# Endogenous Volatility at the Zero Lower Bound: Implications for Stabilization Policy

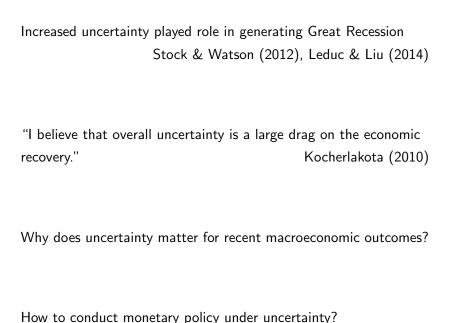
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Increased uncertainty played role in generating Great Recession Stock & Watson (2012), Leduc & Liu (2014)

"I believe that overall uncertainty is a large drag on the economic recovery." Kocherlakota (2010)



Zero lower bound crucial in transmitting effects of uncertainty

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Simple model of nominal price rigidity

Demand-determined output

Monetary policy plays key stabilizing role

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Simple model of nominal price rigidity

Demand-determined output

Monetary policy plays key stabilizing role

Unconstrained central bank can fully stabilize

⇒ Uncertainty about future has no effect

Inability to offset further negative shocks at zero lower bound

Asymmetry lowers mean outcome

Endogenously generates volatility

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Induces precautionary saving by households

Further decline in demand  $\Rightarrow$  Constraint binds for longer

Destabilizing feedback loop

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Further decline in demand  $\Rightarrow$  Constraint binds for longer

Destabilizing feedback loop

Higher uncertainty strengthens feedback

Small uncertainty shocks imply large contractions

### Roadmap

Illustrate mechanism under Taylor (1993)-type rule
Feedback loop may cause equilibrium non-existence

Simple history-dependent rule ensures equilibrium exists
Reduces some fluctuations

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Illustrate mechanism under Taylor (1993)-type rule
Feedback loop may cause equilibrium non-existence

Simple history-dependent rule ensures equilibrium exists
Reduces some fluctuations

Optimal policy can attenuate endogenous volatility
Stabilize distribution of possible outcomes
Commit to responding if bad shocks are realized

### Model Summary

Standard New-Keynesian sticky price model without capital

Shares features with models of Ireland (2003, 2010)

Household consumes, works, & receives firm dividends

Firms employ labor & produce

Quadratic cost of adjusting nominal price

#### Stochastic Processes

Fluctuations in household discount factor (demand shocks)

$$a_t = (1 - \rho_a)a + \rho_a a_{t-1} + \sigma_{t-1}^a \varepsilon_t^a$$

$$\sigma_t^a = (1 - \rho_{\sigma^a})\sigma^a + \rho_{\sigma^a}\sigma_{t-1}^a + \sigma^{\sigma^a}\varepsilon_t^{\sigma^a}$$

Increase in uncertainty captured by increase in  $\sigma^a_t$ 

Harder to forecast  $a_t$  under higher uncertainty

Calibrate & solve nonlinear model using policy function iteration

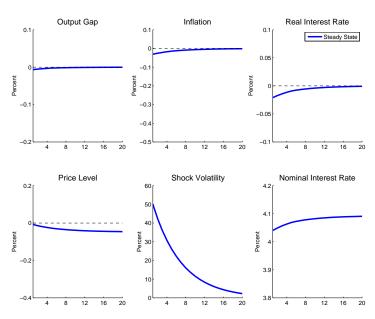
Examine uncertainty shock under two scenarios:

- 1. Steady state
- 2. Zero lower bound

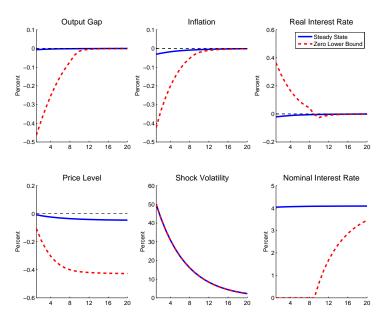
Initially assume monetary policy follows Taylor (1993)-type rule

$$r_t^d = r + \phi_\pi \Big( \pi_t - \pi \Big) + \phi_x x_t$$
 
$$r_t = \max \Big( 0, r_t^d \Big)$$

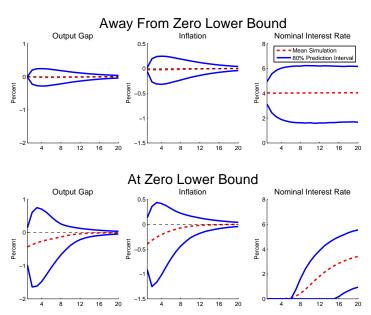
# Impulse Responses to Demand Uncertainty Shock



