

Discussion of

MONETARY POLICY WITH JUDGEMENT

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QUESTION

■ How to use a model together with additional (non-model) info in practical decision-making?

(where model = complete probability distribution over sequences of data)

■ Common problem, also beyond central banks

▲ portfolio management

▲ hedging with derivatives

■ Why not just a bigger model that captures additional info?

▲ info not in form of numbers

▲ small sample (e.g. new environment)

▲ tractability

INCORPORATING JUDGEMENT

- To combine additional info and model structure,
⇒ work with conditional distributions!

1. Optimal Policy

- ▲ usually expressed as function of state variables of model.
- ▲ state variables capture payoff relevant information
⇒ can express optimal policy in terms of conditional distributions of forcing process.

2. Additional Info

- ▲ formulate judgement in terms of conditional distributions of forcing process of model.
⇒ to analyze model with judgement, plug conditional distributions into policy function.

INCORPORATING JUDGEMENT

- Tractable framework: quadratic loss, linear model
 - ▲ certainty equivalence: only conditional means matter for optimal policy
 - ▲ to analyze model with judgement, plug forecasts (at all relevant horizons) into policy function.
 - ▲ no need to spell out law of motion underlying judgement
- Can interpret objective as forecast targetting.
- Finite horizon judgement: approximation results.

DISCUSSION: ALTERNATIVE JUDGEMENTS

■ Communication about alternative judgements

- ▲ to evaluate loss of a policy given some judgement, must spell out evolution of forecasts.

- ▲ tractable in certainty equivalence framework: need 1st and 2nd moments.

■ Integrating lack of confidence in judgement

- ▲ Bayesian approach + certainty equivalence: confidence does not matter.

- ▲ can use tractable model of ambiguity aversion/robustness.

EXAMPLE

- Backward looking model with state variable x_t , instrument i_t :

$$x_{t+1} = \alpha x_t + i_t + z_{t+1}.$$

- Quadratic loss function

$$L = E \left[\sum_{t=0}^{\infty} \beta^t \left(x_{t+1}^2 + \kappa i_t^2 \right) \right].$$

- Usual procedure:

- ▲ explicit law of motion for z , e.g. $z_{t+1} = \gamma z_t + \varepsilon_{t+1}$.

- ▲ state vector (z_t, x_t) captures payoff-relevant info; optimal policy is

$$i_t = I_x x_t + I_z z_t.$$

- But can always rewrite in terms of conditionals:

- ▲ Simplest case: $\kappa = 0$

$$i_t = \alpha x_t + \gamma z_t = \alpha x_t + E_t[z_{t+1}]$$

- ▲ More generally, whole path of forecasts matters

$$i_t = I_x x_t + I_z z_t = \tilde{I}_x x_t + \sum_{j=1}^{\infty} \tilde{I}_z E_t[z_{t+j}].$$

Analogy: consumption function in permanent income model.

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COMMUNICATION ABOUT JUDGEMENT

■ Specialize example: $x_2 = i_0 + i_1 + z_2$; loss function $L = E_0 [x_2^2 + i_1^2]$.

■ Tradeoff: hit target $x_2 = 0$ (offset z_2), but adjust instrument i early!

■ Optimal policy: $i_0 = -E_0 [z_2]$ and $i_1 = \frac{1}{2} (E_0 [z_2] - E_1 [z_2])$.

■ Loss is smaller the more info arrives at date 1:

▲ if no info resolved at 1, then $E_1 [z_2] = E_0 [z_2]$, $i_1 = 0$ and loss

$$L = E \left[\underbrace{(-E_0 [z_2])}_{i_0} + z_2 \right)^2 \right] = \text{var}_0 (z_2).$$

▲ if all info revealed at 1, then $E_1 [z_2] = z_2$, loss L is

$$E_0 \left[\left(\underbrace{-E_0 [z_2]}_{i_0} + \underbrace{\frac{1}{2} (E_0 [z_2] - z_2)}_{i_1} + z_2 \right)^2 \right] + E_0 [i_1^2] = \frac{1}{2} \text{var}_0 (z_2).$$

■ Suppose analysts A and B disagree on mean: $E_0^A [z_2] = (1 + \alpha) E_0^B [z_2]$.

■ If B evaluates A's optimal policy using his own judgement, loss depends on evolution of forecasts!

COMMUNICATION ABOUT JUDGEMENT

- Specialize example: $x_2 = i_0 + i_1 + z_2$; loss function $L = E_0 [x_2^2 + i_1^2]$.
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$$L = E_0^B \left[\underbrace{(-E_0^A [z_0])}_{i_0} + z_2 \right)^2 \Big] = \text{var}_0(z_2) + \alpha^2 E_0^B [z]^2.$$

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- Suppose analysts A and B disagree on mean: $E_0^A [z_2] = (1 + \alpha) E_0^B [z_2]$.

- If B evaluates A's optimal policy using his own judgement, loss depends on evolution of forecasts!

LACK OF CONFIDENCE: A MINMAX APPROACH

- Represent alternative judgments by sets of conditional distributions.

- Describe evolution of sets, e.g. sets of means.

- ▲ Let s_t summarize z_t and additional info.

- ▲ $P_t(s^t) =$ set of distributions of s_{t+1} conditional on $s^t = (s_1, \dots, s_t)$.

- ▲ As before, data $y_t = y_t(s^t)$ depend linearly on the z s.

- Recursive representation of loss

$$L(y; s^t) = \max_{q \in P_t(s^t)} \left\{ y_t' A y_t + \beta E^q \left[L(y; s^t, s_{t+1}) \right] \right\},$$

- Given process $P_t(s^t)$, there is a set Q of probabilities on sequences $\{s_t\}$ such that

$$L(y; s^t) = \max_{Q \in P} E^Q \left[\sum_{t=1}^{\infty} \beta^t y_t' A y_t \right].$$

- Solve minmax problem to find optimal policy!

LACK OF CONFIDENCE: A MINMAX APPROACH

■ Interpretation of min max functional form

- ▲ lack of confidence matters, because worst case forecast drives behavior
- ▲ extreme pessimism? no, simply aggregates judgements while accommodating lack of confidence
- ▲ axiomatic foundations: functional form implied by standard axioms plus uncertainty aversion.

■ Tractability:

- ▲ by Maxmin theorem, can pick policy first, probabilities second.
- ▲ there must be a “supporting” probability $P^* \in Q$ that achieves the max at the optimal solution.
- ▲ optimal policy under P^* must satisfy Svensson formulas!

⇒ 2 Step approach

1. Determine set of conditional forecasts associated with any $P \in Q$.
2. Find sequence of forecasts that maximizes loss under optimal policy.