point of normalization we prefer to define the normalization point in terms of sample averages (geometric averages for growing variables and arithmetic ones otherwise). The nonlinearity of the CES function, in turn, implies that the sample average of production need not exactly coincide with the level of production implied by the production function with sample averages of the right hand variables. Following Klump et al. (2007), we therefore introduce an additional parameter ζ whose expected value is around unity. Hence, we can define $Y_0 = \zeta\bar{Y},\ K_0 = \bar{K},\ N_0 = \bar{N}$; $t_0 = \bar{t}$ and $\pi_0 = \bar{\pi}$ where the bar refers to the appropriate type of sample average. The estimated system, allowing for factor augmentation, is then,

$$r = \left(\frac{\bar{\pi}}{1+\mu}\frac{\zeta\bar{Y}}{\bar{K}}\right)\left[\frac{Y/(\zeta\bar{Y})}{K/\bar{K}}\right]^{\frac{1}{\sigma}} e^{\frac{\sigma-1}{\sigma}\gamma_K(t-\bar{t})} \tag{23}$$

$$\log(w) = \log\left(\frac{(1-\bar{\pi})}{1+\mu}\frac{\zeta\bar{Y}}{\bar{N}}\right) + \frac{1}{\sigma}\log\left(\frac{Y/(\zeta\bar{Y})}{N/\bar{N}}\right) + \frac{\sigma-1}{\sigma}\gamma_N(t-\bar{t}) \tag{24}$$

$$\log\left(\frac{Y}{\bar{Y}}\right) = \log\zeta + \frac{\sigma}{\sigma-1}\log\left[\bar{\pi}\left(\frac{e^{\gamma_K(t-\bar{t})}K}{\bar{K}}\right)^{\frac{\sigma-1}{\sigma}} + (1-\bar{\pi})\left(\frac{e^{\gamma_N(t-\bar{t})}N}{\bar{N}}\right)^{\frac{\sigma-1}{\sigma}}\right] \tag{25}$$

For the estimation of the system we fix parameter $\bar{\pi}$ to its sample average, which is one of the empirical advantages of normalization. We also obtained the results estimating $\bar{\pi}$ freely, but it made no difference to the other relevant parameters.

The system is estimated using two methods. First we use a Generalized Nonlinear Least Squares (*GNLLS*) estimator which is equivalent to a nonlinear SUR model, allowing for cross-equation error correlation. We also used a nonlinear-3SLS (*NL3SLS*) estimator using a constant, a trend, and the first two lags of all the variables as instruments.[32] In both cases we report heteroscedasticity and autocorrelation consistent standard errors.

Initial conditions for the parameters were set as follows: $\zeta(0) = 1$, $\mu(0) = 0.1$. For the other parameters we used the values from the OLS estimation of the FOC for labor (following Thursby (1980)), equation (24). However, we later on (see section (5.5)) use a range of initial values for robustness analysis.

[32]This is a particular case of a more general GMM estimator.

5.3 Estimation results

The results of the two estimation methods for the four specifications are reported in **Tables 5a-b** and **6a-b**. The tables also report, together with the Log-Determinant of the system (our goodness-of-fit measure[33]), an LR test for the null of the specified neutrality against general factor augmentation and, in the NL3SLS case, a *J-test* for instrument validity. We also report ADF-type unit root tests on the residuals of the three equations of the system. Given that we do not know the distribution of the statistic under the no-cointegration null, we use bootstrapped p-values following Park (2003) and Chang and Park (2003).

Both estimation methods show similar patterns across specifications. In all cases, the null of no-cointegration for each equation is rejected according to the bootstrapped p-values. For the NL3SLS estimator, the J-test rejects instrument validity only for the Solow neutral specification. The estimate of the average mark-up parameter, μ, is very close to 0.11 in all cases. The scale parameter, ζ, is practically indistinguishable from unity, again consistent with our priors. The estimate of the substitution elasticity in the factor augmenting specification is 0.6 for the GNLLS estimator and 0.5 for the NL3SLS estimator. Manifestly, these estimates are well below and significantly different from unity. The values for labor-augmenting technical progress in both estimation methods imply an annual growth rate of 1.6% and a non-negligible 0.56-0.72% annual growth rate for capital augmenting technical progress. Thus, technical progress is net labor-saving, but capital-augmenting technical progress cannot be dismissed.

Regarding other specifications, we see that the σ estimates are substantially different from those obtained with general factor augmentation. The point estimate of σ with Hicks neutrality is 0.2-0.3 points higher than the one with factor augmentation. This is consistent with the results from the MC experiment. Although still significantly below one, the Hicks specification biases the estimate of the substitution elasticity towards one. This results from the fact that technical progress contains also a positive capital-augmenting component while it deviates from Hicks-neutrality. The Solow neutral specification leads to an even sharper bias towards Cobb-Douglas. Again, looking back at the results in Table 4 this is consistent with our simulations, which showed that the more the DGP deviates from Solow neutrality, the stronger the bias towards unity. In the case of the Harrod-neutral specification which, together with Hicks-neutral, is most commonly used for estimation, we observe that the results are strongly biased upwards. The NLLS estimator

[33] The preferred specification is the one that minimizes the log-determinant.

yields a very high $\widehat{\sigma} = 1.7$, whilst the NL3SLS estimator yields 1.3. According to our experiments, and for the estimated values of the technical progress parameters, we would expect a much smaller upwards bias if the true σ is in the region of 0.5. However, the density graphs showed that the Harrod-neutral specification can lead to multi-modal distributions. In fact, changing the initial conditions for the estimator in this case led to substantially different results and, in some cases, the nonlinear algorithm was unable to converge. This sensitiveness to initial conditions calls for caution when interpreting the results for this particular specification.

Finally, the log-determinant always showed the same ordering of across specifications. The general factor-augmenting specification had the best fit, followed by Hicks, Solow then Harrod-neutral. In addition, the results from the LR test for the restrictions implied by specific forms of factor augmentation, always reject the restrictions in favor of the general factor augmenting specification. Hence, our results support the use of a more general specification for technical progress and confirm our claim that mis-specification of technical progress can lead to important biases in the estimated substitution elasticity.

Figure 6 plots the model residuals for the four specifications. For the user cost, the three models yield almost the same fit; similarly so for output, although differences widen from 1990 onwards. The main difference emerges in the way the models fit wages, especially for the Harrod-neutral specification (un-surprisingly given its high $\hat{\sigma}$ values).[34] Of course, even if the three models yield similar fit for variables such as output, the implications of the different estimates of the substitution elasticity and technical progress to explain the evolution of factor shares are still markedly different. As we will now see this is also the case for estimates of TFP growth.

5.4 TFP Estimates

We obtained estimates of TFP *growth* arising from (6) and the approximations (7)-(10) and, the simplified approximations, (11) and (12). **Figure 7** plots the GNLLS estimates of TFP separately for each specification (alongside capital deepening).[35] The Hicks-neutral specification, necessarily yields constant growth of TFP, and hence we do not plot it separately. The rest of specifications will always yield increasing or decreasing TFP *growth* (except when linear weight, (12), is used). This

[34]Interestingly, this is a result that Fisher et al. (1977) also obtained in a simulation experiment analyzing production function aggregation. Despite many specifications providing a good fit for output, wages proved much more sensitive to the estimated values of σ.

[35]The NL3SLS ones delivered very similar conclusions.

can be seen in expressions (7) and (11), whose rate of growth is going to be trended owing to the quadratic component. Whether the trend is positive or negative depends on parameter "a", whose sign is a function of whether $\sigma \gtreqless 1$ (except in the Hicks case when the trend is zero).

