Dynamic Correlations, Estimation Risk, and Portfolio Management during the Financial Crisis

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Abstract

We evaluate alternative multivariate models of dynamic correlations in terms of realized out-of-sample Sharpe ratios for an active portfolio manager who rebalances a portfolio of country ETFs on a daily basis. The evaluation period covers the 2008 financial crisis which was marked by soaring volatilities and correlations across international stock markets. We find that the models deliver statistically lower portfolio variances, but not significantly better Sharpe ratios than the naive diversification benchmark strategy. The results clearly show the erosive effects of model estimation risk and transactions costs, the benefits of limiting short sales, and the far greater importance of having access to a risk-free asset whether or not market turbulence is high.

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1 Introduction

The classical theory of portfolio selection pioneered by Markowitz (1952) remains to this day an immensely prominent approach to asset allocation and active portfolio management. The key inputs to this approach are the expected returns and the covariance matrix of the assets under consideration in the portfolio selection problem. According to the classical theory, optimal portfolio weights can be found by minimizing the variance of the portfolio's returns subject to the constraint that the expected portfolio return achieves a specified target value. In practical applications of mean-variance portfolio theory, the expected returns and the covariance matrix of asset returns obviously need to be estimated from the historical data. As with any model with unknown parameters, this immediately gives rise to the well-known problem of estimation risk; i.e., the estimated optimal portfolio rule is subject to parameter uncertainty and can thus be substantially different from the true optimal rule.

The implementation of mean-variance portfolios with inputs estimated via their sample analogues is notorious for producing extreme portfolio weights that fluctuate substantially over time and perform poorly out of sample; see Hodges and Brealey (1972), Michaud (1989), Best and Grauer (1991), and Litterman (2003). A recent study by DeMiguel, Garlappi, and Uppal (2009) casts further doubt on the usefulness of estimated mean-variance portfolio rules when compared to naive diversification. The naive strategy, or $1/N$ rule, simply invests equally across the N assets under consideration, relying neither on any model nor on any data. DeMiguel, Garlappi, and Uppal consider various static asset-allocation models at the monthly frequency and find that the asset misallocation errors of the suboptimal (from the mean-variance perspective) $1/N$ rule are smaller than those of the optimizing models in the presence of estimation risk. See also Jobson and Korkie (1980), Michaud and Michaud (2008), and Duchin and Levy (2009) for more on the issue of estimation errors in the implementation of Markowitz portfolios.

Kirby and Ostdiek (2011) show that the research design in DeMiguel, Garlappi, and Uppal (2009) places the mean-variance model at an inherent disadvantage relative to naive diversification because it focuses on the tangency portfolio which targets a conditional expected return that greatly exceeds the conditional expected return of the $1/N$ strategy. The result is that estimation risk is magnified which in turn leads to excessive portfolio turnover and hence poor out-of-sample performance. Kirby and Ostdiek argue that if the mean-variance model is implemented by targeting the conditional expected return of the $1/N$ portfolio, then the resulting static mean-variance efficient strategies can outperform naive diversification for most of the monthly data sets considered by DeMiguel, Garlappi, and Uppal. Kirby and Ostdiek note, however, that this finding is not robust to the presence of transactions costs.

The correlation structure across assets is a key feature of the portfolio allocation problem since it determines the riskiness of the investment position. It is well known that these correlations vary over time and the econometrics literature has proposed many specifications to model the dynamic movements and co-movements among financial asset returns, which are especially pronounced at the daily frequency. The importance of dynamic correlation modeling is obvious for risk management because the risk of a portfolio depends not on what the correlations were in the past, but on what they will be in the future.

Engle and Colacito (2006) take the asset allocation perspective to measure the value of modeling dynamic correlation structures. Just like DeMiguel, Garlappi, and Uppal (2009) and Kirby and Ostdiek (2011) (and others), they too study the classical asset allocation problem, but with forward-looking correlation forecasts obtained from dynamic correlation models. They consider the realized volatility of optimized portfolios and find that it is smallest when the dynamic correlation model is correctly specified. Their focus, however, is primarily on a setting with only two assets—a stock and a bond—and hence with relatively little estimation risk since few parameters need to be estimated.

In this paper, we further contribute to the literature on portfolio management in the presence of estimation risk by considering an active portfolio manager who uses forecasts from dynamic correlation models to rebalance a portfolio of Exchange Traded Funds (ETFs) on a daily basis. The asset mix consists of ETFs that track broad stock market indices for five countries: the US, UK, Japan, Mexico, and Malaysia. The first three of those are developed markets, whereas the last two are considered emerging markets. This choice of assets (with synchronous returns) allows us to examine whether there is any evidence supporting the decoupling hypothesis in international finance, namely that recently the evolution of emerging stock markets has decoupled itself from the evolution of more developed stock markets. Serban et al. (2007) also consider some dynamic correlation models of daily returns, but present only a limited analysis of Markowitz portfolios comprising three country ETFs over a relatively short and tranquil out-of-sample period.

In addition to the usual plug-in method which simply replaces the covariance matrix by its sample counterpart, we also consider several other popular approaches to forecast the inputs to the portfolio selection problem, ranging from quite parsimonious to highly parametrized ones. The first of these is J.P. Morgan's RiskMetrics, which is a simple exponentially weighted moving average (EWMA) model of return covariances without any parameters to estimate. The other models include the constant conditional correlation (CCC) model of Bollerslev (1990) and some variants of it, the dynamic conditional correlation (DCC) model of Engle (2002), the time-varying correlation (TVC) model of Tse and Tsui (2002), and the recent dynamic equi-correlation (DECO) model of Engle and Kelly (2009). Each of these models builds on a decomposition of the conditional covariance matrix into a product involving the conditional correlation matrix and a diagonal matrix of conditional standard deviations.

The conditional variances (standard deviations) are specified either according to EWMA models or standard generalized autoregressive conditional heteroskedasticity (GARCH) models (Bollerslev 1986). The return innovations are modeled according to a generalized version of the standard multivariate Student-t distribution proposed by Bauwens and Laurent (2005) that allows for asymmetries in each of the marginal distributions. The reason for this choice is the well-known stylized fact about financial returns that they exhibit fat tails and are often skewed; see Cont (2001) for a comprehensive survey of the stylized facts. In order to gain finite-sample statistical efficiency, it is important to base modeling and inference on a more suitable distribution than the multivariate normal.[1] The most sophisticated correlation models we consider have a total of 18 parameters that need to be estimated, even though we follow Engle and Colacito (2006) and make use of the correlation targeting method of Engle and Mezrich (1996) to reduce the dimensionality problem. See Engle (2009) for a discussion of variance and correlation targeting.

Following DeMiguel, Garlappi, and Uppal (2009) and Kirby and Ostdiek (2011), the $1/N$ rule serves as our benchmark strategy for comparison purposes and, as in Kirby and Ostdiek, we level the playing field by using the conditional expected return of the $1/N$ rule as the target expected portfolio return. We hasten to emphasize that our goal is not to advocate the use of the $1/N$ rule as an asset allocation strategy, but merely to use it as a benchmark to assess the performance of the more sophisticated data- and model-dependent strategies. The evaluation period covers the recent financial crisis which was marked by soaring volatilities and correlations across international stock markets. Amid such episodes of turmoil that pervade markets around the world, it becomes especially important to know whether standard models remain useful tools for portfolio management. We consider portfolios that comprise risky assets only and, in addition, we examine the effects of introducing a risk-free security into the asset mix. As in Frost and Savarino (1988), Chopra (1993), and Jagannathan and Ma (2003), we also examine whether the imposition of short-sale constraints leads to improved portfolio performances.

We focus on the realized (ex post) out-of-sample Sharpe ratio as the measure of portfolio performance, which is defined as the ratio of the realized excess return of an investment to its standard deviation. This choice is motivated by the fact that the Sharpe ratio is the most ubiquitous risk-adjusted measure used by financial market practitioners to rank fund managers and to evaluate the attractiveness of investment strategies in general. For example, Chicago-based Morningstar calculates the Sharpe ratio for mutual funds in its Principia investment research and planning software. In order to gauge the statistical

[1] Related evidence is found in Engle and González-Rivera (1991) where it is shown that Gaussian quasi-maximum likelihood estimation of GARCH models is inefficient, with the degree of inefficiency increasing in the severity of departures from normality.

significance of the differences between Sharpe ratios we use the block bootstrap inference methodology proposed by Ledoit and Wolf (2008), which is designed specifically for that purpose. We also examine a simple modification of the Sharpe ratio proposed by Israelsen (2003, 2005) to overcome some shortcomings of the usual measure. To complete our analysis, we finally test for statistical differences in the variances (or, equivalently, in the standard deviations) of returns. It is important to note that the standard deviation of returns is the measure of risk in the Sharpe ratio.

The remainder of the paper is organized as follows. Section 2 presents the modeling framework. It begins with a description of the multivariate distribution of returns, and then moves on to present the volatility specifications and the various correlation specifications that we consider. Details about how the model parameters are estimated are also given in that section. Section 3 presents the in-sample estimation results for each model specification. Section 4 considers the portfolio management problem and presents the out-of-sample portfolio performance results. Section 5 concludes.

2 Modeling framework

2.1 Multivariate distribution

Consider a collection of N assets whose day-t returns are stacked in the $N \times 1$ vector $\boldsymbol{y}_t$. We assume that the daily returns can be represented as

$$\boldsymbol{y}_t = \boldsymbol{\mu} + \boldsymbol{\varepsilon}_t, \tag{1}$$

$$\boldsymbol{\varepsilon}_t = \boldsymbol{H}_t^{1/2} \boldsymbol{z}_t, \tag{2}$$

where $\boldsymbol{\mu}$ is the vector of expected returns, $\boldsymbol{\varepsilon}_t$ is the vector of unexpected returns, and $\boldsymbol{z}_t$ is the vector of innovation terms with conditional moments $E(\boldsymbol{z}_t|\mathcal{I}_{t-1}) = 0$ and $Var(\boldsymbol{z}_t|\mathcal{I}_{t-1}) = \boldsymbol{I}_N$, the identity matrix. Here $\mathcal{I}_t$ represents the information set available up to time t. The conditional covariance matrix of $\boldsymbol{y}_t$ given $\mathcal{I}_{t-1}$ is $\boldsymbol{H}_t$ and the matrix $\boldsymbol{H}_t^{1/2}$ is its Cholesky factorization. In our empirical application we have $N = 5$ risky assets and, as will become clear, the number of parameters to be estimated in each of our model specifications is fairly large. So to make the estimation simpler, we follow the common practice and use auxiliary estimators for unconditional moments. The first instance of this is that we replace $\boldsymbol{\mu}$ in (1) by the vector of sample means, $\bar{\boldsymbol{y}}$. As expected from daily returns, those estimates of the first moments turn out to be very close to zero.

It is well known that financial asset returns exhibit excess kurtosis which is at odds with models that assume a multivariate normal distribution of returns. While the commonly used multivariate Student-t distribution can capture heavy tails, it still restricts returns to be symmetrically distributed around their means. In order to allow for possible return asymmetries, the $\boldsymbol{z}_t$ innovation terms in (2) are assumed to be independent and identically distributed random vectors following the multivariate skewed Student-t distribution of Bauwens and Laurent (2005), which generalizes the standard multivariate Student-t distribution by allowing each marginal distribution to have its own asymmetry parameter. Specifically, the density of $\boldsymbol{z}_t$, given the shape parameters $\boldsymbol{\xi} = (\xi_1, ..., \xi_N)$, $\xi_i > 0$, and the degrees-of-freedom parameter $v > 2$, is given by

$$f(\boldsymbol{z}_t; \boldsymbol{\xi}, v) = \left(\frac{2}{\sqrt{\pi}}\right)^N \left(\prod_{i=1}^{N} \frac{\xi_i s_i}{1+\xi_i^2}\right) \frac{\Gamma(\frac{v+N}{2})}{\Gamma(\frac{v}{2})(v-2)^{\frac{N}{2}}} \left(1 + \frac{\boldsymbol{z}_t^{*\prime} \boldsymbol{z}_t^*}{v-2}\right)^{-\frac{v+N}{2}}, \tag{3}$$

where

$$\begin{aligned}
\boldsymbol{z}_t^* &= (z_{1,t}^*, ..., z_{N,t}^*)', \\
z_{i,t}^* &= (s_i z_{i,t} + m_i)\xi_i^{I_i}, \\
m_i &= \frac{\Gamma(\frac{v-1}{2})\sqrt{v-2}}{\sqrt{\pi}\Gamma(\frac{v}{2})} \left(\xi_i - \frac{1}{\xi_i}\right), \\
s_i^2 &= \left(\xi_i^2 + \frac{1}{\xi_i^2} - 1\right) - m_i^2, \\
I_i &= \begin{cases} -1 & \text{if} \quad z_i \geq -\frac{m_i}{s_i}, \\ 1 & \text{if} \quad z_i < -\frac{m_i}{s_i}. \end{cases}
\end{aligned}$$

Note that m_i and s_i^2 are functions of ξ_i, so they do not represent additional parameters. Here ξ_i^2 determines the skewness in the marginal distribution of $z_{i,t}$. With this specification, the marginal distribution of $z_{i,t}$ is symmetric when $\log \xi_i = 0$ and it is skewed to the right (left) when $\log \xi_i > 0$ (< 0). See Bauwens and Laurent (2005) for the derivation and further discussion of this distribution. It should be clear that this specification also implies the same degree of tail heaviness across marginals, controlled by the parameter v. In a Bayesian setting, Jondeau and Rockinger (2008) consider a more general alternative where each marginal distribution has its own kurtosis parameter. Further discussion of these generalized multivariate Student-t distributions is found in Jondeau, Poon, and Rockinger (2007).

The conditional covariance matrix of $\boldsymbol{y}_t$ is written in the familiar form

$$\boldsymbol{H}_t = \boldsymbol{D}_t \boldsymbol{C}_t \boldsymbol{D}_t, \tag{4}$$

where $\boldsymbol{D}_t = \text{diag}(h_{11,t}^{1/2}, ..., h_{NN,t}^{1/2})$ is a diagonal matrix of conditional standard deviations and $\boldsymbol{C}_t$ is the conditional correlation matrix. The elements of $\boldsymbol{D}_t$ are the square roots of the expected return variances based on $\mathcal{I}_t$, referred to here as the volatilities. The two main volatility specifications that we shall consider are described next.

2.2 Volatility specifications

The most basic conditional variance forecasting specification we consider is the widely popular J.P. Morgan's RiskMetrics, which is an EWMA model that is written as

$$h_{ii,t} = \lambda h_{ii,t-1} + (1-\lambda)\varepsilon_{i,t-1}^2, \tag{5}$$

with $\lambda = 0.94$ for daily returns following the prescription in J.P. Morgan's (1996) technical document. The EWMA model posits today's volatility, $h_{ii,t}$, as a weighted average of the lagged volatility, $h_{ii,t-1}$, and the square of the lagged unexpected return, $\varepsilon_{i,t-1}^2$. Here we follow the common practice and initialize the EWMA recursions with the sample variance so that $h_{ii,1} = \hat{\sigma}_i^2 = T^{-1}\sum_{t=1}^{T}(y_{i,t} - \bar{y}_i)^2$. With no parameters to estimate, the EWMA model in (5) has the clear advantage of simplicity. Furthermore, it does a fairly good job at tracking variance changes. On the flip side, however, it restricts the variance processes to be non-stationary, which is a potential shortcoming if long-run average variances tend to be relatively stable over time.

