EUROPEAN CENTRAL BANK

THE LIQUIDITY EFFECT IN THE EURO AREA

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This version: December 2004

Abstract

This paper presents an analysis of the liquidity effect in the euro area – the link between the availability of aggregate liquidity and the interbank overnight rate. Applying a set of assumptions which also lead to the "martingale property" of the overnight rate, a model linking the latter to the expected *aggregate* liquidity conditions is formulated and calibrated. Different assumptions for how the market is establishing its expectation and uncertainty about the aggregate liquidity conditions are tested for. It is found that the dynamics of the overnight rate is consistent with market participants having, in aggregate, a rather precise expectation about the aggregate liquidity conditions, however, with an apparent uncertainty which by far exceeds the rather limited variance of the error of this expectation. While the model presented here provides a rather good fit of the overnight rate this indicates that further aspects than just the dynamics of the aggregate liquidity conditions need to be included in order to conceptually describe the overnight rate. An EUR 1 billion expected liquidity imbalance on the last day of the maintenance period is estimated to, ceteris paribus, affect the overnight rate by 7 basis points on that day.

JEL Classification numbers: C32, E43, E52

Key Words: overnight rate, liquidity effect

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1 Introduction

The previous century offers several examples of academics and central bankers assuming a causality between the aggregate liquidity held by the banking system with the central bank and economic activity stemming from the so-called money multiplier. However, there seems now to be a wide consensus that the only causality stems from the influence of liquidity on short term interest rates, which accordingly have now been widely acknowledged as the operational target by most central banks in the industrialized world (see, for instance, Bindseil[2004]). Therefore, quantifying and understanding the link between liquidity and short term interest rates – *the* liquidity effect - is not only key to market participants, but also to the monetary policy implementation of central banks, which are steering their domestic interbank overnight rate through their control over the aggregate liquidity position of the banking system vis-à-vis themselves.

This paper analysis the liquidity effect in the euro area. That is, the extent to which, and why, the availability of *aggregate* liquidity in the banking system, i.e. *aggregate* current accounts held with the Eurosystem, is affecting the euro area overnight rate, the EONIA (Euro OverNight Index Average). Effects potentially arising from different idiosyncratic characteristics of *individual* banks, such as individual liquidity positions, individual (and not only aggregate) liquidity shocks, and/or individual liquidity forecasts will not be covered in the empirical analysis. Due to the averaging provision of the reserve requirement system in the euro area and the liquidity management policy followed by the ECB (see ECB [2004]), the liquidity effect has mainly been confined to the days falling after the last MRO allotment of a reserve maintenance period (this MRO is henceforth referred to as "*the last MRO*"). While several previous studies on the liquidity effect (Bindseil&Seitz[2001], Angelini [2002], Prati et al.[2003], Ejerskov et al. [2003], Würtz[2003]), also identify a slight effect before the last MRO, most find it to be negligible in economic terms (see also ECB [2003a]) only triggering movements of the overnight by a maximum of a couple of basis points. Therefore, this study focuses exclusively on the days after the last MRO. On these days the overnight rate is increasingly volatile, fluctuating according to market participants perceptions about whether they will face a liquidity shortage or a surplus (Würtz [2003], Prati et al. [2003], Gaspar et al. [2004]). In the first case they need to take recourse to the marginal lending facility (i.e. to pay a penalty rate) in order to fulfill their reserve requirements, while in the second case they need to place liquidity in excess of their reserve requirements on the deposit facility, again at a penalty rate. We denote the aggregate net recourse to the standing facilities on the last day of the maintenance period as the accumulated aggregate liquidity imbalance.

The above mentioned studies have quantified the liquidity effect in the euro area via different regression techniques, mostly inspired, in some form or the other, by the so-called "martingale property" of the overnight rate (e.g. Hamiltion 1996). Adopting assumptions leading to this property as well as some other assumptions, such as that market participants are homogenous, unaffected by idiosyncratic liquidity shocks and are sharing the same information set about the aggregate liquidity conditions, the present study estimates the liquidity effect by calibrating the parameters of the resulting so-called "aggregate liquidity model" on the overnight rate. The model takes thoroughly into account the specific features of the Eurosystem's operational framework, comprised by its operating procedures (see ECB[2004a]), its allotment policy in open market operations (See ECB[2002]), and its communication policy (see ECB[2000]). This approach has the advantage that it explicitly accommodates the non-linear relationship between the expected accumulated liquidity imbalance and the overnight rate. Moreover, it allows, without introducing too many parameters and/or arbitrary parameterizations, the liquidity effect to depend on several different factors, such as the number of days until the end of the maintenance period and the number of days from the last MRO to the end of the maintenance period. Finally, the resulting parameters can be interpret as variances of different observable liquidity components, to which they can be compared. This yields interesting insights into the validity of the "aggregate liquidity model".

Different assumptions concerning how the market exploits the information provided by the ECB on a daily/weekly basis in order to establish its liquidity forecast are explored, including the signal extraction approach proposed by Bindseil [2001], according to which the market also extracts information from its knowledge about the ECB's allotment policy in MROs. It is noted that the ECB since the launch of the Euro has revised its communication policy on three occasions. From January 1999 to June 2000, it did not provide any information about the liquidity forecasts that was underlying its MRO allotment decisions. From June 2000 to March 2004, it published its forecast of the average autonomous factors, as calculated per the MRO announcement day. Finally it has, since March 2004, also provided this forecast as per the MRO allotment day and in addition made available the so-called benchmark allotment amount (see ECB [2004]). Acknowledging that the liquidity effect may be different in each of these periods, focus is narrowed to the second period. Specifically, the study is based on data for the period June 2000 to December 2003.

The study identifies an increasing liquidity effect towards the last day of the maintenance period, when it amounts to around 7 basis points per one billion euro of expected liquidity imbalance. This effect is slightly larger than the one identified in Würtz[2003] and Bindseil & Seitz [2001]. Via the parameterization of the proposed "aggregate liquidity model", it is possible to explain 57% of the variation of the spread between the EONIA and the key policy rate of the ECB [the *spread*]. Empirical evidence is found that the behavior of the spread is consistent with the market having a quite precise expectation about the accumulated aggregate liquidity imbalance, however, with an uncertainty which by far exceeds the variance of the observed errors of this expectation. Only if allowing uncertainty about the aggregate liquidity conditions to be much higher than what follows from the assumed "aggregate liquidity model", the above fit can be obtained. Applying parameter values which are consistent with the observed uncertainty about the aggregate liquidity conditions yields a liquidity effect which is much higher than the one identified above. Overall, this suggests the conclusion that the "aggregate liquidity model" is in fact not fully capable of *conceptually* describing the dynamics of the spread. Several explanations for this finding can be put forward including the fact that uncertainty about liquidity shocks at the level of *individual* market players, and not only at the aggregate level, also may play a role as suggested e.g. by Gaspar et al. [2004] and woodford[2001]. The finding is, however, also consistent with market participants, in practice, having *heterogeneous* liquidity expectations and/or behaving in a risk averse manner.

No evidence is found that the spread should follow a specific pattern towards the end of the maintenance period, which would otherwise contradict the martingale property. This is consistent with Bindseil et al. [2003], thereby balancing the opposite results of Perez-Quirós & Mendizábal, H.R. [2000] on one hand, and Würtz[2003] and Prati et al.[2003] on the other hand. Finally, this study finds that the so-called natural spread, the difference between the EONIA and the mid point of the corridor, when the liquidity effect is zero, is at around 5 basis points, somewhat higher than the level identified in Würtz(2003).

The paper is structured as follows. Section 2 defines the "aggregate liquidity model", linking the properties of the expected aggregate liquidity imbalance at the end of the maintenance period to the overnight rate. The section also discusses the assumptions of the "aggregate liquidity model", casting some light on the implications for the liquidity effect should these assumptions not hold in practice. Section 3 and 4 describe the more detailed assumptions that we make about the different liquidity components, i.e. items within the central banks balance sheet, that are relevant in the aggregate liquidity model. First, section 3 provides an overview of the different liquidity components and how they together make up the aggregate liquidity imbalance. Moreover, the section presents some specific assumptions about excess reserves and recourse to standing facilities - liquidity components that are normally considered to be less important in this context, because they only add limited uncertainty about the aggregate liquidity imbalance. Section 4 zooms in on the assumptions that we make about the market's forecast of the autonomous liquidity factors, which contribute most uncertainty about the aggregate liquidity imbalance. The section presents two general approaches, as well as the specific variants that are here implemented empirically, for how the market derives this forecast. The two approaches are differing by the extent to which the market is assumed to extract signals from its knowledge about the ECBs allotment policy in MROs. Up till section 5, the paper only conceptually presents, without introducing any parameters, how the overnight rate is linked to the expected liquidity imbalance via the variance of the error of this expectation. A parameterization of the error of the expected liquidity imbalance is introduced in section 6, which also describes other elements of the empirical approach, such as the underlying data-sample, the calibration procedure and the methodology for assessing the goodness of fit. Section 7 presents and interprets the calibrated parameters, while section 8 concludes.

