Annex 1: Heterogeneous autonomous factors forecast

This annex illustrates that the liquidity effect is, ceteris paribus, smaller than predicted by the aggregate liquidity model, if we relax the assumption that individual banks have identical liquidity forecasts. Specifically, it is demonstrated that, if individual banks have heterogeneous liquidity forecasts, then the overnight spread actually prevailing in the market, s^m, is in absolute terms less than the overnight spread, $\bar{s}$, resulting from the properties of the average of the individual forecasts.

For this purpose we consider a one day maintenance period, consisting of two trading sessions, as in the aggregate liquidity model. One (real) trading session before the actual aggregate liquidity imbalance is known by the market and another (trivial) one hereafter. The autonomous factors are assumed to be the only source of uncertainty which affects the overnight rate.

In the first session, market participants are assumed *on average* to expect an aggregate liquidity imbalance $\overline{\varphi}$ at the end of the maintenance period (i.e. the next trading session). This expectation follows from the average of their individual autonomous factors forecasts. The error of the average forecast is given by $\eta \sim n(0, \sigma_\eta)$, whereby in total

$$L_A = \overline{\varphi} + \eta$$

In the second trading session, the overnight spread follows trivially from the actually realized aggregate liquidity imbalance, L_A. If $L_A > 0$, there is an aggregate liquidity surplus, and we assume that all banks will have to "square their liquidity position", i.e. to cover any negative balances or to place any positive balances, at the rate of the deposit standing facility (i.e. the spread equates minus one). Conversely, if $\eta < 0$, there is a shortage, and all banks have to square their liquidity position at the rate of the marginal lending facility (the spread equates plus one).

There are n individual banks, indexed by i. Each bank is making its own forecast of the autonomous factors, resulting in different forecasts, φ_i, of the aggregate liquidity imbalance. Specifically, we assume that,

$L_A = \varphi_i + \eta_i$, where $\eta_i = \eta + \varepsilon_i$; $\varepsilon_i \sim n(0, \sigma_\varepsilon)$; ε_i and η are independent.

It follows immediately, that the error ($\eta + \varepsilon_i$) of the i'th bank's forecast, has a greater variance ($\sigma_\varepsilon + \sigma_\eta$) than the error of the market's average forecast.

Inserting Bank i's expected liquidity imbalance, $\varphi_i = \overline{\varphi} - \varepsilon_i$, into the general equation (2.2) for the overnight rate, results in a *"risk neutral"* spread of:

$$s_i = 1 - 2N\left(\frac{\overline{\varphi} - \varepsilon_i}{\sqrt{\sigma_\eta + \sigma_\varepsilon}}\right)$$, where N is the standard normal cumulative distribution.

Assuming that bank i attaches no costs to being either short or long in the first trading session, this denotes the spread at which the bank is indifferent whether it obtains the liquidity in the first or in the second trading session. However, the assumption that no such costs exist contradicts the existence of a money market equilibrium: banks with different autonomous factors forecasts would trade infinitely large amounts with each other. Therefore, we assume that it is increasingly costly for banks to position themselves differently from zero, the most obvious reason for this being risk aversion. Specifically, we assume that the cost, for each bank, of being either short or long by an amount of x after the first trading session is given by ωx^2, where ω is a parameter (which is unimportant for the present analysis).

Given the spread s^m, actually prevailing in the market during the first trading session, each bank chooses a position, x_i, which maximizes its overall utility. Using the convention that a positive value x_i reflects a long position, we have the following utility function for the i'th bank.

$$U(x_i) = -\omega x_i^2 + \left(s^i - s^m\right)x_i$$

First and second order conditions yield the result that

$$x_i = \frac{s_i - s^m}{2\omega}$$

We then impose the money market clearing constrain that the positions of individual banks must sum to zero, [1] i.e. that $\sum_{i=1}^{n} x_i = 0$

Combing the last two equations, implies that

$$\text{A(1.1)} \qquad \begin{aligned} &\sum_{i=1}^{n} \frac{s_i - s^m}{2\omega} = 0 \quad \Leftrightarrow \\ &s^m = \frac{1}{n}\sum_{i=1}^{n} s_i = \frac{1}{n}\sum_{i=1}^{n}\left(1 - 2N\left(\frac{\overline{\varphi} - \varepsilon_i}{\sqrt{\sigma_\eta + \sigma_\varepsilon}}\right)\right) \end{aligned}$$

Now, compare the average value of the risk neutral spreads in A(1.1) with the spread, $\overline{s}$, resulting from the properties of the average forecast:

$$\text{A(1.2)} \quad \overline{s} = 1 - 2N\left(\frac{\overline{\varphi}}{\sqrt{\sigma_\eta}}\right)$$

In all cases the denominator of the argument for the cumulative normal distribution is larger in A(1.1) than it is in A(1.2). In addition, it follows from the convexity (concavity) of this function for arguments smaller (greater) than zero, that

$$s^m < \overline{s} \quad \text{for } \overline{\varphi} < 0; \text{ and}$$
$$s^m > \overline{s} \quad \text{for } \overline{\varphi} > 0$$

