

# Endogenous Life-Cycle Housing Investment and Portfolio Allocation

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## Abstract

This paper develops a life-cycle portfolio allocation model to address the effects of housing investment on the portfolio allocation of households. The model employs a comprehensive housing investment structure, Epstein-Zin recursive preferences and a stock market entry cost. Furthermore, rather than resorting to calibration we estimate the value of the relative risk aversion and elasticity of intertemporal substitution. The model shows that housing investment has a strong crowding out effect on investment in risky assets throughout the life-cycle. Compared to the housing investment feature, the contribution of using EZ recursive preferences instead of constant relative risk aversion (CRRA) utility is relatively limited in matching the empirical evidence.

*JEL classification:* G0, G11, G12, D10

*Keywords:* Portfolio Choice, Housing, Life-Cycle Model, Real Estate.

# 1 Introduction

Low stock market participation rates and moderate equity holdings for stock market participants are two important empirical observations in the US data. For instance, the 2007 Survey of Consumer Finance (SCF) shows that only 55.3% of US households have direct or indirect holdings of risky assets.<sup>1</sup> Furthermore, data from the Panel Study of Income Dynamics (PSID) for the 1968-2007 period show that the median household direct risky asset holdings and indirect risky asset holdings are zero. Despite this fact, theoretical models with the assumption of the historically prevailing equity premium predict that almost 100% of households should hold risky assets as part of their financial portfolio. This gap between theoretical predictions and empirical observations still poses a great challenge to life-cycle models.

While the primary investment asset for US households is investment in owner-occupied housing, it is generally ignored in portfolio allocation models. It is quite logical that a typical household has a higher priority to invest in housing in order to have an owner-occupied house than investing in the stock market.<sup>2</sup> This fact is probably the main reason for the low stock market participation observed in the data. Cocco (2004) shows that due to the investment in housing, younger and poorer households have less wealth to invest in the stock market. Furthermore, Vestman (2012) shows substantial difference in portfolio allocation of homeowners and renters.

The focus of our research is two-folds. First, this paper incorporates housing investment into a life-cycle asset allocation model to provide an explanation for these two empirical observations: low stock market participation rates and moderate equity holdings for stock market participants. Specifically, we investigate the effects of housing investment on portfolio allocation of households in a life-cycle model through the construction of a fairly comprehensive structural model. This involves

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<sup>1</sup>Risky assets include tax-deferred accounts, directly held stocks, directly held pooled investment funds, bonds, and managed investment accounts or equity in a trust or annuity. Tax-deferred retirement accounts consist of both personally established individual retirement accounts (IRAs) and job-based 401(k) accounts. For detailed information see Bucks, Kennickell, Mach, and Moore (2009).

<sup>2</sup>According to the latest release (2014Q3) of the Census Bureau, the homeownership rate in the US is 64.3 percent. The average homeownership rate between 1980Q1 to 2014Q3 is 65.5 percent with standard deviation of 1.72.

solving and simulating the life-cycle asset allocation model and estimating some crucial parameters. Second, we analyze the effects of having Epstein-Zin (EZ) recursive preferences on households portfolio allocation.<sup>3</sup> The EZ recursive preferences provide the flexibility to disentangle the risk aversion from intertemporal substitution. The main drawback of the commonly used additive utility functions such as the constant relative risk aversion (CRRA) utility function is that RRA, which gives information about how agents deal with uncertainty across possible states of the world, is the inverse of the elasticity of intertemporal substitution (EIS), which is just time preference. In other words, CRRA utility imposes two different roles on the same parameter whereas EZ-type preferences disentangle this relationship.

Housing is different from other financial assets because it serves dual benefits. First, it is a durable consumption good from which owners obtain utility. Second, it also serves as an investment asset that enables owners to hold home equity. Contrary to liquid financial assets such as bonds and stocks, housing investment is illiquid and often highly leveraged. While housing plays an important role in portfolio allocation, it is largely unexplored in the literature because of the difficulty of dealing with various frictions associated with the housing market, such as homeowner/renter distinction, mortgage payment, liquidation cost, moving decision, etc.

Beside housing, our model incorporates some key features in order to better explain the asset allocation profiles of households. The first feature is the use of Epstein-Zin (EZ) preferences where the relative risk aversion (RRA) is disentangled from the elasticity of intertemporal substitution (EIS). The second feature is that stock market participation is endogenously done in each period with an entry cost to the market for the first-time investment (*i.e.*, buying risky assets). The entry cost is widely accepted in the literature even though too little investigation has been done on its magnitude.<sup>4</sup> A computationally easy way of introducing an entry cost is considering it as a fixed proportion of annual labor income as in Gomes and Michaelides (2007), Guvenen (2009b), Guo (2004), and Alan (2006). This entry cost can be considered as the cost of opening a brokerage account, understanding

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<sup>3</sup>See Epstein and Zin (1989) and Epstein and Zin (1991) for details of these recursive preferences.

<sup>4</sup>For example see Alan (2006) and Khorunzhina (2013) for a detailed analysis of the entry cost.

how the market works, and acquiring and evaluating information about the stock market plus the opportunity cost of time. The third feature is that labor supply is inelastic and households receive uninsured labor income in each working period. We use PSID data from 1980-2007 to realistically calibrate the life-cycle labor income process of households. Finally, we incorporate a bequest motive in our model which assumes that households bequest all their financial investments and investments made in housing to their inheritors when they pass away.

This comprehensive model is able to explain the aforementioned empirical observations stated above. More importantly, the model is able to explain them by successfully estimating suitable relative risk aversion and elasticity of intertemporal substitution parameters. We obtain both moderate stock market participation rate as well as moderate equity holding among the stock market participants. Hence, housing investment incorporated into a life-cycle asset allocation model has strong crowding out effects on investment in the risky asset when compared to the no-housing model. The effects are significant throughout the life-cycle.

We further compare the portfolio allocations under the EZ recursive preferences with estimated risk aversion and elasticity of intertemporal substitution parameters to the portfolio allocations under the standard constant relative risk aversion utility function with estimated parameter of relative risk aversion.<sup>5</sup> The results show that under the EZ recursive preferences the life-cycle portfolio allocation profiles matches the empirical evidence better than the same profiles under the CRRA case.

The other results are as follows. The model generates reasonable estimates for the homeownership rate during the working period and a slight over-estimate for the retirement period. Similar to Yao and Zhang (2005). Hu (2005), the model shows that renters have higher share of their financial wealth in the risky asset than homeowners mainly because while homeowners can enjoy the return on their housing investment and hence do not need to rely heavily on risky financial investments, renters need to invest more in the risky financial investment as it is the only investment through which they can

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<sup>5</sup>Note that under CRRA utility assumption elasticity of intertemporal substitution is the inverse of the relative risk aversion. Hence the estimated risk aversion under CRRA automatically gives an estimate for elasticity of intertemporal substitution.

accumulate wealth.

Households wealth allocation throughout the life-cycle has been analyzed in the portfolio allocation literature since the pioneering studies of Merton (1969) and Samuelson (1969). Although it has been ignored by a huge body of the portfolio allocation literature over a very long period of time, there is a gradually growing literature that treats housing as an important determinant of portfolio allocation. Leung (2004) provides a comprehensive literature review of housing and asset pricing. Grossman and Laroque (1990) develop an asset allocation model where infinitely lived households derive utility from a single indivisible durable consumption good. They argue that an adjustment cost for illiquid assets could answer the equity premium puzzle. Flavin and Nakagawa (2008) extend the Grossman and Laroque (1990) model by including both durable and nondurable consumption goods into the utility function. Using a continuous time framework, the paper compares a housing model to a habit persistence model and finds that while both deliver many of the same implications, empirical tests using household level data strongly favor the housing model. Longstaff (2009) studies the implications of illiquid assets in a continuous time asset pricing exchange economy with heterogeneous agents. Villaverde and Krueger (2011) present a general equilibrium model of life-cycle asset allocation to demonstrate the effects of consumer durable goods on consumption and asset allocations. Piazzesi, Schneider, and Tuzel (2007) consider a consumption based asset pricing model where housing is explicitly incorporated into the model both as an asset and as a consumption good. Their paper focuses on the effects of housing-consumption asset pricing models on the predictability of the return on stocks.

The closest articles to our research are Hu (2005), Cocco (2004) and Yao and Zhang (2005). Hu (2005) develops a standard life-cycle portfolio allocation model that provides the flexibility for households to endogenously decide whether to be a renter or a homeowner. The model employs a CRRA utility function with two sets of preference parameters (renters and homeowners) and considers only five time periods where each period corresponds to either 10 or 15 years. Although the paper obtains low levels of investment in the stock market relative to standard models with no housing,

the investment in the stock market is still significantly higher than the empirically observed values. Cocco (2004) analyzes portfolio choice in the presence of housing in a standard life cycle model with a CRRA utility function. Each time period in this model corresponds to five years. The model in that paper assumes that all agents are homeowners. There is no endogenous decision to be a renter or homeowner. The paper concludes that (1) house price risk crowds out stock holding, (2) households have a relatively low stock market participation rate compared to standard models with no housing, and (3) younger and poorer investors have limited financial wealth to invest in the stock market. However it is less successful at matching the share of wealth invested in stocks conditional on participation with predicted values much higher than those observed in the data. The last paper similar to ours is Yao and Zhang (2005). The model in this paper is similar to Cocco (2004) except it allows households to decide between renting or owning a house. Results show that renters invest a higher portion of their financial wealth in the stock market than homeowners. Furthermore, the share of wealth invested in the stock market is greatly higher than empirical observations.

