

A Stochastic Frontier Analysis of Secondary Education Output in Portugal*

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Abstract

This paper contributes to the discussion about the performance and efficiency of Portuguese secondary schools, using the stochastic production frontier analysis. We discuss the impact of school variables and the environment on output (measured by the average national examination scores), as determinants of the shape of technology and degree of efficiency. We find, in particular, that teacher seniority, school size and private management have a positive impact on output. The area where the school is located also plays an important role. Further, results show that schools differ considerably in their efficiency levels, and we also address the use of the stochastic frontier methodology in order to improve the computation of school rankings.

Keywords: Economics of education, stochastic production frontier, technical efficiency
JEL codes: I21

1 Introduction

The scores in the national examinations taken by students at the end of their secondary courses have received much public attention in recent years, partly justified by the importance that they have among the criteria for admission to higher education. Secondary schools have been ranked on the basis of such scores, which provide a way to evaluate in a systematic fashion their relative performance. This paper goes one step forward in the analysis of the

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educational output of Portuguese schools offering secondary courses, measured by the average scores in national examinations. It employs the stochastic frontier methodology to investigate the respective determinants and the degree of efficiency in the utilization of resources. In fact, schools with higher-than-average achievement in exams need not be efficient; one has to take into account the quantity of resources used and the impact of other factors like the environment in which schools operate.

The decision to enroll and invest in schooling is, in the first place, a micro decision of the student and its family that takes into account costs and expected benefits. Such decision has important externalities for the society as whole, as education is believed to be one of the key factors behind sustained increase in labor productivity and long-run economic growth. More generally schooling is expected to design people's values and attitudes so that they are better equipped to participate in society. Thus the quality of education and the amount of resources allocated to it has been since long at the centre of public debate. These issues are of particular concern in the case of Portugal, where the educational attainment indicators lag much behind the OECD average. In fact such situation is often pointed out as the main structural obstacle in the Portuguese catching up process to higher income levels and occurs in spite of public spending with education being, in relative terms, higher in Portugal than in most OECD countries, for some years now. In this context, research on the education production function and efficiency of resource allocation in this area is well justified.

The Stochastic Frontier Analysis (SFA) builds on the microeconomic concept of production function which represents the maximum output attainable given a certain quantity of inputs. Those inputs are defined in a broad sense and may include all variables affecting the production performance. The transposition of this methodology to the field of education is relatively straightforward in theoretical terms but faces important empirical difficulties. They concern, in the first place, the definition of output. What is the relevant output of the educational process? Secondly, there is a multiplicity of factors, beyond school inputs proper, that may influence the learning process such as family background, interaction with colleagues and innate capacities, some of which are very difficult to incorporate into a model. Lastly, the relationship between inputs and output in the educational process is rather complex and can only be summarised imperfectly in a production function. Such difficulties have been addressed in detail in the education economics literature and we will touch upon them in the course of this paper. Some can be lessened by working with comprehensive and disaggregated data: at the level of the school at least, ideally at the level of the student. Others cannot be overcome and call for caution in the interpretation of results.

Production frontier estimates in the field of education have mainly used non-parametric

techniques like the Data Envelopment Analysis (DEA) and the Free Disposable Hull (FDH), sometimes complemented with regression analysis (see, for instance, Bessent et al. (1982), Ray (1991) and Ruggiero (1996)). SFA has been already used in this context as well, like in Mizala et al. (2002). This methodology is more demanding in terms of assumptions, since it requires the specification of a functional form for the production function, but it is less sensitive to the presence of outliers and allows the possibility of making inference about the contribution of inputs. Those aspects led us to favour the SFA for our study.

There are very few production frontier-type applications to education based on micro data for Portugal. Most evidence available in this domain comes from analyses at a rather aggregated level for groups of countries including Portugal, using non-parametric techniques. Such studies present rankings of countries based on performance and input indicators, like in Clements (1999) and Afonso and St.Aubyn (2005). Clements (1999) is nevertheless a valuable reference because it provides a thorough and critical review of the Portuguese educational system. As far as studies based on disaggregated data are concerned, we are only aware of Oliveira and Santos (2005) that applies the FDH to a sample of public secondary schools, measuring output by approval rates. Carneiro (2006) albeit with a different methodology, OLS regression followed by decomposition of variance, investigates the link between student achievement and a number of school and family background variables, exploiting the database from the OECD's Programme for International Student Assessment (PISA) 2000. Our study differs from those just mentioned as far as methodology is concerned, but also in that it measures output through national examination scores and considers almost the full population of schools offering secondary courses in Portugal, both public and private. One of the main strengths of our work is to have put together a dataset with a good coverage in terms of schools, and a reasonable coverage in terms of variables, allowing a microeconomic analysis of secondary education.

This paper is organized as follows. Section 2 presents a brief overview of secondary education in Portugal, while Section 3 describes the stochastic production frontier methodology. Section 4 makes some considerations about the specification of the education production function and data, both in general and for Portuguese secondary schools. Section 5 presents and analyses the econometric results. In that section, we focus on the way school variables and environment determine performance through the education production function. Special attention is devoted to the relative efficiency of schools in public and private sectors. We also address the use of the SFA to rank schools on the basis of estimates of school-specific technical efficiency, in comparison with the rankings simply based on the examination results. Section 6 concludes.

2 Secondary education in Portugal: an overview

In the Portuguese educational system basic compulsory education comprises (since 1986) nine years, divided in three cycles: the first cycle covers the first four years, for children from the age of six onwards, the second cycle covers the 5th and 6th years and the third cycle the 7th through 9th years. Secondary education encompasses three further years of schooling (10th to 12th).¹ Approval in the 12th grade depends partially on the scores in a set of national examinations. Otherwise assessment is made exclusively within the school, even though on the basis of common curricula, which at the secondary level differ according to the field chosen by the student, for example, natural or social sciences, humanities, and arts.

Table 1: Secondary education in Portugal: some enrollment and resource indicators

	Portugal				OECD
	1989/90	1994/95	1999/00	2003/04	2003
Students enrolled*	309,568	457,194	417,705	382,212	-
by school nature (%)					
public schools	92.3	87.5	84.9	82.4	79.5
private schools	7.7	12.5	15.1	17.6	20.5
by courses (%)					
<i>ensino regular</i>	83.8	82.0	87.1	79.5	-
<i>ensino nocturno / recorrente</i>	16.2	18.0	12.9	20.5	-
Total expenditure as % of GDP** (a),(b)	-	-	1.2	1.2	1.4
Expenditure per student					
in US dollars (PPP adjusted)** (a),(b)	-	-	5,422	6,094	6,962
as % of GDP per capita** (a),(b)	-	-	32	34	28
weight of teacher compensation (%)** (a),(c)	-	-	-	78.3	58.4
Teachers per 100 students** (d),(e)	-	-	12.7	13.7	7.9
public schools	-	-	-	13.2	7.7
private schools	-	-	-	16.4	8.6
Teachers with higher university degree* (f),(g)	68.6	75.9	85.8	-	-
Tenured teachers* (f)	56.6	69.7	75.7	82.3	-

Sources: * Ministério da Educação (2003, 2004)

** OECD (2002, 2006)

Notes: (a) Figures for Portugal include only public schools. (b) Portugal's data are for 1999 and 2003. (c) Portugal's and OECD's data are for 2002. (d) Measured in full time equivalents; includes only schools of continental Portugal. (e) Portugal's data are for 2000 and 2004 and OECD's data for 2004. (f) Teachers at the secondary level and third cycle of basic education; includes only public schools of continental Portugal. (g) Tertiary-type A or higher, corresponding to at least four years of university education.

¹ Corresponding to the third level of ISCED (International Standard Classification of Education), also called *upper* secondary education. Schools offering secondary courses in Portugal comprise secondary schools proper, secondary schools with the third cycle of the basic education, and basic schools (second and third cycles of basic education) with secondary courses.

Table 1 presents enrollment and resource indicators for secondary education in Portugal and, in some cases, the corresponding OECD figures. Since 1995 the number of students enrolled in secondary courses has decreased markedly, reflecting mostly population ageing. Note that enrollment rates (percentage of population at typical school age enrolled) have stood approximately unchanged, at almost 60 per cent, over the last decade. Most students attend public schools but over the last years the weight of private schools has increased steadily. Due to high dropout and repetition rates, a significant share of the secondary student population consists of overaged students attending special repeater courses, designated as *ensino recorrente* (former *ensino nocturno*). These are courses with curricula equivalent to those in *ensino regular* which are aimed at students who left school without completing secondary education.