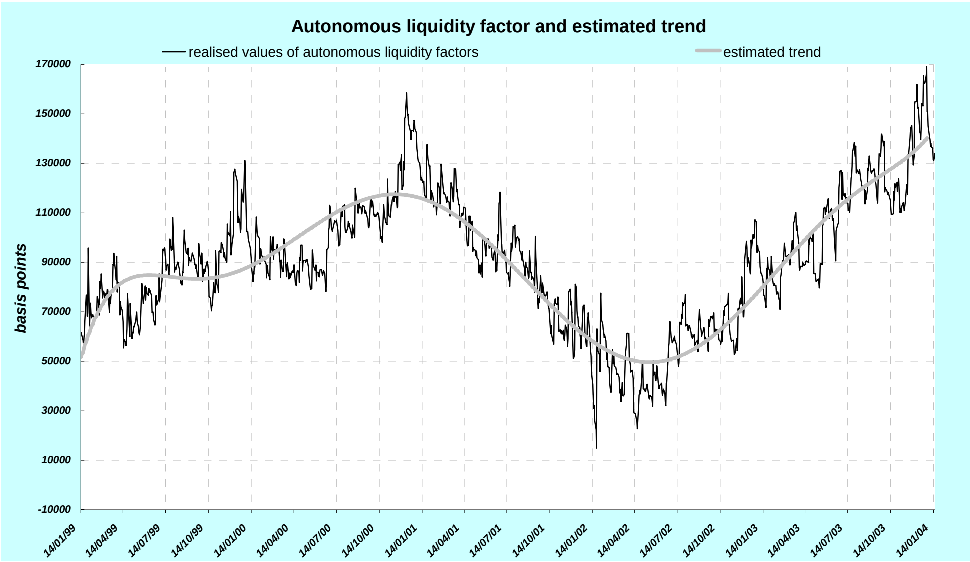
Accordingly, in absolute terms, the overnight spread actually prevailing in the market is always less than the one resulting from the properties of the market's average forecast of the autonomous factors, alone.

[1] This implies that the values of ε for different banks are not independent.

Annex 2: time series forecast of the autonomous factors

This section describes the simple univariate time series model for the autonomous liquidity factors, which we use as one out of three alternative assumptions for how the market is establishing its exogenous autonomous factors forecast. The model is based on a trend component, autoregressive terms and a set of dummies accounting for weekly, monthly, holidays and other effects. It does not distinguish individual autonomous factors (of which banknotes, net assets, and treasury deposits are the most important), but focuses exclusively on the aggregate sum of autonomous factors.[2] Moreover, we make the simplifying assumption that the market can with certainty forecast the trend of the aggregate autonomous factors, which, accordingly, we consider as being non-stochastic. Chart A1.1 depicts the evolution of the aggregated autonomous liquidity factors starting from January 1999 till the end of our sample,[3] and the trend that we identify below.

Chart A1.1: Aggregated autonomous liquidity factors (in euro millions)



The following 5-steps procedure outlines the econometric method used to identify the model.

1 De-trending. We de-trend the aggregated time series of autonomous liquidity factors using a polynomial trend.[4] The de-trended series is stationary and close to normal, exhibiting strong auto-correlation (see Table A1.2). Therefore, it would be natural to proceed with an autoregressive time series model.

2 Model selection. In order to choose the appropriate model, we regress the de-trended series on relevant lags and dummies. The model, providing the best fit and having the superior information criteria is displayed in Table 2. This model contains autoregressive components of order 1 and 2, as well as the

[2] See Bindseil&Seitz [2001] for an example of econometric forecasting models for different components.
[3] We expand our sample backwards in order to have more datapoints underlying the estimation.

following dummy variables (several other were tested for). First we have the following *Calendar effects*: dummies for April, May, and December account for yearly effects; dummies for the first and the second last day of each month, as well as for the last and the fourth last day of each maintenance period, are used for modeling the monthly pattern, while dummies for Tuesday and Thursday account for weekly seasonally. Second, we have the c*ash change over effect*: a dummy variable equal to one during January 2002. Third, a h*oliday indicator*: a dummy variable which is one for each TARGET holiday.

Estimates are presented (see Table A1.2) for the whole data sample, which includes 1260 observations after adjusting endpoints; standard errors and covariances are Newey-West corrected.

3 Forecasting. We start estimating the model outlined above on the sample period starting on 14/01/1999 and ending on the day previous to the announcement day of the first maintenance period with the variable tender rate (17/07/2000), which marks the start of our EONIA data sample.[5] From these estimates we calculate the 1-step ahead, 2-steps ahead, …, 10-steps ahead forecast. Then, we shift the estimation sample further by one observation and create the forecast for the next day and continue this process till we reach the end of our dataset.

4 Adding trend. To get the complete forecast of the autonomous liquidity factors, we add the trend to the previously obtained forecasts for the de-trended values.

5 Calibration to the average published by the ECB. Under the assumption that the forecast of the average autonomous factors published by the ECB contains useful information, we calibrate the time series forecast to this average by adjusting the level of the trend over the relevant forecast horizon. Let Δ denote the difference between the average published by the ECB and the average of the time series forecast on the announcement day (i.e. the average up to the end of the maintenance period). We then shift the level of the trend for each day from the announcement day until the end of the maintenance period by Δ. While in principle this changes the level of the de-trended autonomous factors, and hence the forecasts obtained under point 3, the effect of this is marginal and is ignored for the sake of simplicity.[6] Hence, this calibration procedure simply implies that the 1-step ahead, .., $(N\text{-}t)$-steps ahead forecast obtained in point 4 are on each day, t, shifted by Δ. While this procedure is admittedly somewhat arbitrary, it ensures that the average of the time series forecast as per the announcement day is equivalent to the average published by the ECB, and it turned out to indeed improve the quality of the "raw" time series forecast obtained in point 4.