Our research differs from these studies in several dimensions. First, we develop a comprehensive life-cycle portfolio allocation model that incorporates all features introduced by different studies in the portfolio allocation literature. Among these are housing as an investment and a durable consumption good, an endogenous decision on being a renter or homeowner, a stock market entry cost and an endogenous decision on stock market investment, EZ-type preferences, and a bequest motive. Second, the life-cycle portfolio allocation papers that incorporate housing into the model generally calibrate the parameters of the model. Instead, we estimate two crucial parameters: the relative risk aversion (RRA) and elasticity of intertemporal substitution (EIS). To our knowledge, this paper is the first one in the life-cycle housing-portfolio allocation literature to estimate parameters instead of resorting to calibration. Finally, we explicitly show that the life-cycle conditional risky asset shares are better matched the data when we use EZ recursive preferences (i.e. disentangling the relative risk aversion from the elasticity of intertemporal substitution) than using the CRRA assumption (i.e. impose the restriction that relative risk aversion is the inverse of the elasticity of intertemporal substitution).

The rest of the paper is organized as follows. Section 2 presents the model with all features and defines the optimization problem. The parametrization of the model constitutes Section 3. We discuss estimation of the RRA and EIS parameters in Section 4. Section 5 presents the results of the model, comparative static analysis and comparison of the results of the model with the data. Section 6 provides concluding remarks. The exposition of the solution technique of the life-cycle model is relegated to an Appendix.

## 2 Model

### 2.1 Household Preferences

This model is a discrete time life-cycle model where each period corresponds to one year. As a general convention in the life-cycle literature each year is actually the real age of a household minus 19. We assume that households live for at most  $T$  periods. The probability that a household is alive at age  $t$  conditional on being alive at age  $t - 1$  is  $q_t$ .

In each period, a household derives utility from a constant elasticity of substitution (CES) utility function with nondurable consumption goods,  $C$ , and housing investment (or consumption),  $H$ . Preferences are in the form of Epstein-Zin, where the relative risk aversion (RRA) is disentangled from the elasticity of intertemporal substitution (EIS):

$$V_t = \left\{ u(C_t, H_t)^{\frac{1-\gamma}{\theta}} + \beta (E_t [q_{t+1}V_{t+1}^{1-\gamma} + (1 - q_{t+1})W_{t+1}^{1-\gamma}])^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}} \quad (1)$$

$$\theta = \frac{1 - \gamma}{1 - 1/\psi} \quad (2)$$

$$u(C_t, H_t) = [C_t^v + H_t^v]^{\frac{1}{v}}, \quad (3)$$

where  $\beta$  is the time discount factor,  $\gamma$  is the RRA parameter, and  $\psi$  is the EIS parameter. The intratemporal elasticity of substitution between nondurable consumption goods and housing investment is  $1/(1 - v)$ .  $W_{t+1}$  is the total wealth that a household would bequest if it passes away at age  $t$ .



## 2.2 Labor Income Process

During their working life households supply labor inelastically in each period and receive stochastic labor income  $Y_{it}$ . Define  $y_{it} = \log(Y_{it})$  where  $y_{it}$  has the following process:

$$y_{it} = F_t + u_{it}^l, \quad (4)$$

$$F_t = \beta_0 + \beta_1 age + \beta_2 age^2 + \beta_3 gender + \beta_4 marital\_status + \beta_5 educ, \quad (5)$$

where  $Y_{it}$  is labor income received by household  $i$  at age  $t$ ,  $f(t, Z_{it})$  is a deterministic function of age  $t$  and household characteristics  $Z_{it}$  (education, marital status, and gender). Following Cocco (2004), we assume that the shock to the log of labor income  $u_{it}^l$  composes of both aggregate ( $\eta_t^l$ ) and idiosyncratic ( $\varepsilon_{it}^l$ ) components. We assume that the idiosyncratic component is transitory and independently and identically distributed with mean 0 and variance  $\sigma_{\varepsilon_t^l}^2$  and the aggregate shock has an AR(1) process:

$$\eta_t^l = \rho \eta_{t-1}^l + w_t^l, \quad (6)$$

where  $w_t^l$  is independently and identically distributed with mean 0 and variance  $\sigma_{w_t^l}^2$ . Therefore, during the working period, log labor income is the sum of a hump-shaped deterministic component and two random components, one transitory and one persistent. For simplicity, we assume that the retirement age is deterministic. Households work until period  $K$  where  $K$  corresponds to an age of 65 ( $K = 46$ ). During the retirement periods ( $t > K$ ), households receive a constant and deterministic labor income  $Y_{it} = \xi Y_{iK}$  where  $0 < \xi < 1$  (a fraction of their income during the last year before retiring)<sup>6</sup>.

## 2.3 Housing Investment

Households enter the market as renters in the first period. From the second period on, they endogenously decide either to buy a house and become homeowners or stay in their current house as renters.

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<sup>6</sup>During the whole retirement period  $u_{it}$  becomes  $u_{iK}$ .

A typical homeowner at any time can endogenously decide either (1) to stay in his current house, or (2) to sell his current house and buy a new one, or (3) to sell his house and become a renter. Similarly, a renter can decide either (1) to stay in current rented house, or (2) to rent a new house, or (3) to buy a house and become a homeowner. Housing investment, by its nature, is a lump-sum and illiquid investment. Buying a house requires paying a proportion  $d$  of the house market value as down payment and financing the rest through a mortgage. To capture the illiquidity of housing investment, households incur a liquidation cost equal to a fraction  $\kappa$  of the house market value. Homeowners pay an amount equal to a proportion  $\delta$  of the house's market value for maintenance and depreciation expenses in each period. Renters, on the other hand, don't pay for maintenance and depreciation costs; they only pay an annual rent which is equal to a proportion  $\alpha_R$  of the house's market value. House-related expenses are calculated as a proportion of the house's market value in the literature because it is easy in this way to incorporate them into the model and reduces the computational burden.

Per unit price of housing is denoted by  $P_t^h$ , such that a house of size  $H_i$  has price  $P_t^h H_i$  at time  $t$ . We define  $p_t^h$  as the log of house price,  $p_t^h = \ln(P_t^h)$ , and assume that it follows the following stochastic process:

$$\Delta p_t^h = \mu_h + \varepsilon_t^h. \quad (7)$$

where  $\Delta p_t^h$  follows a stochastic binomial process with the average growth rate of house prices is  $\mu_h$ .

When a household buys a house of size  $H_i$  at time  $t$ , it pays the down payment which is equal to  $dP_t^h H_i$  and finances the rest through a mortgage with a fixed rate  $r^m$ .  $RM_t$  denotes the amount of mortgage debt a household has at age  $t$ . When buying a house, households can borrow up to the house value minus the down payment:

$$RM_t \leq (1 - d) P_t^h H_i. \quad (8)$$

Housing investment constitutes an important part of households wealth. However, because of its illiquidity, this investment cannot be directly used for consumption purpose. As households pay back

their mortgage debt over time, their housing investment (home equity) and total wealth increases while liquid wealth does not necessarily increase. Once households decide to sell their houses by paying the transaction cost, then they transform the housing investment into liquid wealth and can use part of it for consumption.

## 2.4 Financial Assets and Wealth Accumulation

There are only two financial assets that households can invest in: a risky asset and a riskless asset. Return on investment in the risky asset follows the following stochastic process as in Campbell, Cocco, Gomes, and Maenhout (2001), Cocco (2004), and others:

$$R_{t+1}^s - R^b = \mu_s + \varepsilon_{t+1}^s, \quad (9)$$

where  $R_{t+1}^s$  is the gross return on the risky asset at time  $t + 1$ ,  $R^b$  is the constant gross return on the riskless asset,  $\mu_s$  is the excess return from the risky asset over the riskless one (equity premium) and  $\varepsilon_{t+1}^s$  follows a stochastic binomial process. In order to invest in the risky asset, households are required to pay a one-time fixed entry cost. This entry cost can be considered as the cost of opening a brokerage account, understanding how the market works, the cost of acquiring and evaluating information about the risky asset, and the opportunity cost of time. Alan (2006) estimates this cost as approximately 2 percent of annual labor income while Haliassos and Michaelides (2003) obtain a wide range of this cost from 3% to 34% of annual labor income and Gomes and Michaelides (2007) calibrates this fixed cost at 6 percent of annualized labor income. Unlike the risky asset, there is no cost for investing in the riskless asset.