So as an alternative to the EWMA model in (5), the second volatility specification we consider is the GARCH(1,1) model with a stationary solution. This specification, which is often termed the "workhorse of the industry" owing to its immense popularity, takes the form

$$h_{ii,t} = \delta_i + \alpha_i \varepsilon_{i,t-1}^2 + \beta_i h_{ii,t-1}, \tag{6}$$

where the restrictions $\delta_i > 0$, $\alpha_i, \beta_i \geq 0$, and $\alpha_i + \beta_i < 1$ ensure that the conditional variance processes are stationary with unconditional (long-run) variances given by $\sigma_i^2 = \delta_i / \kappa_i$, where $\kappa_i = 1 - \alpha_i - \beta_i$ (Bollerslev 1986). In order to reduce the dimension of the parameter space and the computational complexity of the estimation problem, we use the variance targeting method proposed by Engle and Mezrich (1996). The method takes the models in (6) and rewrites them as

$$h_{ii,t} = \kappa_i \sigma_i^2 + \alpha_i \varepsilon_{i,t-1}^2 + \beta_i h_{ii,t-1}, \tag{7}$$

with $\kappa_i + \alpha_i + \beta_i = 1$, and where the positivity and stationarity constraints become $\kappa_i, \sigma_i^2 > 0$, $\alpha_i \geq 0$, and $\kappa_i + \alpha_i \leq 1$. From (7) we see that today's volatility, $h_{ii,t}$, is a weighted average of the long-run variance, σ_i^2, the square of the lagged unexpected return, $\varepsilon_{i,t-1}^2$, and the lagged volatility, $h_{ii,t-1}$. The parameter κ_i is the weight on the long-run variance in that average. Variance targeting here consists of replacing σ_i^2 in (7) by the sample variance $\hat{\sigma}_i^2$ and then estimating the α_i's and β_i's by maximum likelihood. We also follow the common practice and set the initial value as $h_{ii,1} = \hat{\sigma}_i^2$. See Francq, Horvath, and Zakoian (2009) for further discussion about the method of variance targeting.

With the volatilities in hand, we can then define the vector of standardized (or de-volatilized) returns $\boldsymbol{u}_t = (u_{1,t}, ..., u_{N,t})'$ whose typical element is $u_{i,t} = \varepsilon_{i,t}/\sqrt{h_{ii,t}}$. If a portfolio of the assets comprising $\boldsymbol{y}_t$ is formed with weights $\boldsymbol{\omega}$, then its conditional return variance is $Var(\boldsymbol{\omega}'\boldsymbol{y}_t|\mathcal{I}_{t-1}) = \boldsymbol{\omega}'\boldsymbol{H}_t\boldsymbol{\omega}$. In light of (4), this expression makes clear that the correlation matrix $\boldsymbol{C}_t$ is a key feature of the portfolio management problem.

2.3 EWMA of covariances

Before we turn to the various structures for the conditional correlation matrix $\boldsymbol{C}_t$, it seems natural to consider the simple RiskMetrics model of covariances. This multivariate EWMA (mEWMA) model is given by

$$\boldsymbol{H}_t = \lambda \boldsymbol{H}_{t-1} + (1-\lambda)\boldsymbol{\varepsilon}_{t-1}\boldsymbol{\varepsilon}'_{t-1}, \tag{8}$$

where again $\lambda = 0.94$ as it was in the EWMA model of scalar volatilities. The recursion in (8) is initialized by setting $\boldsymbol{H}_1$ equal to the unconditional covariance matrix of sample returns.

Although it is quite a natural extension of the scalar EWMA approach, the mEWMA model also implies that covariances are not mean-reverting, so if today's covariances are high, then according to (8) they will remain high rather than revert back to their long-run mean values. In the following we present alternatives that yield stationary covariance forecasts.

2.4 CCC specifications

Our starting point for specifying the correlation matrix $\boldsymbol{C}_t$ appearing in (4) is the constant conditional correlation (CCC) model proposed by Bollerslev (1990) in which the conditional correlations between each pair of asset returns are restricted to be constant over time. So in the CCC model, the conditional covariance matrix is defined as

$$\boldsymbol{H}_t = \boldsymbol{D}_t \boldsymbol{C} \boldsymbol{D}_t, \tag{9}$$

where $\boldsymbol{C} = [\rho_{ij}]$ is a well-defined correlation matrix; i.e., symmetric, positive definite, and with $\rho_{ii} = 1$ for all i. As Engle (2009) discusses, it is particularly simple to obtain a well-defined CCC matrix estimate if we use correlation targeting—a direct analogue of the variance targeting method—which consists of replacing $\boldsymbol{C}$ by its empirical counterpart:

$$\bar{\boldsymbol{C}} = \text{diag}\left[\frac{1}{T}\sum_{t=1}^{T}\boldsymbol{u}_\tau\boldsymbol{u}'_\tau\right]^{-\frac{1}{2}}\left[\frac{1}{T}\sum_{t=t}^{T}\boldsymbol{u}_\tau\boldsymbol{u}'_\tau\right]\text{diag}\left[\frac{1}{T}\sum_{t=1}^{T}\boldsymbol{u}_\tau\boldsymbol{u}'_\tau\right]^{-\frac{1}{2}}, \tag{10}$$

the sample correlation matrix of standardized returns.

2.4.1 IND specifications

A special case of the CCC model is the independence correlation structure which simply sets $\boldsymbol{C} = \boldsymbol{I}_N$, an $N \times N$ identity matrix. This benchmark case will be useful to assess the relevance of capturing non-zero correlations when forecasting future returns. We consider two versions of this specification: (i) based on the volatilities from the scalar EWMA models, and (ii) based on the volatilities implied by the GARCH models. In the tables presented below, those cases are labeled as EWMA and IND. A comparison of these two specifications allows an assessment of the value of GARCH modeling.

2.4.2 ECO specification

Rather than imposing zero correlations, a slightly more general specification is to assume that all pairs of returns have the same time-invariant correlation. This equicorrelation (ECO) structure, which is also a restricted case of Bollerslev's CCC model, can be written as

$$\rho = \frac{1}{N(N-1)}\left(\boldsymbol{\iota}'\bar{\boldsymbol{C}}\boldsymbol{\iota} - N\right) = \frac{2}{N(N-1)}\sum_{i>j}\frac{u_{ij}}{\sqrt{u_{ii}u_{jj}}}, \tag{11}$$

where $\boldsymbol{\iota}$ is an N-vector of ones and u_{ij} is the (i, j)-th element of the $\bar{\boldsymbol{C}}$ matrix in (10). The equicorrelation matrix is then

$$\boldsymbol{C}^{ECO} = (1-\rho)\boldsymbol{I}_N + \rho\boldsymbol{J}_N, \tag{12}$$

where $\boldsymbol{J}_N$ is an $N \times N$ matrix of ones.

The reason for considering an equicorrelation structure is that it is well known that unrestricted sample correlations like those in (10) can produce notoriously noisy estimates. In an early study of portfolio selection rules, Elton and Gruber (1973) found that imposing all pairwise correlations to be equal to their average value had the effect of reducing the estimation risk and provided superior asset allocations when compared to a wide range of alternative assumptions. This noise reduction technique also forms the basis of Engle and Kelly's (2009) dynamic version of the equicorrelation model, discussed below.

2.5 DCC specification

The CCC model has a clear advantage of computational simplicity, but the assumption of constant conditional correlations in (9) may be too restrictive for practical portfolio management. The dynamic conditional correlation (DCC) model of Engle (2002) generalizes the CCC model by allowing the correlations to vary over time. It is defined as in (4) with the following correlation matrix:

$$\boldsymbol{C}_t^{DCC} = \text{diag}\left[\boldsymbol{Q}_t\right]^{-\frac{1}{2}} \boldsymbol{Q}_t \text{diag}\left[\boldsymbol{Q}_t\right]^{-\frac{1}{2}}, \tag{13}$$

where diag$[\boldsymbol{Q}_t]$ is a matrix with the same diagonal as $\boldsymbol{Q}_t$ and zero off-diagonal entries. The matrix of quasi-correlations $\boldsymbol{Q}_t$ evolves according to the GARCH-like recursion

$$\boldsymbol{Q}_t = (1-a-b)\bar{\boldsymbol{Q}} + a(\boldsymbol{u}_{t-1}\boldsymbol{u}'_{t-1}) + b\boldsymbol{Q}_{t-1}, \tag{14}$$

where the persistence parameters a and b are non-negative scalars satisfying $a+b<1$ to ensure stationarity of the process. Here we use correlation targeting in (14), as done in Engle (2002); i.e., we set $\bar{\boldsymbol{Q}} = T^{-1}\sum_{t=t}^{T}\boldsymbol{u}_\tau\boldsymbol{u}'_\tau$. So provided that $\boldsymbol{Q}_1$ is positive definite, each subsequent $\boldsymbol{Q}_t$ will also be positive definite (and hence invertible) since it is a weighted average of positive definite matrices. Note that the $\boldsymbol{Q}_t$'s need not be correlation matrices with unit diagonals. It is the rescaling in (13) that converts $\boldsymbol{Q}_t$ into $\boldsymbol{C}_t^{DCC}$, a well-defined correlation matrix for every t.

2.6 TVC specification

An important alternative specification to (13) is the time-varying correlation (TVC) model of Tse and Tsui (2002). There, the correlation dynamics are specified as an ARMA process of the form

$$\boldsymbol{C}_t^{TVC} = (1-a-b)\bar{\boldsymbol{C}} + a\boldsymbol{\Psi}_{t-1} + b\boldsymbol{C}_{t-1}^{TVC}, \tag{15}$$

where the $N \times N$ matrix $\boldsymbol{\Psi}_{t-1}$ is given by

$$\boldsymbol{\Psi}_{t-1} = \text{diag}\left[\frac{1}{m}\sum_{\tau=t-m}^{t-1}\boldsymbol{u}_\tau\boldsymbol{u}'_\tau\right]^{-\frac{1}{2}}\left[\frac{1}{m}\sum_{\tau=t-m}^{t-1}\boldsymbol{u}_\tau\boldsymbol{u}'_\tau\right]\text{diag}\left[\frac{1}{m}\sum_{\tau=t-m}^{t-1}\boldsymbol{u}_\tau\boldsymbol{u}'_\tau\right]^{-\frac{1}{2}}, \tag{16}$$

a rolling sample estimate of the correlation matrix based on standardized returns between times $t-m$ and $t-1$. A necessary condition for $\boldsymbol{\Psi}_{t-1}$ to be positive definite is that $m \geq N$. Here we follow Tse and Tsui (2002) and set $m = N$. The stationarity of $\boldsymbol{C}_t^{TVC}$ is imposed through the constraints $0 \leq a, b \leq 1$ and $a + b < 1$. These restrictions also imply that $\boldsymbol{C}_t^{TVC}$ is a convex combination of $\bar{\boldsymbol{C}}$ in (10), $\boldsymbol{\Psi}_{t-1}$, and $\boldsymbol{C}_{t-1}^{TVC}$. So if $\boldsymbol{C}_1^{TVC}$ is positive definite with unit diagonal elements, then, by recursion, all the successive $\boldsymbol{C}_t^{TVC}$'s will also be well-defined correlation matrices. An apparent difference between the DCC and TVC models is that the DCC dynamics of $\boldsymbol{Q}_t$ in (14) are based on the single lagged term $\boldsymbol{u}_{t-1}\boldsymbol{u}'_{t-1}$ (as in a standard GARCH(1,1) model), whereas the complete specification of the TVC dynamics in (15) depends on a and b, and m in (16).

2.7 DECO specification

A middle ground between the CCC model on one hand and the DCC and TVC models on the other is to maintain the assumption of dynamic correlations, but to restrict all pairwise correlations to be equal. Such an approach would be expected to work well when the pairwise correlations are dominated by a common factor. To examine that possibility, we follow Engle and Kelly (2009) and consider their DCC-based equicorrelation specification. That model uses the DCC matrix in (13) and then sets the equicorrelation parameter ρ_t equal to the average of the pairwise DCC correlations:

$$\rho_t = \frac{1}{N(N-1)}\left(\boldsymbol{\iota}'\boldsymbol{C}_t^{DCC}\boldsymbol{\iota} - N\right) = \frac{2}{N(N-1)}\sum_{i>j}\frac{q_{ij,t}}{\sqrt{q_{ii,t}q_{jj,t}}}, \tag{17}$$

where $q_{ij,t}$ is the (i,j)-th element of the $\boldsymbol{Q}_t$ matrix in (14). The DCC-based equicorrelation (DECO) matrix is then

$$\boldsymbol{C}_t^{DECO} = (1-\rho_t)\boldsymbol{I}_N + \rho_t\boldsymbol{J}_N, \tag{18}$$

where $\boldsymbol{J}_N$ is defined as in the case of the ECO model in (12). Engle and Kelly show that the transformation of $\boldsymbol{C}_t^{DCC}$ in (13) to the equicorrelation structure via (17) and (18) results in a positive definite $\boldsymbol{C}_t^{DECO}$ matrix.

2.8 Model estimation

Let $\boldsymbol{\theta}$ denote a generic parameter vector. In the basic CCC and IND specifications, that vector comprises 16 parameters $(\xi_1, ..., \xi_5, \upsilon, \kappa_1, \alpha_1, ..., \kappa_5, \alpha_5)$. The DCC, TVC, and DECO specifications add two more parameters (a, b) for a total of 18. Given the sample of asset returns $\boldsymbol{y}_1, ..., \boldsymbol{y}_T$, we estimate each model by maximizing its sample log-likelihood function $\mathcal{L}(\boldsymbol{\theta}) = \sum_{t=1}^{T} \log f(\boldsymbol{y}_t|\boldsymbol{\theta}, \mathcal{I}_{t-1})$ with respect to $\boldsymbol{\theta}$, where

$$f(\boldsymbol{y}_t|\boldsymbol{\theta}, \mathcal{I}_{t-1}) = |\boldsymbol{H}_t|^{-1/2} f\big(\boldsymbol{H}_t^{-1/2}(\boldsymbol{y}_t - \boldsymbol{\mu})|\boldsymbol{\theta}, \mathcal{I}_{t-1}\big), \tag{19}$$

subject to the model's positivity and stationarity constraints. The functional form of the multivariate density is given in (3), and the term $|\boldsymbol{H}_t|^{-1/2}$ in (19) is the Jacobian factor that arises in the transformation from $\boldsymbol{z}_t$ to $\boldsymbol{y}_t$. We should emphasize that the non-normal distribution used here does not allow the log-likelihood function to be decomposed as in Engle (2002), Engle and Sheppard (2005), and Engle and Kelly (2009). This means that we cannot employ Engle's two-step estimation approach, but instead we estimate the parameters of the multivariate skewed Student-t distribution together with those of the conditional variances and covariances processes in one step.[2]

3 Estimation results

Our empirical assessment uses daily data on five country ETFs, which are relatively new and already quite popular financial instruments. ETFs hold a fixed number of stocks based on an underlying basket of stocks and, just like common stocks, they are traded on exchanges and can be sold short or bought on margin by both retail and institutional investors. Their popularity stems in large part from the fact that they are low-cost, transparent, and tax-efficient investment vehicles for accessing exposure to the underlying stocks. Since ETFs are actively traded at all times during market hours, deviations of their price from the value of the underlying asset are quickly arbitraged away. Indeed,

[2] The computations were done in Fortran using IMSL. A quasi-Newton method with finite-difference gradients was used for the maximization of the sample log-likelihood function of each model.

a fundamental tenet of ETFs as an asset class is the close adherence of the price of the ETF to the value of it's underlying group of stocks.