Annex 2 compares the quality of this forecasting procedure to that of the other forecasts. Annex 3 investigates more carefully the properties of the errors of this forecast.

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[4] We tested how polynomia of different orders from 2 till 7 could capture the trend of the autonomous liquidity factors and choose a polynomial of order 6.

[5] The model is estimated for weekdays, whereby we have afterwards included non-trading days by simply repeating the value from the previous trading day.

[6] Otherwise a rather complex iterative procedure would be needed. I.e. changing the de-trended time-series would also lead to a change of the estimated parameters, which in turn would lead to a need for a new calibration, and so forth.

Table A1.1: Descriptive statistics for the de-trended autonomous factors (in euro billions)

Mean	0.08	**Std. Dev.**	12.00
Median	0.44	**Skewness**	0.33
Maximum	43.54	**Kurtosis**	3.58
Minimum	-42.10		
		Statistics	**Probability**
Jarque-Bera		41.19	0.00
Augmented Dickey-Fuller test		-8.56	0.00
Autocorrelation of order 1		0.892	

Table A1.2: Estimated Model for Autonomous Liquidity Factors

Variable	***Coefficient***	***Std. Errors***	***t-Statistic***	***Probabil.***
Liquidity factors(-1)	0.87	0.02	50.24	0.00
Liquidity factors(-2)	-2.09	0.42	-4.97	0.00
Dum April	-2.02	0.41	-4.93	0.00
Dum May	-0.96	0.39	-2.45	0.01
Dum December	2.20	0.81	2.73	0.01
Dum first	-7.80	0.82	-9.55	0.00
Dum last	3.84	0.83	4.65	0.00
Dum Tuesday	-1.00	0.28	-3.55	0.00
Dum Thursday	1.25	0.27	4.66	0.00
Dum 23	11.50	1.24	9.30	0.00
Dum 23(-4)	1.86	0.70	2.65	0.01
Dum Jan 02	-10.83	0.69	-15.71	0.00
Dum Target holiday	3.05	1.06	2.88	0.00
R-squared	0.86	**Mean dependent var**		-0.23
Adjusted R-squared	0.86	**S.D. dependent var**		11.89
S.E. of regression	4.44	**Akaike info criterion**		5.83
Sum squared resid	2.46E+04	**Schwarz criterion**		5.88
Log likelihood	-3660.50	**Durbin-Watson stat**		2.10

Annex 3: Basic properties of the autonomous factors forecast errors

Table A2.1 shows the root mean squared daily errors of the different forecasts, for different forecasting horizons and different numbers of calendar days since the MRO announcement till the actual forecasting day. The data set underlying the table has been artificially expanded by pretending that the MRO announcement day could have been on any day after the 8th last calendar day of each of the maintenance periods. Thereby we get for each of the 42 maintenance periods in our sample 8+7+6+5+4+3+2+1=36 different autonomous factors forecasts, each with a different "assumed" announcement day and/or a different number of days since the announcement day till the forecasting day. E.g. when there are 8 days from the announcement till the end of the maintenance period, we have a forecast as per the announcement day, which covers all 8 days, a forecast as per the next day, which covers 7 days etc., such that in total we have 8 different forecasts, covering a horizon ranging from 1 to 8 days. Likewise, when we "pretend" that there are 7 days from the announcement to the end of the maintenance period, we get another 7 forecasts with a horizon ranging from 1 to 7 days.

As a first remark it is seen that the root mean squared errors are generally lower for the ECB forecast than for any of the two other forecasts, thereby evidencing the superiority of this forecast. Only for the longer horizons the quality of, particularly the time series forecast, improves relatively to that of the ECB. Probably this owes to the fact that the trend (see annex 1), which is the most important element for longer horizons, is assumed to be known with certainty in the time series forecast. Somewhat surprisingly, it is also seen that the time series forecast does not, except for the longest horizons, perform better than the simple constant forecast. The quality of the latter is for all horizons declining with the number of days between the announcement and forecasting day. For the time series forecast such a pattern can only be recognized for the T+0 and T+1 horizons.[7] Finally, it is clear from the table that the quality of the ECB forecast does not depend on the number of days since the MRO announcement day.

[7] Note that the pattern of declining forecasting quality of the time series and the constant forecast that are identified in this annex can not be directly compared to the pattern identified in annex 3. This is because annex 3 focuses on the pattern of accumulated errors (i.e. from the forecasting day until the end of the maintenance period), while the present annex focuses on daily errors.