We see that the exact residual method (6) and the Kmenta approximation, (7), give practically identical TFP (minor deviations can be found at the sample extremities). The simple form excluding capital deepening applies wrong trends to the growth rate in TFP in the context of factor-augmenting and Harrod-neutral specifications. Under the Solow neutral specification, however, it works quite satisfactorily. We may conclude that the inclusion of capital deepening is important to capture correctly nonlinearities in TFP growth rates. It is interesting to see that the our favored factor-augmenting case implies accelerating TFP growth especially at the end of the sample.[36] This is compatible with the then observed acceleration of productivity growth (e.g., Basu et al. (2003), Fernald and Ramnath (2004) and Jorgenson (2001)). The Harrod and, in turn, the Solow neutral cases, implies decelerating TFP growth. From our perspective of specification bias, it is illustrating to note that the differences in annualized TFP growth around year 2001 implied by these three specifications are substantial. They range from about 0.8% *per year* for the Harrod-neutral specification, to 1.4% for the factor-augmenting specification.

5.5 Robustness

The nonlinear algorithms can potentially be sensitive to the initialization of the parameter values. In the MC we can set these equal to their true value to abstract from the problem; this is clearly not an option in actual estimation. The model was therefore re-estimated for the initialization range of the substitution elasticity, $\sigma(0) \in [0.2 : 0.05 : 1.2]$. **Figures 8** and **9** present the plots of the sensitivity analysis for both estimation methods. We plot the estimated σ vs. the initialization value and the log-determinant of the system against the estimated σ. For the GNLLS estimator, initial values below 0.5 tend to yield lower $\hat{\sigma}$'s, however, the value always ranges from 0.48 to 0.65, the exception being an initial value of 1, which is an inflexion point for the CES function (see La Grandville and Solow (2009)), and finds an estimated σ close to one. However, the log-determinant for this case is the highest,

[36] This is consistent with the idea that investment in IT led to an economy-wide productivity increase. In our model, however, we do not separate types of capital and so cannot infer anything about the specific source of this acceleration. However, as far as this capital deepening is related with investment in new technologies, our results seem to support the contention that there was a productivity acceleration in the US from the mid-1990s until the early 2000s.

and the model performs substantially worse. For the rest of the initialization points and estimated σ's the log-determinant is quite flat, and usually finds a minimum in the neighborhood of 0.6, as the one reported in Table 5a. Similar conclusions can be reached with the NL3SLS estimator (**Figures 9**), but in this case changes in initial conditions lead to almost no change to the estimated σ (with the exception of an initial value of one). For reasons of space, we only present here the sensitivity analysis for our preferred factor-augmenting specification. Similar results are found for the other specifications with the notable exception of the Harrod-neutral one, as already commented above.

5.6 Some Lessons Learnt

Pulling together the salient points arising from the MC and empirical estimates, we can extract a series of practical lessons about estimation of supply-side systems:

Implications of a priori choices on the nature of technical change. Estimation of the substitution elasticity can be substantially biased if the form of technical progress is mis-specified. For some parameter values, when factor shares are relatively constant, there could be an inherent bias towards Cobb-Douglas, or else Cobb Douglas could be a strong attractor for initial conditions set within its neighborhood.[37] Our empirical results show that the estimated substitution elasticity tends to be significantly lower using a factor augmenting specification and is well below one. We were able to reject Hicks-, Harrod- and Solow-neutral specifications in favor of general factor augmentation with a non-negligible capital-augmenting component.

Beware Cobb Douglas. Situations of near balanced growth may lead to estimation erroneously favoring the unitary elasticity case. This is clear in some cases such as Hicks Neutrality where a unitary bias shrinks the importance of trended capital deepening. Similarly, when seen through the lens of the augmented Kmenta approximation, a unitary elasticity shrinks the impact of quadratic curvature in capital deepening and biased technical change. Furthermore, the MC distributions

[37] All nonlinear estimation requires initial parameter conditions to facilitate algorithmic convergence (see McAdam and Huges-Hallett (1999), Andrle (2010) and the references therein). If the likelihood is unimodal or shows sufficient curvature, the problem of incorrect initial conditions and thus of local minima attenuates. However, we have shown here that multiple equilibria is not just a numerical artefact of certain algorithms but has a parallel in theory (recall the discussion in section (3)).

tended to show a separate mode for the unitary elasticity case, particularly if initial conditions were set within that neighborhood.

There is no simple solution to degenerate Cobb-Douglas estimates, other than some of the practices followed here: discriminating on the basis of global statistical criterion among competing specifications (in our case, the log determinant); varying initial conditions and checking for local maxima; inspecting the great ratios to check for stationarity; and hints for the potential presence of capital-augmenting or non-constant technical progress components (e.g., Klump et al. (2007)).

Aggregate studies favoring Cobb Douglas, though, are rarer than its theoretical dominance might suggest.[38] But there is still arguably a tendency in the literature to report high near-unity substitution elasticities and neglect the role of biases in technical change. Given how useful the analysis of biased technical change has proved (Acemoglu (2009)), this is clearly an error of some proportion.

The Fit of the Production Function versus the Fit of Factor Returns. Figures 6 implicitly make an important, even startling, point. The quite similar production-function residuals suggest that the goodness of fit of production functions appears relatively robust to mis-specified technical neutrality assumptions.[39] The reason is that mis-specification of technical change a under CES production function implies compensating bias in the estimate of the elasticity of substitution.

However, an important qualification (echoing that of Fisher et al. (1977)) is that using an "incorrect" production function may simply shift estimation failures elsewhere. In our case, this arose most clearly in the real wage equations where there is considerable variations in the fit across specifications.[40] This is another reason to follow the system estimation method rather than single-equation approaches.

TFP Growth The dispersion of TFP estimates mirrors (albeit to a more dramatic degree) that of the real wage. Monitoring the level and sources of TFP growth is a key application of the production function literature and a key input into policy debates. Recalling Figures 4 and 5, we see an acceleration in US labor productivity towards the end of the sample driven by capital deepening in combination with technical change. And yet (Figure 7) a Solow-Neutral specification predicts that TFP decelerated rapidly throughout the sample; the factor augmenting case tracked

[38] See, for instance, Table 1 of León-Ledesma et al. (2010).

[39] An early indication of this was given by Willman (2002).

[40] Interestingly, this is exactly what Christoffel et al. (2010) report for their macro-econometric forecasting and simulation model, the NAWM which employs an aggregate Cobb-Douglas production function: good forecasting performance for many real variables (including the output gap) but large and persistent errors in forecasting real wages and the labor share.

TFP growth as being relatively stable prior to an acceleration in the early 1990s; the Harrod Neutral case is the near mirror image of that.

There is an important lesson to be drawn here. Given the discussions in Sections 2 and 5.4, we know that whether the substitution elasticity is above or below unity matters for the transmission of capital deepening and factor-augmenting technical change for TFP's evolution. Getting the substitution elasticity right is hence necessary to correctly estimate TFP growth.

6 Conclusions

The elasticity of substitution between capital and labor is the key parameter that shapes the relationship between factor inputs and output. It plays a fundamental role in models of growth, income distribution, stabilization policy, labor market outcomes, etc. With barely a few exceptions, however, most macroeconomic models work under the assumption of unitary factor substitution. Increasingly, however, empirical evidence favors the more general CES production function. A non-unitary substitution elasticity raises the prospect of identifiable and possibly non-neutral technical change.

We analyzed the effect of mis-specification of technical change on production function estimation. Taking the nonlinear CES function to the data admittedly poses greater difficulties compared to Cobb Douglas. Accordingly, short cuts are often taken, such as the prior imposition of Hicks- or Harrod-neutral technical change. This is either because of the complications arising from the nonlinear CES functional form, or because of theoretical considerations relating to balanced growth. We argue that, when technical progress takes a more general factor augmenting form, mis-specifying technical change can lead to substantial biases. We then provide quantitative evidence using a Monte Carlo experiment and show that, when the substitution elasticity is below (above) unity, the estimated substitution elasticity is biased upwards (downwards). For some parameter values, this bias could potentially tend towards one, the Cobb-Douglas case. When the true factor augmentation of technical progress is either Hicks-, Harrod-, or Solow neutral, estimating the CES as general factor augmenting yields little cost in terms of biases in estimated parameters. We also show that the bias arising in the substitution elasticity estimate can affect estimates of total factor productivity (TFP) growth.

