Panel models with stochastic trends: estimation, forecasting and applications

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Dynamic Panel Data (DPD) model

Consider the $N \times T$ data matrix

$$Y = \{y_{it}\}, \qquad i = 1, \ldots, N, \quad t = 1, \ldots, T.$$

In many panel data settings, we have big N and small T.

To accommodate both panel and dynamic structures in the panel, typically the DPD model is considered

$$y_{it} = c_i + \lambda \cdot y_{i,t-1} + \beta \cdot x_{it} + \varepsilon_{it},$$

with the usual assumptions, and where c_i can be treated as fixed effects or random effects.

DPD model can be extended with time index effects d_t to obtain

$$y_{it} = c_i + d_t + \lambda \cdot y_{i,t-1} + \beta \cdot x_{it} + \varepsilon_{it}.$$

Dynamic Panel Data Model

We propose another treatment of, in particular, the dynamic features in the DPD model.

We consider a model with unobserved factor error structures, see discussions in book of Pesaran (2015), Chapters 26 and 27.

Furthermore, we do not rely on GMM estimation and on the use of instruments. We do (transformed) maximum likelihood.

Hence, most coefficients are estimated via standard regression.

We start with the basic panel regression model

$$y_{it} = \mu_t + c_i + \beta \, x_{it} + \varepsilon_{it},$$

where the lag-dependent variables are removed and the fixed time effects d_t are replaced by the stochastic dynamic process μ_t.

Dynamic Panel Data Model

We have the basic panel model

$$y_{it} = \mu_t + c_i + \beta x_{it} + \varepsilon_{it},$$

where the unobserved component μ_t can be represented by a stationary ARMA process or a nonstationary ARIMA process. Basic examples are the AR(1) process

$$\mu_t = \lambda \mu_{t-1} + \eta_t, \qquad -1 < \lambda < 1,$$

and the random walk process

$$\mu_t = \mu_{t-1} + \eta_t,$$

with η_t being an IID noise term. The overall statistical treatment is similar, whether the process for μ_t is stationary or nonstationary.

Static formulation

We can represent the basic panel model $y_{it} = \mu_t + c_i + \beta\, x_{it} + \varepsilon_{it}$ by the static representation

$$y_{it} = c_i + \beta\, x_{it} + u_{it}, \qquad u_{it} = \mu_t + \varepsilon_{it},$$

where we have that each u_{it} follows an AR(I)MA process.

The model representation in first differences is given by

$$\Delta y_{it} = \beta\, \Delta x_{it} + \Delta u_{it},$$

where we notice that Δu_{it} still follows an AR(I)MA process.

Estimation can be based on both representations, leading to random effects ($\mathrm{E}(x_{it} u_{jt}) = 0$, between/within estimation) or fixed effects ($\mathrm{E}(x_{it} u_{jt}) \neq 0$, differences estimation).

In both cases, basic adjustments are required to allow for the ARIMA process of u_{it}: regression with ARMA errors.

Likelihood approach

We adopt the method of maximum likelihood (ML).

We will argue that even for large N and moderate T, the ML approach is feasible. This development follows closely the approach taken by Hsiao, Pesaran and Tahmiscioglu (2002) in the panel literature, and Harvey and Marshall (1991) and Marshall (1992) in the time series literature.

Our approach provides estimates of time-varying effects such as μ_t. Also forecasts can be generated based on the DPD model with stochastic trends.

Dynamic factor model perspective

The basic DPD model with stochastic trends

$$y_{it} = \mu_t + c_i + b_i\, x_{it} + \varepsilon_{it},$$

can be regarded as a close variant of the dynamic factor model

$$y_{it} = c_i + \gamma_i\, \mu_t + \varepsilon_{it},$$

with "loadings" γ_i, without regression effects, and μ_t representing the dynamic factor, see Engle and Watson (1983), Bai and Ng (2002), Doz et al. (2012) and many many others.

The focus here is on the treatment of the DPD model with stochastic trends.

The inclusion of the loadings γ_i is not made explicit throughout but can be treated.

The generalization to multiple stochastic trends (multiple dynamic factors) is also treated.

Time-varying regression effects

The DPD model can accommodate other time-varying effects. Hence, we also consider time-varying regression coefficients and obtain the DPD model

$$y_{it} = \mu_t + c_i + (\beta_t + b_i)\, x_{it} + \varepsilon_{it},$$

where the time-varying parameters μ_t and β_t follow linear dynamic stochastic processes while c_i and b_i can be treated as fixed or random coefficients.

Panel model with time-varying effects

In our model with time-varying effects

$$y_{it} = \mu_t + c_i + (\beta_t + b_i)\, x_{it} + \varepsilon_{it},$$

the treatment of c_i and b_i can depend on N and T:

- $N << T$: treat c_i and b_i as fixed coefficients
- $N >> T$: treat c_i and b_i as random coefficients, that is

$$c_i \sim NID(c, \sigma_c^2), \qquad b_i \sim NID(b, \sigma_b^2).$$

The maximum likelihood treatment can accommodate both approaches and only small modifications are needed.

A particular concern is identification of coefficients and initialization: c versus μ_1 and b versus β_1.