Expenditure on secondary education as a percentage of GDP in Portugal has stood slightly below the OECD average over the recent period. In 2003, the last year available (OECD (2006)), this figure was 1.2 per cent for Portugal, 0.2 p.p. below the OECD average. A similar picture is given by the expenditure on secondary education *per student*, which is about 15 per cent lower than the OECD average, although this represents a higher-than-average spending effort in relation to the Portuguese GDP per capita (for which the gap to the OECD average is much wider). Considering now the composition of outlays, the most salient fact is the very large weight of teachers' compensation in the total. This is mainly due to an abnormally high headcount for teaching staff in Portugal, as shown by teacher-student ratios much above the OECD average, common to all levels of non-tertiary education. In addition teachers' salaries taken in relation to GDP per capita are comparatively high. In contrast there is evidence that both the other current and capital expenses are quite low in relative terms; for instance, the number of computers per student in 2003 was one of the lowest among OECD countries (OECD (2006)). Clements (1999) presents scattered evidence indicating that the wage bill may have squeezed other inputs like teaching materials and infrastructures. A final aspect worth highlighting is the improvement in the academic qualifications of teachers in public schools since beginning of the nineties, which took place in parallel with a considerable rise in the proportion of teaching staff with tenure.

Educational attainment in Portugal has improved among the recent generations, but remains well below the OECD average. In 2004, the last year for which international comparisons are available, less than 25 per cent of the Portuguese adult population aged 35 to 54 had completed the secondary level of education. Considering the age group 25 to 34 this percentage rose to about 40 per cent, but the corresponding figure for the OECD average was over 75 per cent. Also the performance of Portuguese students in recent international exam-

inations has revealed in general poor competency levels. For example, in the 2003 OECD's PISA for proficiency in mathematics of 15-year-olds, Portugal occupied the 25th position among 29 countries. Proficiency in reading and scientific literacy have been low too.

Given that overall financial input indicators in Portugal are not much below the OECD average (or even above if taken relative to GDP per capita) while performance indicators are generally poor, it should come as no surprise that studies (like the aforementioned by Clements (1999) and Afonso and St.Aubyn (2005)) find that Portugal achieves little with the resources employed. This is even more the case when the input indicators used are based on the number of teachers.

3 The stochastic frontier analysis

3.1 Specification and estimation of the basic model

The basic idea behind the SFA, introduced by Aigner et al. (1977) and Meeusen and van den Broeck (1977), is to add to the production frontier an error term with two components: one that allows for technical inefficiency and another that allows for any random events that may affect individual producers. Since this methodology has been very well described elsewhere (for instance, in Kumbhakar and Lovell (2000)), we confine ourselves here to aspects important for the interpretation of empirical results in the coming sections. We nevertheless devote some attention to the relationship between model specification and the resulting efficiency scores, since this issue has not been so well covered in the literature.

The stochastic production frontier model for cross-sectional data is

$$y_i = f(\mathbf{x}_i; \boldsymbol{\beta}) \cdot \exp(v_i) \cdot TEF_i, \quad (1)$$

where y_i is the output of producer i , $i = 1, \dots, I$, $\mathbf{x}_i$ is a vector of K inputs used by the producer i , $\boldsymbol{\beta}$ is a vector of $K + 1$ technology parameters to be estimated and $f(\mathbf{x}_i; \boldsymbol{\beta})$ is the *deterministic* production frontier. Further $\exp(v_i)$ embodies the random shocks on each producer, being $[f(\mathbf{x}_i; \boldsymbol{\beta}) \cdot \exp(v_i)]$ the *stochastic* production frontier. Finally TEF_i is the output-oriented technical efficiency of producer i , defined as

$$TEF_i = \frac{y_i}{f(\mathbf{x}_i; \boldsymbol{\beta}) \cdot \exp(v_i)}, \quad (2)$$

that is, the ratio of observed output and the maximum feasible output conditional on $\exp(v_i)$. Producer i attains the maximum feasible output if, and only if, $TEF_i = 1$; otherwise $0 <$

$TEF_i < 1$ provides a measure of the shortfall of observed output from the maximum feasible in an environment characterized by $\exp(v_i)$.

In order to estimate the stochastic production frontier model in (1), it is necessary in the first place to specify $f(\cdot)$ further, which is normally assumed to take a Cobb-Douglas or a translog form. Defining $TEF_i = \exp(-u_i)$, with $u_i \geq 0$ to ensure that $TEF_i \leq 1$, and considering a Cobb-Douglas form, the model can be written as

$$\ln y_i = \beta_0 + \sum_k \beta_k \ln x_{k,i} + v_i - u_i, \quad (3)$$

where v_i is assumed to be i.i.d., symmetric and independent of u_i . The error term in this equation, $\varepsilon_i = v_i - u_i$, is composed by a two-sided "noise" term and a nonnegative technical inefficiency term. It follows from $u_i \geq 0$ that ε_i is asymmetric and negatively skewed. Assuming that ε_i is (that is, v_i and u_i are) distributed independently of the $x_{k,i}$'s, the OLS estimation of (3) would yield consistent estimates of the β_k 's but not of β_0 , since $E(\varepsilon_i) \leq 0$.

Model (3) can be estimated by maximum likelihood, upon making assumptions about the distributions of v_i and u_i . This is the estimation method that we employ in our application. The original specification put forward in the literature was the Normal-Half Normal model, which assumed that (a) $v_i \sim iid N(0, \sigma_v^2)$, (b) $u_i \sim iid N^+(0, \sigma_u^2)$ (i.e. as a truncation below at 0 of a normal distribution with mean 0 and variance σ_u^2) and (c) v_i and u_i independent of each other and of the regressors. The Normal-Half Normal specification has been extended to assume more general distributions of u_i . In our application we will use the Normal-Truncated Normal model, the commonest in the empirical literature. In the Normal-Truncated Normal model, the assumption (b) above is replaced by (b') $u_i \sim iid N^+(\mu, \sigma_u^2)$ (i.e. as the truncation below at 0 of a normal distribution with mean μ and variance σ_u^2). The advantage of this generalization is to allow more observations to be farther from zero, since the inefficiency term distribution may have a positive mode (when the mean of the underlying normal distribution is positive).

The log-likelihood function to be maximized is based on the density function $f(\varepsilon_i)$ for a sample of I producers and, prior to maximization, a reparameterization of the type $\sigma^2 = \sigma_u^2 + \sigma_v^2$ and $\gamma = \sigma_u^2 / (\sigma_u^2 + \sigma_v^2)$ is typically introduced. The parameter γ measures the relative importance of σ_v^2 and σ_u^2 . If $\gamma \rightarrow 0$ either $\sigma_v^2 \rightarrow +\infty$ or $\sigma_u^2 \rightarrow 0$: the two-sided error component would dominate and the production frontier could be estimated by OLS. If $\gamma \rightarrow 1$ either $\sigma_u^2 \rightarrow +\infty$ or $\sigma_v^2 \rightarrow 0$: the technical inefficiency component would dominate and one would have a deterministic production frontier without noise.

The parameters (σ^2, γ) are estimated together with the technology parameters in β ,

and the maximum likelihood estimator is consistent with I (number of producers). In the context of SFA, testing the significance of γ assumes particular importance, since if the null hypothesis $\gamma = 0$ were accepted, no stochastic frontier methodology would be necessary and all technology parameters could be consistently estimated by OLS.

Efficiency analysis within this framework typically proceeds with the estimation of producer-specific inefficiency. With the parameters estimates in hand, one can obtain an estimate of the composed error term in (3) for each producer ($\hat{\varepsilon}_i$), which is then used to get the producer-specific efficiency scores. We will use throughout this paper the technical efficiency predictor (TEF_i) proposed by Battese and Coelli (1988) which is based on the expression for $E(\exp(-u_i | \varepsilon_i))$.

The stochastic frontier model has been extended to consider panel data, which opens the possibility of specifying the technology or the efficiency as time-variant. Note that even in the case of panels in which it is more reasonable to assume that they remain constant, there is a gain in using repeated observations on each producer, as the technical efficiency predictors are consistent with T (number of time periods), not with I (number of producers).

3.2 Modelling inefficiency determinants

The SFA makes it possible to estimate the degree of efficiency in the utilization of inputs by producers. In order to gain further insight one may want to carry the analysis one step further and try to relate producer performance with "exogenous" variables, which are not at the discretion of the producer but nevertheless influence the outcome of the production process (in the literature this is sometimes referred to as producer heterogeneity). Such variables could for instance characterize the environment where production takes place. They are not supposed to influence the shape and/or location of the production frontier, but determine how far away the producer is from it.