We then estimate a "normalized" supply side system for the US economy for the 1960:1–2004:4 period. We can reject the Hicks-, Harrod- and Solow-neutral specifications in favor of a factor augmenting one. We find that capital-augmenting technical progress is non-negligible (0.6-0.7% per year). Importantly, the substitution elasticity is found to be around 0.5-0.6, emphatically rejecting Cobb Douglas. That is, our results robustly question the use of Cobb-Douglas production functions for aggregate studies of the US economy in favor of a general CES function. We also provide evidence that the implied TFP growth estimates for the various specifications used is substantially different.

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References

Acemoglu, D. 2002. Technical Change, Inequality and the Labor Market. *Journal of Economic Literature*, 40(1):7–72.

Acemoglu, D. 2009. *Introduction to Modern Economic Growth*. MIT Press.

Andrle, M. 2010. Identification Patterns in Models. Working Paper, Czech National Bank, Research Department.

Antràs, P. 2004. Is the US Aggregate Production Function Cobb-Douglas? New Estimates of the Elasticity of Substitution. *Contributions to Macroeconomics*, 4:Article 4.

Arrow, K. J., Chenery, H. B., Minhas, B. S., and Solow, R. M. 1961. Capital-labor substitution and economic efficiency. *Review of Economics and Statistics*, 43(3):225–250.

Bairam, E. I. 1991. Elasticity of Substitution, Technical Progress and Returns to Scale in Branches of Soviet Industry. *Journal of Applied Econometrics*, 6(1):91–96.

Basu, S., Fernald, J. G., Oulton, N., and Srinivasan, S. 2003. The case of the missing productivity growth, or does information technology explain why productivity accelerated in the U.S. and not in the U.K.? *NBER Macroeconomics Annual*, 18:9–63.

Berndt, E. R. 1976. Reconciling alternative estimates of the elasticity of substitution. *Review of Economics and Statistics*, 58(1):59–68.

Chang, Y. and Park, J. Y. 2003. A sieve bootstrap for the test of a unit root. *Journal of Time Series Analysis*, 24:379–400.

Chirinko, R. S. 2002. Corporate taxation, capital formation, and the substitution elasticity between labor and capital. *National Tax Journal*, 60:339–355.

Chirinko, R. S. 2008. Sigma: The Long and Short of It. *Journal of Macroeconomics*, 30(2):671–686.

Chirinko, R. S., Fazzari, S. M., and Meyer, A. P. 1999. How responsive is business capital formation to its user cost? *Journal of Public Economics*, 74(1):53–80.

Christoffel, K., Coenen, G., and Warne, A. 2010. Forecasting with DSGE Models. In Clements, M. and Hendry, D., editors, *Oxford Handbook on Economic Forecasting.* Oxford University Press, forthcoming.

David, P. A. and van de Klundert, T. 1965. Biased efficiency growth and capital-labor substitution in the US, 1899-1960. *American Economic Review*, 55(3):357–394.

Diamond, P. A. and McFadden, D. 1965. Identification of the Elasticity of Substitution and the bias of Technical Change: An Impossibility Theorem. Working Paper No. 62, University of California Berkeley.

Diamond, P. A., McFadden, D., and Rodriguez, M. 1978. Measurement of the elasticity of substitution and bias of technical change. In Fuss, M. and McFadden, D., editors, *Production Economics, Vol. 2*, pages 125–147. Amsterdam and North Holland.

Duffy, J. and Papageorgiou, C. 2000. A cross-country empirical investigation of the aggregate production function specification. *Journal of Economic Growth*, 5(1):86–120.

Evans, P. 2000. Stylized Facts and their implications for Growth Theory. Mimeo, Ohio State University.

Fernald, J. G. and Ramnath, S. 2004. The Acceleration in U.S. Total Factor Productivity after 1995: The Role of Information Technology. *Economics Perspectives, Federal Reserve Bank of Chicago*, 1Q:52–67.

Fisher, F. M., Solow, R. M., and Kearl, J. M. 1977. Aggregate Production Functions: Some CES Experiments. *Review of Economic Studies*, 44(4):305–320.

Greenwood, J., Hercowitz, Z., and Krussel, P. 1997. Long-Run Implications of Investment-Specific Technical Change. *American Economic Review*, 87:342–362.

Herman, S. W. 2000. Fixed assets and consumer durable goods. Survey of Current Business.

Hicks, J. R. 1932. *The Theory of Wages.* London: Macmillan.

Jones, C. I. 2003. Growth, Capital Shares, and a New Perspective on Production Functions. Mimeo, University of California Berkeley.

Jorgenson, D. W. 2001. Information technology and the U.S. economy. *American Economic Review*, 91:1–32.

Klump, R. and de La Grandville, O. 2000. Economic growth and the elasticity of substitution: Two theorems and some suggestions. *American Economic Review*, 90(1):282–291.

Klump, R., McAdam, P., and Willman, A. 2007. Factor Substitution and Factor Augmenting Technical Progress in the US. *Review of Economics and Statistics*, 89(1):183–92.

Klump, R. and Preissler, H. 2000. CES production functions and economic growth. *Scandinavian Journal of Economics*, 102(1):41–56.

Klump, R. and Saam, M. 2008. Calibration of normalized CES production functions in dynamic models. *Economics Letters*, 99(2):256–259.

Kmenta, J. 1967. On Estimation of the CES Production Function. *International Economic Review*, 8(2):180–189.

Krussel, P., Ohanian, L., Rios-Rull, J. V., and Violante, G. 2000. Capital-Skill Complementarity and Inequality. *Econometrica*, 68(5):1029–1053.

La Grandville, Olivier de. 1989. In Quest of the Slutsky Diamond. *American Economic Review*, 79(3):468–481.

La Grandville, Olivier de. 2009. *Economic Growth: A Unified Approach*. Cambridge University Press.

La Grandville, Olivier de. and Solow, R. M. 2009. Capital-labour substitution and economic growth. In *Economic Growth: A Unified Approach*. Cambridge University Press. chapter 5.

León-Ledesma, M. A., McAdam, P., and Willman, A. 2010. Identifying the Elasticity of Substitution with Biased Technical Change. *American Economic Review*, forthcoming.

Marquetti, A. A. 2003. Analyzing historical and regional patterns of technical change from a Classical-Marxian perspective. *Journal of Economic Behavior and Organization*, 52(2):191–200.

McAdam, P. and Huges-Hallett, A. J. 1999. Nonlinearity, computational complexity and macroeconomic modeling. *Journal of Economic Surveys*, 13(5):577–618.

McAdam, P. and Willman, A. 2008. Medium Run Redux. Working Paper No. 915, European Central Bank.

Orphanides, A. 2003. Monetary policy evaluation with noisy information. *Journal of Monetary Economics*, 50:605–631.

Park, J. Y. 2003. Bootstrap unit root tests. *Econometrica*, 71:1845–1895.

Rapach, D. E. and Wohar, M. E. 2005. Regime changes in international real interest rates. *Journal of Money, Credit and Banking*, 37:887–906.

Rowthorn, R. 1999. Unemployment, wage bargaining and capital-labour substitution. *Cambridge Journal of Economics*, 23(4):413–425.

Sato, R. 2006. *Biased Technical Change and Economic Conservation Laws*. Springer.

Solow, R. M. 1957. Technical change and the aggregate production function. *Review of Economics and Statistics*, 39(3):312–320.

Thursby, J. 1980. Alternative CES estimation techniques. *Review of Economics and Statistics*, 62(2):259–299.

Uzawa, H. 1961. Neutral inventions and the stability of growth equilibrium. *Review of Economic Studies*, 28:117–124.

Willman, A. 2002. Euro area production function and potential output: a supply side system approach. Working Paper No. 153, European Central Bank.

Yuhn, K.-H. 1991. Economic Growth, Technical Change Biases, and the Elasticity of Substitution: A Test of the De La Grandville Hypothesis. *Review of Economics and Statistics*, 73(2):340–346.