Our first ETF is the Standard and Poor's (S&P) Depository Receipt (SPDR) ETF that tracks the S&P 500, a major US stock market index. The four other ETFs comprise Morgan Stanley Capital International (MSCI) iShares for the UK, Japan, Mexico, and Malaysia. These ETFs track their respective MSCI stock market indices, which represent broad aggregations of national equity markets and are the leading benchmarks for international portfolio managers. The five considered ETFs are traded on the New York Stock Exchange and their historical price data are publicly available; their respective ticker symbols are: SPY, EWU, EWJ, EWW, and EWM. Note that the US, UK, and Japan are obviously developed markets, but Mexico and Malaysia are considered emerging markets.

In international finance, the traditional argument for reaping the benefits of international diversification has relied on the presence of low cross-country correlations. While the early literature studied developed markets, our data allows us to follow more recent studies and also examine the benefits offered by emerging markets.[3] We use daily closing prices covering the period from April 1, 1996 to July 7, 2011 and with the price data we computed series of corresponding daily log-returns for each of the ETFs (3842 observations). We use log-returns for estimation purposes only, and then we use the simple returns to assess the portfolio strategies. Note however that at the daily frequency, log-returns are virtually identical to simple returns. Figure 1 plots the relative evolution of the daily ETF closing prices, where each series is divided by its first value so they all begin at a value of 1 dollar on April 1, 1996. It is interesting to observe the co-movements among these prices and how they rebounded after their spectacular plunge in 2008. Figure 2 shows the corresponding five return series multiplied by 100 so they can be read as percentage returns. We can see that volatility changes over time, and the presence of

[3]Early studies on the benefits of international diversification include Solnik (1974) for developed markets and Errunza (1977) for emerging markets. More recent evidence is found in DeSantis and Gerard (1997), Errunza, Hogan, and Hung (1999), Bekaert and Harvey (2000), and Christoffersen et al. (2010).

volatility clustering effects is evident in each series. Comparing Figures 1 and 2 we can also see that volatile periods are generally associated with drops in market values. The clearly noticeable drops in market values during the financial crisis of 2008 seen in Figure 1 along with the marked increases in volatility that can be observed in Figure 2 is the obvious case in point.

Table 1 presents some summary statistics of the percentage returns data. The top portion reports the means, standard deviations, maximum and minimum values, skewness, and kurtosis of each return series. We see that the daily returns are on average very close to zero over the entire sample period, but show wide fluctuations on both the up and downsides. The returns series for the western economies (US, UK, and Mexico) exhibit negative skewness, whereas the eastern ones (Japan and Malaysia) have much higher positive skewness. Moreover, we see that the kurtosis in every case is much higher than it would be if the returns were normally distributed (i.e. 3). The bottom portion of Table 1 displays the sample correlation matrix for these five return series. It is interesting to note the predominant role played by the US stock market. Indeed, each country ETF returns show the highest pairwise correlation vis-à-vis the US ETF returns.

Tables 2–7 show the maximum-likelihood parameter estimates of the IND, ECO, CCC, DCC, TVC, and DECO models, respectively, using the entire sample of returns data. We began by estimating the IND model and retained as final point estimates those that attained the highest value of the log-likelihood function over a grid of initial values. We then used the IND estimates as starting values for the estimation of the other models. In Tables 2–7, the numbers in parentheses are the standard errors associated with each point estimate.

In general, the models reveal that the marginal distributions of ETF return innovations are skewed to the left (since $\log \xi_i < 0$), except in the case of Japan under the ECO and DECO models and in the case of Malaysia under the ECO, CCC, DCC, and DECO models for which they appear more symmetric or even slightly right skewed. The degrees-of-freedom parameter estimate varies from about 4.5 under the IND model to about 8.5 under the TVC model, implying in each case tails far thicker than those of a normal innovation distribution. The GARCH parameter estimates are quite typical of what is usually found with daily returns; i.e., the weights on the long-run variances κ_i and the ARCH parameters α_i are quite small, while the GARCH parameters β_i are much closer to 1 in magnitude.

The estimated a and b parameters in Tables 5–7 determine the persistence over time of the respective model-implied time-varying conditional correlations. The DCC, TVC, and DECO models each imply a great deal of persistence, as evidenced by the values of $a + b$ all close to 1, which in turn implies very slow rates of mean-reversion in correlations. To illustrate these effects, Figures 3–5 show the model-implied correlations between the standardized returns for the five ETFs. The horizontal lines in each plot are the correlations implied by the estimated CCC model, which represent the average innovation correlations. The solid lines in Figures 3–5 are associated with the DCC model, the dashed lines with the TVC model, and the dotted line (which is the same in each plot) with the DECO model. The time-varying innovation correlations are unmistakable in each case. Note how the DCC and TVC models imply very similar patterns over time, while the DECO model exhibits quite a different pattern since it captures the average correlation across all pairs of assets. In general, we observe relative increases in correlations during the 2008 financial crisis. For instance, the US-Japan conditional correlation (in the upper right plot) in Figure 3 spikes up to its highest sample value (around 0.8) during that period. This finding of increasing correlations between stock market indices during volatile periods (typically bear markets) is in line with the inverse relationship between market value and correlations documented in Longin and Solnik (1995, 2001), Ang and Chen (2002), and Engle and Kelly (2009).

Recall that the decoupling hypothesis is the idea that recently the evolution of stock markets in emerging markets has "decoupled" itself from the evolution of more developed stock markets. This notion is clearly not supported by the evidence presented here. Indeed, the correlations between all pairs of indices—developed and emerging—are certainly erratic over short periods of time, but the overall picture that emerges from Figures 3–5 is that they have been trending upward, not downward, over the last decade. This would seem to contradict the notion that emerging stock markets are decoupling themselves from developed stock markets for long periods of time. Christoffersen et al. (2010) reach the same conclusion with weekly returns during the 1973–2009 period.[4]

Our focus, however, is not on the in-sample comparison of these models, but rather on their out-of-sample performance in portfolio management. In the application that follows, all the models, including the EWMA and mEWMA models, are used to produce forecasts for time $t+1$ using only information available up to time t, as would be done in real-time forecasting. For the forecasting exercise we set aside the last 1500 observations—about 6 years.[5] So at time t each model is used to produce the one-day-ahead forecasts $\boldsymbol{\mu}_{t+1|t}$ and $\boldsymbol{H}_{t+1|t}$. The first of these one-day-ahead forecasts is made on July 7, 2005 and every day the returns data available up to that point in time are used to update the forecasts of the next day. The model parameters are re-estimated every 20 trading days using the previous estimates as initial values for the numerical optimization. We iterate this procedure until the day before last in our return sample has been included in the forecasting window.

4 Application to portfolio management

4.1 Management strategies

We consider the problem faced by an active portfolio manager who rebalances a portfolio of ETFs on a daily basis. Here we begin by assuming the absence of a risk-free security, so the portfolio comprises only risky assets. The naive approach to this problem is simply to invest the initial wealth each day equally across the $N = 5$ risky assets under consideration, so the fraction of wealth in each asset is $1/N$. This is referred to as the

[4] Despite the evidence of recent upward tends in correlations, Christoffersen et al. (2010) argue that their findings of very low tail dependence at the end of their sample suggests that there are still benefits from adding emerging markets to a portfolio.

[5] This choice represents the tradeoff we faced between wanting to have a long out-of-sample evaluation period and enough in-sample observations for reliable model estimation.

naive diversification (or $1/N$) rule.[6] It should be noted that the $1/N$ rule doesn't entail any estimation risk since it relies neither on the data nor on any model. And nonetheless DeMiguel, Garlappi, and Uppal (2009) find that this portfolio strategy performs remarkably well in out-of-sample comparisons against more sophisticated approaches based on classical portfolio optimization using monthly returns data. Here we too use the naive $1/N$ rule as our benchmark for comparison purposes.

According to the classical theory of optimal portfolio selection by Markowitz (1952), the mean-variance manager allocates the wealth across the N risky assets so as to minimize the portfolio's variance subject to the constraint that the expected portfolio return attains a specified target, μ_t^*. Without loss of generality, the initial wealth can be normalized to 1 so the portfolio manager's problem is to find the optimal normalized portfolio weights, $\hat{\boldsymbol{\omega}}_t$, as the solution to:

$$\begin{aligned} \min_{\boldsymbol{\omega}_t} \quad & \boldsymbol{\omega}_t' \boldsymbol{H}_{t+1|t} \boldsymbol{\omega}_t \\ \text{s.t.} \quad & \boldsymbol{\omega}_t' \boldsymbol{\mu}_{t+1|t} = \mu_t^*, \\ & \boldsymbol{\omega}_t' \mathbf{1} = 1, \\ & \boldsymbol{\omega}_t \geq 0, \end{aligned} \tag{20}$$

where $\mathbf{1}$ is a vector of ones. The non-negativity constraints $\boldsymbol{\omega}_t \geq 0$ in this formulation of her problem mean that the portfolio manager is prohibited from making short sales. We shall also consider a version of (20) without those short-sale constraints so that the optimal solution, $\hat{\boldsymbol{\omega}}_t$, may contain negative weights (short positions). The portfolio optimization problem in (20) is a standard quadratic programming problem that is readily solved numerically.

In this formulation of the portfolio problem, the manager only allocates wealth to a set of N risky assets. We also consider the case where she has access to a risk-free asset with a zero rate of return, which seems quite realistic given that the portfolio is rebalanced on a daily basis. In the presence of a risk-free asset, the constraint $\boldsymbol{\omega}_t' \mathbf{1} = 1$ is dropped from (20) so the portfolio weights need not sum to one; i.e., $1 - \boldsymbol{\omega}_t' \mathbf{1}$ is the share in the

[6] When we include a safe asset in the investment mix, the naive portfolio rule is in fact $1/(N+1)$ even though we simply talk about the "$1/N$ rule."

risk-free asset. With a zero rate of return, the refuge asset here can be thought of as cash holdings in a bank account. In the reported results, we refer to the portfolios that solve (20) with a specified target μ_t^* as minimum variance (MV) portfolios. Finally, we also consider the special case of the manager restricted to risky assets only and who wishes to minimize the portfolio's variance without a specified target μ_t^*. The corresponding global minimum variance portfolio (GMV) is the solution to the portfolio problem in (20) without the constraint $\boldsymbol{\omega}_t'\boldsymbol{\mu}_{t+1|t} = \mu_t^*$. We consider GMV portfolios with and without short sales. Even though they are somewhat peculiar, these GMV portfolios have the great advantage of taking expected returns out of the optimization problem, allowing a sharp focus on estimation of the covariance matrix.

The quantities that the manager needs to input into (20) are the forecasts $\boldsymbol{\mu}_{t+1|t}$ and $\boldsymbol{H}_{t+1|t}$. The so-called "plug-in" approach simply computes the sample mean $\bar{\boldsymbol{y}}_t$ and covariance matrix of asset returns up to time t and uses those as the inputs to (20). That approach obviously uses the data, but just like the $1/N$ rule it remains model-free. The common plug-in approach is included in our comparisons. The model-based approaches that we consider are those presented in Section 2. Note that the implied forecast $\boldsymbol{\mu}_{t+1|t}$ in each case is the same as that of the plug-in approach: $\bar{\boldsymbol{y}}_t$. So the plug-in, EWMA, IND, ECO, mEWMA, CCC, DCC, TVC, and DECO approaches differ only by their forecasts of the conditional covariance matrix, $\boldsymbol{H}_{t+1|t}$.

The fact that $\boldsymbol{H}_{t+1|t}$ is an estimated quantity gives rise to estimation risk (owing to the uncertainty about the data-generating process) and that risk becomes particularly important when the cost of rebalancing the portfolio is taken into account. Indeed, if transactions are costly, then any attempt to improve asset allocation that leads to an increase in portfolio turnover can worsen the after-transactions-costs portfolio performance. In order to get a sense of the amount of trading required by each portfolio approach, we follow DeMiguel, Garlappi, and Uppal (2009) and Kirby and Ostdiek (2011) and compute the portfolio turnover at time $t+1$, defined as the sum of the absolute values of the trades across the N assets. More specifically, notice that if we let $R_{i,t+1}$ denote the simple return between times t and $t+1$ on asset i, then for each dollar invested in the portfolio at time

t there is $\hat{\omega}_{i,t}(1+R_{i,t+1})$ dollars invested in asset i at time $t+1$. So the share of wealth in risky asset i before the portfolio is rebalanced at time $t+1$ is

$$\hat{\omega}_{i,t^+} = \frac{\hat{\omega}_{i,t}(1+R_{i,t+1})}{\sum_{i=1}^{N}\hat{\omega}_{i,t}(1+R_{i,t+1}) + (1-\sum_{i=1}^{N}\hat{\omega}_{i,t})(1+R_{f,t+1})}$$

and when the portfolio is rebalanced it gives rise to a trade in risky asset i of magnitude $|\hat{\omega}_{i,t+1} - \hat{\omega}_{i,t^+}|$, where $\hat{\omega}_{i,t+1}$ is the optimal portfolio weight on asset i at time $t+1$ (after rebalancing). Here the risk-free return is assumed to be zero; i.e., $R_{f,t+1} = 0$ in each period. So the total amount of turnover (or churning) across all assets in the portfolio is

$$\tau_{t+1} = \sum_{i=1}^{N} |\hat{\omega}_{i,t+1} - \hat{\omega}_{i,t^+}| + \Big|\sum_{i=1}^{N}(\hat{\omega}_{i,t+1} - \hat{\omega}_{i,t^+})\Big|, \tag{21}$$

where the second term on the right-hand side appears only in the presence of a risk-free asset. We shall gauge the magnitude of the turnover measure in (21) relative to its value under the benchmark $1/N$ rule.

It is important to remark that with the naive strategy, $\omega_{i,t} = \omega_{i,t+1} = 1/N$, but ω_{i,t^+} may be different owing to changes in asset prices between times t and $t+1$. If c denotes the proportional transactions cost, then the total cost to rebalance the portfolio is $c \times \tau_{t+1}$. Let $R_{p,t+1} = \sum_{i=1}^{N} R_{i,t+1}\hat{\omega}_{i,t}$ denote the portfolio return from a given strategy before rebalancing occurs. The evolution of wealth invested according to that strategy is then given by

$$W_{t+1} = W_t(1+R_{p,t+1})(1-c\times\tau_{t+1}) \tag{22}$$

and the simple return net of rebalancing costs is $R^c_{p,t+1} = W_{t+1}/W_t - 1$. Since the portfolio $\boldsymbol{\omega}_t$ is formed using only information available at time t and held for one day before being rebalanced at time $t+1$, the return $R^c_{p,t+1}$ represents the one-day *out-of-sample* return. We report portfolio performance results when there are no transactions costs and assuming proportional transactions costs of 1 basis point (bp) for c in (22). Recall that 1 bp $= 10^{-4}$.

The target μ^*_t in (20) needs to be specified. Kirby and Ostdiek (2010) argue that the amount of turnover in the portfolio is very sensitive to the selected target value. So here we follow those authors and set μ^*_t equal to the estimated conditional expected return of the benchmark $1/N$ portfolio; i.e., $\mu^*_t = \bar{\boldsymbol{y}}'_t\mathbf{1}/N$, or $\mu^*_t = \bar{\boldsymbol{y}}'_t\mathbf{1}/(N+1)$ when a safe asset is included. The $1/N$ portfolio is expected to have relatively low turnover, so this choice levels the playing field between the naive approach, the plug-in approach, and the eight model-based approaches. Figure 6 shows the time-series plot of the daily portfolio target returns (in basis points) on each day from July 21, 2005 to July 5, 2011 that the portfolio is rebalanced. We see that the target return values vary between about 0.15 and 3 bps (or between 0.37 and 7.5% in annual terms).