The incorporation of inefficiency determinants into the SFA has initially been done in a kind of second step, after estimating the frontier in the first one, by regressing the TEF_i 's on a vector of producer-specific variables. While this approach may give an informal indication of possible explanatory variables for efficiency, it is econometrically flawed (see Kumbhakar and Lovell (2000)). Several approaches have been suggested in the literature to incorporate appropriately inefficiency effects into the SFA. The one we follow in this paper was introduced by Battese and Coelli (1993), and (in the panel formulation) assumes that u_{it} is a truncation below at zero of a normal distribution with mean $\mu_{it} = \delta_0 + \sum_m \delta_m z_{m,it}$ and variance σ_u^2 , where z_m are producer- and time-specific variables that determine inefficiency. If the δ_m 's are equal to zero, the specification reduces to the Normal-Truncated Normal model above, with

$\mu = \delta_0$. This model with modified assumptions about the distribution of u_{it} can likewise be estimated by maximum likelihood.

The trickiest issue in this context is that, very often, the location of a variable outside the discretion of the producer in the inefficiency mean, as opposed to the production function, is a matter of judgement. For instance, a variable relating to the environment may be nevertheless a determinant of technology. Econometric results also do not always provide guidance. In fact, if a relevant variable is omitted from the production function, producers that "use" it more intensively are likely to appear more efficient. That is, as the efficiency scores were estimated without controlling for such variable (see the next section), the latter may appear to have explanatory power for efficiency. Therefore, in doubtful cases there is merit in testing alternative specifications.

3.3 Model specification and measured technical efficiency

The SFA yields a prediction for the degree of technical efficiency of each unit. The technical efficiency predictor $T\hat{E}F_i$ measures, by definition, efficiency after controlling for (i.e. *net* of) all variables in the production frontier. However, there may be variables that determine the production possibilities, but that we would not like to control for when measuring efficiency.

At this point, one can make a useful (albeit simplifying) distinction between two categories of variables entering the production frontier. The first category refers to inputs proper, whose variation implies a change in the utilization of costly resources from the point of view of the producer. One will always want to control for those by definition of efficiency as a relationship between outcomes and costly inputs. The second category covers variables that influence production, but whose variation does not come at any (visible) cost to the producer. These variables may, for instance, relate to "organization" of production or may be environmental variables of the kind we alluded to in the previous section. When measuring efficiency, for the sake of comparability among units, one will typically want to control for such variables if the producers take them as given. If this is not the case, it is within reach of producers to modify something in order to attain more output, and one will want this to go into inefficiency measurement.

Consider the case in which there are two types of producers (say, type A and type B), and the technology that type B-producers use is such that they always attain more output, for each given combination of inputs. This situation can be modelled by introducing a dummy variable that differentiates both types of producers in (3), which would fit with in the second category of variables above. This amounts to estimating two separate production frontiers, and the efficiency scores ($T\hat{E}F_i$'s) from this model will be measured against each of two,

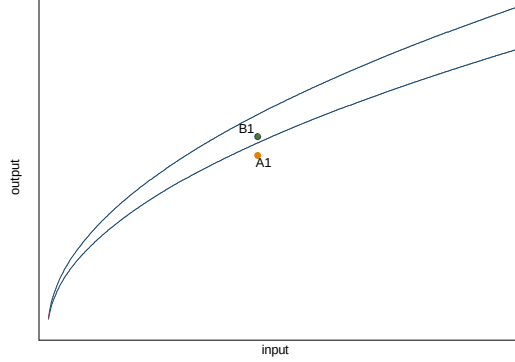


Figure 1: Production frontier for type-A and type-B units, one input case

depending on the type of producer. However, in certain situations, one may be interested in ranking the producers taking the common frontier as benchmark. Figure 1 illustrates this point, for the case of one input (plus the dummy variable) and a deterministic frontier. Unit B_1 vis-a-vis the outer frontier is less efficient than unit A_1 vis-a-vis the inner frontier; however, vis-a-vis a common (outer) frontier, unit B_1 is more efficient than unit A_1 .

In this context, it is useful to introduce a *modified* efficiency predictor, yielding efficiency estimates that do not control for one or more variables in the production function. This allows that the selection of the most appropriate econometric specification be kept separate from the way one wants to measure efficiency.² The modified efficiency predictor ($T\hat{E}F_i^*$) gross of a given variable, say x_k , can be obtained replacing the original estimate for the composed error, $\hat{\varepsilon}_i = \ln y_i - \ln \hat{y}_i$ (omitting t), by $\hat{\varepsilon}_i^* = \ln y_i - \ln \hat{y}_i^*$, where $\hat{y}_i^*$ is calculated taking, in the place of $x_{k,i}$, the value x_k^* that maximizes the contribution of input k to output over the respective sample values for all producers (see Coelli et al. (1999)³).

4 The educational production function

4.1 Conceptual issues

The first step in order to assess school performance using the SFA is to specify a production function. Summarizing the learning processes in a function is quite problematic due to its complexity. Hanushek (1979) and Hanushek (1986) provide a comprehensive discussion of this

²That is, an alternative like omitting the variables from the production function would be obviously incorrect. Also modelling such variables as "exogenous" using the technique in Section 3.2 may be incorrect in many cases, for example, if they were under the discretionarity of the producer.

³Coelli et al. (1999) put forward this idea in a more specific context: estimating the efficiency scores gross of environmental variables, as opposed to specifying them as exogenous, as described in Section 3.2.

topic. The first issue arising is how to measure the outcomes of schooling. Most studies use standardized achievement test scores, however there are quite a few others that use indicators such as school attendance rates (Oliveira and Santos (2005)) and drop rates (Kanep (2004)). Theoretically, the main purpose of education is to develop the skills and knowledge of students in order to make them more productive in the labour market and, in a broader sense, schools have an important role promoting values, contributing to integration processes and preventing social problems. Although the standardized test scores cannot measure such aspects, there is possibly no better indicator available which is comparable.

The second difficult issue concerns the factors determining the educational output. Ideally, the analysis should include not only school inputs, but also family background influences and influence of peers, as well as innate endowments and learning capacities. Many factors influencing the educational production process are hardly observed and measurable, and ultimately difficult to incorporate into a "mechanical" production function. Inputs like innate abilities or peers' influence are difficult to include, because the necessary information is not obtainable or their impact is hard to disentangle. Inputs relating to teachers are typically included using proxies of objective characteristics, like qualifications and experience, but ignoring other non-measurable characteristics that can be important such as communication skills, teaching methods or classroom management. Furthermore the information on some school organizational aspects as curricula, textbooks or school day is limited or otherwise difficult to incorporate into the models. Another shortcoming concerns accuracy of measurement, since for some inputs, in particular those related to the school, one could use a "value added" specification employing measures of cumulative influence over the years. It is fair to say that some of these difficulties are more relevant in the context of disaggregated studies that attempt to model individual student performance, than in the context of modelling performance across schools, as we do. This is, in particular, true for student inputs like innate endowments and learning capacities which should average out at the same level across schools. Studies taking the school as a reference are less informative to the extent they do not consider intra-school heterogeneity, but are less demanding in terms of data.

4.2 School and teacher data⁴

Our research takes the school level as a reference and the output measure is the average score in the 12th grade national examinations. The fact that empirical investigations tend to detect correlation between the level of schooling and post-school achievement offers some

⁴Descriptive statistics of all variables can be found in Tables 4 and 5 of Appendix A. Further details about the variables and their sources are given in Appendix B.

support for concentrating on examination results. On the other hand, national exams in Portugal have an important role as a selection mechanism for further schooling, thus relating directly to "real" outputs. Students, parents and policy makers use them to assess secondary schools' performance and implicitly the quality of education that they provide. We exclude the utilization of indicators such as school completion rates since they are not comparable due to different approval or success criteria across schools. National examinations evaluate student knowledge on specific subjects. There is no set of exams obligatory for all pupils; the requirements depend on the area in which the students are and the post-secondary courses they wish to attend. The utilization of the examination scores for *all* subjects might pose a comparability problem, since the weight of the various disciplines is not uniform among schools. However, the alternative of focusing on one discipline seemed also quite arbitrary, as our input variables relate to the school as a whole. Indeed, leaving out one or more subjects could disregard the strengths of some schools in comparison with others, giving a distorted picture, apart from reducing the size of the sample which would render the average more sensitive to the presence of outliers. Since the source for this variable was the publicly available average scores by school (for the academic years 2003/04 and 2004/05), the use of alternative statistics such as the median was ruled out. However, given that the bulk of our sample is made up of schools with a relatively large number of examinations (93 per cent of schools have more than 100), this does not seem a big concern.