2 The aggregate liquidity model

We make the following assumptions in order to arrive at what we label the "aggregate liquidity model", i.e. a model in which the overnight rate only depends on the commonly known properties of the aggregate liquidity conditions.

A1) Market participants are risk neutral;

A2) Market players have identical expectations about the (unrealized) accumulated liquidity imbalance, L_A, defined as *the actual net recourse to the deposit facility on the last day of the maintenance period*, as well as a common assessment of its higher order moments. Together, these define the perceived probability, P, that the maintenance period will, in aggregate, end on the loose side, i.e. with $L_A>0$;

A3) At the end of the last day of the maintenance period, the aggregate liquidity imbalance is revealed and each bank knows its liquidity position with full certainty. A trivial, "liquidity clearing", trading session then takes place: if there is an aggregate surplus of liquidity (i.e. if $L_A>0$), all banks "square their residual liquidity position", i.e. cover any remaining liquidity need or place any excess liquidity, at the rate of the deposit standing facility. Conversely, if there is an aggregate shortage (i.e. if $L_A<0$) all banks have to square their liquidity position at the rate of the marginal lending facility;

A4) The liquidity management policy of the central bank and the reserve requirement system are such that there is no notable likelihood that credit institutions, before the very last trading day of the maintenance period, go either in overdraft or fulfil there reserve requirements. Together with A1, this implies that reserve holdings on any days within the same maintenance period are perfect substitutes, from which the martingale property of the overnight rate follows;

A5) After the last MRO allotment, there are no expectations in the market about changes of the rates applied to standing facilities within the same maintenance period;

A6) The interbank overnight market is "perfect" without any frictions. I.e. there are no transaction costs, and the market is perfectly liquid.

Under these assumptions, the overnight rate is, at any point in time between the last MRO allotment and the end of the maintenance period, uniquely determined from the perceived probability, P, that there will be an aggregate liquidity surplus. Specifically, one obtains the following simple equation for the spread, s_t, between the overnight rate and the mid point of the corridor set by standing facilities[2], when taking into account that the latter has been two percentage points in the euroarea since April 1999:

(2.1) $$s_t = P_t(L_A < 0) - (1 - P_t(L_A < 0)) = 2P_t(L_A < 0) - 1$$

If this equation would not hold, market participants could perform intertemporal arbitrage. That is they could make a profit by substituting reserve holdings on time t with reserve holdings at the end of the maintenance period (or vice versa).

[2] In the case of the Eurosystem the corridor is symmetric in the sense that its midpoint is equivalent to the key interest rate: the minimum bid rate in MROs.

Yet, as suggested by other empirical work, the overnight spread tends to be slightly positive, even when there are "neutral" liquidity expectations, i.e. when P_t=0.5.[3] This we accommodate by adding a constant, c, to (2.1) such that we have the following general model on the overnight spread after the last MRO allotment.

(2.2) $$s_t = 1 - 2P_t(L_A > 0) + c$$

We will assume that L_A is normally distributed, implying that

(2.3) $P_t = N\left(\dfrac{E_t(L_A)}{\sqrt{\sigma^t_{L_A}}}\right)$, where N is the cumulative standard normal distribution and

$$\sigma^t_{L_A} = E_t(L_A - E_t(L_A))^2$$

We refer to (2.2) as the *aggregate liquidity model*, since it assumes that the overnight spread follows uniquely from the properties of the expected aggregate liquidity imbalance, which are known and shared by all market participants. Moreover, we define *the liquidity effect*, Γ_t, as the marginal change of the spread implied by a marginal change in the expected liquidity imbalance when the latter is zero:

(2.4) $$\Gamma_t = \left.\frac{\partial S_t}{\partial E_t(L_A)}\right|_{E_t(L_A)=0}$$

It follows immediately from the properties of the normal distribution that Γ_t is a decreasing function in $\sigma^t_{L_A}$, expressing the uncertainty about the expected aggregate liquidity imbalance.

To what extent, however, are the assumptions A1 to A6 realistic for the euro area interbank market?

First, it is acknowledged that A5 is consistent with the observed behavior of the ECB during the sample period under consideration, and that A6 is pretty standard in most empirical literature, being consistent with the well developed and integrated infra structure of the unsecured euro area money market (see for instance ECB[2003b]). Anecdotal evidence also suggests that euro area banks are normally acting in a risk neutral manner, simply aiming to fulfil their reserve requirements whenever this is cheapest. Note, however, if this assumption is not valid, the liquidity effect is likely to be lower than stipulated by (2.2), because market participants will then require a risk premium in order to perform the underlying intertemporal arbitrage. That is a risk premium for accepting increased exposure to the overnight rate in the last moment of the

[3] Several reasons for such a "natural spread" can be put forward. After the last MRO of the maintenance period, it is probably mostly related to asymmetric opportunity costs of the standing facilities. That is, some additional costs that are only incurred when taking recourse to the marginal lending facility and not when taking recourse to the deposit facility. Such additional costs include the cost of collateral and probably also a perceived connotation of negative window dressing towards the central bank. Moreover, there may be a slight structural recourse to the deposit facility, such that if L_A=0, there is in fact still a liquidity need to be covered.

maintenance period, when it is maximum volatile: either it is at the rate of the marginal lending facility or it is at the rate of the deposit facility.

Although some authors question the perfect substitutability of reserves on individual days as implied by A4 (e.g. Perez-Quirós & Mendizábal, H.R. [2000] and Prati et al. [2003]), this assumption seems rather reasonable for the euro area, due to the comparatively high level of reserve requirements, and is in fact also empirically well supported (see for instance ECB [2003a], Würtz [2003], and Bindseil et al. [2002]).

A2 and A3 may appear to be somewhat more controversial, in the sense that they represent obvious simplifications of reality, which are nevertheless rather important for the liquidity effect.

Consider at first A3. If, as opposed to this assumption, "short" and "long" banks can not square liquidity positions with each other, but are forced to only do this via standing facilities, uncertainty about individual liquidity positions, and not only about the aggregate liquidity position (the sum of the individual positions), will play a role. It follows, for instance, from Woodford[2001] how this for a given amount of uncertainty $\sigma^t_{L_A}$ about the aggregate liquidity position, reduces the liquidity effect that otherwise follows from the "aggregate liquidity model": if banks do not know for sure their liquidity position before the close of the market (which here means just before the "trivial trading session"), they will be more (less) inclined to hold liquidity buffers against this uncertainty, when the overnight rate decreases (increases) towards the level of the deposit (marginal lending) standing facility. This introduces elasticity into the liquidity demand at the time of the trivial trading session, and therefore all other equal, dampens the effect of an aggregate liquidity shock.[4] On the other hand, some factors support that A3 is a fair simplification for the case of the euro area interbank market. Due to the high degree of integration of this market, and related to that the state of development of the payment systems, it seems to be a fair assumption that the overnight interbank market remains liquid until banks' uncertainty about their end of day position vanishes to insignificant levels. This is consistent with the fact that the recourses to standing facilities which are *not* triggered by an aggregate liquidity imbalance [5] as well as excess reserves are normally quite small (see ECB[2002]). This is further discussed in the next section.

The assumption A2, that all market participants share the same unique forecast of the aggregate liquidity imbalance, is not easy to verify up front. Only on the MRO announcement day, the ECB is (during the sample period) making publicly available its aggregate liquidity forecast, thereby providing a natural guide to the forecast of each market player. On the subsequent days, the ECB does not publish updated forecasts, covering the period all until the end of the maintenance period, but publishes only the ex post key liquidity

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[4] That is, if there is an aggregate surplus the overnight rate will, even in the trivial trading session, be larger than the rate of the deposit facility, and the other way around, if the there is a liquidity deficit, it will be less than the rate of the marginal lending facility.

[5] For instance, at the end of the maintenance period there is a small recourse to the marginal lending facility, even though there is an aggregate liquidity surplus, and the other way around, there is also a small recourse to the deposit facility when there is an aggregate surplus. Before the last day of the maintenance period, there is also some recourse to standing facilities, mostly by banks

figures for the aggregate liquidity conditions on the preceding day. Table 1 shows the regular schedule for the ECB's daily and weekly releases of information on the aggregate liquidity conditions. After the MRO announcement, in the absence of a publicly available forecasts, market participants need to establish their own forecasts, taking into account the available sources of information. In practice, these forecasts inevitably rely on different, and to some extent private, sources of information arising for instance from banks' interaction in the interbank market.[6] Thus, there are sensible reasons to believe that market participants after the MRO announcement day do in fact have *heterogeneous* liquidity forecasts. Nevertheless, also not knowing the extent of the heterogeneity of individual banks' forecasts, we will proceed under the assumption that the forecasts are indeed homogenous, being established solely on the basis of the information published by the ECB together with forecasting models which are assumed to be adopted by all market participants. A violation of A2 will, just like violations of A1 and A3, (under some further reasonable assumptions) lead to a smaller liquidity effect for a given amount of uncertainty about the (average) aggregate liquidity conditions. This is shown in annex 1.