### 2.4.1 Wealth Accumulation of Renters

In this model, there are two types of households: renters and homeowners. Depending on their homeownership status, households have different budget constraints and cash-on-hand structures. The

liquid wealth ( $LW$ ) of a typical household who was renter at age  $t - 1$  is the sum of investments made at age  $t - 1$  in both risky ( $S$ ) and riskless ( $B$ ) assets:

$$LW_t = R^b B_{t-1} + R_t^s S_{t-1}. \quad (10)$$

We refer to the sum of liquid wealth and labor income as cash-on-hand. At age  $t$ , the cash-on-hand is  $X_t \equiv LW_t + Y_t$ . A typical renter uses this cash-on-hand to decide whether to buy a house by paying the down payment and begin to pay annual mortgage payments or just stay in a rental property and pay an annual rent. He also decides on consumption expenditure, whether to pay the fixed entry cost for investment in the risky asset (if it has not been paid yet) and decides the portfolio composition among the different financial assets:

$$X_t = C_t + S_t + B_t + FIX_t \alpha_F Y_t + (1 - HR_t) [\alpha_R P_t^h H_t] + HR_t [M_t + dP_t^h H_t] \quad (11)$$

where  $S_t$  is the total investment made in the risky asset and  $B_t$  is the total investment made in the riskless asset at age  $t$ .  $FIX_t$  is a dummy variable that takes the value of 1 if a renter pays the fixed entry cost at age  $t$ , and 0 otherwise. The term  $\alpha_F Y_t$  represents the fixed entry cost which is the proportion  $\alpha_F$  of the annualized labor income at age  $t$ .  $HR_t$  is a dummy variable that takes the value of 1 if a household become homeowner at time  $t$  and it is 0 otherwise.  $M_t$  is the household's annual mortgage payment at time  $t$ .

#### 2.4.2 Wealth Accumulation of Homeowners

The liquid wealth structure of a typical homeowner at age  $t$  is the same as the liquid wealth structure of a renter at time. Similarly, the cash-on-hand of homeowners at age  $t$  is the sum of liquid wealth and labor income  $X_t \equiv LW_t + Y_t$ . A typical homeowner decides in every period whether to stay in his current house or sell the current house and buy another one or sell the current house and move to a rental property. Once he sells his house, he would use his cash-on-hand plus the proceeds from

selling the house to either buy a new house (upgrade or downgrade) by paying the down payment or move to a rental property and pay rent. He would then decide on consumption expenditure and investment in financial assets using the remaining cash-on-hand. On the other hand, if he decides to stay in the current house, then he would pay the annual mortgage payment (if the mortgage debt is not yet finished) and use the remaining cash-on-hand for consumption expenditure and investments in financial assets. Furthermore, households pay for the maintenance and depreciation expense which is equal to proportion  $\delta$  of the house's market value. It follows that the budget constraint of households takes the following form:

$$\begin{aligned}
X_t = & C_t + S_t + B_t + FIX_t \alpha_F Y_t + HR_t (1 - MS_t) [M_t + \delta P_t^h H_{t-1}] + \\
& HR_t MS_t [dP_t^h H_t + M_t + \delta P_t^h H_{t-1} - ((1 - \kappa) P_t^h H_{t-1} - RM_t)] + \\
& (1 - HR_t) MS_t [\alpha_R P_t^h H_t - ((1 - \kappa) P_t^h H_{t-1} - RM_t)], \tag{12}
\end{aligned}$$

where  $MS_t$  is a dummy variable that takes the value of 1 if a homeowner moves to a new house or rental property and 0 if he stays in his current house. The term  $(1 - \kappa) P_t^h H_{t-1}$  denotes the amount a homeowner receives if he sells his house, adjusted for the liquidation cost.  $RM_t$  is the total mortgage debt a homeowner has at time  $t$  and  $M_t$  is the annual mortgage payment, which consists of both the annual interest payment  $MI_t$  and the annual principal payment  $MP_t$ . The mortgage debt and annual mortgage payments have the following processes. In each period,  $MI_t$  is calculated from remaining mortgage balance:

$$M_t = MP_t + MI_t, \tag{13}$$

$$RM_t = RM_{t-1} - MP_t. \tag{14}$$

The mortgage interest payments are set such that the mortgage is paid back over a period of 25 years, given the mortgage rate  $r^m$ .

Households are assumed to satisfy the non-negativity constraints on consumption and housing investment and short-sale constraints on the risky and the riskless asset. They cannot have negative amounts of consumption and housing investment. Furthermore, households cannot borrow at the riskless rate in order to invest in the risky asset.

$$C_t \geq 0, S_t \geq 0, B_t \geq 0, H_t \geq 0, \forall t \quad (15)$$

Furthermore, we define the correlations between the shocks to returns on investment in the risky asset and the persistent shocks to labor income as  $\rho_{sl}$ , the shocks to returns on investment in the risky asset and the shocks to returns on housing prices as  $\rho_{sh}$ , and finally the shocks to returns on housing investment and the persistent shocks to labor income as  $\rho_{hl}$ .

It is common in the recent life-cycle portfolio allocation models to incorporate a bequest motive in order to match the skewness of the wealth distribution.<sup>7</sup> Conditional on being alive at age  $t - 1$ , a household could pass away with probability  $1 - q_t$ . We assume that households bequest their total liquid wealth and all investments made in housing to their inheritors at any period they pass away:

$$W_t = R^b B_{t-1} + R_t^s S_{t-1} + P_t^h H_{t-1} - RM_t. \quad (16)$$

## 2.5 Optimization Problem

Before defining the optimization problem and the value function, we list the state and the control variables in a compact form. The state variables are age ( $t$ ), liquid wealth ( $LW_t$ ), risky asset participation status ( $IFIX_t = 1$  if the fixed cost has been paid,  $= 0$  otherwise), homeownership status ( $O_t = 1$  if homeowner,  $= 0$  otherwise), the size of house owned from the previous period ( $H_{t-1}$ ), the remaining mortgage debt ( $RM_t$ ) and the lagged labor income shock. We denote the state variables by  $\Omega$ , where  $\Omega_t = \{t, LW_t, IFIX_t, O_t, H_{t-1}, RM_t, u_{i,t-1}\}$ . The control variables are consumption ( $C_t$ ),

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<sup>7</sup>For example see Laitner (2002), Gomes and Michaelides (2005), Gourinchas and Parker (2002), and Yao and Zhang (2005).

the size of house a household choose at age  $t$  ( $H_t$ ), the risky asset participation decision ( $FIX_t$ ), and the share of financial investment in the risky asset ( $s_t$ ). We denote the control variables by  $\Psi$ , where  $\Psi_t = \{C_t, H_t, FIX_t, s_t\}$ . Given the stochastic labor income, stochastic house price evolution and stochastic risky asset returns, the household's optimization problem is then

$$V_t(\Omega_t) = \max_{\Psi_t} \left\{ u(C_t, H_t)^{\frac{1-\gamma}{\theta}} + \beta \left( E_t [q_t V_{t+1}(\Omega_{t+1})^{1-\gamma} + (1 - q_t) W_{t+1}^{1-\gamma}] \right)^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}} \quad (17)$$

subject to dynamics, restrictions and budget constraints in equations (4) to (16).

### 3 Parameterization

Two crucial parameters (RRA and EIS) will be estimated but we must calibrate the remaining parameters of the model. Tables 1 displays the list of parameters that are calibrated and their values.

Each period in the model corresponds to one year. Households are assumed to enter the market at age of 20 and live for at most 80 additional years. The age-dependent labor income process is based on PSID data from 1980-2007. In order to obtain a random sample of US households, we dropped families that were part of the Survey of Economic Opportunities.<sup>8</sup> We treat retirement period labor income as a constant fraction of labor income of the last working period as in Gomes and Michaelides (2005) and Cocco (2004). The permanent component of labor income, equation (5), is a function of households' age and age squared, as well as dummy variables for education (1 if the head of the household has at least 12 years of education, 0 if not), marital status (1 if married and 0 if not), and gender (1 if head of household is male and 0 if female). We define labor income widely enough to account for endogenous ways of self insurance against labor income shocks. Labor income includes total labor income, unemployment compensation, social security, total transfers, child support, and

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<sup>8</sup>The original PSID sample was drawn from two independent sources: an over-sample of roughly 2000 poor families selected from the Survey of Economic Opportunities (SEO), and a nationally-representative sample of roughly 3000 families from all states. Since the SEO is over-sample of low income families, it isn't a random sample of the US population.

other welfare of both the head of the family and his spouse if present. Following the procedure in Cocco (2004), we estimate the standard deviations of both aggregate and transitory shocks and the persistence parameter of labor income using the same PSID data from 1980 to 2007. We estimate the persistence parameter  $\phi$  as 0.82 which is in line with the findings of Guvenen (2009a) and estimate the standard deviation of  $\varepsilon_{it}^{l_1}$  and  $\varepsilon_{it}^{l_2}$  at 0.1632% and 0.3272% respectively. These estimates are in line with the parameters in Campbell, Cocco, Gomes, and Maenhout (2001) and Cocco (2004), and Cocco, Gomes, and Maenhout (2004).