Panel model with stochastic trends

The panel model with a *single* explanatory variable x_{it} and time-varying effects

$$y_{it} = \mu_t + c_i + (\beta_t + b_i)\, x_{it} + \varepsilon_{it},$$

can be represented as a regression model for

$$y_t = (\, y_{1t}\,,\, y_{2t}\,,\, \ldots, y_{Nt}\,)',$$

and given by

$$y_t = X_t \delta + u_t,$$

with

$$X_t = [I_N, \mathrm{diag}(\, x_{1t}\,,\, x_{2t}\,,\, \ldots, x_{Nt}\,)], \qquad \delta = (c_1, \ldots, c_N, b_1, \ldots, b_N),$$

and

$$u_t = \mathbf{1}\mu_t + x_t\, \beta_t + \varepsilon_t, \qquad x_t = (\, x_{1t}\,,\, x_{2t}\,,\, \ldots, x_{Nt}\,)'.$$

Regression model formulation

For the basic panel model $y_{it} = \mu_t + c_i + (\beta_t + b_i)\, x_{it} + \varepsilon_{it}$ and its vector representation

$$y_t = X_t \delta + u_t,$$

with

$$X_t = [I_N, \mathrm{diag}(\, x_{1t}\,,\, x_{2t}\,,\, \ldots, x_{Nt}\,)], \qquad \delta = (c_1, \ldots, c_N, b_1, \ldots, b_N).$$

For a fully pooled model, $c_i = c$ and $b_i = b$, we have $X_t = [\mathbf{1}, x_t]$ and $\delta = (c, b)'$. The error

$$u_t = \mathbf{1}\mu_t + x_t\, \beta_t + \varepsilon_t, \qquad x_t = (\, x_{1t}\,,\, x_{2t}\,,\, \ldots, x_{Nt}\,)'.$$

implies that

$$\mathrm{Var}(u_t) = \Omega_0(X, \psi), \qquad \mathrm{Cov}(u_t, u_{t-j}) = \Omega_j(X, \psi), \qquad j = 1, 2, \ldots.$$

To accommodate the heteroskedastic autocovariance structure of u_t, we apply generalized least squares (GLS) to

$$y = X\delta + u, \qquad u \sim NID(0, \Omega), \qquad \Omega = \Omega(X, \psi).$$

Generalized Least Squares

The application of GLS to

$$y = X\delta + u, \qquad u \sim NID(0, \Omega), \qquad \Omega = \Omega(X, \psi).$$

with the error

$$u_t = \mathbf{1}\mu_t + x_t\,\beta_t + \varepsilon_t, \qquad x_t = (\,x_{1t}\,,\, x_{2t}\,,\, \ldots, x_{Nt}\,)'.$$

faces a complicated covariance structure for Ω, making the treatment of GLS infeasible because it requires e.g. the Choleski decomposition $\Omega = LL'$ such that OLS can be applied to

$$v_y = V_X\delta + v_u, \qquad v_u \sim NID(0, I),$$

where $(v_y, V_X, v_u) = L^{-1}(y, X, u)$.

However, it turns out that these Choleski calculations are easily done recursively by the Kalman filter.

State space representation of error

The error expression

$$u_t = \mathbf{1}\mu_t + x_t\,\beta_t + \varepsilon_t, \qquad x_t = (\,x_{1t}\,,\,x_{2t}\,,\,\ldots,x_{Nt}\,)'.$$

can be represented in state space form, that is

$$u_t = Z_t\,\alpha_t + \varepsilon_t, \qquad \alpha_t = A\,\alpha_{t-1} + \eta_t,$$

and initial condition $\alpha_1 \sim NID(a, P)$. In case of the u_t expression, we have

$$Z_t = (\mathbf{1}, x_t), \qquad \alpha_t = (\mu_t, \beta_t)',$$

and matrix $A = A(\psi)$ is determined by the dynamic properties given to μ_t and β_t.

We notice that u_t is a $N \times 1$ vector and can be of high dimension while the state vector α_t is of low dimension (here two elements).

GLS via Kalman filter

The Kalman filter can be applied to the state space model

$$u_t = Z_t \alpha_t + \varepsilon_t, \qquad \alpha_t = A \alpha_{t-1} + \eta_t,$$

and will effectively carry out the Choleski decomposition $\Omega = LL'$ and the corresponding transformation

$$v_u = L^{-1} u.$$

When we replace u by y and each column of X consecutively, each application of the Kalman filter for a different "u", will lead to the computation of

$$(v_y, V_X) = L^{-1}(y, X),$$

which are used in the OLS calculations applied to

$$v_y = V_X \delta + v_u, \qquad v_u \sim NID(0, I),$$

which delivers the GLS estimation of δ.

GLS and MLE via Kalman filter

The Kalman filter can only be applied to the state space model

$$u_t = Z_t\alpha_t + \varepsilon_t, \qquad \alpha_t = A\alpha_{t-1} + \eta_t,$$

when "system" matrices $A = A(\psi)$, $\text{Var}(\varepsilon_t;\psi)$ and $\text{Var}(\eta_t;\psi)$ are known. For a any given value of ψ, we can treat these as known.

We then estimate ψ, via the full maximum likelihood estimator

$$\widehat{\psi}_{ML} = \arg_\psi \max \left\{ -\frac{NT}{2}[\log 2\pi + \log|F| + (v'F^{-1}v)]\right\}$$

where $v = v_y - V_X\widehat{\delta}_{GLS}$ and $F =$Var(v_u) is also obtained from the Kalman filter. The MLE $\widehat{\psi}$ is obtained via numerical optimization. Function evaluation relies on Kalman filter and OLS computations.

Panel model with stochastic trends: discussion

- We propose a "transformed" likelihood-based approach to dynamice panel data models with time-varying effects or "unobserved factor error structures" using Kalman filtering
- This approach is somewhat similar to Hsiao, Pesaran and Tahmiscioglu (2002)
- Our implementation is different: it relies on Kalman filtering
- The use of the Kalman filter to "transform" the data before carrying out a regression goes back to Rosenberg (1973), and is well recognised in the work of Harvey (1989), de Jong (1988, 1991) and Durbin and K (2012).