It remains to discuss how we evaluate portfolio performance. For that we choose the out-of-sample realized Sharpe ratio because it is the most ubiquitous risk-adjusted measure used by financial market practitioners to rank fund managers and to evaluate the attractiveness of investment strategies in general. We use an expanding-window procedure to compare the portfolio performances. Let T denote the total number of returns under consideration in the data set and let t_1 be the first day of portfolio formation. With daily rebalancing, we obtain a time-series of out-of-sample returns $R^c_{p,t}$ for $t = t_1 + 1, ..., T$. In our empirical assessments, we consider three out-of-sample periods: (i) July 22, 2005 to July 6, 2011, (ii) July 22, 2005 to July 15, 2008, and (iii) July 16, 2008 to July 6, 2011. The first of those represents the entire out-of-sample evaluation period with 1499 returns, and the second and third periods each comprise 749 return observations.[7] As in Brownless, Engle, and Kelly (2009), we consider September 2008—the month in which Lehman Brothers filed for Chapter 11 bankruptcy protection—as the beginning of the financial crisis, so the period from July 22, 2005 to July 15, 2008 represents our pre-crisis subsample. The Sharpe ratio of strategy j is then computed as $\text{SR}_j = \bar{R}^c_{p,j}/\sqrt{\hat{\sigma}^2_j}$, where $\bar{R}^c_{p,j}$ is the average of the returns to strategy j computed over $t_1 + 1, ..., T$ and $\hat{\sigma}^2_j$ is the corresponding out-of-sample variance.[8] We also report the average portfolio turnover $\bar{\tau}_j$, computed as the out-of-sample average of (21) for each strategy.

The Sharpe ratio works well as a performance gauge when excess returns are positive. In that case, higher the Sharpe ratio, the better; and at a given level of excess return,

[7]Note that one observation is lost when computing $R^c_{p,t}$, the returns net of rebalancing costs.

[8]When the portfolio comprises risky assets only, this quantity is sometimes referred to as the information ratio in the portfolio management literature.

lower the standard deviation of return, the better. When excess returns are negative, however, the Sharpe ratio yields unreasonable performance rankings. For instance, consider two portfolios that achieve the same negative excess return, but with different standard deviations. According to the usual Sharpe ratio, the one with the larger standard deviation would be ranked higher! In recognition of this shortcoming, Israelsen (2003, 2005) proposes to modify the Sharpe ratio as follows:

$$\text{SR-m}_j = \begin{cases} \bar{R}^c_{p,j}/\sqrt{\hat{\sigma}^2}, & \text{if } \bar{R}^c_{p,j} \geq 0, \\ \bar{R}^c_{p,j} \times \sqrt{\hat{\sigma}^2}, & \text{if } \bar{R}^c_{p,j} < 0. \end{cases}$$

So when returns are negative, a portfolio with a smaller standard deviation gets ranked higher than one that took more risk. Whereas when returns are positive, the Sharpe ratio and its modified version yield the same performance ranking. The modified Sharpe ratio is thus entirely consistent with the basic principle of risk-adjusted returns, which is that higher risk is only preferable if accompanied by higher (net) return.[9]

To assess the statistical significance of the differences between the Sharpe ratio of the benchmark $1/N$ strategy ($\text{SR}_{1/N}$) and those of the plug-in and model-based strategies, we use a bootstrap inference method. Specifically, we test the equality of Sharpe ratios according to the block bootstrap proposed in Ledoit and Wolf (2008) which is designed to accommodate serially correlated and heteroskedastic time series of returns. The null hypothesis is $H_0 : \text{SR}_j - \text{SR}_{1/N} = 0$ for which we compute a two-sided p-value using Ledoit and Wolf's studentized circular block bootstrap with block size equal to 20 and 1000 bootstrap replications.[10] Following the suggestion in Ledoit and Wolf (2008), we use the same methodology to test for statistical differences between the modified Sharpe ratios and the variances (or standard deviations) of returns.

[9] As Israelsen (2005) notes, there is an odd feature of the modified Sharpe ratio: its magnitude can be quite large. So its interest here is mainly as a ranking criterion.

[10] We also tried other block sizes and found the results to be essentially the same as those reported here.

4.2 Portfolio performance results

Tables 8 and 9 report the portfolio performance results over the entire out-of-sample period (July 22, 2005 to July 6, 2011) when the portfolio comprises only risky assets (Table 8) and when a risk-free security is part of the asset mix (Table 9). Tables 10 and 11 show the corresponding results over the pre-crisis period from July 22, 2005 to July 15, 2008, and Tables 12 and 13 pertain to the later period from July 16, 2008 to July 6, 2011. The results for the benchmark $1/N$ strategy are given in the leading row of each table. When the investment mix comprises risky assets only (Tables 8, 10, and 13), the results for the MV portfolios are shown in Panel A while the GMV portfolio results are given in Panel B of the tables. Note that Tables 9, 11, and 13 only report MV portfolio results, since the GMV solution in those cases is the trivial portfolio with 100% in the risk-free asset and no risky positions. Within the panels, the rows refer to the plug-in and model-based strategies, with and without short selling. Note also that for the portfolios of risky assets only (again Tables 8, 10, and 13), the short selling constraints never bind for the EWMA and IND strategies. This follows by construction since those models assume that the risky assets are uncorrelated. The reported out-of-sample portfolio results include the annualized percentage mean return, the annualized standard deviation (Std Dev) of returns, the annualized Sharpe ratio (SR), the annualized modified Sharpe ratio (SR-m), and the average turnover (Turn) over the out-of-sample period.[11] The entries in the columns labeled "p-val" are respectively the two-sided bootstrap p-values for tests of the equality of the strategy's variance (or equivalently of its standard deviation), Sharpe ratio, and modified Sharpe ratio with those of the benchmark $1/N$ rule. Finally, each table shows the results when there are no transactions costs and assuming proportional transactions costs of 1 bp.

Table 8 shows that when the portfolio comprises risky assets only over the entire out-of-sample period, the plug-in and the model-based strategies achieve significantly lower

[11] The annualized mean return is computed here as the daily mean return times 252 and the annualized standard deviation of returns, SR, and SR-m are obtained by multiplying their daily counterparts by the square root of 252.

portfolio variances than the $1/N$ rule at least in the absence of transactions costs (p-values < 0.10). The one noticeable exception is the mEWMA when short sales are allowed with p-values of 0.14 in Panel A and 0.11 in Panel B. When a 1 bp transactions cost is introduced, however, the plug-in strategy ceases to yield significantly lower portfolio variances in all cases. In fact, it is only the IND strategy which continues to yield lower portfolio variances in every case. Indeed all the other model-based strategies do not consistently perform better than the benchmark in terms of portfolio variance, even under the GMV approach. Table 8 also shows that the plug-in and model-based strategies yield worse Sharpe and modified Sharpe ratios than the $1/N$ rule across the board. This is not surprising for the MV portfolios given the usual tradeoff one would expect between a portfolio's mean return (reward) and its standard deviation (risk), at least in the absence of transactions costs. In many instances, these performance measures are significantly worse than the benchmark. We see that the effects of short sale restrictions can be quite important. For the MV portfolios in Panel A, as well as the GMV ones in Panel B, prohibiting short selling makes the SR and SR-m measures not significantly lower than what is achieved through naive diversification.

These results are in line with the findings of Jagannathan and Ma (2003) who show that imposing a short-selling constraint amounts to shrinking the extreme elements of the covariance matrix and thereby stabilizes the portfolio weights. As in DeMiguel, Garlappi, and Uppal (2009) though, we find that even restraining short sales is not sufficient to completely mitigate the error in estimating the covariance matrix and thus to provide better portfolio Sharpe ratios than the naive $1/N$ rule, which ignores the data altogether.

Table 8 already shows how the usual Sharpe ratio can sometimes be misleading. For instance, the plug-in strategy with a 1 bp transactions cost has a SR of -0.28 and as such would be ranked better than the DCC strategy with a SR of -0.49. But in fact the negative returns to both strategies are about the same, and we see that the plug-in strategy with a standard deviation of 34.15 was far riskier than the DCC one with a standard deviation of 20.37. In that case, the modified Sharpe ratio correctly ranks the DCC-based strategy above the plug-in approach.

The portfolio performance results change dramatically as soon as the manager has access to a risk-free asset, even if it yields a zero return. Comparing Panel A of Table 8 with Table 9, we clearly see that the introduction of a risk-free asset in the investment mix results in portfolio variances that are significantly lower than the benchmark. Indeed, the p-values *never* exceed 1%. Furthermore, in the presence of a risk-free asset, the Sharpe ratios delivered by the plug-in and model-based strategies continue to be lower than that of the $1/N$ rule, but the differences are unmistakably nowhere statistically significant. The only apparent exceptions occur with the EWMA and IND strategies in the presence of a transactions cost (p-values ≤ 0.11), which shows the importance of capturing conditional correlations when forecasting future returns. The following quote from Amenc and Martellini (2011) summarizes well this finding about the risk-free asset:

> Any attempt at "improving" portfolio diversification techniques, either by introducing sophisticated time- and state-dependent risk models, or by extending them to higher-order moments, is also equally misleading if the goal is again to hope for protection in 2008-like market conditions. When there is simply no place to hide, even the most sophisticated portfolio diversification techniques are expected to fail.

Table 8 shows what happens when "there is no place to hide" (i.e. when the portfolio manager can only take risky positions) and Table 9 shows that even if the risk-free rate of return is zero, having access to a safe-haven investment vehicle provides valuable protection against downside risk. Indeed, the fact that the statistically significant negative performance measures in Table 8 cease to be significantly different vis-à-vis the $1/N$ rule (in Table 9) once the constraint $\boldsymbol{\omega}_t'\mathbf{1} = 1$ is dropped from (20) is truly remarkable.[12] It is also interesting to compare in Tables 8 and 9 the effects of having access to a risk-free asset on portfolio stability. We already saw in Table 8 that prohibiting short sales generally

[12]Note that relaxing the constraint is the main effect here, since the two series of portfolio target returns (in Figure 6) are very close. We further confirmed this by performing the portfolio exercises with a risk-free asset in the mix (so N=6) but targeting the return implied by the 1/5 rule. The results were virtually identical to those reported in Tables 9, 11, and 13.

reduces portfolio turnover. Looking now at Table 9 we see that access to a risk-free asset has an even greater effect. In every instance the portfolio turnover is reduced to a value less than 1, far below the 1.67 turnover value of the $1/N$ strategy. This in turn explains the relative portfolio performance improvements seen in Table 9 when transactions are costly.

Tables 10 and 11 show the corresponding analysis during our pre-crisis period from July 22, 2005 to July 15, 2008. During this period the performance of the $1/N$ rule is even better, with an annualized Sharpe ratio of 0.66 in the absence of transactions costs in Tables 10 and 11, and of 0.14 (0.38) with a 1 bp transactions cost in Table 10 (Table 11). These can be attributed to the lower variance of returns during this period. This relatively calmer period also translates into greater variance and covariance predictability, as evidenced by the significantly lower portfolio variances of the plug-in and model-based strategies. Without transactions costs, the corresponding p-values are all less than 0.01 (reported as zero in the tables). Again not surprisingly this translates into lower portfolio mean returns and significantly lower SR and SR-m performance measures. As in Table 8 though, these findings are not robust for all the strategies in the face of transactions costs. The significantly lower portfolio variances (p-values $< 1\%$) and the often times not significantly worse SR and SR-m measures vis-à-vis the $1/N$ rule are re-established in Table 11 by allowing the manager to take risk-free positions.

The results for the crisis period from July 16, 2008 to July 6, 2011 are reported in Tables 12 and 13. As we would expect, we see from those tables that the performance of the $1/N$ portfolio strategy deteriorates during the financial crisis. This, however, is not mainly due to lower mean returns but rather to their increased variances and correlations. Indeed, comparing Tables 10 and 12 we see that the mean return on the $1/N$ portfolio remains about the same but its standard deviation increases from 17.54 to 30.02. The risk-return tradeoff becomes evident in Table 12. Again comparing Tables 10 and 12 we see during the crisis period nearly doubled portfolio standard deviations and this is accompanied by higher portfolio mean returns across all strategies, at least when there are no transactions costs. That relationship, however, gets eroded when transactions are costly. With even just a 1 bp transactions cost, many portfolios have higher variances and lower mean returns during the crisis.

In Table 12 we notice that the plug-in and model-based strategies do not achieve significantly lower portfolio variances owing to the fact that return variances and correlations are less predictable in turbulent times. In sharp contrast to what we see in Tables 8 and 10, the model-based portfolio strategies are clearly not significantly different from those of the $1/N$ rule. And this holds under both the MV and GMV approaches, with and without transactions costs, and whether or not short sales are allowed. In some cases, even though not significantly so, we see the model-based strategies yielding higher Sharpe ratios than the naive portfolio with a SR of 0.12. The only exception occurs when transactions are costly and that is the CCC portfolio whose better SR performance of 0.31 achieves a p-value of 0.07 under the MV approach without short selling. In that same column we also see two other low p-values of 0.09 and 0.07 for the mEWMA and DCC strategies, but those are associated with negative Sharpe ratios under the GMV approach which ignores expected returns.

So what do we learn from all these portfolio performance comparisons? For starters, the significantly poorer performance of the model-based strategies over the entire out-of-sample period (from July 22, 2005 to July 6, 2011) in Table 8 is driven by the pre-crisis subsample ending on July 15, 2008 (Table 10). Indeed from July 16, 2008 to July 6, 2011, the model-based strategies are generally not statistically different from the $1/N$ rule (Tables 12 and 13). Recall also that the portfolio strategies are extremely sensitive to whether a risk-free asset is available. Comparing Table 8 versus 9, and 10 versus 11, shows that this constraint plays a far more important role than the short-sale constraints. Indeed, the effects of limiting short sales on the statistical significance of the portfolio performances is tiny compared to the effects of relaxing the constraint that the portfolio weights (of the risky assets) sum to one, which changes the performance results from much worse to not statistically different from the $1/N$ rule. This is not to say that limiting the amount of short selling is not important. In all the tables presented here, we see that prohibiting short sales generally yields more stable portfolios with lower turnover. The ability to shelter funds plays an additional role, allowing the manager to significantly lower the portfolio variance no matter which strategy is employed. Table 13 shows how this becomes even more important during high-turbulence crisis periods.

To better understand the role of the risk-free asset here, recall that our portfolio manager is targeting the $1/N$ expected return and can get there ex ante either on the capital market line (CML) or on the frontier of risk assets only. Suppose for a moment that the active manager cannot take short positions. Is she then better off hitting the target expected return across risky and risk-free assets rather than just the risky assets? When positive returns are expected, it is better ex ante to be on the CML as this has the maximum Sharpe ratio. Our results show that, relative to the $1/N$ rule, the ex post benefit is a significantly lower standard deviation, even though the ex post Sharpe ratio itself is not significantly higher.

5 Conclusion

We have conducted an empirical assessment of alternative time-series models for conditional variances and correlations for the purpose of active portfolio management with daily rebalancing. In addition to the usual plug-in method which simply replaces the covariance matrix by its sample counterpart, we also considered the RiskMetrics EWMA models and several other popular models to forecast the inputs to the portfolio selection problem. We considered portfolios of five country ETFs which results in a total of 18 parameters that need to be estimated for the most sophisticated correlation models we consider, even though we make use of correlation targeting to reduce the dimensionality problem. Following DeMiguel, Gralppi, and Uppal (2009) and Kirby and Ostdiek (2011), the $1/N$ rule serves as our benchmark strategy for comparison purposes and, as in Kirby and Ostdiek, the conditional expected return of the $1/N$ rule is used as the target expected portfolio return. We also considered minimum variance portfolios without an expected return target.