We calibrate several parameters related to housing. Yao and Zhang (2005) and Hu (2005) set the required down payment rate at 20% of the house value while Cocco (2004) sets it at 15%. We set the down payment rate as 20% of the house market value. To be consistent with previous studies, we set the annual rental rate of houses at 6% of the market value of the rental property, and annual maintenance and depreciation cost at 1% of the house's market value.<sup>9</sup> We set  $v = 0.33$  so that the intratemporal elasticity of substitution between nondurable consumption and housing,  $1/(1 - v)$ , is 1.5 as in Piazzesi, Schneider, and Tuzel (2007).

We next discuss the value of the parameters related to housing prices in equation (7). In order to obtain annual growth rate of house prices, we estimate equation (7) using the Case-Shiller index. The annual average real growth rate of housing prices is 1.9% for the Case-Shiller index with a standard deviation of 5.72%. However, part of the price increase is due to quality improvement which cannot be accounted for using Case-Shiller index. Therefore, we use 1% instead of 1.9% in order to take into account the increase due to quality changes of houses. Accordingly, we set  $\mu_h = 0.01$  and  $\sigma_h = 0.0572$ .

We set the time discount factor  $\beta$  to 0.95. The real risk free rate  $R^b$  is 3%. We set the annual mean return on the risky asset at 9% which means a 6% equity premium  $\mu_s$  with a standard deviation of 20%.

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<sup>9</sup>According to Housing Statistics of the United States, in 2000 the median price specified housing units in the United States is \$119,600 and monthly median rent is \$602. Then a rough calculation shows an annual rent of 6.1% of house value.



There is no consensus in the literature on the size of correlations between shocks to the returns on the risky asset, persistent shocks to labor income, and housing price shocks. Flavin and Yamashita (2002) use PSID data and find that there is almost no correlation between return on investment in the risky asset and the rate of growth of housing prices. Although the authors find that the city-level Case-Shiller index conflicts with PSID results, they claim that the PSID data are nationwide while city level data aren't and so set  $\rho_{sh} = 0$ . Cocco (2004), Hu (2005), and Yao and Zhang (2005) also assume  $\rho_{sh}$  is 0. However, in the baseline model we assume at annual frequency a 0.20 positive correlation between shocks to return on risky asset and shocks to housing investment return. The correlation between persistent shocks to labor income and shocks to the return on investment in the risky asset  $\rho_{sl}$  is calibrated in Campbell, Cocco, Gomes, and Maenhout (2001) to be positive while ranging from 0.3 to 0.5 depending on education of households. However it is set to zero in Cocco (2004), Hu (2005), and Yao and Zhang (2005). We set the correlation coefficient between persistent shocks to labor income and shocks to the return on investment in the risky asset to 0.10. Finally, we assume 0.075 correlation between persistent shocks to labor income and shocks to the return on housing investment  $\rho_{hl}$  in the baseline case. The same correlation is calibrated to 0.5 in Cocco (2004) and 0.2 Yao and Zhang (2005) while Hu (2005) assumes no correlation.

In general, the mortgage rate is higher than the constant return on the riskless asset because mortgages bear a long-term interest rate risk and a default risk. Since our model doesn't have interest rate risk and default risk, we set the interest rate on mortgage loans same to the return on the riskless asset at 3%. This implies that paying the mortgage debt by  $x$  amount is equivalent to increasing riskless asset investment by  $x$  amount. Housing investment (home equity) increases by less than  $x$  amount because part of the mortgage payments goes to the interest component of the payment. Households pay their mortgage debt over 25 years and the mortgage payment variables ( $MP_t$ ,  $MI_t$ ) are computed accordingly. The one-time fixed-entry cost for investment in the risky asset is set at 5% of annual labor income. Finally, we parameterized the conditional survival probabilities  $q_t$  from the mortality

Table 1: Baseline Parameter Values

Description	Parameter	Value
Time discount factor	$\beta$	0.95
Gross return on the riskless asset	$R^b$	1.03
Equity premium	$\mu_s$	0.06
Liquidation cost	$\kappa$	0.10
Intratemporal elasticity of substitution	$\nu$	0.33
Rental rate	$\alpha_R$	0.06
Mortgage rate	$r^m$	0.03
Fixed entry cost	$\alpha_F$	0.05
Down payment	$d$	0.20
Depreciation and maintenance	$\delta$	0.01
Average growth rate on housing prices	$\mu_h$	0.01
Std. of persistent shock to labor income	$\sigma_{l_2}$	0.1632
Std. of temporary shock to labor income	$\sigma_{l_1}$	0.3272
Std. of shocks to return on housing inv.	$\sigma_h$	0.057
Std. of shocks to return on risky asset inv.	$\sigma_s$	0.20
Retirement income factor	$\xi$	0.66
Persistence parameter of labor income shocks	$\phi$	0.82
Correlations between shocks		
- return on housing and return on risky asset inv.	$\rho_{sh}$	0.20
- return on housing investment and labor income	$\rho_{hl}$	0.075
- return on risky asset inv. and labor income	$\rho_{sl}$	0.10

tables produced by the National Center for Health Statistics.<sup>10</sup>

There is no analytical solution to this problem. Therefore we use numerical approximation based on the value function iteration, starting from the last period  $T$  and move backward in time. The details are given in the Appendix.

## 4 Estimation

Solving this structural model numerically is extensively time consuming, so we are able to estimate only two parameters: the relative risk aversion  $\gamma$  and the elasticity of intertemporal substitution  $\psi$ . The remaining parameters are calibrated to the values discussed above. These two parameters play an

<sup>10</sup>[http://www.cdc.gov/nchs/data/nvsr/nvsr58/nvsr58\\_21.pdf](http://www.cdc.gov/nchs/data/nvsr/nvsr58/nvsr58_21.pdf), Table 1.

important role in obtaining life-cycle asset allocation profiles; different values of these two parameters lead to different life-cycle allocations. Furthermore, there is no clear consensus in the literature on the exact values of these parameters. For instance, Vissing-Jorgensen (2002) suggests that limited asset market participation is important for the estimation of the elasticity of intertemporal substitution. She estimates the EIS for stockholders to be around 0.3-0.4 and for bond holders around 0.8-1. On the other hand, Guvenen (2003) uses an EIS for stockholders close to 1 while for nonstockholders close to 0.1. Hall (1988) finds that the EIS is unlikely to be much higher than 0.1. However, Hansen and Singleton (1982) and Vissing-Jorgensen and Attanasio (2003) estimate the EIS as greater than 1. They also estimate relative risk aversion between 5-10. In a life-cycle portfolio allocation model, Gomes and Michaelides (2005) uses different values of RRA and EIS and conclude that life-cycle portfolio allocations are very sensitive to the values of RRA and EIS. Because these two parameters play critical roles and because there is no consensus in the literature on the exact values for them, it is important to estimate these parameters.

We estimate our model using a minimum distance estimator. See for example Greene (2008, Chapter 15). Let us denote by  $w_{i,t}$  the value taken by variable  $i$  at age  $t$  over the life-cycle. Let us also denote by  $g_{i,t}(\gamma, \psi)$  the predicted value generated by the model for variable  $i$  at age  $t$  for a given value of the parameters  $\gamma$  and  $\psi$ .<sup>11</sup> A consistent estimator of the RRA and EIS parameters is then given by

$$(\hat{\gamma}, \hat{\psi}) = \arg \min_{\gamma, \psi} \sum_{i=1}^N \sum_{t=1}^K (w_{i,t} - g_{i,t}(\gamma, \psi))^2, \quad (18)$$

where  $N$  is the number of variables used in the minimum distance estimation.

This estimation procedure is similar to the impulse-response matching approach where the estimates are chosen by minimizing the distance between the empirical impulse-response function and the impulse-response function generated by a model (see Inoue and Kilian (2013) and Hall, Inoue, Nason, and Rossi (2012) and the numerous references therein). Although similar, it is more simple

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<sup>11</sup>We simulate a very large number of life-cycle paths (50,000) and use the average as the predicted value.

because we directly match the data ( $w_{i,t}$ ) and the prediction from the model. We do not have to deal with the complications associated to the estimation of the empirical impulse-response function.

The life-cycle data we use for the estimation are investment in the risky asset as a share of total financial assets, risky asset market participation rate, and homeownership status because these three are the main life-cycle profiles we are interested in. We use SCF 2007 for the data because SCF is probably the most comprehensive survey on US households financial assets.

We define total investment in risky assets as the sum of stock mutual funds, bond mutual funds, mortgage-backed bonds, corporate bonds, publicly traded stocks and foreign bonds. On the other hand, riskless asset are composed of checking accounts, certificate of deposits, government-backed bond mutual funds, US Government bonds, saving bonds, municipal bonds, and cash and call money accounts. For some assets, households are asked how they are invested: either in risky, or riskless, or split between them. Depending on the answers we classify IRA-Keogh accounts, pensions and saving-money market accounts either risky, riskless, or split between both.