Panel model with stochastic trends: discussion

- A clear advantage of the Kalman filter approach is that the "transformation" does not only make GLS feasible, it also allows for likelihood evaluation and ...
- ... for *signal extraction*: the estimation of time-varying effects (state vector) using corresponding smoothing methods.
- And it allows for Forecasting !!
- This approach is valid in cases of treating c_i and b_i as fixed effects or as random effects.
- The implementation requires some further discussion.

Panel model with time-varying effects

- In cases of treating c_i and b_i as fixed effects, we obtain an X matrix with more columns which necessitates more transformations by the Kalman filter. Notice that only the "mean" part of the Kalman filter needs to be repeated, leading to huge computational gains.
- Depending on the application and organization of the data, the methods can also treat "group" estimation.
- However 1, when N is big, we will resort to random effects, $c_i \sim NID(c, \sigma_c^2 I_N)$ and $b_i \sim NID(b, \sigma_b^2 I_N)$. The means c and b will be treated by GLS as pooled coefficients. The "variance" part will lead to a more complicated variance matrix for ε_t, from $\sigma_\varepsilon^2 I_N$ to

$$\sigma_\varepsilon^2 I_N + \sigma_c^2 11' + \sigma_b^2 x_t x_t'.$$

  The non-diagionality of $\mathrm{Var}(\varepsilon_t)$ can be treated by the Kalman filter straightforwardly.
- However 2, ...

Panel model with time-varying effects

When Kalman filter is applied to a multivariate model with big N, we have two options:

(a) filtering and updating, equation by equation

(b) collapsing the observation vector, via transformation

These options can also be done jointly.

In case of (a), we are avoiding inverting $N \times N$ matrices in the Kalman filter. It can only be done straightforwardly when $\text{Var}(\varepsilon_t)$ is diagonal, otherwise further transformations are needed.

In case of (b), the dimension of the observation vector is reduced to the dimension of the state vector (here 2×1).
The computations become slightly more involved but huge computational gains are the reward.

For more detailed discussions on this, see K and Durbin (2004) and Jungbacker and K (2015).

Review of our model

In our dynamic panel data model with time-varying effects

$$y_{it} = \mu_t + c_i + (\beta_t + b_i)\, x_{it} + \varepsilon_{it},$$

where we treat both μ_t and β_t as random walk processes, that is

$$\mu_t = \mu_{t-1} + \eta_t, \qquad \beta_t = \beta_{t-1} + \kappa_t,$$

and we will treat c_i and b_i as random coefficients, that is

$$c_i \sim NID(c, \sigma_c^2), \qquad b_i \sim NID(b, \sigma_b^2).$$

The identification issues are resolved by the initializations:
$\mu_1 = \beta_1 = 0$, fixed.
Hence, estimates of c and b are interpreted as those in first year.
Alternatively, we can set these to $\mu_T = \beta_T = 0$.

Illustrations

To illustrate this panel approach, we present two case studies:

- US Philips-Curve using inflation time series of US States. This is joint work with Marente Vlekke (CPB Netherlands Institute for Economic Policy Analysis, The Hague).
- Global Measure of Ethane using weather station data around the globe. This is joint work with Marina Friedrich (VU), Yicong Lin (VU), Emmanuel Mahieu (ULiège) and Stephan Smeekes (Maastricht U).

In both cases $T > N$, but the methods do not prohibit their use in cases with $T << N$.

US Phillips-Curve, 1978-2017

This illustration is motivated by recent publication of Hazell, Herreño, Nakamura and Steinsson in their paper "The Slope of the Phillips Curve: Evidence from U.S. States", in *Quarterly Journal of Economics*, August 2022.

Basic Phillips-Curve regression model is given by

$$\pi_t = c + bU_t + \gamma \pi_t^E + \epsilon_t,$$

where π_t is an inflation measure, U_t is output gap and π_t^E is expected inflation in the long-term. This model is heavily adopted with mixed results and findings.

Issue 1: identification problem due to covarying U_t and π_t^E; see discussion in Mavroeidis, Plagborg-Moller and Stock (2014).

Issue 2: classic simultaneity problem in distinguishing demand / supply shocks.

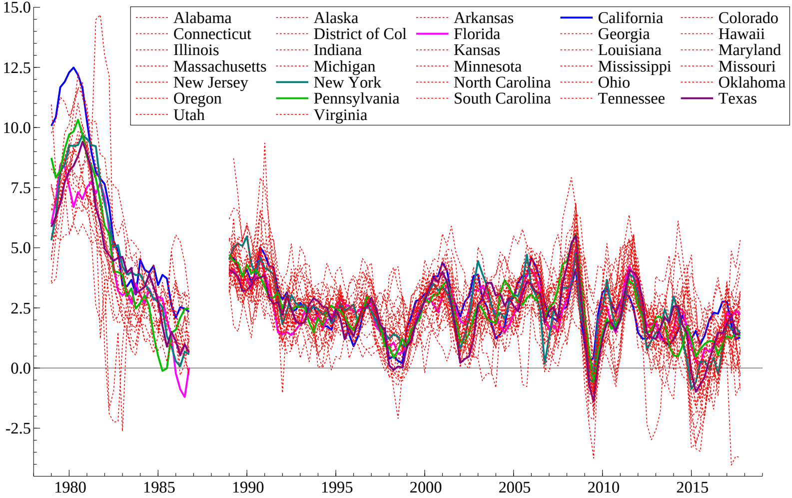
US Phillips-Curve, 1978-2017

This illustration is motivated by recent publication of Hazell, Herreño, Nakamura and Steinsson in their paper "The Slope of the Phillips Curve: Evidence from U.S. States", in *Quarterly Journal of Economics*, August 2022.