As it is often the case in education production context, our output measure may be affected by different degrees of difficulty of examinations across years. The average score for all schools is about 8 per cent higher in 2004/05 than in the year before (see Table 4 of Appendix A), while the variance is similar in both years. Figure 2 presents the density⁵ of scores for the two years (divided by 200, the maximum), indicating a shift to the right of the distribution as a whole in 2004/05.

Our school data comprise the number of students (split between *ensino regular* and *recorrente*), teachers and classes, all variables relating only to secondary courses, and whether the school has private or public management. Arguably we are lacking some measure of capital, for instance regarding school facilities. The school data were supplied by the *Gabinete de Informação e Avaliação do Sistema Educativo* and cover for most variables the academic years 2003/04 and 2004/05. Concerning teacher data we dispose of information about seniority, age, academic background, tenure (only for public schools) and average wage, for the school as a whole. For these variables the source was the 2^o *Recenseamento Geral da Administração Pública* (2nd General Government Census) for the public schools and the *Quadros de Pes-*

⁵Density functions throughout this paper were computed using the Epanechnikov kernel function.

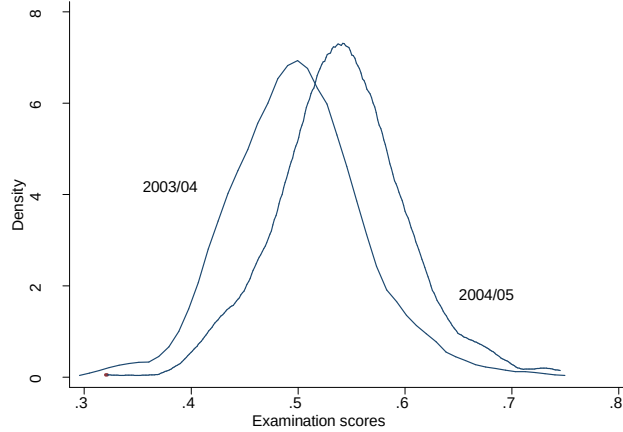


Figure 2: Examination scores in 2003/04 and 2004/05

soal survey for the private.⁶ Unfortunately the last General Government Census dates from December 1999, corresponding to the academic year 1999/2000 (the December 2005 Census was not yet available at the time of the writing of this paper). For the sake of comparability, the *Quadros de Pessoal* survey of 1999 (October) was taken. Therefore, data on teacher characteristics relate to a different academic year from the output and school inputs, and we take them on board under the hypothesis that the relative position of schools regarding such characteristics of the teaching staff has not changed substantially in the intervening three to four years. We consider this to be a tenable assumption.

Our set of regressors includes, concerning school data, the average number of teachers per 100 students as an input quantity measure. However, this ratio alone provides an imperfect measure of the teaching effort put into the educational production process, since teaching duties (in hours per week) are not uniform across teachers and full time equivalents are not available. On the one hand, tenured teachers in public schools may be relieved of teaching duties for several reasons.⁷ On the other hand, due to the decline of the student population over the last decade, it may also happen that there is an excess of teaching staff allocated to some public schools for their needs. While it is reasonable to assume that other tasks carried out by teachers also contribute positively to school performance, their contribution

⁶ *Quadros de Pessoal* is an annual mandatory employment survey collected by the *Ministério do Trabalho e da Solidariedade Social*, and covers private sector employees.

⁷ Apart from particular cases like disability and performing administrative tasks at school, teachers are entitled to a progressive reduction of weekly teaching hours starting when they become 40 and reach 10 years of seniority. This reduction can add up to a maximum of 8 hours (out of a normal workload of 20 hours at the secondary level).

is likely to fall short of that of teaching. Therefore we include in the regression the variable teacher-class ratio, as an indicator for the degree of "intensity" in the utilization of teaching staff. The number of classes may also be interpreted as a proxy of capital, since it captures the availability of basic facilities, in case this is a binding constraint.

There is a general perception that students are not equally involved in their educational project. Students in *ensino recorrente* perform on average worse than their counterparts in *ensino normal*, and thus it is important to control for the relative weight of both groups in schools. Further we include in the regression an indicator for the secondary education production scale (a ranking based both on the number of students and on the number of teachers), given that our input quantity variable is a relative measure and does not have a scale dimension. We also consider a dummy variable to differentiate private institutions.

Concerning teacher data, seniority in the job is not defined using the same criteria in both sources used. While in the general government census this variable is measured as the number of years as a civil servant, for teachers akin to number of years teaching in public schools, in *Quadros de Pessoal* it corresponds to the number of years working in the particular school they are. We take on board in the regression the teacher average age, available in both sources, as a proxy of average seniority. Note that for public schools these variables are highly correlated ($r = 0.98$), and we believe that this is also the case, albeit possibly to a lesser extent, for private schools. Teacher average wage is not considered, given the high colinearity with seniority, in particular for public schools. We tested the significance of other candidates to enter the regression, namely, the proportion of teachers with an university degree but results pointed to non-inclusion. In fact such proportion is likely to have increased much over the last fifteen years (in line with the evidence in Section 2) and shows currently reduced variability (Table 4 of Appendix A). The share of teachers with tenure (available only for public schools) also presented little explanatory power, possibly for the same reason.

As to the school coverage, as a rule, our results were based on a sample of about 500 schools of continental Portugal out of a population of less than 600 in which national examinations took place.⁸

⁸ Apart from missing data, we eliminated 12 observations with abnormal values for the teacher-student ratio which we conjectured that could be errors. Our sample is more complete for public schools than for private. Taking 2004/05, the coverage figures are, respectively, 92 and 66 per cent. However, a number of private schools missing did not appear in the examination rankings we used, presumably because of the reduced number of exams taking place there.

4.3 Environmental data

One can expect that the region where schools are located influences the attainment levels. In the public debate about school performance in Portugal it is often stated that schools in predominantly rural areas have worse outcomes than their counterparts in more developed urban centres. Looking at the density function of examination scores (two-year average) in poorer and richer municipalities⁹, depicted in Figure 3, one indeed gets the impression that such a regularity exists. The density function of scores for schools in more developed areas is comparatively shifted to the right and, in particular, schools with very bad results, at the bottom of score distribution, come predominantly from poorer regions, and the opposite happens at the top of the distribution.

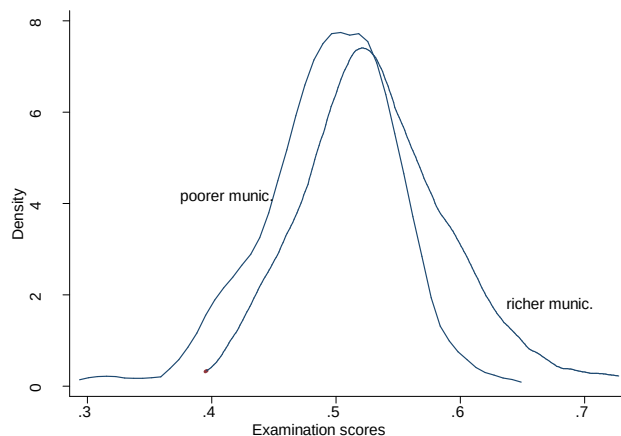


Figure 3: Examination scores in poorer and richer municipalities

In order to study the impact of school location on output, the educational production frontier must include environmental variables. We considered environmental variables at the municipality (*concelho*) level, assuming that three characteristics could be relevant in school performance measurement context: health conditions, education level and living standard. In a first stage, in order to capture such dimensions we chose, respectively, a Health Status Index elaborated by Santana et al. (2004), the average years of schooling and the Purchase Power Index (*Indicador per capita do Poder de Compra Concelhio 2004*, INE (2004)), all by municipality.

When we carried out preliminary tests in order to get an idea about the explanatory power of these variables, we verified that they were not significant as a whole. As to the Health

⁹The criterion to separate between poorer and richer municipalities was the average value for the *Indicador per capita do Poder de Compra Concelhio 2004* (see below).

Status Index, this reflected the small sample variance (see Table 5 of Appendix A), signalling rather homogeneous health conditions across municipalities. Therefore, we did not consider it. For the other two variables there was a colinearity problem ($r = 0.89$), that we tried to overcome bringing in alternative indicators. We chose the average household electricity consumption (actually, one of the variables on which the Purchase Power Index is based) for the living standard, and the illiteracy rate for the education level, featuring a smaller degree of correlation ($r = -0.62$). Those two were the environmental regressors taken on board in our estimation (for the last year available - see Appendix B).

An important drawback we faced in terms of data availability was the lack of information on the family background of students (we address this issue in detail in Section 5.3).