Tables

Table 1. Parameter values for the Monte Carlo

Parameter	Description	Values
π_0	Distribution parameter	0.4
σ	Substitution elasticity	0.5, 0.9, 1.3
γ_K	Capital-Augmenting Technical Progress*	0.00, 0.005, 0.01, 0.015, 0.02
γ_N	Labor-Augmenting Technical Progress*	0.02, 0.015, 0.01, 0.005, 0.00
η	Labor growth rate	0.015
κ	Capital growth rate	See text
$Y_0^* = N_0$	Normalization values for output and labor	1
K_0	Normalization value for capital	π_0 / r_0
r_0	Normalization value for the user cost	0.05
α	Capital Trend Absorption in r	0.50
$\sigma_{\varepsilon_t^N}, \sigma_{\varepsilon_t^K}$	Standard Error, Labor and Capital DGP shock	0.10
$\sigma_{\varepsilon_t^{\Gamma^K}}$	Standard Error, Capital-Augmenting Technical Progress shock	0.01 for $\gamma_K = 0$; 0.05 for $\gamma_K \neq 0$
$\sigma_{\varepsilon_t^{\Gamma^N}}$	Standard Error, Labor-Augmenting Technical Progress shock	0.01 for $\gamma_N = 0$; 0.05 for $\gamma_N \neq 0$
$\sigma_{\varepsilon_t^w}$	Standard Error, Real Wage shock	0.05
$\sigma_{\varepsilon_t^r}$	Standard Error, Real Interest Rate shock	0.10
T	Sample Size	50
M	Monte Carlo Draws	5,000

Note: *, $\gamma_N + \gamma_K = 0.02$

Table 2. Monte Carlo results. Hicks-neutral specification

	$\sigma = 0.5$	$\sigma = 0.9$	$\sigma = 1.3$
		$\gamma_K = 0.00, \gamma_N = 0.02$	
σ^m	0.8670	0.9893	1.0458
10% : 90%	0.7679 : 1.0078	0.9135 : 1.0850	0.9460 : 1.1467
γ^m	0.0120	0.0120	0.0121
		$\gamma_K = 0.005, \gamma_N = 0.015$	
σ^m	0.6966	0.9603	1.1442
10% : 90%	0.6151 : 0.8609	0.8617 : 1.0773	0.9989 : 1.3009
γ^m	0.0109	0.0110	0.0110
		Benchmark $\gamma_K = \gamma_N = 0.01$	
σ^m	0.5198	0.9144	1.3084
10% : 90%	0.4688 : 0.5940	0.8068 : 1.0550	1.1177 : 1.5612
γ^m	0.0101	0.0100	0.0100
		$\gamma_K = 0.015, \gamma_N = 0.005$	
σ^m	0.7257	0.8992	1.5341
10% : 90%	0.6009 : 0.9180	0.7921 : 1.0253	1.2407 : 2.0363
γ^m	0.0099	0.0092	0.0090
		$\gamma_K = 0.02, \gamma_N = 0.00$	
σ^m	0.9597	0.9814	1.5198
10% : 90%	0.8060 : 1.4691	0.8617 : 1.1450	1.2185 : 2.1014
γ^m	0.0081	0.0082	0.0077

Table 3. Monte Carlo results. Harrod-neutral specification

	$\sigma = 0.5$	$\sigma = 0.9$	$\sigma = 1.3$
	Benchmark $\gamma_K = 0.00, \gamma_N = 0.02$		
σ^m	0.5206	0.8998	1.2949
10% : 90%	0.4815 : 0.5568	0.8045 : 1.0183	1.0962 : 1.5780
γ^m	0.0198	0.0201	0.0200
	$\gamma_K = 0.005, \gamma_N = 0.015$		
σ^m	0.5873	0.9155	1.2535
10% : 90%	0.5317 : 0.7315	0.8290 : 1.0276	1.0581 : 1.4891
γ^m	0.0163	0.0186	0.0177
	$\gamma_K = \gamma_N = 0.01$		
σ^m	0.8187	0.9726	1.1109
10% : 90%	0.7100 : 0.9642	0.8686 : 1.0889	0.9149 : 1.3236
γ^m	0.0171	0.0171	0.0158
	$\gamma_K = 0.015, \gamma_N = 0.005$		
σ^m	0.9503	1.0091	0.9299
10% : 90%	0.8517 : 1.1824	0.9196 : 1.1308	0.7776 : 1.1582
γ^m	0.0156	0.0153	0.0149
	$\gamma_K = 0.02, \gamma_N = 0.00$		
σ^m	1.0670	1.0315	0.8504
10% : 90%	0.9370 : 1.3455	0.9469 : 1.1329	0.7213 : 0.9992
γ^m	0.0128	0.0134	0.0140

Table 4. Monte Carlo results. Solow-neutral specification

	$\sigma = 0.5$	$\sigma = 0.9$	$\sigma = 1.3$
		$\gamma_K = 0.00, \gamma_N = 0.02$	
σ^m	0.7685	1.0049	1.0007
10% : 90%	0.7122 : 0.9988	0.9651 : 1.0431	0.9530 : 1.0403
γ^m	0.0212	0.0299	0.0301
		$\gamma_K = 0.005, \gamma_N = 0.015$	
σ^m	0.8946	0.9943	1.0338
10% : 90%	0.7220 : 0.9676	0.9483 : 1.0393	0.9794 : 1.0802
γ^m	0.0275	0.0276	0.0271
		$\gamma_K = \gamma_N = 0.01$	
σ^m	0.8360	0.9808	1.0872
10% : 90%	0.7485 : 0.9258	0.9282 : 1.0348	1.0215 : 1.1465
γ^m	0.0262	0.9980	0.0241
		$\gamma_K = 0.015, \gamma_N = 0.005$	
σ^m	0.6911	0.9561	1.1682
10% : 90%	0.5715 : 0.8117	0.8875 : 1.0263	1.0754 : 1.2556
γ^m	0.0217	0.0228	0.0219
		Benchmark $\gamma_K = 0.02, \gamma_N = 0.00$	
σ^m	0.5274	0.9138	1.3080
10% : 90%	0.4764 : 0.5722	0.8332 : 1.0006	1.1809 : 1.4385
γ^m	0.0201	0.0201	0.0201

Table 5a. GNLLS estimates of the normalized system, US, 1960:1-2004:4

	Parameter Estimate	Standard error	T-ratio
	Factor Augmenting specification		
	Parameter Estimate	Standard error	T-ratio
σ	0.6301	0.0407	15.4674
γ_N	0.0040	0.0002	19.4031
γ_K	0.0018	0.0004	4.9988
ζ	0.9988	0.0055	183.024
μ	0.1072	0.0090	11.8900
Log-determinant	-21.172		
FOC_K ADF	-2.6350	[0.0112]	
FOC_N ADF	-3.4668	[0.0001]	
CES ADF	-2.9666	[0.0028]	
	Hicks Neutral specification		
σ	0.8000	0.0426	18.7548
γ	0.0034	0.0001	25.9222
ζ	1.0013	0.0065	153.539
μ	0.1106	0.0085	12.9320
Log-determinant	-21.137		
FOC_K ADF	-2.6524	[0.0072]	
FOC_N ADF	-3.6366	[0.0000]	
CES ADF	-3.0749	[0.0012]	
Hicks vs. Factor Augmenting	6.6112	[0.0101]	

Notes: p-values in squared parentheses. Auto-correlation and heteroskedasticity robust standard errors reported. The p-values for the residual ADF (co-integration) tests were obtained from 2,500 bootstrap draws.

Table 5b. GNLLS estimates of the normalized system, US, 1960:1-2004:4. (Contd.)

	Parameter Estimate	Standard error	T-ratio
	Harrod-Neutral Specification		
σ	1.7020	0.3565	4.7739
γ	0.0046	0.0002	23.2089
ζ	1.0035	0.0080	124.740
μ	0.1106	0.0059	18.6950
Log-determinant	-20.936		
FOC_K ADF	-2.7551 [0.0044]		
FOC_N ADF	-2.5239 [0.0100]		
CES ADF	-3.1665 [0.0000]		
Harrod vs. Factor augmenting	45.3494 [0.0000]		
	Solow-Neutral specification		
σ	0.9504	0.0171	55.6004
γ	0.0131	0.0005	26.3396
ζ	1.0045	0.0072	140.008
μ	0.1131	0.0085	13.3727
Log-determinant	-21.111		
FOC_K ADF	-2.6490 [0.0064]		
FOC_N ADF	-3.3522 [0.0000]		
CES ADF	-3.0052 [0.0032]		
Solow vs. Factor augmenting	11.7390 [0.0000]		

Notes: see Table 5a.