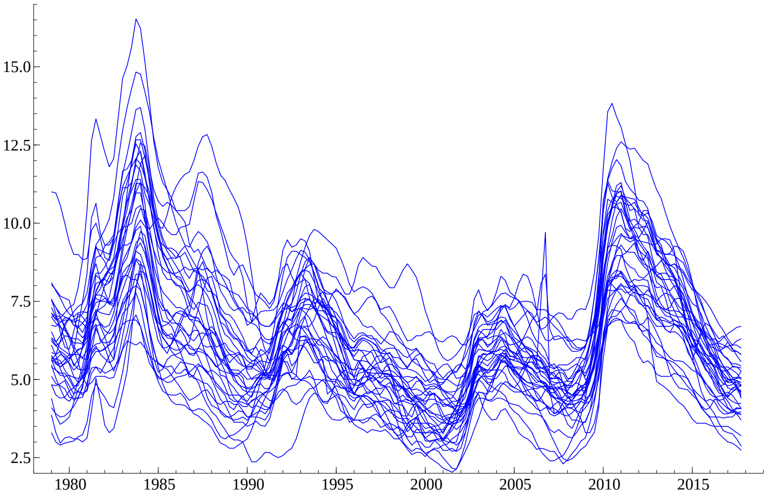
Similar to Fitzgerald and Nicolini (2014) and McLeay and Tenreyro (2019), they use regional data and overcome a simultaneity problem: central banks cannot offset regional demand shocks using a single national interest rate.

They use US States data and make it available on their websites.

Quarterly Inflation by US States, 1978-2017



Quarterly Unemployment by US States, 1978-2017



Country panel for US Inflation

The first dynamic panel data model is with time-varying level only

$$y_{it} = \mu_t + c_i + b_i \, x_{i,t-1} + \varepsilon_{it},$$

where $c_i \sim NID(c, \sigma_c^2)$, $c_i \sim NID(b, \sigma_b^2)$ and μ_t is modeled as a random walk process.

The second dynamic panel data model is with two time-varying effects

$$y_{it} = \mu_t + c_i + (\beta_t + b_i) \, x_{i,t-1} + \varepsilon_{it},$$

where both μ_t and β_t are random walk processes.

Notice, we include lagged unemployment as output gap measure.

Equation (19) in Hazell et al (2022)

The empirical Phillips Curve model of Hazell et al (2022) is

$$\pi_{it} = \mu_t + c_i + b \cdot U_{i,t-4} + d \cdot p_{i,t-4} + \epsilon_{it},$$

for US states $i = 1, \ldots, N$ and quarterly time index $t = 1, \ldots, T$,

- π_{it} is a non-tradeable inflation in US State i and quarter t;
- U_{it} is the unemployment rate (SA, from BLS / Local Area Unempl Stat);
- p_{it} is a relative price variable.

We allow to have a time-varying slope for unemployment:

$$\pi_{it} = \mu_t + c_i + (b + \beta_t) \cdot U_{i,t-4} + d \cdot p_{i,t-4} + \epsilon_{it},$$

This model fits our general framework perfectly.

Equation (19) in Hazell et al (2022)

Given the general model,

$$\pi_{it} = \mu_t + c_i + (b + \beta_t) \cdot U_{i,t-4} + d \cdot p_{i,t-4} + \epsilon_{it},$$

and in spirit of Table 1 of Hazell et al (2022), we consider four special cases:

(i) $\pi_{it} = c + b \cdot U_{i,t-4} + d \cdot p_{i,t-4} + \epsilon_{it}$

(ii) $\pi_{it} = c_i + b \cdot U_{i,t-4} + d \cdot p_{i,t-4} + \epsilon_{it}$

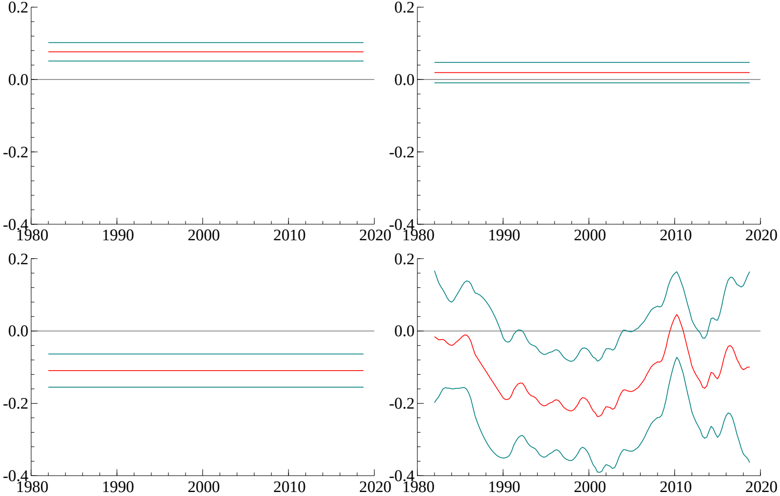
(iii) $\pi_{it} = \mu_t + c_i + b \cdot U_{i,t-4} + d \cdot p_{i,t-4} + \epsilon_{it}$

(iv) $\pi_{it} = \mu_t + c_i + (b + \beta_t) \cdot U_{i,t-4} + d \cdot p_{i,t-4} + \epsilon_{it}$

Estimates of Slope of Phillips Curve

		No Fixed Effects	No Time Effects	Lagged Unempl	TV Lagged Unempl
State effect			✓	✓	✓
Stoch trend				✓	✓
Stoch slope					✓
		(i)	(ii)	(iii)	(iv)
1978 -	*coef*	0.1030	-0.0005	-0.1551	-0.1389
	s.e.	0.0159	0.0177	0.0237	0.2266
	-llik	9705.6	9613.0	8158.1	8140.3
1982 -	*coef*	0.0766	0.0192	-0.1094	-0.0997
	s.e.	0.0130	0.0144	0.0234	0.1346
	-llik	8240.6	8154.6	7504.4	7501.4

Estimates of Slope of Phillips Curve



Equation (19) with Instrumental Variable

In the original Eq 19 of Hazell et al (2022),

$$\pi_{it} = \mu_t + c_i + b \cdot U_{i,t-4} + d \cdot p_{i,t-4} + \epsilon_{it},$$

- lagged unemployment rate $U_{i,t-4}$ is treated as *endogenous*;
- lagged relative price $p_{i,t-4}$ is treated as *exogenous*.