5 Empirical results

5.1 Specification

We estimated cross section (2004/05) and panel (2003/04 and 2004/05) versions of the different specifications described in this section. Firstly, the measurement of output differs from one year to the other and, although we include an extra regressor in the panel to capture such effect, it is important to single out the cross section results which are not affected by it. Secondly, some regressors are only available for one year¹⁰ and we wanted to check the robustness of results to the inclusion of such variables with the same value for both years in the panel.

We first considered a stochastic production frontier without environment (and with time-invariant inefficiency) given by

$$\ln y_{it} = \beta_0 + \beta_1 \frac{Te_{it}}{St_{it}} + \beta_2 \frac{Te_{it}^2}{St_{it}^2} + \beta_3 \frac{Te_i}{Cl_i} + \beta_4 \ln A_i + \beta_5 \ln S_{it} + \beta_6 P_{it} + \beta_7 R_{it} + \beta_8 Y_t + v_{it} - u_i \quad (4)$$

where it refers to the i th school in period t ; y is the average score in the national examinations; $\frac{Te}{St}$ and $\frac{Te^2}{St^2}$ are the number of teachers per 100 students and its square¹¹; $\frac{Te}{Cl}$ is the number of teachers per class; A is average age of teachers; S is a measure of school size; P is a dummy variable which takes on value 1 if the school is private and 0 if the school is public; R is the share of the student population in *ensino recorrente*; Y is a dummy variable which takes on

¹⁰That is, teachers per class (2004/05) and average teacher age (1999/00).

¹¹In what follows we refer to this variable indifferently as "teachers per 100 students" and "teacher-student ratio".

value 0 if the observation is for 2003/04 and 1 if the observation is for 2004/05 (meaning that in the panel the constant varies with time). Variables v and u are defined as described in the Section 3.

We followed a log-linear specification for the teacher-student ratio, average age and school size, in order to allow for a decreasing marginal contribution to output¹², while the coefficients of the remaining variables are semi-elasticities. However, for the teacher-student ratio, the specification in logs was approximated by a quadratic function, corresponding to the first two terms of the respective series expansion. This ratio could not enter the regression in logs because, otherwise, it would have been impossible to disentangle the impacts of the number of teachers and the number of students as inputs and as indicators of school size. Note that $\ln \frac{Te}{St} = \ln Te - \ln St$, and these two variables (rather correlated with each other, $r = 0.90$) are highly colinear with the school size which is based on them.

As to the environmental regressors, that is, the living standard ($LivSt$) and education ($Educ$) indicators by municipality, the theoretical developments in the SFA described in Section 3.2 suggest modeling them in the one-sided error component. The fact that schools do not have control over the environment would speak for such a treatment. This is nevertheless a debatable assumption since in the traditional education production modelling, socioeconomic characteristics enter the production function (see Coelli et al. (1999) for a similar discussion in another context). Therefore, we estimated both alternatives, the first one consisting of an extended version of equation (4) encompassing the socioeconomic variables. The second one with those variables in the mean of the distribution underlying u_i

$$\mu_i = \delta_0 + \delta_1 LivSt_i + \delta_2 Educ_i. \quad (5)$$

The inclusion of time intends to capture effects specific to a given academic year, notably a different degree of difficulty of examinations. Time can be modeled as a shift (parallel in logs) in the production frontier, like in (4). Similarly to the preceding paragraph, time can also be specified as a determinant of inefficiency against a common frontier, as an additional variable in (5). We checked the sensitivity of results to both possibilities.

5.2 Estimated stochastic production frontier

The estimation results for the different specifications are shown in Table 2 (cross section) and 3 (panel). All models were estimated by maximum likelihood using FRONTIER 4.1 (Coelli

¹²A heavier translog-type specification for these variables was checked, but the additional terms were not statistically significant in general, bringing no gain in comparison to a more parsimonious model.

(1996)).

Table 2: Stochastic frontier estimates, cross sectional data

		Without environment	Environment in production	Environment in inefficiency
Production frontier	Teachers/100 St.	0.014 (2.8)	0.014 (2.8)	0.013 (2.5)
	(Teachers/100 St.) ² ^(a)	-0.037 (-2.4)	-0.038 (-2.5)	-0.037 (-2.4)
	Teachers/classes	-0.024 (-2.3)	-0.023 (-2.3)	-0.021 (-2.0)
	ln Age	0.369 (7.6)	0.225 (4.1)	0.243 (4.1)
	ln Size	0.036 (4.6)	0.025 (3.3)	0.026 (3.1)
	Private school	0.096 (6.9)	0.065 (4.8)	0.069 (4.7)
	Share <i>ensino rec.</i>	-0.071 (-3.0)	-0.057 (-2.4)	-0.073 (-2.6)
	Living std.	-	0.022 (2.5)	-
	Education ^(a)	-	-0.438 (-3.4)	-
	Constant	3.142 (19.2)	3.725 (18.4)	3.678 (17.4)
	Living std.	-	-	-0.087 (-1.4)
	Education ^(a)	-	-	0.665 (1.9)
	Mu/ delta0	-0.351 (-1.7)	-0.321 (-0.9)	0.118 (1.4)
	Sigma ²	0.037 (2.9)	0.032 (1.7)	0.014 (2.7)
Distributions of u and v	Gamma	0.834 (13.6)	0.812 (7.3)	0.579 (3.1)
	Sigma u	0.176	0.161	0.091
	Sigma v	0.078	0.078	0.078
Efficiency scores	Average $\widehat{TEF}_i$	0.94	0.94	0.93

Notes: Results based on a cross-section of 490 schools (2004/05), except for the average age (1999/00). Environmental variables are for the last year available (see Appendix B). t-ratios in brackets.

(a) Coefficient multiplied by 100.

Table 3: Stochastic frontier estimates, panel data

		Without environment	Environment in production	Envir./ time in inefficiency
Production frontier	Teachers/100 St.	0.004 (1.4)	0.004 (1.4)	0.009 (2.6)
	(Teachers/100 St.) ² ^(a)	-0.011 (-1.1)	-0.011 (-1.1)	-0.027 (-2.2)
	Teachers/classes	-0.008 (-1.7)	-0.011 (-2.2)	-0.014 (-2.8)
	ln Age	0.327 (7.4)	0.162 (3.4)	0.212 (5.3)
	ln Size	0.043 (6.6)	0.031 (5.2)	0.034 (6.2)
	Private school	0.112 (7.7)	0.085 (6.2)	0.073 (6.3)
	Share <i>ensino rec.</i>	-0.111 (-4.5)	-0.079 (-3.2)	-0.103 (-5.4)
	Time	0.078 (21.8)	0.078 (21.7)	-
	Living std.	-	0.018 (2.3)	-
	Education ^(a)	-	-0.604 (-5.3)	-
	Constant	3.348 (22.2)	4.028 (23.3)	3.782 (25.7)
Distributions of u and v	Time	-	-	-0.092 (-7.8)
	Living std.	-	-	-0.031 (-3.2)
	Education ^(a)	-	-	0.477 (4.2)
	Mu/ delta0	0.179 (6.6)	0.172 (6.0)	0.193 (5.2)
	Sigma ²	0.011 (7.4)	0.010 (4.7)	0.010 (13.9)
	Gamma	0.711 (22.0)	0.715 (25.4)	0.373 (2.4)
	Sigma u	0.088	0.086	0.062
	Sigma v	0.056	0.054	0.081
	Efficiency scores			
	Average $\hat{TEF}_i$	0.83	0.84	0.88 ^(b)

Notes: Results based on a panel of 502 schools (986 observations), 2003/04 and 2004/05, except for teachers/classes (2004/05) and the average age (1999/00). Environmental variables are for the last year available (see Appendix B). t-ratios in brackets.

(a) Coefficient multiplied by 100. (b) Predictors in this model depend also on t , so the average is over i and t .

The null hypothesis of absence of random technical inefficiency ($\gamma = 0$) is rejected in the different specifications and thus the stochastic frontier model seems quite appropriate for the data. The point estimate of γ is similar across the models without inefficiency effects. In the models with environment (and time) modelled in the one-sided error component, the estimate of γ goes down, as some heterogeneity previously captured by σ_u now goes into the producer- (and time-) specific mean. In the cross section, the parameter μ , or the δ 's in the model with producer-specific mean, are statistically not significant or barely significant, pointing to a Normal-Half Normal specification. By contrast the panel results indicate a positive mode of the distribution of u (mean of the underlying normal distribution). This is in line with a higher measured efficiency level in the cross section, consistent with that mode being zero, and a larger σ_u in order to "accommodate" the less efficient observations.