Table A2.1: Root mean squares of the daily errors of the three forecasts								
	Number of calendar days from the MRO announcement day till the forecasting day							
	0	1	2	3	4	5	6	7
1 step ahead forecast (T+0)								
ECB forecast	1.38	0.83	0.90	0.98	1.13	1.08	1.01	1.13
Constant forecast	4.92	4.72	5.00	5.51	6.26	7.21	8.23	10.23
Time-series forecast	4.55	5.86	7.02	7.59	7.98	8.40	8.19	10.06
2 steps ahead forecast (T+1)								
ECB forecast	1.45	1.46	1.51	1.66	1.74	1.62	2.19	
Constant forecast	4.72	5.00	5.51	6.26	7.21	8.23	10.23	
Time-series forecast	4.61	5.86	6.59	7.03	7.32	6.85	6.69	
3 steps ahead forecast (T+2)								
ECB forecast	2.13	2.16	2.33	2.41	2.51	3.03		
Constant forecast	5.00	5.51	6.26	7.21	8.23	10.23		
Time-series forecast	5.27	5.83	5.91	6.26	5.58	4.71		
4 steps ahead forecast (T+3)								
ECB forecast	3.30	3.59	3.35	3.49	4.56			
Constant forecast	5.51	6.26	7.21	8.23	10.23			
Time-series forecast	6.24	6.44	6.38	4.88	3.77			
5 steps ahead forecast (T+4)								
ECB forecast	5.06	4.56	4.82	6.07				
Constant forecast	6.26	7.21	8.23	10.23				
Time-series forecast	7.14	7.13	5.49	3.59				
6 steps ahead forecast (T+5)								
ECB forecast	6.00	6.06	6.52					
Constant forecast	7.21	8.23	10.23					
Time-series forecast	8.32	6.10	3.81					
7 steps ahead forecast (T+6)								
ECB forecast	7.80	7.60						
Constant forecast	8.23	10.23						
Time-series forecast	7.70	4.43						
8 steps ahead forecast (T+7)								
ECB forecast	9.24							
Constant forecast	10.23							
Time-series forecast	6.11							

Annex 4: the incorporation of special events.

The following summarizes how the models have occasionally been amended in order to cope with situations in which the flow of information on the aggregate liquidity conditions deviated from the "regular" time schedule in table 1 of section 2.

<u>Publication of the ECB's autonomous factors forecast *on* the allotment day</u>

Whenever the ECB's autonomous factors forecast on the allotment day was exceptionally different from the forecast on the announcement day, the ECB published the latter forecast together with the MRO allotment.[8] This was done in the last MRO allotments of the maintenance periods ending on 23 ***December 2001*** and 23 ***November 2003***.

Within the signal extraction model, this can be interpret as containing explicit information on the accumulated difference between the ECB's autonomous factors forecast on the allotment day and the market's forecast on the announcement day. Hence, we incorporate this additional information into the signal extraction model by adding the following equation to the signal extraction problem for these two maintenance periods:

$$\frac{1}{N}\Sigma^{\varepsilon} = \overline{\varphi}_{\text{allotment}} - \overline{\varphi}_{\text{announcement}}$$

Within the benchmark model (with time series and constant forecast), the additional information is simply incorporated by re-calibrating the forecasts as per the MRO allotment day.

<u>Publication of the ECB's autonomous factors forecast *later than* on the allotment day</u>

In the morning of the *settlement* day of the last MRO – i.e. after the last MRO allotment - of the maintenance period ending on 23 ***September 2003***, an exceptionally large recourse to the deposit facility of EUR 6.5 billion was recorded. At the same time, however, the ECB witnessed a loosening of its autonomous factors forecast which, in accumulated terms, was almost of a similar magnitude. So as to avoid that the large recourse to the deposit facility would lead to strong premature volatility of the overnight rate, the ECB re-published its forecast of the average autonomous factors on the MRO *settlement* day (17 September).

Within the signal extraction model, this can be interpret as information on the expected sum of η, i.e. the expected accumulated error of the ECB's autonomous factors forecast on the MRO allotment day. We incorporate this information by adding the following equation to the signal extraction for this maintenance period:

$$e_1 \cdot \left(1 + \alpha_c^{\eta}(N-2)\right) = 6.5$$

[8] This rule only applied to the sample period underlying this study. Since March 2004 the ECB has in each MRO published its average autonomous factors forecast on both announcement and allotment days.

Within the benchmark model (with time series and constant forecast), the additional information is simply incorporated by re-calibrating the forecasts.

Fine-tuning operation on the last day of the maintenance period.

On the last day of the maintenance period ending on ***23 May 2003***, the ECB carried out a fine-tuning operation, so as to offset the impact of an exceptionally large accumulated net recourse to the deposit facility of EUR 9 billion that had occurred after the last MRO allotment (20 May).

As it is unclear to which extent the EONIA for that day actually reflected the ECB's decision to fine-tune, which was only announced at 10:30 am., the observation for this particular day has been excluded from the sample period.

Underbidding in the last MRO allotment and subsequent fine-tuning

Due to some rather peculiar circumstances, an underbidding occurred in the last MRO of the maintenance period ending on 23 ***December 2002***. So as to partially restore the resulting liquidity deficit, the ECB carried out a fine-tuning operation in the morning of the MRO settlement day (18 December), without, however, publishing a new forecast of the average autonomous factors.

This situation has been accommodated by pretending that the fine-tuning operation (and not the MRO itself) was the last operation of that maintenance period, and the liquidity target refers to the one of the fine-tuning operation and not of the MRO. In the same vein, ε, i.e. the difference between the market's autonomous factors forecast on the announcement day and the ECB's forecast on the allotment day, covers in this case two days of updating of the ECB's forecast. While in principle, this should lead to a greater variance of ε, we have for the sake of simplicity disregarded this aspect, and assumed the variance to be the same as in other maintenance periods.

Exceptional information about excess reserves.