As Hazell et al, we instrument lagged unemployment as in Bartik (1991).

The instrument is a *shift-share* variable and is constructed using employment shares of individual industries at each US State i: $Bartik_{it}$.

In our implementation of 2SLS, we apply Eq 19 (with/without β_t), replace π_{it} by $U_{i,t-4}$ on LFS and $U_{i,t-4}$ by $Bartik_{i,t-4}$ on RHS.

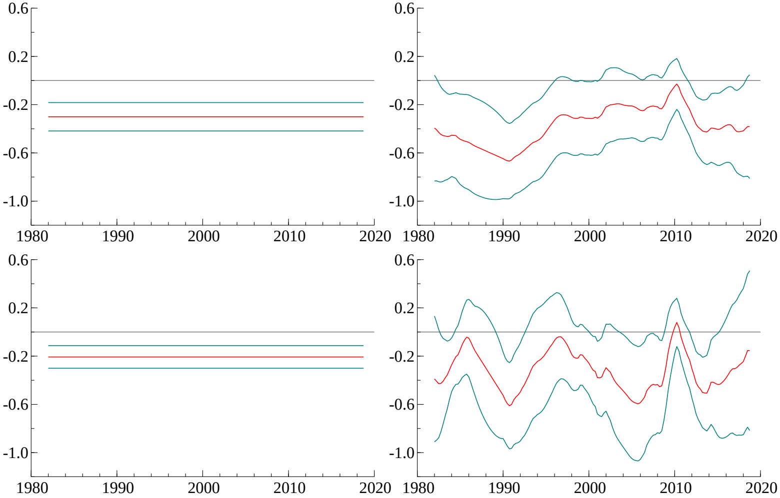
After estimation, the in-sample fit for $U_{i,t-4}$ is denoted by $\widehat{U}_{i,t-4}$ and replaces $U_{i,t-4}$ in the original Eq 19 for π_{it}.

Due to data availability and construction of IV, we can only estimate the model with IV for the sample 1982 – 2017.

Slope Estimates of Phillips Curve 1982 -

		Kalman Filter		IV Kalman Filter
	Lagged	TV Lagged	Tradable	TV Tradable
	Unempl	Unempl	Demand IV	Demand IV
State	✓	✓	✓	✓
TV Level	✓	✓	✓	✓
TV Slope		✓		✓
	(iii)	(iv)	(v)	(vi)
coef	-0.1094	-0.0997	-0.3008	-0.3819
s.e.	0.0234	0.1346	0.0600	0.2191
-llik	7504.4	7501.4	7502.0	7499.3

IV Slope Estimates Phillips Curve 1982 -



Concluding Remarks on Inflation

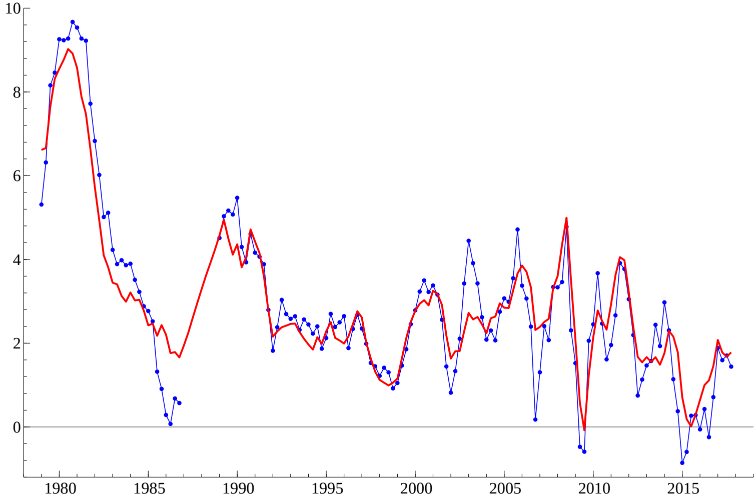
We have replicated the rigorous analysis of Hazell et al (2022) in estimating the highly anticipated Slope of the Phillips Curve.

The stochastic trend (replacing the time fixed effects) and the time-varying slope coefficient lead to a significantly better fit.

Also the residual diagnostics look much better for model with stochastic trends. Hence, there is less need for ad-hoc corrections of standard errors of estimates and related test statistics.

This panel model can be explored further. For example, we can assess its workings for individual US States.

New York State Inflation Fit, 1978-2017



Aim: extracting common trends from Ethane measures

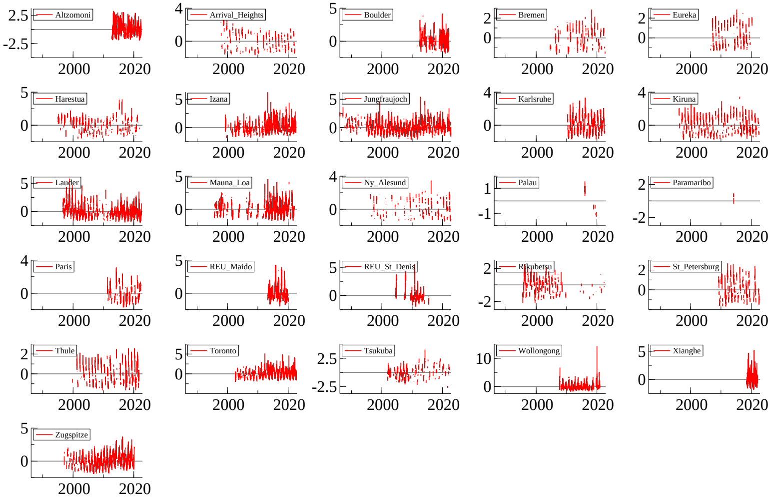
We have Ethane measures from $N = 26$ stations around the globe.