The impacts of school variables have the same signs and similar magnitudes in the panel and cross section formulations. These also give the same message as to the significance of the variables, with the important exception of the teacher-student ratio that is only clearly significant in the cross section. In general, the evidence for the panel should be more reliable for it is based on more information (note that the global variance $\hat{\sigma}^2$ is lower in this formulation, for corresponding specifications). However, as the teacher-class is only available for 2004/05, the panel formulation is also more prone to measurement errors in this regressor. More robust evidence, by re-estimating the model for a longer panel with more information, would be necessary.

The dummy for the academic year is equally significant in the production frontier and in the inefficiency effects.¹³ The same holds for the socioeconomic variables, in the panel formulation, while the cross section suggests the first alternative as more adequate. Overall the data do not seem to support clearly either of the modelling alternatives. However, for practical reasons, we retain in the subsequent analysis the specification considering environment and time in production, as it yields efficiency scores net of those variables (see Section 3.3), lending itself better to school comparisons and elaboration of rankings.

The inclusion of the environmental regressors does not modify the adequacy of the models as a whole, nor changes the significance of the other variables, but it leads to a reduction of the magnitude of some responses, as spelled out below. Note that the sign of such regressors in the production frontier is the opposite of that in the inefficiency term, and rightly so, because they determine the maximum output level in the former, and the deviation from it in the latter.

¹³This should not come as a surprise, since the omission of this variable in the technology may make it appear to have explanatory power for inefficiency (see discussion at the end of Section 3.2).

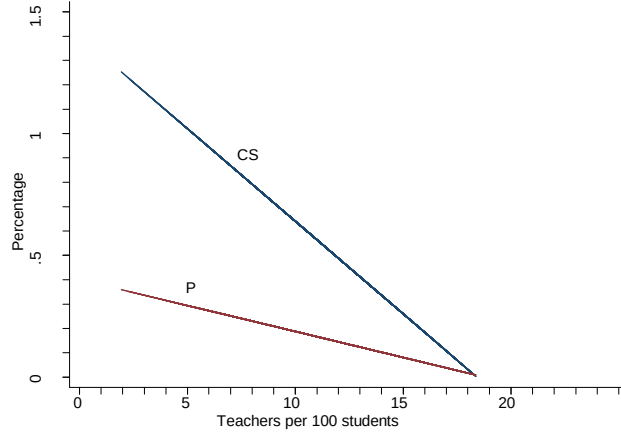


Figure 4: Marginal impacts of the variation in the teacher-student ratio

As said, the impact of the number of teachers per 100 students on output is significant in the cross section, but the evidence is weaker for the panel. Since the quadratic term is negative, we obtain a positive, but marginally decreasing, response of output to this variable. The teacher-class ratio has a negative impact on outcomes, meaning that schools whose teachers are comparatively less engaged in teaching have poorer performance. The inclusion of the teacher-class ratio is important in order to get a more precise estimate of the output response to the number of teachers per 100 student, and affects the interpretation of the latter variable. The impact *ceteris paribus* of a rise in the teacher-student ratio due, say, to an increase in the number of teachers, assumes that the average relationship between teachers and classes is kept, that is, "new" teachers are engaged in teaching to the same degree as the older. Thus the number of classes increases proportionally, or the number of students per class (class size) goes down. Figure 4 shows the marginal output responses for the panel and the cross section.¹⁴ The latter is considerably higher, although the point where they approach zero almost coincides in both samples. It is also possible to estimate the (much smaller) impact on performance of a positive variation in the number of teachers not accompanied by a change in the class size, which takes into account the offsetting effect of the rise in the teacher-class ratio.¹⁵

We have seen in Section 2 that in Portugal, over the last decade, there has been a fall in the number of students in secondary courses, implying a strong increase in the teacher-

¹⁴This impact becomes negative after a certain point, following from the parabolic shape for the contribution of the teacher-student ratio that we are imposing. As said above, the parabola intends to proxy a logarithmic shape, for which that contribution would decay to zero.

¹⁵This is given by $(\hat{\beta}_1 + \hat{\beta}_3 \frac{St}{Cl}) + 2\hat{\beta}_2 \frac{Te}{St}$.

student ratio in some schools. Given the low flexibility to move teachers with tenure across schools, the positive impact on output that should have ensued might have been restricted to the schools affected by the phenomenon and with relatively high figures for the class size (in parallel with the reduction in that variable). Schools already featuring small class sizes had a reduced margin to obtain output gains from further reductions. For those, as there are government regulations that fix a minimum value for that size¹⁶, the decrease in the number of students was most likely accompanied by a reduction in the number of classes. In our specification this would be captured by a rise in the teacher-class ratio, offsetting the impact of the change in the teacher-student ratio.

We found mixed evidence about the significance of the teacher-student ratio. Hanushek (1986) provides a survey of much econometric work in this area, most of which finds no significant impact of the teacher-student ratio on output. Hanushek points out, as a possible reason for this, the fact that the relationship between the number of students and teachers, or students and classes, is often subject to regulations, which reduce much the sample variability. As said, there are such regulations in Portugal, applying to the class size, but the variable shows sample variability (Table 4 in Appendix A).

Teacher seniority, proxied by age, appears important for educational output in all models considered. The relevance of variables measuring "quality" of teachers, as opposed to those measuring "quantity", is well documented in the literature. However, controlling for the influence of environmental variables on output, the estimated elasticity goes down both in the cross section (from 0.369 to 0.225) and in the panel (from 0.327 to 0.162). In fact, beyond the pure effect of teacher experience, this regressor is most likely capturing an additional impact concerning the region where the school is located. Clements (1999) states that there is «a systematic movement of teachers from less desirable areas to developed urban centers», as they become more experienced. The smaller impact of experience, correcting from the fact that more developed regions tend to attract more senior teachers, is likely to be more accurate.

We now compare the gain in output (in percentage points) at the estimated frontier of an increase in both the number of teachers per 100 students and teacher seniority from the current level where the school is, to the sample level yielding maximum output. This is depicted in Figure 5, taking the results for the panel. The vertical lines indicate the sample median of the variables. Figure 5 shows that, for the teacher-student ratio, a great proportion of schools operate at levels where output gains are very low (contrary to the average age). This result holds also for the cross section. Such evidence is likely to reflect the abnormally

¹⁶ As a general rule, 24 pupils (and a maximum of 28)(see *Despacho 13 765/2004 do Ministro da Educação*).

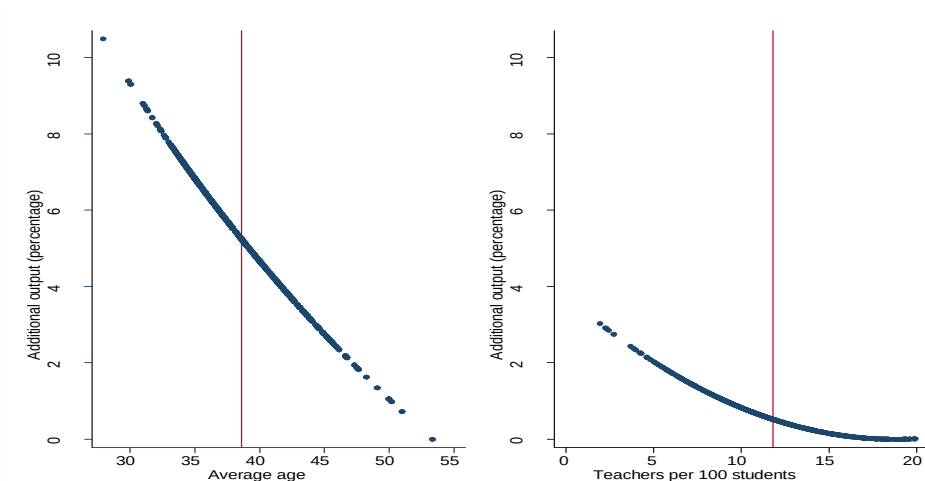


Figure 5: Additional output from increases in the utilization of resources

high value of the teacher-student ratio in Portuguese schools vis-a-vis international standards.

The share of *ensino recorrente* influences negatively the outcomes, as expected. School size, measuring the dimension of that part of school devoted to secondary education, appears as a positive determinant of output. This speaks for concentrating resources in fewer schools, whenever possible, a result which is in line with other empirical work. For instance, Rainey and Murova (2003) and Mizala et al. (2002) also find scale economies. The impact of the private school dummy on educational output is quite large and precisely estimated across the specifications considered, meaning that we are able to estimate separate frontiers for the two groups. When location is taken into account, correcting for the prevalence of private schools in relatively more developed regions,¹⁷ private institutions feature output gains between 6.5 and 8.5 per cent. That result is, as always, conditional on the variables included in the regression and this point deserves a more detailed discussion, so we come back to it in a separate section below.

As regards the environmental variables, the living standard and education regressors are significant and the estimated impacts go in the expected directions. This indicates that such variables influence school output, in line with the conclusions reached by Oliveira and Santos (2005), although they use a diverse methodology and other variables.