Therefore, comparing the actually observed variance of the error of the forecast of L_A with the variance that is consistent with the observed values of the spread, given the "aggregate liquidity model", yields interesting insights into the underlying assumptions. To the extent that the latter variance turns out to be larger than the former variance, such that the liquidity effect is lower than predicted by the aggregate liquidity model, the more relevant seem to be the effects of risk aversion (violation of A1), idiosyncratic liquidity shocks after the close of the market (violation of A3), or heterogeneous liquidity forecasts (violation of A2). Yet, of course, a finding that the properties of the assumed aggregate liquidity forecast is inconsistent with the overnight spread, can also just indicate that the market is in reality applying a different (homogenous) forecast. Hence, we test different possibilities for how the market establishes its liquidity forecast, and try also to assess whether more than one of these forecasts are relevant at the same time in explaining the overnight spread, which will of course also indicate an obvious breach with A2.

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with very small reserve requirements and/or limited access to the interbank market. ECB[2002], confirms that these so-called "individual recourses" are normally of limited magnitude.

[6] When attempting to square out their individual liquidity imbalances, banks find out whether it is difficult or easy to find a counterpart with an opposite position and accordingly learn whether the market, in aggregate, seems to be short or long.

As an example of an other source of information which in practice seems to play a role in the market's assessment of P, one can mention the market's experience about the level of the overnight rate and the accumulated liquidity imbalance at the end of preceding maintenance periods. These seem to a large extent, to affect the market's expectations for the end of the prevailing maintenance period. That is if the overnight rate was relatively high/low at the end of previous maintenance period, a similar pattern is expected for the prevailing maintenance period.

Table 1: Regular timetable for relevant events and information releases up to and after the last MRO allotment.

Day (time)	**1** (15.30)	**2** (9.15)	**2** (11.15)	**3** (9.15)	**4** (9.15)	………	Last day, **N**, of MP (9.15 am)
Event	MRO Announcement	None	MRO allotment	MRO settlement	None	None	None
Information released by the ECB	Forecast of the ***daily average autonomous factors.***	Key liquidity numbers for day 1	Allotment amount	Key liquidity numbers for day 2	Key liquidity numbers for day 3	Key liquidity numbers for day t-1	Publication of liquidity numbers for t day N-1

Key liquidity numbers are: aggregate recourse to standing facilities, aggregate current accounts; reserve requirements and outstanding open market operations (historical numbers are also available).

Before proceeding to the definitions and assumed properties of the different components which are making up L_A, and which accordingly defines P and the overnight spread, a few remarks about conventions and notation.

- For a given maintenance period, we will by *N* denote the total number of days from (and including) the announcement day of the last MRO till the end of the maintenance period, and by $t \in \{1,...,\mathrm{N}\}$ a time-index tracking the number of ***calendar*** days since the MRO announcement day, which is day 1. That is by "day t", we mean the day which falls t-1 calendar days after the announcement of the last MRO.
- When we, for a vector x, write Σ^{x}, we refer to the sum of the elements of this vector.
- Unless specifically mentioned, we will not distinguish trading and non-trading days, but will assume that the market acts as if the balance sheet of the Eurosystem could also change on non-trading days. This simplifying assumption turned out, if anything, to even improve the empirical results. Finally, we adopt the notation that a **top**-symbol to any of the vectors introduced below indicates the day for which the vector is defined, while a **sub**-symbol refers to the corresponding element of the vector.
- While, of course the empirical implementation of the below models take into account the occasional deviations from the "regular timetable" shown above in table 1 (See ECB[2004a] and ECB[2004b]), the following presentation assumes, for the sake of simplicity, that this sequence is indeed followed in each maintenance period.

3 The relevant liquidity components

Consider now, one by one, the liquidity components, which are affecting L_A, and which are revealed to the market through the flow of information summarized in Table 1. Each vector is defined per maintenance period.

Aggregate liquidity needs of the banking system

a   a (N×1) vector containing the actual autonomous factors for each day since the MRO announcement day till the end of the maintenance period. Only the autonomous factors up till time t-1 are known at time t. The next section describe two fundamentally different assumptions, which are both implemented empirically, concerning how the market is actually establishing its autonomous factors forecast.

rr   a scalar denoting, the sum of the accumulated reserve requirements over the whole maintenance period. The accumulated reserve requirements are given by the daily average reserve requirements (published by the ECB) multiplied by the number of days in the maintenance period. Reserve requirements are always known with certainty after the last MRO.

xr   a scalar denoting excess reserves, which are defined as the difference between the accumulated amount of reserve holdings and the accumulated amount of reserve requirements. In principle, the existence of excess reserves is at odds with the assumptions A3 and A4, which imply that market participants would, as a minimum, place any excess liquidity on the deposit facility, and not leave it as unremunerated excess reserves on their current account. However, taking also into account that excess reserves are in the Eurosystem relatively limited (ECB(2002)), rather constant through time, and independent of the level of interest rates (see Bindseil et al (2003)), we treat them here as an *exogenous* liquidity need, which does not reflect uncertainty about actual end of day positions that are, in this context, sufficient to challenge the two assumptions. As mentioned above, the empirical results cast some more light on the apparent validity of these assumptions. Excess reserves are not known, neither by the ECB nor by the market, until after the end of the maintenance period. For the sake of simplicity, we assume that the market forecasts excess reserves by applying the simple model published in ECB[2002],[7] and that the market assumes the ECB to apply the same forecast when taking its allotment decision.

Aggregate supply of liquidity to the banking system through open market operations

m   a scalar denoting the allotment amount in the last MRO of the maintenance period.

$\tilde{m}$ a (N×1) vector containing, for each day since the MRO announcement till the end of the maintenance period, the amount of other outstanding open market operations. This comprises the MRO allotted in the previous week, the outstanding longer term refinancing operations, other open market operations, and until the settlement of the last MRO: the MRO that was allotted two weeks earlier.

[7] According to this "model", daily average excess reserves are EUR 0.64 billion when the maintenance period ends on a weekday, EUR 0.71 billion if the period ends on a Saturday, and EUR 0.9 billion if it ends on a Sunday.

We assume that the ECB, after the last MRO, will not conduct any open market operations with settlement in the same maintenance period. Therefore, after the allotment of the last MRO, the total liquidity supply through open market operations, given by $\tilde{m}$ and m, is assumed to be known with certainty by the market. The handling of the two exceptional cases in which the ECB in fact conducted fine-tuning operations are further discussed below.

Recourse to standing facilities

Before the last day of the maintenance period, some exogenous supply (demand) for liquidity may result from counterparties' so-called "individual" recourse to standing facilities, normally owing to minor unexpected idiosyncratic liquidity shocks. The $((N-1)\times 1)$ vector, L, contains the daily net recourse to the deposit facility (i.e. recourse to the deposit facility less recourse to the marginal lending facility) from the MRO announcement day till the penultimate day the maintenance period. As discussed in section 2, such recourse is in principle at odds with the assumption A4, that an unexpected liquidity shock can not lead to an overdraft or a fulfillment of reserve requirements before the end of the maintenance period. However, relatively large recourses to standing facilities, particularly in accumulated terms, may occur for other reasons than unexpected liquidity shocks.[8] Moreover, while consistently with A4, the likelihood of daily overdrafts or early fulfillment of reserve requirements may be sufficiently small to not affect the overnight rate (again this is further discussed below, in the light of the empirical results), the accumulated effect on the aggregate liquidity imbalance can indeed be sufficient to affect the overnight rate. Therefore, it is important to include the individual recourse to standing facilities, that has occurred since the MRO allotment, into the calculation of the expected liquidity imbalance. We make the assumption that the individual net recourse to standing facilities is zero on the last day of the maintenance period so that the accumulated liquidity imbalance, L_A, indeed becomes equivalent to the observed net recourse to the deposit facility on that day.[9] All elements of L have an expected value of zero.

How the different components sum up to L_A

The following "liquidity table" summarizes how the banking system's daily reserve holdings, dr_t, results, through the balance sheet identity, from the liquidity needs, the supply of liquidity through open market operations, and the recourse to standing facilities. The liquidity table also shows, how the accumulated liquidity imbalance, then results from the condition that the accumulated amount of daily reserve holdings over a maintenance period must be equivalent to the accumulated amount of reserve requirements and excess reserves. In this regard, we define sr as the sum of reserve holdings from the first day of the maintenance period till the day before the announcement of the last MRO.