We divide the stations according to their location in the Northern or Southern hemisphere: $N_{north} = 21$ and $N_{south} = 5$

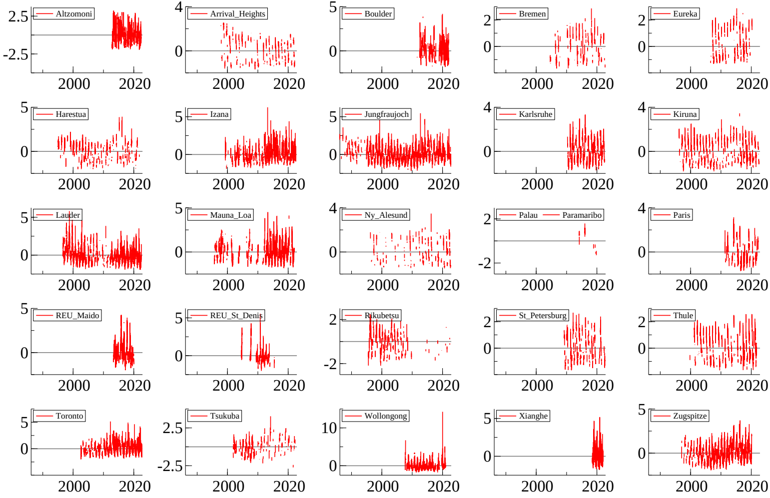
The stations are located at different altitudes: we have demeaned the individual time series.

Let's have a look at the time series in our $N \times T$ data matrix.

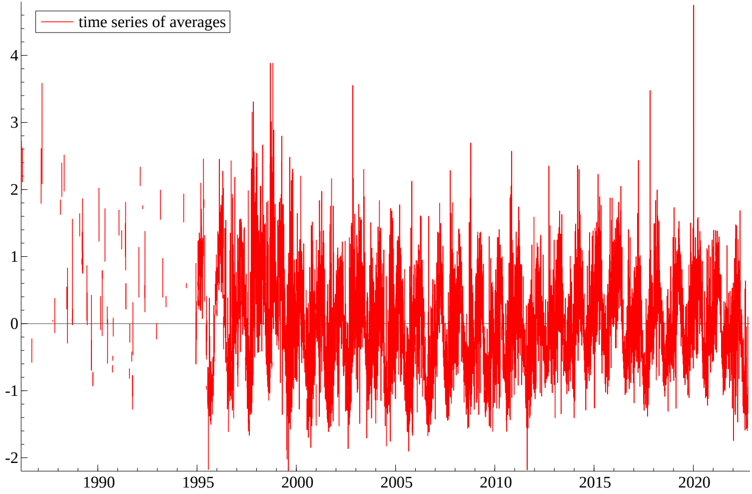
Ethane measures: data



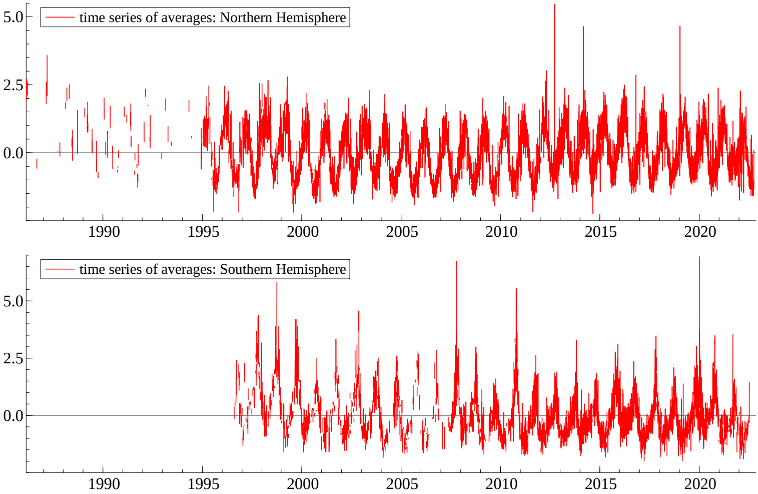
Ethane measures: data



Ethane measures: cross-sectional average



Ethane measures: cross-sectional average



Univariate Decompositions

We first start with considering each time series separately : univariate analysis

We consider the unobserved component time series model for a particular time series i:

$$y_t = \mu_t + \psi_t + \varepsilon_t, \qquad t = 1, \dots, T,$$

for $i = 1, \dots, N$, that is y_t is a time series of Ethane measures from a particular station.

The components represent a long-term trend (μ_t) and summer/winter seasonal effect (ψ_t) and short-term noise (ε_t).

Univariate Decomposition: trend

We consider the unobserved component time series model for a particular time series i, the *level plus seasonal* model:

$$y_t = \mu_t + \psi_t + \varepsilon_t, \qquad \varepsilon_t \sim NID(0, \sigma_\varepsilon^2),$$

for $t = 1, \ldots, T$.