In the "cash-change-over" maintenance period which ended on 23 ***January 2002***, the ECB forecasted excess reserves to be unusually high. This was communicated to the market, however, without giving the actual value of the ECB's forecast.

We make the rather rough assumption that the market, on the basis of this information, adjusted its forecast for excess reserves such that it was equivalent to the actual ex post value of EUR 1.7 billion (as a daily average).

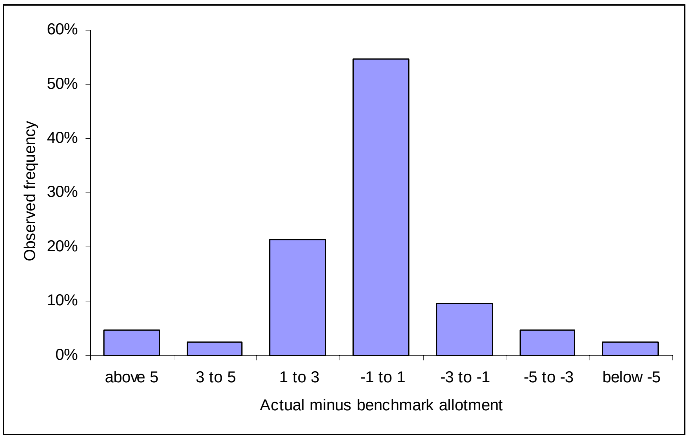
Annex 5: Sample estimates of the variance parameters.

This annex describes how sample estimates of the parameters describing the variance of the ECB's liquidity target, the different autonomous factors forecasting errors/updates as well the residual variance have been obtained. The methodology used aim to ensure that the sample estimates capture the most important properties of the different covariances in the context modeling the overnight rate. These properties are: 1) the dependency of the variance of the accumulated errors on the number of days till the end of the maintenance period, and 2) the correlation between errors on individual days. Table A3.1 below summarizes the sample estimates that we have identified, while the different charts illustrates the extent to which these estimates capture the observed properties. The observed properties are calculated using the same artificially expanded data set as in Annex 2.

Variance of γ, the ECB's liquidity target

The sample estimate of the variance, σ^{γ}, is estimated directly from the available observations (see Chart A3.1) at EUR 19.1 billions.[9] The variance of the ECB's accumulated liquidity target for a given maintenance period then results by multiplying with the square of the number of days from the settlement day till the end of the maintenance period.

Chart AZ.1 Empirical distribution of the ECB's liquidity target (in EUR billions).



Variance of μ (and η), the error of the autonomous factors forecasts

Concerning the sample estimates of the parameters for the autonomous factors forecast errors, we first estimate the parameters which describe the variance of the error, μ^1, on the MRO announcement day. These parameters $\left(\alpha_c^{\mu}, \alpha_e^{\mu}, \alpha_\rho^{\mu}\right)$ are different for each of the three autonomous factors forecast, although it is recalled that the variances of the accumulated errors of the three forecasts as per the announcement day are identical due to the calibration to the ECB forecast.

Let $\overline{\sigma}_N^\mu$ denote the observed variance of the accumulated error of the forecast from the announcement day till the end of the maintenance period when this period spans N days. Chart A3.2 shows how this observed variance, which is calculated directly from the available forecasts,[10] logically declines with N for each of the three forecasts.

Let then $\overline{\alpha}_N^\mu$ denote the observed dependency of the average forecast error in the remaining N-1 days on the forecast error on day 1, i.e. on the announcement day. Specifically, $\overline{\alpha}_N^\mu$ is found from the following regression, which is estimated for each forecast and for each value of $N \in [2;8]$:[11]

$\frac{1}{N-2}\sum_{i=2}^{N}\mu_i^1 = \overline{\alpha}_N^\mu \cdot \mu_1^1 + \pi$, where π is an iid error term.

We then approximate $\alpha_c^\mu, \alpha_e^\mu, \alpha_\rho^\mu$ by minimizing the following loss function, whereby the first part expresses the difference between the observed and the fitted dependency of forecast errors on individual days, while the second part denotes the difference between the observed and the fitted variance of the accumulated errors :

$$\mathrm{Loss}\left(\alpha_c^\mu, \alpha_e^\mu, \alpha_\rho^\mu\right) = \sum_{N=2}^{8}\left(\overline{\alpha}_N^\mu - \left(\frac{\alpha_c^\mu \alpha_e^\mu - \frac{1}{N}\alpha_\rho^\mu}{\alpha_e^\mu + \alpha_\rho^\mu}\right)\right)^2 + \sum_{N=1}^{8}\left(\frac{\overline{\sigma}_N^\mu - \alpha_e^\mu \sum_{i=1}^{N}\sum_{j}^{N}\psi_{i,j}}{N^2}\right)^2$$

where $\psi = \theta\theta'$, and it is recalled that $\theta = \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ \alpha_c^\mu & 1 & 0 & \cdots & 0 \\ \alpha_c^\mu & \alpha_c^\mu & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \alpha_c^\mu & \alpha_c^\mu & \alpha_c^\mu & \cdots & 1 \end{pmatrix}$ and $\dim(\theta) = N$.