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[8] For instance individual banks may aim to alter the aggregate liquidity conditions and hence the overnight rate by taking large recourse to standing facilities. See Ewerhart et al. [2004].

[9] Without this assumption we would not be able to distinguish the aggregate liquidity imbalance from the individual recourse to standing facilities on the last day of the period.

Liquidity table	Other OMOs	+	Last MRO	−	Aut. factors	−	Net deposict facility	=	reserve holdings	=	required & excess reserves (not yet fulfilled)
MRO ann. day	$\tilde{m}_1$	+	0	−	a_1	−	L_1	=	dr_1		
MRO all. day	$\tilde{m}_2$	+	0	−	a_2	−	L_2	=	dr_2		
MRO settl. day	$\tilde{m}_3$	+	m	−	a_3	−	L_3	=	dr_3		
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$		
Last day of MP	$\tilde{m}_N$	+	m	−	a_N	−	$(0+\mathbf{L_A})$	=	dr_N		
Totals	$\Sigma^{\tilde{m}}$	+	$n \cdot m$	−	Σ^a	−	$(\Sigma^L + \mathbf{L_A})$	=	$\sum_{i=1}^{N} dr_i$	=	$rr + xr - sr$

From this it follows that

$$L_A = \Sigma^{\tilde{m}} + n \cdot m - \Sigma^a - \Sigma^L + sr - rr - xr \tag{3.1}$$

Accordingly, the expected value of L_A follows from the forecast of the autonomous factors, excess reserves, and the individual recourse to standing facilities. In the same vein, the variances of the errors of these individual forecasts also define the variance of the difference between the expected and the actual value of L_A. For the sake of simplicity, and avoiding too many parameters, we do not explicitly model the uncertainty stemming from individual recourse to standing facilities and excess reserves, and the following proceeds as if there was no such uncertainty (yet, it will be explained in section 5.2 how we have *implicitly* modeled this uncertainty). Under this assumption, the only uncertainty about the aggregate liquidity imbalance stems from the uncertainty about the autonomous factors. The following section describes two different approaches for how the market is establishing its autonomous factors forecast. Both are implemented empirically.

4 Two approaches for the autonomous factors forecast

4.1 The benchmark approach

In the first and simplest approach for the autonomous factors forecast (which we refer to as the "*benchmark approach*"), the market is assumed to have available on each day an *exogenous* forecast φ^t of the autonomous factors up till the end of the maintenance period. This forecast comprises all relevant information and has the error $\mu^t = a - \varphi^t$. The expected liquidity imbalance follows trivially by inserting this forecast into the liquidity table, while the variance of the error of this expectation is given by the variance, $\overline{\sigma}_N^{\mu^t}$, of the accumulated error, Σ^{μ^t}. As detailed in section 2 the overnight spread then follows from these two moments of L_A.

4.2 The signal extraction approach

In the other, somewhat more complicated approach, which we refer to as the *"signal extraction approach"*, the market is only assumed to have available an exogenous autonomous factors forecast on the MRO announcement day, the day on which the ECB publishes the average of its own autonomous factors forecast. The exogenous forecast of the market is denoted by φ^1. On the subsequent days the market *implicitly* establishes an autonomous factors forecast, updating φ^1 by extracting signals from the ECB's decision to allot m, knowing that this decision is based on 1) an autonomous factors forecast different from φ^1 [10] (see ECB[2002]), and 2) a certain liquidity target of the ECB, i.e. a certain target value for L_A. Specifically we define

- ε a $(N\times1)$ vector of differences between the ECB's forecast of the autonomous factors on the MRO *allotment* day, φ^{ECB}, and the market's forecast, φ^1, of the autonomous factors as per the MRO *announcement* day. Hence, $\varepsilon = \varphi^{ECB} - \varphi^1$.
- η a $((N\text{-}1)\times1)$ vector containing the same day, the one-day ahead, the two-days ahead, … the (N-1) days ahead error of the ECB's forecast of the autonomous factors on the MRO allotment day. Hence, $\eta = a - \varphi^{ECB}$
- γ a scalar denoting the liquidity target of the ECB. That is the expected net recourse to the deposit facility resulting from φ^{ECB}, the ECB's autonomous factors forecast on the *allotment* day. γ is assumed, *from the point of view of the market* to follow a gausian white noise process with zero mean and variance σ_γ. [11] Moreover, γ is assumed to be set by the ECB with a pure quantitative target in mind and not also with an (for the market: unobservable and stochastic) interest rate target, r^*, in mind. [12]

From these definitions it follows that

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[10] Note that it is most natural to focus on the difference to the market's forecast on the announcement day and not to that on the allotment day, because the ECB (in the sample period) only published its autonomous factors forecast on the MRO announcement day. As explained below, we assume that the market fully adopts this forecast.

[11] Due to the ECB's rounding of the allotment amount in billions of euro, γ is in principle not normally distributed. However, in practice, the assumption of a normal distribution is reasonable, and our attempts to relax this assumption did not lead to any changes of parameter estimates nor of the goodness of fit.

[12] Otherwise a non-linear and complex relationship between γ and r^* would have to be taken into account, in order to fully represent the information available to the market (See Bindseil[2001]), and the linear signal extraction presented in the following would not be sufficient.

(4.1) $\quad a = \varphi^1 + \varepsilon + \eta$, and

(4.2) $\quad L_A = \gamma - \Sigma^{\eta}$

Moreover, defining the benchmark allotment amount, m^{bench}, [13] as the allotment which on the basis of φ^1 and L_1 (the individual net recourse to the deposit facility on the MRO announcement day) results in an expected liquidity imbalance of zero, we have the identity that

(4.3) $\quad m - m^{bench} = \dfrac{\gamma + \Sigma^{\varepsilon}}{N-2}$,

On the basis of its knowledge about the linear structures expressed by (4.1) and (4.3), the market is then assumed to perform *linear signal extraction* in order to calculate expected values of the unobserved variables (ε, η and γ), given the observed variables (a, φ^1, m and m^{bench}), and on the basis of that implicitly update its forecast of the autonomous factors. The implicitly updated autonomous factors forecast on time t follows as $\varphi^t = \varphi^1 + E_t(\varepsilon + \eta)$, and the expected liquidity imbalance can then either be calculated by inserting this into the liquidity table, or alternative from (4.2).

In order to perform linear signal extraction, the market is assumed to know, or at least to have some perception of, the covariance ($\sigma_\gamma, \sigma_\varepsilon, \sigma_\eta$) of the unobservable variables, which we assume to be mutually independent. Specifically, knowing the linear relationship ($Z = AX$) between observable (Z) and unobservable variables (X), which follows from equation (4.1) and (4.3), the expected value of the unobservable variables is calculated by minimizing the expected mean squared error. That is:

(4.4)
$$Z = \begin{pmatrix} m - m^{market}_{bench} \\ a_1 - \varphi^1_1 \\ a_2 - \varphi^1_2 \\ \cdots \\ a_{t-1} - \varphi^1_{t-1} \end{pmatrix} = \begin{pmatrix} \frac{1}{N-2} & \frac{1}{N-2} & \frac{1}{N-2} & .. & \frac{1}{N-2} & 0 & .. & 0 \\ 0 & 1 & 0 & .. & 0 & 0 & .. & 0 \\ 0 & 0 & 1 & .. & 0 & 1 & .. & 0 \\ .. & .. & .. & .. & .. & .. & .. & .. \\ 0 & 0 & 0 & .. & 1 & 0 & .. & 1 \end{pmatrix} \cdot \begin{pmatrix} \gamma \\ \varepsilon_1 \\ \varepsilon_2 \\ .. \\ \varepsilon_N \\ \eta_2 \\ .. \\ \eta_{t-1} \end{pmatrix} = AX$$

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[13] It follows directly from the liquidity table that the benchmark allotment amount can be calculated as

$$m^{market}_{bench} = \frac{rr + xr - sr + L_1 + \Sigma^{\varphi^1} - \Sigma^{\tilde{m}}}{N-2}$$

This definition is similar to the one applied by the ECB, who, since March 2004 (i.e. after the end of the sample period underlying this study), has been publishing the benchmark allotment, both when taking into account the autonomous factors forecast on the announcement day and on the allotment day.