Data used for the estimation include households that report all demographic information (i.e. age, gender, marital status, education background) and necessary data for the estimation. We average-out the data for each age and further smooth-out the lifetime profiles through kernel regression because the raw data are not smooth enough for the estimation. Kernel regression, as a non-parametric regression method, is appropriate when the relation between the dependent and the independent variables is clearly not linear. Define  $\bar{y}_t^a$  as the age-dependent averaged data for any one of the variables, then the goal of doing a kernel regression is to estimate the following functional form,

$$\bar{y}_t^a = m(x_t) + e_t, \tag{19}$$

where  $m(x_t)$  isn't specified and  $x_t = t$ . To estimate  $m(x_t)$  we use a kernel  $K_h(x)$  constructed from the normal distribution,

$$K_h(x) = \frac{1}{h\sqrt{2\pi}} e^{-\frac{x^2}{2h^2}}, \tag{20}$$

which yield the following kernel regression estimator:

$$\hat{m}_h(x_o) = \frac{\sum_{t=1}^T K_h(x_t - x_o) \bar{y}_t^a}{\sum_{t=1}^T K_h(x_t - x_o)}. \quad (21)$$

We set the bandwidth equal to  $h = 0.2\hat{\sigma}_x$  where  $\hat{\sigma}_x$  is the sample standard deviation of  $x_t$ . We perform this kernel regression separately for each series. The resulting predicted values are used as the observed data  $w_{i,t}$  when performing the minimum distance estimation. Figure 1 displays the raw data and the kernel-smoothed data for the estimation.

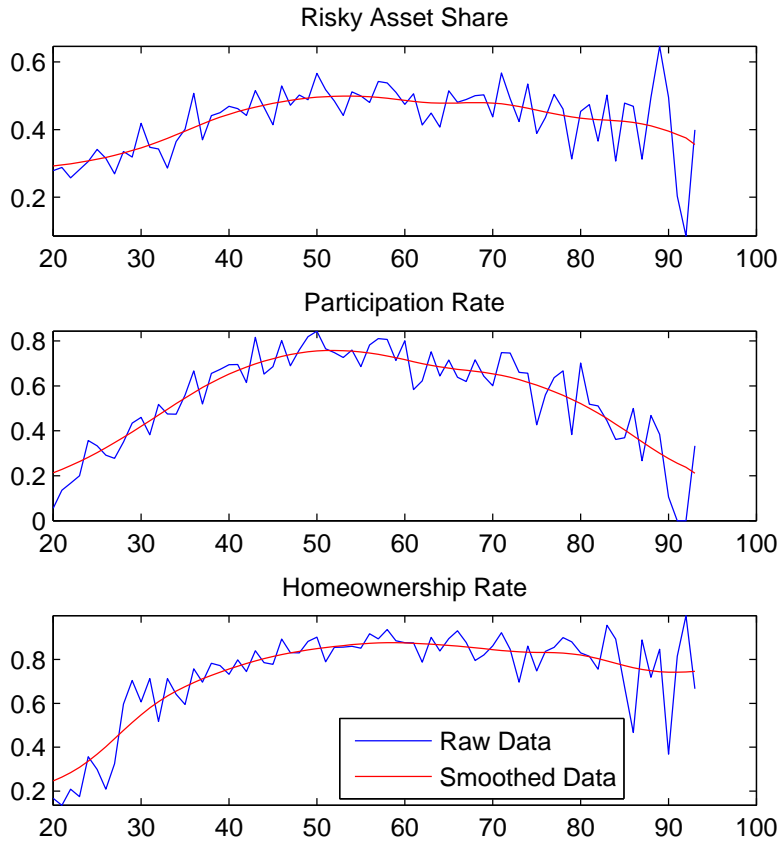


Figure 1: Raw and Smoothed Data for the Estimation

## 5 Results

### 5.1 Model with Housing Investment

The EZ recursive preferences provides us with the flexibility of estimating RRA different than the inverse of the EIS. The estimated value for RRA ( $\hat{\gamma}$ ) and EIS ( $\hat{\psi}$ ) are respectively 3.78(0.0954) and 0.34(0.2423) with the standard errors in the parentheses. These estimates are moderate and are in line with the literature. Having estimated these parameters within acceptable intervals, we use the corresponding optimal solution to simulate the corresponding life-cycle risky asset and housing investment profiles of households.

We display in Panel A Table 2 wealth accumulation profiles for the simulated baseline model and for the empirical data from the SCF. The model shows that households' financial wealth increases during the working life and decreases during the retirement period. It is simply because once households retire, their labor income decreases substantially and they use the investment in financial accounts to compensate the decrease in the labor income and to have a smooth consumption profile over the life-cycle. Total wealth also shows a similar picture but the decrease during the retirement period is limited as households do not cash in on their home equity or are not allowed to use reverse mortgage in the model. The lower panel of the table shows the wealth accumulation profiles in the SCF data over the life-cycle. Similar to the model, households accumulate both financial wealth and total wealth during the working period and decumulate to some extent during the retirement period.

Panel B of the table displays the differences between the wealth accumulation profiles of risky asset market participants and non-participants. The total wealth and the financial wealth of participants is higher than that of non-participants. However, the non-participants have more investment in housing (more home equity) than homeowners because in the absence of risky asset, non-participants prefer to have more investment in the housing investment to compensate, to some extent, the lack of risky asset investment. The financial wealth of non-participants is quite low as they can only invest in the riskless asset with relatively low return.

Table 2: Wealth Accumulation Profiles

Age	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
<b>Panel A</b>								
<b>1. Baseline Model</b>								
Total Wealth	0.88	2.49	3.29	4.07	5.57	6.34	6.81	6.79
Financial Wealth	0.28	0.77	1.97	2.52	3.26	2.32	1.41	0.58
<b>2. SCF</b>								
Total Wealth	0.16	2.27	3.48	4.33	4.69	4.22	2.61	
Financial Wealth	0.05	1.57	2.51	3.23	3.57	3.39	2.16	
<b>Panel B</b>								
<b>1. Participants of the Risky Asset Market</b>								
Total Wealth	0.99	2.64	3.51	4.34	5.84	7.7	9.07	7.9
Financial Wealth	0.57	1.30	2.64	3.31	4.38	4.94	5.01	3.04
<b>2. Non-Participants of the Risky Asset Market</b>								
Total Wealth	0.79	2.27	2.66	3.25	4.88	5.18	5.98	6.55
Financial Wealth	0.04	0.07	0.08	0.11	0.13	0.06	0.08	0.09
<b>Panel C</b>								
<b>1. Homeowners</b>								
Total Wealth	1.83	3.25	3.52	3.96	5.62	6.41	6.86	6.79
Financial Wealth	0.23	0.44	0.64	0.95	2.19	2.26	1.25	0.40
<b>2. Renters</b>								
Total Wealth	0.31	1.27	3.08	4.18	4.96	4.11	5.44	6.60
Financial Wealth	0.31	1.27	3.08	4.18	4.96	4.11	5.44	6.60

Total Wealth is the sum of financial wealth and home equity, where the financial wealth is composed of investment in risky and riskless financial assets. These reported wealth numbers are normalized by labor income.

The last panel displays the wealth accumulation profiles depending on the homeownership status. Homeowners are wealthier than renters throughout the life-cycle while renters hold more risky asset than homeowners. Due to their housing investment, homeowners do not need to invest heavily on the risky asset. However, in the absence of housing investment, renters need to invest more in the risky asset in order to accumulate more wealth both for the retirement and for the bequest motive.

Table 3 shows life-cycle portfolio allocation and homeownership status of households for the model with baseline parameters (Tables 1). The first row of the table shows the share of risky asset within all financial assets for the population while the second row displays the same share conditional on investing in the risky asset. The third and the fourth rows display the risky asset market

Table 3: Life-Cycle Profiles - Baseline Model

Age	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Risky Asset Share (Uncond.)	0.15	0.24	0.41	0.49	0.47	0.37	0.31	0.3
Risky Asset Share (Cond.)	0.33	0.47	0.63	0.66	0.7	0.72	0.65	0.62
Participation Rate	0.39	0.51	0.64	0.75	0.68	0.52	0.48	0.49
Homeownership Rate	0.32	0.7	0.73	0.74	0.74	0.78	0.86	0.94

Risky asset share is the share of risky asset investment over all financial assets conditional on investing in the risky asset.

participation rate and homeownership rate, respectively.

The first two rows of Table 3 show that households gradually increases the share of risky asset investment within total financial assets during the working period reaching the highest unconditional share of 49% and highest conditional share of 70% before retirement and then decrease these shares during the retirement. The main reason for the change in the portfolio from high but stochastic return risky asset investment to low and constant return riskless asset investment during the retirement is because as people get older it would be difficult to compensate any big negative shock to risky asset investment. Therefore, they gradually transfer their financial wealth from the risky asset to riskless one.

One can argue that during early years of working period, it would be optimal to invest the remaining wealth in the risky asset after households make housing investment and consumption decisions. However, there are some reasons that keep households away from investing all remaining wealth only in the risky asset. During the early ages, labor income is the only source of wealth for households and it is low compared to middle-aged households labor income. Once households pay either regular mortgage payment or rent out of labor income, they left with lower labor income for consumption and financial investment. After the consumption decision, they need to put some of this labor income in risk-free asset for precautionary reasons. If they would invest bulk of this remaining labor income in the risky asset and then face with big negative shocks to both labor income and risky asset investment, they might not be able to pay mortgage payments or rent or they would left with too little income for



consumption. Therefore, for early years, it is optimal for households to invest bulk of their financial wealth in the riskless asset and gradually increase the share of risky asset over the working period.