In order to highlight the impact on efficiency scores of controlling for school location, Figure 6 presents the resulting density functions for schools in poorer and richer municipalities

¹⁷ Roughly three quarters of the private schools are located in municipalities in the upper quartile of the Purchase Power Index.

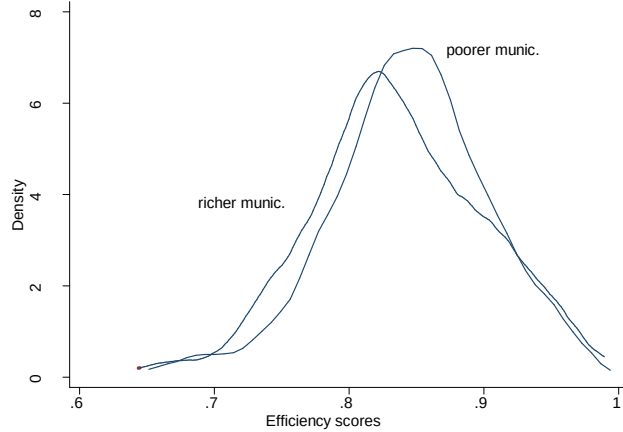


Figure 6: Efficiency scores in poorer and richer municipalities

(taking the panel results¹⁸). They overlap much more now than in Figure 3, since the environmental variables are capturing the original gap between the densities of examination scores.

5.3 What can we say about efficiency of public *vs.* private schools?

Clements (1999) stated in his conclusions that «preliminary evidence (...) suggests that private schools may be more efficient than their public sector counterparts, as they achieve higher success rate with higher student-teacher ratios. (...) Additional work is needed to address whether these differences simply reflect the more favourable status of students in private schools, rather than greater efficiency *per se*». As far as school performance in national examinations is concerned, the general perception is that private schools outdo those in the general government. Looking at the distribution of examination scores (two-year averages) for schools in the public and private sectors (chart on the left side of Figure 7), one sees that such a perception is not entirely justified.

Private schools have indeed better results at the upper percentiles of the distribution, but at the intermediate and lower percentiles the results in both groups are similar. The average score in both types of schools is also close (0.51 in public schools, against 0.54 in private). As public attention typically focus on the schools with the best and worst scores, that is, at the top and at the bottom of the distribution, this tends to emphasize somewhat the achievement

¹⁸This point, made with the aid of the efficiency scores density for the panel, could also be illustrated with the corresponding density for the cross section (like others in the remainder of the paper). Note that comparing with Figure 6, such density is more (positively) asymmetric with the respective mode closer to 1, as μ is not significant.

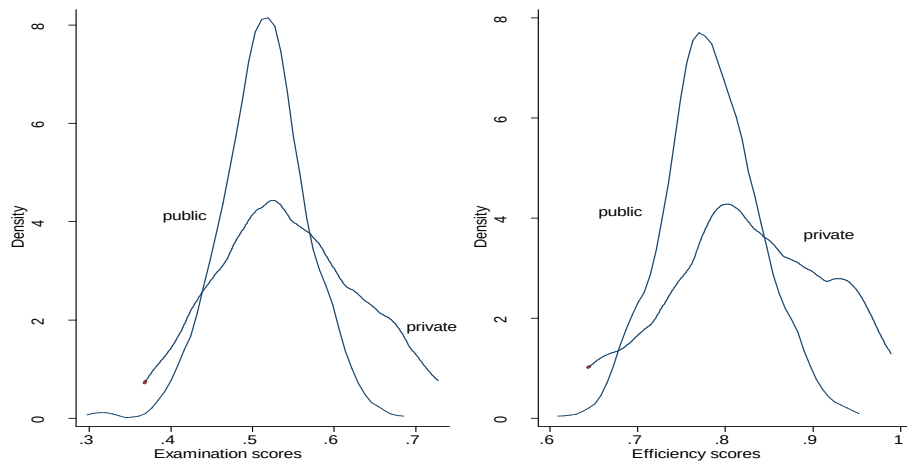


Figure 7: Examination scores (left) and efficiency scores (right), in public and private schools

of private schools.

Concerning the use of resources, figures in Table 4 (Appendix A) indicate that schools in public and private sectors employ a similar level, as far as one can measure by the teacher-student ratio, although private institutions employ teachers "more intensively", allowing them to have lower class sizes. The average teacher age is almost identical in both groups. After controlling for the use of resources and other school and environment variables, we have seen in the preceding section that empirical results clearly indicate that private schools are more efficient. Similar conclusions can be reached by considering the density of the modified efficiency predictor, gross of the impact of the private management dummy (see Section 3.3), in public and private schools. The chart on the right side of Figure 7 (based on the results for the panel) shows a clearer difference at the various percentiles between the two groups.

It is worth noting that if we had run a regression with financial inputs (like average expenditure per student), instead of physical, the efficiency gap between private and public schools would have been most likely larger, for average teachers' salary in general government (in 1999) was considerably higher than in the private sector (Table 4 in Appendix A).

The second important issue in this context is the extent to which the results we have just outlined are accounted for by the fact that students attending private schools typically come from households with higher social status.¹⁹ Carneiro (2006) presents evidence pointing to a

¹⁹ Another relevant issue we became aware of was that sometimes private schools might have student selection mechanisms that could bias the results in their favour. As we did not have detailed information, we could not

strong importance of family background variables as determinants of educational outcomes in Portugal, and this is very much in line with the empirical findings of studies for other countries. Unfortunately information on the socioeconomic background of students who took the examinations (or more generally of the students attending a given school) was not available. Actually, for this type of insights, it would be important to focus on a lower aggregation level - that of the student.²⁰ By considering school averages one already loses information on intra-school variance, very important in this context.

We conjecture that controlling for the family background of students would reduce the magnitude of the private sector dummy, but it would not obliterate its significance. In fact, in the first place, about 1/4 of students in private schools attend institutions privately run but financed by government (OECD (2006)), and for those the household background argument does not hold. Secondly, such argument is normally put forward in connection with private schools featuring outstanding results, at the very top of the distribution of examination scores. However, observing that distribution for private schools, one sees considerable dispersion, with a number of institutions ranking very low in terms of performance. In the light of the findings in Carneiro (2006), mentioned above, for those schools there seems to be no reason to assume *a priori* that their students come from advantaged households.

5.4 A proposal for an efficiency ranking of Portuguese secondary schools

In the SFA efficiency is assessed (and producers ranked) controlling for the variables in the production frontier. However, as explained in Section 3.3, it is possible to calculate a modified predictor which does not control for some of those variables in the measurement of efficiency. We now apply this idea to the construction of a ranking of Portuguese secondary schools. Out of the explanatory variables in our regression, in terms of Section 3.3 terminology, the teacher-student ratio and average teacher age would fit in with the first category i.e. those to be always controlled for when measuring efficiency. As to the remaining variables, one would like to net their effect only if producers do not have discretion over them. In the context of ranking Portuguese secondary schools, a difficulty now arises coming from the fact that public schools have much less autonomy than their private counterparts. Meaning that there are production decisions feasible from the point of view of the Ministry of Education, but not of individual public schools. Only the Ministry of Education can for example modify the rules governing teacher hiring and allocation to schools, teaching duties and, theoretically,

pursue this.

²⁰This would be possibly the most interesting extension of the research in this paper. We are aware of data of that kind in the framework of OECD's PISA, but covering a much smaller sample of schools than ours.

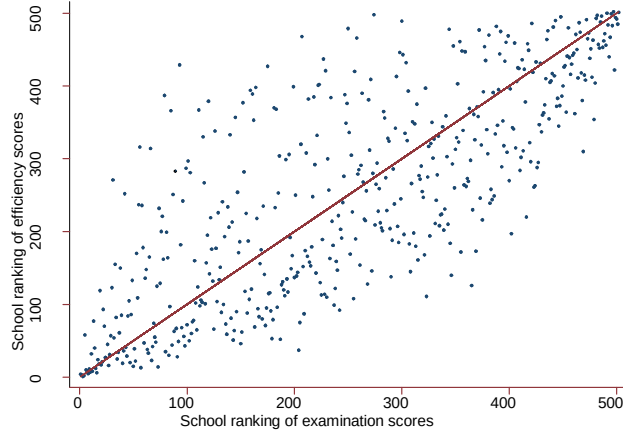


Figure 8: Original and proposed rankings of schools

impose management practices closer to private sector. Such decisions would typically alter features captured by the teacher-class ratio and the private school dummy. By contrast in the case of the share of *ensino recorrente* and school size, neither the school has influence on the demand (number and type of students) directed to it, nor the Ministry can interfere with such variables beyond a small extent. As to the environmental variables, they are of course given.