Chart A3.2 shows that the estimated parameters (see Table A3.1) fit relatively well the observed variance of the accumulated error of the forecasts on the MRO announcement day, for different numbers of days between this day and the end of the maintenance period. The fit is almost identical for the three forecasts, the difference of which only shows up in the correlation structure of the daily errors, which is shown in Chart A3.3 to A3.5. Only for the constant and the time series forecast these charts show a clear pattern and the parameter α_ρ^μ, capturing the extent to which the accumulated sum of the autonomous factors forecast is

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[9] Excluding maintenance periods with underbidding the variance was only EUR 3.0 billion, while considering only these periods it amounted to EUR 123.1 billion.

[10] It is recalled that the data set underlying this (and the other calculations of observed variances in this annex) is artificially expanded as explained in Annex 2.

[11] Hence, each regression is based on 42 observations, i.e. the number of maintenance periods in the sample.

wrongly distributed on individual days, is only significant for these two forecast. The fact that the estimate of this parameter is higher for the time series forecast than for the constant forecast, indicates that the time-series forecast is in fact even worse than is the constant forecast in capturing the distribution of the autonomous factors on individual days.

Chart A3.2 Variance of the accumulated error of the autonomous factors forecast on the MRO announcement day, as a function of the number of days covered by this forecast.

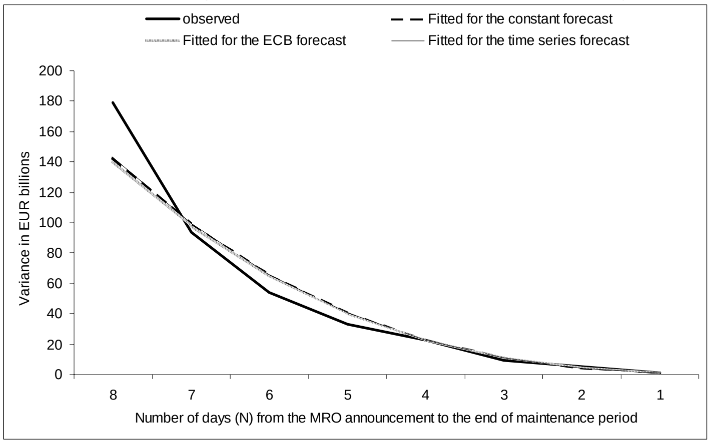


*Chart A3.3 Observed and fitted dependency between the error on the announcement day and the average error on subsequent days for the **Time-series forecast***

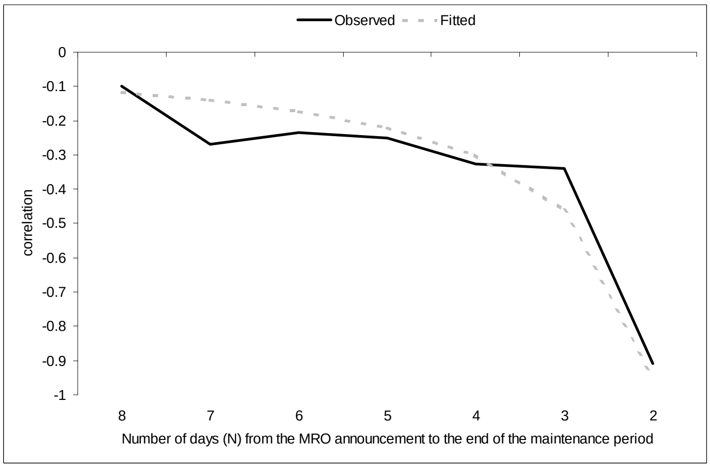


Chart A3.4 Observed and fitted dependency between the error on the announcement day and the average error on subsequent days for the ***Constant forecast***

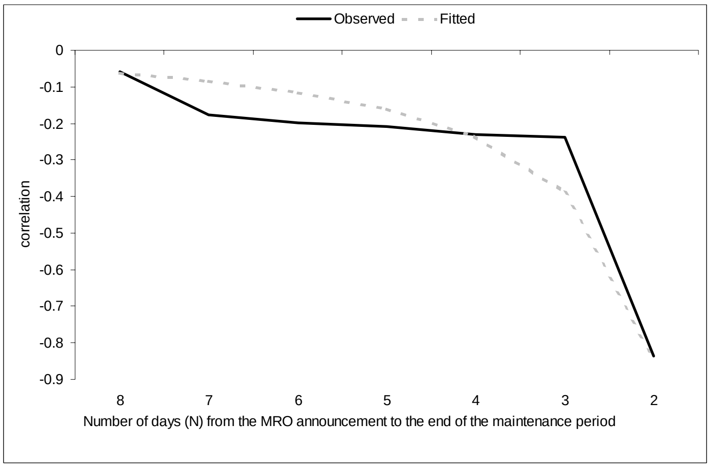
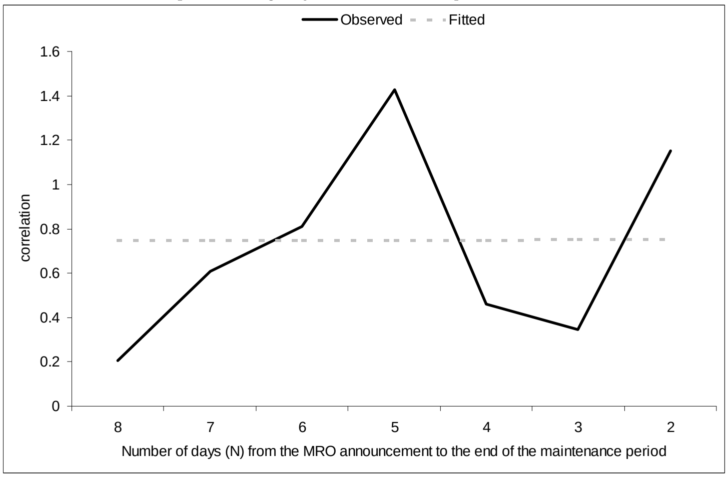