Table 6a. NL3SLS estimates of the normalized system, US, 1960:1-2004:4

	Factor Augmenting specification		
	Parameter Estimate	Standard error	T-ratio
σ	0.4914	0.0254	19.3835
γ_N	0.0041	0.0002	27.6293
γ_K	0.0014	0.0002	6.4195
ζ	0.9979	0.0042	239.210
μ	0.1049	0.0088	11.8532
Log-determinant	-21.153		
J-test	36.216 [0.5060]		
FOC_K ADF	-2.5744 [0.0064]		
FOC_N ADF	-3.2001 [0.0001]		
CES ADF	-2.9582 [0.0028]		
	Hicks Neutral specification		
σ	0.7845	0.0425	18.4753
γ	0.0034	0.0001	25.8856
ζ	1.0019	0.0065	153.178
μ	0.1115	0.0087	12.7974
Log-determinant	-21.134		
J-test	17.035 [0.9987]		
FOC_K ADF	-2.6346 [0.0096]		
FOC_N ADF	-3.6622 [0.0000]		
CES ADF	-3.0984 [0.0016]		
Hicks vs. Factor Augmenting	2.7272 [0.0986]		

Notes: see Table 5a.

Table 6b. NL3SLS estimates of the normalized system, US, 1960:1-2004:4. (Contd.)

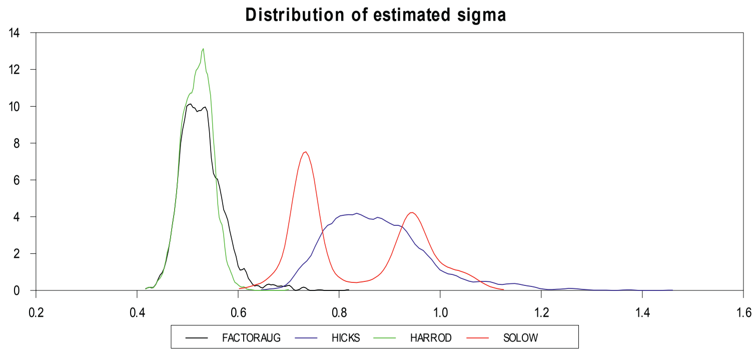
	Harrod-Neutral Specification		
	Parameter Estimate	Standard error	T-ratio
σ	1.3382	0.2159	6.1970
γ	0.0047	0.0002	23.5301
ζ	1.0005	0.0080	125.110
μ	0.1083	0.0067	16.1153
Log-determinant	-20.917		
J-test	24.4660 [0.9563]		
FOC_K ADF	-2.7669 [0.0056]		
FOC_N ADF	-2.7090 [0.0060]		
CES ADF	-3.0390 [0.0024]		
Harrod vs. Factor augmenting	44.481 [0.0000]		
	Solow-Neutral specification		
σ	0.9497	0.0172	55.3298
γ	0.0131	0.0005	26.3164
ζ	1.0046	0.0072	139.546
μ	0.1133	0.0085	13.3537
Log-determinant	-21.111		
J-test	14993.2 [0.0000]		
FOC_K ADF	-2.6457 [0.0068]		
FOC_N ADF	-3.3533 [0.0016]		
CES ADF	-3.0073 [0.0040]		
Solow vs. Factor augmenting	7.2402 [0.0071]		

Notes: see Table 5a.

Figures

Figure 1. Distribution of estimated σ. True σ = 0.5.

$\gamma_K = 0.00, \gamma_N = 0.02$



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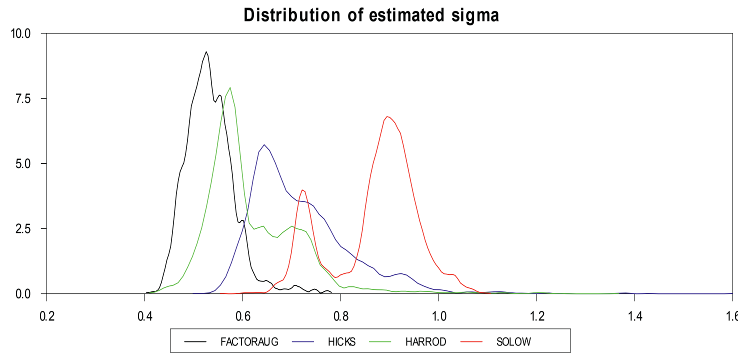

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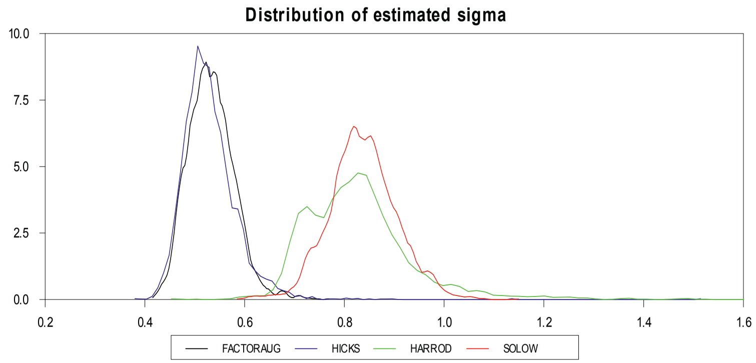


$\gamma_K = 0.02, \gamma_N = 0.00$

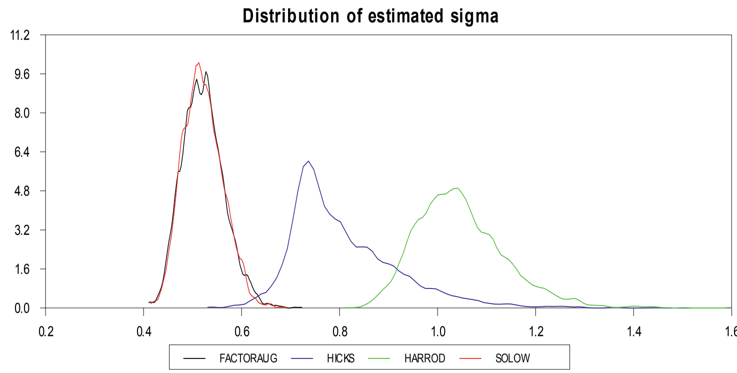
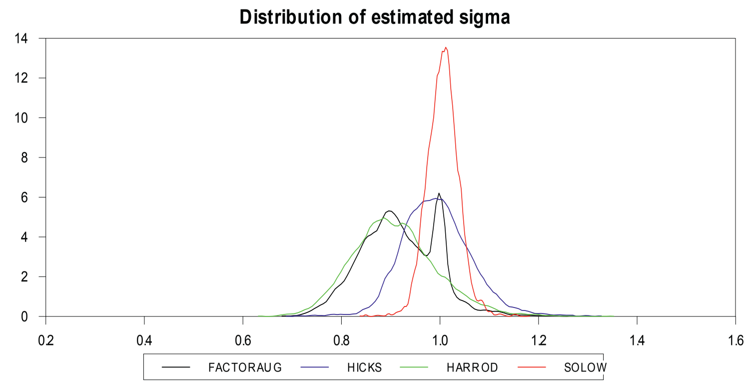
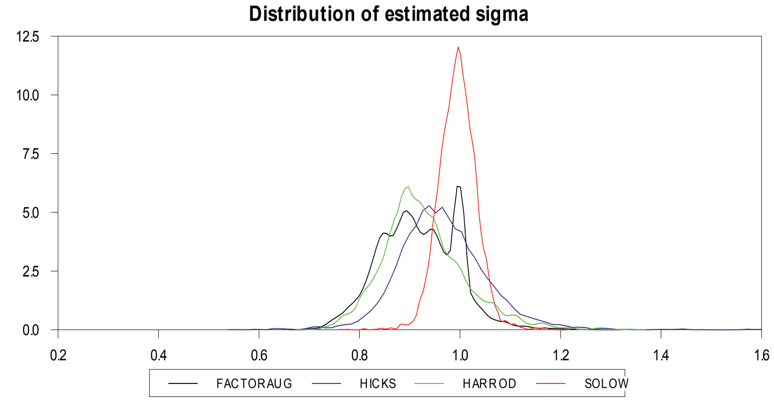


Figure 2. Distribution of estimated σ. True σ = 0.9.