Minimizing the expected mean squared error, $E\left((X - E(X|Z)) \cdot (X - E(X|Z))'\right)$, yields

(4.5) $\quad E(X|Z) = B'Z$ where $B = (AE(XX')A')^{-1} AE(XX')$ and

(4.6) $\quad E(X - E(X|Z))^2 = B'AE(XX')A'B - 2B'AE(XX') + E(XX')$,

(4.7) $$E(XX') = \begin{pmatrix} \sigma_\gamma & 0 & 0 \\ 0 & \sigma_\varepsilon & 0 \\ 0 & 0 & \sigma_\eta \end{pmatrix}$$

The signal extraction at time t provides expected values and variances of γ and the first t-1 entries of ε and η. The calculation of expected values and variances of the remaining N-t+1 entries follow from σ_ε and σ_η. From this the overnight rate is calculated from (2.2).

In principle one could imagine a combination of the benchmark and the signal extraction approach: on each day the market has available an exogenous autonomous factors forecast as envisaged by the benchmark approach, while at the same time the market extracts additional information from the ECB's allotment decision as envisaged by the signal extraction approach. For instance, the market could each day establish an exogenous forecast of the autonomous factors by mechanically executing a time series model, and on the basis of its knowledge about the covariance of the error of this forecast, as well as its knowledge about the covariance of ε, η and γ (the ECB's allotment policy), extract further information about the autonomous factors. This would give the market a non-zero expected value of the error of its exogenous autonomous factors forecast, which in the benchmark case is otherwise assumed to have a zero mean. In this case the dimension of the signal extraction problem increases (equations corresponding to $\varphi^t - \varphi^1 = \varepsilon + \eta - \mu^t$ have to be added to (4.4)). However, the results of this combined approach did not give any better results than the simple signal extraction approach, and we therefore refrain from elaborating further on this approach and the results hereof.

4.3 Exogenous autonomous factors forecast

Each of the two modeling approaches are implemented with different assumptions concerning how the market establishes the exogenous forecasts, φ, of the autonomous factors. The following describes three such assumptions as well as for which of the two modeling approaches they appear to be relevant, and accordingly have been implemented. Note, in this regard, that a given exogenous forecast leads to different actual forecasts when applied in the two modeling approaches. In the benchmark approach the exogenous forecast is applied directly on each day, while in the benchmark approach the exogenous forecast as per the announcement day is used on each day, corrected with the term $E_t(\varepsilon + \eta)$. Therefore, all combinations of modeling approach and exogenous autonomous factors forecast represent unique models on the spread that are not nested into each other, and in which the parameter values will as a rule be different (the parameterizations are described below).

Note that for all forecasts we make sure that they, on the MRO *announcement day* (and not necessarily on other days), result in the same average forecast of the autonomous factors as the average, $\overline{\varphi}$, published by the ECB. That is $\frac{1}{N}\Sigma^{\varphi_t^1} = \overline{\varphi}$. The three exogenous forecasts are:

1. **Constant forecast**. First, we suppose that the market does not make any assumption about the future path of the autonomous liquidity factors. On time t, the forecast for each of the reaming N-t days of the maintenance period is simply equivalent to the average value published by the ECB.

   The application of this very simple forecast is most intuitive for the signal extraction approach in which the market is assumed to only have available an exogenous forecast on the MRO announcement day, and to implicitly update this forecast as it learns more about the ex post autonomous factors. Yet, we will also implement this forecast for the benchmark approach.

2. **Time series forecast**. Second, we assume that the market, *on each day,* establishes its forecast of the future path and the level of the autonomous factors by combining a simple time series forecast with the forecast of the average autonomous factors published by the ECB. The time series model is based on the aggregate daily autonomous factors, taking into account a trend, autoregressive terms and a set of dummies, accounting for different calendar effects. Annex 2 describes the model in detail, and how the forecast of this model is combined with the forecast published by the ECB.

   This forecasting procedure appears most logic in the context of the benchmark approach, which indeed assumes that the market relies on a *daily* exogenous forecast of the autonomous factors. Yet, we also test it for the case of the signal extraction approach. In this case, only the forecast as per the MRO announcement day is relevant, providing market participants with a "yard stick" of the path of the autonomous factors until the end of the maintenance period.

3. **ECB forecast.** Third, we assume that the market on each day possesses the same forecast as the one applied by the ECB in its daily liquidity management routines. This forecast seems only relevant in the context of the benchmark model, and not in the context of the signal extraction model: if the market has indeed readily available the same forecast as the ECB, the market would not gain any additional information by extracting signals from the ECB's allotment decisions as envisaged by the signal extraction approach.

On the basis of our data sample, which is described below, Annex 3 provides an overview of the mean squared errors of the different forecasts for different forecast horizons and different number of days between the actual forecasting day and the MRO announcement. It is clear from the annex that the ECB forecast is generally much better than the two other forecasts. Related to that, the quality of the two other forecasts declines with the number of days since they have been calibrated to the ECB forecast.

5 Empirical approach

When taking the observable liquidity components, including the exogenous autonomous factors forecast as given, the only parameters linking [via (2.1)] the aggregate liquidity conditions to the overnight spread, are those describing the variance of the different liquidity components. After a brief description of the data underlying the empirical analysis, the following describes how these variances have been parameterized, and thereafter how parameter estimates have been obtained through a calibration procedure. This is followed by a short presentation of the methodology used for assessing the quality of the fit of the different models.

5.1 Data

We concentrate our study on the maintenance periods ranging from the introduction of the variable rate tender in July 2000 to the end of December 2003. Consequently, our dataset consists of 42 maintenance periods, sharing an identical 2% width of the corridor set by the standing facilities, and (normally) the same sequence of information made available by the ECB via wire services (see Table 1). However, in some of these maintenance periods, the ECB gave additional information to the market, such as an update of its forecast of the average autonomous factors on the allotment day, which implies that the signal extraction problem was slightly different than the one described above under the assumption of the "regular" maintenance period. Annex 4 describes these situations in detail, as well as how the regular signal extraction problem was amended accordingly. Focus is limited to the days falling at or after the settlement day of the last MRO.[14] In total we have 117 datapoints for the EONIA rate.

In addition we have on each day available, or have constructed, the above mentioned autonomous factors forecasts, including the one of the ECB. The latter allows us to calculate a unique time-series consisting in the exact liquidity target of the ECB in each MRO-allotment decision. Moreover, we have of course time-series for the other liquidity components described in the previous section. This includes current account holdings, recourse to standing facilities and reserve requirements. All quantities mentioned below are in EUR billions.

[14] The MRO *allotment* day is not included because it is unclear which information set is actually priced into the EONIA on that day: the information set available before the announcement of the MRO allotment at 11.15, or the information set available hereafter.

5.2 Parameterization

This subsection explains which parameterization of the covariance matrices of the different exogenous autonomous factors forecasts has been chosen for the benchmark and the signal extraction approach respectively. At the end it is also explained how a so-called "residual variance term" has been included, also with a view to capture uncertainty about excess reserves and individual recourse to standing facilities, which is not modeled explicitly.

The parameterization for a given forecast needs to be so that it spans the actually observed properties of the forecast errors. Specifically, it is key for both approaches that the parameterization allows to fit, for different values of N and t, the observed variance of the accumulated error over the horizon N-t. In addition, it is for the signal extraction approach key to allow for a satisfactory fit of the of the observed correlation between forecast errors for individual days, since this is used to calculate the daily updates of the forecasts. If the parameterization would not allow to well describe the observed properties of the forecast errors, the models would from the outset be inconsistent. Since the three exogenous autonomous factors forecasts have each different properties and they are updated differently in each of the two approaches, different parameterizations are, a priori, needed for each approach and for each exogenous forecast.

Yet, the same parameterization turned out to be able to provide a good fit of the observed covariance for *each* of the three forecasts as per the MRO *announcement day*, irrespectively of the number of days, N, until the end of the maintenance period. On this day, each of the forecasts are calibrated to the forecast published by the ECB and therefore has the same accumulated error. Only the correlation between errors on individual days is different. In the following we first describe and motivate this parameterization, and presents hereafter, how it is amended to also cover the forecast errors on other than the MRO announcement day.

The variance of the (1×N) vector, μ^1, of daily errors of the exogenous autonomous factors forecast *on the MRO announcement day*, the only day on which the signal extraction approach assumes the market to have available such a forecast, is assumed to be given by the sum of two elements, which are described by three parameters, $\alpha_e^\mu, \alpha_\rho^\mu$, and α_c^μ, that are in turn different for each of the three exogenous forecasts:

(5.1) $\mu^1 = \theta \cdot e + \omega \cdot \rho$,

where e and ρ are vectors of independent and identical normally distributed innovations with variance α_e^μ and α_ρ^μ respectively. The N×N matrices, θ and ω, are defined as:

$$\theta = \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ \alpha_c^\mu & 1 & 0 & \cdots & 0 \\ \alpha_c^\mu & \alpha_c^\mu & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \alpha_c^\mu & \alpha_c^\mu & \alpha_c^\mu & \cdots & 1 \end{pmatrix} \quad \omega = \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ \frac{-1}{N-1} & 1 & 0 & \cdots & 0 \\ \frac{-1}{N-1} & \frac{-1}{N-2} & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{-1}{N-1} & \frac{-1}{N-2} & \frac{-1}{N-3} & \cdots & 0 \end{pmatrix}$$

The first part, $\theta \cdot e$, of the error can be interpret as capturing the element that is due to an error in the forecast of the *overall level* of the autonomous factors all until the end of the maintenance period. Accordingly, this part of the error affects the variance of the accumulated error, Σ^{μ^1}. In contrast, the second part, $\omega \cdot \rho$, can be interpret as representing the errors on individual days, which are not due to a wrong assessment of the level of the autonomous factors, but simply reflects a wrong distribution of the forecast on individual days. These daily errors offset each other and does not affect the accumulated error.