A possible dynamic specification for μ_t is a basic local level model:

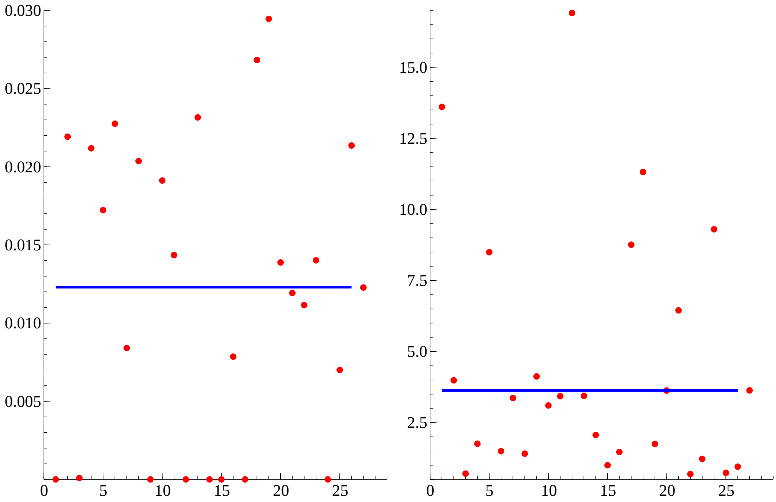
$$\mu_{t+1} = \mu_t + \eta_t, \qquad \eta_t \sim NID(0, \sigma_\eta^2),$$

where the value of σ_η^2 determines the variation in the trend.

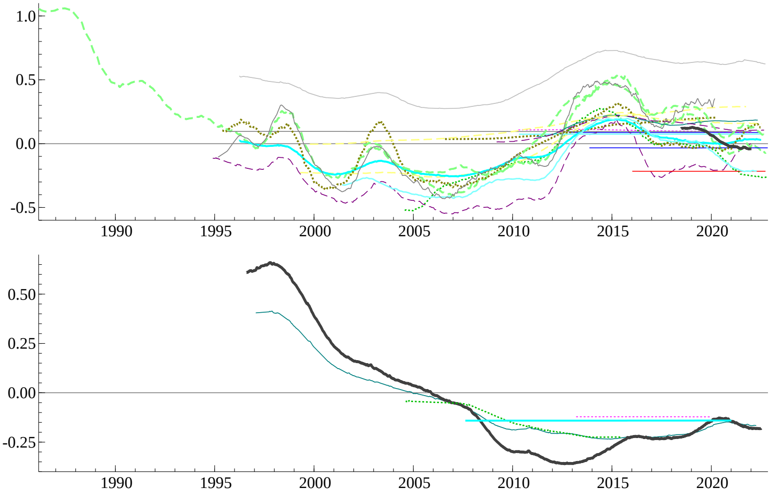
An appropriate dynamic specification for the seasonal effect ψ_t, typically based on multiple stochastic seasonal sine/cosine waves.

The parameters and components are estimated by placing the model in state space form and applying Kalman filter methods: see Harvey 1989, Durbin and Koopman 2012.

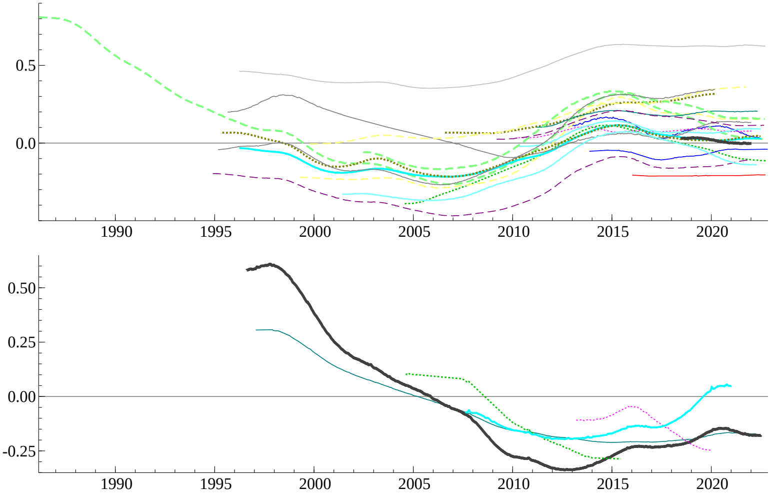
Parameter estimates



Smooth estimates of the level



Smooth estimates of the level: pooled pars



Panel with stochastic level and seasonal

In climate studies, interest focuses on the common level or the "global" Ethane measure.

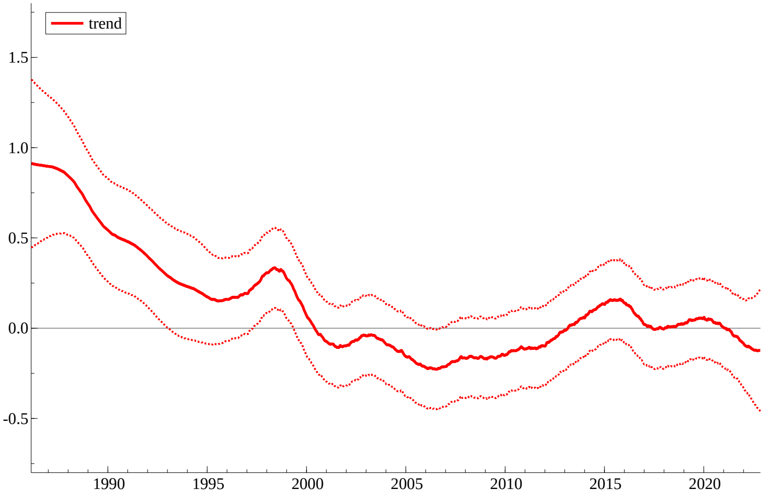
We consider a country panel model with stochastic trend (and seasonal), that is

$$y_{it} = \lambda_i^{\mu}\,\mu_t + \lambda_i^{\psi}\,\psi_t + \varepsilon_{it}, \qquad t = 1, \ldots, T,$$

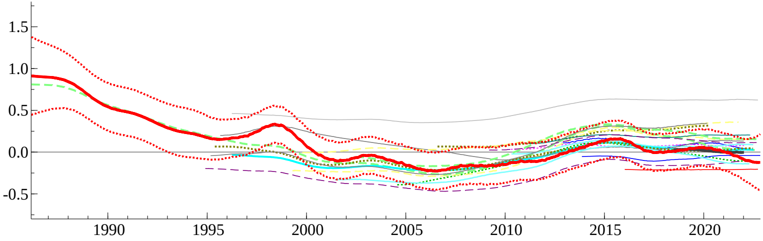
The weights or loadings λ_i^{μ} and λ_i^{ψ} are estimated by maximum likelihood using state space methods.