Chart A3.5 Observed and fitted dependency between the error on the announcement day and the average error on subsequent days for the ***ECB forecast***



The parameter α_q^μ captures the declining quality of the forecasts (in accumulated terms) after the MRO announcement day. This parameter was only needed for the constant and the time-series forecast owing to the "hybrid" nature of these two forecasts. The ECB forecast is not of a "hybrid" nature, and can be assumed to be constructed in exactly the same way on each day. Therefore, the variance of the accumulated error of this latter forecast is independent of the number of days since the MRO announcement, and can on any day be fully described by the two parameters, α_c^μ and α_e^μ, as explained in section 5.2. Specifically, these two parameters then also describe the variance of the error, η, of the ECB's forecast on the MRO allotment day. The sample estimate of the parameter, α_q^μ (see table A3.1) is obtained for the constant and the time-series forecast by minimizing another loss function. In this loss function $\overline{\sigma}_{N,t}^\mu$ denotes the *observed* variance of the accumulated error of the relevant forecast t days after the announcement day in the case of N calendar days from the latter till the end of the maintenance period. Again this variance can be directly calculated from the observed forecast errors.

$$\text{Loss}\left(\alpha_q^{\mu}\right) = \sum_{N=2}^{8}\sum_{t=1}^{N}\left(\frac{\overline{\sigma}_N^{\mu}\cdot t^{\alpha_q^{\mu}} - \overline{\sigma}_{N,t}^{\mu}}{N^2}\right)^2$$

Chart A3.6 and A3.7 show the observed and the fitted pattern of the “declining quality” of the time series and the constant forecast after the MRO announcement day. It is interesting to note that the quality of the time series forecast actually declines substantially more than does the quality of the constant forecast.

Chart A3.6 Observed and fitted variance of the accumulated error of the ***time series forecast*** *as a function of the number of days since the MRO announcement, considering different number of days from the MRO announcement to the end of the maintenance period.*

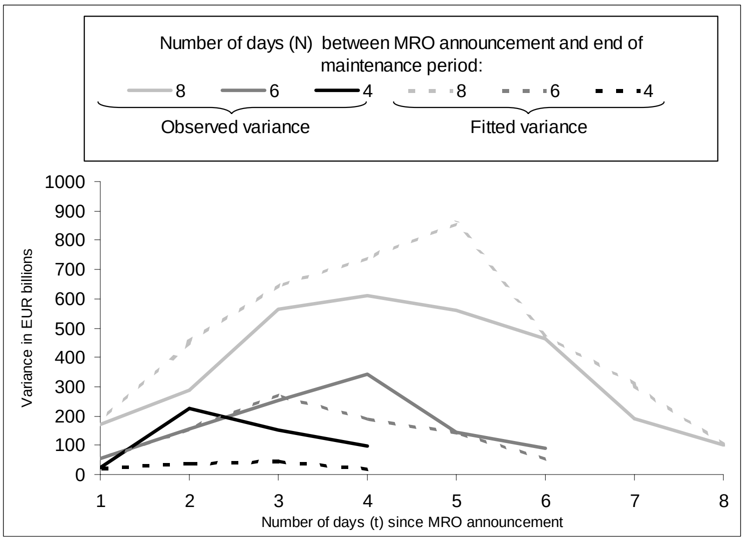
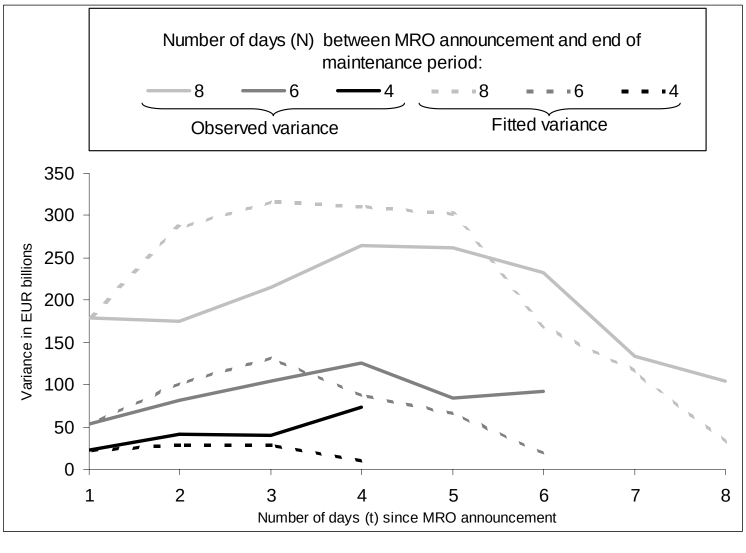


Chart A3.7 Observed and fitted variance of the accumulated error of the ***constant forecast*** *as a function of the number of days since the MRO announcement, considering different number of days from the MRO announcement to the end of the maintenance period.*



Variance of ε, the difference between the markets forecast on announcement day and the ECB's forecast on the allotment day.