It follows immediately from (5.1) that the covariance matrix for the error of the autonomous factors forecast *on the announcement day* is given by

(5.2) $\quad \operatorname{Var}\left(\mu^1\right) = \alpha_e^\mu \cdot \theta\theta' + \alpha_\rho^\mu \cdot \omega\omega'$

On the basis of this we derive the parameterizations of the relevant covariance matrices on other than the MRO announcement day.

First, consider the benchmark approach, which, in contrast to the signal extraction approach, assumes that the market has available an exogenous autonomous factors forecast on each day and not only on the announcement day. Therefore, we need a parameterization which can describe the covariance matrix of the ((N-t) ×1) vector μ^t of forecast errors for $t \geq 1$. As a first observation it is noted that only the variance of the *accumulated* error, Σ^{μ^t}, of the forecast on day t, and not the covariance between errors on individual days, matters in the benchmark approach. Therefore, we can in this approach leave out the element $\omega \cdot \rho$ from (5.1). Now consider the three different exogenous factors forecast.

For the ECB forecast the specification in (5.2) turned out to not only be able to fit the observed variance for any values of N, but also for any value of N-t. Hence, for *the benchmark model with ECB forecast*, we only need two parameters, $\alpha_e^\mu, \alpha_c^\mu$, in order to describe the variance of the autonomous factors forecast error on any day, t, after the last MRO:

(5.3) $\quad \operatorname{Var}\left(\mu^t\right) = \alpha_e^\mu \cdot \theta\theta'$, where θ is defined as in (5.2), but with dimension (N-t)×(N-t) instead of N×N.

For the benchmark models with constant and time series forecasts of the autonomous factors, one more parameter is, however, needed. This stems from the fact that these forecast are "hybrids" in the sense that they, on other than the announcement day, are a mixture of the ECB forecast and some other forecast.

Hence, by construction these two forecasts have the same accumulated error as the ECB forecast on the MRO announcement day, while on subsequent days their quality is, as mentioned above, normally lower than that of the ECB. This we accommodate by introducing another parameter, α_q^{μ}, which allows the forecast quality to deteriorate with the number of days, t, since the announcement day. Hence, in the case of the *benchmark model with either time series or constant forecast*, we have three parameters, $\alpha_e^{\mu}, \alpha_c^{\mu}$, and α_q^{μ} describing the variance of the errors of the exogenous autonomous factors forecast.

(5.4) $\quad \mathrm{var}\left(\mu^t\right) = \alpha_e^{\mu} \cdot t^{\alpha_q^{\mu}} \cdot \theta\theta'$, where θ is defined as in (5.3)

Second, consider the signal extraction approach, in which we need parameterizations for ε and for the non-zero values of η.[15] By definition $\mathrm{Var}(\eta)$ is given by $\alpha_e^{\mu} \cdot \theta\theta'$ as estimated for the ECB-forecast. Since, in addition, we assume independence between ε and η and know that $\varepsilon + \eta = \mu$, the parameterization of $\mathrm{var}(\varepsilon)$ follows from the fact that $\mathrm{var}(\varepsilon) = \mathrm{var}(\mu) - \mathrm{var}(\eta)$. Acknowledging that this variance can strictly speaking not be entirely described by the parameterization in (5.2), because the first value of η is zero, we will for numerical convenience make the assumption that the variance of ε can indeed be described by (5.2), whereby the parameters for ε (i.e. $\alpha_c^{\varepsilon}, \alpha_e^{\varepsilon}, \alpha_{\rho}^{\varepsilon}$) are of course normally different from those of μ.

Residual variance

In the models presented above, the overnight spread follows uniquely from the properties of the relevant autonomous factors forecast and the natural spread. However, in practice, uncertainty about excess reserves and future individual recourse to standing facilities are also relevant within the aggregate liquidity model. In order to capture this uncertainty, we add in each model a term to the variance of the expected liquidity imbalance, which decreases towards the end of the maintenance period[16]. This "residual variance" term is, as explained in the following, also used as a gauge for the extent to which the assumptions underlying the aggregate liquidity model are valid, as discussed in section 2. Hence, for each model the total variance of the error of the expected liquidity imbalance is given by:

$\sigma_{L_A}^t = \mathrm{var}\left(\Sigma\mu^t\right) + \alpha^{\pi} \cdot (N - t)$, where α^{π} is a parameter to be estimated.

[15] It is recalled that the first element of η is zero, because the autonomous factors for the announcement day are known with certainty on the allotment day.

[16] In addition to this decreasing term, it would also appear reasonable to add a constant. This would be consistent with the fact that only uncertainty about the accumulated individual recourse to standing facilities decreases towards the end of the maintenance period, while uncertainty about excess reserves is independent of the number of days until the end of the maintenance period.

Table 2 summarizes the relevant parameters for each of the 5 unique modes that have been calibrated. Annex 5 explains how the so-called sample estimates of the variance parameters for the different forecasts as well as for the residual variance have been obtained. The sample estimates of the parameters are defined as those which provide the best fit of the actually observed properties of the different liquidity components. It is clear from the annex that the chosen parameterization actually allows for a satisfactory fit of the observed variances and covariances of the errors of all three forecasts, for different combinations of N and N-t. Accordingly, a potentially dissatisfactory fit of the overnight rate should not be due to unrealistic constrains imposed by the parameterizations.

Table 2: Parameters in each of the calibrated models.

		Modelling approach	
		Benchmark	Signal extraction
Exogenous Autonomous factors forecasting approach	Constant	$c, \alpha_c^\mu, \alpha_e^\mu, \alpha_q^\mu, \alpha^\pi$	$c, \alpha_c^\varepsilon, \alpha_e^\varepsilon, \alpha_\rho^\varepsilon, \alpha_c^\eta, \alpha_e^\eta, \sigma_\gamma, \alpha^\pi$
	Time series	$c, \alpha_c^\mu, \alpha_e^\mu, \alpha_q^\mu, \alpha^\pi$	$c, \alpha_c^\varepsilon, \alpha_e^\varepsilon, \alpha_\rho^\varepsilon, \alpha_c^\eta, \alpha_e^\eta, \sigma_\gamma, \alpha^\pi$
	ECB	$c, \alpha_c^\mu, \alpha_e^\mu, \alpha^\pi$	N/A

5.3 Calibration procedure

Parameter estimates are for each model obtained by calibrating the observed liquidity components to the overnight spread by use of (2.1). When allowing the parameters to vary freely, we obtain an estimate of the so-called *unconstrained* parameters, providing the best measure of the actually observed liquidity effect. Conversely, when preventing the parameters from deviating "too much" from their sample estimates, we obtain the so-called *constrained* parameter estimates, which, of course, may not necessarily provide an optimal fit of the overnight rate. Comparing the goodness of fit (of the overnight rate) resulting from the two parameter estimates yields, as discussed in section 2, interesting insights into the validity of the aggregate liquidity model.

Specifically, the models are calibrated via a least squared error approach,[17] using the following general loss function, consisting in a weighted average of the mean squared error of the spread and the mean squared difference between the calibrated parameters ($P^{calibrated}$) and their sample estimates (P^{sample}).

[17] We don't make assumptions about the distribution of the error of the estimated spread, thereby ruling out the application of a maximum likelihood procedure.

(5.5) $$\text{loss} = \omega \frac{\sum_{i=1}^{M} (\text{Spread}_i^{\text{calibrated}} - \text{Spread}_i^{\text{observed}})^2}{M} + \frac{(1-\omega)}{X} \sum_{j=1}^{X} \frac{(p_j^{\text{calibrated}} - p_j^{\text{sample}})^2}{p_j^{\text{calibrated}}}$$

where M is the total number of observations, X is the total number of parameters, and $\omega \in \{1;0.5\}$ is a weight parameter.