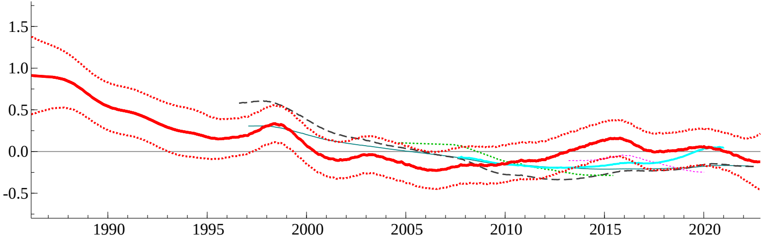
The loadings can also be set a-priori from information generated from the univariate analysis.

Pooled level: a global Ethane measure



Pooled level: a global Ethane measure





Multiple stochastic levels and seasonals

Next we consider Northern H and Southern H jointly,

and consider the country panel model with multiple (two) stochastic trends (and seasonals), that is

$$y_t = \Lambda^\mu (\mu_t^N, \mu_t^S)' + \Lambda^\psi (\psi_t^N, \psi_t^S)' + \varepsilon_t, \qquad t = 1, \ldots, T,$$

The weight or loading matrices have dimension $N \times 2$ and select the appropriate trend and seasonal for the Northern or Southern hemispheres.

Bivariate Level specification

The bivariate level specification is

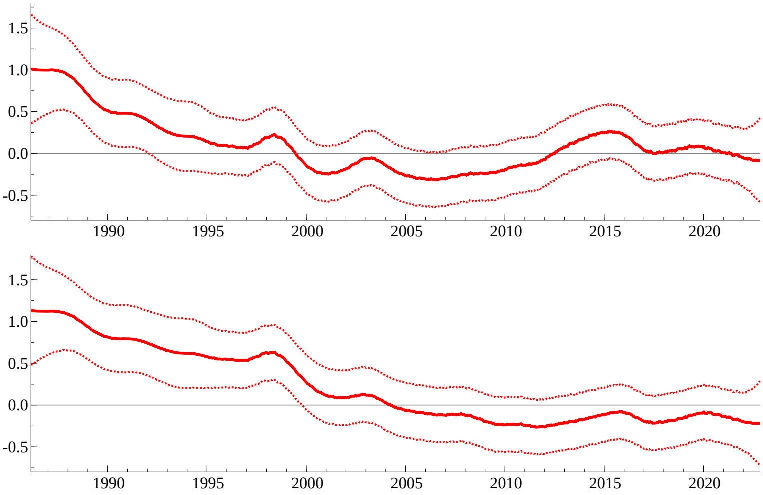
$$(\mu_{t+1}^N, \mu_{t+1}^S)' = (\mu_t^N, \mu_t^S)' + (\eta_t^N, \eta_t^S)', \qquad (\eta_t^N, \eta_t^S)' \sim N(0, \Sigma_\eta),$$

where Σ_η is a 2×2 covariance matrix and is estimated by MLE, together with other parameters.

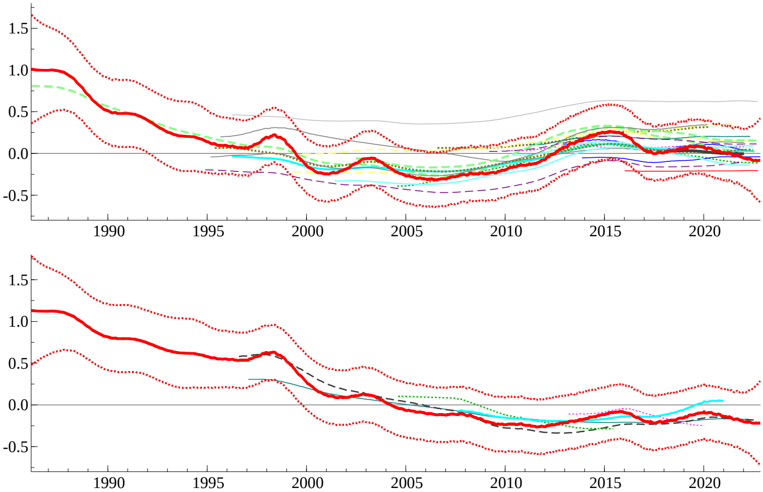
The estimate of the correlation between η_t^N and η_t^S is $\widehat{\rho}_\eta = 0.72$.

The global Ethane measures for the Northern and Southern H have much in common but are not exactly the same.

Two pooled levels: North / South H



Two pooled levels: North / South H



Discussion

Our aim is to construct a global index for Ethane.

We plan to extend the analysis further.

For example, the variance matrix ε_t can be spatially specified as $\sigma_\varepsilon^2 V$ where the $N \times N$ matrix V reflects correlations that are determined by the distances between the stations.

Also, other covariates such as *altitude* and *temperature* can be considered.

We further need to investigate whether the country panel model is useful for forecasting.

Concluding Remarks

Our aim has been to argue that alternative dynamic panel data model can be feasible and effective.

All inference is likelihood-based, exact, and has no approximations.

We plan to extend the analysis further based on the presented framework.