Our empirical assessment reveals that in the three years leading up to the recent financial crisis, the model-based strategies were in general statistically better ex-post in terms of portfolio variance than the $1/N$ rule when the portfolio manager could only take risky positions. During the financial crisis, the portfolio variances can not be statistically distinguished from the benchmark. All that changes when a risk-free asset becomes part of the investment mix. Indeed, before and during the crisis it is the presence of the safe-haven asset that allows the plug-in and model-based strategies to achieve significantly lower portfolio variances. This helps explain the good performance results in Serban et al. (1997) and Engle and Colacito (2006), for instance, who also consider some models of variances and correlations for asset allocation, but always with a risk-free asset present.

Our results with the $1/N$ rule as the benchmark provide further evidence that the desire to elaborate highly parametrized multivariate conditional heteroskedasticity and correlation models is necessarily accompanied by greater estimation risk. And that parameter uncertainty can easily translate into quite poor out-of-sample risk-adjusted portfolio performances, even during relatively tranquil markets. Indeed, in the absence of transactions costs our results reveal a clear risk-return tradeoff in the sense that the lower portfolio variances achieved through the model-based strategies are accompanied by lower mean returns. So the models do not deliver significantly better Sharpe ratios than does naive diversification. Even a very small 1 bp transactions cost is enough to break down the risk-return tradeoff and turn negative the Sharpe ratios for the plug-in and model-based strategies. This sheds more light on the fragility of mean-variance optimizing portfolios. The evidence presented here clearly show the relative benefits of limiting short sales to stabilize the portfolio weights and the far greater importance for the portfolio manager of being able to take a risk-free position, even if it yields a zero return. This is an interesting result with practical implications because often times institutional portfolio managers are constrained to be fully invested in their asset classes, without the flexibility of turning to the risk-free asset. Of course, retail investors already have that flexibility.

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Table 1: Summary statistics of daily country ETF log-returns

	Developed markets			Emerging markets	
	US	UK	Japan	Mexico	Malaysia
Mean	0.025	0.021	-0.006	0.053	0.015
Std Dev	1.327	1.597	1.658	2.131	2.174
Max.	13.552	15.771	14.660	19.460	17.526
Min.	-10.366	-12.839	-10.945	-18.721	-13.502
Skewness	-0.038	-0.132	0.342	-0.007	0.460
Kurtosis	11.902	11.557	8.917	11.265	11.684
			Correlation matrix		
US	1				
UK	0.718	1			
Japan	0.597	0.555	1		
Mexico	0.670	0.584	0.501	1	
Malaysia	0.381	0.353	0.351	0.377	1

Notes: The data consists of daily log-returns (in percentages) on country ETFs for the US, UK, Japan, Mexico, and Malaysia. The first three of those are developed markets, whereas the last two are considered emerging markets. The entire sample period comprises 3842 return observations from April 2, 1996 to July 6, 2011.

Table 2: Parameter estimates of the IND model

	Developed markets			Emerging markets	
	US	UK	Japan	Mexico	Malaysia
Shape parameters					
ξ_i	0.9134	0.9753	0.9834	0.9187	0.9946
	(0.0370)	(0.0095)	(0.0324)	(0.0124)	(0.0084)
Degrees-of-freedom parameter					
v	4.5740				
	(0.1893)				
GARCH parameters					
κ_i	0.0075	0.0076	0.0152	0.0149	0.0050
	(0.0046)	(0.0041)	(0.0087)	(0.0078)	(0.0019)
α_i	0.0722	0.0503	0.0602	0.0636	0.0592
	(0.0135)	(0.0087)	(0.0127)	(0.0167)	(0.0159)
β_i	0.9202	0.9419	0.9245	0.9214	0.9357
	(0.0098)	(0.0057)	(0.0065)	(0.0050)	(0.0024)
Log-likelihood	55328.0				

Notes: This table shows the maximum-likelihood parameter estimates of the independent correlation structure model based on the entire sample of returns data. The numbers in parentheses are standard errors.

Table 3: Parameter estimates of the ECO model

	Developed markets			Emerging markets	
	US	UK	Japan	Mexico	Malaysia
Shape parameters					
ξ_i	0.8903	0.9677	1.0096	0.9361	1.0057
	(0.0204)	(0.0256)	(0.0245)	(0.0228)	(0.0209)
Degrees-of-freedom parameter					
v	6.4947				
	(0.3657)				
GARCH parameters					
κ_i	0.0053	0.0075	0.0122	0.0088	0.0035
	(0.0059)	(0.0074)	(0.0076)	(0.0004)	(0.0011)
α_i	0.0593	0.0526	0.0569	0.0540	0.0453
	(0.0103)	(0.0089)	(0.0088)	(0.0094)	(0.0080)
β_i	0.9352	0.9397	0.9308	0.9371	0.9511
	(0.0089)	(0.0093)	(0.0097)	(0.0094)	(0.0067)
Log-likelihood	57756.8				

Notes: This table shows the maximum-likelihood parameter estimates of the equicorrelation model based on the entire sample of returns data. The numbers in parentheses are standard errors.

Table 4: Parameter estimates of the CCC model

	Developed markets			Emerging markets	
	US	UK	Japan	Mexico	Malaysia
Shape parameters					
ξ_i	0.8952	0.9723	0.9981	0.9510	1.0134
	(0.0198)	(0.0216)	(0.0224)	(0.0211)	(0.0231)
Degrees-of-freedom parameter					
v	6.9063				
	(0.4610)				
GARCH parameters					
κ_i	0.0070	0.0086	0.0125	0.0103	0.0031
	(0.0017)	(0.0053)	(0.0057)	(0.0035)	(0.0029)
α_i	0.0564	0.0516	0.0612	0.0552	0.0509
	(0.0066)	(0.0050)	(0.0052)	(0.0055)	(0.0052)
β_i	0.9364	0.9397	0.9261	0.9344	0.9459
	(0.0094)	(0.0096)	(0.0102)	(0.0099)	(0.0073)
Log-likelihood	58012.6				

Notes: This table shows the maximum-likelihood parameter estimates of the constant conditional correlation model based on the entire sample of returns data. The numbers in parentheses are standard errors.

Table 5: Parameter estimates of the DCC model

	Developed markets			Emerging markets	
	US	UK	Japan	Mexico	Malaysia
Shape parameters					
ξ_i	0.9020	0.9812	0.9891	0.9461	1.0284
	(0.0216)	(0.0248)	(0.0245)	(0.0235)	(0.0249)
Degrees-of-freedom parameter					
v	7.0141				
	(0.4791)				
GARCH parameters					
κ_i	0.0056	0.0057	0.0117	0.0076	0.0030
	(0.0046)	(0.0061)	(0.0016)	(0.0047)	(0.0018)
α_i	0.0453	0.0392	0.0512	0.0466	0.0465
	(0.0134)	(0.0115)	(0.0112)	(0.0126)	(0.0101)
β_i	0.9489	0.9549	0.9369	0.9456	0.9504
	(0.0197)	(0.0194)	(0.0181)	(0.0199)	(0.0140)
DCC persistence parameters					
a	0.0096				
	(0.0055)				
b	0.9895				
	(0.0054)				
Log-likelihood	58601.3				

Notes: This table shows the maximum-likelihood parameter estimates of the dynamic conditional correlation model based on the entire sample of returns data. The numbers in parentheses are standard errors.

Table 6: Parameter estimates of the TVC model

	Developed markets			Emerging markets	
	US	UK	Japan	Mexico	Malaysia
Shape parameters					
ξ_i	0.9966	0.9988	0.9973	0.9986	0.9988
	(0.0230)	(0.0264)	(0.0225)	(0.0237)	(0.0208)
Degrees-of-freedom parameter					
v	8.4999				
	(0.6648)				
GARCH parameters					
κ_i	0.0051	0.0058	0.0099	0.0072	0.0025
	(0.0006)	(0.0005)	(0.0088)	(0.0013)	(0.0011)
α_i	0.0578	0.0535	0.0588	0.0554	0.0545
	(0.0054)	(0.0010)	(0.0033)	(0.0045)	(0.0027)
β_i	0.9370	0.9405	0.9312	0.9372	0.9429
	(0.0053)	(0.0054)	(0.0063)	(0.0057)	(0.0044)
TVC persistence parameters					
a	0.0128				
	(0.0221)				
b	0.9867				
	(0.0327)				
Log-likelihood	58539.9				

Notes: This table shows the maximum-likelihood parameter estimates of the time-varying correlation model based on the entire sample of returns data. The numbers in parentheses are standard errors.

Table 7: Parameter estimates of the DECO model

	Developed markets			Emerging markets	
	US	UK	Japan	Mexico	Malaysia
Shape parameters					
ξ_i	0.8912	0.9682	1.0096	0.9364	1.0067
	(0.0207)	(0.0266)	(0.0244)	(0.0241)	(0.0216)
Degrees-of-freedom parameter					
v	6.4947				
	(0.3185)				
GARCH parameters					
κ_i	0.0056	0.0073	0.0129	0.0084	0.0041
	(0.0016)	(0.0022)	(0.0135)	(0.0022)	(0.0011)
α_i	0.0601	0.0516	0.0549	0.0521	0.0464
	(0.0094)	(0.0079)	(0.0075)	(0.0086)	(0.0073)
β_i	0.9343	0.9410	0.9320	0.9394	0.9493
	(0.0136)	(0.0141)	(0.0142)	(0.0142)	(0.0111)
DECO persistence parameters					
a	0.0246				
	(0.0221)				
b	0.9750				
	(0.0110)				
Log-likelihood	58291.9				

Notes: This table shows the maximum-likelihood parameter estimates of the dynamic equicorrelation model based on the entire sample of returns data. The numbers in parentheses are standard errors.

Table 8: Portfolio of risky assets only: July 22, 2005 to July 6, 2011

	No transactions costs								Transactions costs = 1 bp						
Strategy	Mean	Std Dev	p-val	SR	p-val	SR-m	p-val	Turn	Mean	Std Dev	p-val	SR	p-val	SR-m	p-val
$1/N$	11.08	24.56	1.00	0.45	1.00	0.45	1.00	3.25	2.87	24.72	1.00	0.11	1.00	0.11	1.00
Panel A: MV portfolios															
Short sales allowed															
Plug-in	6.75	23.28	0.00	0.29	0.07	0.29	0.07	6.49	-9.62	34.15	0.44	-0.28	0.30	-1.30	0.55
EWMA	8.12	23.21	0.01	0.35	0.01	0.35	0.01	3.92	-1.76	24.52	0.82	-0.07	0.25	-0.17	0.37
IND	8.36	23.33	0.00	0.36	0.02	0.36	0.02	2.39	2.31	23.49	0.00	0.09	0.83	0.09	0.83
ECO	5.37	21.87	0.01	0.24	0.02	0.24	0.02	2.59	-1.16	22.10	0.00	-0.05	0.13	-0.10	0.28
mEWMA	2.31	19.70	0.14	0.11	0.29	0.11	0.29	9.16	-20.80	25.43	0.89	-0.81	0.01	-2.10	0.11
CCC	6.18	21.76	0.03	0.28	0.06	0.28	0.06	2.37	0.20	21.84	0.02	0.01	0.35	0.01	0.35
DCC	2.31	20.11	0.05	0.11	0.04	0.11	0.04	4.89	-10.03	20.37	0.06	-0.49	0.00	-0.81	0.02
TVC	0.74	20.25	0.03	0.03	0.01	0.03	0.01	3.97	-9.27	20.41	0.03	-0.45	0.00	-0.75	0.01
DECO	0.55	20.20	0.08	0.02	0.01	0.02	0.01	6.03	-14.66	21.70	0.03	-0.67	0.00	-1.26	0.02
Short sales prohibited															
Plug-in	7.67	23.43	0.00	0.32	0.12	0.32	0.12	3.61	-1.43	24.45	0.67	-0.05	0.27	-0.13	0.36
EWMA	8.12	23.21	0.01	0.35	0.01	0.35	0.01	3.92	-1.76	24.52	0.82	-0.07	0.25	-0.17	0.37
IND	8.36	23.33	0.00	0.36	0.02	0.36	0.02	2.39	2.31	23.49	0.00	0.09	0.83	0.09	0.83
ECO	6.81	22.12	0.01	0.30	0.06	0.30	0.06	2.93	-0.59	22.45	0.00	-0.02	0.22	-0.05	0.35
mEWMA	5.27	21.58	0.01	0.24	0.07	0.24	0.07	1.80	0.73	21.64	0.01	0.03	0.52	0.03	0.52
CCC	7.38	22.02	0.02	0.33	0.14	0.33	0.14	1.97	2.41	22.11	0.01	0.11	0.95	0.11	0.95
DCC	6.17	21.62	0.02	0.28	0.09	0.28	0.09	2.32	0.31	22.00	0.00	0.01	0.43	0.01	0.43
TVC	5.51	21.61	0.02	0.25	0.05	0.25	0.05	1.96	0.55	21.77	0.01	0.02	0.45	0.02	0.45
DECO	5.59	21.59	0.03	0.26	0.05	0.26	0.05	2.97	-1.89	22.72	0.07	-0.08	0.26	-0.17	0.37
Panel B: GMV portfolios															
Short sales allowed															
Plug-in	6.07	22.77	0.00	0.26	0.08	0.26	0.08	6.81	-10.93	32.27	0.44	-0.33	0.27	-1.40	0.54
EWMA	6.97	22.46	0.02	0.31	0.00	0.31	0.00	2.71	0.13	22.57	0.01	0.01	0.13	0.01	0.13
IND	7.56	22.68	0.03	0.33	0.00	0.33	0.00	2.72	0.70	22.82	0.01	0.03	0.27	0.03	0.27
ECO	3.50	20.53	0.03	0.17	0.01	0.17	0.01	4.03	-6.66	23.26	0.59	-0.28	0.13	-0.61	0.34
mEWMA	-1.83	18.79	0.11	-0.09	0.12	-0.13	0.19	12.08	-32.30	25.87	0.75	-1.24	0.00	-3.31	0.04
CCC	3.93	20.37	0.03	0.19	0.02	0.19	0.02	3.22	-4.19	20.53	0.01	-0.20	0.01	-0.34	0.08
DCC	0.11	18.80	0.03	0.01	0.03	0.01	0.03	5.89	-14.74	19.26	0.03	-0.76	0.00	-1.12	0.01
TVC	-1.79	18.83	0.03	-0.09	0.02	-0.13	0.06	11.54	-30.91	34.07	0.48	-0.90	0.00	-4.18	0.18
DECO	-2.18	19.01	0.06	-0.11	0.05	-0.16	0.13	15.40	-41.02	58.77	0.44	-0.69	0.03	-9.56	0.45
Short sales prohibited															
Plug-in	7.07	23.22	0.00	0.30	0.09	0.30	0.09	4.33	-3.86	25.09	0.72	-0.15	0.18	-0.38	0.33
EWMA	6.97	22.46	0.02	0.31	0.00	0.31	0.00	2.71	0.13	22.57	0.01	0.01	0.13	0.01	0.13
IND	7.56	22.68	0.03	0.33	0.00	0.33	0.00	2.72	0.70	22.82	0.01	0.03	0.27	0.03	0.27
ECO	4.49	20.75	0.03	0.21	0.02	0.21	0.02	3.96	-5.51	23.51	0.65	-0.23	0.19	-0.51	0.41
mEWMA	2.84	19.96	0.08	0.14	0.08	0.14	0.08	1.69	-1.42	20.00	0.07	-0.07	0.30	-0.11	0.39
CCC	5.13	20.73	0.03	0.24	0.05	0.24	0.05	5.51	-8.78	28.66	0.63	-0.30	0.33	-0.99	0.57
DCC	4.06	20.18	0.06	0.20	0.07	0.20	0.07	5.11	-8.84	28.90	0.59	-0.30	0.30	-1.01	0.55
TVC	2.62	20.04	0.07	0.13	0.04	0.13	0.04	2.58	-3.87	20.26	0.05	-0.19	0.07	-0.31	0.14
DECO	3.27	20.01	0.08	0.16	0.08	0.16	0.08	1.86	-1.42	20.20	0.06	-0.07	0.31	-0.11	0.42

Notes: This table reports the daily out-of-sample portfolio performances of the benchmark $1/N$ strategy, the plug-in strategy, and 8 model-based strategies. Note that in this case with risky assets only, the short selling constraints never bind for the EWMA and IND strategies. The portfolio return statistics are the annualized percentage mean, the annualized standard deviation (Std Dev), the annualized Sharpe ratio (SR), the annualized modified Sharpe ratio (SR-m), and the average turnover (Turn). For each portfolio strategy, three p-values (p-val) are reported. These correspond to tests of the equality of the strategy's standard deviation, Sharpe ratio, and modified Sharpe ratio with those of the benchmark $1/N$ strategy. Values less than 0.01 are reported as zero.