Weights of 1 and 0.5 are considered. The weight of 1 results in unconstrained parameter estimates. We define the constrained parameters to be those resulting from the weight of 0.5, when a "punishment" is introduced whenever the parameters are deviating too much from their sample estimates. However, why in the latter case only consider the somewhat arbitrary choice of a weight of 0.5, and hence still allow the parameters to deviate somewhat from their sample estimates, instead of choosing a weight of 0? What we are looking for, by investigating the constrained parameters, is a gauge for the extent to which the observed liquidity effect can actually be explained within the aggregate liquidity model. The parameters which are in reality relevant are those *perceived* by the market in each individual MRO. Obviously, these perceived parameters can in practice be rather different from the sample estimates derived here (see Annex 5), and may particularly not be constant, but varying from MRO to MRO. On the other hand, if indeed the "aggregate liquidity model" is relevant, it would be clearly unreasonable to believe that the parameters perceived by the market should in the long run, on average, be far away from the sample estimates which the market learns about in the course of time. When testing different possibilities, we found that a weight of 0.5 seems to successfully prevent "extreme" parameter values, ensuring that they are quite close to the sample estimates.

Only the parameters describing the variance of explicitly modeled quantities (i.e. γ, ε, η, and μ) are included in the second part of the loss function (i.e. the part receiving the weight (1-ω)). The constant, c, for the natural spread, and the parameter, α^{π}, for the residual variance, are both omitted. While, there is obviously no reasonable way in which to constrain the natural spread, one could also have constrained the parameter, α^{π}. However, having in mind that this uncertainty is in any event comparatively limited (see annex 5), we refrain from constraining this parameter, using instead its unconstrained value (in the case of otherwise constrained parameters) as a gauge for the validity of the aggregate liquidity model. Large values of this parameter indicates that the assumptions (particularly A1, A2 and A3) underlying the aggregate liquidity model may be put into question. Related to that, leaving this parameter unconstrained, also allows to draw the rather interesting conclusion that particularly the benchmark model with constrained parameters seems to provide a good fit of the spread, as long as we just allow for this additional term in the variance. This is further illustrated and elaborated below.

In the following, we report the estimated parameters resulting from both the case of constrained and unconstrained parameters.[18]

5.4 Methodology for assessing "goodness of fit"

We primarily measure the goodness of fit of a given model through the extent to which it can explain the EONIA spread. This can not be assessed by simply comparing the loss functions of the different models, which contains different parameters and weights, but should rather rely on the mean squared error of the fitted spread. Yet, as a direct alternative to that, we regress, for each of the models, the fitted spread on the realized spread, and subsequently calculate and compare the adjusted in-sample coefficients of determination, i.e. the adjusted R^2. [19]

For each model, we also calculate the adjusted R^2 when either excluding, or limiting focus to, the following two sub-samples, when by far the largest absolute values of the spread have been realized: 1) the last day of the maintenance period; and 2) the maintenance periods in which underbidding occurred without the ECB subsequently deciding on a rate cut. This provides insight into the robustness of the models, i.e. whether they only fit the spread on days with rather extreme volatility of the overnight rate or whether they can also explain the spread on other, more quite, days.

Additionally, the intercept and the slope of the above regression gives information on the extent to which a given model is biased. Only if the intercept and the slope are insignificantly different from zero and one, respectively, a model is unbiased for all values of the spread.[20] Therefore, for each model we also make a Wald test for the joint hypothesis that the intercept is zero and the slope is one.

All the relevant regressions, R^2, and Wald tests are reported in Table 3. Moreover, Chart 1 depicts scatter plots of the realized and fitted spread for a selected number of models.

Finally, in order to further check the parameters stability and the absence of structural breaks, we make a cross-validation. Using the independence property of every maintenance period for the model estimation, we sequentially exclude one maintenance period from the data sample and check that parameter estimates remain at the same level. The results of this are reported in Chart 6. Finally, Table 4 shows some basic properties of the residuals. In line with other studies using daily data (e.g. Würtz[2003]), the residuals turned out to be highly correlated. Accounting for this with a moving average does not qualitatively affect

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[18] In the case of unconstrained parameters (i.e. ω=1) for the signal extraction approach, which contains up to 8 parameters (without moving average), numerical problems arose, because of problems in identifying $\alpha_1^\varepsilon, \alpha_2^\varepsilon$ and σ_γ. Therefore, the following term was in this case, without any signs of a deterioration of the fit, added to the loss function: $\frac{(\sigma^{\gamma^{\text{calibrated}}} - \sigma^{\gamma^{\text{sample}}})^2}{\sigma^{\gamma^{\text{calibrated}}}}$

[19] By adjusting the R^2, we account for the two free parameters in the above regression. As further explained below, these parameters should, however, not contribute any explanatory power if the models are unbiased.

[20] If the intercept is significantly positive (negative), while the slope is one, then the corresponding model systematically overshoots (underestimates) the spread. If the slope is significantly greater (less) than one, while the intercept is zero, then the corresponding model overshoots (underestimates) the spread, particularly for large values.

any of the below conclusions. Nevertheless, for the sake of completeness, we have also reported the estimated parameters obtained with such a moving average (see Table 3).

6 Results

The following highlights the main conclusions that can be drawn from the estimated parameters.

Which model has the best explanatory power?

It follows from Table 3 that the benchmark approach with the ECB forecast provides by far the best fit of the spread after the last MRO allotment, with an adjusted R^2 of 57%. This R^2 is higher than what can be obtained when considering more simple regression models such as in Würtz(2003) and Bindseil&Nautz(2001),[21] suggesting that the modeling framework applied here is more precisely capturing the dynamics of the spread after the last MRO allotment.

Both the benchmark and the signal extraction approach with constant and time series forecast (i.e. a total of four different models) give the same, relatively modest, adjusted R^2 of around 30% to 35%. A priori, this suggests that the additional information resulting from the extraction of signals from the ECB's allotment decisions, as envisaged in the signal extraction approach, does not add any explanatory power. Also the fact that this approach, as it follows by comparing Chart 2a and 2b (particularly the lines for the sample estimates), let the variance depend on the total number of days N, between the MRO announcement and the end of the maintenance period,[22] does apparently not bring anything. The superior performance of the benchmark approach with the ECB forecast, on which focus of the following is mainly concentrated, indicates that the forecast mostly applied by the market is closer to that of the ECB's than to the rather poor constant and time series forecasts.

Yet, the question is whether any of the other models may capture some elements of the overnight spread that the benchmark approach with the ECB forecast does not catch. In order to investigate this, we regress at the same time the spreads resulting from a benchmark model and some other model, on the observed overnight spread. We then see whether the "other" model in question, to a significant extent, contain information which is complementary to that of the benchmark model[23]. The outcome of these regressions is shown in Table 5. The table, confirms the above finding that none of the signal extraction models add significantly to the explanatory power of the benchmark approach with ECB forecast. However, both the

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[21] Implementing variants of these models, without any moving average and/or lagged values of the spread, yields rather moderate values of the R^2 – in the magnitude of 30%.

[22] The larger the number of days between the announcement of the last MRO and the end of the maintenance period, the greater the variance.

[23] We are aware of the high correlation between spreads, fitted by the different models; the purpose of the regression is to explore the significance of the regression coefficients. That is, if "the other" model contains additional information, which is not replicated by the benchmark model, then the corresponding regression coefficient must be significant and the coefficient of the determination of the regressions should be higher than the R^2 of single models.

benchmark approach with time series forecast and constant forecast seem to (slightly) complement the benchmark approach with ECB forecast.[24] A priori this contradicts A2, that all market participants should have identical liquidity forecast.

The validity of the models: consistency between estimated and observed variances.

Thanks to large increases in the residual variance (i.e. α^{π}) to levels far exceeding the sample estimate for uncertainty about excess reserves and individual recourse to standing facilities, the fit of the models did, in general, not deteriorate when the parameters were constrained, although of course the point estimate of the different parameters changed substantially. While the sample estimate of α^{π} is as low as 0.37, it varies in the case of constrained parameters between 111 for the benchmark approach with ECB forecast and 1123 for the benchmark approach with time series forecast (see Table 3). This clearly shows that the variance of the error of the expected liquidity imbalance resulting from the unconstrained parameters [the *calibrated* variance] is by far exceeding the actually *observed* variance resulting from the sample estimates [the *observed* variance]. Taking the benchmark model with the ECB forecast as an example, the total calibrated variance is around 50 times greater than the observed variance following from this forecast (see Chart 2), and it is also much greater than the observed variance of the errors of any of the other autonomous factors forecasts. 7 days before the end of the maintenance period, the total calibrated variance is for the benchmark model with ECB forecast at around EUR 5000 billion, while the total observed variance is only around EUR 150 billion. On the last day of the maintenance period, the same numbers amount to around EUR 150 and EUR 3 billion respectively. Hence, while the liquidity imbalance actually expected by the market seems to be well in line with the ECB forecast, the market acts as if the uncertainty about this expectation is much larger than the actual variance of the errors of the ECB forecast. Also for the other 4 models, the estimated variances turned out to be much greater than the observed variances. This points to the conclusion that only the first and not the second moment of the aggregate liquidity conditions, resulting from the forecasts applied here, is consistent, within the aggregate liquidity model, with the observed behavior of the overnight spread.