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# Expected Distributions of Possible Outcomes After Shock



### Interactions Between Uncertainty & Zero Lower Bound

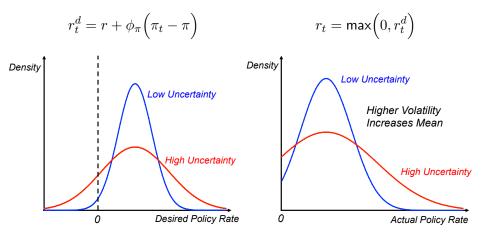
Two distinct mechanisms

- 1. Precautionary saving & precautionary labor supply
- 2. Contractionary bias in nominal interest rate distribution

Contractionary bias emerges under standard Taylor-type rule

Misses inflation target on average due to zero lower bound

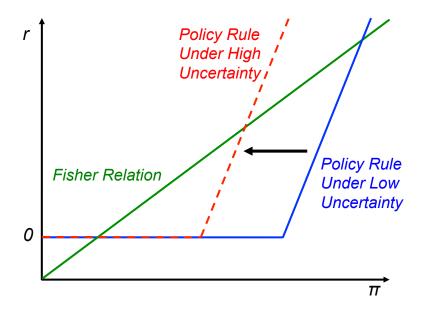
# Contractionary Bias & Distribution of Policy Rates



More volatile inflation ⇒ Higher volatility in desired rates

⇒ Raises expected mean of **actual** rates

# General-Equilibrium Effects of Contractionary Bias



# Removing the Contractionary Bias

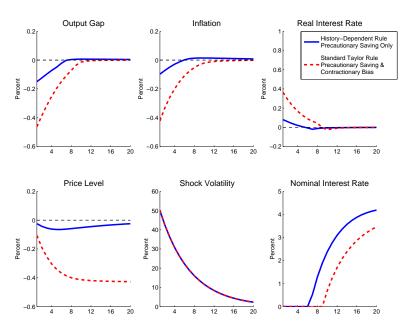
Simple history-dependent rule eliminates bias

$$r_t^d = r + \phi_\pi \Big( \pi_t - \pi \Big) + \phi_x x_t + \phi_{pl} \Big( p_t - p^* \Big)$$
 
$$r_t = \max \Big( 0, r_t^d \Big)$$

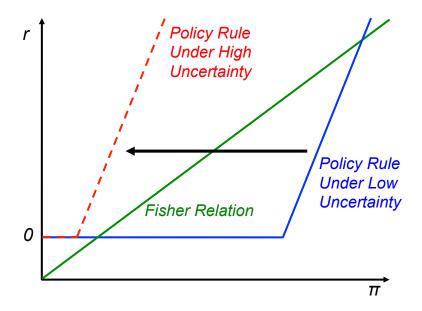
Higher-than-desired rates offset with lower rates in future

Removes bias by achieving inflation target on average

# Decomposing the Impulse Responses



# Contractionary Bias Can Cause Equilibrium Non-Existence



### Should We Remove the Contractionary Bias?

Non-existence occurs for calibration needed to match data

Use history-dependent rule in calibration

Minimum deviation that removes bias

Assumes central bank prevents disequilibrium in actual economy

Continues to respond to economy at zero lower bound

#### Are uncertainty shocks a key driver of output and inflation?

Choose steady state & uncertainty shock volatility to match:

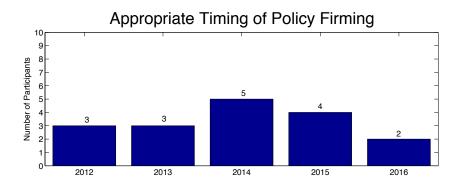
- 1. Unconditional volatility:  $x_t$ ,  $\pi_t$ ,  $r_t$
- 2. Stochastic volatility:  $x_t$ ,  $\pi_t$ ,  $r_t$
- 3. Number quarters at zero lower bound