Table 9: Portfolio of risky assets and a risk-free asset: July 22, 2005 to July 6, 2011

	No transactions costs								Transactions costs = 1 bp						
Strategy	Mean	Std Dev	p-val	SR	p-val	SR-m	p-val	Turn	Mean	Std Dev	p-val	SR	p-val	SR-m	p-val
$1/N$	9.23	20.47	1.00	0.45	1.00	0.45	1.00	1.67	5.01	20.49	1.00	0.24	1.00	0.24	1.00
Short sales allowed															
Plug-in	1.24	6.08	0.00	0.20	0.39	0.20	0.39	0.79	-0.74	6.08	0.01	-0.12	0.23	-0.02	0.43
EWMA	1.49	8.75	0.01	0.17	0.14	0.17	0.14	0.92	-0.84	8.75	0.01	-0.09	0.06	-0.03	0.27
IND	1.72	8.91	0.01	0.19	0.17	0.19	0.17	0.93	-0.63	8.91	0.01	-0.07	0.08	-0.02	0.26
ECO	1.35	5.61	0.00	0.24	0.49	0.24	0.49	0.82	-0.71	5.61	0.01	-0.12	0.27	-0.02	0.42
mEWMA	1.34	5.29	0.00	0.25	0.67	0.25	0.67	0.91	-0.96	5.29	0.00	-0.18	0.36	-0.02	0.51
CCC	1.45	5.56	0.01	0.26	0.56	0.26	0.56	0.80	-0.56	5.56	0.01	-0.10	0.34	-0.01	0.47
DCC	0.89	5.17	0.01	0.17	0.52	0.17	0.52	0.85	-1.26	5.17	0.01	-0.24	0.25	-0.02	0.43
TVC	0.60	5.21	0.00	0.11	0.44	0.11	0.44	0.86	-1.57	5.21	0.01	-0.30	0.18	-0.03	0.45
DECO	1.53	5.24	0.00	0.29	0.71	0.29	0.71	0.82	-0.55	5.24	0.00	-0.10	0.38	-0.01	0.46
Short sales prohibited															
Plug-in	2.56	9.10	0.01	0.28	0.30	0.28	0.30	0.75	0.67	9.10	0.01	0.07	0.29	0.07	0.29
EWMA	2.66	10.38	0.01	0.25	0.13	0.25	0.13	0.96	0.24	10.38	0.01	0.02	0.06	0.02	0.06
IND	2.77	10.45	0.01	0.26	0.14	0.26	0.14	0.96	0.34	10.45	0.01	0.03	0.11	0.03	0.11
ECO	2.51	8.96	0.01	0.28	0.27	0.28	0.27	0.82	0.43	8.96	0.01	0.05	0.21	0.05	0.21
mEWMA	2.55	8.49	0.01	0.30	0.36	0.30	0.36	0.77	0.60	8.49	0.01	0.07	0.32	0.07	0.32
CCC	2.43	8.74	0.01	0.27	0.29	0.27	0.29	0.79	0.44	8.74	0.01	0.05	0.24	0.05	0.24
DCC	2.60	8.51	0.01	0.30	0.40	0.30	0.40	0.76	0.68	8.51	0.01	0.08	0.34	0.08	0.34
TVC	2.51	8.51	0.01	0.29	0.39	0.29	0.39	0.76	0.59	8.51	0.01	0.07	0.30	0.07	0.30
DECO	2.50	8.56	0.01	0.29	0.38	0.29	0.38	0.77	0.55	8.56	0.01	0.06	0.31	0.06	0.31

Notes: See footnote of Table 8.

Table 10: Portfolio of risky assets only: July 22, 2005 to July 15, 2008

	No transactions costs								Transactions costs = 1 bp						
Strategy	Mean	Std Dev	p-val	SR	p-val	SR-m	p-val	Turn	Mean	Std Dev	p-val	SR	p-val	SR-m	p-val
$1/N$	11.69	17.54	1.00	0.66	1.00	0.66	1.00	3.62	2.56	17.81	1.00	0.14	1.00	0.14	1.00
Panel A: MV portfolios															
Short sales allowed															
Plug-in	4.75	14.75	0.00	0.32	0.06	0.32	0.06	3.79	-4.81	16.41	0.50	-0.29	0.23	-0.31	0.35
EWMA	5.88	15.42	0.00	0.38	0.01	0.38	0.01	2.22	0.29	15.46	0.00	0.02	0.38	0.02	0.38
IND	6.57	15.57	0.00	0.42	0.01	0.42	0.01	2.91	-0.77	16.01	0.00	-0.05	0.36	-0.05	0.41
ECO	1.15	14.01	0.00	0.08	0.01	0.08	0.01	2.30	-4.66	14.07	0.00	-0.33	0.05	-0.26	0.14
mEWMA	-5.52	13.13	0.00	-0.42	0.02	-0.28	0.04	13.34	-39.19	26.12	0.48	-1.50	0.00	-4.06	0.13
CCC	0.92	14.03	0.00	0.07	0.00	0.07	0.00	3.05	-6.76	14.21	0.00	-0.47	0.02	-0.38	0.07
DCC	-3.71	12.97	0.00	-0.28	0.01	-0.19	0.01	4.83	-15.90	13.21	0.00	-1.20	0.00	-0.83	0.02
TVC	-4.71	13.03	0.00	-0.36	0.00	-0.24	0.02	4.36	-15.72	13.19	0.00	-1.19	0.00	-0.82	0.03
DECO	-4.41	13.30	0.00	-0.33	0.01	-0.23	0.01	8.76	-26.52	17.17	0.81	-1.54	0.00	-1.80	0.02
Short sales prohibited															
Plug-in	6.11	15.06	0.00	0.40	0.09	0.40	0.09	4.59	-5.47	16.81	0.53	-0.32	0.17	-0.36	0.25
EWMA	5.88	15.42	0.00	0.38	0.01	0.38	0.01	2.22	0.29	15.46	0.00	0.02	0.38	0.02	0.38
IND	6.57	15.57	0.00	0.42	0.01	0.42	0.01	2.91	-0.77	16.01	0.00	-0.05	0.36	-0.05	0.41
ECO	3.56	14.42	0.00	0.24	0.01	0.24	0.01	2.93	-3.84	14.81	0.00	-0.26	0.09	-0.23	0.18
mEWMA	2.02	14.11	0.00	0.14	0.02	0.14	0.02	2.02	-3.08	14.25	0.00	-0.21	0.16	-0.17	0.29
CCC	2.86	14.42	0.00	0.19	0.01	0.19	0.01	2.42	-3.25	14.65	0.00	-0.22	0.12	-0.19	0.20
DCC	2.41	14.07	0.00	0.17	0.03	0.17	0.03	2.68	-4.34	15.01	0.00	-0.29	0.15	-0.26	0.25
TVC	1.94	14.08	0.00	0.13	0.02	0.13	0.02	1.76	-2.50	14.21	0.00	-0.17	0.22	-0.14	0.33
DECO	2.93	14.03	0.00	0.21	0.05	0.21	0.05	1.76	-1.50	14.08	0.00	-0.11	0.30	-0.08	0.38
Panel B: GMV portfolios															
Short sales allowed															
Plug-in	4.64	14.49	0.00	0.32	0.07	0.32	0.07	10.04	-20.67	35.24	0.45	-0.58	0.29	-2.89	0.59
EWMA	4.78	15.13	0.00	0.32	0.00	0.32	0.00	3.09	-3.03	15.32	0.00	-0.19	0.05	-0.18	0.09
IND	5.87	15.41	0.00	0.38	0.00	0.38	0.00	2.91	-1.46	15.54	0.00	-0.09	0.12	-0.09	0.20
ECO	-0.29	13.75	0.00	-0.02	0.00	-0.01	0.00	2.33	-6.18	13.82	0.00	-0.44	0.02	-0.34	0.10
mEWMA	-8.10	12.77	0.00	-0.63	0.01	-0.41	0.02	13.18	-41.35	26.30	0.36	-1.57	0.00	-4.31	0.07
CCC	-0.74	13.52	0.00	-0.05	0.00	-0.04	0.01	3.59	-9.81	13.79	0.00	-0.71	0.00	-0.53	0.05
DCC	-5.07	12.58	0.00	-0.40	0.00	-0.25	0.02	4.83	-17.25	13.03	0.00	-1.32	0.00	-0.89	0.03
TVC	-6.68	12.61	0.00	-0.53	0.00	-0.33	0.01	7.08	-24.55	17.23	0.79	-1.42	0.00	-1.68	0.06
DECO	-6.78	13.05	0.00	-0.52	0.00	-0.35	0.01	7.76	-26.36	15.51	0.13	-1.70	0.00	-1.62	0.02
Short sales prohibited															
Plug-in	5.82	14.93	0.00	0.39	0.07	0.39	0.07	5.08	-7.01	19.15	0.62	-0.36	0.34	-0.53	0.52
EWMA	4.78	15.13	0.00	0.32	0.00	0.32	0.00	3.09	-3.03	15.32	0.00	-0.19	0.05	-0.18	0.09
IND	5.87	15.41	0.00	0.38	0.00	0.38	0.00	2.91	-1.46	15.54	0.00	-0.09	0.12	-0.09	0.20
ECO	1.28	14.10	0.00	0.09	0.00	0.09	0.00	2.50	-5.03	14.39	0.00	-0.35	0.05	-0.28	0.16
mEWMA	-1.34	14.06	0.00	-0.09	0.01	-0.07	0.01	1.53	-5.19	14.09	0.00	-0.36	0.06	-0.29	0.15
CCC	1.39	14.11	0.00	0.09	0.01	0.09	0.01	8.75	-20.68	31.28	0.43	-0.66	0.16	-2.56	0.59
DCC	0.59	13.94	0.00	0.04	0.01	0.04	0.01	8.62	-21.16	32.40	0.42	-0.65	0.15	-2.72	0.58
TVC	-0.76	13.94	0.00	-0.05	0.01	-0.04	0.01	2.67	-7.50	14.29	0.00	-0.52	0.02	-0.42	0.08
DECO	-0.30	14.02	0.00	-0.02	0.00	-0.01	0.00	2.33	-6.18	14.50	0.00	-0.42	0.05	-0.35	0.12

Notes: See footnote of Table 8.

Table 11: Portfolio of risky assets and a risk-free asset: July 22, 2005 to July 15, 2008

	No transactions costs								Transactions costs = 1 bp						
Strategy	Mean	Std Dev	p-val	SR	p-val	SR-m	p-val	Turn	Mean	Std Dev	p-val	SR	p-val	SR-m	p-val
$1/N$	9.74	14.61	1.00	0.66	1.00	0.66	1.00	1.66	5.54	14.63	1.00	0.38	1.00	0.38	1.00
Short sales allowed															
Plug-in	1.92	6.59	0.00	0.29	0.30	0.29	0.30	0.94	-0.46	6.59	0.00	-0.07	0.22	-0.01	0.38
EWMA	2.34	8.88	0.00	0.26	0.04	0.26	0.04	1.05	-0.30	8.89	0.00	-0.03	0.04	-0.01	0.15
IND	2.66	8.96	0.00	0.29	0.04	0.29	0.04	1.04	0.03	8.97	0.00	0.00	0.04	0.00	0.04
ECO	1.45	6.01	0.00	0.24	0.30	0.24	0.30	0.99	-1.04	6.01	0.00	-0.17	0.19	-0.02	0.37
mEWMA	1.73	5.82	0.00	0.29	0.57	0.29	0.57	1.06	-0.93	5.82	0.00	-0.16	0.40	-0.02	0.46
CCC	1.50	6.02	0.00	0.25	0.37	0.25	0.37	0.98	-0.98	6.02	0.00	-0.16	0.23	-0.02	0.39
DCC	0.94	5.59	0.00	0.16	0.40	0.16	0.40	1.00	-1.57	5.59	0.00	-0.28	0.26	-0.03	0.40
TVC	0.82	5.66	0.00	0.14	0.37	0.14	0.37	1.02	-1.74	5.66	0.00	-0.30	0.24	-0.04	0.41
DECO	1.62	5.59	0.00	0.29	0.50	0.29	0.50	0.99	-0.87	5.59	0.00	-0.15	0.36	-0.02	0.45
Short sales prohibited															
Plug-in	4.51	9.02	0.00	0.50	0.31	0.50	0.31	0.93	2.16	9.02	0.00	0.24	0.43	0.24	0.43
EWMA	3.34	9.18	0.00	0.36	0.05	0.36	0.05	1.06	0.65	9.18	0.00	0.07	0.06	0.07	0.06
IND	3.48	9.23	0.00	0.37	0.07	0.37	0.07	1.05	0.81	9.23	0.00	0.08	0.07	0.08	0.07
ECO	3.60	8.65	0.00	0.42	0.21	0.42	0.21	1.02	1.03	8.65	0.00	0.12	0.20	0.12	0.20
mEWMA	3.79	8.19	0.00	0.46	0.43	0.46	0.43	1.00	1.26	8.19	0.00	0.15	0.41	0.15	0.41
CCC	3.63	8.54	0.00	0.42	0.25	0.42	0.25	1.01	1.09	8.54	0.00	0.13	0.21	0.13	0.21
DCC	3.90	8.23	0.00	0.47	0.43	0.47	0.43	0.98	1.41	8.23	0.00	0.17	0.38	0.17	0.38
TVC	3.72	8.24	0.00	0.45	0.38	0.45	0.38	0.99	1.23	8.23	0.00	0.15	0.34	0.15	0.34
DECO	3.60	8.31	0.00	0.43	0.33	0.43	0.33	1.00	1.08	8.31	0.00	0.13	0.32	0.13	0.32

Notes: See footnote of Table 8.