As a consequence, the liquidity effect resulting from the unconstrained parameter estimates is much smaller than the one resulting from the sample estimates (see Chart 4). Using the latter, and again taking the benchmark model with the ECB forecast as an example, the spread would on the last day of the maintenance period reach the borders of the interest rate corridor already with a liquidity imbalance of around EUR 3 billion. According to the unconstrained parameter estimates, such an imbalance would only affect the spread by around 20 basis points. As discussed in section 2, this finding puts the assumptions A1, A2 and A3 into question.

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[24] When regressed on the actual spread together with the fitted spread from the benchmark model with ECB forecast, the two other benchmark models improve the R^2 by around 8%, i.e. the R^2 of the benchmark model with ECB forecast of 57% increases to 65%. This suggests that, at least part of the market, assumes the rather naive time series and constant forecast.

Chart 3 gives a more detailed overview of the observed liquidity effect, i.e. the one identified via unconstrained parameters. It is seen that the liquidity effect increases towards the end of the maintenance period, which is a result of the declining uncertainty about the accumulated liquidity imbalance (see Chart 2). The liquidity effect is for the benchmark approach with the ECB forecast, somewhat higher than it has been found to be in other studies. On the last day of the maintenance period, it is estimated to be 0.07, while on the penultimate day, and the day before that, it amounts to around 0.038 and 0.026, respectively. This compares to the estimate of 0.044 in Würtz[2003] and 0.022 in Bindseil & Nautz (2001) for the liquidity effect on the last day of the period. For the other, more noisy models, the liquidity effect was lower, varying between 2 and 4 on the last day of the maintenance period.

The natural spread

The natural spread (i.e. the constant) varies substantially across the different models, ranging from around 2 basis points in the benchmark model with constant forecast to around 15 basis points in the signal extraction model with constant forecast. Moreover, the natural spread is rather non-robust with regards to outliers, particularly underbidding maintenance periods (see Chart 6a and 6e). In the case of the benchmark model with ECB forecast, the estimate of the natural spread is 5.7 basis points when considering the whole sample period, while it varies between 4 and 7 basis points when sequentially leaving out one maintenance period (Chart 6e).

Although, the above mentioned (unreasonable) estimate of 15 basis points is probably due to the exceptionally bad performance of the particular model in underbidding maintenance periods (see the R^2 reported in Table 3), the above illustrates that the estimation of the natural spread after the last MRO allotment is noisy, highly dependent on the actual specification of the model. Yet, the bulk of the estimates for the natural spread identified here seem to concentrate around the level of 5 basis points, i.e. close to the level identified by the benchmark model with the ECB forecast. There were for none of the models any evidence that the natural spread should decline or increase towards the end of the maintenance period, which would otherwise violate the martingale property. This was confirmed by several (insignificant) attempts to parameterize such patterns and by the fact that the level of the residuals do not seem to follow any specific pattern, but are on each day fairly concentrated around zero (see Chart 5 for some examples).

Robustness of the models

All models yield somewhat different results when separately considering each of the sub samples, consisting either of underbidding episodes only, the last day of the maintenance periods only, or the remaining "more quite" days only. This, however, does not appear to be detrimental to the assessment that the parameter estimates are generally rather robust.

Firstly, the R^2 is higher when considering the two "volatile" sub samples, comprising the underbidding episodes and the last day of the maintenance periods (see Table 3). For instance, the benchmark approach with ECB forecast and constrained parameters has an R^2 of 61% when only considering the last day of the maintenance periods and 52% on all other days. Considering only underbidding maintenance periods the R^2 is 49%, while in all other (but these) periods, it amounts to only 30%. It is, however, not surprising that the models mainly manage to explain the spread on days where the signal from the aggregate liquidity conditions is known to be strongest. On days which are neither the last of the maintenance period nor falling after an underbidding earlier in the period, the signal from the models considered here is clearly less pronounced, and the (modest) fluctuations of the overnight rate are therefore probably mainly driven by other factors, such as the market's experience from the previous maintenance periods, etc. This is also consistent with the persistence in the residuals (see table 4), and the fact that fitting a moving average can boost the R^2 up to 70%.

Secondly, the Wald tests in Table 3 show that all models turned out to be biased when only considering underbidding maintenance periods. In these maintenance periods, the overnight spread was probably mainly on a high level, because of a pre-perception in the market that the ECB would indeed have a tight liquidity target. Hence, even if the liquidity conditions were in fact not that tight, the market was nevertheless convinced about the opposite, and the overnight spread was systematically higher than identified by the models. However, attempts to fit additional parameters for the underbidding maintenance periods did not lead to any practical changes of the other parameters. Moreover, when considering all observations at the same time and/or limiting focus to the last day of the maintenance periods, there is no longer evidence of any basis. Therefore, we conclude that the bias of the models in underbidding maintenance periods, should not lead to mistrust in the parameter estimates.

The cross validations, examples of which are shown in charts 6, show that, as expected, the constrained parameter estimates are generally more robust than the unconstrained estimates. While the parameters are somewhat sensitive to the underbidding episodes in February, April, and October 2001 and in December 2002,[25] they seem generally to be rather robust, only changing moderately if single maintenance periods are left out.

7 Conclusion

This paper has investigated the extent to which the aggregate liquidity conditions can explain the dynamics of the euro area overnight rate after the last MRO-allotment. For that purpose a model has been formulated, making the classical assumptions leading to the martingale property of the overnight rate, as well as the assumptions that market participants (A1) are risk neutral, (A2) can clear all possible liquidity shocks with each other before the close of the market, and finally (A3) share the same information set. Different approaches for how the market forecasts the aggregate liquidity conditions are explored. According to the so-called benchmark approach, the market is assumed to have available each day an exogenous forecast of the autonomous liquidity factors, while in the signal extraction approach this forecast is implicitly adjusted with additional information that can be extracted from the ECB’s allotment policy. Within these two general approaches, the relevance of three specific autonomous factors forecasts is tested investigated: a naive constant forecast, a time series forecast, based on a simple time series model, and finally the ECB forecast, i.e. the one assumed by the ECB in its daily liquidity management. The resulting different models are calibrated using parameterizations that are carefully selected with a view to ensure that they can indeed describe the actually observed variance of the different liquidity components, thereby allowing for an interpretation within the aggregate liquidity model. We investigate the extent to which the models are consistent with the underlying assumptions by comparing the observed variances of the liquidity components to the variances resulting from the calibrated parameters. Using a sample period ranging from June 2000 to December 2003, the following conclusions are reached.

[25] As discussed above, this applies particularly to the natural spread (see chart 8b and 9b).

First, the dynamics of the overnight rate is consistent with the market having a rather precise forecast of the autonomous factors (i.e. the forecast applied by the ECB) but is clearly inconsistent with the observed variance of the errors of this forecast. That is, the level of uncertainty about the accumulated liquidity imbalance which, within the aggregate liquidity model, is consistent with the observed overnight rate, clearly exceeds what can be justified by the errors of this relatively precise liquidity forecast. This points to the conclusion that some of the assumptions behind the aggregate liquidity model (most likely A1, A2, or A3) are not valid for the case of the euro area.

Second, even though the assumptions underlying the aggregate liquidity model may be questioned, this model can, via the benchmark approach and only 3 parameters, explain 57% of the variation of the overnight spread after the last MRO. Assuming the high-quality ECB forecast of the autonomous factors, an EUR 1 billion expected liquidity imbalance impacts, within the benchmark approach, the overnight rate by 7 basis points on the last day of the maintenance period. Assuming the other more noisy autonomous factors forecasts, the liquidity effect turned out, on the last day, to be only between 2 and 4 basis points.

Third, within the modeling framework and parameterizations used here, it turned out that the signal extraction approach did not add any additional explanatory power to the much simpler benchmark approach. Accordingly, it seems that the market does not extract any significant information from the ECB's allotment policy. Nevertheless, applied within the benchmark approach, the rather naive constant and time series forecasts were found to provide some information about the accumulated liquidity imbalance, which are complementary to the ECB-forecast. This is consistent with the idea that market participants, in practice, have heterogeneous liquidity forecasts. Obviously this challenges A3.

Fourth, bearing in mind that the estimate of the constant term is somewhat noisy, the natural spread is found to be at around 5 basis points after the last MRO allotment. No violations of the martingale property of the overnight rate could be found.

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