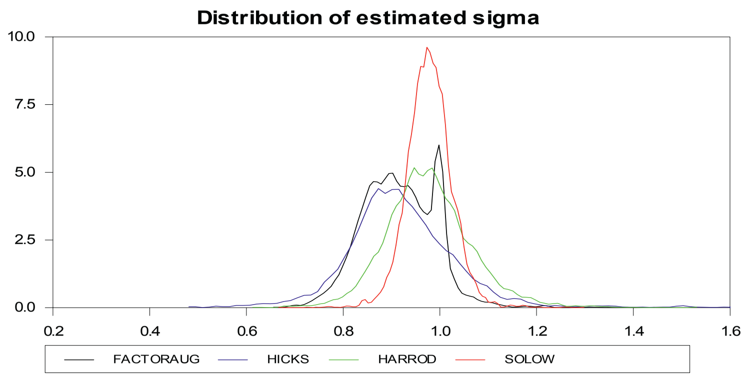
$\gamma_K = 0.00, \gamma_N = 0.02$



$\gamma_K = 0.005, \gamma_N = 0.015$



$\gamma_K = \gamma_N = 0.01$



$\gamma_K = 0.02, \gamma_N = 0.00$

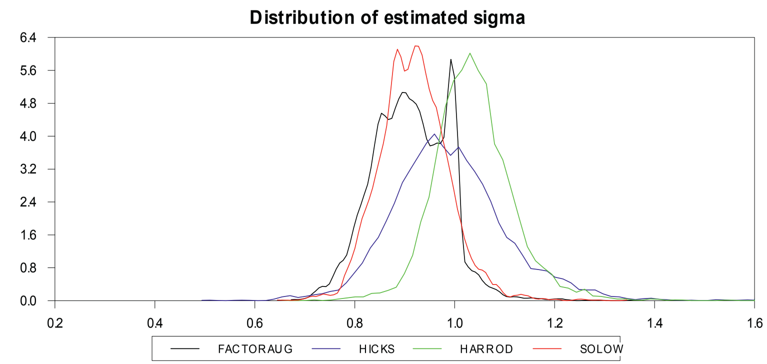
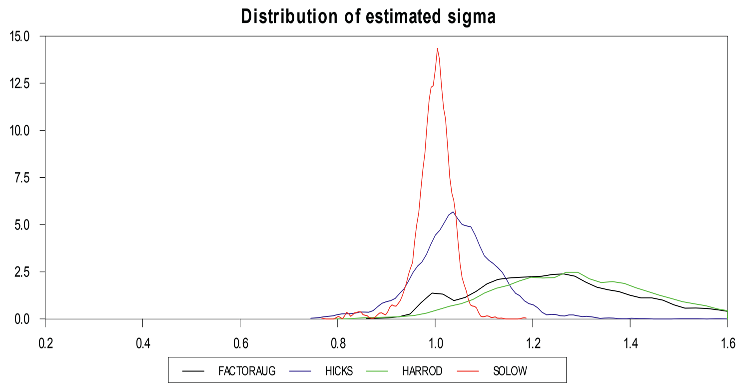
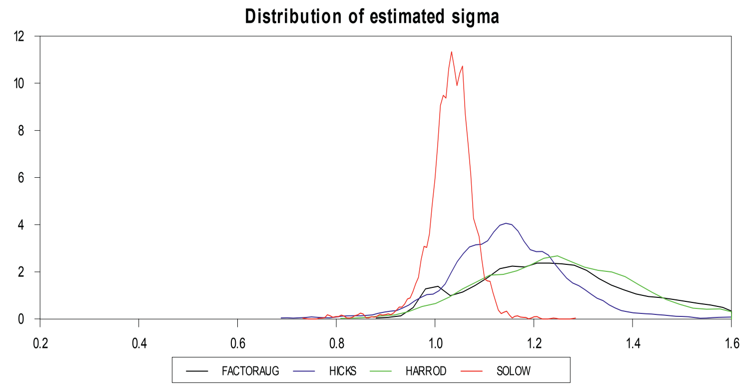


Figure 3. Distribution of estimated σ. True σ = 1.3.

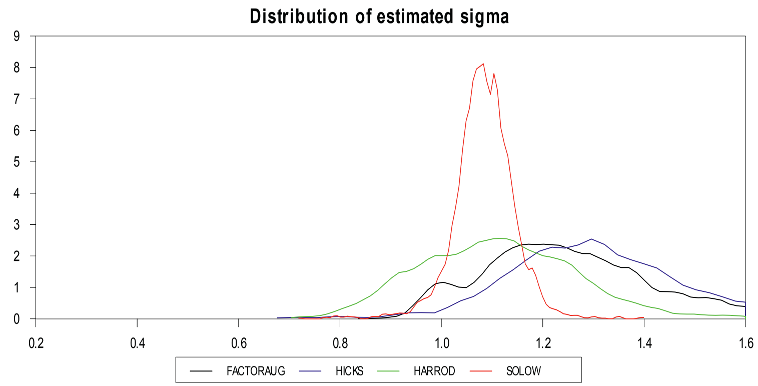
$\gamma_K = 0.00, \gamma_N = 0.02$



$\gamma_K = 0.005, \gamma_N = 0.015$



$\gamma_K = \gamma_N = 0.01$



$\gamma_K = 0.02, \gamma_N = 0.00$

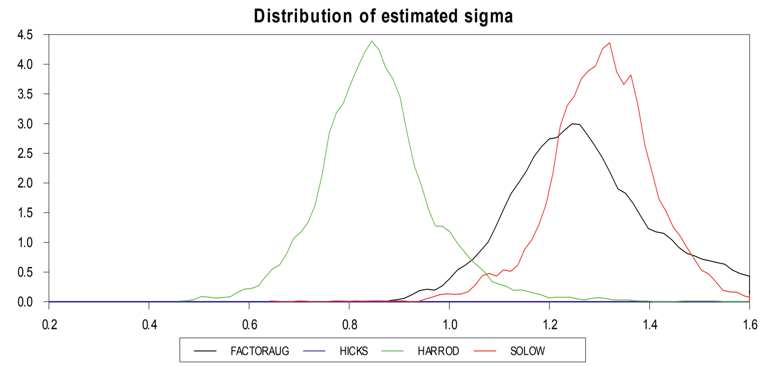


Figure 4. Great ratios for the US economy.

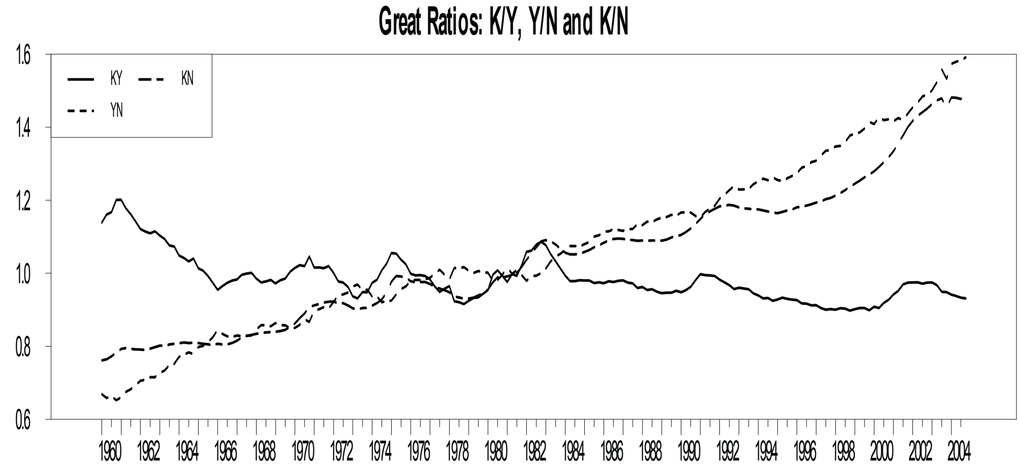


Figure 5. Real wages, productivity, and real user cost.