Table 12: Portfolio of risky assets only: July 16, 2008 to July 6, 2011

	No transactions costs								Transactions costs = 1 bp						
Strategy	Mean	Std Dev	p-val	SR	p-val	SR-m	p-val	Turn	Mean	Std Dev	p-val	SR	p-val	SR-m	p-val
$1/N$	11.02	30.02	1.00	0.36	1.00	0.36	1.00	2.89	3.73	30.12	1.00	0.12	1.00	0.12	1.00
Panel A: MV portfolios															
Short sales allowed															
Plug-in	9.43	29.46	0.16	0.32	0.70	0.32	0.70	9.20	-13.79	45.47	0.41	-0.30	0.49	-2.48	0.59
EWMA	10.93	29.01	0.19	0.37	0.77	0.37	0.77	5.63	-3.27	31.07	0.60	-0.10	0.34	-0.40	0.46
IND	10.72	29.12	0.15	0.36	0.97	0.36	0.97	1.88	5.97	29.13	0.14	0.21	0.16	0.21	0.16
ECO	10.18	27.60	0.14	0.37	0.97	0.37	0.97	2.88	2.91	27.93	0.13	0.10	0.87	0.10	0.87
mEWMA	10.71	24.59	0.19	0.43	0.89	0.43	0.89	5.00	-1.90	24.72	0.17	-0.07	0.68	-0.18	0.67
CCC	12.06	27.41	0.16	0.44	0.43	0.44	0.43	1.70	7.77	27.43	0.17	0.28	0.15	0.28	0.15
DCC	9.01	25.33	0.16	0.35	0.94	0.35	0.94	4.96	-3.51	25.62	0.15	-0.13	0.27	-0.36	0.38
TVC	6.81	25.52	0.14	0.26	0.63	0.26	0.63	3.57	-2.21	25.69	0.16	-0.08	0.33	-0.22	0.43
DECO	6.14	25.29	0.15	0.24	0.61	0.24	0.61	3.32	-2.23	25.44	0.17	-0.08	0.41	-0.22	0.47
Short sales prohibited															
Plug-in	9.91	29.54	0.14	0.33	0.74	0.33	0.74	2.63	3.26	30.23	0.85	0.10	0.92	0.10	0.92
EWMA	10.93	29.01	0.19	0.37	0.77	0.37	0.77	5.63	-3.27	31.07	0.60	-0.10	0.34	-0.40	0.46
IND	10.72	29.12	0.15	0.36	0.97	0.36	0.97	1.88	5.97	29.13	0.14	0.21	0.16	0.21	0.16
ECO	10.67	27.78	0.13	0.38	0.84	0.38	0.84	2.94	3.25	28.11	0.13	0.11	0.95	0.11	0.95
mEWMA	9.09	27.08	0.14	0.33	0.80	0.33	0.80	1.58	5.11	27.12	0.12	0.18	0.69	0.18	0.69
CCC	12.53	27.62	0.15	0.45	0.33	0.45	0.33	1.52	8.69	27.64	0.14	0.31	0.07	0.31	0.07
DCC	10.61	27.17	0.12	0.39	0.83	0.39	0.83	1.97	5.63	27.27	0.14	0.20	0.55	0.20	0.55
TVC	9.71	27.15	0.16	0.35	0.93	0.35	0.93	2.17	4.23	27.33	0.16	0.15	0.81	0.15	0.81
DECO	8.88	27.14	0.16	0.32	0.74	0.32	0.74	4.18	-1.68	28.90	0.59	-0.06	0.46	-0.19	0.56
Panel B: GMV portfolios															
Short sales allowed															
Plug-in	8.16	28.77	0.04	0.28	0.47	0.28	0.47	3.59	-0.56	29.04	0.03	-0.02	0.36	-0.06	0.45
EWMA	9.73	27.96	0.14	0.34	0.70	0.34	0.70	2.33	3.84	28.02	0.12	0.13	0.86	0.13	0.86
IND	9.83	28.16	0.13	0.35	0.69	0.35	0.69	2.53	3.43	28.31	0.11	0.12	0.98	0.12	0.98
ECO	7.89	25.59	0.15	0.31	0.67	0.31	0.67	5.73	-6.56	29.87	0.97	-0.22	0.46	-0.77	0.57
mEWMA	5.01	23.31	0.19	0.21	0.76	0.21	0.76	11.00	-22.75	25.46	0.15	-0.89	0.09	-2.29	0.20
CCC	9.23	25.45	0.10	0.36	0.98	0.36	0.98	2.85	2.02	25.57	0.13	0.08	0.80	0.08	0.80
DCC	5.98	23.42	0.12	0.25	0.72	0.25	0.72	6.96	-11.59	23.93	0.10	-0.48	0.07	-1.10	0.17
TVC	3.72	23.47	0.12	0.16	0.49	0.16	0.49	16.02	-36.71	45.04	0.49	-0.81	0.11	-6.56	0.50
DECO	3.02	23.52	0.13	0.12	0.60	0.12	0.60	23.06	-55.17	81.72	0.43	-0.67	0.16	-17.89	0.63
Short sales prohibited															
Plug-in	8.98	29.27	0.02	0.30	0.51	0.30	0.51	3.59	-0.08	29.89	0.72	-0.00	0.44	-0.01	0.61
EWMA	9.73	27.96	0.14	0.34	0.70	0.34	0.70	2.33	3.84	28.02	0.12	0.13	0.86	0.13	0.86
IND	9.83	28.16	0.13	0.35	0.69	0.35	0.69	2.53	3.43	28.31	0.11	0.12	0.98	0.12	0.98
ECO	8.30	25.75	0.12	0.32	0.73	0.32	0.73	5.43	-5.40	29.99	0.98	-0.18	0.49	-0.64	0.57
mEWMA	7.62	24.49	0.15	0.31	0.81	0.31	0.81	1.86	2.93	24.54	0.13	0.12	0.98	0.12	0.98
CCC	9.47	25.71	0.12	0.37	0.99	0.37	0.99	2.29	3.69	25.80	0.10	0.14	0.90	0.14	0.90
DCC	8.17	24.91	0.11	0.32	0.83	0.32	0.83	1.61	4.10	24.94	0.12	0.16	0.84	0.16	0.84
TVC	6.64	24.69	0.15	0.27	0.65	0.27	0.65	2.48	0.37	24.85	0.15	0.01	0.62	0.01	0.62
DECO	7.46	24.60	0.16	0.30	0.77	0.30	0.77	1.39	3.94	24.62	0.14	0.16	0.87	0.16	0.87

Notes: See footnote of Table 8.

Table 13: Portfolio of risky assets and a risk-free asset: July 16, 2008 to July 6, 2011

	No transactions costs								Transactions costs = 1 bp						
Strategy	Mean	Std Dev	p-val	SR	p-val	SR-m	p-val	Turn	Mean	Std Dev	p-val	SR	p-val	SR-m	p-val
$1/N$	9.18	25.01	1.00	0.36	1.00	0.36	1.00	1.68	4.95	25.04	1.00	0.19	1.00	0.19	1.00
Short sales allowed															
Plug-in	0.71	5.54	0.02	0.13	0.64	0.13	0.64	0.63	-0.88	5.53	0.02	-0.16	0.46	-0.02	0.65
EWMA	0.87	8.63	0.01	0.10	0.38	0.10	0.38	0.80	-1.16	8.63	0.03	-0.13	0.27	-0.04	0.48
IND	1.03	8.86	0.02	0.11	0.41	0.11	0.41	0.82	-1.05	8.86	0.02	-0.11	0.28	-0.03	0.53
ECO	1.34	5.18	0.01	0.26	0.83	0.26	0.83	0.64	-0.28	5.18	0.02	-0.05	0.60	-0.01	0.69
mEWMA	0.96	4.70	0.02	0.20	0.79	0.20	0.79	0.77	-0.99	4.70	0.01	-0.21	0.51	-0.02	0.70
CCC	1.49	5.05	0.02	0.29	0.90	0.29	0.90	0.61	-0.06	5.05	0.02	-0.01	0.69	-0.00	0.69
DCC	0.87	4.70	0.01	0.18	0.75	0.18	0.75	0.71	-0.92	4.70	0.02	-0.19	0.51	-0.02	0.68
TVC	0.41	4.72	0.02	0.08	0.63	0.08	0.63	0.71	-1.37	4.72	0.03	-0.29	0.38	-0.02	0.67
DECO	1.51	4.87	0.02	0.31	0.92	0.31	0.92	0.66	-0.15	4.87	0.01	-0.03	0.71	-0.00	0.72
Short sales prohibited															
Plug-in	0.83	9.19	0.02	0.09	0.25	0.09	0.25	0.57	-0.60	9.19	0.01	-0.06	0.25	-0.02	0.48
EWMA	2.21	11.46	0.02	0.19	0.34	0.19	0.34	0.85	0.06	11.46	0.02	0.01	0.33	0.01	0.33
IND	2.31	11.56	0.01	0.20	0.38	0.20	0.38	0.87	0.12	11.56	0.01	0.01	0.34	0.01	0.34
ECO	1.60	9.28	0.02	0.17	0.42	0.17	0.42	0.63	0.01	9.28	0.02	0.00	0.40	0.00	0.40
mEWMA	1.44	8.80	0.02	0.16	0.44	0.16	0.44	0.54	0.08	8.80	0.01	0.01	0.46	0.01	0.46
CCC	1.36	8.95	0.02	0.15	0.38	0.15	0.38	0.57	-0.08	8.95	0.02	-0.01	0.42	-0.00	0.54
DCC	1.44	8.79	0.01	0.16	0.42	0.16	0.42	0.53	0.09	8.79	0.02	0.01	0.46	0.01	0.46
TVC	1.43	8.80	0.02	0.16	0.43	0.16	0.43	0.54	0.07	8.80	0.01	0.01	0.45	0.01	0.45
DECO	1.54	8.82	0.02	0.17	0.44	0.17	0.44	0.55	0.15	8.83	0.02	0.02	0.44	0.02	0.44

Notes: See footnote of Table 8.

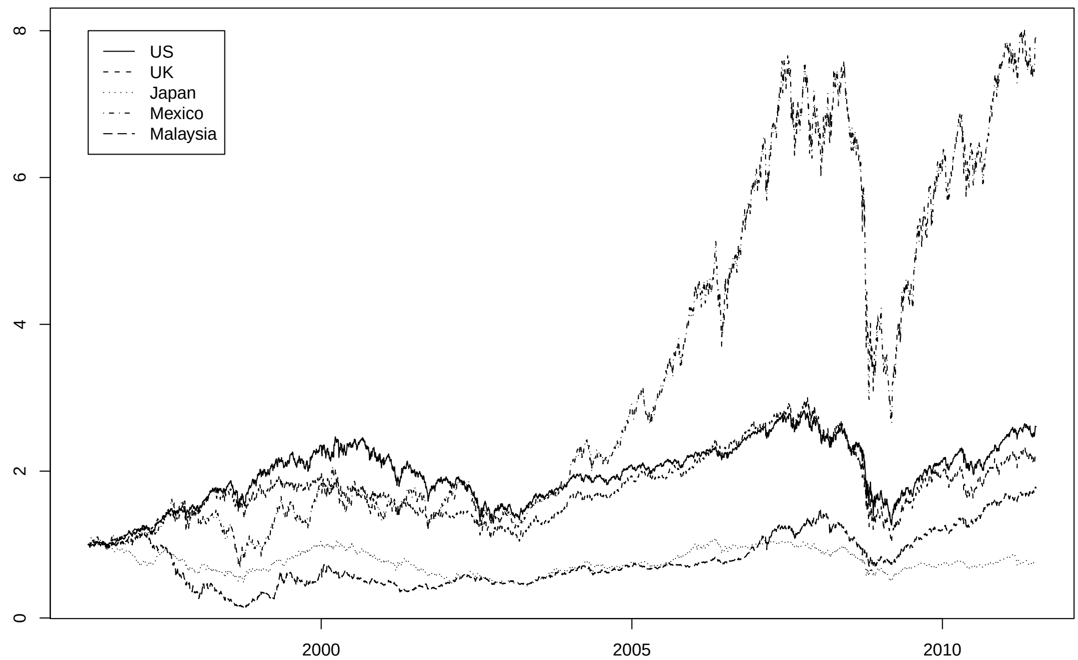

Figure 1: Time-series plot of the (relative) daily ETF closing prices from April 1, 1996 to July 6, 2011 (3843 observations). The series are divided by their first value, so they each begin at a value of 1 dollar.

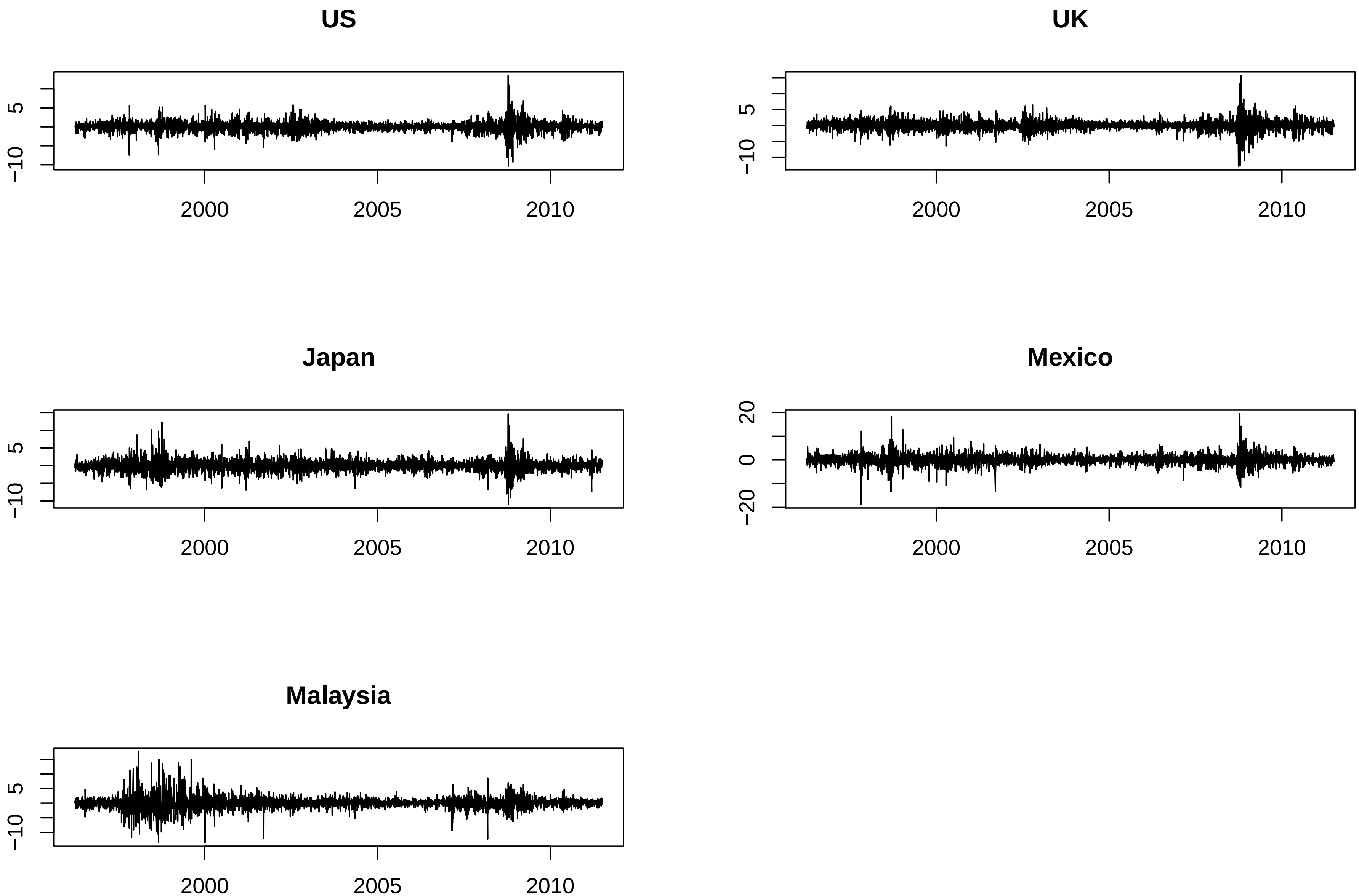

Figure 2: Time-series plots of the daily log-returns (in percentages) on 3 developed market ETFs (US, UK, Japan) and 2 emerging market ETFs (Mexico, Malaysia) from April 2, 1996 to July 6, 2011 (3842 observations).