Compare with 1984 - 2013 data

Assess fit via bootstrapped small-sample confidence intervals

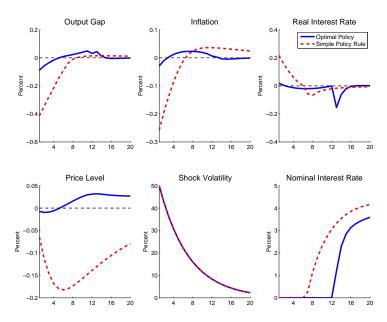
	Data	В	aseline Model
Moment	1984 - 2013	Mean	Confidence Interval
Unconditional Volatility			
x	2.52	1.70	(0.89, 3.00)
$\pi$	0.98	1.03	(0.62, 1.58)
r	2.91	2.42	(1.70, 3.28)
Stochastic Volatility			
x	0.77	0.73	(0.28, 1.57)
$\pi$	0.49	0.40	(0.19, 0.77)
r	0.74	0.72	(0.41, 1.16)
Quarters at Zero Lower Bound	20	13	(2, 29)

# Survey of FOMC Participaints from January 2012

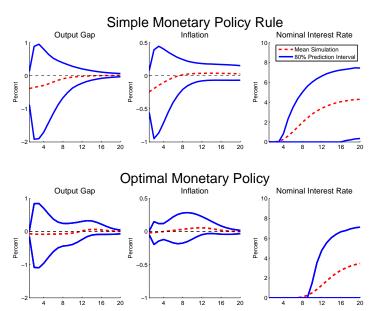


	Data	Constant Shock Volatility	
Moment	1984 - 2013	Mean	Confidence Interval
Unconditional Volatility			
x	2.52	0.93	(0.72, 1.21)
$\pi$	0.98	0.62	(0.49, 0.77)
r	2.91	1.89	(1.53, 2.29)
Stochastic Volatility			
x	0.77	0.23	(0.12, 0.38)
$\pi$	0.49	0.14	(0.09, 0.22)
r	0.74	0.40	(0.25, 0.58)
Quarters at Zero Lower Bound	20	5	(0, 13)

# Impulse Responses Under Optimal Monetary Policy



# Possible Outcomes Under Optimal Monetary Policy



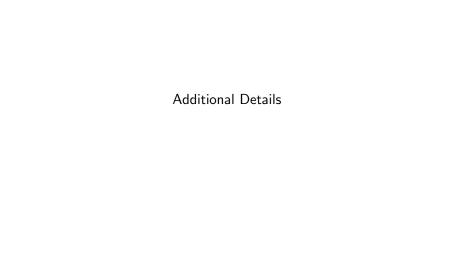
	Simple Rule	C	Optimal Policy
Moment	Mean	Mean	Confidence Interval
Unconditional Volatility			
x	1.70	0.51	(0.03, 1.39)
$\pi$	1.03	0.14	(0.01, 0.42)
r	2.42	2.65	(1.87, 3.60)
Stochastic Volatility			
x	0.73	0.39	(0.03, 1.00)
$\pi$	0.40	0.10	(0.01, 0.29)
r	0.72	0.78	(0.46, 1.24)
Quarters at Zero Lower Bound	13	19	(2, 44)

#### Conclusions

Zero lower bound crucial in transmitting effects of uncertainty

Form of monetary policy reaction function is crucial

Policy must commit to responding if bad shocks are realized



# Representative Household

Household maximizes lifetime utility from consumption and leisure

$$\max \, E_t \sum_{i=0}^\infty a_{t+i} \beta^i \left( \frac{C_{t+i}^\eta (1-N_{t+i})^{1-\eta}}{1-\sigma} \right)^{1-\sigma}$$

Household budget constraint

$$C_t + \frac{B_t}{P_t R_t} \le \frac{W_t}{P_t} N_t + \frac{B_{t-1}}{P_t} + \frac{D_t}{P_t}$$

Household stochastic discount factor

$$M_{t+1} = \left(\beta \frac{a_{t+1}}{a_t}\right) \left(\frac{C_{t+1}^{\eta} (1 - N_{t+1})^{1-\eta}}{C_t^{\eta} (1 - N_t)^{1-\eta}}\right)^{1-\sigma} \left(\frac{C_t}{C_{t+1}}\right)$$

# Representative Goods-Producing Firm

Firm i chooses  $N_t(i)$  and  $P_t(i)$  to maximize cash flows

$$\max E_t \left\{ \sum_{s=0}^{\infty} M_{t+s} \left( \frac{D_{t+s}(i)}{P_{t+s}} \right) \right\}$$

Definition of firm cash flows

$$\frac{D_t(i)}{P_t} = \left[\frac{P_t(i)}{P_t}\right]^{1-\theta} Y_t - \frac{W_t}{P_t} N_t(i) - \frac{\phi_P}{2} \left[\frac{P_t(i)}{\Pi P_{t-1}(i)} - 1\right]^2 Y_t$$