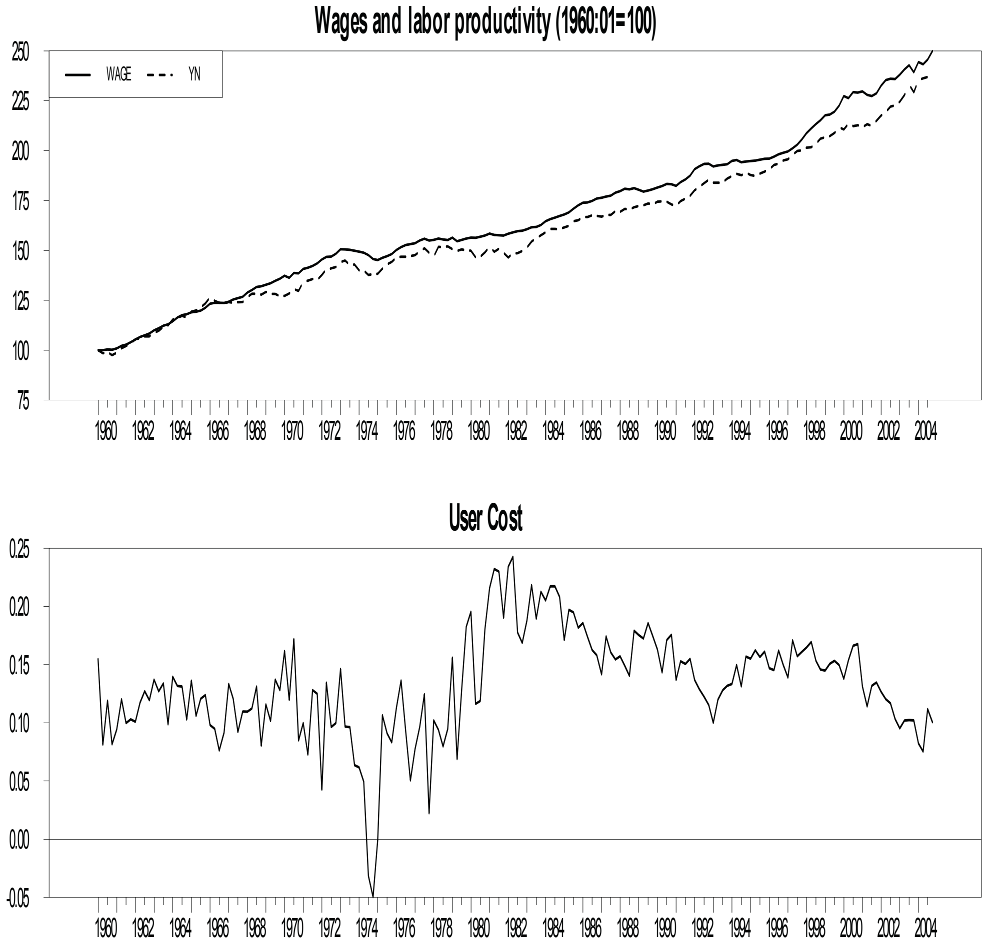
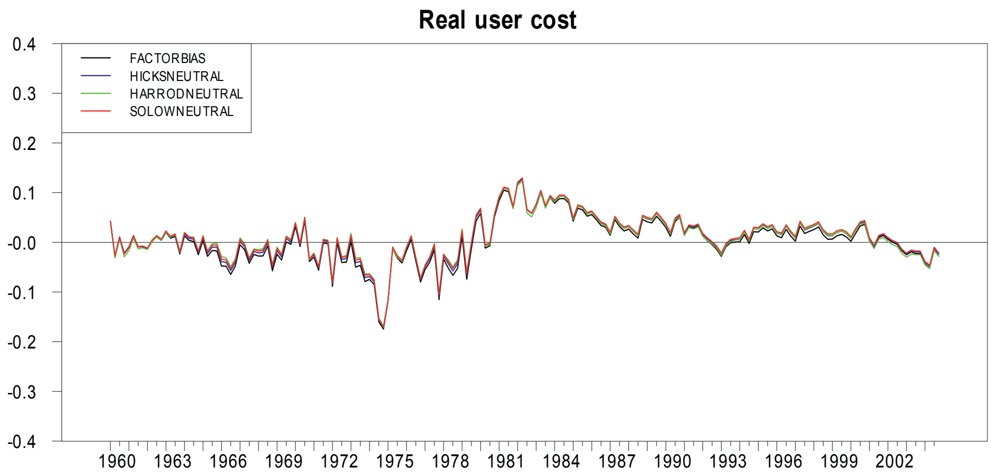
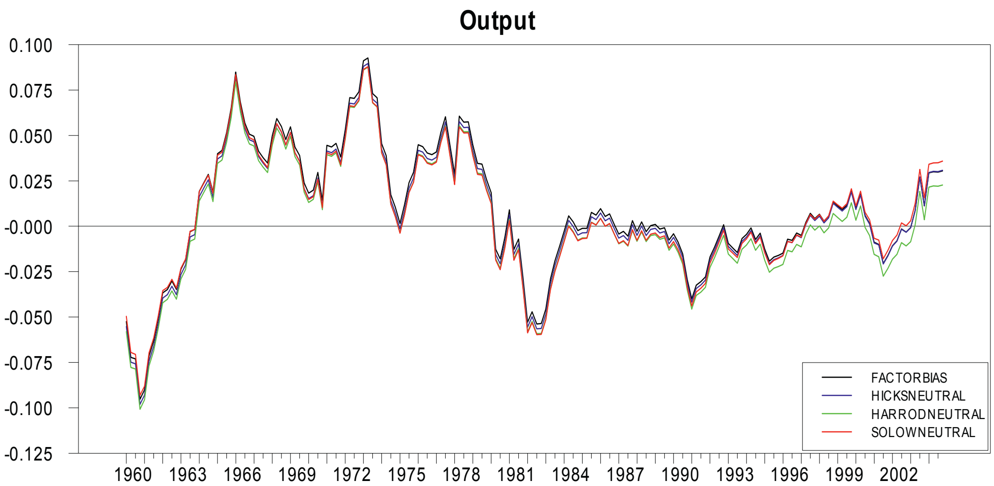


Figure 6. Residuals for the user cost, w and Y equations: four specifications (NLGLS).