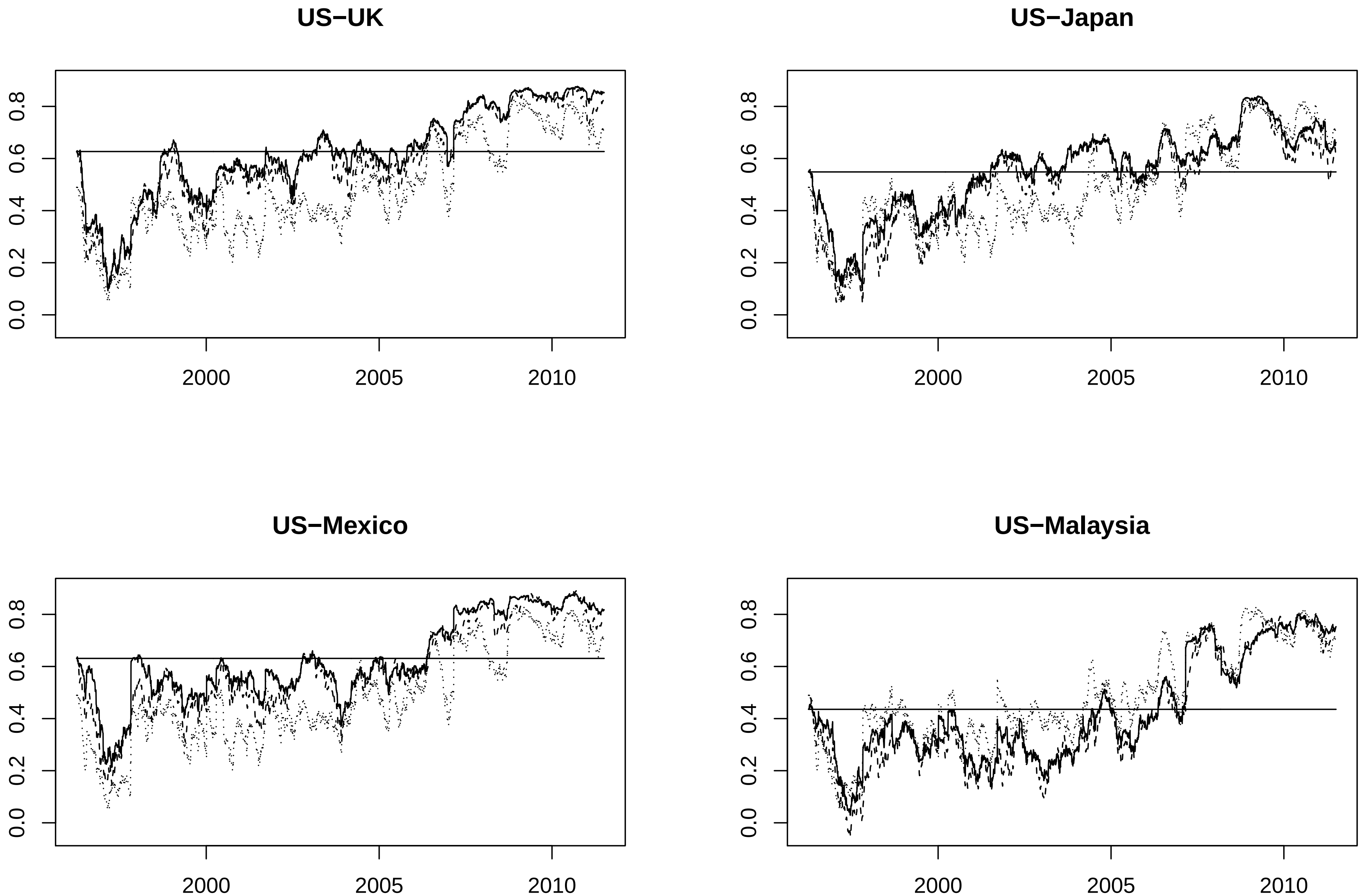

Figure 3: Time-series plots of the model-implied conditional correlations between the standardized returns on the ETFs for the US and the UK (upper left), the US and Japan (upper right), the US and Mexico (lower left), and the US and Malaysia (lower right). The horizontal lines correspond to the CCC model, the solid lines correspond to the DCC model, the dashed lines correspond to the TVC model, and the dotted line (which is the same in each plot) corresponds to the DECO model.

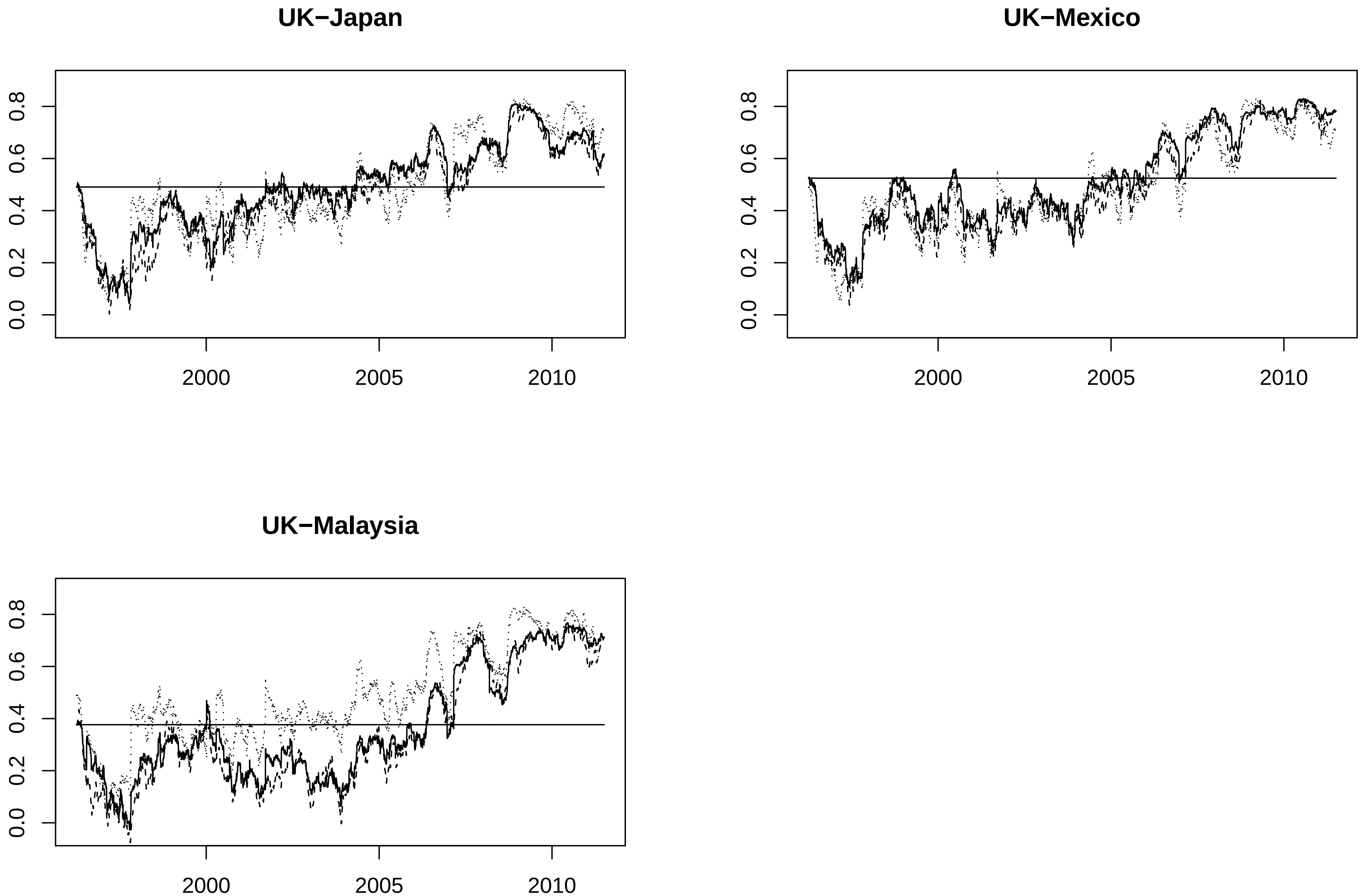

Figure 4: Time-series plots of the model-implied conditional correlations between the standardized returns on the ETFs for the UK and Japan (upper left), the UK and Mexico (upper right), and the UK and Malaysia (lower left). The horizontal lines correspond to the CCC model, the solid lines correspond to the DCC model, the dashed lines correspond to the TVC model, and the dotted line (which is the same in each plot) corresponds to the DECO model.

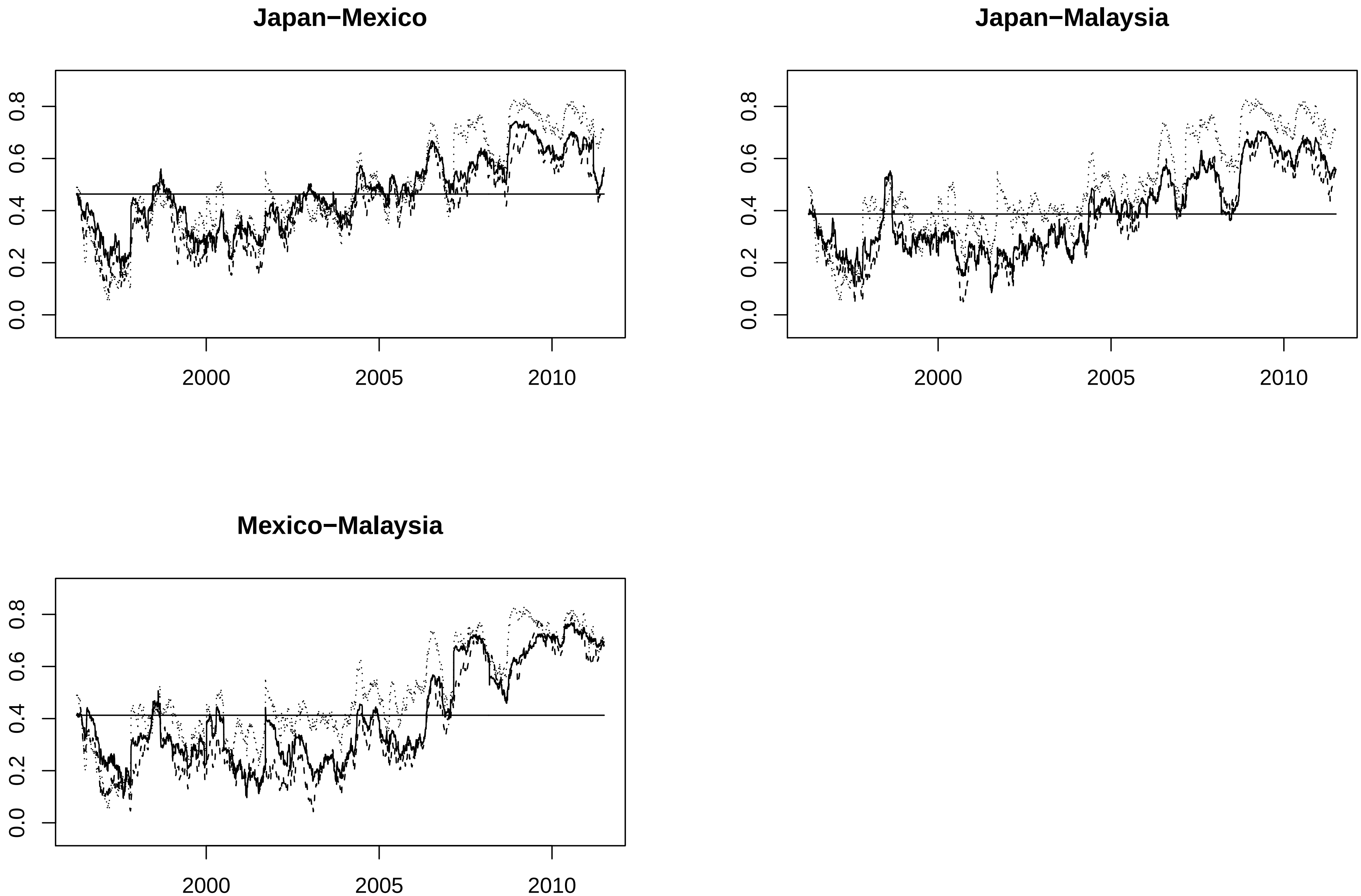

Figure 5: Time-series plots of the model-implied conditional correlations between the standardized returns on the ETFs for Japan and Mexico (upper left), Japan and Malaysia (upper right), and Mexico and Malaysia (lower left). The horizontal lines correspond to the CCC model, the solid lines correspond to the DCC model, the dashed lines correspond to the TVC model, and the dotted line (which is the same in each plot) corresponds to the DECO model.

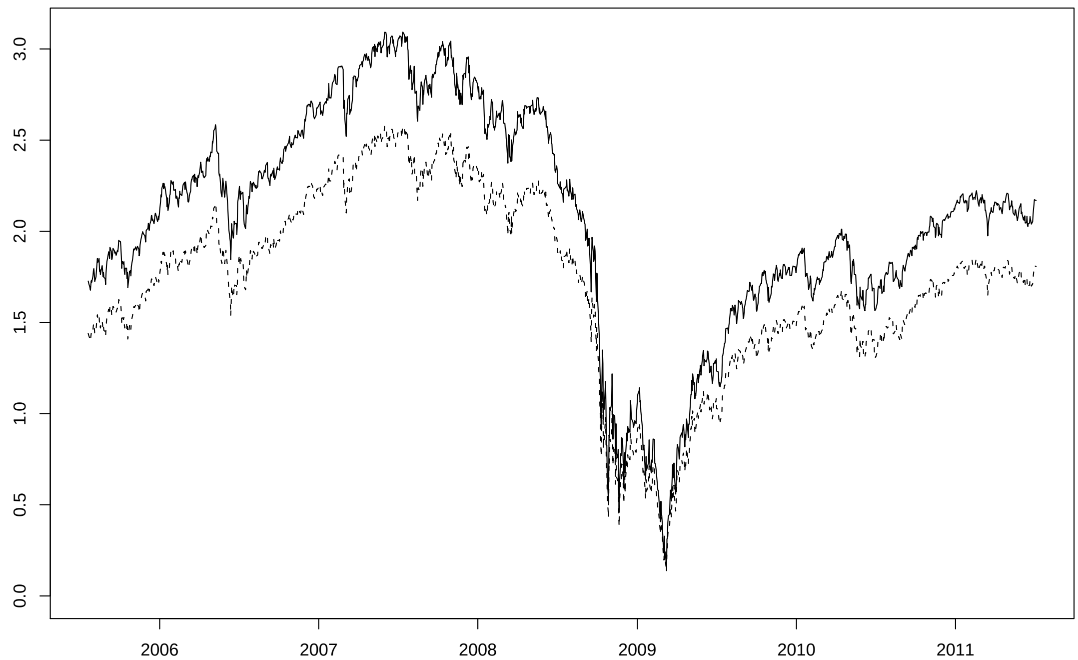

Figure 6: Time-series plot of the daily portfolio target returns (in basis points) from July 21, 2005 to July 5, 2011. The solid line is the target return when the portfolio comprises risky assets only and the dashed line is the target return when the asset mix includes a risk-free security.