Quadratic cost of changing nominal price  $P_t(i)$ 

$$\frac{\phi_P}{2} \left[ \frac{P_t(i)}{\prod P_{t-1}(i)} - 1 \right]^2 Y_t$$

Cobb-Douglas production function subject to fixed costs

$$Y_t(i) = N_t(i) - \Phi$$

# Aggregation & National Income Accounting

All users of final output assemble the final good  $Y_t$  using the range of varieties  $Y_t(i)$  in a CES aggregator

$$Y_t = \left[ \int_0^1 Y_t(i)^{\frac{\theta - 1}{\theta}} di \right]^{\frac{\theta}{\theta - 1}}$$

Aggregate production function

$$Y_t = N_t - \Phi$$

National income accounting

$$Y_t = C_t + \frac{\phi_P}{2} \left( \frac{\Pi_t}{\Pi} - 1 \right)^2 Y_t$$

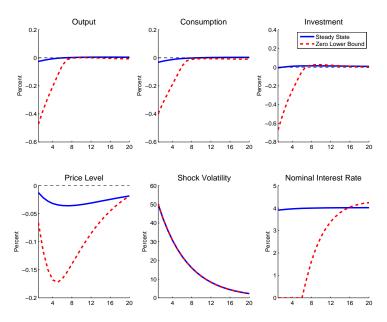
### Model Summary

Model summarized by consumption Euler equation and NK Philips Curve

$$1 = \mathbb{E}_t \left\{ M_{t+1} \left( \frac{R_t}{\Pi_{t+1}} \right) \right\}$$

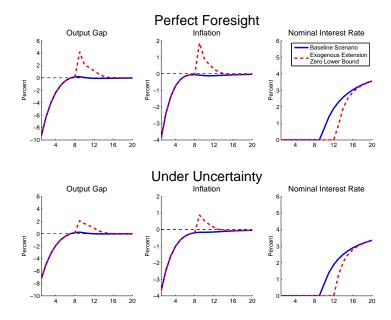
$$\phi_P \left( \frac{\Pi_t}{\Pi} - 1 \right) \left( \frac{\Pi_t}{\Pi} \right) = (1 - \theta) + \theta \Xi_t$$
$$+ \phi_P E_t \left\{ M_{t+1} \frac{Y_{t+1}}{Y_t} \left( \frac{\Pi_{t+1}}{\Pi} - 1 \right) \left( \frac{\Pi_{t+1}}{\Pi} \right) \right\}$$

# Impulse Responses to Uncertainty Shock with Capital



	Data	No Zero Lower Bound	
Moment	1984 - 2013	Mean	Confidence Interval
Unconditional Volatility			
$\overline{x}$	2.52	1.24	(0.80, 1.79)
$\pi$	0.98	0.83	(0.55, 1.17)
r	2.91		
Stochastic Volatility			
$\overline{x}$	0.77	0.41	(0.22, 0.70)
$\pi$	0.49	0.28	(0.28, 0.48)
r	0.74		
Quarters at Zero Lower Bound	20		

### Our Solution to the "Forward Guidance Puzzle"



# Two Steady States & Numerical Convergence

#### Gavin, Keen, Richter, Throckmorton (2015)

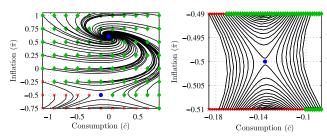


Fig. 16. Convergence paths to the steady-state equilibria (circles) in the deterministic version of Model 1. A diamond denotes an initial conjecture that converges to the positive inflation steady state, and a cross denotes an initial conjecture that asymptotically converges to a corner solution where there is no consumption.

### Numerical Convergence Under Uncertainty

Gavin, Keen, Richter, Throckmorton (2015)

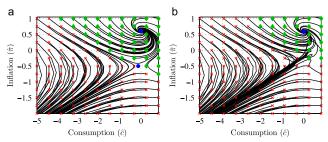


Fig. 18. Convergence paths to steady states (circles) for the perfect foresight and stochastic models when  $\beta_{-1} = \overline{\beta}$ . A diamond denotes an initial conjecture that converges to the positive inflation steady state, and a cross denotes an initial conjecture that asymptotically converges to a corner solution where there is no consumption. (a) Perfect foresight Model 1, (b) stochastic Model 1.