The third row of Table 3 shows the proportion of the population that has paid the fixed entry cost and is actively investing in the risky asset. The participation rate follows a pattern similar to the pattern of the risky asset share. During the working life, this share monotonically increases from 39% to 75% and then decreases during the retirement period to less than 50%.

Looking at the first three rows of Table 3, it is clear that the model is able to generate a moderate level of risky asset market participation rate and moderate share of risky asset holding for the whole life-cycle. As we will show later, this is mainly due to the housing investment and slightly due to the assumption of EZ recursive preferences, through which we are able to disentangle the relative risk aversion from the elasticity of intertemporal substitution.

The baseline model finally shows that households have a strong preference for homeownership. Homeownership rate continuously increases from 32% to 94% over the life cycle in the model. As we will see in the subsequent sections, this rate is slightly higher than the rate we observe in the SCF data for the retirement group. However, for the other age groups the model has generated reasonable homeownership rates.<sup>12</sup> Among many reasons for the over-estimation of homeownership rates for old people, we can think about at least two reasons. First, the model does not include compulsory moving that can happen because of many reasons such as health reasons, retiring, divorce, etc. The compulsory moving is high for retired people while it is low for the working-age people. Second, the model does not include reverse mortgaging which could lead some households to cash in on their houses at some point in time during retirement.

## 5.2 Model Without Housing Investment

Next, we compare the aforementioned model to the same model with no housing investment (NHI) in order to quantitatively analyze and measure the effects of housing investment on portfolio allocation.

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<sup>12</sup>For instance in Yao and Zhang (2005) the homeownership rate is estimated to be 100% from age of 35 to 95.

Table 4: Life-Cycle Profiles - No Housing Investment

Age	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Risky Asset Share (Uncond.)	0.75	0.96	0.98	0.98	0.96	0.83	0.67	0.53
Risky Asset Share (Cond.)	0.77	0.97	0.98	0.98	0.96	0.88	0.78	0.68
Participation Rate	0.88	1	1	1	1	0.94	0.86	0.78

Risky asset share is the share of risky asset investment over all financial assets conditional on investing in the risky asset.

First, it is worth making clear the main features of the NHI model. The NHI model includes all features of the main model except those related to housing investment. In the NHI model, there is no housing investment, no house price risk, no mortgage payments and no homeownership status. All other parameter values are the same, including RRA and EIS. Households make decisions on consumption expenditure and on investment in financial markets (riskless and risky assets). Table 4 shows the life-cycle portfolio allocation profiles of households for the NHI model.

There are significant changes in life-cycle wealth allocation profiles between these two models. It is clear that housing investment has a strong crowding out effect on investment in the risky asset. The first row of the table shows that the risky asset market participants invest almost all of their financial wealth in the risky asset during the working period. As the highest share was at most 70% in the baseline case, we observe that housing investment in this setup leads to significant decrease in both conditional and unconditional risky asset investment during the working period. Households reduce their risky asset investment during the retirement in the NHI model similar to the baseline model but the share are still higher than the baseline model.

Compared to the baseline model, we observe higher participation rate during the working period. Between ages of 30-70 all households have investment in the risky asset, but the rate decreases during the retirement period. The decrease in the participation rate during the retirement is simply because households do not want to face any big negative shock to their risky asset investment during the retirement period as they would have less time to compensate big losses due to such negative shocks in the absence of housing investment which can be liquidated in such severe liquidity constraint cases.

Therefore, the number of households investing in the risky asset decreases during this period.

One main reason for observing a long lasting crowding out effect of housing investment in the baseline model is the duration of mortgage debt. Most of the households become homeowners and begin to pay their mortgage debt between the ages of 20-30. They pay back their mortgage debt over 25 years. When reaching ages around 50, a majority of households are homeowners with less mortgage debt. They can then make investments in financial assets without much concern about their housing investment because of very low or no mortgage debt. However, we also should take into account the fact that some households may want to sell their relatively small house and buy a larger ones. Hence, the negative effects of mortgage debts on financial asset investment could last to extended periods. Furthermore, while the burden of mortgage debt decrease substantially as households get older, they are getting closer to the end of their life and are left with less time to compensate a potential big negative shock to their investment .

### **5.3 Comparison of the Data and the Model with Housing**

In this part of the paper, we investigate the extent to which the portfolio allocation profiles in the model match the data. We use Survey of Consumer Finances (SCF) 2007 since SCF provides the most comprehensive information on financial wealth of households in the US. The data collection process and the definition of each variable is explained in detail in the discussion covering the estimation of the model.

Table 5 displays the unconditional and conditional risky asset shares, the risky asset participation rate, and the homeownership rate obtained from the SCF data. Similar to the baseline model in Table 3, the empirical evidence shows that households increase their risky asset investment during the working period and decreases during the retirement period. Furthermore the model-estimated unconditional and conditional risky asset shares over the life-cycle match the empirical counterpart reasonably well. In many previous life-cycle portfolio allocation models, the portfolio shares invested in the risky asset were always significantly higher than the empirical evidence. For example,

Table 5: Life-Cycle Profiles - SCF Data

Age	20-30	30-40	40-50	50-60	60-70	70-80	80-90	
Risky Asset Share (Uncond)	0.32	0.38	0.46	0.49	0.46	0.43	0.4	0.38
Risky Asset Share (Cond)	0.43	0.47	0.52	0.54	0.52	0.52	0.49	0.46
Participation Rate	0.52	0.64	0.75	0.8	0.77	0.73	0.64	0.58
Homeownership Rate	0.27	0.61	0.78	0.85	0.87	0.84	0.76	0.69

Risky asset share is the share of risky asset investment over all financial assets conditional on investing in the risky asset.

see Campbell, Cocco, Gomes, and Maenhout (2001), Cocco (2004), Cocco, Gomes, and Maenhout (2004), Gomes and Michaelides (2007), Hu (2005), Yao and Zhang (2005) among many. The gap between the predictions of life-cycle models and the empirical evidence is called the “portfolio allocation puzzle” which is a kind of flip side of the “equity premium puzzle” of Mehra and Prescott (1985). Therefore, it is important to note that the model is able to match the data fairly well.

The third row of Table 5 shows the risky asset participation rate for the SCF data. Both the baseline model and the SCF data displays an increasing participation rate during the working period while a decreasing rate for the retirement period. Furthermore, the participation rate in the baseline model is fairly close but lower than the empirical counterpart throughout the life-cycle. Overall, the comparison between the baseline model and the SCF data indicates that the model has successfully address both the moderate rate of stock market participation rate and moderate level of equity holding of stock market participants.

When compared to the SCF data, the model estimates the homeownership rate for the working period years fairly well but overestimates homeownership rate for the late years of the life-cycle. Two possible reasons for the over-estimation of the homeownership rate during the late years are as follows. First, the model does not incorporate involuntarily moving due to many reasons including retirement, health issues, divorce. The involuntarily moving is higher during the retirement period than during the working period. Second, the model does not cover the reverse mortgage feature, which is a form of loan that uses house as a collateral and which is possibly higher for the elderly

people.

## 5.4 Comparative Statics

In this section we discuss the effects of some of the parameters of our model on the life-cycle portfolio allocations. Specifically, we investigate the effects of house price risk and the size of the entry cost. We also compare the life-cycle portfolio allocation profiles of homeowners and renters in order to analyze how different these profiles are.

The baseline model assumes that house prices are affected by stochastic shocks. In order to analyze the effect of the uncertainty of house prices on households' portfolio allocation decisions, we set different values for the standard deviation of house prices ranging from 0% to 20% while keeping the mean real growth rate of house prices constant at 1% as in the baseline case. Table 6 displays the life-cycle portfolio allocation and the homeownership rates for different degrees of house price uncertainty.

Table 6 shows that while keeping the expected return on housing investment constant, as the risk associated with house prices decreases, we observe clear increases in the homeownership rate starting from middle-age forward. For young households, however, we observe the reverse, as housing investment become more risky, relatively many young households are induced to buy house. The distinction between the behavior of young and non-young households on housing investment stems from the fact that young households are more inclined towards the risky assets compared to the other households, while young households perceive more risky housing more appealing asset, non-young households perceive less risky housing more appealing asset.

As house price become more risky, during the first ten years, the participation rate decreases to some extent as more young households substitute from risky asset investment to housing investment. However, for the subsequent periods, we observe increases in the participation rate as well as the risky asset share within total financial assets due to the fact that housing is now more risky and less appealing asset, and hence households prefer to hold more risky financial assets with higher expected

return than risky housing investment with relatively lower expected return.

These findings can be linked to housing bubbles in the sense that removing uncertainty about the future behavior of house prices leads to more investment and sometimes over investment in housing, which in our case crowds out risky asset investment. Therefore, this model, although it is not a general equilibrium model, can somewhat show that if households are certain about future increases in house prices, there would be switches from financial investment to housing investment at both extensive and intensive margin.