Firstly, it is recalled that ε and η are independent and that $\varepsilon = a - \varphi^1 - \eta$. From this it follows that the correlation structure of ε needs to be identical to that of $a - \varphi^1 = \mu^1$. Hence the sample estimates for α_c^ε and α_ρ^ε are set identical to those for α_c^μ and α_ρ^μ for the relevant autonomous factors forecast. Yet, the overall level of the variance of ε, as determined by α_e^ε, is obviously different from that of μ. We explore the following identity in order to find the sample estimate of α_e^ε:

$$\varepsilon = \mu^1 - \eta$$

Due to the calibration of the market's forecast on the announcement day (i.e. day 1), we know that the variance of the accumulated value of μ^1 must be identical to that of the ECB's forecast on the announcement day. Using again the assumption of independence between ε and η, and denoting the variance of the accumulated value of ε by $\overline{\sigma}_N^\varepsilon$, we then have established that

$$\overline{\sigma}_N^\varepsilon = \overline{\sigma}_N^\eta - \overline{\sigma}_{N-1}^\eta$$

We then obtain the sample estimate of α_e^ε by minimizing the following loss function, in which $\tilde{\sigma}_N^\varepsilon$ denotes the observed variance of the accumulated value of ε when there are N trading days from announcement till the end of the maintenance period, while $\overline{\sigma}_N^\eta$ denotes the variance fitted for η when there are N trading days till the end of the maintenance period:

$$\text{Loss}\left(\alpha_e^\varepsilon\right) = \sum_{N=2}^{8}\left(\frac{\tilde{\sigma}_N^\varepsilon - \left(\overline{\sigma}_N^\eta - \overline{\sigma}_{N-1}^\eta\right)}{N^2}\right)^2$$

The "residual" variance term, α^π.

A sample estimate of the residual variance, $t^2 \cdot \alpha^\pi$, which is intended to cover uncertainty about excess reserves and individual recourse to standing facilities, can be easily derived from the observed variance of these two liquidity components. The observed variance of the sum of the excess reserves forecast error (i.e. the error of the simple model assumed in the main text) and the accumulated individual net recourse to the deposit facility when there are t days to the end of the maintenance period, is denoted by $\overline{\sigma}_t^\pi$. A sample estimate of the parameter α^π is then found by regressing the observed variance, $\overline{\sigma}_t^\pi$, on t^2, i.e. the squared number of days left till the end of the maintenance period. This exercise results in an estimate of α^{π} at 0.37. The observed variance is shown together with the fitted variance in Chart A3.8.[12]

Chart A3.8 Observed and fitted variance of the sum of the excess reserves forecast error and the accumulated individual net recourse to standing facilities until the of the maintenance period.

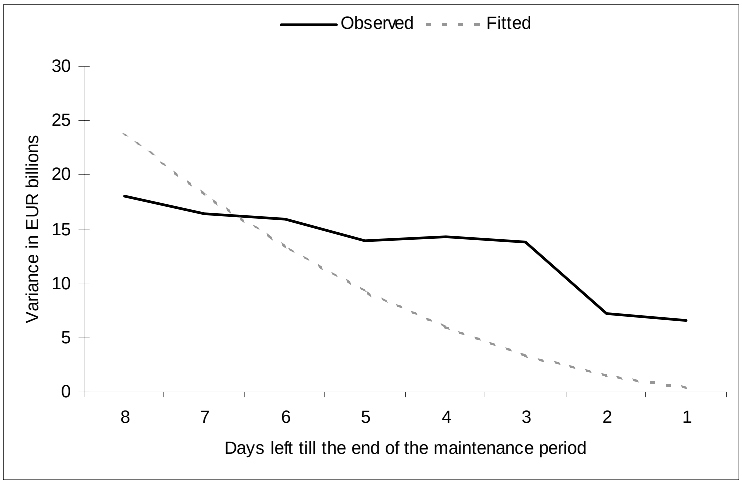


Table A3.1: Sample estimates of variance parameters (in EUR billions)

Parameter		Autonomous factors forecasting procedure		
		Constant forecast	ECB forecast	Time series forecast
Autonomous factors Forecast error (μ)	Overall level (α_e^{μ})	1.02	1.10	1.02
	Correlation of persistent part of the error (α_c^{μ})	0.79	0.75	0.80
	Variance of errors offset before end of MP (α_ρ^{μ})	10.83	0.00	34.82
	Decreasing quality after ann. Day (α_q^{μ})	1.61	N/A	2.26
ECB forecast errors on the allotment day (η)	Overall level of (α_e^{η})	N/A	1.10	N/A
	Correlation of persistent part of the error (α_c^{η})	N/A	0.75	N/A
Difference between the markets' forecast on announcement day and the ECB's forecast on allotment day (ε)	Overall level (α_e^{ε})	0.45	N/A	0.45
	Correlation of persistent part of the error (α_c^{ε})	0.79	N/A	0.80
	Variance of errors offset before end of MP ($\alpha_\rho^{\varepsilon}$)	10.83	N/A	34.82

[12] This variance is calculated from our data sample. For each maintenance period, we have 8 observations ($t \in [1.,,,8]$) of the sum of the excess reserves error and the accumulated individual recourse to standing facilities in the remaining days of the maintenance period (i.e. accumulated from time t to the end of the maintenance period, N (on which t=8). Only the accumulated sum of individual recourse depends on t.