From a policy perspective, the relevant efficiency scores are the ones that allow for the possibility of the Ministry of Education to introduce reforms in the system. It follows from the discussion in the preceding paragraph that they can be obtained using the modified predictor gross of the effect of the teacher-class ratio and the private school dummy. This methodology leads to a measured efficiency level of 0.77 (panel) and 0.90 (cross section). Such levels are below the initial ones presented in Tables 3 (0.84) and 2 (0.94) in which inefficiency is net of different intensities in the utilization of teachers and the private management effect. Figure 8 presents the loci of individual schools in the ranking resulting from the original examination scores (two-year average) and the ranking on the basis of the efficiency scores computed with the proposed methodology. It shows that the proposed ranking entails considerable changes in comparison with the original one (the 45° line indicates the schools whose position is unchanged).

6 Conclusions

In this paper we studied the determinants of educational output of 502 Portuguese secondary schools and provided an estimate of the respective technical efficiency level. Output was measured by average school scores in the 12th grade national examinations. SFA models with school, teacher and environmental variables were estimated. We considered two alternatives for modelling the latter, as determinants of production technology or heterogeneity in inefficiency. Using the SFA, we put forward a proposal for an efficiency ranking for Portuguese secondary schools, useful from a policy perspective.

Our study faced empirical difficulties. Some of them resulted from the transposition of the educational process into a production function, and others from the lack of information on important dimensions of schools. One of the main challenges of this work was to assemble a dataset that covered most Portuguese secondary schools, including a reasonable number of variables explaining educational output. Variables measuring the family background of students could not be included and teacher characteristics were summarised by a seniority indicator.

Our analysis points to the existence of technical inefficiency: examination scores could be on average about 10 to 20 per cent higher for the current level of resources. This value is measured against a production frontier that takes as a benchmark the most efficient Portuguese schools (implicitly establishing a target). It would be interesting to apply the same techniques to a panel of schools from several countries, in order to compare the different degrees of technical inefficiency. The cross-country analysis of global performance and input indicators suggests that the Portuguese benchmark might be inefficient when compared to other countries. In this case the level of measured inefficiency of the Portuguese schools would be higher.

Results indicate that "quality" of teachers has more effect on output than "quantity". The variation in the number of teachers per student tends to have less influence on output than differences in their characteristics proxied by seniority. On the other hand, there seems to be a high proportion of schools operating at teacher-student levels where output gains are very small. This should reflect, in particular, the fact that many schools have lost students over the recent years and did not adjust the number of teachers. Therefore, enhancing the flexibility in the allocation of teachers could free resources without a noticeable effect on scores. Part of the reduction in outlays so obtained could be applied on non-personnel spending items, in which Portugal ranks very low in international comparisons. Such added flexibility would also be the way to achieve a greater uniformity in the average class size across schools. As

far as the school network is concerned, we found evidence of scale economies in secondary education, indicating potential gains from the concentration of resources.

Our study demonstrates that there is a sizeable influence of geographical location of schools on outcomes. The schools located in municipalities featuring higher living standards and education levels achieve comparatively more.

Comparison of examination scores for schools in the general government and in the private sector shows that, although the latter do slightly better on average, their individual performance is quite heterogeneous. After resources and other outcome determinants have been taken into account, the better performance of privately managed schools becomes more evident. Efficiency analysis reveals average output gains of 7 to 8 per cent, even if this figure may be overestimated as it was not possible to disentangle the influence on it of the socioeconomic background of students. Such results highlight the importance of transposing to schools in the general government some management practices common in the private sector, like teacher accountability and merit rewarding mechanisms.

7 Appendices

A Stochastic frontier dataset: some descriptive statistics

Table 4: School variables

	all schools				public schools		private schools	
	mean	std.dev.	q25	q75	mean	std.dev.	mean	std.dev.
Output								
Average score in exams	104	13	96	112	103	12	109	18
in 2004	100	12	92	107	99	11	105	18
in 2005	108	12	101	115	107	11	113	17
Inputs								
School data								
Teachers per 100 st.	12.4	4.0	9.9	14.2	12.3	3.7	12.8	5.5
Teachers per class	2.8	0.8	2.3	3.3	2.9	0.8	2.3	0.8
Students per class	22.4	4.0	20.3	24.8	22.8	3.6	19.6	5.3
Size (ranking)	-	-	-	-	321	147	138	104
Share ens. recorrente	0.17	0.17	0.00	0.28	0.18	0.16	0.14	0.24
Teacher data								
Aver. age (years)	38.8	4.0	35.7	41.6	39.0	3.9	37.9	4.5
Aver. wage (euro)	1393	258	1233	1567	1453	215	1046	211
Share univ. educ.	0.97	0.04	0.96	0.98	0.98	0.03	0.93	0.07

Notes: Statistics for most variables based on 986 observations (years 2003/04 and 2004/05) for 502 schools (427 public, 75 private); teachers per class and students per class available only for the year 2004/05 and average teacher age for the year 1999/00. Average teachers' wage and teacher education based on 501 schools (427 public, 74 private) for the year 1999/00. See the text and Appendix B for the definition of variables.

Table 5: Environmental variables

	mean	std.dev.	q25	q50	q75
Health status (index)	223	19	212	228	238
Purchase power (index)	73	27	57	67	81
Average schooling (years)	6.1	0.9	5.4	5.9	6.6
Hous. elect. cons. (Kw/h)	2.0	0.6	1.6	2.0	2.4
Illiteracy rate (%)	12.7	5.2	8.7	11.8	16.4

Notes: Statistics for environmental variables based on data for 241 municipalities of continental Portugal. See the text and Appendix B for the definition of variables and years they refer to.

B Data and sources

B.1 Output

Average Score of national examinations. Published by Expresso and elaborated by Socinova (Gabinete de Investigação em Sociologia da Universidade Nova) with information from the Ministério da Educação, for 555 schools (academic year 2003/04) and 545 schools (academic year 2004/05), comprising almost 238 thousand exams. Considers the exams taken in June and July by internal (i.e. not self-proposed) students.

B.2 Inputs

Number of teachers per 100 students. Created with data from GIASE (Gabinete de Informação e Avaliação do Sistema Educativo). Considers all students ("ensino regular" and "ensino recorrente") and teachers at the secondary level, for each school. Data cover the academic years 2003/04 and 2004/05.

Number of teachers per class. Created with data from GIASE. Considers all teachers and classes at the secondary level, for each school. Data cover only the academic year 2004/05.

Share of the student population in "ensino recorrente". Created with data from GIASE. Data cover the academic years 2003/04 and 2004/05.

School size. Created with data from GIASE. Calculated as a ranking variable, obtained as the mean of two other rankings, one of the number of students and another of the number of teachers. Data cover the academic years 2003/04 and 2004/05.

Private school dummy. Created with data from GIASE. Dummy variable which takes on value 1 if the school is private and 0 if the school is public. Data cover the academic years 2003/04 and 2004/05.

Average teacher age. Created with teachers' individual data from the "2º Recenseamento Geral da Administração Pública" (2nd General Government Census), for public schools, and "Quadros de Pessoal" survey of 1999, for private schools. Schools with less than 7 teachers were excluded. Covers 55,380 teachers in public schools and 4,853 in private.

Average wage of teachers. Created with teachers' individual data from the "2º Recenseamento Geral da Administração Pública" (2nd General Government Census), for public schools, and "Quadros de Pessoal" survey of 1999, for private schools. Covers 49,719 teachers in public schools and 4,928 in private.

Proportion of teachers with university education. Created with teachers' individual data from the "2º Recenseamento Geral da Administração Pública" (2nd General Government Census), for public schools, and "Quadros de Pessoal" survey of 1999, for private schools. Covers 55,191 teachers in public schools and 4,817 in private.

Purchase power index, by municipality. Published every two years in the series "Estudo sobre o Poder de Compra Concelhio" by the Instituto Nacional de Estatística (INE). Obtained by applying factor analysis to a range of living standard indicators. The last available year is 2004 (INE (2004)).

Average years of schooling, by municipality. Created on the basis of statistics from INE on the share of the population in each educational level, considering the years of schooling necessary to complete it. The last available year is 2001.

Health status index, by municipality. Data underlying the health indicator presented in Santana (2004), one of the partial indicators (health, demography, supply of health care, utilization of health care, social and economic) used to assess global health conditions. Data are for 2001.

Average household electricity consumption, by municipality. The source is INE. The last year available is 2002.

Illiteracy rate, by municipality. The source is INE. The last year available is 2001.

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