The baseline case assumes that households pay a one-time fixed entry cost the first time they decide to invest in the risky asset. It covers the cost opening a brokerage account, the cost of understanding how the market works, and the cost of acquiring and evaluating information about the stock market. Similar to Alan (2006), Gomes and Michaelides (2007), Guvenen (2009b), and Guo (2004), this cost is a fixed proportion of households annual labor income.

By using different size of this cost as a share of labor income, we analyze the impact of it on households financial investment decisions. We observe that as the size of fixed-cost increases, the participation rate initially slightly decreases as expected, but as time passes the the participation rate increases. The increase in later years reflect the motivation that households want to compensate both the losses stemming from the late participation to the risky asset market and the higher cost of the participation and hence direct more resources to the risky asset investment as the size of the participation cost increases. We further observe as households reach the terminal period, the magnitude of the effect of the size of the fixed cost decreases.

We next investigate the life-cycle profiles of homeowners and renters. The model assumes that all households are renters in the initial period. From the second period on and in each period, households endogenously make a homeownership decision. For instance, a homeowner at age  $t$  has three options regarding homeownership: (1) stay in the current house, (2) move to a new house, (3) move to a rental property. Similarly, a renter decides to either stay in the current rental property or move to a bigger or smaller rental property or buy a house and become a homeowner. Note that a typical household

Table 6: Life-Cycle Profiles - No House Price Risk

Age	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
<b>Panel A: Risky Asset Share (Uncond.)</b>								
Std. of House Price								
0.20	0.19	0.48	0.69	0.83	0.82	0.72	0.58	0.46
0.16	0.18	0.35	0.58	0.74	0.73	0.65	0.52	0.42
0.12	0.27	0.27	0.48	0.6	0.58	0.47	0.37	0.31
0.08	0.14	0.23	0.39	0.45	0.43	0.35	0.29	0.29
0.04	0.23	0.29	0.39	0.43	0.4	0.32	0.27	0.26
0.00	0.15	0.25	0.35	0.43	0.37	0.26	0.21	0.21
<b>Panel B: Risky Asset Share (Cond.)</b>								
Std. of House Price								
0.20	0.44	0.7	0.84	0.91	0.92	0.91	0.82	0.72
0.16	0.43	0.56	0.76	0.85	0.87	0.88	0.79	0.7
0.12	0.59	0.51	0.7	0.77	0.79	0.81	0.73	0.63
0.08	0.33	0.44	0.64	0.69	0.78	0.88	0.8	0.67
0.04	0.48	0.56	0.66	0.71	0.79	0.92	0.86	0.65
0.00	0.32	0.5	0.63	0.56	0.7	0.91	0.85	0.61
<b>Panel C: Participation Rate</b>								
Std. of House Price								
0.20	0.36	0.68	0.82	0.91	0.9	0.8	0.71	0.63
0.16	0.38	0.61	0.76	0.87	0.84	0.74	0.66	0.61
0.12	0.42	0.53	0.69	0.78	0.74	0.58	0.51	0.5
0.08	0.39	0.51	0.6	0.65	0.57	0.39	0.36	0.44
0.04	0.43	0.52	0.58	0.6	0.52	0.35	0.31	0.42
0.00	0.41	0.49	0.56	0.77	0.6	0.28	0.25	0.38
<b>Panel D: Homeownership Rate</b>								
Std. of House Price								
0.20	0.49	0.69	0.46	0.3	0.37	0.52	0.69	0.88
0.16	0.47	0.72	0.56	0.4	0.44	0.56	0.72	0.89
0.12	0.36	0.73	0.64	0.59	0.64	0.77	0.85	0.93
0.08	0.32	0.7	0.73	0.73	0.74	0.79	0.87	0.95
0.04	0.26	0.66	0.73	0.74	0.76	0.82	0.89	0.96
0.00	0.25	0.64	0.77	0.80	0.80	0.85	0.90	0.96

Risky asset share is the share of risky asset investment over all financial assets conditional on investing in the risky asset. For the last age group, the data comes from ages from 91 to 94.

Table 7: Participation Rate for Different Entry Costs

	Age	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Risky Asset Share (Uncond)	Fixed Cost								
	0	0.31	0.29	0.35	0.39	0.37	0.29	0.24	0.25
	0.02	0.31	0.29	0.35	0.4	0.37	0.29	0.24	0.26
	0.04	0.3	0.29	0.35	0.4	0.37	0.29	0.24	0.26
	0.06	0.31	0.3	0.36	0.4	0.38	0.3	0.25	0.26
	0.08	0.31	0.3	0.4	0.45	0.42	0.34	0.28	0.28
	0.10	0.3	0.3	0.4	0.45	0.43	0.35	0.29	0.29
Fixed Cost Risky Asset Share (Cond)	0	0.65	0.57	0.6	0.65	0.74	0.87	0.81	0.63
	0.02	0.64	0.57	0.6	0.65	0.74	0.87	0.8	0.63
	0.04	0.64	0.56	0.6	0.65	0.74	0.87	0.8	0.63
	0.06	0.65	0.57	0.61	0.66	0.75	0.87	0.81	0.63
	0.08	0.65	0.57	0.65	0.7	0.78	0.89	0.82	0.65
	0.10	0.65	0.57	0.66	0.7	0.78	0.89	0.82	0.65
	Fixed Cost Participation Rate	0	0.44	0.52	0.57	0.61	0.51	0.33	0.3
0.02		0.43	0.52	0.58	0.61	0.52	0.33	0.3	0.42
0.04		0.43	0.52	0.58	0.61	0.52	0.33	0.3	0.42
0.06		0.43	0.53	0.58	0.62	0.52	0.34	0.3	0.43
0.08		0.43	0.53	0.61	0.64	0.56	0.39	0.35	0.45
0.10		0.42	0.53	0.61	0.65	0.56	0.39	0.35	0.45
Fixed Cost Homeownership Rate		0	0.32	0.72	0.77	0.78	0.79	0.84	0.9
	0.02	0.32	0.71	0.77	0.78	0.79	0.84	0.89	0.96
	0.04	0.32	0.71	0.77	0.78	0.79	0.84	0.9	0.96
	0.06	0.31	0.7	0.77	0.77	0.78	0.83	0.89	0.96
	0.08	0.32	0.71	0.72	0.72	0.74	0.8	0.87	0.95
	0.10	0.31	0.7	0.71	0.72	0.74	0.8	0.87	0.95

Risky asset share is the share of risky asset investment over all financial assets conditional on investing in the risky asset. Fixed cost payment rate is the rate of the population that have already paid the entry cost.



can be a renter for some periods and a homeowner for some other periods. The homeowners part of the table shows the life-cycle profiles for households when they are homeowners. On the other hand, the renter part of the table shows the life-cycle profiles for households when they are renters. For example, assume that a household was a renter for the first 15 years, and endogenously became a homeowner for the next 30 years, and then again endogenously became a renter for the rest of his life. Then the life-cycle profile for the first 15 years will be used for obtaining renters' life-cycle profiles, the next 30 years will be used to for obtaining homeowners' life-cycle profile, and finally remaining life-cycle profiles will again be used for obtaining renters' life-cycle profile.

Table 8 presents the life-cycle profiles for both homeowners and renters. The results show that renters have higher participation rate to the risky asset market than homeowners. The renters' participation rate increases from 49% to 100% and stays there for the rest of the life-cycle whereas the highest participation rate of homeowners is 65% around ages between 50-60. The unconditional and conditional risky asset shares of renters are also quite higher than that of homeowners throughout the life-cycle. These results are similar to the results in Yao and Zhang (2005) and Hu (2005) who also find that renters' risky asset investment share is larger than homeowners' risky asset investment share. The comparison between homeowners and renters also indicate the crowding out effect of housing investment on households risky asset investment.

As we mentioned earlier, renters are not lifetime renters in this model. While some households are renter for some periods, they can endogenously become homeowners for some other periods by paying the down payment. Therefore, renters portfolio allocation may still be oriented toward home purchases. In order to analyze the impact of the potential of home purchases on renters portfolio allocation, we can compare the portfolio allocation profiles presented in Table 8 to the portfolio allocation households in the no housing case presented in Table 4. The no housing model is the same with the baseline model except that households have no housing investment. So we can think of it a model with renters who would never become homeowner.

In the absence of housing investment model, in which households do not expect homeownership

Table 8: Life-Cycle Profiles - Homeowners vs Renters

Age	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
<b>Homeowners</b>								
Risky Share(Uncond.)	0.12	0.15	0.21	0.31	0.28	0.21	0.22	0.28
Risky Share(Cond.)	0.37	0.33	0.4	0.47	0.5	0.54	0.56	0.61
Participation Rate	0.31	0.45	0.52	0.65	0.57	0.38	0.39	0.46
<b>Renters</b>								
Risky Asset Share (Uncond.)	0.18	0.47	0.91	1	0.99	0.97	0.85	0.85
Risky Asset Share (Cond.)	0.37	0.7	0.97	1	0.99	0.98	0.85	0.85
Participation Rate	0.49	0.66	0.94	1	1	1	1	1

Risky asset share is the share of risky asset investment over all financial assets conditional on investing in the risky asset. For the last age group, the data comes from ages from 91 to 94.

in future, until 50s households hold proportionally higher share of risky asset compared to the model in which households do expect potential homeownership in the future.