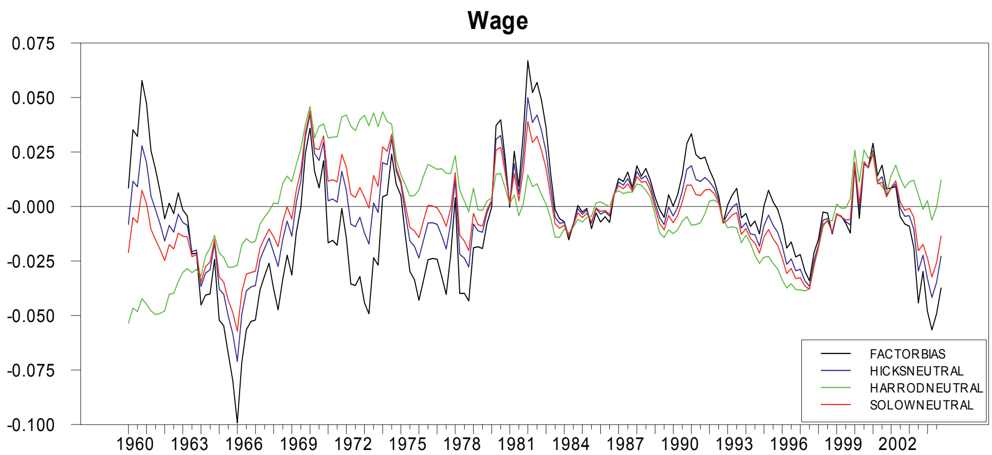


Figure 7. Total Factor Productivity and K/N Ratio Growth (GNLLS)

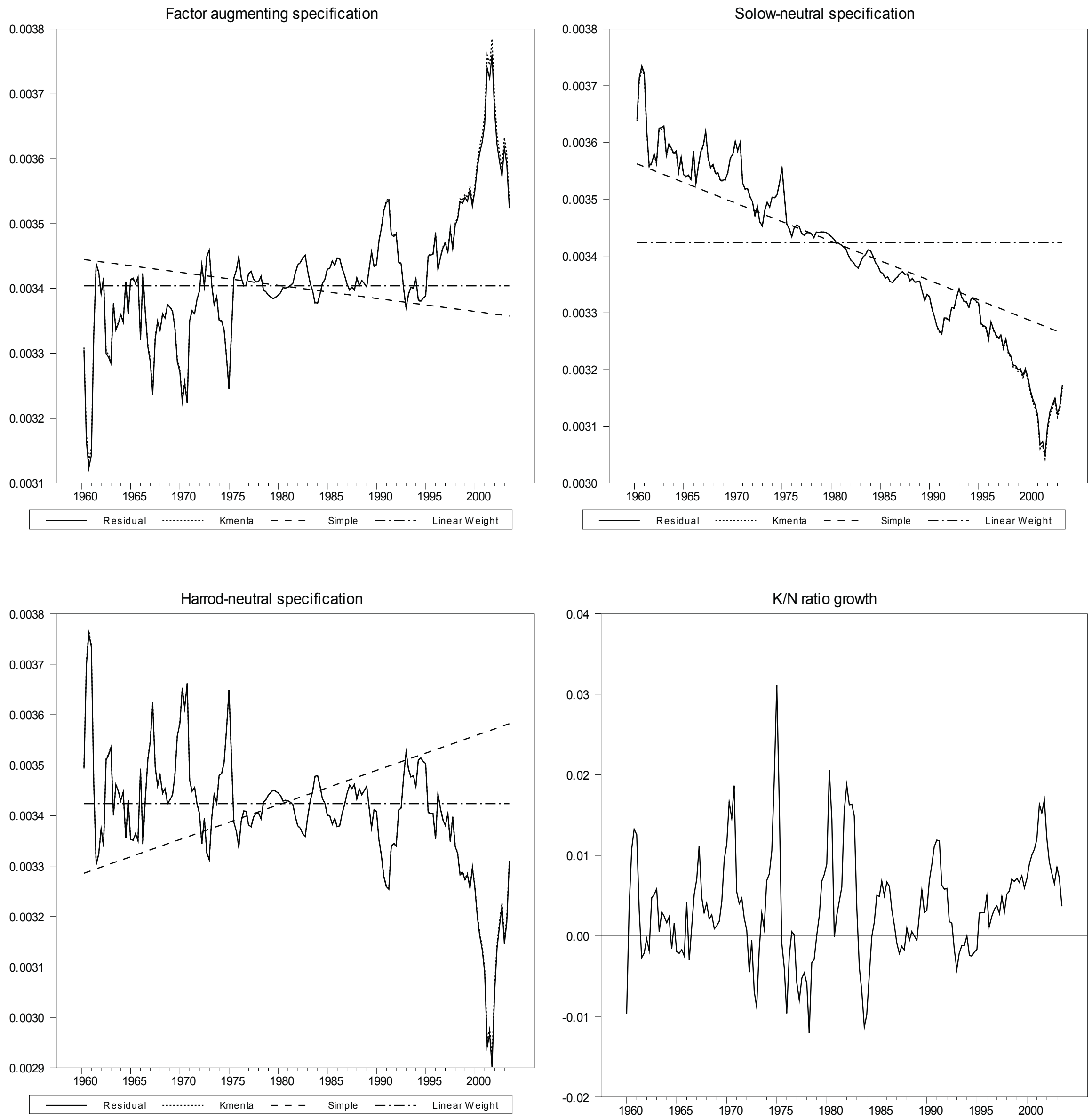
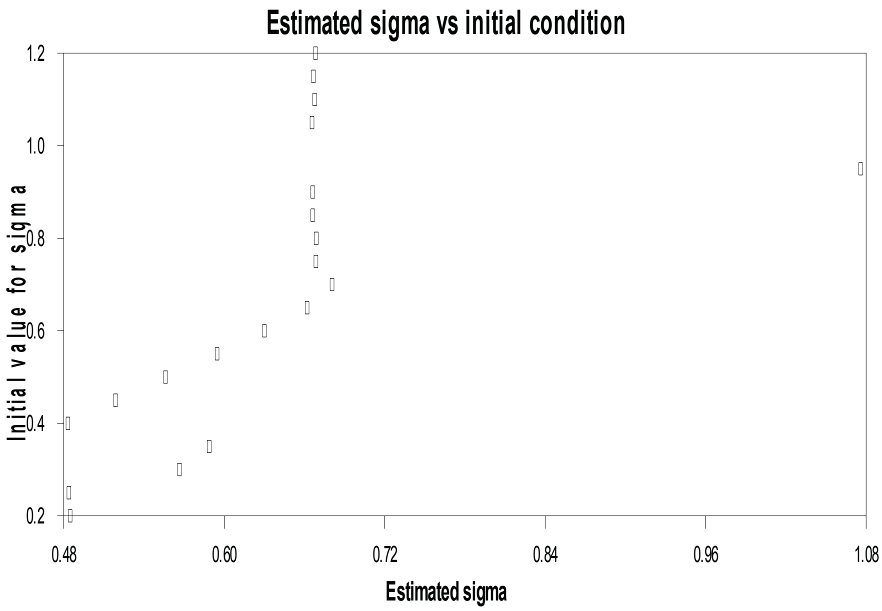


Figure 8. Sensitivity to changes in initialization of σ (GNLLS).



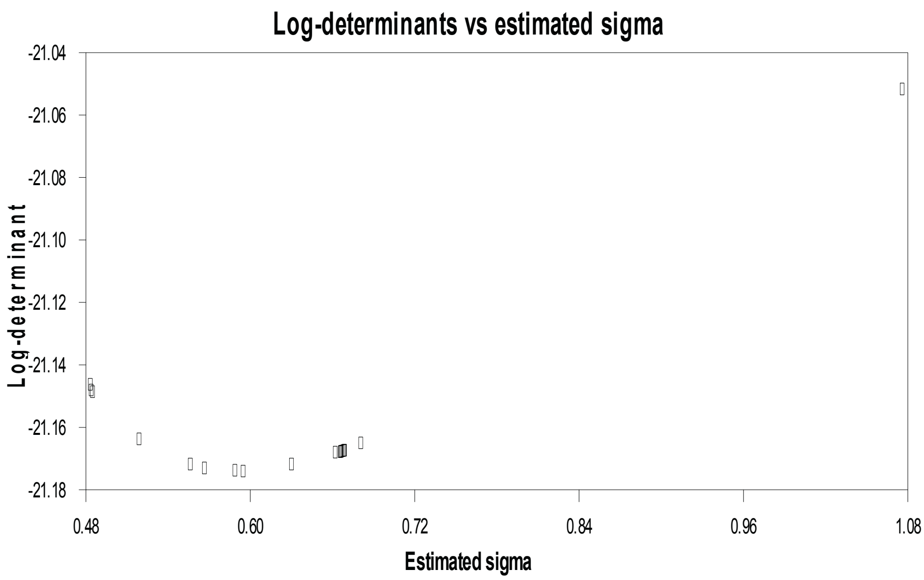


Figure 9. Sensitivity to changes in initialization of σ (NL3SLS).