The gap between these two models for young and middle-aged households is due to the fact that renters with the potential of being homeowner might be saving the down payment for future home purchase. Therefore, the possibility of being homeowner in the future makes young and middle-aged renters relatively more risky averse in their financial asset investment compared to the people who cannot be homeowner at all.

## 5.5 Results for Additive Utility

We next investigate the impact of using EZ's recursive utility instead of the more traditional additive utility functions. In the Epstein-Zin preferences, if we set  $\gamma = 1/\psi$  (the relative risk aversion being equal to the inverse of the elasticity of intertemporal substitution), then it reduces to a CRRA utility function. We can first test the hypothesis  $H_0 : \gamma = 1/\psi$  against  $H_1 : \gamma \neq 1/\psi$  to see if our data is informative enough to be able to reject CRRA utility. We can do so by computing a  $t$  statistic,  $t = (\hat{\gamma}\hat{\psi} - 1)/se(\hat{\gamma}\hat{\psi})$ , where the standard error  $se(\hat{\gamma}\hat{\psi})$  is computed with the delta method. We find that  $se(\hat{\gamma}\hat{\psi}) = 0.0961$  and  $t = 2.9681$ . Using a 5% significance level, we can reject  $H_0 : \gamma = 1/\psi$  and the assumption of CRRA utility.

Table 9: Life-Cycle Profiles - CRRA with  $\gamma = 3.14$  and  $\psi = 0.3185$

Age	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Risky Asset Share (Uncond)	0.16	0.36	0.54	0.65	0.59	0.44	0.47	0.51
Risky Asset Share (Cond)	0.35	0.61	0.72	0.75	0.75	0.75	0.8	0.86
Participation Rate	0.4	0.58	0.74	0.87	0.78	0.59	0.58	0.6
Homeownership Rate	0.28	0.6	0.75	0.72	0.7	0.76	0.77	0.93

Risky asset share is the share of risky asset investment over all financial assets conditional on investing in the risky asset. For the last age group, the data comes from ages from 91 to 94.

Below, we report lifetime profiles for the CRRA utility function. We re-estimate the parameters of the model with CRRA restriction. The estimated parameters in this case are  $\gamma = 3.14$  and  $\psi = 0.3185$  respectively. The aim of this case is to see the contribution of disentangling the risk aversion from the inverse of intertemporal substitution. In other words, we want to see the contribution of having EZ-type recursive preferences on households life-cycle portfolio allocation.

Comparing Table 9 to the baseline case in Table 3, it seems that the unconditional and conditional risky asset shares under the CRRA assumption are higher than the shares under the EZ recursive preferences in the baseline case. Furthermore, comparing to the data in Table 5 the unconditional and conditional risky shares under the EZ preferences in the baseline model match the empirical evidence better than the same shares under CRRA utility assumption. On the other hand, the CRRA model have estimates for the participation rate and the homeownership rate that are similar to the EZ case.

The estimation procedure minimizes the distance between the life-cycle profiles of the model and the data. Therefore, we do expect to obtain life-cycle profiles under both EZ preferences and the CRRA utility assumption to be close to the data. However, the restriction under the CRRA assumption that setting the elasticity of intertemporal substitution to the inverse of the relative risk aversion leads to the deviation of the portfolio allocation of the model from the empirical data compared to the EZ recursive model in which this restriction is relaxed. Therefore, the relatively better results for the conditional risky asset share in terms of matching the empirical evidence support the use of use of EZ recursive preferences rather than the restricted CRRA in these models.

## 6 Conclusion

In this study, we focus on the joint effects of housing investment and EZ recursive preferences on portfolio allocation in a fairly rich life-cycle model. The importance of housing investment for households is high since housing constitutes an important part of households' portfolios. It is both an investment asset which enables owners to hold home equity, and a durable consumption good from which households derive utility. In order to fully understand the effects of housing investment on portfolio allocation, we developed a fairly comprehensive life-cycle asset allocation model where many important features are taken into account.

The main results of the model can be summarized as follows. First, we show that housing investment has strong crowding out effect on investment in risky assets and this effect is observable throughout the life-cycle. Early in life, households labor income is relatively low and they are willing to be homeowners by paying a down payment and keep paying annual mortgage payments. So they are liquidity constraint and put some of their labor income mostly in the riskless form due to precautionary motives. Their risky asset share increases gradually over the working life. Hence, owner-occupied housing is a substitute for investment in risky assets. Even after accumulating enough wealth to pay the fixed cost and begin investing in risky assets, the portfolio's share of risky assets is still at lower levels than predicted by the model with no housing investment. Second, the unconditional and conditional risky share over the life-cycle under the EZ preferences matches the empirical data better than the same share under the CRRA utility form while we observe similar patterns for the participation rate and homeownership rate for these two cases. Third, the effect of the presence of housing investment on households portfolio allocation is larger than the effect of disentangling the relative risk aversion from the intertemporal elasticity of substitution through using EZ recursive preferences.

We further find that, in the absence of house price risk, households invest more in housing and the homeownership rates goes up. Similarly the share of investment in the risky assets goes down because households enjoy higher and riskless return on housing investment on top of the utility that they

generate from housing. We further find that the size of the fixed entry cost on risky asset investment has limited impact on portfolio allocation.

Some extensions of the model for future research include focusing more on the real estate side of the model by analyzing the size and the effects of the liquidation cost, introducing an exogenous mandatory moving and selling of houses (job relocation, old age, health issues, etc.), and allowing households to default on their mortgage. Furthermore, estimating the size of the fixed entry cost and introducing more realistic house price dynamics are left for future research.

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## Appendix

We begin by discretizing the state space and variables over which the choices are made. As each period is one year, age ( $t$ ) is discrete state variables taking 81 values and mortgage debt ( $RT$ ) takes 25 values. We discretize



liquid wealth using ( $LW_t$ ) using 17 grid points and use denser grids for low levels of liquid wealth to capture the higher curvature of the value function at this region and we use 11 grid points to discretize the house size ( $H_t$ ). The random variables (i.e. shocks to labor income process, shocks to return on risky asset, and shocks to return on housing investment) take two possible values. The other state variables, risky asset market participation status ( $IFIX_t$ ) and homeownership status ( $O_t$ ) also take two possible values. All variables are normalized by the permanent component of labor income ( $F_t$ ) and use lower case for the normalized variables.<sup>13</sup>

At each point in the state space, the risky asset participation decision is done by comparing the value function conditional on having paid the fixed cost with the value function conditional on having not yet paid the fixed cost. Similarly, the homeownership decision (e.g. house buying or selling decision) is done by comparing the value function conditional on being a renter with the value function conditional on being a homeowner. In both comparisons, adjustments for the payment of the fixed cost of risky asset participation and costs accrued from buying/selling a house (e.g. down payment, annual mortgage payment, liquidation cost etc.) are taken into account respectively.

$$\begin{aligned}
v_t(lw_t, h_{t-1}, rm_t, ifix_t, o_t) = & \\
& \max_{0,1} \{ v_t(lw_t, h_{t-1}, rm_t, ifix_t = 0, o_t = 0), \\
& v_t(lw_t, h_{t-1}, rm_t, ifix_t = 1, o_t = 0), \\
& v_t(lw_t, h_{t-1}, rm_t, ifix_t = 0, o_t = 1), \\
& v_t(lw_t, h_{t-1}, rm_t, ifix_t = 1, o_t = 1) \}
\end{aligned} \tag{22}$$

where  $ifix_t = 1$  and  $ifix_t = 0$  denotes whether the one-time fixed participation cost is paid or not paid, and  $o_t = 1$  and  $o_t = 0$  denotes whether the household choose to homeowner or renters at time  $t$  respectively.

We use backward induction to solve the problem because this is finite-time problem. In period  $T + 1$ , the policy functions are determined by the bequest motive. The value function in this period coincides with the utility function, which is the bequest function. In every period prior to  $T + 1$ , we obtain the utility function for different combinations of housing, consumption, and other state and choice variables. Then the value function for a typical time  $t$  is equal to the utility function of that period plus the discounted expected continuation value

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<sup>13</sup>The model has a state space dimension of 24,235,200.

$(E_t [V_{t+1}])$ . If the continuation value doesn't lie on the state space grid, we compute the value function using cubic spline interpolation. This backward induction process is iterated from age T to 1.

$$\begin{aligned}
 v_t(lw_t, h_{t-1}, rm_t, fix_t, o_t) = & \\
 & \max_{c_t, h_t, s_t} \left\{ u(c_t, h_t)^{\frac{1-\gamma}{\theta}} + \right. \\
 & \left. \beta (E_t [q_{t+1} v_t(lw_{t+1}, h_t, rm_{t+1}, fix_{t+1}, o_{t+1}) + (1 - q_{t+1}) w_{t+1}^{1-\gamma}]^{\frac{1}{\theta}})^{\frac{\theta}{1-\gamma}} \right\}
 \end{aligned} \tag{23}$$

Once we compute the value function of all the alternatives, we choose the one that maximizes the value function over all choice variables. The optimum policy rules for consumption, housing, and investment in financial assets correspond to ones that maximize the value function.